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Abstract

The Go/No-Go paradigm is being used in various clinical applications frequently. Besides, the sequential sampling models have achieved much attention for modeling the underlying processes of decision making. One of the most successful sequential sampling models that is used also for modeling the Go/No-Go paradigm is the drift diffusion model (DDM). The major assumption of the drift diffusion model is that the process of information accumulation is terminated by reaching the boundaries. But this assumption has been argued during the last years and some studies have confirmed that the accumulation process continues after making a decision (i.e. accumulator reaches the boundaries). The main constraint of the drift diffusion model for modeling the Go/No-Go paradigm is that it can not capture the ongoing information accumulation after making a decision. This is important in the Go/No-Go paradigm because when the participants decide to no-go they should wait until finishing the trial and it is a good time for continuing the information accumulation. In this paper, we are going to develop a variation of the drift diffusion model which is able to capture continuing the information accumulation after making a no-go decision. The developed model is based on substituting the lower boundary with a reflecting boundary. This paper aims to introduce an alternative model for Go/No-Go paradigm and after presenting a theoretical discussion on the model behavior, mean first passage time, and the first passage time distribution of the model, it is compared with the previous DDM model.

Keywords: Sequential Sampling Models; Go/No-Go Task; Reflecting Boundary; Drift Diffusion Model;

Introduction

There are many experiments in the clinical setup which are based on Go/No-Go paradigm such as continues performance task (Karalunas et al., 2018). The main difference of the Go/No-Go paradigm with the 2-alternative forced-choice paradigm (2-AFCP) is that in the 2-AFCP the participants should do an action for both decisions (press a button for each decision) but in the Go/No-Go paradigm the participants should just do an action for the go decision and for the no-go decision they should do nothing (Gomez, Ratcliff, & Perea, 2007). Therefore this paradigm provides a good framework for diagnosing and rehabilitating impulsive behaviors, self-control, and inhibitory control (Huang-Pollock et al., 2017). Despite these clinical applications, the Go/No-Go paradigm can not provide detailed information about the response time distribution of No-Go decisions (Ratcliff, Huang-Pollock, & McKoon, 2018). Thus, the main measurements of the Go/No-Go paradigm are the go accuracy, no-go accuracy, and go reaction time (Nejati, Salehinejad, Nitsche, Najian, & Javadi, 2020).

On the other hand, the sequential sampling models can shed light on the underlying processes of the decision making and provide additional information about what happens in the brain during making a decision (Ratcliff, Smith, Brown, & McKoon, 2016). There are various sequential sampling models that can capture different underlying processes. For example race diffusion model (pure parallel processing), leaky competing accumulator model (lateral inhibition) (Usher & McClelland, 2001), and drift diffusion model (forward inhibition) (Ratcliff et al., 2016). But all models have the same basic assumption which is starting sampling from a random starting point z and continue the information accumulation with a constant rate v until reaching the threshold of the decision a (i.e. 0 and a in the DDM) (Ratcliff et al., 2016). DDM has been also used for modeling the Go/No-Go paradigm. In 2007, a DDM model has been introduced for the Go/No-Go tasks (Gomez et al., 2007). In this model, the researchers estimate the parameters of the Go/No-Go DDM model based on the performance of participants on 2-choice tasks. But this framework is not suitable for clinical applications because the participants should complete both 2-choice and Go/No-Go tasks and it takes time. It is so important because usually, the Go/No-Go paradigm is utilized for patients with some impulsive behavior and also hyper activities. Thus, they can no participate in many clinical experiments. So, a new DDM framework has been introduced by Ratcliff et al. (2018) which has the ability the estimating the parameters of the DDM based on only the performance of the participants on Go/No-Go tasks. There are several ways to model the Go/No-Go paradigm using DDM. The simplest one is considering only one threshold for the model (single boundary) (Gomez et al., 2007). In contrast, there are some other ideas for modeling the Go/No-Go paradigm that includes two thresholds in the DDM (i.e. the no-go threshold is implicit) and the model assumes the decision maker waits until the next trial by hitting the no-go threshold. In these two threshold models, the assumption is that there is a go bias in the response pattern of the participants and the DDM should capture this go bias. For this purpose, three assumptions have presented: a) Starting point bias should be located closer to the go threshold, b) A constant value should be added to the rate of accumulation for both go and no-go decisions which cause an equal reduction in the no-go drift rate and increase in go drift rate, and c) The reduction in the rate of accumulation for no-go decision is greater than the increase for the rate of accumulation for the go decision (Gomez et al., 2007; Ratcliff et al., 2018). Additional to the mentioned patterns which indicate the strategy of Go/No-Go is different from 2-alternative forced choice, the non-decision time of these two paradigms is different which is the result of the difference in the execution time (Ratcliff et al., 2018).

Recently Evans, Dutilh, Wagenmakers, and van der Maas (2020) have highlighted double responding phenomena as a constraint of the sequential sampling models including drift diffusion model. Based on this phenomenon, the decision maker continues the sampling of information after making a decision and may change his/her decision after responding (Pleskac & Busemeyer, 2010; Rabbitt & Vyas, 1981). "Double responding" (Evans et al., 2020), "partial error" (Burle, Spieser, Servant, & Hasbroucq, 2014), "error correcting" (Rabbitt, 2002), and "change of mind" (Rabbitt & Rodgers, 1977) are the keywords in the literature that point to the same phenomenon which is continuing the information accumulation process after making a decision. This phenomenon is studied in 2-AFCP but it is reasonable to explore it in the Go/No-Go paradigm. Because, in the Go/No-Go paradigm, when the participants decide to no-go they should do nothing for a while, and during this waiting time, the accumulation process is still alive and can accumulate more information. So, it is time for changing the decision. Thus there is enough evidence for motivating us to develop a model for Go/No-Go paradigm based on this phenomenon. In other words, the main motivation of this research is developing a sequential sampling model for the Go/No-Go paradigm that has the ability to capture double responses.

The rest of this paper is organized in the following order: in the Model section, first, the lack of previous Go/No-Go DDMs are discussed, and then a new DDM based on using a reflecting boundary for the no-go threshold is presented. After that, the mathematical properties of the model and the parameter estimation of the model are presented in this section. In the last parts of this section, the behavior of the model in the parameter space and parameter recovery of the model are illustrated. The Behavioral Data Analysis section contains the fitting results of the model on a behavioral data set. In the next section, a discussion on the model is presented and finally, in the last section, a conclusion is presented.

Model

From the computational point of view, the difference between the Go/No-Go paradigm and 2AFCP is that no data is recorded for the no-go decision and it is considered that if the button is not pressed during a finite time interval, the decision is no-go. The assumption of DDM for the no-go decision is that when the accumulator hits the lower boundary, the decision maker decides to no-go and the information accumulation process is finished (Ratcliff et al., 2018; Gomez et al., 2007). But the no-go decision can be the result of making no decision. Moreover, the go decision can be the result of a double responding process (i.e. first decide to no-go and then decide to go). Thus, some processes are not captured by the DDM models of Go/No-Go.

One of the missing cases is the one that the accumulator does not reach the go or no-go thresholds, and the decision maker does not make any decision during the time interval. In this case, the response which is recorded is no-go but the DDM model assumes all the processes of the no-go decision are hit with the lower boundary. Therefore, the model should capture alive processes at the end of the time interval and consider them as the no-go response. The other missing process that should be included in the model is mind changing. When the decision maker decides to press the button (go decision), by pressing the button, the trial is finished and the stimulus disappears from the monitor. In this case, if the decision maker wants to change his/her decision, any data is not recorded and we can not model this case without additional information. But in the case of the no-go decision, the decision maker should wait until the end of the time interval. Thus, there is enough time to continue the accumulation process. By continuing the accumulation process, three scenarios are possible. The first one is sampling for the benefit of the no-go decision and the decision of the decision maker does not change. The second one is sampling towards the go decision and there is enough time to reaching to the go threshold and the decision maker changes his/her decision. So, in this case, a double responding phenomenon has occurred but no data is recorded for the first decision. The last scenario is sampling toward the go threshold but there is not enough time for reaching the go threshold and the time interval finishes before the second decision. In this case, the recorded data is the no-go response but the decision maker is not confident about his/her decision (Pleskac & Busemeyer, 2010). The confidence index can be used as a measure of how often the double responding phenomenon is occurred in a Go/No-Go paradigm. Thus, it could shed light on the underlying process of decision making in a Go/No-Go paradigm.

Reflected boundary drift diffusion model

The traditional drift diffusion models have two absorbing boundaries (i.e. lower and upper threshold). These absorbing boundaries cause terminating the process when the accumulator hits to one of these boundaries. The main idea behind the developing reflected boundary drift diffusion model (RBDDM) is substituting the lower absorbing boundary with a reflecting boundary. The difference between the absorbing boundary and the reflecting boundary is that the process does not stop by hitting the reflecting boundary. Thus if the formulation of the standard diffusion process is as below:

$$\begin{cases} X(0) = z, \\ X(t+\Delta t) = X(t) + v\Delta t + e\sqrt{\Delta t} \quad e \sim \mathcal{N}(0,1), \\ stop, \quad X(t) \geq a \; or \; X(t) \leq 0 \end{cases}, \tag{1}$$

where e is the noise of accumulation, and the parameters 'z' is the starting point bias, v is the rate of information accumulation, and a is the threshold of making a decision. Additional to these three parameters, a parameter t_0 should be added to the model for non-decision time (i.e. encoding and motor time). Based on the mentioned formulation for the traditional DDM, the RBDDM can be formulated as below:

$$\begin{cases} X(0) = z, \\ X(t+\Delta t) = X(t) + v\Delta t + e\sqrt{\Delta t} \quad e \sim \mathcal{N}(0,1), \\ X(t) = 0, \quad X(t) < 0, \\ stop, \quad X(t) \geq a \; or \; t = t_{end} \end{cases} \tag{2}$$

where t_{end} is the termination time of the trial and it is considered as input of the model. So, the RBDDM has four free parameters (v, a, z, t_0) and one input parameter (t_{end}). The reflecting boundary prevents from finishing the process when the accumulator hits to it until termination time. In other words, the reflecting boundary behaves like a wall. Figure (1) gives a good intuition about the RBDDM. Some sample paths of the main processes of the RBDDM is illustrated in this figure. These processes include go decision with and without double responding occurrence, and no-go response with and without making no-go decision.

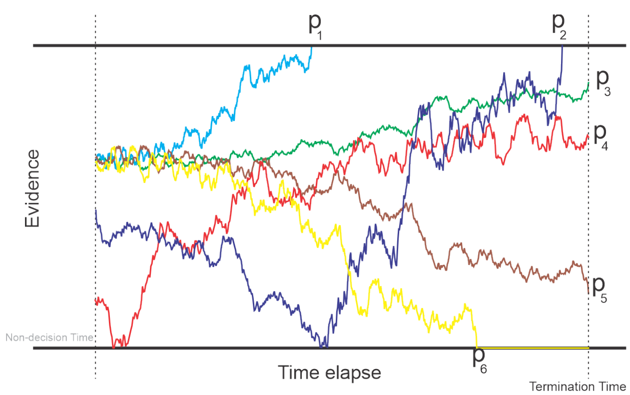

Figure 1: The graph of six sample paths of the various processes that can be generated by the RBDDM. p_1: the process of go response without double responding phenomenon, p_2: the process of go response with a double responding phenomenon, p_3: the process of no-go response without making a decision and positive drift rate, p_4: the process of no-go response after hitting to the no-go boundary and reflecting, p_5: the process of no-go response without making a decision and negative drift rate, p_6: the process of the no-go response with no-go decision.

It is worth mentioning that the RBDDM could be considered as a general form of the Go/No-Go DDM because the previous models only generate the processes p_1 and p_6, but the RBDDM not only generates them but also generates some additional plausible processes such as $p2$, p_3, p_4, and p_5. Additionally, the RBDDM can also model the aforementioned go bias using the staring point bias parameter. On the other hand, the nature of using a reflecting boundary for the no-go threshold and an absorbing boundary for the go threshold makes a go bias. Because even the accumulator starts accumulating from the lower boundary ($z = 0$) there is a chance for reaching the upper boundary before the termination time. This property of the RBDDM will discuss in the next part more.

Mathematical properties

The aim of this part is to mention some mathematical properties of RBDDM. The expected exit time from $(0,a)$ starting at $0 < z < a$, is the first property which is mentioned. $T(z)$ denotes this mean first passage time of the process starts for z, and it is the solution of the following differential equation (Gardiner et al., 1985):

$$vT'(z) + \frac{1}{2}T''(z) = -1, \tag{3}$$

with respect to the following boundary conditions:

$$T(a) = 0, \quad T'(0) = 0. \tag{4}$$

The solution of (3) subject to (4) is as follow:

$$T(z) = \frac{1}{v}\left(a - z + \frac{1}{2v}(e^{-2va} - e^{-2vz})\right). \tag{5}$$

Based on this solution, it is concluded that the mean first passage time of the process starting from the lower boundary is $T(0) = \frac{1}{v}\left(a + \frac{1}{2v}(e^{-2va} - 1)\right)$, and if $v > 0$ then we can conclude that $T(0) > 0$, which means there is a chance for go response even $z = 0$. But in the DDM if $z = 0$, then there is no chance for go response, so, it implies that the reflecting boundary adds a go bias to the RBDDM. On the other hand, by tending z to a (i.e. $z \longrightarrow a$), $T(z)$ tends to zero.

The other important property of the RBDDM which should be presented is the first passage time distribution of the process. If the process starts from z, the probability that the process be still alive after time t is denoted by $G(z,t)$ and can be obtained by (Gardiner et al., 1985; Grasman, Onno, et al., 1999):

$$\frac{\partial}{\partial t}G(z,t) = v\frac{\partial}{\partial x}G(z,t) + \frac{1}{2}\frac{\partial^2}{\partial z^2}G(z,t), \tag{6}$$

with the following initial condition,

$$G(z,0) = \begin{cases} 1, & 0 < z < a, \\ 0, & o.w \end{cases}, \tag{7}$$

and the following boundary conditions:

$$G(a,t) = 0, \quad \frac{\partial}{\partial x}G(x,t)|_{x=0} = 0. \tag{8}$$

Since the process is homogeneous, so $G(z,t)$ also determines the first passage time the process starts from z. Thus, by approximating the solution of this Fokker-Plank equation, the first passage time distribution of the RBDDM could be obtained. But it is important to mention that there is some analytical solution for the first passage time. The analytical form of survival probability of this process can is as follows (Goel & Richter-Dyn, 2016; Dybiec, Gudowska-Nowak, & Hänggi, 2006):

$$S(t) = \frac{2}{a} e^{\frac{v(-2z-vt)}{2}} \sum_{n=0}^{\infty} \left[\frac{\frac{(2n+1)\pi}{2a}(-1)^n e^{va} - v}{v^2 + (\frac{(2n+1)\pi}{2a})^2} \times \cos[\frac{(2n+1)\pi z}{2a}] \times e^{-\left(\frac{(2n+1)\pi}{2a}\right)^2 \frac{t}{2}} \right]. \tag{9}$$

So, by considering the survival probability distribution as above, the cumulative first passage time distribution, $F(t)$, and first passage time density function, $f(t)$, can be obtained easily by:

$$F(t) = 1 - S(t), \quad f(t) = -\frac{d}{dt} S(t).$$

Moreover, the probability of the go and no-go responses can be define as follows. Figure (2) illustrates the proportion of the go response as the function of t_{end}. As it is obvious the proportion of go response increases by increasing the termination time, t_{end}.

$$P_{no-go} = S(t_{end}), \quad P_{go} = 1 - P_{no-go}.$$

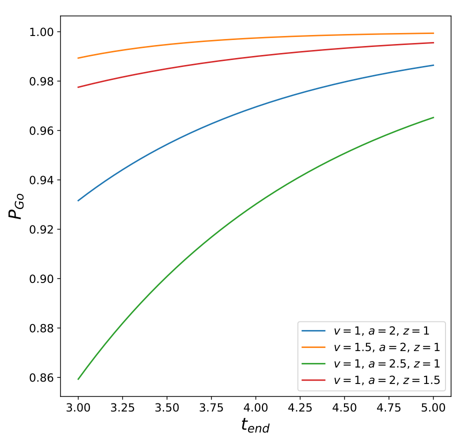

Figure 2: Plot of go proportion as a function of t_end for different parameter values.

Qualitative model comparison

Two main models which are used to model Go/No-Go paradigm are DDM (Ratcliff et al., 2018) and also single accumulator race diffusion model (RDM) (Trueblood, Endres, Busemeyer, & Finn, 2011). Figure (3) illustrates the behaviors of these models besides the RBDDM. RBDDM behaves similarly to the DDM when it has high drift rate values or high boundary separation values. But it tends to the behavior of the RDM when the relative the starting point approaches to the lower boundary.

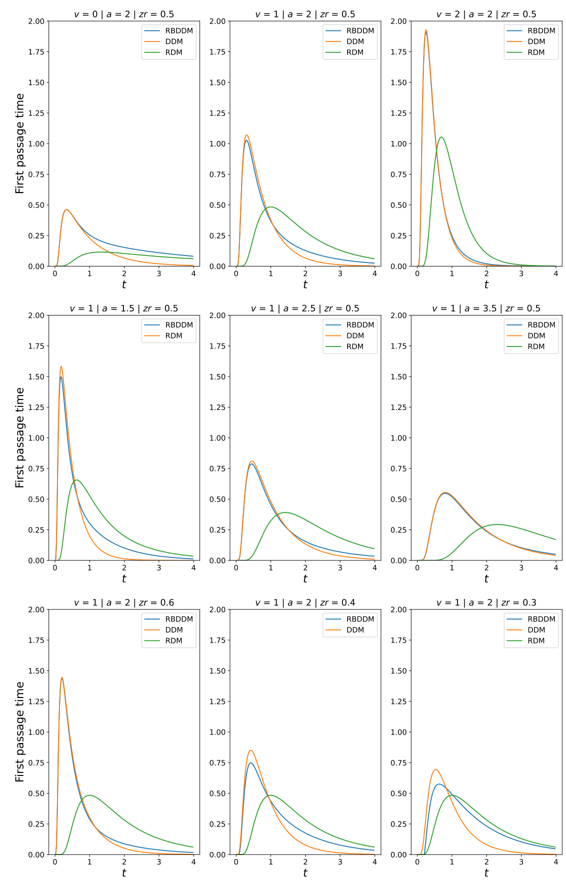

Figure 3: Behavior of first passage time distribution of RBDDM, DDM, and RDM for different parameter values.

Parameter recovery

Parameter recovery is one of the important parts of a cognitive model. If the parameters of a model do not recover well, the psychologists can not inference based on the obtained values for the parameters (i.e the obtained values are not interpretable). Thus, the parameters of a good model should recover well. Various procedures of the parameter recovery have been developed for the sequential sampling models such as the maximum likelihood (Heathcote, Brown, & Mewhort, 2002), Chi-squared (Ratcliff & Tuerlinckx, 2002), PDE method (Voss & Voss, 2008), hierarchical Bayesian computing (Wiecki, Sofer, & Frank, 2013), and Monte-Carlo simulation (Chandrasekaran & Hawkins, 2019). But by the growth of the model complexity, obtaining the exact likelihood function for the models becomes harder and the previous methods can not be utilized and some more powerful methods are needed. Recently, deep learning algorithms have been used for estimating the parameters of the sequential sampling models and several network architectures have been developed by the researchers (Fengler, Frank, Govindarajan, & Chen, 2020). The foundation of utilizing a deep learning method for estimating the parameters of a model is simulating the behavior of the model in the parameter space and constructing a training data-set based on sampled data. Then, the networks learn the behavior of the model in the parameter space and map it to the parameter values of the model. The developed networks consist of a part for feature extraction which could be an auto-encoder (Radev, Mertens, Voss, Ardizzone, & Köthe, 2020) or several convolutional layers (Radev, Mertens, Voss, & Köthe, 2020). In our case, we have utilized a deep inference neural network (Radev, Mertens, Voss, & Köthe, 2020) with six convolutional layers for estimating the parameters of the model. The training data set includes 99000 sample data consist of 99000 RBDDM experiments with 100 trials and 35640 sample data consist of 35640 RBDDM experiments with 200 trials. These numbers of trials are chosen because the minimum number of trials for the most of Go/No-Go paradigms in the clinical setup is 100. But there are some clinical experiments such as CPT that need much more trials. The simulated experiments have different parameters that are sampled with the following priors:

$$v \sim \mathcal{HN}(1,3),$$

$$a \sim Gamma(2,2),$$

$$zr \sim Uniform(0,1),$$

$$t_0 \sim \mathcal{HN}(0.3,0.3),$$

where zr is the relative starting point bias and defines $zr = \frac{z}{a}$. Moreover, the number of go trials varies between 50 to 100 and is obtained by $100 \times Uniform(0.5,1)$. The go trials are simulated with a positive drift rate (v) and the no-go trials are simulated with a negative drift rate ($-v$). Also, the response time for the no-go responses is considered equal to the termination time and the conditions of each trial are stored too ($+1$ for go trials and -1 for no-go trial). Therefore, the accuracy of each trial can be obtained by comparing the response time and the condition. For the simulation of the training data set, the time step is considered $\Delta t = 0.001$, and all simulations are done with termination time $t_{end} = 3$. Each convolutional layer of the deep inference network has 2 channels and the filter size of the layers are 64, 64, 128, 128, 128, 216 respectively. Figure (4) and Figure (5) illustrate the quality of parameter recovery of the model with 100 and 200 trials.

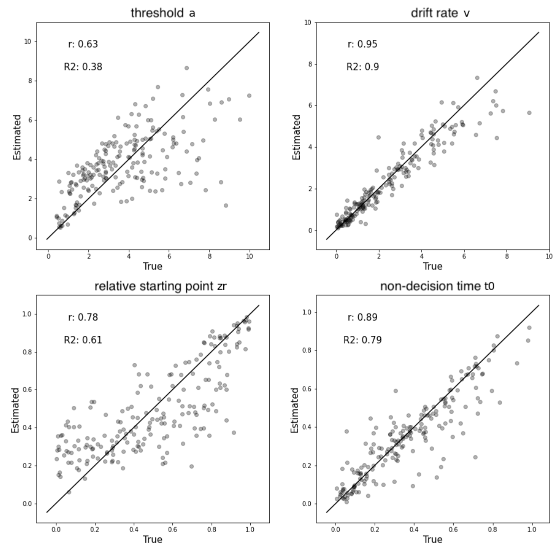

Figure 4: The plot of precision of parameter recovers for RBDDM with 100 trials.

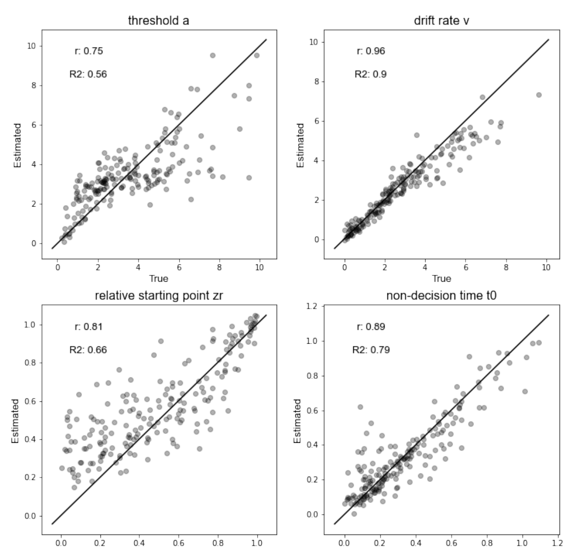

Figure 5: The plot of precision of parameter recovers for RBDDM with 200 trials.

As you can see, the drift rate, non-decision time, and the relative starting point bias parameters can be recovered very well in both data. Additionally, the recovery precision of the threshold parameter is acceptable with 100 but it improves when the number of trials increases.

Behavioral Data Analysis

In order to test the performance of the RBDDM on behavioral data, it is fitted on the collected data from twenty-four children, between 7–12 years old (M = 9.25, SD=1.53) who participated in a stop-signal task. All participants were diagnosed with ADHD symptoms by a professional child psychiatrist according to the fifth version of Diagnostic and Statistical Manual of Mental Disorders (Edition et al., 2013). In the stop-signal task, a plane appears at the middle of the screen in one of the four possible orientations (up, down, left, and right), as the go stimulus. The participants are instructed to press corresponding arrow keys on the keyboard as quickly and accurately as possible. In 50 out of 100 trials, a beep sound is presented which serves as the stop signal, and participants have to deny the response. Both DDM and RBDDM have been fitted on this data set which resulted as reported in Table (1).

Table 1: Estimated parameters for DDM and RBDDM.

Model	a	v_{go}	v_{no-go}	zr	t_0	BIC
DDM	1.74 (0.24)	0.94 (0.52)	0.35 (0.26)	0.60 (0.08)	0.62 (0.23)	10722
RDDM	2.92 (0.87)	0.53 (0.19)	–	0.59 (0.10)	0.24 (0.06)	**4484**

The obtained BIC for the DDM and RBDDM yields that the RBDDM is fitted on this data much better than the RBDDM. The fitting procedure of the DDM is as same as (Ratcliff et al., 2018) and the RBDDM is fitted by the mention convolutional neural network in this paper.

Discussion

There are several discussion points about the RBDDM. The processes that it captures, go bias modeling, fitting procedure, and separate drift rate for go and no-go responses are the main discussion points.

The type of processes that the model can capture is the first discussion point. The presented model can generate several processes that are not generated by traditional Go/No-Go DDM. The main three processes that are not generated by the DDM and the RBDDM has the ability to generate them are a) the accumulator approaches the upper boundary but it does not reach to the upper boundary b) the accumulator approaches the lower boundary and it does not reach to lower boundary c) the accumulator reaches the lower boundary and reflects. On the other hand, the previous Go/No-Go DDMs assume that the no-go decision is made whenever the accumulator hits the lower boundary but in RBDDM, all alive processes at the end of the time interval are considered as the no-go response.

The second point of discussion is the way to model the go bias. Previously three strategies have been introduced for capturing the go bias in the model including starting point bias, the bias in the drift rates, and bias and reduction in the drift rates. But here, we have used only one drift rate for both go and no-go decisions and the go bias is captured by adding a reflecting boundary to the model and also locating starting point bias closer to the go threshold.

The third point of discussion addressed the fitting procedure. The previous fitting procedure that was used for fitting DDM on Go/No-Go data, is the quantile chi-squared method. By utilizing this framework, all the no-go responses represent by one quantile and the go responses are presented by five or nine quantiles. It is important to mention that the fitting procedure can add some ad-hoc assumptions to the model. For example in the case of using the quantile chi-squared method for the Go/No-Go data, considering only one quantile for the no-go responses yields the no-go responses have a uniform distribution which is not a realistic assumption. In contrast, the RBDDM considers all alive processes at the termination time as the no-go response and represents them by termination time, which does not add an ad-hoc assumption to the model. Because the assumption is the decision maker should persist on his/her no-go decision until the termination time. As illustrated in this paper, the RBDDM can be fitted better than the DDM model on the behavioral data.

The last thing that should be discussed, is considering the same drift rate for go and no-go responses. As mentioned before, for capturing the go bias, the Go/No-Go DDMs use separate drift rates for the go and no-go decisions. In fact, the Go/No-Go DDMs include two separate drift rates in the model and estimate them simultaneously. But in RBDDM, it is assumed that the go drift rate is negative of the no-go drift rate. This assumption adds one limitation to the model which is the conclusion of the estimating problem in the no-go drift rate in RBDDM. One of the solutions for this issue that provides a possibility for overcoming this problem is utilizing joint modeling. For example, it is possible to estimate the no-go response time distribution by using eye-tracking or EEG techniques which are out of the scope of this study and could be the topic of some future researches (Turner, Forstmann, Steyvers, et al., 2019).

Conclusion

In this paper, a new drift diffusion model for the Go/No-Go paradigm was introduced which is based on substituting the lower absorbing boundary of the traditional drift diffusion model with a reflecting boundary. The model can be considered as the general form of the previous Go/No-Go DDMs. Moreover, the mathematical properties of the model were discussed. A formulation for the mean first passage time and a partial differential equation for the first passage time distribution of the model were presented. Finally, the parameter estimation of the model based on simulation study was illustrated and it was shown that the parameters of the model can recover well.

Code and Data Availability

All codes and the data of this paper are available online at `https://osf.io/chvqm/`

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