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August 1969

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STRAGGLING AND PARTICLE IDENTIFICATION IN SILICON DETECTORS*

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August 1969

ABSTRACT

The distribution functions for the straggling of charged particles in silicon detectors are given in a comprehensive graphical form. Approximate quantum mechanical corrections have been applied to the Vavilov functions.

A simple identification procedure for use with a digital computer is described.

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I. Introduction

A system frequently used for the identification of charged particles of kinetic energy T consists of one or more thin silicon detectors ("ΔT counters") in which the particles experience energy losses Δ, and a final detector ("T counter") thick enough to absorb the total residual energy of the incident particles[1]. The resolution of this system is limited by straggling, dominated by the effect in the ΔT counter. If different particles of the same incident kinetic energy T have overlapping straggling curves in the thin detector, it will not be possible to identify particles having energy losses in the region of overlap (Figs. 1-3).

It is often assumed that, for large absorber thicknesses t, the straggling curve is Gaussian[2]. This is only an approximation (see Figs. 5-7). A better approximation is given by the Vavilov theory[3], which has been shown to be fairly accurate for the description of the energy loss of charged particles in silicon[4-6].

The observation of pulse heights in a thin detector is described by the probability density function $f(\Delta, t)$, but the determination of the overlap of the energy loss functions requires a knowledge of the distribution function $\Phi(\Delta, t) = \int_0^{\Delta} f(\Delta', t)\, d\Delta'$. Notice that physicists frequently use the expression "distribution function" to describe $f(\Delta, t)$. If the mean energy losses of particles A and B are called $\overline{\Delta}_A$ and $\overline{\Delta}_B$ respectively, it will be found for an energy loss Δ_1 defined by $\overline{\Delta}_A < \Delta_1 < \overline{\Delta}_B$ that a fraction

$$P_A = 1 - \Phi_A(\Delta_1, t) \tag{1}$$

of all particles A will exceed the energy loss Δ_1, while a fraction

$$P_B = \Phi_B(\Delta_1, t) \tag{2}$$

of all particles B will have energy losses smaller than Δ_1.

The distribution functions are given and discussed in this paper. Also, some comments are made about the use of on-line computers in particle identification.

II. The Distribution Functions

The probability that charged particles will experience energy losses between Δ and $\Delta + d\Delta$ in traversing an absorber of thickness t is given by the probability density function $f(\Delta, t)$, which also depends on the charge ze and the velocity $v = \beta c$ of the incident particle. The discussion in this paper is based on the Vavilov theory[3], with some corrections brought about by the use of quantum mechanical collision cross section[7, 8] instead of the $1/E^2$ cross section used by Vavilov. An extensive discussion has been given by Seltzer and Berger[9]. Their nomenclature is used for this discussion. It has recently been found[10] that, in the stopping power, there is a charge dependence over and above the z^2 term usually assumed. It presumably would appear in the theory with the use of higher Born approximations, and would influence the straggling somewhat at low energies. No correction for this effect is included here.

The distribution functions required for the determination of the overlap of the straggling functions of different particles in a given ΔT counter are defined in terms of the modified Vavilov functions[7, 8]:

$$\Phi(\Delta, t) \equiv \int_0^{\Delta} f(\Delta', t)\, d\Delta' \,, \tag{3}$$

with $$\Phi(\infty, t) \equiv \int_0^{\infty} f(\Delta', t)\, d\Delta' = 1 \,.$$

The parameters z and β are implicity included in Eq.(3). The accuracy of $f(\Delta, t)$ is limited due to uncertainties in the quantum mechanical corrections (including the deviations from the z^2 behavior); the numerical integration in Eq. (3) is accurate only to about 0.2%. The overall accuracy of Φ is estimated to be about 0.01 for $\Phi > 0.3$, and about 0.005 for $\Phi < 0.3$. For the present application to silicon absorbers, the following reduced variables have to be calculated:

$$\sigma^2 = 78.22\times t\, z^2 (1 - \beta^2/2)(1 + q)\, Q' /(1 - \beta^2) \quad \mathrm{keV}^2, \tag{4}$$

$$\xi = 0.07654\times t\, z^2 /\beta^2 \quad \mathrm{keV}\,, \tag{5}$$

$$\kappa = 7.490\times 10^{-5}\times t\, z^2 (1 - \beta^2) /\beta^4 \,, \tag{6}$$

where particles of charge ze, rest mass M, and initial velocity $v = \beta c$ are incident on a silicon detector of thickness t (in mg/cm^2); q is the quantum mechanical correction, given approximately by

$$q = 0.00186\,\beta^{-2} \ln(102\beta^2 + 0.746) \tag{7}$$

$$\text{for } 0.0005 < \beta^2 < 0.0075,$$

$$q = 0.0009\,\beta^{-2} \ln(306\,\beta^2) \qquad \text{for } 0.0075 < \beta^2 \,; \tag{7'}$$

Q' is a factor caused by the increase in straggling due to the spread in energy of the particle beam[11]; with $T_1 = T - \overline{\Delta}$,

$$Q' = (T/T_1)^{1/3} \text{ for } T_1/T > 0.4 \quad , \qquad B \approx 2.3 \quad ,$$

$$Q' = 0.99 \quad (T/T_1)^{1/2} \qquad T_1/T > 0.4 \quad , \qquad B \approx 3.5 \quad , \tag{8}$$

$$Q' = 0.985 \quad (T/T_1)^{2/3} \qquad T_1/T > 0.6 \quad , \qquad B \approx 6.9 \quad ,$$

where B is the stopping number (Fig. 4).

Here σ is the standard deviation of the straggling function, and ξ and κ are the parameters of the Vavilov theory[9]. Furthermore, the mean energy loss has to be calculated:

$$\overline{\Delta} = \int_0^\infty f(\Delta, t)\,\Delta d\Delta = tS = 2\,\xi B \quad , \qquad \text{keV} \tag{9}$$

where S (in keV cm^2/mg) is the stopping power[12-14] and B the stopping number (see Fig. 4). The velocity should be calculated with

$$\beta^2 = \tau\,(\tau + 2)\,/\,(\tau + 1)^2 \,, \quad \text{with} \quad \tau = T/Mc^2 \,. \tag{10}$$

Notice that $\sigma^2 = \xi^2 \,(1 - \beta^2/2)\,/\kappa$ except for the corrections given by q and Q'. The probability densities $f(\Delta, t)$ are given in Ref. 9 in terms of the parameters κ and β^2 and as a function of the dimensionless energy-loss variable

$$\lambda = \bar{\lambda} + (\Delta - \overline{\Delta})\,/\,\xi \;, \tag{11}$$

where $\qquad \bar{\lambda} = 0.577216 - 1 - \beta^2 - \ln\kappa \;.$

For present purposes, another dimensionless energy-loss variable is more suitable:

$$p = (\Delta - \overline{\Delta})\,/\,\sigma \,. \tag{12}$$

The parameters κ and β^2 are kept unchanged.

Equation (3) can be rewritten in terms of these parameters as

$$\Phi(p, \kappa, \beta) = \int_0^p f(p', \kappa, \beta)\, dp'. \tag{13}$$

Distribution functions have been calculated by numerical integration of $f(p', \kappa, \beta)$, and are given in Figs. 5-7 for three values of β^2. The dependence on β^2 is quite small, and it will not be necessary to use interpolation for β^2.

The functions presented here do not include corrections for effects connected with the operation of silicon detectors, e.g., electronic noise, inhomogeneity of detector thickness, counting statistics, and channeling.

Since the function Φ does not change very much for $\kappa > 6$, the exact choice of κ for $T_1/T < 0.8$ is not very critical. It is seen, though, than even for $\kappa = 10$ there is a difference between the straggling function and a Gaussian curve (see Fig. 6).

Note that the "skewness parameter" γ_3 in Fig. 4 of Ref. 11 is related to κ in a simple way: $\kappa \approx 1/(4\gamma_3^2)$. For $\beta^2 = 0$, the expression is exact.

A Fortran program VPLOT, giving graphs similar to Figs. 1-3 and both $f(\Delta, t)$ and $\Phi(\Delta, t)$, is available from the author.

It should be noted that it may frequently be easier to calculate the moments of an experimental straggling function and compare them with the theoretical moments[12] than to calculate a Vavilov function.

III. Approximate Expressions and Examples

For estimates at low and moderate energies, the following simplified expressions can be used for the parameters:

$$\sigma = 9\ z\sqrt{t} \qquad \text{keV},$$

$$\xi = 36\ z^2\ t\ A/T \qquad \text{keV},$$

$$\kappa = 16.3\ z^2\ t\ A^2/T^2 ,$$

$$\beta^2 = 0.00214\ \ T/A ,$$

where T in (MeV) is the kinetic energy, and A the atomic number of the incident ion. If the energy loss $\overline{\Delta} = (T - T_1)$ in the ΔT counter amounts to more than 10% of T , σ should be multiplied by $\sqrt{Q'}$ from Eq. (8).

Examples.

1. In Fig. 1, to determine the fraction Φ_{300} of protons exceeding an energy loss $\Delta = 300$ keV, calculate p , using σ from the figure: $p = (300-235)/40.4 = 1.61$. From Fig. 6, for $\kappa = 0.215$, we obtain $\Phi \approx 93\%$, showing that 7% of the protons suffer energy losses of more than 300 keV.

2. Consider deuterons and tritons of 40 MeV. To calculate the fraction of particles appearing beyond the energy loss $\Delta_m = (\overline{\Delta}_d + \overline{\Delta}_{tr})/2$ as a function of detector thickness t (mg/cm^2), we have, from Ref. 14, $S_d = 20.2$ keV cm^2/mg, $S_{tr} = 27.7$ keV cm^2/mg, from Ref. 12, $\beta_d^2 = 0.0414$, $\beta_{tr}^2 = 0.0279$, $\kappa_d = 0.0419\ t$, $\kappa_{tr} = 0.0934\ t$, $p_{tr} = -p_d$. We get

	t (in mg/cm^2)			
	3	10	30	100
$\bar{\Delta}_d$ (keV)	60.6	202	606	2020
$\bar{\Delta}_{tr}$ (keV)	83.1	277	831	2770
Δ_m (keV)	71.8	239.5	718	2395
σ (keV)	15.9	29.0	50.0	89.5
p_d	0.71	1.29	2.24	4.2
κ_d	0.126	0.419	1.26	4.2
κ_{tr}	0.28	0.934	2.80	9.3
Φ_d (%)	82	89	97.7	99.5+
$1-\Phi_{tr}$(%)	28	9	1.9	0.2-

IV. The Use of On-Line Computers in Particle Identification

Different approaches have been used for the identification of charged particles using the ΔT-T system described in the introduction[15-19] (see, e.g., Ref. 2 for a review).

The following method seems to be the simplest and also the most accurate for use with digitial computers, if a few hundred words of fast memory are available to store range-energy tables for the different particles to be identified. Basically, the method consists of a table look-up of the ranges associated with the energies T and $T_1 = T - \Delta$. The range difference $r_M = R_M(T) - R_M(T_1)$ for an arbitrarily chosen particle of mass M is compared with the detector thickness t. If

$$(t - t_{\ell}) \leq r_M \leq (t + t_u)$$

the mass M of the detected particle is as assumed. For $r_M < (t - t_{\ell})$ or $r_M > (t + t_u)$ the range table for another particle has to be used. The determination of appropriate values of t_{ℓ} and t_u is described later. A suitable sequencing of the table look-up has to be chosen. To achieve the fastest operation, it is necessary to use as table entries the ranges associated with the energies corresponding to the center of each channel of the pulse-height analyzer. If the analyzing systems for the two counters have different calibration constants, the pulse height in one of them has to be converted into the equivalent pulse height in the other. It is then possible to use the pulse heights directly as the index for the range tables.

Sample Program

The pulse height from the T counter is called JT, from the ΔT counter, JD. The ratio of the calibration constants C_T (in keV/channel) of T and C_D of ΔT is $CA = C_T/C_D$. The range table RA has been calculated previously for three different particles in such a way that one has $RA(JEA) = R[T(JEA)]$, where $T(JEA) = C_D \cdot JEA$, and $RA(JEA+1) = R[T(JEA)+C_D]$. In other words; the range tables are listed for energies equal to the channel width multiplied by an integer. The lower and upper limits, $t - t_{\ell} = PIL$ and $t + t_u = PIU$, for each particle have to be determined either from diagrams corresponding to Figs. 1-3 or from the curves in Fig. 5 of Ref. 14. If the detector thickness t is not known accurately, it can be determined experimentally in preliminary test runs.

```
      SUBROUTINE XIDENT(JD,JT,ID)
      COMMON/RD/ RA(3,450), PIL(3),PIU(3),CA
    1 JEA=INT(CA*FLOAT(JT))
      JEF=JEA+JD
      DO 5 M=1,3
    2 TA=RA(M,JEF)-RA(M,JEA)
      IF (TA-PIL(M)) 7,3,3
    3 ID=M
      IF (TA-PIU(M)) 9,9,5
    5 CONTINUE
    7 ID=99
    9 RETURN
      END
```

This program is especially simple for $C_T = C_D$: statement 1 can be eliminated, and in each test for a mass, three subtractions are necessary, and two comparisons.

In a program using the relation

$$R_M(T_M) = (M/z^2)\ R_p(m_r \times T_M) \ ,$$

where R_p is the proton range and $m_r = 938.259(\mathrm{MeV})/Mc^2$, the two products $m_r T_M$ and $m_r(T_M - \Delta)$, and also $(M/z^2)(R_T - R_{T_1})$, have to be calculated before the comparison can be made. Notice that a considerable simplification can be introduced if $R_p = C\ T^{\alpha}$, with a constant α for a certain energy range. Then

$$R_M(T_M) = (M/z^2)\, C\, m_r^{\alpha} T^{\alpha}$$

and $\langle r \rangle = \langle R_M(T_M) - R_M(T_M - \Delta) \rangle = (M/z^2)\, C \,\langle m_r^{\alpha} \left[T_M^{\alpha} - (T_M - \Delta)^{\alpha} \right] \rangle$

Since $\langle r \rangle = t$, the experimental coefficient

$$\langle \left[T_M^{\alpha} - (T_M - \Delta)^{\alpha} \right] \rangle / t = z^2 / (M \langle m_r^{\alpha} \rangle)$$

can be used to determine the identity of the particle[2]. The method breaks down if α is not constant (See Fig. 3 of Ref. 14). Since T_M^{α} can of course also be obtained in a table look-up, this method is simpler if α can be considered to be constant.

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REFERENCES

1. F. S. Goulding, D. A. Landis, J. Cerny, and R. H. Pehl, Nucl. Instr. and Meth. 31 (1964) 1.
2. D. D. Armstrong, J. G. Berry, E. R. Flynn, W. S. Hall, P. W. Keaton, Jr., and M. P. Kellog, Nucl. Instr. and Meth. 70 (1969) 69.
3. P. V. Vavilov, Soviet Physics JETP 5 (1957) 749.
4. H. D. Maccabee, M. R. Raju, and C. A. Tobias, Phys. Rev. 165 (1968), 469.
5. J. J. Kolata, T. M. Amos, and H. Bichsel, Phys. Rev. 176 (1968), 484.
6. D. W. Aitken, W. L. Lakin, and H. R. Zulliger, Phys. Rev. 179 (1969) 393.
7. Hans Bichsel, University of Southern California Report USC-136-147 (Jan. 1969).
8. P. Shulek, B. M. Golovin, L. A. Kulyukina, S. V. Medved', and P. Pavlovich, Soviet J. Nucl. Phys. 4 (1967) 400.
9. S. M. Seltzer and M. J. Berger, Publication 1133, Nat. Acad. Sci. - Nat. Res. Council (second printing, 1967), section 9.
10. H. H. Andersen, H. Simonsen, and H. Sørensen, Nucl. Physics A125 (1969) 171.
11. C. Tschalär, Nucl. Instr. and Meth. 61 (1968) 141 and 64 (1968) 237.
12. Hans Bichsel, American Institute of Physics Handbook 3rd ed. (McGraw-Hill, to be published); also USC-136-150, July 1969.

13. D. J. Skyrme, Nucl. Instr. and Meth. 57 (1967) 61.

14. Hans Bichsel and Christoph Tschalär, Nuclear Data A3 (1967) 343, also UCRL-17663 Rev., 1969.

15. R. H. Stokes, J. A. Northrop, and K. Boyer, Rev. Sci. Instr. 29 (1958) 61.

16. L. Wåhlin, Nucl. Instr. and Meth. 14 (1961) 281.

17. F. S. Goulding, D. A. Landis, J. Cerny, and R. H. Pehl, IEEE Trans. N.S. 13, No. 3 (1966) 514.

18. P. R. Alderson and K. Bearpark, Nucl. Instr. and Meth. 62 (1968) 217.

FIGURE CAPTIONS.

Fig. 1. Calculated energy losses of 40-MeV protons, deuterons, and tritons in a silicon detector of thickness t = 20 mg/cm^2. The same numbers of particles of each kind are incident. The mean energy loss $\overline{\Delta}$ is quite different from the most probable energy loss, especially for the protons. If the cross-over point of the curves is selected for the separation of protons and deuterons (Δ = 327 keV), about 2.7% of the protons will be identified as deuterons, while about 1.5% of the deuterons will appear as protons. Similarly, for Δ = 480 keV, about 3.6% of the deuterons will appear as tritons and 2.6% of the tritons will appear as deuterons. It should be noted that, with the assumed system, it is not possible to distinguish between protons and deuterons in the overlapping region (approximately 300 to 420 keV).

Fig. 2. Energy loss of 100-MeV p, d, tr in a silicon detector with thickness 20 mg/cm^2. For protons, with $\overline{\Delta}$ = 117 keV, the most probable energy loss is about 93 keV. About 14% of the protons will lose energies in excess of 145 keV, about 7% of deuterons will have smaller energy losses. About 5% of the protons suffer energy losses exceeding 200 keV; 23% of the deuterons exceed energy losses of 218 keV (intercept of d and tr curves), and 7.5% of the tritons fall below this point. A detector this thin would obviously not be practical for particle identification at this energy.

Fig. 3. Energy-loss distribution for 100-MeV ^{3}He and ^{4}He ions; 7.2% of the ^{3}He will experience energy losses larger than 1213 keV (crossover point of the curves), and 4.3% of the ^{4}He ions will be found below this point.

Fig. 4. The reduced stopping number B for protons in silicon. $\beta^2 \approx 2T/Mc^2$. The stopping power is given by

$$S = 0.1531\ z^2\ B/\beta^2 \qquad \text{MeV cm}^2/\text{g},$$

the mean energy loss by

$$\bar{\Delta} = 153.1\ z^2\ t\ B/\beta^2 \qquad \text{keV}.$$

The curve is semi-empirical, and applies approximately for other particles. For $\beta^2 \geq 0.04$

$$B = \ln\left[5891\ \beta^2 / (1 - \beta^2)\right] - \beta^2 - 0.0019/\beta^2 .$$

For $\beta \leq 0.024\ z^{2/3}$ the nuclear charge is partly shielded by atomic electrons, and z has to be replaced by z^*, given approximately by[12)]

$$z^* = z\left[1 - \exp(-1.316\ x + 0.1112\ x^2 - 0.065\ x^3)\right],$$

where $x = 100\ \beta/z^{2/3}$ and $z \geq 2$. For $x \leq 0.27$, the theory does not apply. For the solid curve, a charge state correction has to be applied for all particles. The dashed curve is B corrected for the charge state of the proton.

Fig. 5. Contour lines for straggling curves, modified for quantum mechanical corrections, for $\beta^2 = 0.04$ (protons of about 20 MeV). The ratio $p(\Phi, \kappa) = (\Delta_q - \bar{\Delta})/\sigma$ is plotted for different values of Φ, as a function of κ. Φ is the fraction of particles experiencing

energy losses less than Δ_q, Eq. (1). The accuracy of the numbers is about 3%. The location of the mean is given by p = 0.

Fig. 6. Contour lines for $\beta^2 = 0.1$ (protons of about 50 MeV). $\kappa = \infty$ is a Gaussian.

Fig. 7. Contour lines for $\beta^2 = 0.2$ (protons of about 110 MeV).

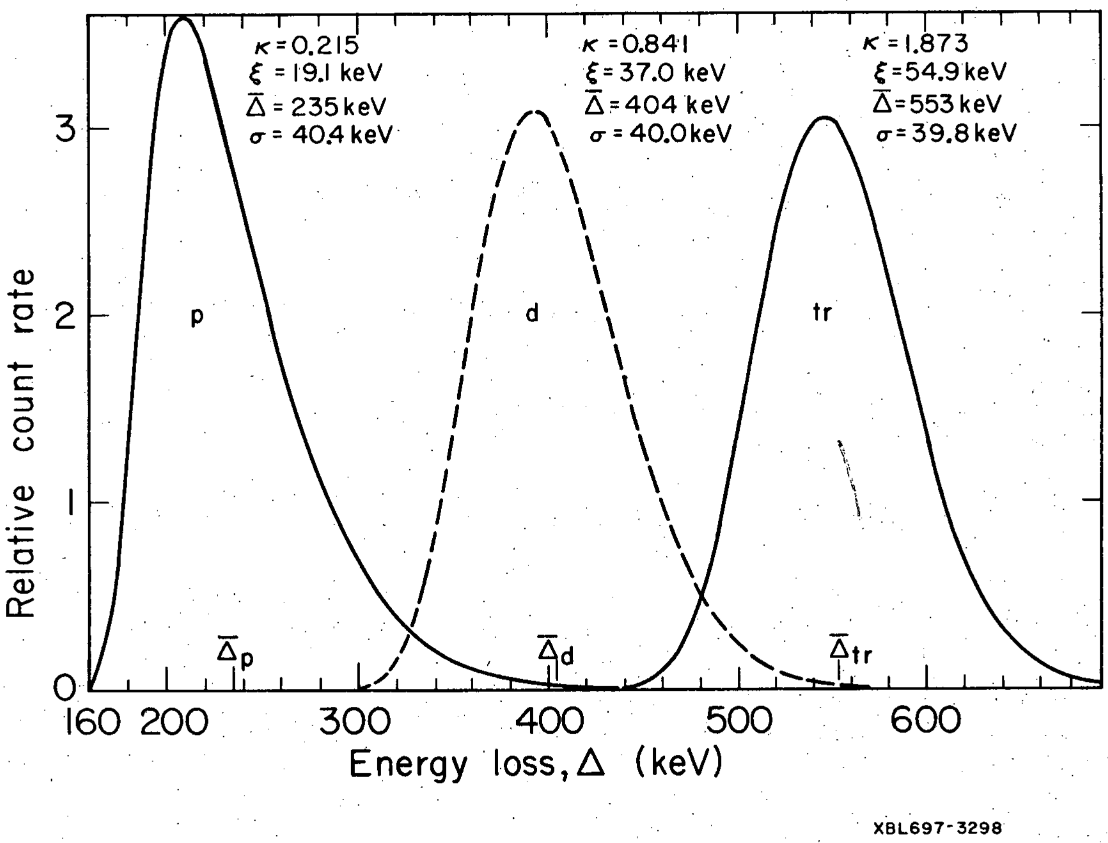
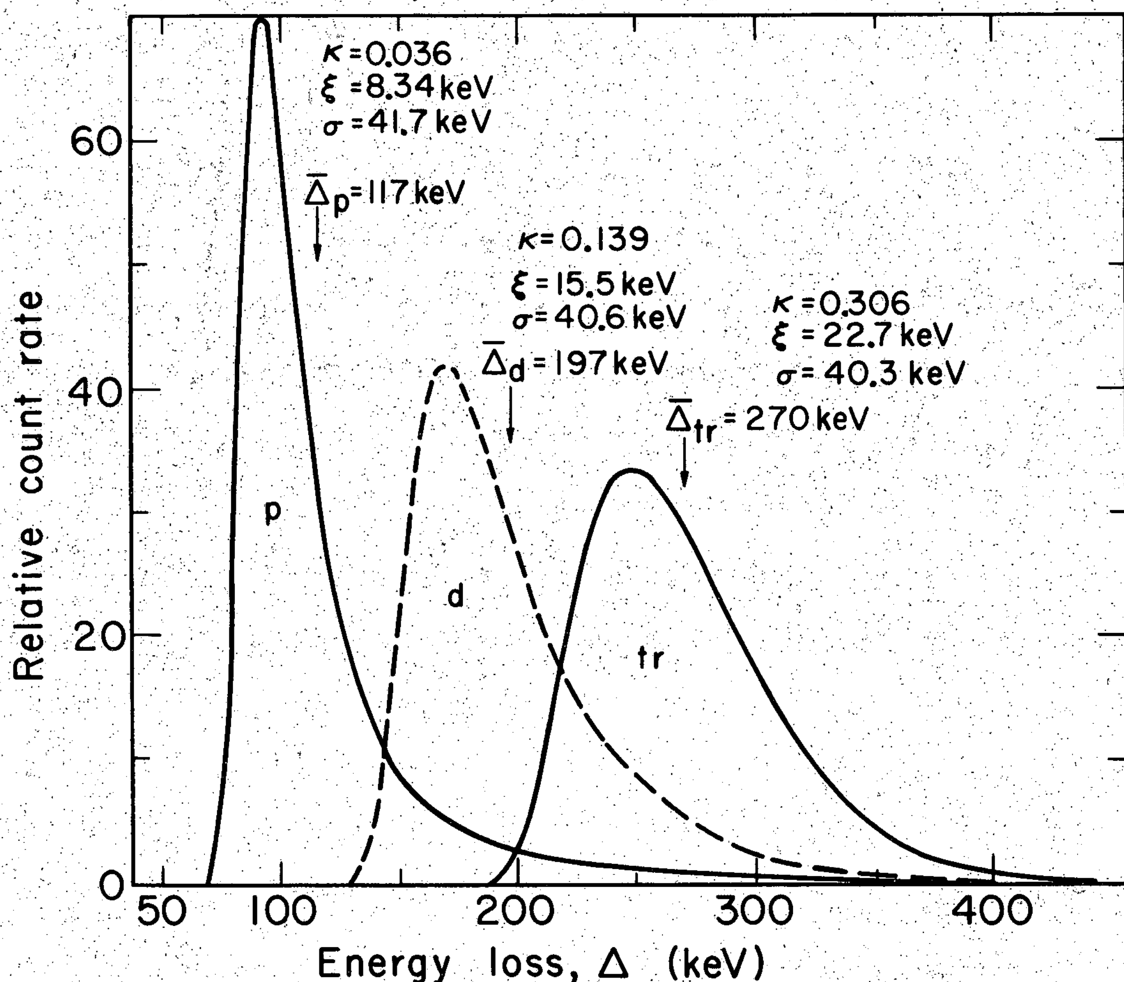
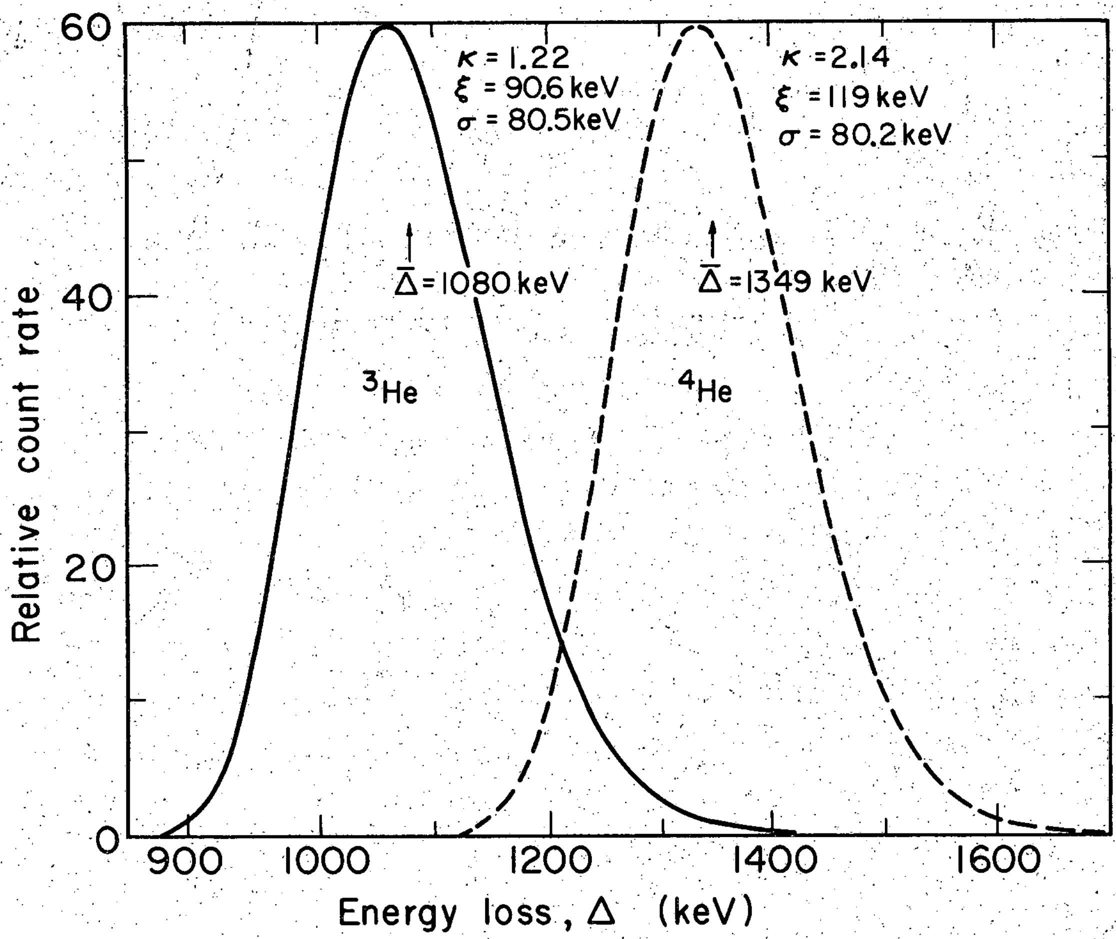

κ = 0.215
ξ = 19.1 keV
Δ̄ = 235 keV
σ = 40.4 keV
κ = 0.841
ξ = 37.0 keV
Δ̄ = 404 keV
σ = 40.0 keV
κ = 1.873
ξ = 54.9 keV
Δ̄ = 553 keV
σ = 39.8 keV
p
d
tr
Δ̄p
Δ̄d
Δ̄tr
Relative count rate
0
1
2
3
160 200
300
400
500
600
Energy loss, Δ (keV)
XBL697-3298

Fig. 1



XBL697-3299

Fig. 2



XBL697-3300

Fig. 3

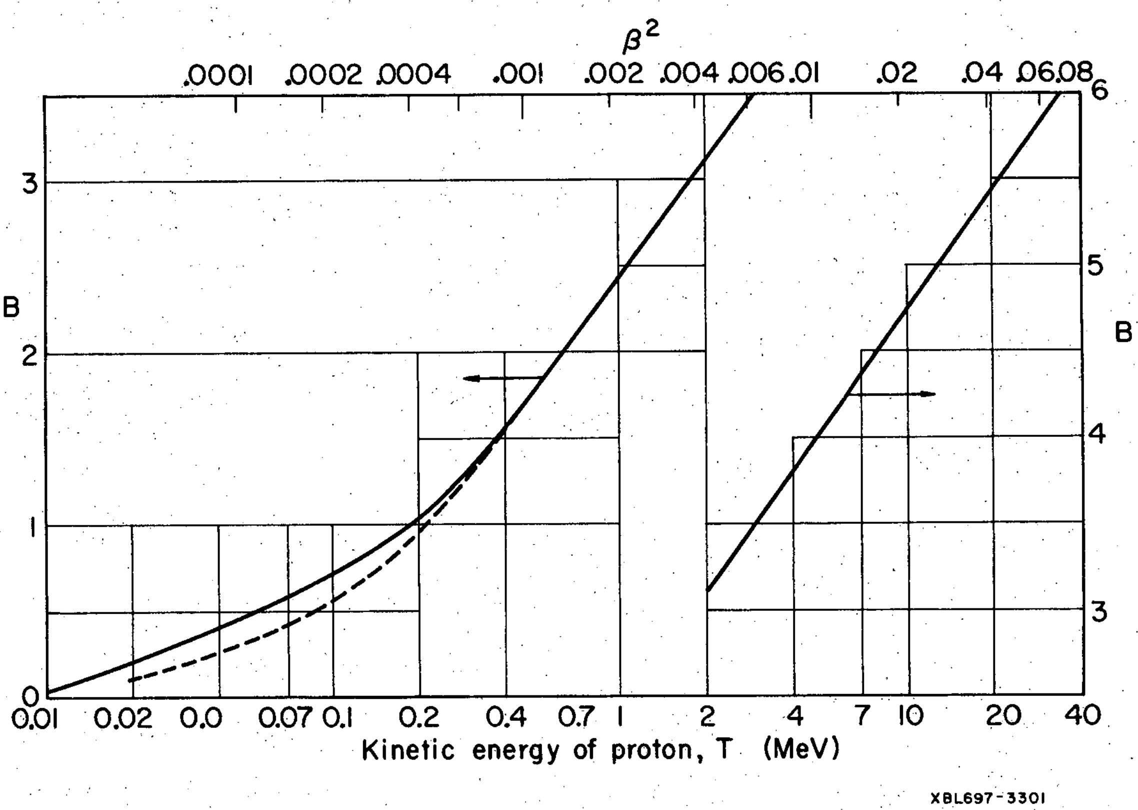
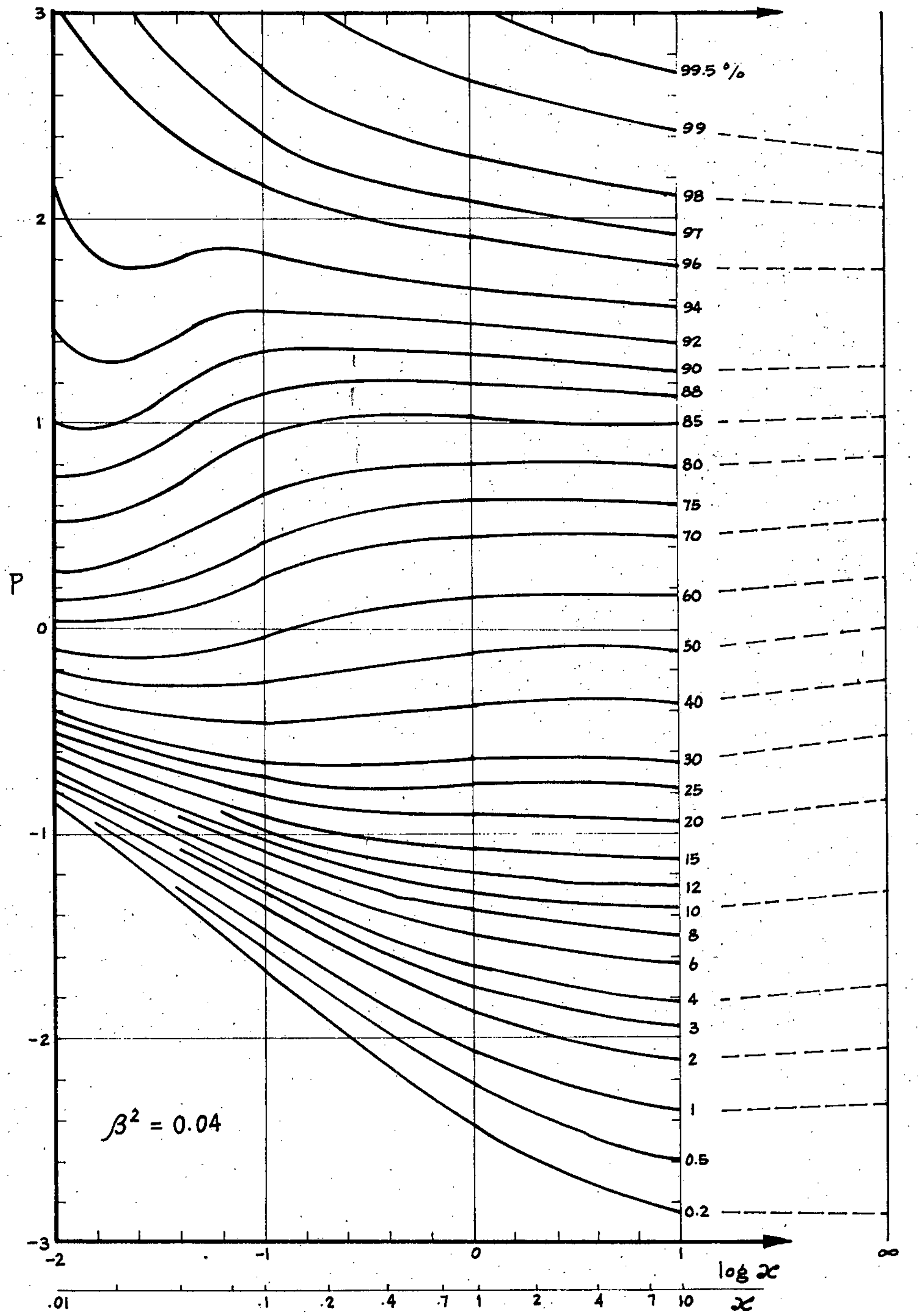

β²
.0001 .0002 .0004 .001 .002 .004 .006 .01 .02 .04 .06 .08
B
0 1 2 3
B
3 4 5 6
0.01 0.02 0.0 0.07 0.1 0.2 0.4 0.7 1 2 4 7 10 20 40
Kinetic energy of proton, T (MeV)
XBL697-3301

Fig. 4



XBL 696-755

Fig. 5

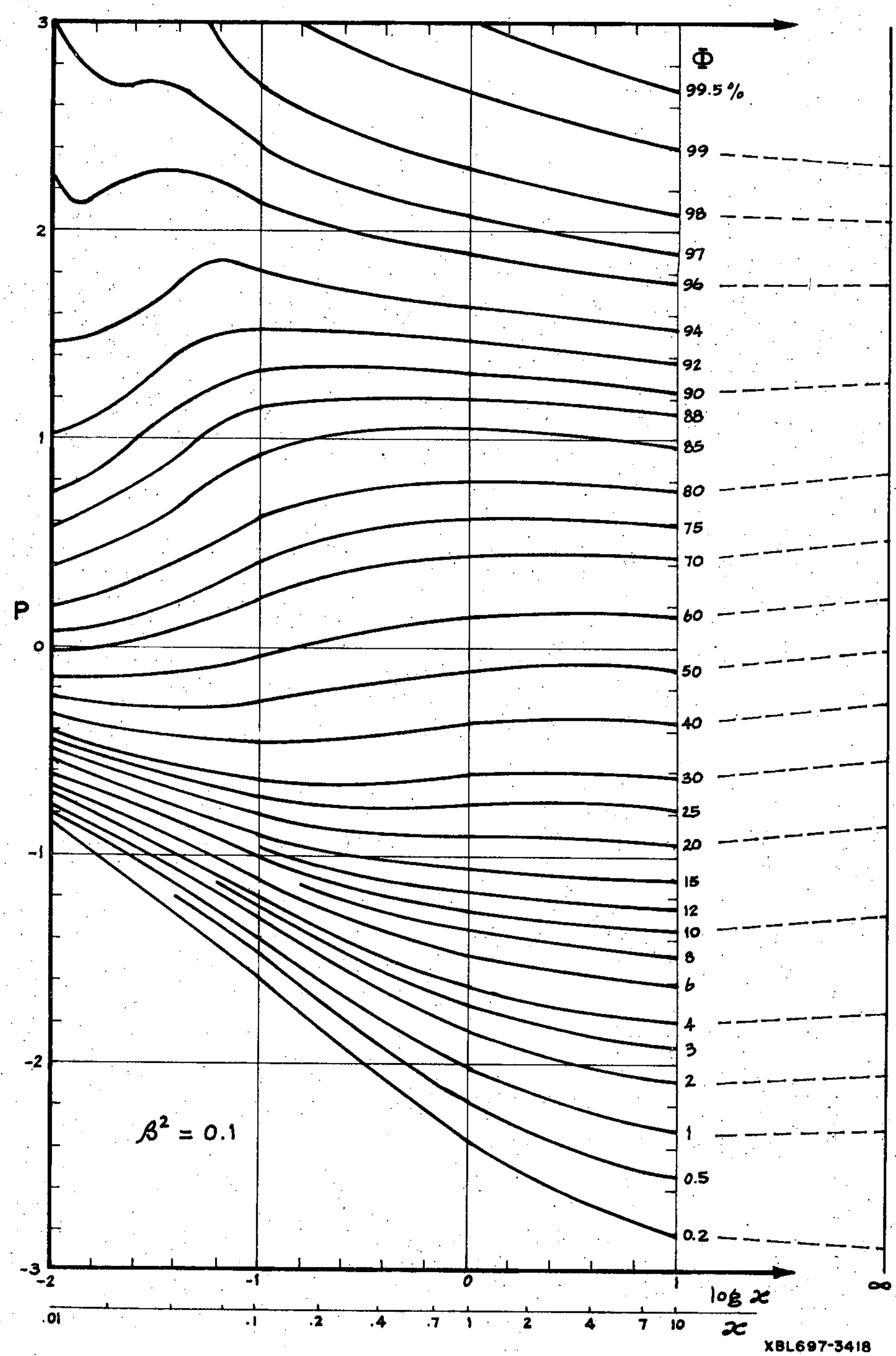
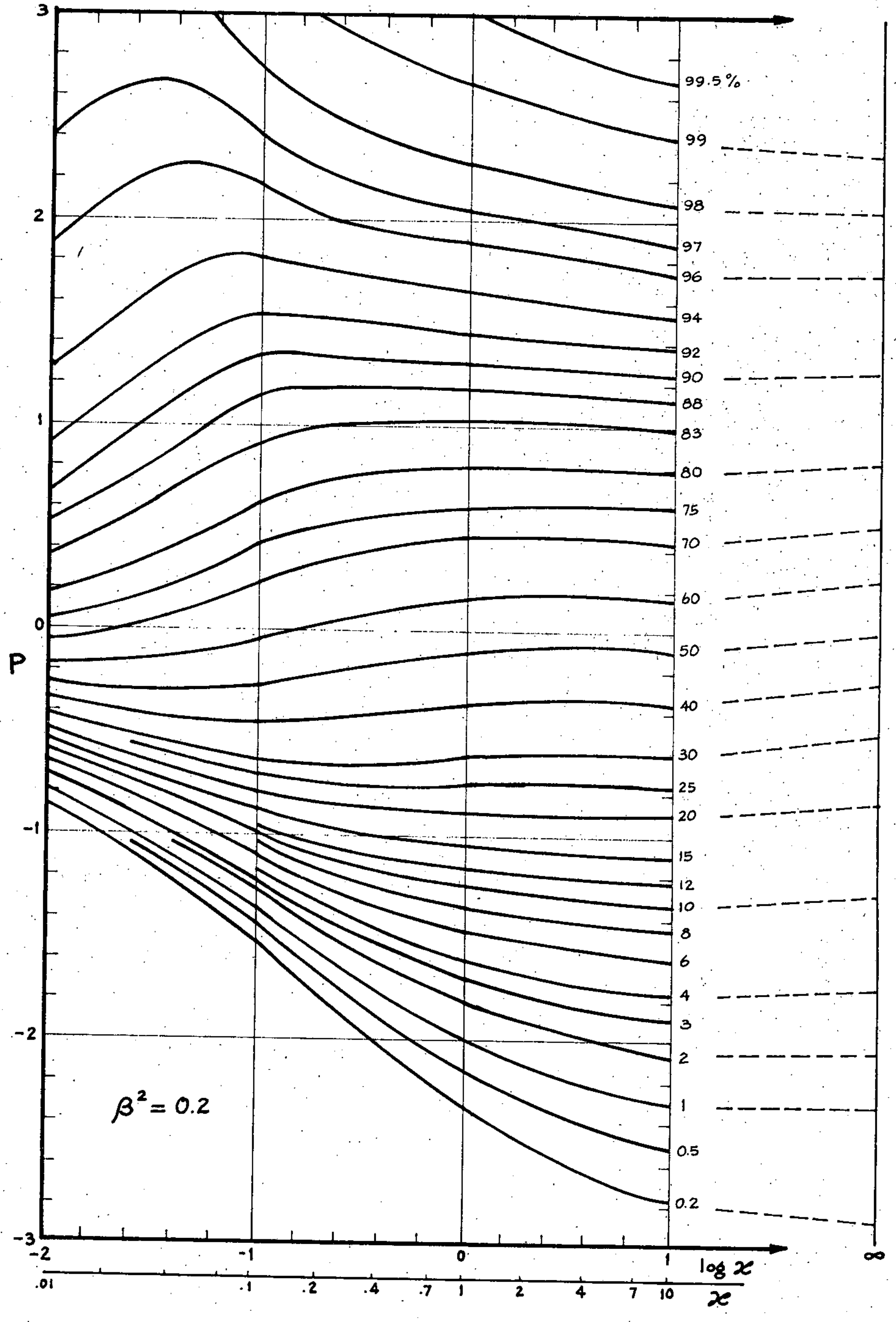

Φ
99.5 %
99
98
97
96
94
92
90
88
85
80
75
70
60
50
40
30
25
20
15
12
10
8
6
4
3
2
1
0.5
0.2
P
3
2
1
0
-1
-2
-3
$\beta^2 = 0.1$
-2
-1
0
1
log x
∞
.01
.1
.2
.4
.7
1
2
4
7
10
x
XBL697-3418

Fig. 6



XBL 696-757

Fig. 7

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