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Abstract

An approximate treatment of the effect of recoil in single nucleon transfer in heavy ion reactions is outlined. It is shown that the effect of recoil is to remove the restrictions on the orbital angular momentum transfer. The effect of recoil is shown to depend upon the energy of the projectile becoming more significant at higher projectile energies, and for the case where the neutron binding to the residual nucleus is small, it is more important for smaller final binding energies. A simple expression is obtained for the recoil amplitude and cross-section for p-wave projectiles.

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1. Introduction

Recently, there has been a large amount of experimental data available on single and multinuclear transfer reactions induced by heavy ions. It was first pointed out by Breit, Hull, and Gluckstern[1]) that nucleon transfer reactions at energies below the coulomb barrier could be treated quantitatively because of the possibilities of giving a classical description for the trajectories of the heavy ions. The theory has been extended by Breit and Ebel[2]). Buttle and Goldfarb[3]) proposed a distorted wave Born approximation which could be applied to the case where the energy of the projectile was close to the coulomb barrier. Schmittroth, Tobocman, and Golestaneh[4]), using a method developed by Sawaguri and Tobocman[5]), improve upon the formalism of Buttle and Goldfarb by treating the nuclear form factor more accurately. The transition amplitude in the distorted wave Born approximation (DWBA) involves a six dimensional integration. Buttle and Goldfarb[3]) introduced an approximation of neglecting terms of the order of the ratio M_n/M_c, where M_n is the mass of the transferred nucleon and M_c is the mass of the nuclear core, which enabled them to rewrite the integral in a form resembling the transition amplitudes in the DWBA treatment of inelastic nucleon-nucleus scattering or in the zero-range deuteron stripping theory. (In spite of the similarity, a zero range approximation is not implied in the theory of Buttle and Goldfarb[3]) or that of Tobocman et al.[4]). We shall refer to the approximation as the no-recoil approximation. A number of single nucleon transfer reactions have been analyzed[6]) using the theory of Buttle and Goldfarb, and the spectroscopic factors extracted from these reactions have been found to be consistent with with those from light projectile induced reactions.

One of the consequences of the no-recoil approximation is a severe limitation obtained on the allowed orbital angular momentum transfer. In particular, if the transferred particle occupies pure single particle states in the projectile and the residual nucleus, characterized by the respective orbital and total angular momenta $(\ell_1 j_1)$ and $(\ell_2 j_2)$, the transfer angular momentum L satisfies the relations,

$$|j_1-j_2| \leq L \leq j_1+j_2 \tag{1.1a}$$

$$|\ell_1-\ell_2| \leq L \leq \ell_1+\ell_2 \tag{1.1b}$$

$$\ell_1+\ell_2 + L = \text{even} \quad . \tag{1.1c}$$

If $j_1 = 1/2$, as in the case of $(^{16}O, ^{15}N)$ or $(^{16}O, ^{15}O)$ reactions, one obtains one single L characterizing the nuclear form factor.

Greider[7]) has pointed out the importance of the effects of recoil on the angular distributions in heavy-ion single nucleon transfer reactions. The exact treatment of recoil would necessitate the numerical computation of the six dimensional integral. This has been done by Kamamuri and Yoshida[8]). Recently, Buttle and Goldfarb[9]) have considered the effect of recoil in trying to explain the post-prior discrepancy. Their approximate method of including the recoil effect changes the wave numbers of the incident projectile and that of the outgoing particle. They do not, however, consider the other important effect of recoil, which is the violation of the selection rules, eq. (1.1). The magnitude of the predicted cross section is very sensitive to the value of L. Hence, one could experimentally determine the importance of the recoil corrections.

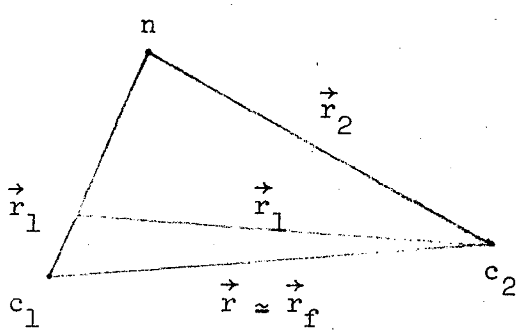
We consider the case where the target nucleus is very massive relative to the projectile. Hence the important recoil effect will be the inclusion of the terms of the order M_n/M_{a_1} and neglecting those of order M_n/M_{c_2}. Where M_{a_1} is the mass of the projectile, M_{c_2}, that of the target, and M_n, that of the transferred particle.

2. The Formalism

We shall use the notation of Buttle and Goldfarb[3]). The reaction we consider is

$$a_1+c_2 \equiv (c_1+n) + c_2 \to a_2+c_1 \equiv (c_2+n) + c_1 \tag{2.1}$$

The co-ordinate system is shown in fig. 1.



$$\vec{r}_2 = \vec{r}_1 - \vec{r}$$
$$\vec{r}_i = \vec{r} - \frac{M_n}{M_{a_1}} \vec{r}_1 \tag{2.2}$$
$$\vec{r}_f \simeq \vec{r}$$

The transition amplitude is given by

$$\begin{aligned}\mathcal{T}_{fi}\,(\vec{k}_f,\vec{k}_i) = {} & \Theta^{(1)}_{j_1\ell_1}\,\Theta^{(2)}_{j_2\ell_2} \sum_{\lambda_1\nu_1\rho_1}\ \sum_{\lambda_2\nu_2\rho_2} \langle \ell_1\lambda_1\ 1/2\ \nu_1 | j_1\rho_1\rangle \\ & \times \langle j_1\rho_1 c_1\gamma_1 | a_1\alpha_1\rangle \langle j_2\rho_2 c_2\gamma_2 | a_2\alpha_2\rangle \langle \ell_2\lambda_2\ 1/2\ \nu_2 | j_2\rho_2\rangle \\ & \times \int d^3r \int d^3r_1\ \chi^{(-)*}(\vec{k}_f,\vec{r})\ \Psi^*_{\ell_2\lambda_2}(\vec{r}_2)\ V_{c_1n}(r_1)\ \Psi_{\ell_1\lambda_1}(\vec{r}_1) \\ & \times\ \chi^{(+)}(\vec{k}_i,\vec{r}_i)\end{aligned} \tag{2.3}$$

where the letters a_1, a_2, c_1, c_2 represent the spins of the respective nuclei represented by the same letters, while α_1, α_2, γ, and γ_2 represent their respective z-components. If the bound states of the neutron in the projectile and the residual nucleus are not pure single particle states, one should allow for a sum on the angular momenta $(j_1\ell_1)$ and $(j_2\ell_2)$. If the mass ratio M_n/M_{a_1} is sufficiently small compared to unity, one could make a Taylor expansion of the initial distorted wave $\chi^{(+)}(\vec{k}_i,\vec{r}_i)$, i.e.,

$$\chi^{(+)}(\vec{k}_i,\vec{r}_i) = \chi^{(+)}(\vec{k}_i,\vec{r}) - \frac{M_n}{M_{a_1}} \vec{r}_1 \cdot \vec{\nabla}\, \chi^{(+)}(\vec{k}_i,\vec{r}) \tag{2.5}$$

where we have terminated the series after the term of order M_n/M_{a_1}. Up to order M_n/M_{a_1}, the transition amplitude could be written as

$$\mathcal{J}_{fi}(\vec{k}_f,\vec{k}_i) = \mathcal{J}^{(0)}_{fi}(\vec{k}_f,\vec{k}_i) - \frac{M_n}{M_{a_1}} \mathcal{J}^{(1)}_{fi}(\vec{k}_f,\vec{k}_i) \tag{2.6}$$

where $\mathcal{J}^{(0)}_{fi}$ is the no-recoil amplitude considered by Buttle and Goldfarb. We use the approximation that

$$\vec{\nabla}\, \chi^{(+)}(\vec{k}_i,\vec{r}) \cong i\, \vec{k}_i\, \chi^{(+)}(\vec{k}_i,\vec{r}) \tag{2.7}$$

which is the correct relation for a plane wave. The recoil amplitude is then given by

$$\mathcal{J}^{(1)}_{fi}(\vec{k}_f,\vec{k}_i) = ik_i \left(\frac{4\pi}{3}\right)^{1/2} \Theta^{(1)}_{j_1\ell_1}\, \Theta^{(2)}_{j_2\ell_2} \sum_{\substack{\lambda_1 \nu\, \rho_1 \\ \lambda_2\, \rho_2}} \langle \ell_1\lambda_1\, 1/2\, \nu | j_1\rho_1\rangle$$

$$\times \langle j_1\rho_1 c_1\gamma_1 | a_1\alpha_1\rangle \langle \ell_2\lambda_2\, 1/2\, \nu | j_2\rho_2\rangle \langle j_2\rho_2 c_2\gamma_2 | a_2\alpha_2\rangle$$

$$\times \int d^3r \int d^3r_1\, \chi^{(-)*}(\vec{k}_f,\vec{r})\, \Psi^*_{\ell_2\lambda_2}(\vec{r}_2)\, r_1 Y_{10}(\vec{r}_1)\, V_{c_1 n}(r_1)$$

$$\times \Psi_{\ell_1\lambda_1}(\vec{r}_1)\, \chi^{(+)}(\vec{k}_i,\vec{r}) \tag{2.8}$$

where we have chosen $\vec{k}_i$ as the z-axis and have assumed $V_{c_1 n}$ to be spin-independent.

We shall first calculate the nuclear structure form factor. Following Buttle and Goldfarb[3]), we assume that the bound state wave function $\Psi_{\ell_2\lambda_2}(\vec{r}_2)$ could be represented by its asymptotic form,

$$\Psi_{\ell_2\lambda_2}(\vec{r}_2) \cong N_2\, h^{(1)}_{\ell_2}(i\chi_2 r_2)\, Y_{\ell_2\lambda_2}(\vec{r}_2) \tag{2.9}$$

where $h^{(1)}_{\ell_2}(z)$ is a spherical Hankel function of the first kind and of order ℓ_2. Using the expansion of the spherical Hankel function in terms of spherical Hankel and Bessel functions[4]), we obtain

$$\int d^3r_1\, \Psi^*_{\ell_2\lambda_2}(\vec{r}_2)\, r_1\, Y_{10}(\vec{r}_1)\, V_{c_1 n}(r_1)\, \Psi_{\ell_1\lambda_1}(\vec{r}_1$$

$$= N_2\sqrt{4\pi} \sum_{\substack{\ell\ell^1\\ \lambda\lambda^1}} (-1)^{1/2(\ell+\ell_2-\ell')} \left[\frac{(2\ell+1)(2\ell_1+1)(2\ell_2+1)3}{4\pi(2\ell'+1)^2}\right]^{1/2}$$

$$\times \langle \ell\lambda\ell_2\lambda_2|\ell'\lambda'\rangle \langle \ell 0\ell_2 0|\ell' 0\rangle \langle \ell_1\lambda_1|0|\ell'\lambda'\rangle \langle \ell_1 0|0|\ell' 0\rangle$$

$$\times\, h^{(1)}_{\ell}(i\chi_2 r)\, Y_{\ell\lambda}(\vec{r})\, B_{\ell'\ell_1} \tag{2.10}$$

where the factor $B_{\ell'\ell_1}$ is defined by

$$B_{\ell'\ell_1} = \int_0^\infty r_1^3 dr_1\, j^*_{\ell^1}(i\chi_2 r_1)\, V_{c_1 n}(r_1)\, U_{\ell_1}(r_i) \tag{2.11}$$

Using the recurrence relations for the spherical Bessel functions, one can show that

$$B_{\ell'\ell_1} = i\left(\frac{d}{d\chi_2} - \frac{\ell_1}{\chi_2}\right) A_{\ell_1} \qquad \text{for } \ell' = \ell_1+1 \tag{2.12a}$$

$$= -i\left(\frac{d}{d\chi_2} + \frac{\ell_1+1}{\chi_2}\right) A_{\ell_1} \qquad \text{for } \ell' = \ell_1-1 \tag{2.12b}$$

The factor A_{ℓ_1}, which also appears in the no-recoil approximation, is given by

$$A_{\ell_1} = \int_0^\infty r_1^2 dr_1\, \mathcal{J}^*_{\ell_1}(i\chi_2 r_1)\, V_{c_1n}(r_1)\, U_{\ell_1}(r_1) \tag{2.13}$$

For reactions whose Q value is close to zero, if we use the form for A_{ℓ_1} given by[4])

$$A_{\ell_1} \cong (-1)^{\ell_1} \frac{\hbar^2}{2M_n} N_1\, \chi_2^{\ell_1} (\chi_1)^{-\ell_1-1} \quad , \tag{2.14}$$

it can be verified that

$$B_{\ell'\ell_1} = 0 \qquad \text{for } \ell' - \ell_1+1 \tag{2.15a}$$

$$= -\frac{i(2\ell_1+1)}{\chi_2} A_{\ell_1} \qquad \text{for } \ell' = \ell_1-1 \tag{2.15b}$$

In the Appendix, we have shown that eq. (2.15) are valid as long as $\chi_2 R_1 < \ell_1$, where R_1 is the radius of the projectile.

In particular, for p-wave projectiles like ^{16}O or ^{12}C, this implies that if the neutron is captured in the residual nucleus with small binding energy, $\ell' = 0$, and from the vector addition coefficient $\langle \ell 0 \ell_2 0 | \ell' 0 \rangle$, we obtain the condition that $\ell = \ell_2$. The recoil amplitude is thus characterized by the transfer angular momentum which is the orbital angular momentum of the captured particle in the residual nucleus. Secondly, it can be verified that the parity of ℓ is opposite that of the transfer orbital angular momentum appearing in the no-recoil approximation. The differential cross-section for the recoil term is

$$\frac{d\sigma}{d\Omega}\Big|_{recoil} = \frac{\mu_{a_1}\ \mu_{c_1}}{(2\pi\hbar^2)^2} \times \frac{k_f}{k_i} \left(\frac{M_n}{M_{a_1}}\ \frac{k_i}{\chi_2}\right)^2 \left(\Theta^{(1)}_{j_1\ell_1}\ \Theta^{(2)}_{j_2\ell_2}\ N_2\ \sqrt{4\pi}\right)^2$$
$$\times \frac{(2a_2+1)}{(2c_2+1)} \times \frac{3}{2}\,(2\ell_2+1)\ \sum_{LM}\Bigg| \frac{\langle 10\ell_2 M|LM\rangle}{(2L+1)^{1/2}}\, U(1j_1\ell_2 j_2;1/2\ L)$$
$$\times\ t_{\ell_2 M}\Big|^2 \tag{2.16}$$

where

$$t_{\ell_2 M} = \int d^3r\ \chi^{(-)*}(\vec{k}_f,\vec{r})\ h^{(1)}_{\ell_2}(i\chi_2 r)\ Y_{\ell_2 M}(\hat{r})\ \chi^{(+)}(\vec{k}_i,\vec{r}) \tag{2.17}$$

and

$$U(abcd;ef) = [(2\ell+1)(2f+1)]^{1/2}\ W(abcd;ef) \tag{2.18}$$

where W(abcd;ef) is a Racah coefficient. The above expressions are valid for $\chi_2 R < 1$ and $\ell_1 = 1$. In the general case both $\ell' = \ell_1 - 1$ and $\ell' = \ell_1 + 1$ would be permitted. Inspection of the form factor, eq. (2.10) shows that we obtain the following selection rules

$$|\ell' - \ell_2| \le \ell \le \ell' + \ell_2, \tag{2.19a}$$

$$\ell' + \ell^2 + \ell = \text{even} \tag{2.19b}$$

$$|\ell_1 - 1| \le \ell' \le \ell_1 + 1$$

$$\ell_1 + \ell + 1 = \text{even} \tag{2.19d}$$

which implies that $\ell' = \ell_1 \pm 1$, and

$$|\ell_1 - 1 - \ell_2| \leq \ell \leq \ell_1 + \ell_2 + 1 \tag{2.20a}$$

and

$$\ell + \ell_2 + \ell_1 = \text{odd} \tag{2.20b}$$

Equations (2.20a) and (2.20b) represent the only selection rules on the transfer angular momentum, ℓ. The above results are due to a first order treatment of the recoil term. It can be realized that our result is an approximation to the method suggested by Dodd and Greider[10]) where one writes

$$\chi^{(+)}(\vec{k}_i, \vec{r}_i) \cong \chi^{(+)}(\vec{k}_i, \vec{r}) \exp(i\, \vec{q}\cdot\vec{r}_1) \tag{2.21a}$$

where

$$\vec{q} = \frac{M_n}{M_{a_1}} \vec{k}_i \quad . \tag{2.21b}$$

If we expand the plane wave,

$$\exp(i\, \vec{q}\cdot\vec{r}_1) = \sum_{n=0}^{\infty} \frac{(i\, \vec{q}\cdot\vec{r}_1)^n}{n!} \tag{2.22}$$

We will have all the powers of r_1 appearing in integrals of the form

$$B_{\ell' L \ell_1} = \int_0^\infty r_1^{L+2}\, d_1^r\, j^*_{\ell'}(i\chi_2 r_1)\, V_{cn}(r_1)\, U_{\ell_1}(r_1) \tag{2.23}$$

It can be shown that for p-wave projectiles as long as the general form of A_1 is of the type given by eq. (2.14) or eq. (A7), the only nonvanishing integral corresponds to $\ell' = 0$. Hence, the first order expression obtained remains valid even for higher energies as long as we consider transitions to states of very low binding energy in the residual nucleus.

In the case of strongly bound final states, the validity of using the Taylor expansion would depend upon the parameter $\left(\frac{M_n}{M_{a_1}} \frac{k_i}{\chi_2}\right)$. As long as the parameter is considerably smaller than unity, our approximation would be justifiable. When the parameter becomes large it is preferable to use an exact computation similar to that of Kamamuri and Yoshida[8]) or the expression given by eq. (22a) in the paper of Sawaguri and Tobocman[5]).

3. Conclusion

We have presented an approximate first order treatment of the recoil effect in heavy ion single nucleon transfer reactions. We have shown that it violates the selection rules on the transfer angular momentum. Unlike the usual belief that it violates only the parity selection rule, eq. (1.1c), an exact calculation will violate all the selection rules, eq. (1.1), and there is no limitation on the ℓ-transfer. We have been able to show that for very low binding energies of the nucleon in the residual nucleus, a considerable simplification is obtained. In particular, in the case of the p-wave projectiles, the transfer angular momentum for the recoil term is specified uniquely by the orbital angular momentum of the captured nucleon in the residual nucleus, independent of the detailed nature of the projectile. This is in contrast to the no-recoil amplitude which distinguishes between p-wave projectiles such as ^{16}O or ^{12}C.

In a later paper, numerical results of the recoil terms will be presented.

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Appendix

The Evaluation of $A_\ell = \int_0^\infty r^2 dr\, j_\ell^*(i\chi_2 r)\, V(r)\, U_\ell(r)$

We shall make the assumption that the binding interaction is approximated by a square well potential of range R_1, where R_1 is the radius of the projectile. The integral is then given by

$$A_\ell = V_0\, C \int_0^{R_1} r^2 dr\, j_\ell^*(i\chi_2 r)\, j_\ell(qr) \tag{A1}$$

where

$$q^2 = \frac{2M_n}{\hbar^2}(V_0 - B_1) = \frac{2M_n}{\hbar^2} V_0 - \chi_1{}^2 \tag{A2}$$

where V_0 is the depth of the potential and B_1 is the binding energy of the nucleon in the projectile and C is a normalization constant.

$$A_\ell = \frac{-V_0 C}{(q^2+\chi_2^2)}\, W(j_\ell^*(i\chi_2 r),\, j_\ell(qr))\big|_{r=R_1} \tag{A3}$$

where the Wronskian $W(a,b)$ is defined by

$$W(a,b) = a\frac{db}{dr} - b\frac{da}{dr} \tag{A4}$$

At the radius R_1, the internal wave function has to be matched to an exponentially decaying solution, i.e.,

$$C\, j_\ell(qR_1) = D\, h_\ell^{(1)}(i\chi_1 R_1) \tag{A5}$$

Thus, we obtain

$$A_\ell = -\frac{V_0 D}{(q^2+\chi_2^2)}\, W(j_\ell^*(i\chi_2 R_1),\, h_\ell^{(1)}(i\chi_1 R_1)) \tag{A6}$$

If χ_2 is small and $\chi_2 R_1 < \ell_1$, we can show that

$$A_\ell \cong - \frac{V_0 D}{q^2} \frac{(i\chi_2 R_1)^\ell}{(2\ell+1)!!} \times (-i\chi_1)\, h^{(1)}_{\ell+1}(i\chi_1 R_1) \tag{A7}$$

which is valid for all values of χ_1. Using eq. (A7) it immediately follows that

$$B_{\ell+1,\ell} = i\left(\frac{d}{d\chi_2} - \frac{\ell}{\chi_2}\right) A_\ell = 0 \quad . \tag{A8}$$

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