

Lawrence Berkeley National Laboratory

Recent Work

Title

PONDERCMOTIVE STABILIZATION OF FLUTE MODES IN MIRRORS: FEEDBACK CONTROL AND NUMERICAL RESULTS

Permalink

<https://escholarship.org/uc/item/1vn2t7hg>

Author

Similon, P.L.

Publication Date

1986-08-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

RECEIVED
LAWRENCE
BERKELEY LABORATORY

OCT 28 1986

LIBRARY AND
DOCUMENTS SECTION

Submitted to Physical Review Letters

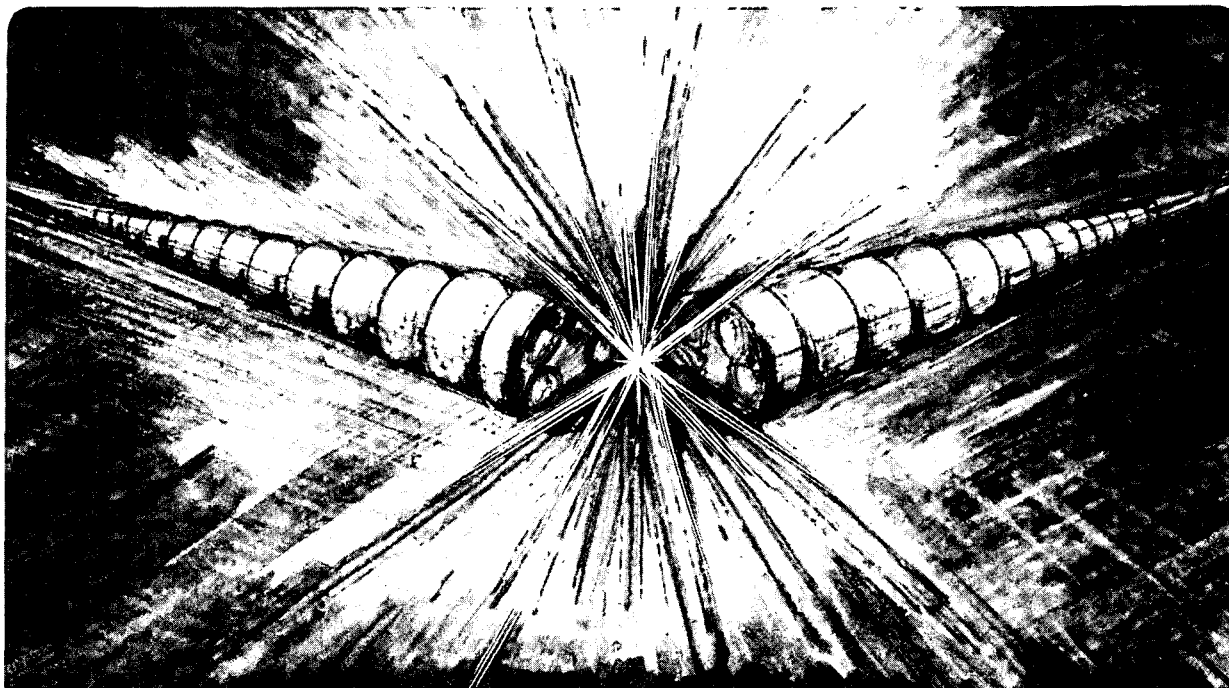
PONDEROMOTIVE STABILIZATION OF FLUTE MODES IN MIRRORS: FEEDBACK CONTROL AND NUMERICAL RESULTS

P.L. Similon

August 1986

For Reference

Not to be taken from this room



LBL-21626
e.1

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

**Ponderomotive Stabilization of Flute Modes in Mirrors.
Feedback Control and Numerical Results.**

P. L. Similon

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720,
and
Laboratory of Plasma Studies
Cornell University
Ithaca, New York 14853.

August, 1986

**Ponderomotive Stabilization of Flute Modes in Mirrors.
Feedback Control and Numerical Results.**

P. L. Similon

Lawrence Berkeley Laboratory

University of California

Berkeley, California 94720,

and

Laboratory of Plasma Studies

and Cornell Theory Center

Cornell University

Ithaca, New York 14853.

Abstract

Ponderomotive stabilization of rigid plasma flute modes is numerically investigated using a variational principle, for a simple geometry, without eikonal approximation. While the near-field of the studied antenna can be stabilizing, the far-field has a small contribution only, because of large cancellation by quasi-mode coupling terms. The field energy for stabilization is evaluated and is a non-negligible fraction of the plasma thermal energy. A new antenna design is proposed, and feedback stabilization is investigated. Their use drastically reduces power requirements.

PACS numbers: 52.35.Mw, 52.35.Py, 52.55.Jd

Stabilization of the plasma flute instability in axisymmetric magnetic mirrors has been investigated for plasma confinement and controlled thermonuclear fusion. In particular, stabilization by high-frequency rf fields has been the object of much experimental and theoretical research¹⁻⁸.

The results reported here make use of a variational approach introduced earlier^{3,7}, specialized to a simple geometry: a nonuniform slab, with antenna. The numerical results lead to important considerations on the design and requirements of ponderomotive stabilization. Although the model assumes a cold plasma dielectric tensor and considers only perpendicular electric field, it makes no eikonal approximation, and offers a clear picture of the basic phenomena.

The main results are summarized as follows. First, the self-consistent reaction of the fields to the plasma displacement plays an essential role: the propagating rf field (the "far-field" of the antenna) does not stabilize the rigid motion of the plasma (the "m=1 flute mode"). Only the near-field is stabilizing, even though its energy content is normally lower than the far-field. Because of momentum balance, stabilization can only be achieved by rf fields which are "backed up" by some conducting structure, such as antennas, conducting limitors (which create their own near-field by induction), or by walls. Second, in the usual case when the rf frequency ω is close to the ion gyrofrequency Ω_i , the electron ponderomotive contribution is opposed to the ion contribution, inside the plasma column (i.e. at high enough densities). As a result, rf fields decaying radially away from the antenna do not stabilize internal interchange modes. Stabilization of the rigid flute mode occurs only at the edge of the plasma, where density is low and ion ponderomotive force dominates. Finally, evaluation of rf energy necessary for stabilization shows that it is comparable to the energy released by the plasma through interchange. In a fusion reactor regime, the power required to maintain such rf energy levels is prohibitively large, unless the Q-factor of the system antenna-plasma reaches exceeding large values.

Major improvement of this situation can however be reached with two modifications proposed here. First, feedback control can enhance the rf response to plasma perturbations, and be used for stability. Second azimuthally localized antennas are very favorable, since they can locally communicate momentum to the plasma, which greatly increases the overall efficiency.

I report results on feedback control, when the antenna currents are allowed to vary, for instance because of the rf generator finite impedance. By appropriately choosing the impedances one can adjust the rf field response to react in a stabilizing way to plasma interchange modes. The variational formalism is extended to this case by including the reactive energy of the generator circuitry in the ponderomotive energy.

A complete description of the oscillation-center formalism which leads to the variational formulation used here has appeared in previous articles^{3,7}. It was shown that the stability variational principle is composed of four parts : a) the usual magnetohydrodynamic term ΔW_{MHD} which contains the destabilizing interchange terms due to unfavorable average curvature in axisymmetric mirrors; b) an equilibrium ponderomotive force term ΔW_p , which has the form of an interchange term; c) a ponderomotive magnetization term ΔW_M , which is small in the present case because of small plasma diamagnetism (5%) and which will not be discussed further; and d) a self-consistent rf field response term ΔW_A , proportional to the second variation of the ponderomotive energy $\delta^2 V$, and involving the response function of the rf wave operator.

The self-consistent calculation reported here considers the model of a plasma slab, uniform in the y- and z- directions, with equilibrium density profile $n(x)$ and magnetic field profile $B(x)z$ (Fig. 1) . The density is $n(x) = n_0 \exp[-(x/w)^2/2]$, with width $w = 0.6 a$, and the field is $B(x) = B_0[1 - \beta n(x)/2n_0]$, with a diamagnetic well equal to $\beta/2 = 5\%$. The antenna is modelled by a pair of antiparallel current sheet : the rf currents of amplitude I are directed along the y-axis, at $\omega = 1.1 \Omega_i$. Since the antenna is localized in the ("parallel") z direction, the rf field emission has a

k_{\parallel} -spectrum. The plasma is furthermore surrounded by conducting walls.

For this model, the wave equations are solved, the ponderomotive force density is evaluated, as well as the terms of the variational principle^{3,7}. The calculation of ponderomotive forces and wave propagation uses the perpendicular cold plasma dielectric tensor^{7,9}. The antenna is characterized by impedance L_p as a function of k_{\parallel} [Fig. 2(a)]. One recognizes the component of the far-field, at resonance (i.e. at $k_{\parallel} = k_r = 0.47k_A$, where the Alfvén wavevector $k_A = \Omega_i/v_A = 4.0 a^{-1}$). The near-field includes the other spectral components and is obtained by a weighted integration over k_{\parallel} , with the exclusion of the resonant region.

The stabilization properties to a rigid displacement ξ of the plasma slab are summarized in Fig. 2(b), the relevant curve being the sum $\Delta W_p + \Delta W_A$. It must be noted that, for most of the spectrum, those terms are separately much greater in magnitude than their sum. The large cancellation, which illustrates the importance of self-consistency, is relatively strongest for the far-field, i.e., close to $k_{\parallel} = k_r$. The conclusion is that although far-fields can have an appreciable amplitude, they do not contribute to stabilization.

The near-fields are illustrated in Fig. 3(a). The rf vector potential field¹⁰ (A_x, A_y) decays in the plasma, on an Alfvén length scale k_A^{-1} . The ponderomotive force density $F(x)$ and the interchange term $G(x)$ (which gives information on the stabilization of internal modes: $\Delta W_p = \int \xi^2(x)G(x)dx$) are illustrated in Fig. 3(b). They are negative inside the plasma column, due to electron ponderomotive force³. Only the outer edge of the plasma gives stabilizing contribution. Those rf fields will therefore not stabilize internal modes (where $\xi(x)$ is confined inside). Flat pressure profiles will result, if finite ion Larmor radius effects do not stabilize those modes. As ω approaches Ω_i , the stabilizing part is pushed more and more to the low density region (where $k_{\parallel}^2 v_A^2 / \omega^2 \leq [2(\omega - \Omega_i) / \Omega_i]^{-1/2}$), which leads to a rather pathological stabilization possibly linked to the instability observed experimentally¹.

The far-field modes are illustrated in Fig. 3(c). They are characterized by an amplitude inside the plasma much larger than for near-field modes, at equal current excitation. Fig. 3(d) shows the corresponding functions F and G. We have mentioned already that self-consistent effects cancel most of those apparently favorable data.

There is a simple reason why propagating modes (i.e. far-field) do not provide stabilization of the rigid plasma motion : those modes are entirely supported by the plasma column, and, as the plasma suffers a displacement, they move with the plasma, with no change of energy. This interpretation shows that this conclusion holds for plasma models much more general than the one considered here, both with respect to dielectric properties, and with respect to geometry. The propagating modes could in principle interact with the rigid wall surrounding the plasma, and provide some stabilization, but I do not favor this scheme for two practical reasons: wall dissipation enhances energy losses, and in a fusion device, the necessary distance between wall and plasma will greatly reduce the wall stabilization efficiency. In conclusion, for stabilization, one should use the near-field of the antenna, and possibly the near-field induced in conducting limiters.

The results for a given localized antenna are obtained by weighting the curves (Fig. 2) with the k_{\parallel} components of the antenna current intensity. An antenna designed to suppress the energy-consuming propagating mode (at k_r) will have large Q (the singularity in the impedance curve, Fig. 2(a), corresponds to energy convected away by those modes⁹). For example for this plasma slab, two parallel antennas axially separated by a distance $2\pi/k_r$, and conducting opposing currents will satisfy this criterion (the k_r component of the excitation current vanishes).

An estimate of the amount of rf energy follows from Fig. 2(a), $E_{rf} = (1/4)L_p|I|^2 \approx 0.09(4\pi/c^2)a|I|^2$. Similarly, from Fig. 2(b), $\Delta W_p + \Delta W_A \approx 0.15(4\pi/c^2) \times a|I|^2(\xi/a)^2$. Stabilization of the interchange mode requires $\Delta W_{MHD} + \Delta W_p + \Delta W_A \geq 0$, where $\Delta W_{MHD} = -(\gamma_{MHD})^2 \xi^2 \int m n dx$, and where γ_{MHD} is the flute growth rate in absence

of rf. Typically, for an axisymmetric mirror, $\gamma_{MHD}^2 \approx \kappa v_i^2 / a_p$, with κ the unfavorable average curvature, v_i the ion thermal velocity, and a_p the pressure gradient length. These equations lead to the value of current and rf free energy necessary for stabilization. The stability condition becomes here $E_{rf} \geq 0.70 m_i a^2 \gamma_{MHD}^2 a n_0$, which is a nonnegligible fraction $\kappa a^2 / a_p$ of the total plasma thermal energy.

The power loss incurred for sustaining this amount of rf energy is conveniently expressed in terms of the antenna Q , as $\omega E_{rf} / 2\pi Q$. In a fusion reactor regime, those losses are prohibitively large, unless Q is very large (of the order of $5.0 \cdot 10^5$ for a 20kG magnetic field and for rf energy circulation time of one second). The conclusion is that the configuration above (or for an axisymmetric plasma, a single loop antenna) is not favorable.

Feedback control largely improves the situation. We consider first passive feedback: the impedance of the generator is finite, and is chosen so that the antenna currents react properly to plasma motions. The result of the analysis is that the modified expression of ΔW_A must include the free energy of the rf field stored in the generator circuit, represented here by a current source J in parallel with an impedance $-i\omega L_g$. In the formulas, one must substitute for $V = -(1/4)L_p |I|^2$ the new value of ponderomotive energy $V' = -(1/4)J^*(L_g - L_g(L_g + L_p)^{-1}L_g)J$. The antenna impedance $-i\omega L_p$ is to be considered as a function of the plasma parameters, and in particular, of its position^{3,7}. The second variation of the energy becomes $\delta^2 V' = -(1/4)[I^* \delta^2 L_p I - I^* \delta L_p (L_g + L_p)^{-1} \delta L_p I]$, where $I = (L_g + L_p)^{-1} L_g J$ is, as before, the equilibrium value of the antenna current, and where use has been made of the relation $\delta I = -(L_g + L_p)^{-1} \delta L_p I$. We have previously discussed the first term of $\delta^2 V'$ which gives the results shown in Fig. 2. The second term is due to the variation of the antenna currents. One can expect this term to be large and stabilizing, provided the inductances L_p and L_g are properly matched.

If tried on the present configuration, the result is disappointing: for a displacement

ξ of the plasma, because of symmetry, the variation δL_p vanishes at equilibrium, so that it is not possible to exploit this kind of feedback. The reason is clear : the symmetry of the antenna prevents an effective directed action of the rf field on the plasma (the same conclusion applies to a full-loop antenna surrounding an axisymmetric plasma). The remedy directly follows : one must allow the current variation to be different on each side of the plasma, so that the rf forces increase on the side towards which the plasma tends to move, and decrease on the other side. For such multi-antenna system, the generalized expression of $\delta^2 V'$ is identical to the one given above, where L_p and L_g are reinterpreted as inductance matrices, and I and J as current arrays. Assuming the antennas weakly coupled, one gets $\delta L_p \approx 4|I|^{-2} \xi \int_0^a F(x) dx$, which, from Fig. 3(b) is here $\delta L_p \approx 0.06(16\pi a/c^2)(\xi/a)$. The modified ΔW_A is roughly a factor $0.2(L_p + L_g)^{-1} a \delta L_p / \delta \xi$ greater than before. The requirement on the antenna Q is proportionally reduced. The maximum value of $\|(L_g + L_p)^{-1}\|$ is of course limited by dissipation (their imaginary part).

We propose an antenna design favorable for axisymmetric plasma stabilization (Fig. 4). It has four quarter-turn antennas, each connected independently to the generator(s). The azimuthal localization of the ponderomotive force density allows a directed response to plasma displacements. The distance L is chosen to filter out the propagating mode ($L = 2\pi k_r^{-1}$), to give large Q . The generator impedances are adjusted as described above for stabilizing feedback. Note that this particular geometry still preserves the axisymmetry of the equilibrium, rf fields included.

Finally, we note the further possibility of active feedback: a set of detectors registering plasma motion and controlling the antenna currents may allow very sensitive reaction to plasma displacement. If the current I is forced to vary with ξ , the corresponding value of $\delta^2 V'$ is¹¹ $-(1/4)I^* \delta L_p \delta I$. It is equivalent to a high- Q circuitry, the sensitivity $I^{-1} \delta I / \delta \xi$ taking place of $(L_g + L_p)^{-1} \delta L_p / \delta \xi$. Assuming high sensitivity, the power requirement can again be considerably lowered. This system, combined with

an antenna of the type of Fig. 4, probably offers the best features for stabilization.

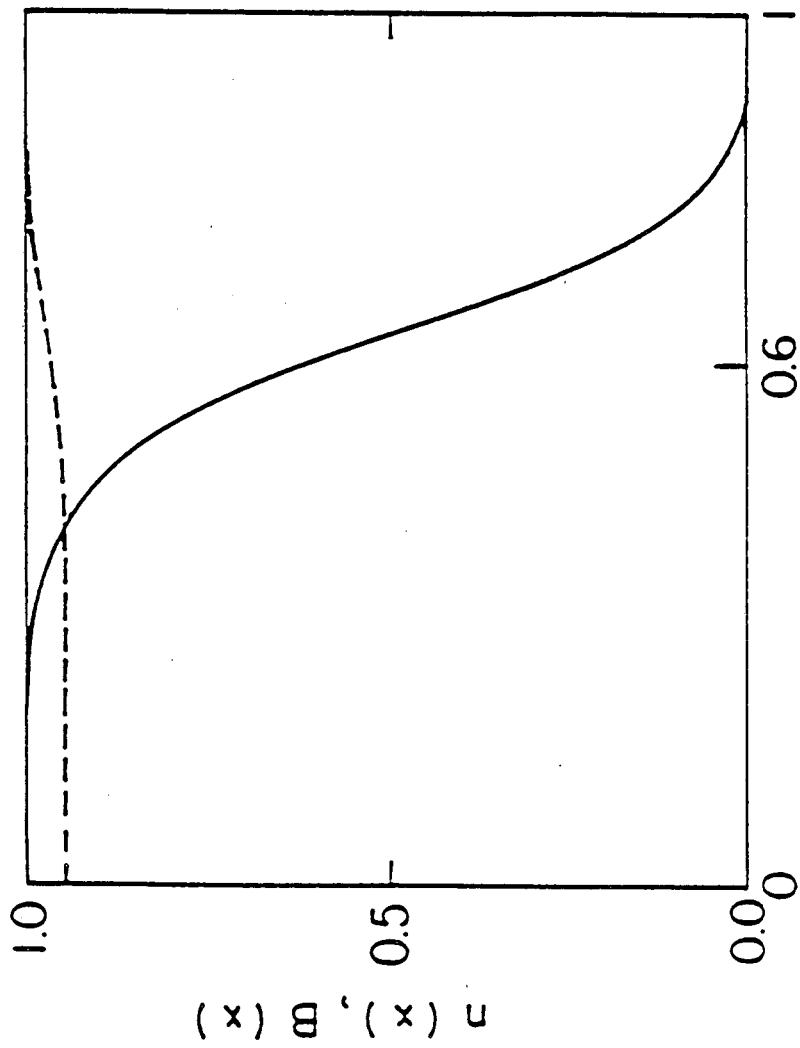
I am grateful to Allan N. Kaufman for interesting discussions. This research was supported in part by DOE contract DE-AC03-76 SF00098 (Lawrence Berkeley Laboratory), and by the College of Engineering, Cornell University. The numerical results were obtained at the Institute for Theoretical Physics in Santa Barbara (April 1985), under NSF grant PHY82-17853, supplemented by funds from NASA.

References

1. J.R. Ferron, N. Hershkowitz, R.A. Breun, S.N. Golovato, and R. Goulding, Phys. Rev. Lett. 51, 1955 (1983).
2. Y. Yasaka and R. Itatani, Phys. Rev. Lett. 56, 2811 (1986).
3. P.L. Similon, and A.N. Kaufman, Phys. Rev. Lett. 53, 1061 (1984).
4. D.A. D'Ippolito and J.R. Myra, Phys. Fluids 28, 1895 (1985).
5. B.I. Cohen and T.D. Rognlien, Phys. Fluids 28, 2194 (1984).
6. P.L. Similon, A.N. Kaufman, and D.D. Holm, Phys. Lett. A 106, 29 (1984).
7. P.L. Similon, A.N. Kaufman, and D.D. Holm, Phys. Fluids 29, 1908 (1986).
8. D.A. D'Ippolito and J.R. Myra, Phys. Fluids 29, 2599 (1986).
9. T.H. Stix, *The Theory of Plasma Waves*, (McGraw-Hill, New York, 1962).
10. In the radiation gauge used here, the rf electric field is $\mathbf{E} = -i(\omega/c)\mathbf{A}$.
11. Although the equations have lost their self-adjoint property, it is still permissible to use the variational principle perturbatively (the ponderomotive terms are small compared to the terms in ΔW_{MHD}).

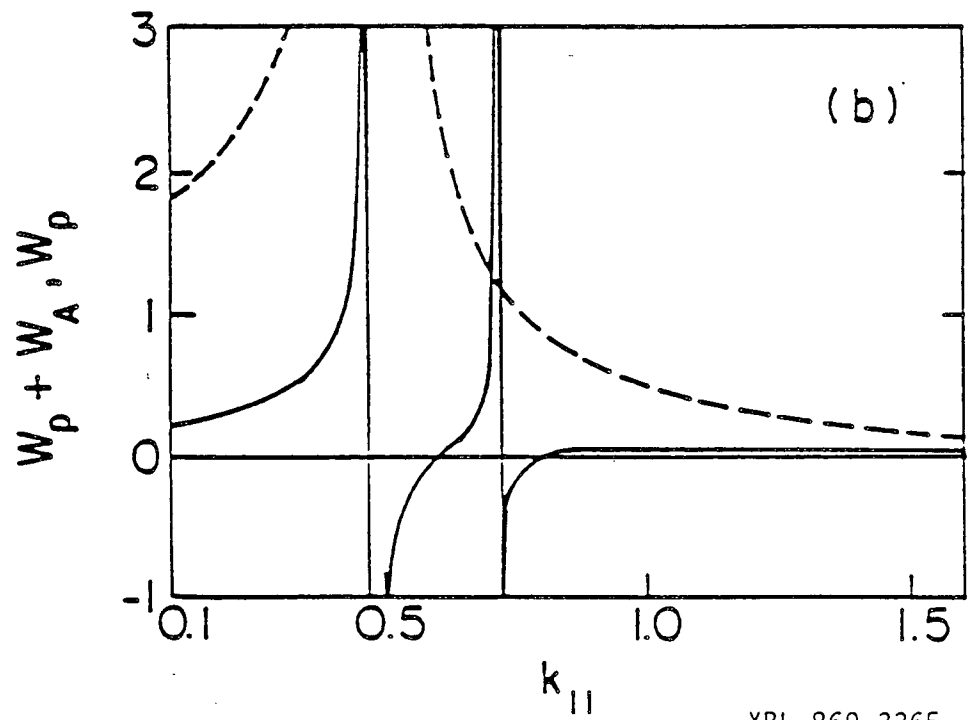
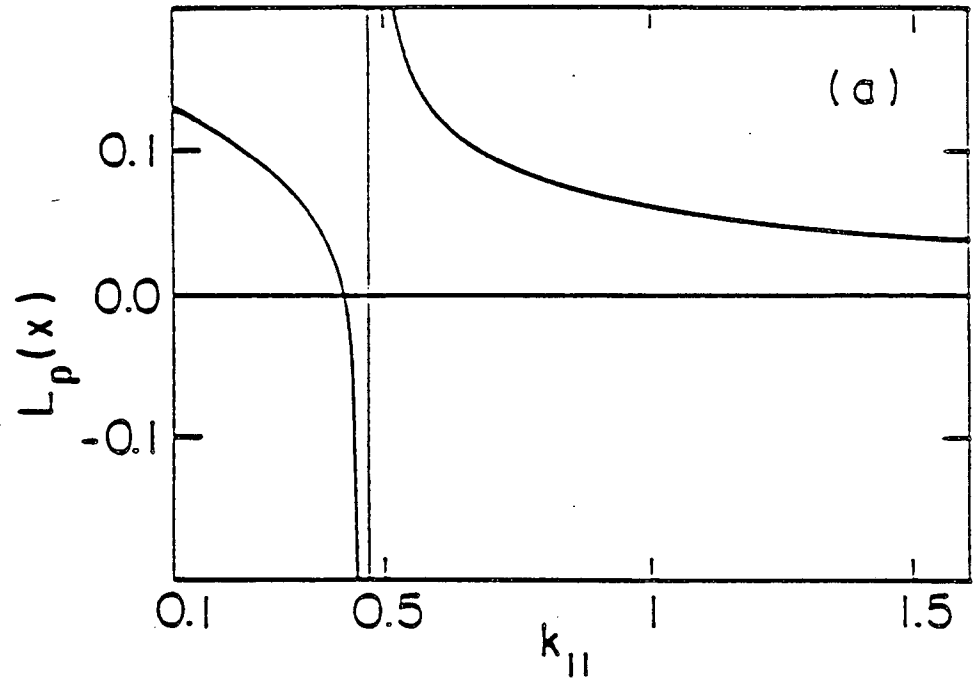
Figure Captions

- FIG. 1. Equilibrium density (solid line) and magnetic field (dashed line) profiles normalized to their maximum values. The center of the symmetric plasma slab is at $x = 0$, the antenna at $x = \pm a = \pm 1$, and the wall at $x = \pm 5a$.
- FIG. 2. (a) Antenna inductance L_p (normalized to $16\pi a/c^2$) in function of k_{\parallel} (normalized to $k_A = 4a^{-1}$), for $\omega = 1.1\Omega_i$, showing the resonance at $k_{\parallel} = k_r = 0.47k_A$. (b) Total rf contribution $\Delta W_p + \Delta W_A$ (solid line) versus ponderomotive contribution only ΔW_p (dashed line), normalized to $(4\pi/c^2)a|I|^2(\xi/a)^2$.
- FIG. 3. (a) Profiles of the rf field A_x (solid line) and A_y (dashed line), normalized to their maximum value, for $\omega = 1.1\Omega_i$ and $k_{\parallel} = k_A$. (b) Ponderomotive force density $-F$ (dashed line) (positive $-F$ is inwards and stabilizing) and interchange term G (solid line), normalized to $(4\pi/c^2)|I|^2$ and $(4\pi/c^2)a^{-1}|I|^2$. (c) and (d) Same as above, for $k_{\parallel} = 0.3k_A$.
- FIG. 4. Four quarter-turn antennas, with $L = 2\pi k_r^{-1}$. The arrows show the phases of the equilibrium rf currents.



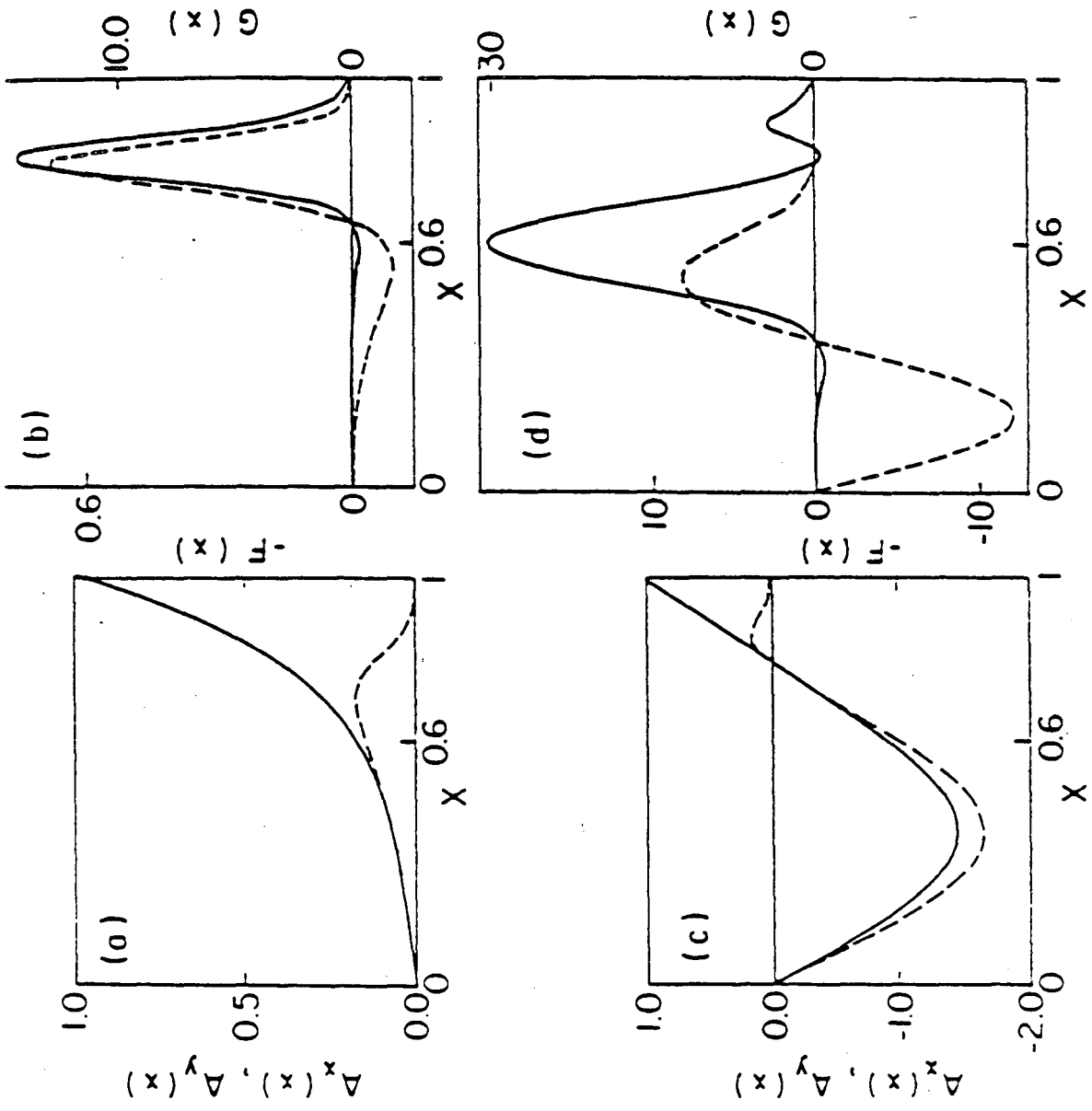
XBL 869-3364

Fig. 1



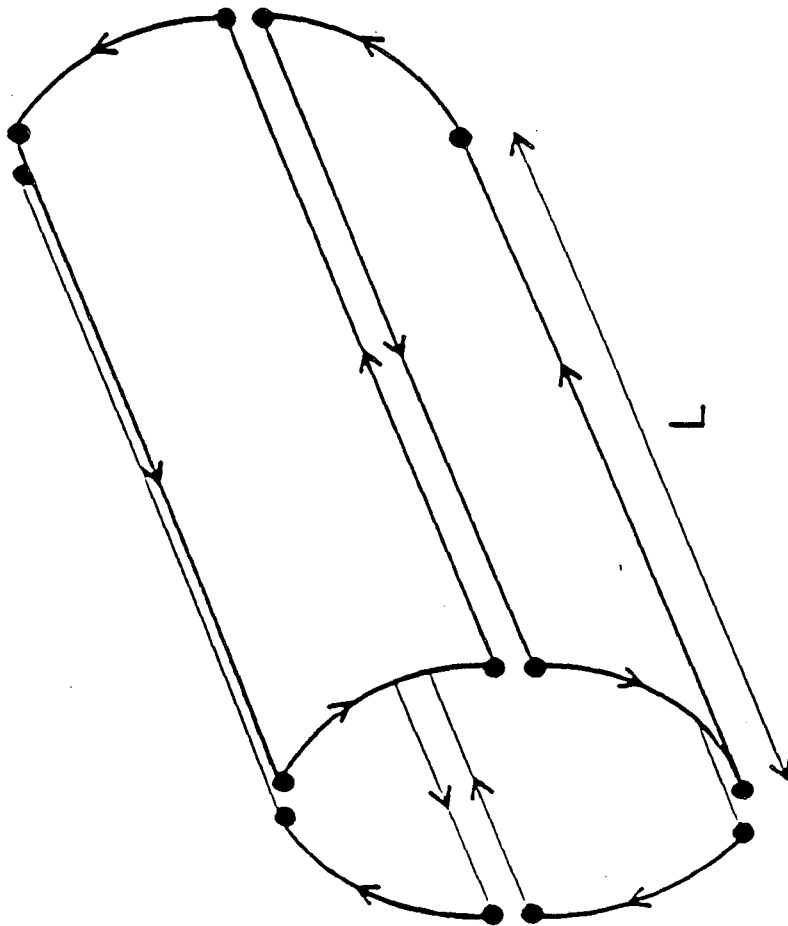
XBL 869-3365

Fig. 2



XBL 869-3366

Fig. 3



XBL 869-3367

Fig. 4

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

*LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720*