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UNITARIZATION OF THE DUAL RESONANCE AMPLITUDE. III. GENERAL RULES FOR THE ORIENTABLE AND NONORIENTABLE MULTILOOP AMPLITUDES

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UNITARIZATION OF THE DUAL RESONANCE AMPLITUDE. III. GENERAL RULES FOR THE ORIENTABLE AND NONORIENTABLE MULTILOOP AMPLITUDES*

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ABSTRACT

We complete the KSV unitarization scheme for the dual resonance amplitude by presenting the nonorientable and overlapping multiloop amplitudes. We also present rules for writing down arbitrarily mixed planar, nonplanar, nonorientable, and overlapping multiloop amplitudes by inspection.

I. INTRODUCTION

This paper is the last in a series of three papers on the unitarization of the Veneziano amplitude.[1-5] Remarkably, the nonorientable and overlapping loop amplitudes retain most of the essential features of the integrand of the planar and nonplanar multiloop amplitudes. Each nonorientable loop is distinguished by the fact that its projective transformation is loxodromic (i.e., its multiplier is negative). The overlapping loop amplitudes have hyperbolic projective transformations (as in the case with the planar and nonplanar loops); the difference lies in the fact that the two pairs of invariant points are found to have overlapping orderings.

Fortunately, the momentum-dependent factors in the integrand are independent of the arrangement of the various loops in a multiloop amplitude. We give simple rules for writing down by inspection the factors raised to the $\alpha_0 - 1$ power, which are sensitive to the topology of the diagram.

II. GENERAL MULTILOOP FORMULAS

A. The Nonorientable Multiloop Amplitude

In our previous papers,[1-3] we went into considerable detail in deriving the planar and nonplanar multiloop amplitudes by joining the factorized legs of the N-reggeon tree as well as Sciuto three-reggeon vertices. Because all multiloop amplitudes share many of the same characteristics, we shall present only the nonorientable and overlapping loop formulas without the derivation.

In Ref. 3, we found that the particular factorized tree used to calculate the multiloop amplitudes was often not the most desirable one from the viewpoint of obtaining symmetry among all quark loops and in discussing the limits of integration. In Fig. 1, for example, we display the factorized tree which was used in the calculation of the nonplanar multiloop amplitude. To restore the symmetry among all the quark loops, however, we found it convenient to move all external lines lying on the outer quark loop away from the loops. In Ref. 3 we found that these external variables, when bunched together, are confined to lie between the invariant points of the product of all projective transformations taken in order (see Fig. 2). The symmetry among all quark loops is restored, because the Koba-Nielsen variables associated with external legs trapped within a loop are confined to lie between the invariant points of that loop, whereas the variables associated with the outer quark loop are confined to lie between the invariant points of the product of all projective transformations.

Essentially all features of the multiloop amplitude remain the same if we go from Fig. 1 to Fig. 2, except for the factors raised to the power $\alpha_0 - 1$ and the region of integration.

Using the notation of Refs. 1, 2, and 3, we let y_i represent each external leg and y_β and y_α the invariant points of the $(\alpha\beta)$ loop. As we move external legs lying on the outer quark loop away from the loops, the critical $\alpha_0 - 1$ factors, which are sensitive to the quark topology, change from

$$\prod_{(\alpha\beta)=\{\mathcal{L}\}} \times \left[\frac{(y_{\beta+1} - y_\beta)(R_{\beta\alpha}(y_{\beta+1}) - y_{\alpha-1})}{\left(y_\beta - R_{\beta\alpha}(y_{\beta+1})\right)}\right]^{\alpha_0-1} \left[\frac{(y_{\beta-1} - y_\beta)(R_{\beta\alpha}(y_{\beta-1}) - y_{\alpha+1})}{\left(y_\beta - R_{\beta\alpha}(y_{\beta-1})\right)}\right]^{\alpha_0-1} \tag{2.1}$$

to

$$\left[\frac{\left(y_0 - x^{(2)}\right)\left(\hat{R}(y_0) - y_{\alpha-1}\right)}{\left(x^{(2)} - \hat{R}(y_0)\right)}\right]^{\alpha_0-1} \prod_{(\alpha\beta)=\{\mathcal{L}\}} \left[\frac{(y_{\beta-1} - y_\beta)\left(R_{\beta\alpha}(y_{\beta-1}) - y_{\alpha+1}\right)}{\left(y_\beta - R_{\beta\alpha}(y_{\beta-1})\right)}\right]^{\alpha_0-1}$$

$$\times \left[\prod_{(\alpha\beta)=\{\mathcal{L}\}} X_{\alpha\beta}\, X_{\hat{R}}^{-1}\right]^{\frac{1}{2}(\alpha_0-1)}, \tag{2.2}$$

where $\hat{R} \equiv R_{\beta\alpha} R_{\delta\gamma} \cdots R_{\lambda\sigma}$ and $x^{(2)} = \hat{R}^{-\infty}(z)$. [Notice that we have dropped all factors like $(y_i - y_{i+1})^{\alpha_0-1}$ for simplicity.]

(X_i is the multiplier of R_i. Also, $x_{\alpha\beta}^{(2)} = y_\alpha = R_{\beta\alpha}^{\infty}(z_1)$, $z_1 \neq x_{\alpha\beta}^{(1)}$ **and** $x_{\alpha\beta}^{(1)} = y_\beta = R_{\beta\alpha}^{-\infty}(z_2)$, $z_2 \neq x_{\alpha\beta}^{(2)}$)

The factor raised to the $\frac{1}{2}(\alpha_0 - 1)$ power is projectively invariant and symmetric in all quark loops.[6] It arises only when <u>all</u> external lines are removed from the region between two or more loops.

Now assume that we add an extra twist operator to Fig. 1 for each loop, so that it now describes the nonorientable multiloop amplitude. Applying the techniques used in Refs. 1 and 2, we find that the projective transformation describing each loop is loxodromic (i.e., it has a negative multiplier lying between zero and minus one) and that the $\alpha_0 - 1$ factors change slightly from the analogous factors given in (2.1):

$$\prod_{(\alpha\beta)=\{\mathcal{L}\}} \times \left[\frac{(y_{\beta+1}-y_\beta)\left(R_{\beta\alpha}(y_{\beta+1})-y_{\alpha+1}\right)}{\left(y_\beta - R_{\beta\alpha}(y_{\beta+1})\right)}\right]^{\alpha_0-1} \left[\frac{(y_{\beta-1}-y_\beta)\left(R_{\beta\alpha}(y_{\beta-1})-y_{\alpha-1}\right)}{\left(y_\beta - R_{\beta\alpha}(y_{\beta-1})\right)}\right]^{\alpha_0-1} . \quad (2.3)$$

Notice that the only difference between the nonplanar and nonorientable factors is that we have exchanged the roles of some of the y's.

In the nonplanar case, lines trapped between the invariant points of the same loop remain there, though lines occurring within nonorientable loops are free to move out, due to duality. Mathematically, this corresponds to:

Nonplanar case: $\quad y_{\beta+1} < y_\beta < y_\alpha \rightarrow y_\beta < y_\alpha < R_{\beta\alpha}(y_{\beta+1})$

and $\quad y_\beta < y_{\beta-1} < y_{\beta-2} < y_\alpha \rightarrow y_\beta < y_{\beta-2} < R_{\beta\alpha}(y_{\beta-1}) < y_\alpha;$

Nonorientatable case:

$$y_{\beta+1} < y_\beta < y_{\beta-1} < y_\alpha \rightarrow y_\beta < y_{\beta-1} < R_{\beta\alpha}(y_{\beta+1}) < y_\alpha$$

and $$y_\beta < y_{\beta-1} < y_{\beta-2} < y_\alpha \rightarrow y_\beta < y_{\beta-2} < y_\alpha < R_{\beta\alpha}(y_{\beta-1}). \qquad (2.4)$$

In the nonplanar case, the projective transformation is hyperbolic, and therefore can send external lines past the invariant points. In the nonorientatable case, the transformation is loxodromic, so R^2 is responsible for sending external lines past loops.

As before, Fig. 1 is an awkward configuration to discuss the limits of integration for the nonorientable case. Assume that we have pushed all external lines away from the loops, as in Fig. 2 (notice that the lines situated between the various invariant points no longer exist). Since R_i^2 is responsible for sending external lines past the ith loop, we expect that the product of the squares of all projective transformations, taken in order, will send a line completely around the diagram. We thus find that the analogous factors in Fig. 2 are

$$\left[\frac{\left(y_0 - x^{(2)}\right)\left(\bar{R}(y_0) - y_{\alpha-1}\right)}{\left(x^{(2)} - \bar{R}(y_0)\right)}\right]^{\alpha_0-1} \left[\prod_{(\alpha\beta)=\{\mathcal{L}\}} X_{\alpha\beta}^{\ 2}\, X_{\bar{R}^{-1}}\right]^{\frac{\alpha_0-1}{2}}, \qquad (2.5)$$

where $\bar{R} \equiv R_{\beta\alpha}^{\ 2} R_{\delta\gamma}^{\ 2} \cdots R_{\lambda\sigma}^{\ 2}$ and $x^{(2)} \equiv \bar{R}^{-\infty}(z)$.

As in Refs. 2 and 3, we expect the region of integration for Fig. 2 in the nonorientable case to be

$$U_2 = \Big\{ x^{(2)} \leq x^{(1)}_{\sigma\lambda} \leq x^{(2)}_{\sigma\lambda} \leq \cdots \leq x^{(1)}_{\alpha\beta} \leq x^{(2)}_{\alpha\beta} \leq x^{(1)} \leq$$

$$\leq R_{\beta\alpha}{}^2 \cdots R_{\lambda\sigma}{}^2(y_0) \leq y_{\alpha-1} < y_{\alpha-2} \leq \cdots \leq y_1 \leq y_0 \leq x^{(2)}$$

$$\text{and} \quad x^{(1)} \leq \overline{R}(\overline{y}) \leq y_0 \leq \overline{y} \leq x^{(2)} \Big\} ; \tag{2.6}$$

$\overline{y}$, which is arbitrary, is introduced to eliminate periodicities, exactly as in Refs. 2 and 3.

All other factors in the integrand of the nonorientable loop amplitude are identical to the planar case (except for the 1-X's).

B. Overlapping Multiloop Amplitude

The double overlapping loop amplitude can be calculated by joining the $(\alpha\beta)$ and $(\gamma\delta)$ pairs as shown in Fig. 3. As in the case with the nonorientable loop, all the essential features of the integrand remain the same as in the nonplanar case, except for the $\alpha_0 - 1$ factors. A direct calculation reveals that, for Fig. 3, we have the same form as the nonplanar case, except that the invariant points of different loops are allowed to overlap. Duality allows us to successively move external lines past the loops; mathematically, this corresponds to

$$y_{\delta+1} < y_\delta < y_\beta < y_\gamma < y_\alpha \to y_\delta < y_\beta < y_\gamma < R_{\delta\gamma}(y_{\delta+1}) < y_\alpha \to$$

$$\to y_\delta < y_\beta < R^{-1}_{\beta\alpha} R_{\delta\gamma}(y_{\delta+1}) < y_\gamma < y_\alpha \to$$

$$\to y_\delta < R^{-1}_{\delta\gamma} R^{-1}_{\beta\alpha} R_{\delta\gamma}(y_{\delta+1}) < y_\beta < y_\gamma < y_\alpha \to$$

$$\to y_\delta < y_\beta < y_\gamma < y_\alpha < R_{\beta\alpha} R^{-1}_{\delta\gamma} R^{-1}_{\beta\alpha} R_{\delta\gamma}(y_{\delta+1}) . \tag{2.7}$$

We are then free to bunch all external lines, as in Fig. 4. In this case, we find that the operator $R_1 R_2^{-1} R_1^{-1} R_2$ is responsible for sending external lines past loops 1 and 2. Again, from projective invariance, we find that the $\alpha_0 - 1$ factors become

$$\left[\frac{\left(y_0 - x^{(2)}\right)\left(R(y_0) - y_{S-3}\right)}{\left(x^{(2)} - R(y_0)\right)}\right]^{\alpha_0 - 1} \left[X_1{}^2 X_2{}^2 X_{R^{-1}}\right]^{\frac{\alpha_0 - 1}{2}} , \tag{2.8}$$

where $R \equiv R_1 R_2^{-1} R_1^{-1} R_2$ and $x^{(2)} \equiv R^{-\infty}(z)$.

As in the previous cases, we find that the limits of integration for Fig. 4 can be expressed as

$$x^{(2)} \leq x^{(1)}_{R_2} \leq x^{(1)}_{R_1} \leq x^{(2)}_{R_2} \leq x^{(2)}_{R_1} \leq x^{(1)}$$

$$\leq R(y_0) \leq y_{S-3} \leq y_{S-4} \leq \cdots \leq y_1 \leq y_0 \leq x^{(2)}$$

and $$x^{(1)} \leq R(\bar{y}) \leq y_0 \leq \bar{y} \leq x^{(2)} \quad . \tag{2.9}$$

We end our discussion by stating that the linear dependence correction factor is $(1 - X)$ for each planar loop, $(1 - X)^2$ for each nonplanar and overlapping loop, and $(1 + |X|)^2$ for each nonorientable loop.

III. ILLUSTRATIVE EXAMPLE

We shall illustrate our principles, which apply to all multiloop diagrams, by writing down the amplitude for a specific example. We shall find it convenient to make certain dual manipulations on the multiloop diagram such that (a) no loop ever occurs within another loop, (b) external legs confined between nonorientable and overlapping loops are removed, (c) all external legs not confined within nonplanar loops are bunched together, (d) no more than two loops ever overlap. In this configuration, all multiloop diagrams can be reduced to planar, nonplanar, nonorientable, and double overlapping loops placed in series along a chain.

Now consider the diagram corresponding to Fig. 5, which contains two planar, two nonplanar, one double overlapping, and one nonorientable loops. Applying our techniques, we know that the operator $\hat{R}^{-1} \equiv R_7^{-2} R_6^{-1} R_5 R_6 R_5^{-1} R_4^{-1} R_3^{-1} R_2^{-1} R_1^{-1}$ is responsible for moving external lines past the region occupied by the loops. Therefore, we find

$$\int \prod_{i=1}^{7} d^4k_i \int_{U_1} \prod_{i=1}^{7} dX_i \int_{U_2} \prod_{i=1}^{S} dy_i \prod_{j=1}^{7} dx_j^{(1)}\, dx_j^{(2)}$$

$$\times \quad \frac{(y_a - y_b)(y_b - y_c)(y_c - y_a)}{dy_a dy_b dy_c} \prod_{\{Q\}} (1 - X_Q)^{-4}$$

$$\times \quad (1 - X_1)(1 - X_2)(1 - X_3)^2(1 - X_4)^2(1 - X_5)^2(1 - X_6)^2(1 + |X_7|)^2$$

Equation continued on next page

Equation continued.

$$\prod_{\substack{i\neq 1\\(i\neq\ell,m,S)}}^{S} (y_i - y_{i+1})^{\alpha_0 -1} \left\{\frac{[y_{\ell+1} - R_3(y_m)][x_3^{(1)} - y_m]}{[x_3^{(1)} - R_3(y_m)]}\right\}^{\alpha_0 -1}$$

$$\times \left\{\frac{[y_{m+1} - R_4(y_S)][x_4^{(1)} - y_S]}{[x_4^{(1)} - R_4(y_S)]}\right\}^{\alpha_0 -1} \left\{\frac{[y_\ell - \hat{R}(y_1)][x^{(2)} - y_1]}{[x^{(2)} - \hat{R}(y_1)]}\right\}^{\alpha_0 -1}$$

$$\times \prod_{\mathcal{L},\mathcal{L}''} \prod_{\substack{i,j=1\\(n=0,i\neq j)}}^{S} \prod_{n=0}^{\infty} \left\{y_i - [R^{\pm}]^{(n)}_{\mathcal{L},\mathcal{L}'}(y_j)\right\}^{-\frac{1}{2}k_i k_j}$$

$$\times \left\{X_1 X_2 X_3 X_4 X_5{}^2 X_6{}^2 X_7{}^2 X_{\hat{R}-1}\right\}^{\frac{\alpha_0 -1}{2}}$$

$$\times \prod_{i=1}^{S} \prod_{\substack{\mathcal{L},\mathcal{L}',\mathcal{L}''\\(\mathcal{L}'\neq\mathcal{L}'')}} \prod_{n=0}^{\infty} \left\{\frac{y_i - [R^{\pm}]^{(n)}_{\mathcal{L},\mathcal{L}'}(x^{(2)}_{\mathcal{L}''})}{y_i - [R^{\pm}]^{(n)}_{\mathcal{L},\mathcal{L}'}(x^{(1)}_{\mathcal{L}''})}\right\}^{-k_i k_{\mathcal{L}''}}$$

$$\times \prod_{\substack{\mathcal{L},\mathcal{L}',\mathcal{L}'',\mathcal{L}'''\\(\mathcal{L}\neq\mathcal{L}',\mathcal{L}'''\neq\mathcal{L}'')}} \prod_{n=0}^{\infty} \left\{\frac{x^{(1)}_{\mathcal{L}} - [R^{\pm}]^{(n)}_{\mathcal{L}'\mathcal{L}''}(x^{(1)}_{\mathcal{L}'''})}{x^{(1)}_{\mathcal{L}} - [R^{\pm}]^{(n)}_{\mathcal{L}'\mathcal{L}''}(x^{(2)}_{\mathcal{L}'''})}\right\}^{-\frac{1}{2}k_{\mathcal{L}}\cdot k_{\mathcal{L}'''}}$$

$$\times \prod_{\substack{\mathcal{L},\mathcal{L}',\mathcal{L}'',\mathcal{L}'''\\(\mathcal{L}\neq\mathcal{L}',\mathcal{L}'''\neq\mathcal{L}'')}} \prod_{n=0}^{\infty} \left\{\frac{x^{(2)}_{\mathcal{L}} - [R^{\pm}]^{(n)}_{\mathcal{L}',\mathcal{L}''}(x^{(2)}_{\mathcal{L}'''})}{x^{(2)}_{\mathcal{L}} - [R^{\pm}]^{(n)}_{\mathcal{L}'\mathcal{L}''}(x^{(1)}_{\mathcal{L}'''})}\right\}^{-\frac{1}{2}k_{\mathcal{L}}\cdot k_{\mathcal{L}'''}} ,$$

$$U_2 = [x^{(2)} \le x_7^{(1)} \le x_7^{(2)} \le x_6^{(1)} \le x_5^{(1)} \le x_6^{(2)} \le x_5^{(2)} \le x_4^{(1)}$$

$$\le y_S \le y_{S-1} \le \cdots \le y_{m+2} \le y_{m+1} \le R_4(y_S)$$

$$\le x_4^{(2)} \le x_3^{(1)} \le y_m \le y_{m-1} \le \cdots \le y_{\ell+2} \le y_{\ell+1}$$

$$\le R_3(y_m) \le x_3^{(2)} \le x_2^{(1)} \le x_2^{(2)} \le x_1^{(1)} \le x_1^{(2)}$$

$$\le x^{(1)} \le y_\ell \le y_{\ell-1} \le \cdots \le y_2 \le y_1 \le \hat{R}^{-1}(y_\ell) \le x^{(2)}]$$

and $x_4^{(1)} \le \bar{y}_4 \le y_S \le R_4(\bar{y}_4) \le x_4^{(2)}$

and $x_3^{(1)} \le \bar{y}_3 \le y_m \le R_3(\bar{y}_3) \le x_3^{(2)}$

and $x^{(1)} \le \bar{y}_{\hat{R}} \le y_\ell \le \hat{R}^{-1}(y_{\hat{R}}) \le x^{(2)}$,

where $x_i^{(1)} \equiv R_i^{-\infty}(z_1), \qquad z_1 \ne x_i^{(2)}$,

$$x_i^{(2)} \equiv R_i^{\infty}(z_2), \qquad z_2 \ne x_i^{(1)},$$

$$x^{(1)} = \hat{R}^{\infty}(z_3), \qquad z_3 \ne x^{(2)},$$

$$x^{(2)} = \hat{R}^{-\infty}(z_4), \qquad z_4 \ne x^{(1)}.$$

Q = set of all **closed paths** = set of all group elements generated by products of $R_i^{\pm}$ such that cyclic permutations and inverses of such permutations of previously included members are excluded.

$[R^{\pm}]^{(n)}_{\mathcal{L},\mathcal{L}'}$ = set of all open paths = set of all group elements generated by products of $R_i^{\pm}$, such that first ~~(last)~~ element is $R^{\pm}_{\mathcal{L}}$, last is $R^{\pm}_{\mathcal{L}'}$, and n represents the number of R's in the product.

U_1 is determined implicitly from the conditions on U_2.

In passing, we note that the renormalization program[7] is complicated by the fact that the invariant points can overlap in a general multiloop amplitude. The results for the divergence of the N-loop planar amplitude[8,9] do not apply in this case (or in the case where we have loxodromic transformations).

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FOOTNOTES AND REFERENCES

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6. Going from (2.1) to (2.2) is complicated by the fact that the momentum-dependent factors contribute α_0 factors if we let $y \to R(y)$. Equation (2.2) has been rigorously verified for those factors raised to the minus one power. The "proof" of (2.2) for the α_0 terms is indirect: suppose we had sewn two different multiloop diagrams, one with m external legs between two loops and one with only $m - 1$ such external legs $(m > 1)$. Since we know the form of both amplitudes, the substitution $y \to R(y)$ (which should remove one of the m legs) does not contribute any $\frac{\alpha_0 - 1}{2}$ factors to the $m - 1$ case. But the momentum-dependent factors are insensitive to whether we have m legs or one. The $\frac{\alpha_0 - 1}{2}$ factors, which arise only when <u>all</u> legs are removed from between two loops, then do not receive any contribution from the momentum-dependent factors.

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FIGURE CAPTIONS

Fig. 1. The general nonplanar double loop amplitude.

Fig. 2. The ordering of the variables in a general nonplanar multiloop amplitude (notice that variables located between different loops have been moved via duality).

Fig. 3. The general overlapping double loop amplitude.

Fig. 4. The ordering of the variables in the overlapping double loop amplitude (notice that variables located within the loops have pushed out via duality).

Fig. 5. An illustrative example: a multiloop amplitude containing two planar, two nonplanar, one overlapping, and one nonorientable loops.

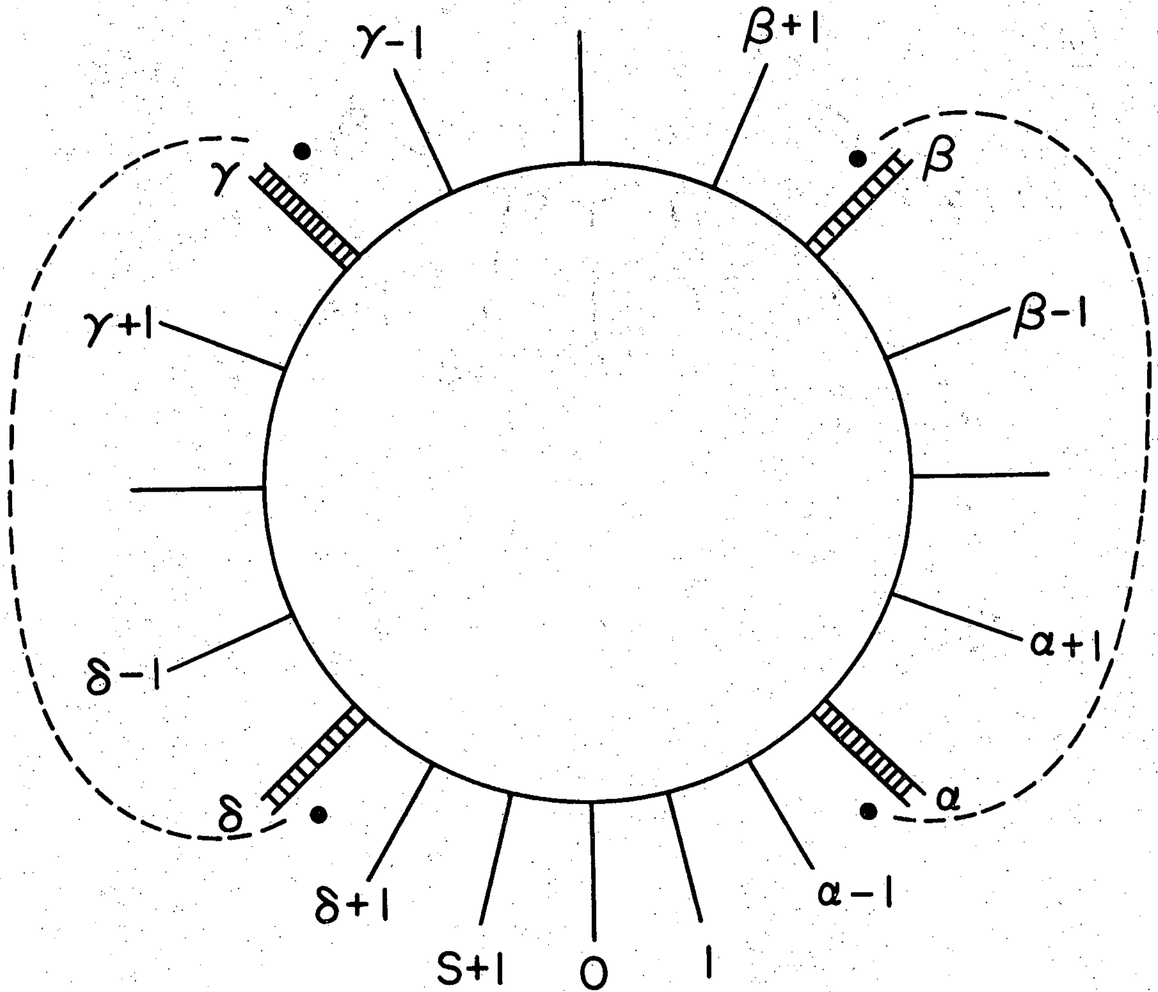
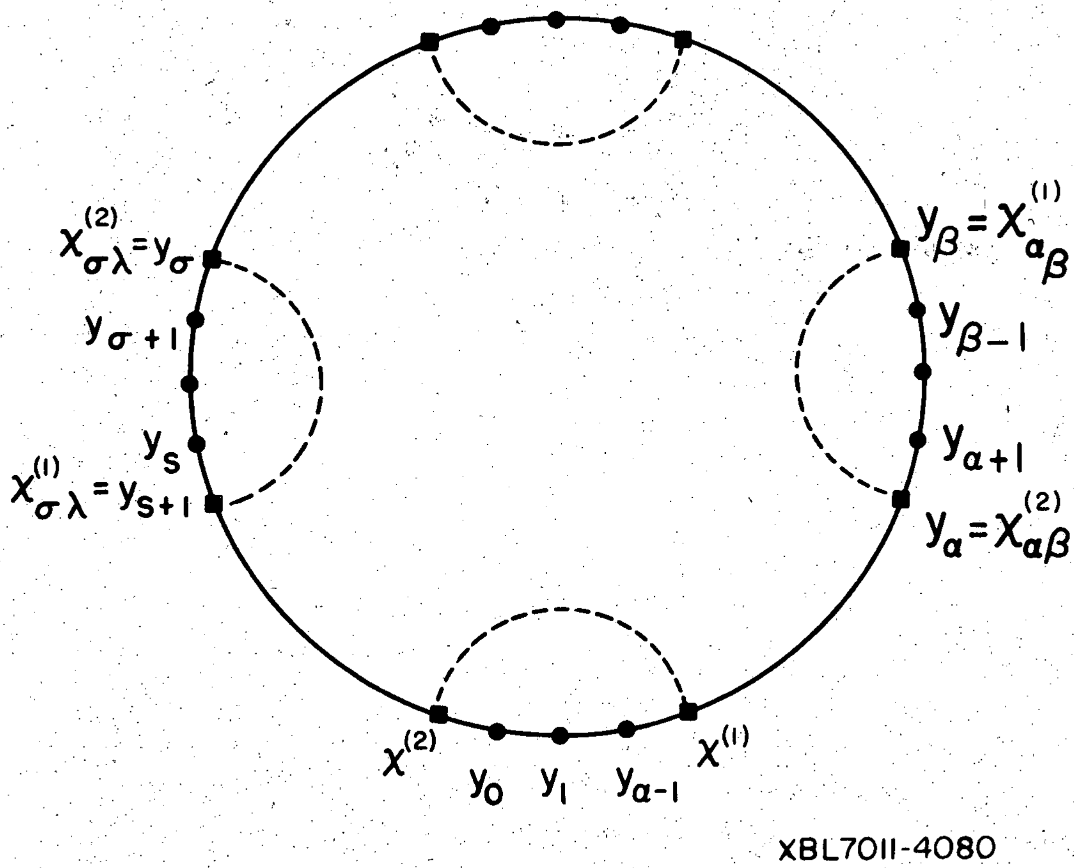
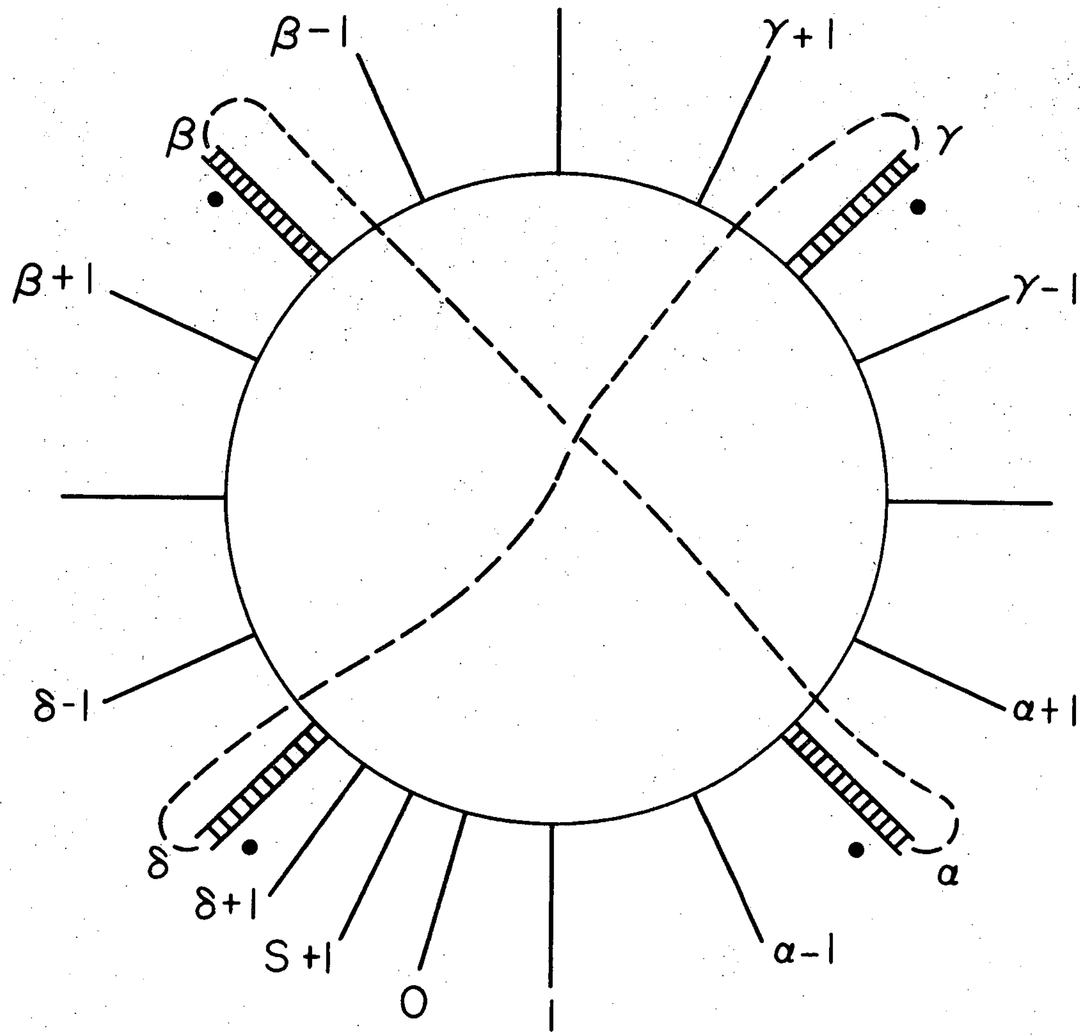


Fig. 1.



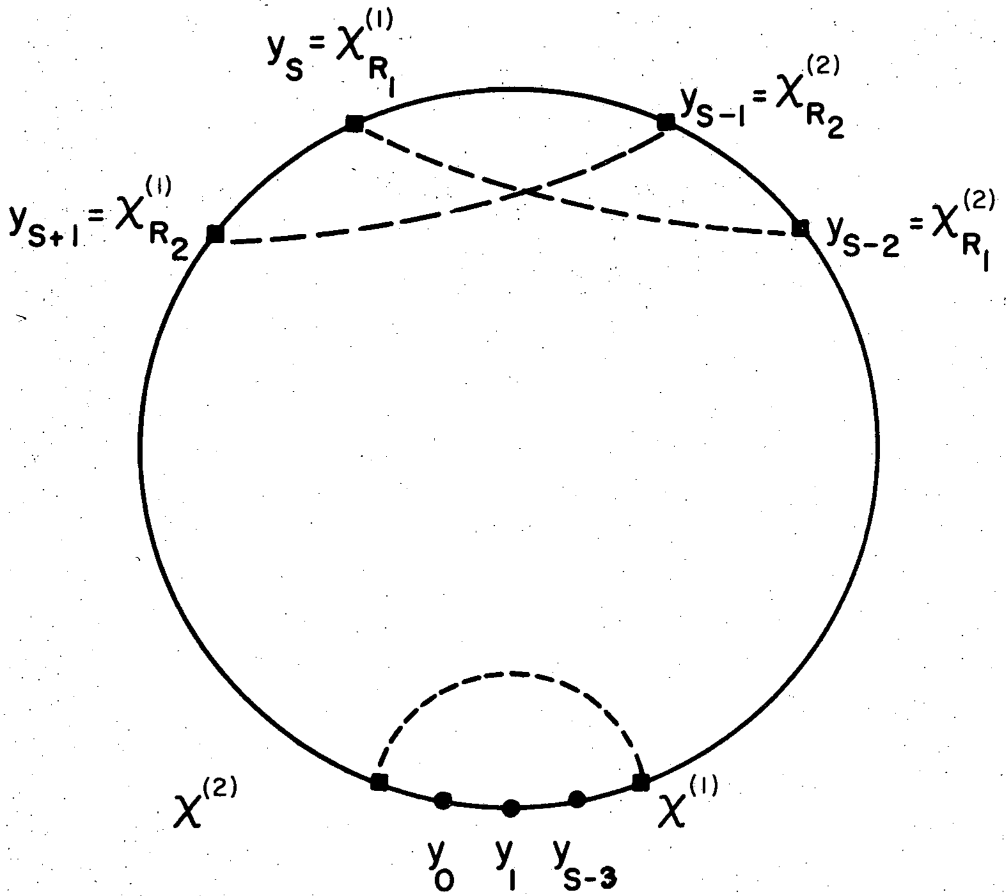
$\chi^{(2)}_{\sigma\lambda} = y_\sigma$
$y_{\sigma+1}$
y_s
$\chi^{(1)}_{\sigma\lambda} = y_{s+1}$
$y_\beta = \chi^{(1)}_{\alpha\beta}$
$y_{\beta-1}$
$y_{\alpha+1}$
$y_\alpha = \chi^{(2)}_{\alpha\beta}$
$\chi^{(2)}$
y_0
y_1
$y_{\alpha-1}$
$\chi^{(1)}$
XBL7011-4080

Fig. 2.



XBL7011-4082

Fig. 3.



$y_S = \chi_{R_1}^{(1)}$
$y_{S-1} = \chi_{R_2}^{(2)}$
$y_{S+1} = \chi_{R_2}^{(1)}$
$y_{S-2} = \chi_{R_1}^{(2)}$
$\chi^{(2)}$
$\chi^{(1)}$
y_0
y_1
y_{S-3}
XBL7011-4081

Fig. 4.

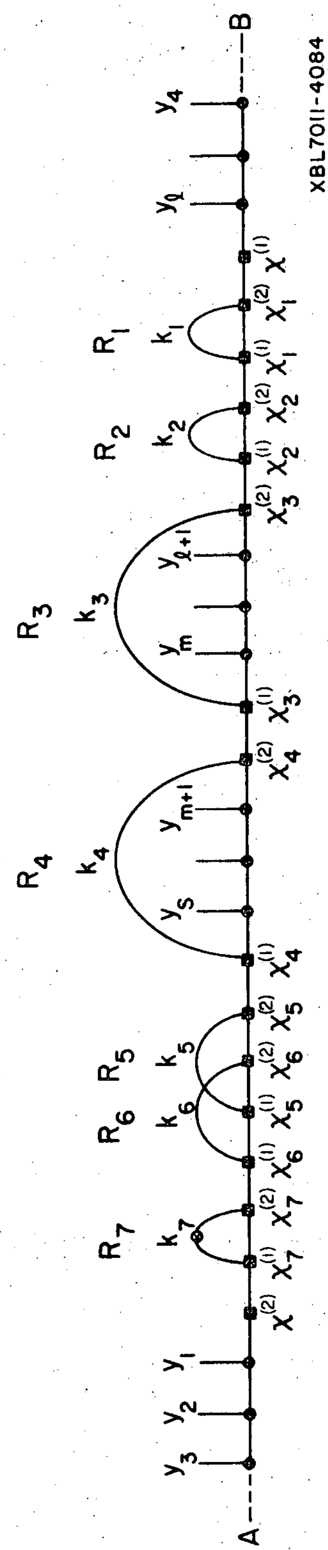

A
B
y_3 y_2 y_1
$X^{(2)}$
R_7 k_7 $X_7^{(1)}$ $X_7^{(2)}$
R_6 k_6 $X_6^{(1)}$ $X_6^{(2)}$
R_5 k_5 $X_5^{(1)}$ $X_5^{(2)}$
R_4 k_4 $X_4^{(1)}$ $X_4^{(2)}$ y_s y_{m+1}
R_3 k_3 $X_3^{(1)}$ $X_3^{(2)}$ y_m $y_{\ell+1}$
R_2 k_2 $X_2^{(1)}$ $X_2^{(2)}$
R_1 k_1 $X_1^{(1)}$ $X_1^{(2)}$
$X^{(1)}$
y_ℓ y_4
XBL7011-4084

Fig. 5.

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