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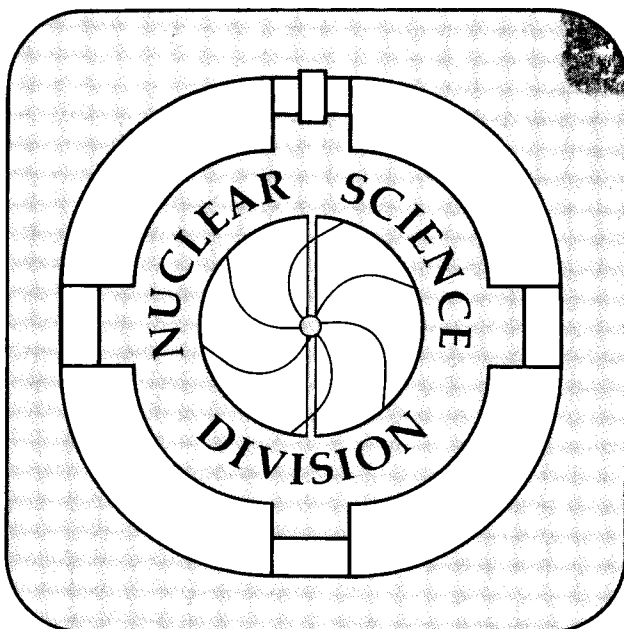# Lawrence Berkeley Laboratory

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The Charge and Mass Dependence of Nuclear Interaction Cross Sections

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NUCLEAR SCIENCE DIVISION

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The Charge and Mass Dependence of Nuclear Interaction Cross Sections [†]

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Abstract

The Thomas-Fermi model of Seyler and Blanchard [1] is employed for the purpose of calculating the interaction cross sections of nuclei as a function of charge and mass. Comparison is made with the Bevalac experiments of Tanihata et al. [2].

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1 Introduction

The current availability of exotic-isotope beams at the Bevalac [2] and at GANIL [3] has made it possible to measure interaction cross sections σ_I for nuclei far from stability. It was observed that separate projectile and target interaction radii ($R_I(p)$ and $R_I(t)$) can be very accurately parameterized by assuming that

$$\sigma_I(p,t) = \pi[R_I(p) + R_I(t)]^2 . \tag{1}$$

This separability suggests that these radii can be used to characterize each nucleus uniquely.

Especially for the light nuclei, where most of the measurements have been made, one expects to see interesting variations in these radii because specific shell model orbitals are involved. For example, a just completed shell is expected to be especially compact, while mid-shell orbitals are expected to be larger than average or to produce deformations. Such effects can be identified by looking for differences between statistical model predictions and the measured values. Neither the liquid drop model nor the droplet model [4] are suitable for making this sort of comparison because they do not contain any means for predicting the dependence of the nuclear surface properties on neutron excess.

In order to overcome this limitation and other similar problems that may arise in treating light nuclei at the limits of stability, we have undertaken a series of self-consistent, Thomas-Fermi calculations using a phenomenological interaction similar to that of Seyler and Blanchard [1]. In the next section these calculations are briefly described. The following sections describe how this model was used to calculate the interaction radii and how they compare with the measured values. In the final section specific differences are discussed and some remarks are made concerning the general trends of the calculations.

2 The Thomas-Fermi method and the Seyler-Blanchard force

The phenomenological, momentum-dependent, two-body interaction of Seyler and Blanchard [1] has been employed in general studies of saturating two

component systems [4]), for predicting nuclear masses and sizes [5]), for studying nuclei at finite temperatures in equilibrium with their associated vapor [6]), and for a detailed study of the behavior of the surface energy of a two-component system [7]). The nuclear properties are obtained by minimizing the energy of a system of particles whose kinetic energy distribution is obtained from the density by the Thomas-Fermi assumption and whose potential energy is calculated with the phenomenological Seyler-Blanchard interaction. The Euler equation that results is solved by computer iteration.

We found that it was necessary to modify the original Seyler-Blanchard interaction slightly in order to obtain satisfactory agreement with the measured charge distributions while retaining agreement with measured values of the nuclear masses.[†] The interaction that was used for two like (l), or unlike (u), nucleons with separation r and relative momentum of magnitude p (where p is in units of the Fermi momentum of standard $N = Z$ nuclear matter) was

$$V(r,p) = -\frac{C}{4\pi a^3} \frac{e^{-r/a}}{r/a} [\alpha_{l,u} - \beta_{l,u} p^2 + \gamma_{l,u}/p] , \qquad (2)$$

with the parameter values

$$
\begin{array}{llr}
C = 455.46 \text{ MeV fm}^3 , & a = 0.59542 \text{ fm} & \\
\alpha_l = 0.74597 , & \alpha_u = 2.86331 & \\
\beta_l = 0.25255 , & \beta_u = 1.23740 & (3) \\
\gamma_l = 0.21329 , & \gamma_u = 0.0 . &
\end{array}
$$

These values were determined by a fit to nuclear masses [8]) and they lead to the following nuclear properties:

$$
\begin{array}{lll}
\text{radius constant of nuclear matter} & r_0 = & 1.13 \text{ fm} \\
\text{volume energy} & a_1 = & 16.527 \text{ MeV} \\
\text{symmetry energy} & J = & 31.375 \text{ MeV} \quad (4) \\
\text{surface energy} & a_2 = & 20.268 \text{ MeV} \\
\text{compressibility} & K = & 301.27 \text{ Mev.}
\end{array}
$$

[†]We are currently engaged in an extension of the Seyler-Blanchard, Thomas-Fermi approach to the calculation of fission barriers as a function of angular momentum. One of the consequences of this project will be a more precisely determined set of force parameters.

3

Since we are mainly concerned here with the spatial extent of the nuclear density distribution we imposed an additional restriction on the determination of the force parameters by demanding that the diffuseness of our calculated charge distributions should be in substantial agreement with that obtained from electron scattering experiments [9]. (See Fig. 1.) In addition the properties of pure neutron matter implied by our choice of parameters were adjusted to be in close agreement with the work of Friedman and Pandharipande [10]. (See Fig. 2.)
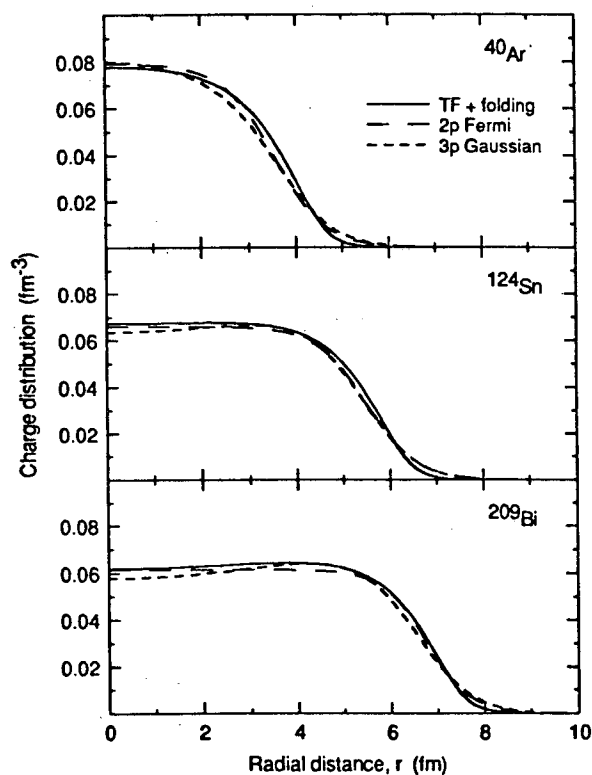
Fig. 1   The calculated charge distribution, which is obtained by folding the proton charge distribution ($\langle r^2 \rangle^{1/2}$ equals 0.85 fm) into the results of the Thomas-Fermi calculation, is compared with the measured charge distributions in a number of cases. In each case comparison is made with a two parameter Fermi distribution and a three parameter "Gaussian" distribution whose parameters have been determined from fits to electron scattering angular distributions. (For more details see ref. [9].)

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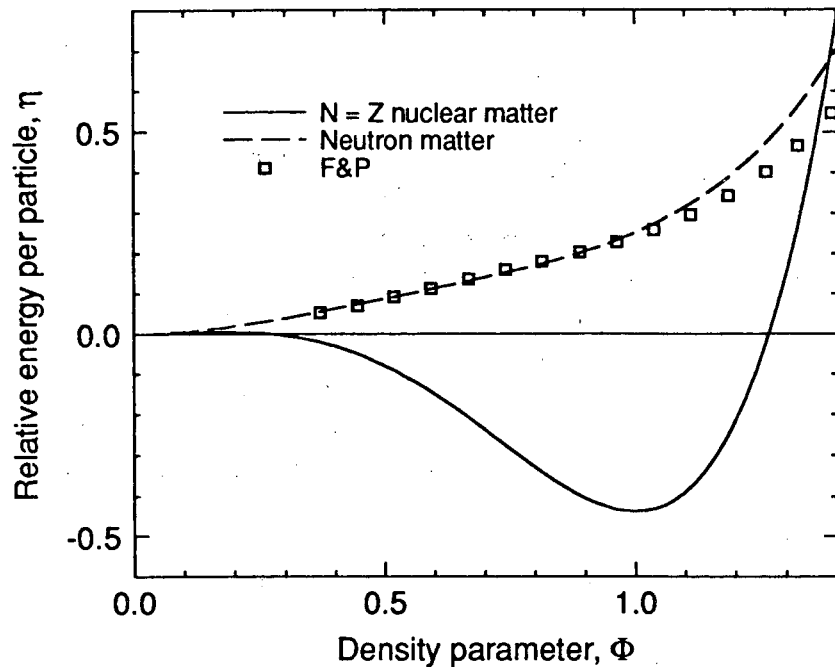
Fig. 2   The calculated energy per particle of nuclear matter, η, is plotted versus the density parameter, Φ, (proportional to the Fermi momentum) for the two cases of $N = Z$ nuclear matter and pure neutron matter. (These quantities are in units of the 37.68 MeV calculated Fermi energy and the 266.0 MeV/c Fermi momentum of $N = Z$ nuclear matter.) The open squares correspond to the detailed microscopic calculation of the properties of pure neutron matter by Friedman and Pandharipande [10]).

3 Calculation of Interaction Radii

The value of the interaction radius R_I for comparison with the measurements was obtained from the Glauber theory calculation,

$$\pi R_I^2 = 2\pi \int_0^\infty \left[1 - \exp\left(-\bar{\sigma} \int_{-\infty}^{+\infty} \varrho\,(b,z)\,\mathrm{d}z\right)\right] b\,\mathrm{d}b\,, \tag{5}$$

where z is the longitudinal and b the transverse (or impact parameter) coordinate in a cylindrical coordinate system oriented along the beam direction. The particle number density $\varrho\,(b,z)$ is obtained from the Thomas-Fermi calculation described in the previous sections. In this expression $\bar{\sigma}$ is 85% of the elementary nucleon-nucleon cross-section, $\sigma = 40$ mb, appropriate for the 790 MeV bombarding energy. This reduction is thought to be associated with the fact that the interaction cross section in nuclear matter is expected to be smaller than the free nucleon value. This was pointed out already in ref. [2]) and the specific value of the reduction factor we have used was suggested there. The agreement that we find between our calculated values of R_I and the measured ones gives additional support to this choice.

Figure 3 shows how our calculated values of R_I (represented by the open circles) compare with the measured values for isotopes of helium, lithium, beryllium and boron. As expected the strong binding of ^{4}He causes it to be somewhat smaller than the Thomas-Fermi prediction. Both ^{11}Li and ^{14}Be are larger than would be expected by smooth extrapolation along the isotopic sequence. It is interesting to note that both of these nuclei lie slightly beyond the neutron drip line predicted by our particular choice of parameters in the Thomas-Fermi calculations. This fact lends support to the view that the existence of these nuclei is due to quantum effects, in particular, to pairing.

We have recently shown [11]) that quantum penetration into the forbidden region can produce surprisingly large probabilities for finding a loosely bound nucleon outside the nucleus. Tanihata [12]) has further shown that there is a definite relationship between the separation energy of the last neutron and the deviation of the measured interaction radius from the Thomas-Fermi model predictions (using a model similar to the one presented here). This deviation has the expected behavior, in that, the observed value of the inter-

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action radius increases as the separation energy decreases.
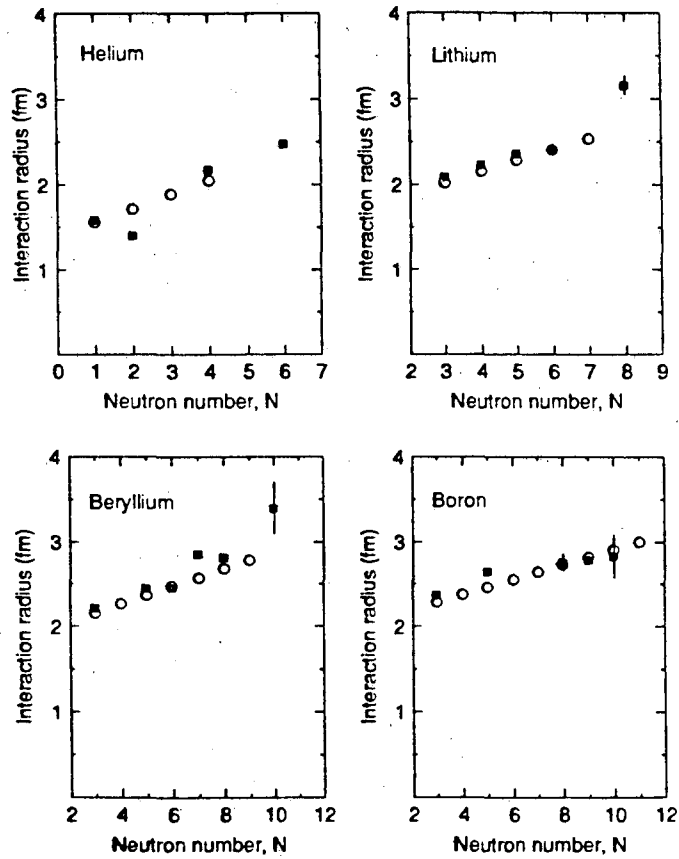

Fig. 3 The interaction radii R_I are plotted against the mass number A for different isotopic sequences. The solid squares represent the measured values from the work of Tanihata et al. [2] for isotopes of helium, lithium, beryllium and boron. The interaction radii calculated from the Thomas-Fermi density distributions and eq. (5) are indicated by the open circles in each figure.

In Table 1 we have listed the calculated values of the interaction radii for the isotopes of all the elements from helium to neon. The actual location of the limits of stability predicted by the model may change slightly when the values of the coefficients used in the model are further refined.

TABLE 1

Calculated interaction radii for the isotopes of all the elements from helium to neon.

N\Z	He 2	Li 3	Be 4	B 5	C 6	N 7	O 8	F 9	Ne 10
1	1.57								
2	1.73	1.90	2.08						
3	1.90	2.03	2.16	2.29					
4	2.06	2.16	2.27	2.38	2.49				
5		2.29	2.37	2.47	2.56	2.66	2.75		
6		2.41	2.48	2.56	2.64	2.72	2.81	2.89	
7		2.54	2.58	2.65	2.72	2.79	2.87	2.95	3.02
8			2.69	2.74	2.80	2.87	2.94	3.00	3.07
9			2.79	2.83	2.88	2.94	3.00	3.06	3.13
10				2.92	2.96	3.01	3.07	3.12	3.18
11				3.01	3.04	3.08	3.13	3.19	3.24
12					3.12	3.16	3.20	3.25	3.30
13					3.20	3.23	3.27	3.31	3.35
14					3.28	3.30	3.33	3.37	3.41
15						3.38	3.40	3.43	3.47
16						3.45	3.47	3.49	3.53
17							3.53	3.56	3.58
18							3.60	3.62	3.64
19								3.68	3.70
20								3.74	3.76
21								3.80	3.81
22									3.87
23									3.93
24									
25									

4   Conclusion

As the first step in a major program of applying the Thomas-Fermi method to the prediction of macroscopic nuclear properties we have calculated interaction radii of exotic-isotopes for comparison with the values measured by Tanihata et al. The parameters of the force were chosen to reproduce nuclear binding energies and the charge distributions measured by electron scattering. The resulting density distributions for the isotopes of interest were calculated and interaction radii inferred using a simple Glauber theory approach and an effective nucleon-nucleon cross section.

Originally we thought that even the anomalously large interaction radii of ^{11}Li and ^{14}Be might be reproduced by the theory since the diffuseness of the density distribution increases as the drip-line (limit of stability against particle emission) is approached. The trend, while present, was not sufficient to explain the observed values of R_I. We hope that more measurements at the limit of stability will be made so it can be determined whether the sudden increase in size is associated with some macroscopic effect that has not yet been properly treated or whether it is associated with the discreteness of the system and the filling of specific shell model states.

In Fig. 4 we have plotted the ratio of our interaction radii to the quantity $1.13\ A^{1/3}$ against mass number for the isotopes of carbon and neon. In each case the most stable isotope is indicated by a solid square. By and large it is clear that the nuclei are more compact the more stable they are, and that the relative size tends to increase as one moves in either direction toward the limits of stability.
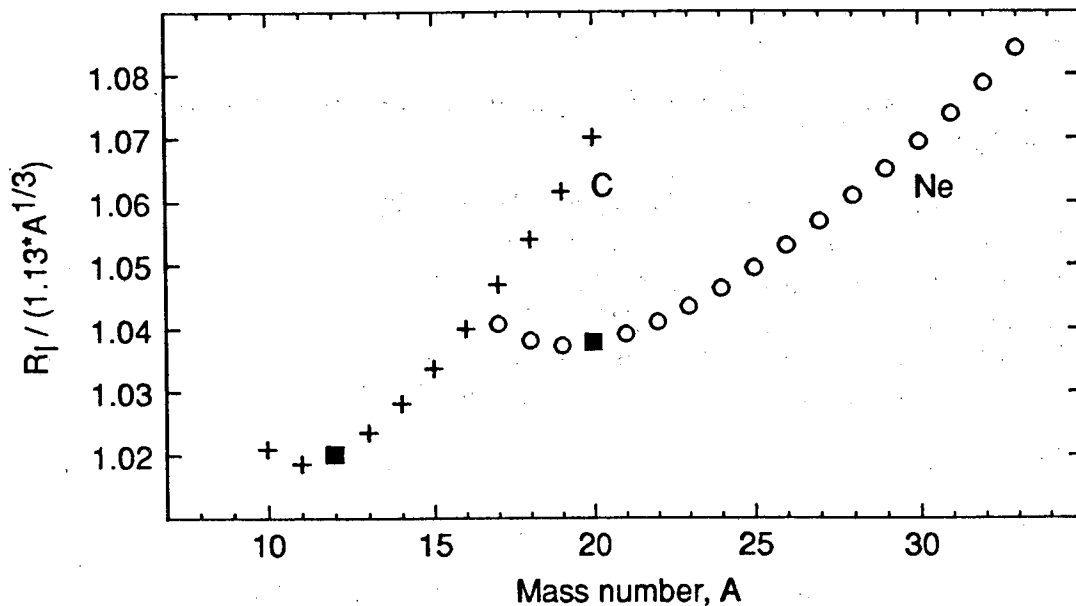
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Fig. 4   The ratio of our calculated interaction radii to the liquid drop model value of $1.13\ A^{1/3}$ is plotted versus mass number A for the isotopes of carbon and neon. In each case the most stable isotope is indicated by a solid square.

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