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Direction Of Arrival (DOA) Estimation Using Array Signal Processing

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by

Ashmika Dhabale

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Thesis Committee:

Dr. Yingbo Hua, Chairperson
Dr. Sheldon Tan
Dr. Nael Abu-Ghazaleh

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The Thesis of Ashmika Dhabale is approved:

Committee Chairperson

University of California, Riverside

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ABSTRACT OF THE THESIS

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by

Ashmika Dhabale

Master of Science, Graduate Program in Electrical Engineering

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Dr. Yingbo Hua, Chairperson

Array signal processing is an important field in Signal Processing. It has various applications in the field of communication, radar, sonar as well as speech processing. Array signal processing plays a crucial role when it comes to direction of arrival(DOA) estimation using spectral analysis. The objective is to discuss the methods used to estimate the signal arriving from a desired direction in the presence of noise and interfering signals. The sensor array collects spatial samples of propagating wave fields, which are processed using different spectral analysis algorithms. We discuss the array models on which direction finding algorithms are based. We review the non-parametric methods and subspace-based algorithms for DOA estimation. Non-parametric algorithms includes Conventional beamforming and Capon's method whereas under subspace-based algorithm we discuss the MUSIC method. We review the performance and limitations of these methods. A user interface in Labview is used for analyzing the performance. This thesis provides an overview of Conventional beamforming method, Capon's method and the MUSIC method for DOA estimation.

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Chapter 1

Introduction

Array Signal Processing deals with processing signals received by the antenna array, strengthening the message signals, mitigating the noise signals and estimating some of the signal parameters. Sensor Array Signal Processing have been an important part in analyzing received data at the sensor end [2]. Sensor array can control the beam flexibility with a high signal gain and strong ability for handling interference as compared to traditional signal sensors. Sensor array hence gives better resolution and performance in signal reception and parameter estimation. Thus one of the important use of Array Signal Processing is in Direction of Arrival (DOA) estimation and it has a variety of applications in wireless communication, sonar, tracking of objects, radio direction finding, etc.

We discuss the problem of locating the signal sources by using array sensors in this thesis. This problem deals with analyzing the energy distribution with source position representing high concentration of energy in space. We discuss the model for the output signal of the receiving sensor array. The source location problem then turns into a parameter estima-

tion problem with such a model. Under certain assumptions which are discussed later in this chapter, the parameter that characterizes source locations is direction of arrival(DOA). Direction of Arrival(DOA) estimation thus determines the angle of arrival of a spatially propagating signal at the receiver array.

For Direction of Arrival estimation the following assumptions are made in the thesis[11].

1. Point source- The signal source is taken as a point source. The direction from the antenna array can thus be estimated uniquely.
2. Narrowband signal- This is to ensure that all the array elements in the receiver can capture the signal at the same time
3. Array assumptions- The sources are assumed to be situated in the far field of the array. The sources and the sensors in array are assumed to be in the same plane. Also, the propagation medium is assumed to be homogeneous so that the waves arriving at the array can be considered planar.

Under these assumptions, the source location can be characterized by just using direction of arrival (DOA) parameter. Fig. 1.1 describes the source setup following the above mentioned assumptions.

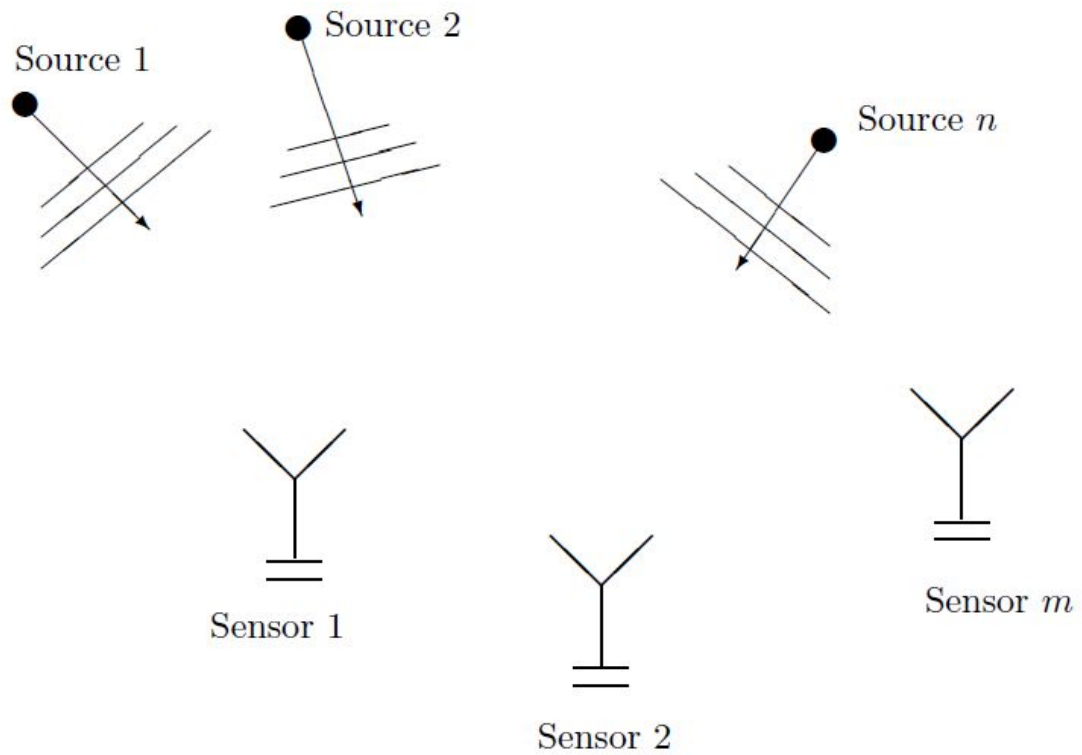


Figure 1.1: The setup of source location problem

There are three ways of finding the Direction of Arrival (DOA) [5]:

1. Spectral-based algorithm
 - Conventional beamformer
 - Capon's beamformer
2. Subspace- based algorithms
 - Multiple signal classification (MUSIC)
 - Extension to MUSIC
3. Parametric algorithms

- Deterministic Maximum Likelihood Method
- Stochastic Maximum Likelihood Method

We have discussed the non-parametric based and subspace based algorithms in this thesis. Spatial spectral estimation achieves space signal parameter by using arrays. In spatial filtering the data from the array sensors is combined linearly in order to foster the main message component and attenuate the interference as well as the signals coming from different directions. There are three parts in the spatial spectrum: the incident signal space, array receiver and then the parameter estimation processor. We can name the three stages as shown in fig. 1.2 as target stage, observation stage and the estimation stage [4].

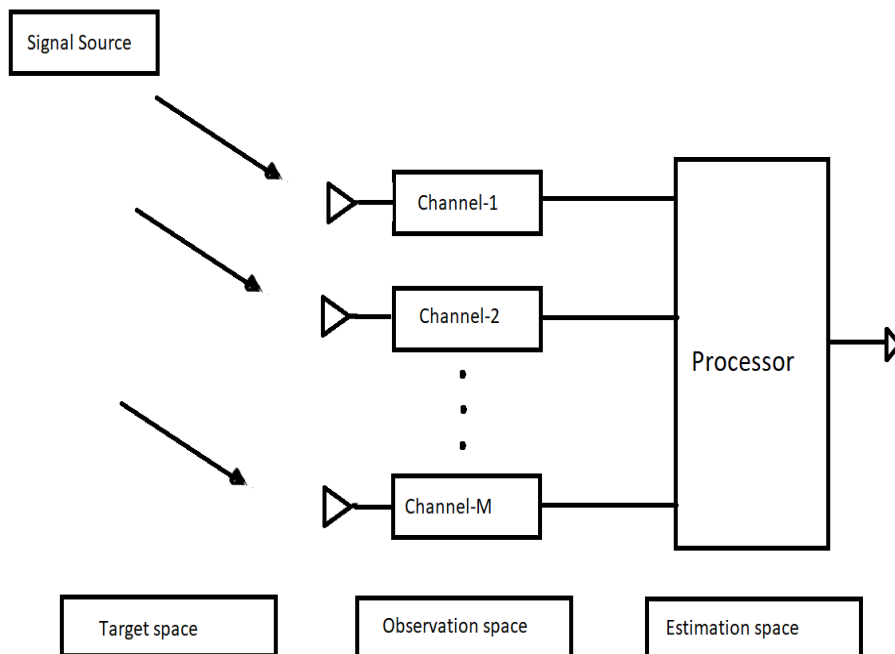


Figure 1.2: System structure of DOA estimation

Target stage comprises of signal source parameters which are estimated using certain methods in spatial spectrum estimation system. The observation stage then receives the data which may contain signal characteristics, some space characteristics and also some features of space array element. The signal characteristics could be azimuth, distance, polarization, etc. The space characteristics involves noise, miscellaneous signals and interference. The influence of spatial array elements can give the data related to mutual coupling, channel inconsistency, frequency band inconsistency, etc. In the estimation stage spatial spectrum estimation techniques are used to extract the parameters of the signal. Spatial spectrum basically deals with the energy distribution of signals in all spatial directions.

Subspace-based algorithms on the other hand use observations from eigen decomposition of the co-variance matrix into signal and noise subspace[3].

DOA estimation result depends on the signal source as well as the propagating medium. There are some factors which affect the estimation results like number of array elements, number of snapshots, SNR and coherence of the signal source. Generally when array parameters are the same better resolution can be obtained by more array elements. Snapshots are the number of samples in time domain which aids in DOA estimation. SNR and coherent signal sources can also affect DOA estimation directly.

Chapter 2

Array model

2.1 Uniform Linear Array

DOA estimation algorithms discussed in this thesis are simulated using uniform linear array structure assumption. Consider the array of uniformly spaced m identical sensors on a line as shown in figure 2.1. This is uniform linear array (ULA)[11].

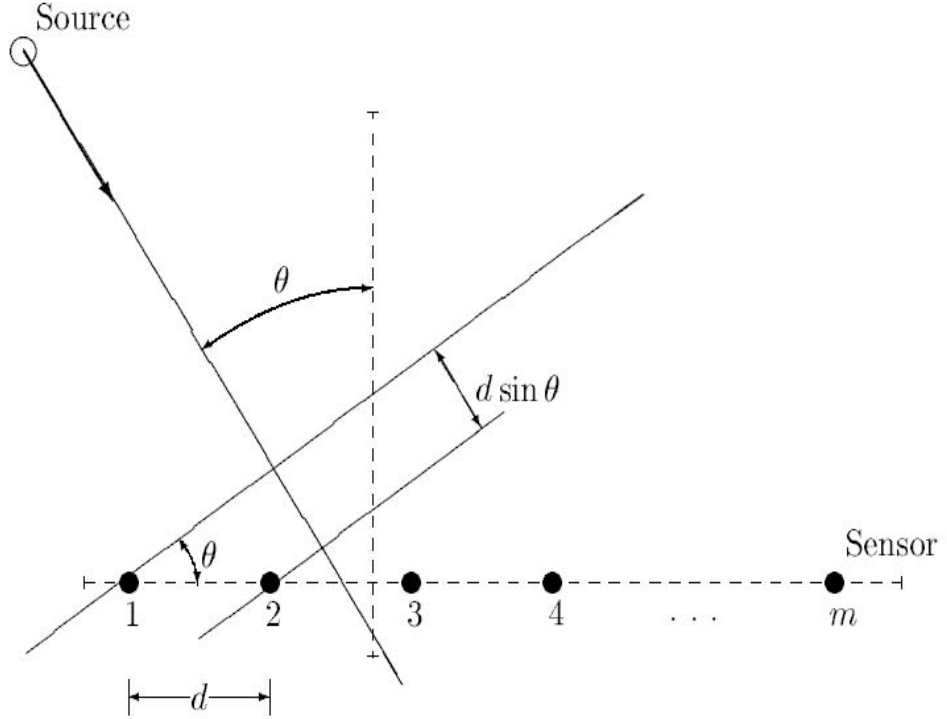


Figure 2.1: Uniform Linear Array

Let d be the distance between two array elements and θ be the direction of arrival of the signal measured anticlockwise to the normal to the line of sensors. Using the first sensor in the array as reference and planar wave hypothesis we can write

$$\tau_k = (k - 1) \frac{d \sin \theta}{c} \quad \text{for } \theta \in [-90^\circ, 90^\circ] \quad (2.1)$$

where c is the propagation velocity of the incoming signal. Also from [11] we know that array transfer vector or direction vector is:

$$a(\theta) = [1 \quad e^{-i\omega_c \tau_2} \quad \dots \quad e^{-i\omega_c \tau_m}]^T \quad (2.2)$$

Using equations 2.1 and 2.2 we get:

$$a(\theta) = [1, e^{-i\omega_c d \sin\theta/c}, \dots, e^{-i(m-1)\omega_c d \sin\theta/c}]^T \quad (2.3)$$

The limitation of ULA is that θ is restricted to $[-90^\circ, 90^\circ]$. If sources are located symmetric with respect to array line it will give identical delay which cannot be distinguished from one another. In order to remove this ambiguity sensors are used which pass signals whose DOA are in the range of $[-90^\circ, 90^\circ]$. Let λ denote wavelength of the signal which is distance travelled by the carrier in one period of time

$$\lambda = c/f_c, \quad f_c = \omega_c/2\pi \quad (2.4)$$

Define:

$$f_s = f_c \frac{d \sin\theta}{c} = \frac{d \sin\theta}{\lambda} \quad (2.5)$$

and

$$\omega_s = 2\pi f_s = \omega_c \frac{d \sin\theta}{c} \quad (2.6)$$

Now we can rewrite the equation as:

$$a(\theta) = [1e^{-i\omega_s} \dots e^{-i(m-1)\omega_s}]^T \quad (2.7)$$

This is called a Vandermonde vector which is completely analogous to the vector made from the uniform samples of the sinusoidal signals $\{e^{-i\omega_s t}\}$. So we can see that ω_s is the spatial frequency. So if we sample the sinusoidal signal with frequency ω_c we need to choose sampling frequency f_o to avoid aliasing effects. By Shannon sampling theorem:

$$f_o \geq 2f_c \quad (2.8)$$

so

$$T_o \leq \frac{T_c}{2} \tag{2.9}$$

where T_o is the sampling period and T_c is the period of the continuous sinusoidal signal.

Now for the case of uniform linear array vector $a(\theta)$ is uniquely defined that is without any spatial aliasing if and only if ω_s has the following constraints:

$$|\omega_s| \leq \pi \tag{2.10}$$

Equation is equivalent to

$$|f_s| \leq \frac{1}{2} \iff d|\sin\theta| \leq \frac{\lambda}{2} \tag{2.11}$$

We can see that the condition on d depends upon θ but we have no knowledge about the direction of arrival of the source signal. We know that the worst delay occurs for $\theta = 0^\circ$. So for the equation 2.11 to hold for any θ we can write the condition as:

$$\boxed{d \leq \frac{\lambda}{2}} \tag{2.12}$$

Equation 2.12 basically says that the distance d should be smaller than half of the signal wavelength in ULA.

2.2 Narrowband sensor array model

Approaches to direction finding problem using array based models work with different properties as compared to energy detectors. We are more concerned with the relative amplitudes and sensor phases which are a function of the direction of arrival of the propagating signal. The behaviour of the direction finding techniques is dependent on the

narrow-band model described in this chapter. The narrow band model illustrates the data received at the sensor outputs as a function of direction of arrival, signal waveform and characteristics of the sensors. Along with this the direction finding methods are dependent on the structure of the sensor array. The geometry of the array affects vector $\mathbf{x}(n)$ of output signals $x_1(n), \dots, x_M(n)$ from individual sensors as seen in figure 2.2 [2].

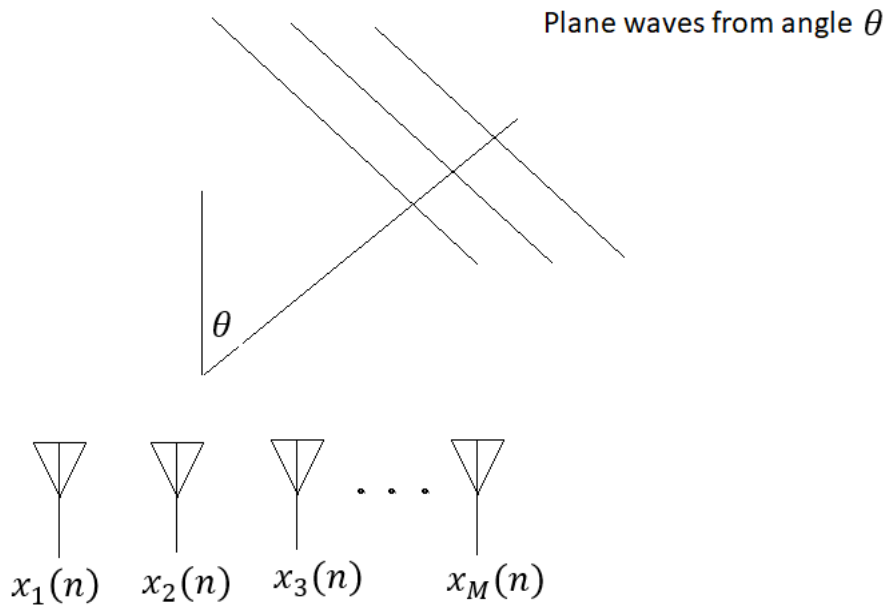


Figure 2.2: Example of a sensor array receiving a plane wave arriving from angle θ

In this chapter a simple case of array receiving sinusoidal signal is considered. Lets consider an analytic signal $exp(j(\omega t + \phi))$ corresponding to a sine wave with phase ϕ , frequency ω and angle of arrival θ with respect to the array. For the sake of simplicity we assume the array sensor and the signal source to be co-planar so that the direction of arrival of the signals can be described with a single angle. The signal received at the two ends of the array only differs by a delay if the propagating medium does not affect it significantly. The dependence of the delay on the stricture of the array sensors and on the angle of arrival

can be determined.

Figure 2.2 is referred for understanding this concept. Let us take the array to constitute of M sensors with co-ordinates $(q_1, r_1), \dots, (q_M, r_M)$. The delay t_m of the signal at m -th sensor relative to the signal at origin can be expressed as $t_m = -[q_m \sin(\theta) + r_m \cos(\theta)]/c$ where θ is measure clockwise from the r axis and c is the speed of light. As we have taken the signal to be sinusoidal the propagation delay is equivalent to a phase shift of $\psi_m = -\omega t_m$, which can be seen as multiplication by $\exp(j\psi_m)$. The signal received can thus be written in vector form as:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} e^{j\psi_1} \\ e^{j\psi_2} \\ \vdots \\ e^{j\psi_M} \end{bmatrix} e^{j(\omega t + \phi)} \quad (2.13)$$

where

$$\psi_m(\omega, \theta) = [q_m \sin(\theta) + r_m \cos(\theta)]\omega/c \quad (2.14)$$

The output of the sensors would change if the sensors have differing directional and frequency dependent characteristics. This can be modelled by making changes to elements of vector in equation 2.13.

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} g_1(\omega, \theta) e^{j\phi_1(\omega, \theta)} e^{j\psi_1(\omega, \theta)} \\ g_2(\omega, \theta) e^{j\phi_2(\omega, \theta)} e^{j\psi_2(\omega, \theta)} \\ \vdots \\ g_M(\omega, \theta) e^{j\phi_M(\omega, \theta)} e^{j\psi_M(\omega, \theta)} \end{bmatrix} e^{j(\omega t + \phi)} = \mathbf{a}(\omega, \theta) e^{j(\omega t + \phi)} \quad (2.15)$$

where $g_m(\omega, \theta)$ and $\phi_m(\omega, \theta)$ denotes the gain and phase of the m -th sensor. This model can be extended for a more generalized case where multiple non-sinusoidal signals might

arrive at the array by decomposing it in frequency domain and using linear superposition:

$$\begin{aligned}
\mathbf{x}(\omega) &= \sum_{l=1}^L \mathbf{a}(w, \theta_l) s_l(\omega) + \mathbf{i}(\omega) \\
&= [\mathbf{a}(w, \theta_1) \cdots \mathbf{a}(w, \theta_L)] [s_1(\omega) \cdots s_L(\omega)]^T + \mathbf{i}(\omega) \\
&= \mathbf{A}(\omega, \Theta) s(\omega) + \mathbf{i}(\omega)
\end{aligned} \tag{2.16}$$

where L signals $s_1(\omega) \cdots s_L(\omega)$ arrive from angles $\theta_1, \cdots, \theta_L$ and $\mathbf{i}(\omega)$ represents the noise components. The array data is linear with respect to the signals and in frequency domain it is linear with respect to the vector $\mathbf{a}(\omega, \theta)$ (array response vector).

It is advantageous to reject signal components and noise that lie outside a narrow frequency band of interest. Prior knowledge of centre frequencies and bandwidths of the signals of interest should be available to know the narrow band of interest. The array response vector $\mathbf{a}(\omega, \theta)$ would be approximately constant with respect to ω over the band of interest over all angles θ if the band is sufficiently narrow. By this assumption we can model the array data in time domain by dropping the dependence on ω .

$$\begin{aligned}
\mathbf{x}(t) &= \sum_{l=1}^L \mathbf{a}(\theta_l) s_l(t) + \mathbf{i}(t) \\
&= [\mathbf{a}(\theta_1) \cdots \mathbf{a}(\theta_L)] [s_1(t) \cdots s_L(t)]^T + \mathbf{i}(t) \\
&= \mathbf{A}(\Theta) s(t) + \mathbf{i}(t)
\end{aligned} \tag{2.17}$$

where $s(t)$ and $\mathbf{i}(t)$ are analytic signal. The spatial characteristics of the array response can be modelled assuming that the signals $s_l(t)$ are not sinusoidal. In most of the algorithms sampled complex envelope of the array data is used. The complex envelope of a bandlimited analytic signal can be procured by doing a complex down-conversion which can be written same as in equation 2.17. The only difference is that $\mathbf{x}(t)$, $s(t)$ and $\mathbf{i}(t)$ denote the complex

envelopes of the array data, the signal and the noise respectively, and t is replaced by n :

$$\mathbf{x}(n) = \sum_{l=1}^L \mathbf{a}(\theta_l) s_l(n) + \mathbf{i}(n) = \mathbf{A}(\Theta) s(n) + \mathbf{i}(n) \quad (2.18)$$

The array response vector is estimated using calibration data but also can be known analytically in case of uniform linear array with identical sensors. In general in the application of direction finding methods a search over the array response vector would be needed. The array response vector should also be non-ambiguous as the direction finding methods need to distinguish between different signals. There can be two types of cases-

1. The array response vector due to signal coming from two different direction is same and the direction finding methods are not able to distinguish them.
2. In case of ambiguous array response even one signal can be incorrectly estimated to have come from an angle different from the true direction of arrival angle.

This is rank 1 type of ambiguity. If there exist angles $\theta_1 \cdots, \theta_K$ such that corresponding array vectors are linearly dependent only then a rank $K-1$ ambiguity will exist. The study of rank ambiguity issues in direction of arrival estimation can be seen in [12]. In order to detect the angle of arrival of signals with precision lack of such ambiguities are desirable. In case of uniform linear array which is generally used, the spacing between the two elements should be less than half of the wavelength of the highest frequency in the receiver band. This requirement can be interpreted as a spatial analogue of the Nyquist sampling criteria. The direction finding methods are discussed in the following chapter based on this narrow-band model.

Chapter 3

Conventional beamforming algorithm

3.1 Introduction

Beamforming is the process of signal of interest that come from only one particular direction and impinges different sensors at the receiver. The resultant signal gives a higher strength because of coherent combining. Coherent combining is done after proper phase compensation at each sensor. Thus, the resultant gain of the sensor would look like a large dumbbell shaped lobe aimed in the direction of interest [5] This important concept is used in different communication, voice and sonar applications.

3.2 Mathematical model for conventional beamforming

Conventional beamforming method for finding DOA uses the basics of spatial filtering. A detailed study of beamforming method can be seen in [13]. An array of sensors are used here as receivers as they can be steered electronically instead of the mechanical complications in a fixed antenna. Weighing factor operation is done to linearly combine the sensor output and get a single output signal $y(n)$. The basic model can be found in [2].

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \quad (3.1)$$

where superscript H denotes the Hermitian operation and the response of this spatial filter can be described by its effective antenna pattern $P(\theta)$

$$P(\theta) = |\mathbf{w}^H \mathbf{a}(\theta)|^2 \quad (3.2)$$

$P(\theta)$ is the average power of the response from spatial filter when unity power signal arrives from angle θ . The gains and phases in \mathbf{w} are taken such that proper beams and nulls could be created. The effective array pattern are quite different from the individual components as it could be seen that the gain of individual sensors would be just a horizontal line when plotted as a function of DOA of the signal. The total average power in the output can thus be expressed as-

$$\begin{aligned} P_{total} &= \left\langle |y(n)|^2 \right\rangle_N \\ &= \mathbf{w}^H \langle \mathbf{x}(n) \mathbf{x}^H(n) \rangle_N \mathbf{w} \\ &= \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \end{aligned} \quad (3.3)$$

where $\langle \cdot \rangle_N$ is the time averaging over N time samples and \mathbf{R}_{xx} is the spatial auto-correlation matrix of the data in equation (3.3). This matrix contains information about the array

response vectors.

$$\begin{aligned}
\mathbf{R}_{xx} &= \langle \mathbf{x}(n)\mathbf{x}^H(n) \rangle_N \\
&= \langle (\mathbf{A}(\Theta)\mathbf{s}(n) + \mathbf{i}(n))(\mathbf{A}(\Theta)\mathbf{s}(n) + \mathbf{i}(n))^H \rangle_N \\
&\rightarrow \mathbf{A}(\Theta)\mathbf{R}_{xx}\mathbf{A}^H(\Theta) + \mathbf{R}_{ii} \quad \text{for } N \rightarrow \infty
\end{aligned} \tag{3.4}$$

In beamforming angular sections of the region are scanned using the basic narrow band -signal model. The weight \mathbf{w}_b co-ordinates the phases on the incoming signal say coming from θ_0 . When vector $\mathbf{w}_b = \mathbf{a}(\theta_0)$ the antenna pattern will have the maximum gain from the direction θ_0 . This happens because the weighing factor will add things constructively and will give the required sum of amplitude. This comes from Cauchy-Schwarz inequality:

$$|\mathbf{w}^H \mathbf{a}(\theta_0)|^2 \leq \|\mathbf{w}\|^2 \|\mathbf{a}(\theta_0)\|^2 \tag{3.5}$$

The equality holds true if and only if \mathbf{w} is proportional to $\mathbf{a}(\theta_0)$ for all vectors \mathbf{w} . So the effective pattern 3.2 will have a global maxima at θ_0 in absence of a rank-1 ambiguity. The direction of arrival angles are decided based on the local maxima of the received power. So basically the beam monitors an angular region for each θ the average power $P_b(\theta)$ of the steered array is measured.

$$\begin{aligned}
P_b(\theta) &= \langle |y_b(n)|^2 \rangle_N \\
&= \langle |\mathbf{w}_b^H \mathbf{x}(n)|^2 \rangle_N \\
&= \mathbf{w}_b^H \mathbf{R}_{xx} \mathbf{w}_b \\
&= \mathbf{a}^H(\theta) \mathbf{R}_{xx} \mathbf{a}(\theta)
\end{aligned} \tag{3.6}$$

\mathbf{R}_{xx} would be same for the data so this would reduce the computations for different regions under consideration. This helps to reduce the blind spots for detecting the transient signals.

3.3 Simulation results

Simulation result of some cases have been shown below. Fig 3.1 shows the simulation result for signals arriving from 50 and 80 degrees. The other details include carrier frequency to be around 0.3 GHz, total number of antennas are 10, the number of snapshots are taken around 1000 and the spacing of the elements in the array is kept as half the value of the wavelength. Also the SNR or signal to noise ratio is considered to be around 5.

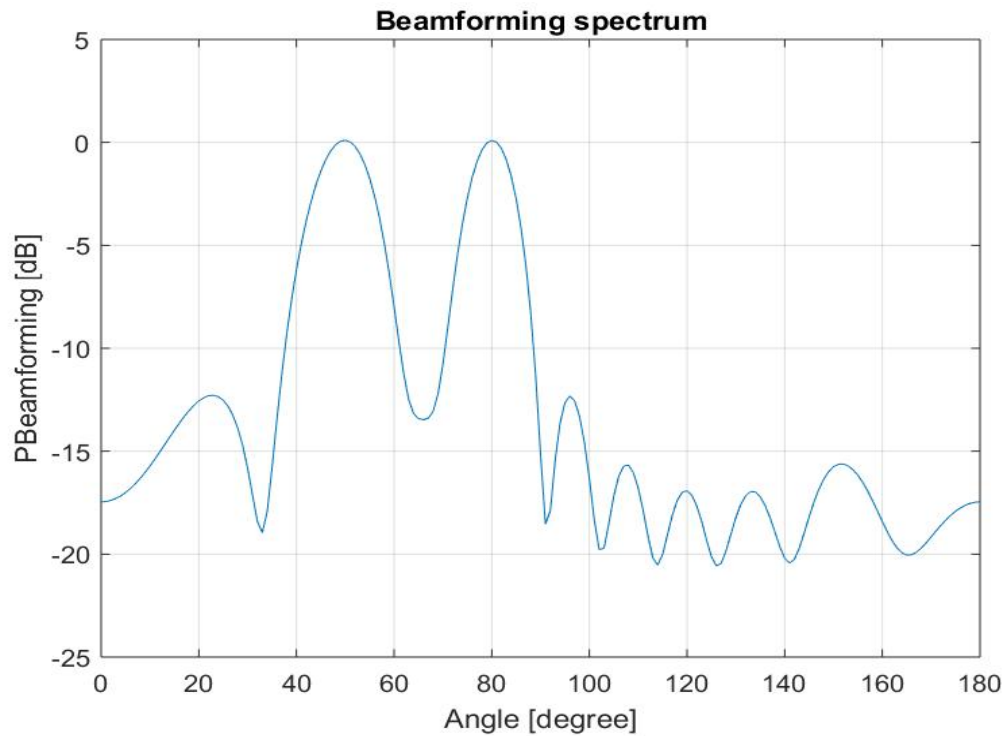


Figure 3.1: Average output power obtained by beamforming method for two signals arriving from 50 and 80 degrees

Detection of multiple signals are still a concern as a strong unwanted signal can give a wrong estimation result. Detection of closely spaced sources wont be that effective as well. The trade off in this to decrease the beam width for more accuracy which can be

done by increasing the number of sensor arrays. But this will again challenge the physical limitations of the receiver and would also require high computation of data. Fig 3.2 shows the simulation result for signals arriving from 70 and 80 degrees. The other details remain same that includes carrier frequency to be around 0.3 GHz, total number of antennas are 10, the number of snapshots are taken around 1000 and the spacing of the elements in the array is kept as half the value of the wavelength. Also, the SNR is around 5.

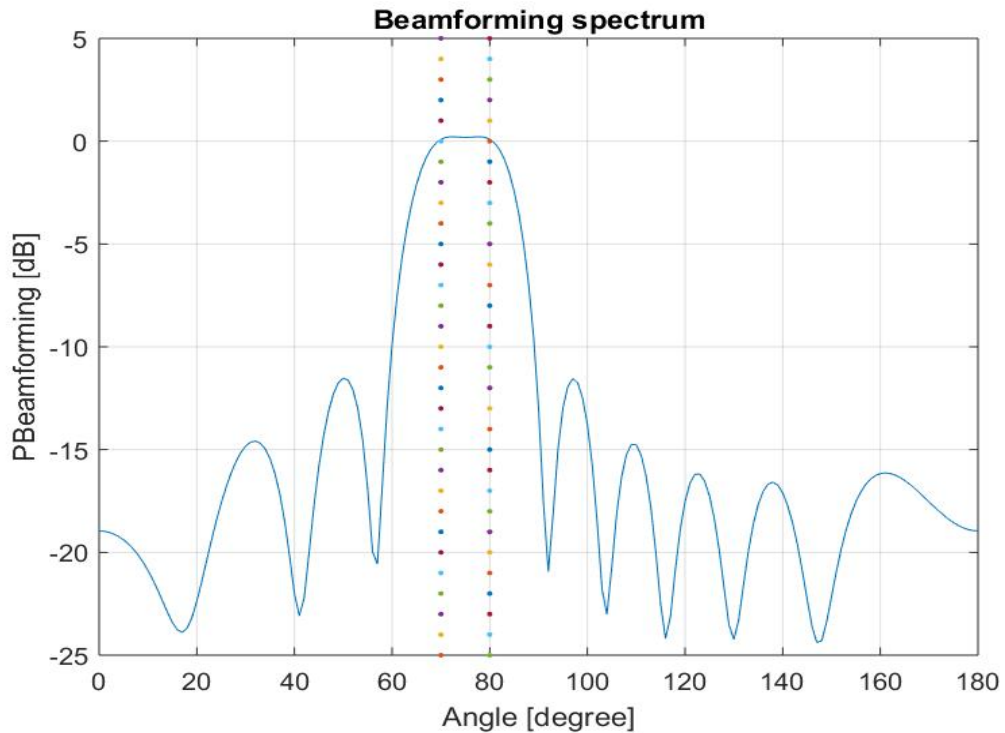


Figure 3.2: Average output power obtained by beamforming method for two closely spaced signal sources arriving from 70 and 80 degrees

It can be seen that no clear peak is obtained in the case of closely located signal sources. This creates a problem of missing detection of a signal as the two signals would be detected as one.

Chapter 4

Capon algorithm

4.1 Introduction to Capon's method

As we have seen that in beamforming the general inclination is to believe that the direction from where the power is arriving will give us the strongest beam. Degrees of freedom available are generally one less than the number of sensors. With the perception stated above all the degrees of freedom are used to bolster the beam strength in the look direction. The thought in this being that maximum power would be achieved when look direction coincides with the true DOA.

The problem arises when there are more than one signal. A meticulous way is to use some degrees of freedom to form a beam in the look direction. Now the remaining should be used to form nulls in other directions which would take care of other signals. If we minimize the output power and inhibit the beam in the look direction or maintain unity gain we can prevent the trivial solution $w=0$. This can also fulfill the need to form nulls in the directions from where other signals are coming. Thus Capon method uses degrees of freedom for two

tasks. One is to minimize the array processor output power. Other is to contain the gain as unity in the look direction.

$$\min_w \langle |y(n)|^2 \rangle_N \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta) = 1 \quad (4.1)$$

This method is also called as MVDR- Minimum variance distortionless response. This name can be justified as minimum variance is achieved by minimizing the average power and simultaneously maintaining unity gain in the look direction gives you a distortionless signal.

4.2 Mathematical model

Given a look direction the variance of the array output signal $y(n)$ is minimized and the signal from the look direction with no distortion that is unity gain and no phase shift is passed. The weighing vector $w_c(\theta)$ would be [2] :

$$\mathbf{w}_c(\theta) = (\mathbf{a}^H(\theta) \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta))^{-1} \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta) \quad (4.2)$$

Capon's method will search over the all thetas to find the direction for which the measured power received is maximum:

$$\begin{aligned} P_c(\theta) &= \mathbf{w}_c^H(\theta) \mathbf{R}_{xx} \mathbf{w}_c(\theta) \\ &= (\mathbf{a}^H(\theta) \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta))^{-1} \end{aligned} \quad (4.3)$$

Though we are not finding the maximum likelihood estimator of θ but this algorithm is still coined as Capon's maximum likelihood method. This is because $P_c(\theta)$ is the maximum likelihood estimate of the signal power of a signal which is arriving from angle θ in presence of temporal white Gaussian noise having arbitrary spatial characteristics. The power is

calculated using the estimated auto-correlation matrix. This is nothing but the point wise maximum likelihood estimate of the angular density of received power. We can see the performance improvement when compared to the conventional beamforming algorithm.

4.3 Simulation results

The same examples as we have seen with the conventional beamforming method have been duplicated here and the results are pretty evident. Figure 4.1 shows the simulation result for signals arriving from 50 and 80 degrees with carrier frequency 0.3 GHz, total number of antennas 10, SNR around 5, number of snapshots taken 1000 and the spacing of the elements in the array is kept as half the value of the wavelength. We get two sharp peaks at the desired angle of arrival in this case.

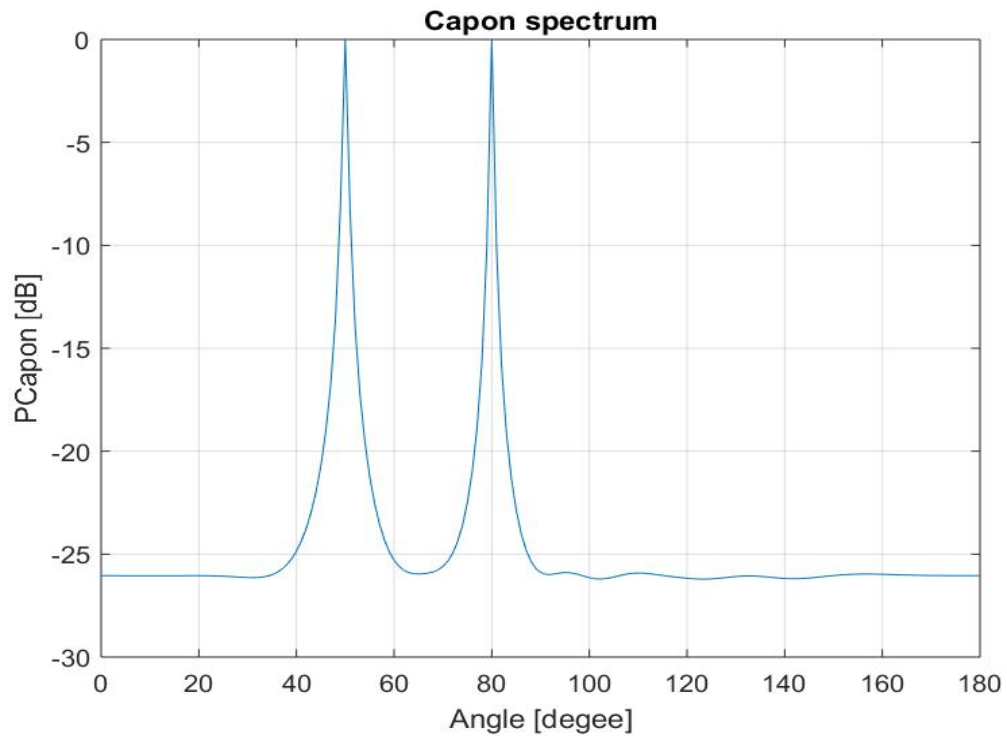


Figure 4.1: Average power output of Capon’s method for two signals arriving from 50 and 80 degrees

Fig 4.2 shows the simulation result for signals arriving from 70 and 80 degrees with carrier frequency 0.3 GHz, total number of antennas 10, SNR around 5, number of snapshots taken 1000 and the spacing of the elements in the array is kept as half the value of the wavelength. In figure 4.2 it can be seen that the detection of closely spaced signal sources as done earlier in figure 3.2 where the difference between the placement was of 10 degrees is now detected with distinct peaks owing to the better resolution of Capon’s beamformer method.

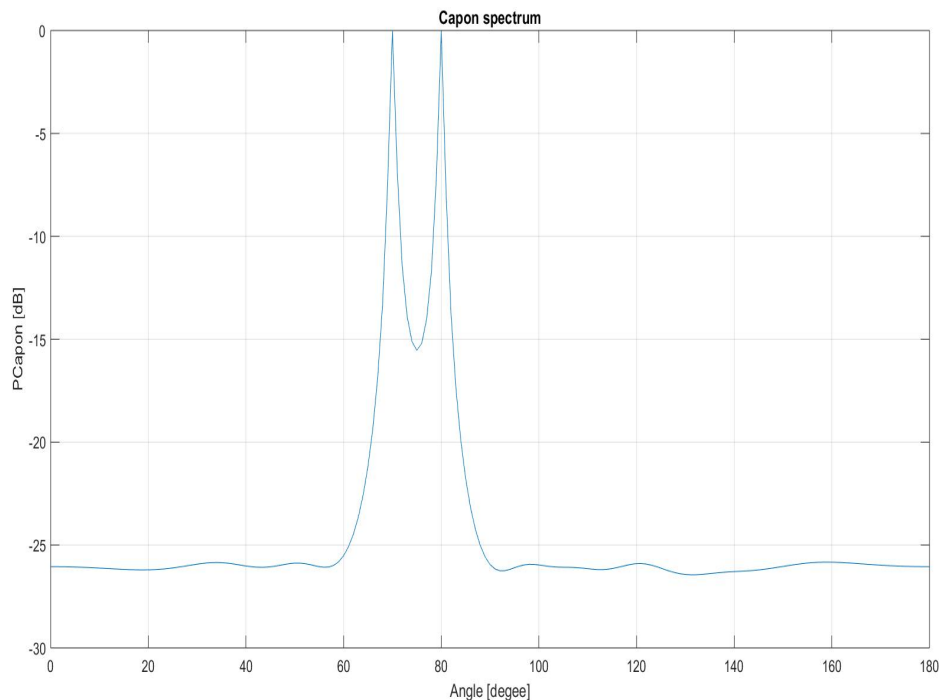


Figure 4.2: Average power output of Capon’s method for two signals arriving from 70 and 80 degrees

Capon’s method also succumbs once we start expecting more accuracy and go below the range of 10 degrees spacing of signal sources. Also, if other signals are correlated with the our propagating signal of interest it creates problems. This happens because in Capon’s method that correlation is unintentionally used to reduce the processor output power. The processor cancels the signal of interest without having to spatially null the signal. Hence though we have unity gain in the direction of the desired signal, we get a low output power in such cases and the method gives wrong results. Fig 4.3 shows the simulation result for signals arriving from 35 and 40 degrees with carrier frequency 0.3 GHz, total number of antennas 10, SNR around 5, number of snapshots taken 1000 and the

spacing of the elements in the array is kept as half the value of the wavelength. In fig 4.3 we can see that when the signal sources are less than 10 degrees apart it becomes difficult to get two distinguished peaks.

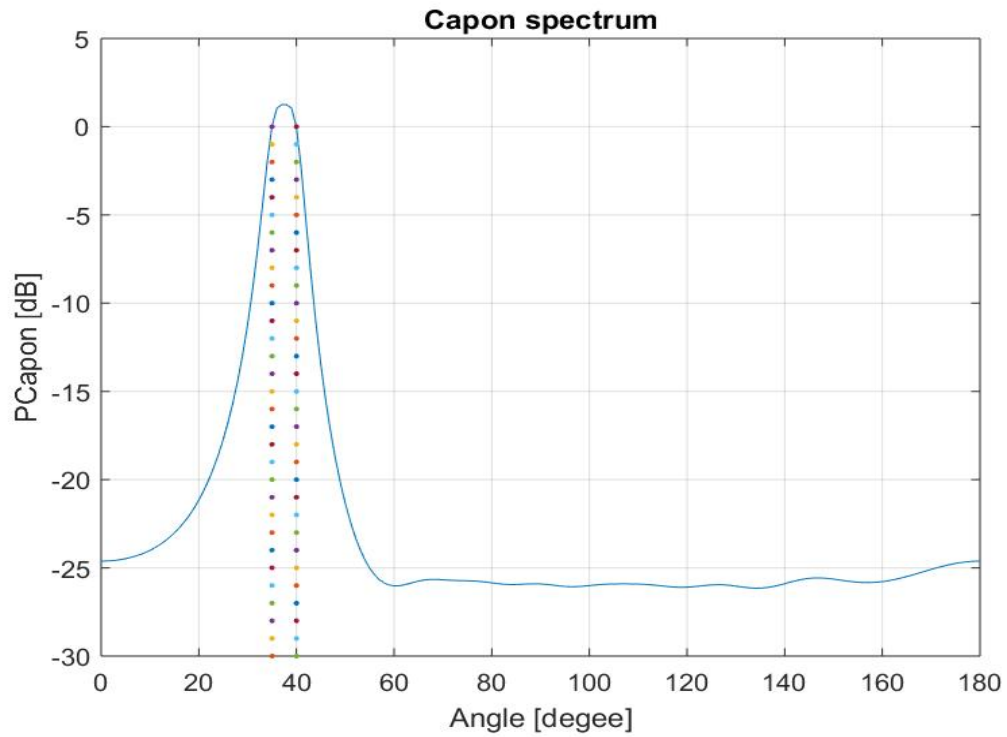


Figure 4.3: Average power output of Capon's method for two signals arriving from 35 and 40 degrees

If two closely spaced sources have signals which are perfectly co-related this method will not spatially null either signal as it can just add them in a destructive way to reduce final power output. The peaks would just merge to give a single peak somewhere in between the true direction of arrival angle.

Chapter 5

Super- resolution array based approach

The earlier methods failed mostly in cases where the sources are closely located or the incoming signals are highly correlated. A constant trade off goes on in those algorithms between forming a strong beam in the look direction, attenuating other signals and maintaining low side lobe to attenuate noise. This happens because the received data auto correlation matrix structure is overlooked. The matrix constitutes full rank auto correlation matrix of the noise and unwanted signals and a low rank auto correlation matrix of desired signal components. The earlier algorithm also fails to detect the difference between when a signal is rejected due to spatial nulls and when it is rejected due to destructive combination of correlated signals. This conditions are encountered generally in multipath propagation and signal jamming situations. These super resolution algorithms jointly estimate the parameters for all signals in contrast to the previous algorithms. This is done by taking

advantage of the structure of auto correlation matrix. In addition to it rather than directly working on the data these algorithms might also use beamforming and nulling in subspaces of the received data. The conclusions are obtained from spatial filtering algorithm but these concepts are more of a fit for subspace fitting.

5.1 Introduction to MUSIC algorithm

The proposal of Multiple Signal Classification(MUSIC) opened a new era for spatial spectrum estimation. The promotion of the structure of algorithm characterized rise and progress and became critical for theoretical system of spatial spectrum.

The intention is to conduct characteristics decomposition for the co-variance matrix of the processor output data. This results in a signal subspace which is orthogonal to the noise subspace. The direction of arrival is estimated using these orthogonal subspace.

5.2 Mathematical Model for MUSIC

Let us consider a model. \mathbf{R}_{xx} is the auto correlation matrix of the received data. We can get the desired signal components by subtracting \mathbf{R}_{ii} that is auto correlation matrix of the interference from \mathbf{R}_{xx} . Given a weight vector \mathbf{w} the output power coming from the desired signal can be expressed as:

$$\begin{aligned} P &= \mathbf{w}^H (\mathbf{R}_{xx} - \mathbf{R}_{ii}) \mathbf{w} \\ &= \mathbf{w}^H \mathbf{A}(\Theta) \mathbf{R}_{ss} \mathbf{A}(\Theta)^H \mathbf{w} \end{aligned} \tag{5.1}$$

in a similar fashion the sum of the average powers of the K outputs due to the desired signals would be:

$$P_{total} = \sum_{k=1}^K P_k = tr[\mathbf{W}^H(\mathbf{R}_{xx} - \mathbf{R}_{ii})\mathbf{W}] \quad (5.2)$$

where $tr[\cdot]$ denotes the matrix trace operation and $\mathbf{W} = [\mathbf{w}_1 \cdots \mathbf{w}_k]$. Just as when we limit the narrowness of the transition band of an FIR filter by applying pass band and stop band simultaneously, the resolution here can be limited by simultaneous operation of beamforming and null-steering. Lets take an example of nulling all signals simultaneously. Now as power P is of the desired signal space it can be seen as if no noise signals are present. Lets say M is the number of sensors and L the number of signals. SO if less than M signals are present which are not completely correlated, \mathbf{W} can be selected such that it would be able to null all the signals simultaneously. As the signals are not fully correlated they can be rejected by only forming spatial nulls. We can get K weight vectors where $K=M-L$ as they would be linearly independent of each other. We want a $M \times K$ matrix E where each column \mathbf{w}_k , for $k = 1, \dots, K$ nulls all the desired signal. This is achieved by subjecting the condition that the columns of \mathbf{W} are linearly independent while minimizing the total average power P_{total} . This condition can also be seen as $\mathbf{W}^H\mathbf{W} = \mathbf{I}$. Basically \mathbf{W}_{null} is the solution of-

$$\begin{aligned} \min_{\mathbf{W}} P_{total} \\ \Rightarrow \min_{\mathbf{W}} [\mathbf{W}^H(\mathbf{R}_{xx} - \mathbf{R}_{ii})\mathbf{W}] \quad \text{subject to} \quad \mathbf{W}^H\mathbf{W} = \mathbf{I} \end{aligned} \quad (5.3)$$

The direction of arrival estimation ca now be done by searching over θ for those array vectors $\mathbf{a}(\theta)$ for which

$$\|\mathbf{W}_{null}^H\mathbf{a}(\theta)\|^2 = 0 \quad (5.4)$$

Basically we searching for the directions where a null is seen simultaneously in all $K=M-L$ vectors. Similarly we can choose W_{beam} for which we get the maximum total average output power P_{total} . This corresponds to simultaneous beamforming in all desired directions. So the solution of equation (5.5) where $K=L$ would give us W_{beam} .

$$\begin{aligned} \max_W P_{total} \\ \Rightarrow \max_W \text{tr}[\mathbf{W}^H(\mathbf{R}_{xx} - \mathbf{R}_{ii})\mathbf{W}] \quad \text{subject to} \quad \mathbf{W}^H\mathbf{W} = \mathbf{I} \end{aligned} \quad (5.5)$$

The direction of arrival angle estimation can be done by searching over θ for array vector $\mathbf{a}(\theta)$ for which $\|\mathbf{W}_{beam}^H \mathbf{a}(\theta)\|^2$ showcases a beam formation and is maximized. As we can see we need to find \mathbf{W}_{null} or \mathbf{W}_{beam} . The solution of these optimization problems are needed. It can be calculated in terms of eigenvalues and eigenvectors of the signal only spatial auto correlation matrix [6].

$$\mathbf{W}_{null} = [\mathbf{e}_{L+1} \cdots \mathbf{e}_M] \quad (5.6)$$

and

$$\mathbf{W}_{beam} = [\mathbf{e}_1 \cdots \mathbf{e}_L] \quad (5.7)$$

where \mathbf{e}_m are eigenvectors defined by:

$$(\mathbf{R}_{xx} - \mathbf{R}_{ii})\mathbf{e}_m = \lambda_m \mathbf{e}_m \quad (5.8)$$

and are ordered according to associated eigenvalues λ_m which are real and non-negative.

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L \geq \lambda_{L+1} = \cdots = \lambda_M = 0 \quad (5.9)$$

The eigenvalue problem can be expressed with noise auto correlation matrix being described within an unknown multiplicative constant σ^2 such that $\mathbf{R}_{ii} = \sigma^2 \mathbf{Q}_{ii}$ as:

$$\mathbf{R}_{xx} \mathbf{e}_m = \lambda_m \mathbf{Q}_{ii} \mathbf{e}_m \quad (5.10)$$

where

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_L \geq \lambda_{L+1} = \dots = \lambda_M = \sigma^2 \quad (5.11)$$

Thus MUSIC algorithm can be seen as having the following steps. The first step is to estimate the auto correlation matrix \mathbf{R}_{xx} using finite N. Then equation (5.10) is solve and the DOA can be found by either the maxima of

$$\|\mathbf{W}_{beam}^H \mathbf{a}(\theta)\|^2 \quad (5.12)$$

or the minima of

$$\|\mathbf{W}_{null}^H \mathbf{a}(\theta)\|^2 \quad (5.13)$$

The results obtained from equations (5.12) and (5.13) yields the same results and choice of either would give the same resolution.

5.3 The principle and implementation of MUSIC algorithm

MUSIC has the capacity to estimate DOA of multiple signals. These estimations are precise and are applicable for short data circumstances as well.

5.3.1 Implementation steps

While using this method we need to take care of certain restrictions so that the results are precise. The conditions for MUSIC algorithm to function properly are:

1) The number of signals say L should be less than the total number of sensors which we have taken as M.

2) Within a given multiplicative constant we need to know \mathbf{R}_{ii} .

3) The auto correlation matrix \mathbf{R}_{ss} which belongs to the signal should have full rank which in this case would be L.

The problems faced if these requirements are not met are:

- 1) We cannot find the weight vectors which would be able to null the signals.
- 2) The modified data space wont be just signal data space in which either beamforming or nulling occurs.
- 3) Also signals can get nullified without being spatially nullified.

A major difference in MUSIC and previous spatial filtering based algorithm seen is here the weight vectors first and then finds out the beams or nulls. In contrast to this in the earlier method for every direction a new weight vector was computed. Basically in MUSIC all desired signals are jointly processed and then processing is done to locate the DoA. Whereas the previous algorithms would estimate DoA of one signals and while doing so for the next signal all these other signals are not considered. This in return gives us better performance as compared to Capon's method or the conventional beamforming method. Nevertheless the implementation steps are described in figure 5.1.

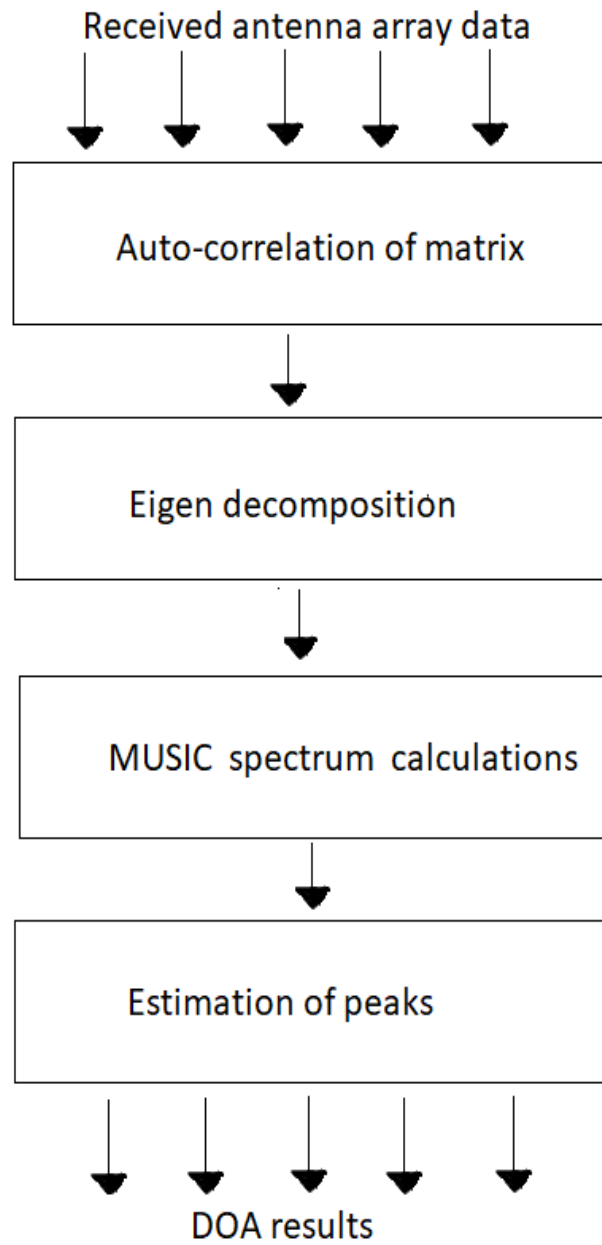


Figure 5.1: Implementation steps for MUSIC algorithm

5.3.2 Simulation results

Figure 5.2 shows the simulation result similar to previous ones for signals arriving from 40 and 80 degrees with carrier frequency 0.3 GHz, total number of antennas 10, SNR around 5, number of snapshots taken 1000 and the spacing of the elements in the array is kept as half the value of the wavelength. Figure 5.2 shows sharp peaks for signals arriving from multiple sources.

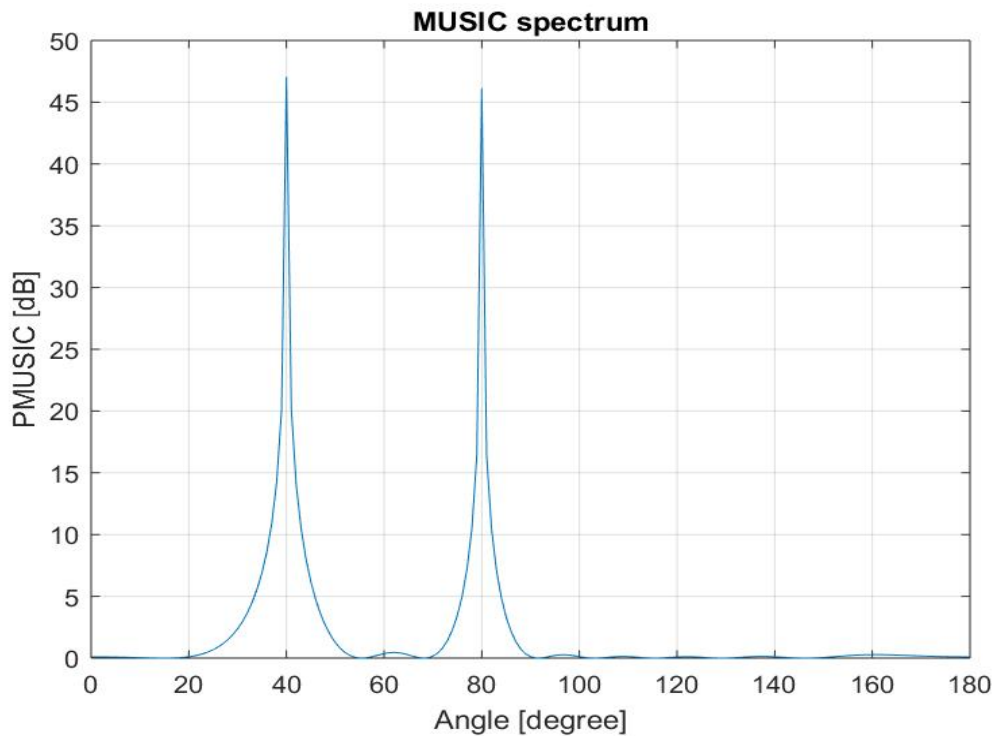


Figure 5.2: Average power output of MUSIC method for two signals arriving from 40 and 80 degrees.

Fig 5.3 shows the simulation result for signals arriving from 30 and 40 degrees with carrier frequency 0.3 GHz, total number of antennas 10, SNR around 5, number of snapshots taken 1000 and the spacing of the elements in the array is kept as half the value

of the wavelength. Figure 5.3 demonstrates better results for closely spaced sources which in this case in a range of 10 degrees than the earlier results in figure 3.2 and also in figure 4.2

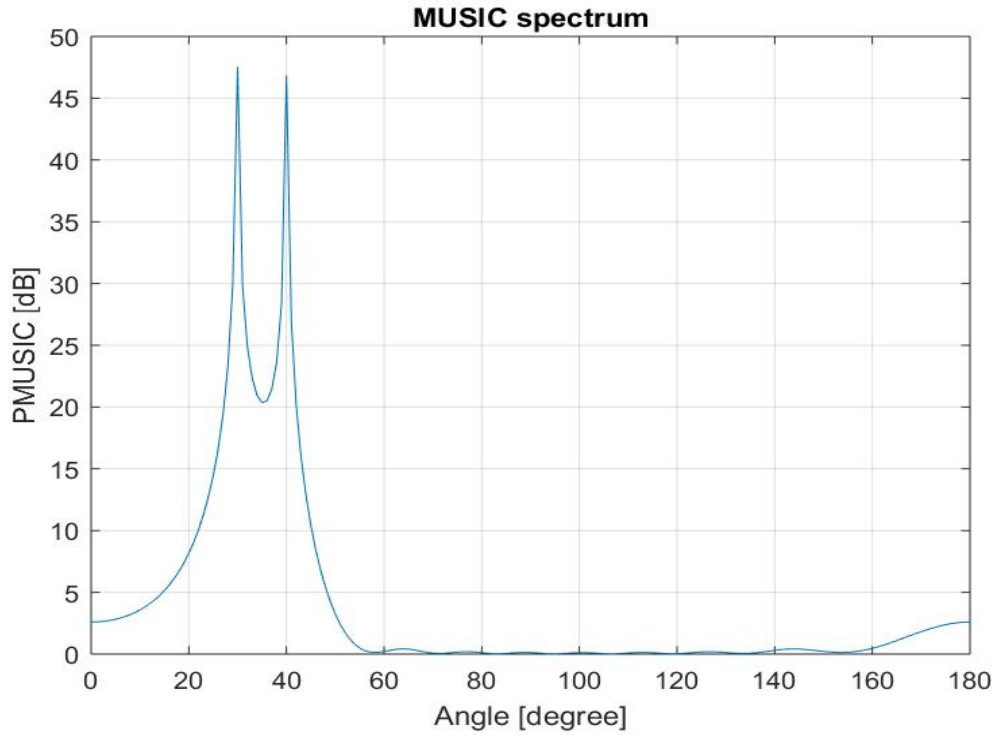


Figure 5.3: Average power output of MUSIC method for two signals arriving from 30 and 40 degrees.

Fig 5.4 shows the simulation result for signals arriving from 35 and 40 degrees with carrier frequency 0.3 GHz, total number of antennas 10, SNR around 5, number of snapshots taken 1000 and the spacing of the elements in the array is kept as half the value of the wavelength. The earlier drawback with very closely spaced sources which was not resolved in simulation result shown in figure 4.3 have been removed using MUSIC methods as shown in figure 5.4. The range of the signal sources here is of less than 10 degrees.

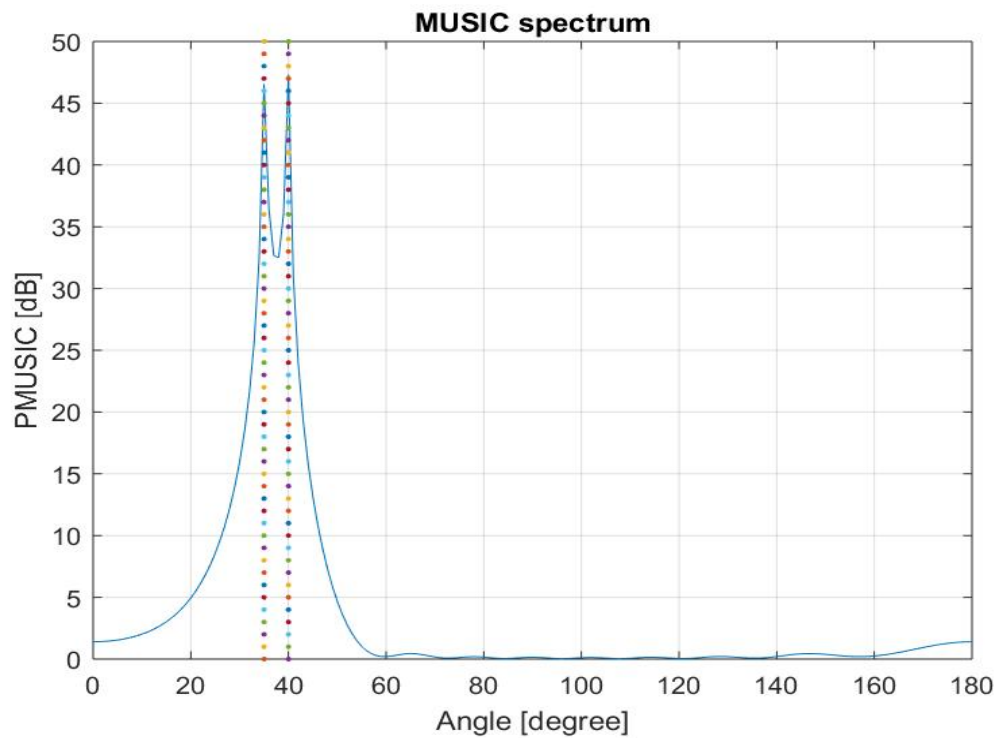


Figure 5.4: Average power output of MUSIC method for two signals arriving from 35 and 40 degrees.

MUSIC method also fails when you have fully co-related signals or multipath cases. Several approaches like spatial smoothing have been done to overcome this. The approach of root-MUSIC can also be seen in [7].

Figure 5.5 shows the user interface created in labview to analyze the three methods discussed in this thesis.

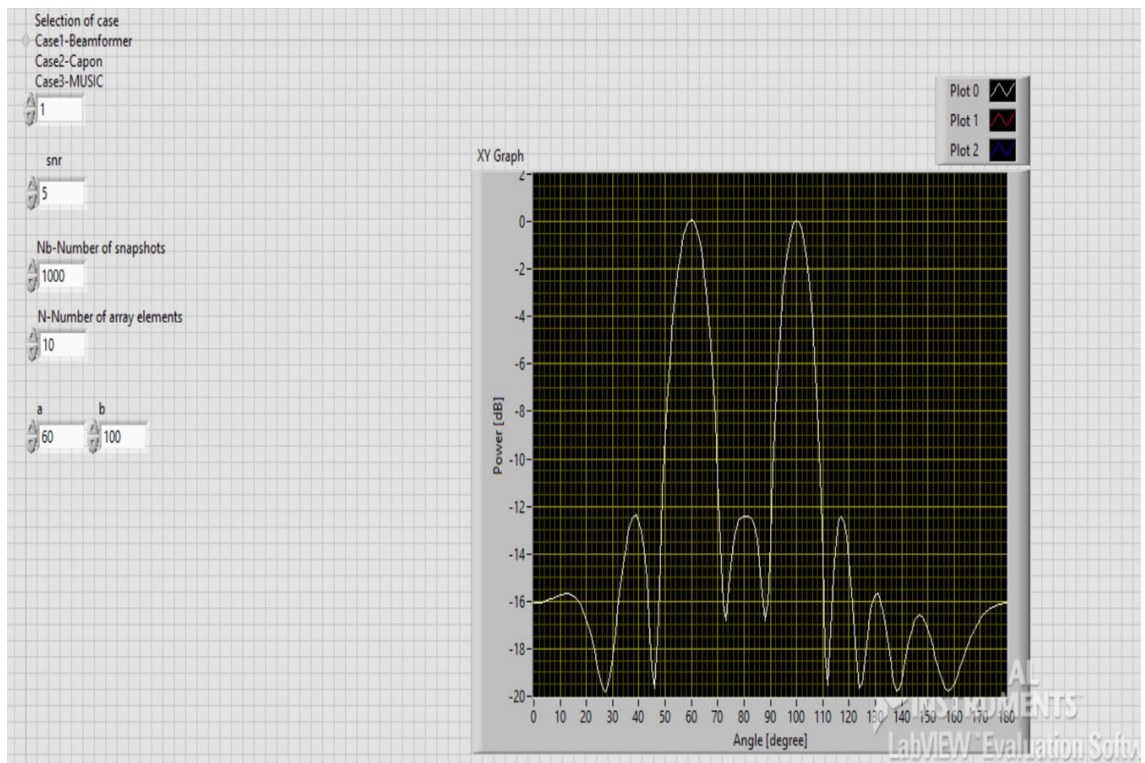


Figure 5.5: User interface in Labview

Chapter 6

Conclusions

In this thesis, the theory of Array Signal Processing is reviewed for direction of arrival estimation problem. We have discussed spectral analysis for solving the direction finding problem. In non-parametric methods, Conventional beamforming method and Capon's method has been discussed. The performance and limitations of these algorithms have been reviewed by providing MATLAB simulations. MUSIC method which is a subspace-based algorithm has been discussed in this thesis. We review the performance of MUSIC using MATLAB simulations and compare it's resolution with non-parametric methods discussed earlier. The limitations of MUSIC are also discussed later. It is also inferred that direction of arrival estimation can be done with better accuracy if the SNR is adequately high or the data collection time is sufficiently long and the signal model is accurate. All the algorithms discussed in this thesis are also reviewed using a user interface in Labview.

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