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MODELS FOR RISKY INVESTMENT

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The models presented attempt to evaluate the return on investment for alternative investments, and hence, investment portfolios. The emphasis is on computationally efficient rather than purely optimal methods of determining the portfolio composition and its return on investment. The methods also provide the return on any given investment when risk is explicitly considered.

The model utilizes some behavioral assumptions regarding the admissability of risks as the basis for the use of a bounded utility function with expected value as the decision criteria. The model is then extended to the case of equity investment, and a computational algorithm which considers the risk posture in terms of its joint probabilities is presented.

MODELS FOR RISKY INVESTMENT*

The problem of reducing risk in a given set of situations, namely, real estate and security portfolio analysis, cannot be accomplished through simple diversification. If each of the potential investments has an "expected" or "nominal yield," we can describe any variation from this "yield" by a distribution $h(x_{ij}, P(x_j))$. The correlation of returns or risk involved in a group of investments prevents the elimination of risk through diversification.[1,2]

The models presented evaluate the return on investment of various portfolios. The conditions under which large investors may rationally use expected value as the decision criterion are described and a computational model has been developed.

The following variables are used in formulating the general model:

i = the investment alternatives, where $i \varepsilon I$

j = the non-routine events having an effect on any $i \varepsilon I$; where $j \varepsilon J$

t = the period (e.g., years, months, etc.) $t = 1, \ldots, p$

C = the capital to be added to the investor's portfolio

C_i = the amount to be invested in the ith investment (as dollars or a proportion of the total capital added).

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*Based upon research done while the author was a Graduate Research Fellow in the Real Estate Research Program, UCLA. The author is indebted to Karl Borch and William N. Kinnard, Jr. for their helpful comments on earlier versions. I am, of course, responsible for any errors which remain.

[1]See Markowitz (19), p. 5, for a further discussion of this problem with regard to security analysis.

[2]Models for security and real estate portfolio analysis which do not assume independence of returns from separate investments are very similar to the more sophisticated models of capital budgeting. See Weingartner (26).

If $\Sigma C_i = 1 = C$, all New Capital will be invested, and hence the entire portfolio will be invested.

$P(x_j)$ = the probability of the jth event

x_{ij} = the consequence (loss or risk) involved in the ith investment given the jth event occurs (a value measure)

$x_{ij}P(x_j)$ = the expected value of the jth event for the ith investment

$E(x_i) = \sum_{j=1}^{n} x_{ij}P(x_j)$ = the expected variation from the contracted payment for the ith investment. This might be viewed as the measure of risk from the ith investment for a decision maker whose utility for money is linear for the values (x_{ij}) for all $j \varepsilon J$

$E(x_{ij}) = h(x_{ij},P(x_j)) = \int_0^1 x_{ij}\phi(x_j)$ for non-independent risks

$g(x_k,P(x_k))$ = a distribution representing the investor's risk policy where x_k is an undesirable consequence, and $P(x_k)$ is the maximum acceptable probability with which the investor will accept the possibility of x_k occurring

$F^o(x_j) = g(x_{ij},P(x_j)) = g'(x_j,\phi(x_j))$ for $\bigcap_j P(x_j) \neq 0$, $j \varepsilon J$

$F^o(x_k) = g(x_k,P(x_k)) = g'(x_k, \phi(x_k))$

P_{it} = the ith investment's contracted payment to principal and interest in year t

$P_{it} = \left[P'_{it} \vdots P''_{it} \right]$ where the partition separates payment to interest (P'_{it}) and payment to principal, sinking fund, or return of capital (P''_{it}).

It is clear that the replacement of random parameters by their expected values results in a solution which is feasible, but not necessarily optimal.

(It is necessary to find the solution to the appropriate stochastic programming problem to guarantee optimality and this is usually difficult.)[3] However, the additional restrictions developed in the model coupled with the number and type of investments undertaken by the decision maker should provide sufficient conditions for optimality on this point.

The models presume the investor has the ability to evaluate the risks in terms of consequences and probability distributions for each of the alternative investments. For the case of real estate portfolio analysis, this will require that the investor have a history of default, foreclosure, and renegotiation of mortgage terms, in terms of geographical, spatial, income producing, and other economic characteristics (e.g., type of property, type of construction, competitive environment, et. al.). There has been sufficient success in evaluating consumer credit to indicate that the development of these histories would be feasible.[4] The Smith (22) model uses a system with "bad account probabilities" as a basis for the development of a risk measure. His method uses the attributes of the borrower (age, sex, occupation, etc.) as the parameters in the risk measure. Although his model uses a risk index as its measure, its index is developed from the

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[3] Hadley (9), p. 181.

[4] Smith (22), pp. 327-340. Also, he notes a number of recent studies among his references. (Isaac, Earl J., "Statistical Problems in the Development of Credit Scoring Systems," Numerical Pointing Plans for Evaluating Consumer Credit Risks, Philadelphia Credit Bureau, Philadelphia, 1963, pp. 38-48; _____ and Forgy, Edward W., "The Development of Numerical Credit Evaluation Systems," Journal of the American Statistical Association, Sept. 1963, pp. 799-806; Wells, Henry L., "New Customer Credit Pointing System," Numerical Pointing Plans for Evaluating Consumer Credit Risks, Philadelphia Credit Bureau, Philadelphia, 1963, pp. 4-21; Zaegel, R.J., "Experience with a Credit Scoring System," Numerical Pointing Plans for Evaluating Consumer Credit Risks, Philadelphia Credit Bureau, Philadelphia, 1963, pp. 22-37.

analysis, in a Bayesian sense, of the marginal, conditional and joint probabilities.[5]

The Model

$$\underset{r}{\text{Max}} \sum_{i=1}^{m} \left(-C_i + \sum_{t=1}^{p} \frac{P_{it} - E_t(x_{ij})}{(1 + r_i)^t} \right)$$

Subject to

$$F^o(x_j) < F^o(x_k) \text{ at all points}$$

$$\Sigma C_i = 1 = C$$

Where max r implies an ordering of the feasible investments and acceptance of investments with monotonically non-increasing r_i, $(r_1 > r_2 > \ldots r_n)$, where r_i is the return on any given investment until $\Sigma C_i \cong 1 \cong C$. (The approximately equal allows us to forego immediately a discussion of the end point conditions.)

The author is aware of the arguments regarding internal-rate-of-return methods and the asserted superiority of present value[6] and annual-net-benefit methods. Hirshleifer (15) states the latter two methods are equivalent and logically satisfactory alternatives. And, although internal-rate-of-return will normally lead to the same conclusions as

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[5] Smith (22), pp. 329-330 and 338-339.

[6] See Lorie and Savage (17) for the conditions under which internal-rate-of-return will not maximize present value of the firm's net worth. Some other expositions favoring present value methods are Alchian (1), Hirshleifer (15), and Hirshleifer (14), however, this latter article does describe some of the limitations of present value methods under uncertainty. Robinson (20) is among the contemporary supporters of internal-rate-of-return methods.

the latter two methods it is not logically equivalent and does on occasion lead to non-unique results.[7]

However, the recent efforts of the author in working with models using an annual-net-benefits schema led to undesirable results. With the help of T. Gillings,[8] tests of an annual-net-benefits model were made using his special analog computer. In these tests the model gave ambiguous results. The model was of the general form

$$\left[A_i = -C_i + \sum_{t=1}^{p} \frac{(P_{it} - E(x_{it}))}{(1 + r_o)^t} \right]$$

where r_o = the discounting rate, cost of capital or rate of return with certainty for this lender.

The ambiguity results when the investments compared 1) were of differing time periods, and 2) the alternative calculation of the internal-rate-of-return (r) was two to three times the discount rate (r_o). This was verified first by using the analog computer and then by repeating the calculations using digital equipment. In these cases it was felt that the model had a definite bias towards investments with longer lives.

Reassessment and evaluation of the conditions under which models of this type might be used led to the selection of an internal-rate-of-return formulation over present value formulations although the author realizes its limitations.

The obvious assumption which first must be made is that a continuous

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[7] See Hirshleifer (15), p. 156.

[8] Visiting Professor of Business Administration, University of California, Berkeley. The tests were performed in April, 1966.

stream of approximately equivalent investment alternatives exists over time, implying

$$I_1 \cong I_2 \cong \ldots \cong I_t \ .$$

Although this at first appears untenable, many investors faced with the problem of investing large amounts of funds on a continuous basis find that the long-run nature of their commitments precludes any substantial gains being made through variations in the market. And, in fact, the costs in extra work and effort, coupled with reduced return from generally more liquid investments, far outweighs any possible gains to be made in this type of speculative activity. Perhaps the largest problem of these investors when attempting to take advantage of variations in market rates over time is that they will have to place any funds held over and an equivalently large addition of new funds. Since their ability to select a diversified portfolio so that they may reduce risk and still maintain a high return may already be severely taxed, the probability that these additional funds may have to be placed in investments of lesser quality or return greatly increases.

In the use of rate-of-return models the second problem to be faced is non-uniqueness. Lorie and Savage (17) have constructed a classic example of non-uniqueness in the evaluation of the possible replacement of an oil well pump by a more efficient pump. In this case we have:

(1) an initial outlay (new pump)

(2) subsequent cash inflows

(3) a net cash outlay (absence of oil because of more rapid exhaustion due to higher capacity pump)

Here, the reference point is assumed to be the production level using

the less efficient pump and not some zero measure.[9] Although it is obvious that the above conditions will lead to non-unique results, the use of expected values for the stochastic variables and a real zero measure should eliminate non-unique solutions for our proposed model. It might be noted that Lorie and Savage (17) along with many other writers[10] point out the rare occurrence of non-unique rates (r) in cases which can be considered economically interesting; i.e., those cases in which

$$-1.0 < r < +\infty$$

where -1.0 = total loss of capital invested.[11]

A Description of the Model

The nature of the investment decision can probably be best represented by an illustration of the types of investment alternatives which will make up the portfolio.

An example of a set of possible investments might be:

i	i^{th} investment alternative	C_i	R_i	p
12	Shopping Center - Daly City	3.2 million	7.5%	(1978)
21	Office Building (13 story) St. Louis	4.1 million	7.3%	(1983)
7	High Rise Apartment - San Diego	2.6 million	6.9%	(1986)
.				
.	. . .	. . .	. . .	. . .
.				
1	U.S. Government Bonds	300 million 250 million	3.9% 3.8%	(1986) (1983)

[9] See Lorie (17), pp. 229-239.

[10] Even Hirshleifer (15) a severe critic of rate-of-return admits the rarity of non-uniqueness.

[11] See Hillier (13), p. 486.

The firm would attempt to max r by selecting from the ordered set of alternative investments until

$$\sum_{j=1}^{m'} C_i = C = 1 \text{ for } i \varepsilon I;\ i = 1,\ldots,m',\ldots m$$

where m' is the last i which is to be entered in the portfolio.[12] It is highly unlikely that the capital required for the last investment i = m' would be exactly equal to the capital remaining. Therefore, it will be necessary to set up a decision role to handle the end point conditions. Hirshleifer (14), Baumol and Quandt (3), and Weingartner (26) all propose methods to handle these conditions.

One simple decision rule (α_o) might be:

(1) borrow enough to complete the portfolio $\Sigma C_i = C$ if and only if $r_{m'} > r_o(b)$, where $r_o(b)$ = the marginal borrowing rate for the investor.

(2) Invest in riskless short-term government investments; e.g., various saving and loan associations in $10,000 increments or short-term government bonds, $r_o(s)$, until $\Sigma C_i = C$ if $r_{m'} < r_o(b)$ and the analyst anticipates that better opportunities will be available in the near future (this can be viewed as the alternative to holding cash).

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[12] An alternative formulation might be

$$\sum_{i=1}^{m} \gamma_i C_i = C = 1, \text{ where } \gamma_i = \begin{cases} 1, \text{ when } i \leq m',\ r_i \geq r_{m'} \\ 0, \text{ when } i > m',\ r_i < r_{m'} \end{cases}$$

See Weingartner (26) for an integer programming formulation.

(3) Invest in long-term government bonds, $r_o(1)$, until $\Sigma C_i = C$; if $r_{m'} < r_o(1)$, or if $r_o(1) < r_{m'} < r_o(b)$ and opportunities do not appear lucrative enough to postpone long-term placement.

To the degree the alternative C_i have r_i (returns) that are approximately equal this problem will assume greater importance and more complex decision rules (α) will have to be developed.

The Risk Policy

The firm (investor) who seeks to increase his return on investment must ordinarily take increasing risks in order to increase the net return on its portfolio. These risks in the real estate investment market will range from foreclosure and the sale of the property at some value less than the remaining investment to a revision of the interest rate downwards or simply late payments, and the accompanying loss of a few days' interest or the cost of administrative activity resulting in a less than anticipated realization.

Our investor, being a large investor, will probably be able to "self-insure" for most risks. However, for those extreme losses which he is not willing to sustain he has two possible alternatives given the structure of the model presented:

(1) He can sell participation shares in the investment in order to reduce his possible loss (x_{ij}). This operates much like reinsurance, or

(2) He can decide not to accept this alternative. This is the primary method used to eliminate overly large possible

losses.[13,14,15]

Given that our investor uses method (1) wherever practical there may still be some losses he shall be unwilling to sustain. It appears that he will be willing to face larger risks (x_{ij}) if and only if the probability of occurrence $P(x_j)$ is considerably reduced. If we can assume there is no perfectly certain state of the world except the complete set (Ω, all possible states of the world) with probability $P(\Omega) = 1$ and the null set ($\emptyset$) with probability $P(\emptyset) = 0$, then there must exist some probability $P(x_j) > 0$, no matter how small, that a given x_j occurs. Now determine the investor's risk policy $F^o(x_k)$ in terms of x_k and $P(x_k)$ or

$$F^o(x_k) = g(x_k, P(x_k))$$

where the following table represents a risk policy for a firm with one billion dollars in assets, and ten per cent capital and earned surplus.

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[13] See Hertz (10) for a discussion of a variation of this method.

[14] See Markowitz (19), Baumol (2), and Borch (4) for alternative means of handling risk in portfolio selection. However, all of these means must be applied to a proposed complete portfolio and not directly to a single investment. We feel that the above methods are conceptually superior; however, the opportunity for practical application of expected gain, variance-confidence limit models for the large investment market is severely limited due to the complexity of the risk posture.

[15] Another alternative not considered by this model would be to develop a utility function for our investor and allow him to max E(U) over, all $i \in I$. This again suffers from computational handicaps and we are unable to use this type of model for valuation purposes.

k	k^{th} event	x_k in dollars	$P(x_k)$
1	Bankruptcy	500 million	.00000000001
2	Reduction of size of firm and its activities	100 million	.0000000001
.			
.	...	...	...
.			
g	Investigation of an alternative	50,000	.80

This last event makes it possible for the firm to perform research and conduct investigations into properties, situations, methods, etc.

The investor will not make any investment wherein for any $j \in J$

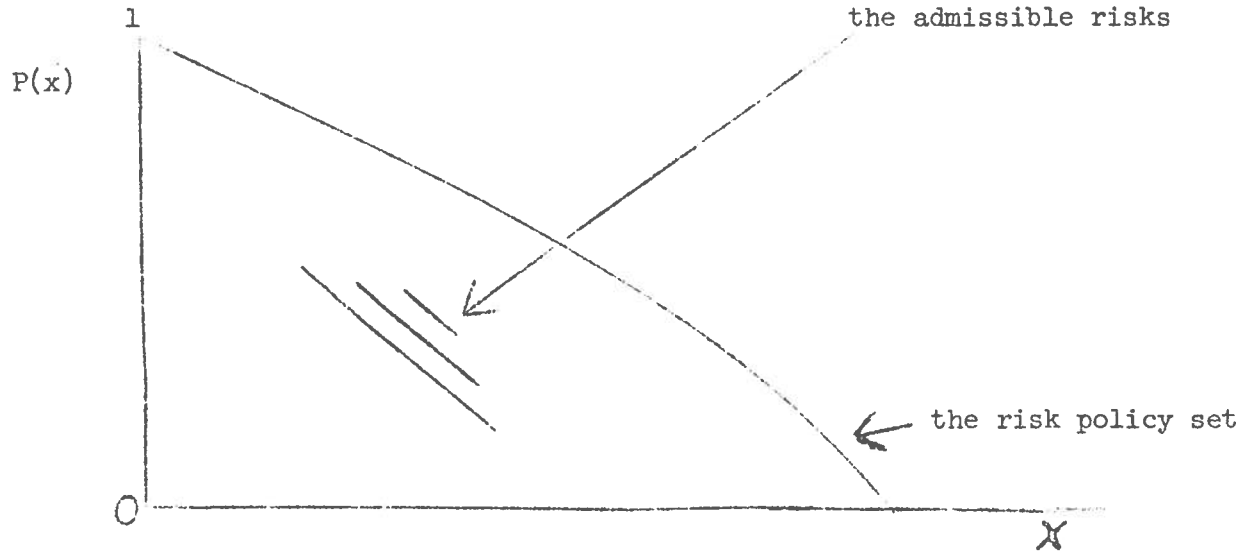
$$x_{ij} \geq x_k \text{ and } P(x_j) = P(x_k)$$

or $$x_{ij} = x_k \text{ and } P(x_j) \geq P(x_k)$$

so we have a policy

reject $_i$ $\quad g(x_{ij}, P(x_j)) \geq g(x_k, P(x_k))$ for any $i \in I$.

The policy set can be described as



or we consider only those $i \varepsilon I$ where all $j \varepsilon J$ are in the admissible set and $r_i > 0$. The above description presumes that each $j \varepsilon J$ are exclusive to i. In the next section we shall relax that assumption.

Non-Independent $P(x_j)$

In the beginning of this paper we stated that if it were possible to select all of our portfolio so that every investment was independent of each other with respect to both risks and returns, it would be possible to effectively eliminate all risk through simple diversification. We should then expect that a given event j may affect more than one $i \varepsilon I$. If the product of the investment vector $i_{(mxl)}$ and $j_{(lxn)}$ the risk vector is computed, this yields a matrix $ixj_{(mxn)}$.

The risk posture for independent $P(x_j)$ is defined as

1. $h(x_{ij}, P(x_j))$ for $\bigcap jP(x_j) = 0$ for all $j \varepsilon J$

Now we define for non-independent $P(x_j)$

2. $\phi_i(x_j) = \bigcup jP_i(x_j)$ for all $j \varepsilon J$

and then

3. $x_j = \sum_i x_{ij}$ and $\phi(x_j) = \bigcap_i \phi_i(x_j)$ for all $i \varepsilon I$

then if $\bigcap_j P(x_j) \neq 0$ for all $j \varepsilon J$ the risk posture is now more generally defined as $h(x_j, \phi(x_j))$ where

4. $h(x_j, \phi(x_j)) = \int_0^1 x_j \phi(x_j)$ and the admissibility requirement holds in the form $F^o(x_j) = g'(x_j, \phi(x_j)) < g'(x_k, \phi(x_k)) = F^o(x_k)$ if, in fact

$$F^o(x_j) \geq F^o(x_k)$$

we will remove the investment with the smallest r_i from the portfolio until

$$F^o(x_j) < F^o(x_k).$$

Obviously, for any single i;

$$g(x_{ij}, P(x_j)) = F_i^o(x_j) < F^o(x_k)$$

or i would not be in the admissible set.

Then the optimal policy, given the model presented, would be to expand the investment vector representing the existing portfolio by the candidate investment having the largest r_i, accept (i) only if the new portfolio is admissible, reject r_i if not, and commence another iteration until

$$\Sigma C_i = C = 1 \quad \text{where } \mathcal{C} \text{ may be the total portfolio or the available increment,}$$

i.e., all the capital has been allocated. This is much the same policy followed implicitly by sound risk managers when they determine to invest or not to invest in a given geographical area, type of income producing property, or a security.

The Model for Equity Investment

Equity investment in real estate is a part of the portfolio of many life insurance firms,[16] and the large insurance company's characteristics meet all of the assumptions made concerning the investor that are in the general model. Of particular importance is the ability to evaluate risk in terms of consequences and probability distributions. Since this is their stock in trade on the selling (insurance) side, it would seem reasonable to require this on the buying (investment opportunities) side.

In order to expand the lending model to include equity investment we need only to define a deterministic variable which allows for additional future outlays D_v, where $v \varepsilon V$ (such as repairs and maintenance for income

[16] See Snider (23).

property) and to expand the range of x_j to $-C \leq x_j \leq +\infty$ and thereby allow for an appreciation of value of our investment. (Since we can lose no more than all of our capital, this forms our lower bound.)

Then the model

$$\underset{r}{\text{Max}} \sum_{i=1}^{m} \left(C_i - \sum_{t=1}^{p} \frac{\left(\left[P'_{it} \vdots P''_{it} \right] + E_t(x_{ij}) - \sum_{v=1}^{q} t \; D_{iv} \right)}{(1 + r_i)^t} \right)$$

Subject to the following constraints

(1) $F^o(x_j) < F^o(x_k)$ for all $i \varepsilon I$, and $j \varepsilon J$

(2) $\Sigma C_i = 1 = C$

(3) $C_i \geq 0$ No negative investment

(4) $D_{iv} \geq 0$

(5) $-C \leq \left[P'_{it} \vdots P''_{it} \right] = P_{it} \leq +C$ this allows the lender to advance additional funds to a project

(6) $-C \leq x_j \leq +\infty$

(7) $r_i^* \geq r_o$ where r_i^* = the interest rate of investments to be included in the portfolio, and r_o the riskless rate

(8) The same rule for $\max_r$ that appears on page 3 is used.[17]

The following is an operational means using a computer model of determining the portfolio.

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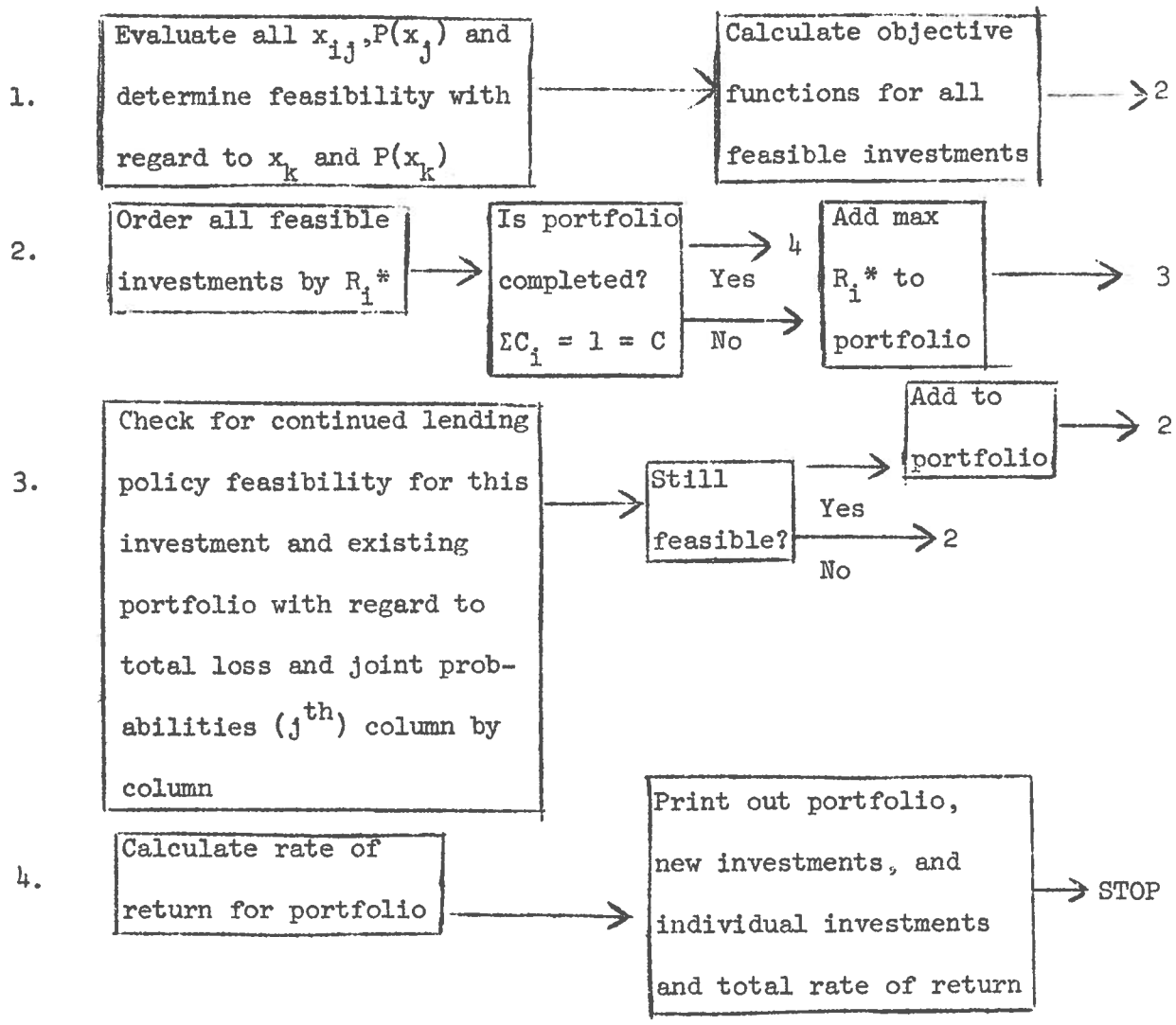
[17] If each of the ith investments are evaluated sequentially from a monotonically non-increasing set $\{r_i\}$, an investment $(i + n)$ with $(r_{i+n}, C_{i+n}, F^o_{i+n}(x_j))$ can be entered into the portfolio and there is no guarantee that an investment $i + q$ with $(r_{i+q}, C_{i+q}, F^o_{i+q}(x_j))$ may not be rejected when $r_{i+q} + \delta = r_{i+n}$ and δ is incrementally small $C_{i+q} > C_{i+n}$, and $F^o_{i+q}(x_j) \leq F^o(x_k)$, but if $F^o(x_{i+n} + x_{i+q}) \geq F^o(x_k)$, $(i+q)$ can be rejected although its inclusion and $(i+n)$'s exclusion might result in an alternative portfolio with a higher r.

Operational Techniques

Any model for which the necessary computations cannot be performed in an economically feasible manner will suffer greatly. This model has been designed so that its computational requirements may be easily satisfied on existing digital computers. The use of column and row operations reduces the need for fast storage, and makes feasible the storage of the results of and calculations for many operations on tape without excessively long run times.

The design of the model lends itself to description of its process through the use of flow charts.



Evaluation of the Model

There exists a distinct possibility that the model described may not be optimal in the sense that there may be portfolios with either a greater return and equal variance or equal return and a smaller variance. But, computations involved in the calculation of the Markowitz type models for investors with a risk posture similar to the large scale real estate investment portfolio seem at present to be so great as to fail from excessive time to perform the calculations and cost considerations. If an efficient algorithm can be established for expected value-variance models, it should be at least as efficient as a criterion as the model we have described.

However, the model presented does provide feasible if not optimal solutions. Since the model presented seems to be computationally very efficient, and since there is a high likelihood that the model will provide nearly optimal answers with reasonable computation costs, the net benefit to the user may be greater than any presently known version of an expected value-variance model. It does seem that the development of measures of risk in terms of variance is a practical possibility. But, an attempt to develop an analytical method to replace the direct calculation from the $P(x_j)$ is beyond the scope of this paper.

It should be noted that the model's failures lie mainly in the area of interdependence of returns and the handling of the non-exclusive risk probability distributions. But it does not appear that any other models have a more effective means of coping with both these problems.

Continued work is being performed to empirically determine the degree of applicability of the behavioral assumptions pertaining to inadmissibilit

of investment alternatives. Completion of this work and additional research into the cost and value of continued search for the optimal solution will be necessary to determine the appropriateness of the model.

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