Title
HOW FAST DOES THE MULTIPLICITY OF PARTICLE PRODUCTION IN P-P COLLISIONS REALLY INCREASE WITH PRIMARY ENERGY?

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ABSTRACT

Multiplicity distributions of particles created in p-p collisions with $E_{\text{lab}} = 50$-200 GeV are shown to contain at least one Poisson component which becomes dominant at multiplicities $k \geq 1.5$ ($k$). The mean values of both this component and of the (relatively low multiplicity residue) increase like $E_{\text{lab}}^{1/2}$; the relative weight of the Poisson component decreases approximately like $E_{\text{lab}}^{-0.3}$. If these trends continue beyond the available accelerator energies, one might expect the mean multiplicity to approach an $E_{\text{lab}}^{1/2}$ law beyond 10 TeV.
How fast does the multiplicity of particle production in p-p collisions really increase with primary energy?

It has been well-known for a long time that the dependence of the mean multiplicity, m, of particles produced in high energy p-p collisions on the primary laboratory energy $E_o$ is much weaker than $\sim E_o^{1/2}$, the fastest rise allowed kinematically or implied e.g by Heisenberg’s original theory.[1] At the same time the rise is faster than $\sim \ln E_o$ as predicted by scaling models [2] and even somewhat faster than the $E_o^{1/4}$ dependence predicted by thermodynamical-hydrodynamical considerations [3], [4] implying a Lorentz contracted production volume.

This paper is concerned with the problem of predicting the evolution of m with $E_o$ at super-high energies (say, beyond 10 TeV), not just from an extrapolation of fits to values of m observed in the energy range covered by present accelerators, but by using regularities observed in the structure of multiplicity distributions (abbreviated hereafter as MD), too. It turns out that—provided these regularities continue to hold at very high $E_o$—a very fast increase, $\sim E_o^{1/2}$ is asymptotically expected.

In order to emphasize the Poisson-like shape of MD’s † it is preferable to use instead of the probability $W(k)$ for observing k secondaries the quantity

$$Y(k) = \ln[k! W(k)]$$

(1)

Obviously, if $W(k)$ is a Poisson distribution (PD) of mean, say, a, $Y$ is linear in k:

† Hereafter we will be concerned only with negative multiplicities $k = (n_{ch} - 2)/2$ where $n_{ch}$ is the total multiplicity of charged secondaries; k refers to created particles only.
Fig. 1 shows the MD at \( E_0 = 69 \) GeV \([5]\) on a \( Y \) vs. \( k \) plot. The MD shape is obvious. However, with increasing energy the shape becomes more complicated.

Assume that \( W(k) \) is a superposition of two MD's, say, \( W_1 \) and \( W_2 \) with means \( m_1 \) and \( m_2 > m_1 \).

\[
W(k) = (1 - \alpha) W_1 (k) + \alpha W_2 (k) \quad (3)
\]

where one component, say \( W_2 \), is Poisson and all we know about \( W_1 \) is that it practically dies out beyond some value \( k_c \) of \( k \). Then, beyond \( k_c \),

\[
Y \approx -m_2 + \ln \alpha + \ln m_2 \quad (4)
\]

i.e., we get again a straight line with slope \( \ln m_2 \) but with an intercept depending on both \( m_2 \) and the relative weight \( \alpha \) of the Poisson component.

\[\text{†} \text{As has been done in Ref. [6] for mathematical expediency, but without presenting compelling evidence for Poisson components!}\]
Fig. 2

This is well seen in fig. 2 which shows the MD at 300 GeV, [7] the statistically best measured sample to date \((10^4\) events).

The fact that this pattern is consistently repeated over the whole range of accelerator energies is shown in fig. 3, which displays the scaling variable \(Z\), defined as

\[
Z = \frac{m + \frac{y}{m}}{m} \text{,} \quad (5)
\]

as a function of

\[
X \equiv \frac{k}{m} \text{.} \quad (6)
\]

for all available data from 50 to 2100 GeV. On such a plot PD would be a straight line of slope 1 passing through the origin.

As can be seen, beyond \(x \sim 1.5\) the points (drawn from both HBC exposures between 50 and 405 GeV and ISR results between 500 and 2100 GeV equivalent laboratory energy) do indeed cluster along a straight line confirming up to the highest available energy the presence of a PD component \((W_2)\). As to the residue \((W_1)\), it can be shown that, in spite of the apparently high statistics gathered to this day, the accuracy is insufficient to define its shape. A PD is not excluded, although a more complicated structure...
may be indicated by the HBC-data.† However, the accuracy is just sufficient to estimate the mean values \( m_2 \) and \( m_1 \) (the Poisson component and the residue).

These values are plotted against \( E_0 \) (di-log plot) in fig. 4, together with the overall mean \( m \) of \( k \).

As can be seen, the energy dependence of both \( m_1 \) and \( m_2 \) can be well parameterized as \( \sim E_0^\delta \) with \( \delta \) close to 1/2 (the fitted values are \( \delta = 0.54 \pm 0.03 \) for \( m_2 \) and—understandably with lower accuracy—\( \delta = 0.57 \pm 0.13 \) for \( m_1 \)).

It thus appears that p-p collisions can be regarded as a mixture of (at least) two types of events, each of which produces particles with a multiplicity law \( \sim E_0^{1/2} \).

This can be reconciled with the relatively slow variation of the mean of the mixture, namely \( m \) (indeed, for \( m \), \( \delta = 0.35 \pm 0.01 \), if it is parameterized in the same way as \( m_1 \) and \( m_2 \)) only if \( \alpha \) (the relative weight of \( W_2 \)) decreases with \( E_0 \). Within the statistical

† The largest uncertainty resides in \( W_1(0) \) because of the systematical effects connected with estimation of the elastic contribution.
and systematic uncertainties of the data, $\alpha$ appears to decrease like $E_0^{-(0.3 \pm 0.1)}$. If this trend continues at very high energies, $W_1$ should become dominant beyond, say, 50 TeV (at $\sim 400$ GeV $W_1$ and $W_2$ have comparable weights) and there one might expect $m$ to increase like $E_0^{1/2}$. This may be essential in understanding the development of extensive air showers in the early stages of their evolution.

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References