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RECENT DEVELOPMENTS IN THE THEORY OF WEAK INTERACTIONS Lecture Notes I through VII

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Nicola Cabibbo

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RECENT DEVELOPMENTS IN THE THEORY OF WEAK INTERACTIONS

Nicola Cabibbo

March 17, 1964

(Lectures for experimental physicists)

The series will probably consist of eight lectures. There will be no lecture on Tuesday, April 28 (Washington meeting). Please, look out for possible schedule changes.

These lectures are mainly intended for experimentalists. The emphasis will be on physical concepts, and I will try to keep the mathematics to an acceptable level. Some knowledge of the elements of field theory (free fields, meaning of Feynman graphs, etc.) would be desirable. Any comment or suggestion is welcome. The following arguments will be covered:

1. The A-V theory-- its experimental basis.
2. Conserved vector current theory.
3. Neutrino reactions.
4. Lepton processes;--muon-electron universality and the two neutrinos.
5. Goldberger and Treiman relations--quasi conserved currents.
6. Universality and SU_3 symmetry.
7. Non-leptonic decays--again SU_3 symmetry.

I. Introduction

Let us begin with a rapid view of the field, to introduce some of the problems which will be discussed later in detail.

The theory of weak interaction was born around 1932-33 with the neutrino hypothesis (Pauli) and the first successful theory of beta decay (Fermi). After the rapid development which followed the discovery of parity non-conservation, the present scheme for weak interactions was formulated in 1957-58 (The V-A theory of Feynman and Gell-Mann, Marshak and Sudarshan).

According to this theory the Hamiltonian for weak interactions is of the current-current type:[1]

$$H(x) = \frac{G}{\sqrt{2}} (J_\lambda J_\lambda^+) . \tag{1.1}$$

J_λ is a current sum of a leptonic part J_λ^ℓ and a part J_λ^s which operates on strong interacting particles:

$$J_\lambda = J_\lambda^s + J_\lambda^\ell . \tag{1.2}$$

We know the explicit form of J_λ^ℓ in terms of the electron, muon and neutrino fields:[2]

$$J_\lambda^\ell = (\bar{\nu}_\mu \gamma_\lambda (1 + \gamma_5) \mu) + (\bar{\nu}_e \gamma_\lambda (1 + \gamma_5) e) . \tag{1.3}$$

From the term in (1.1) in which J_λ^ℓ couples with itself we get the Hamiltonian for muon decay:

$$\frac{G}{\sqrt{2}} (\bar{\mu} \gamma_\lambda (1 + \gamma_5) \nu_\mu)(\bar{\nu}_e \gamma_\lambda (1 + \gamma_5) e) \tag{1.4}$$

We can evaluate the muon lifetime, in terms of G, and from the experimental value obtain

$$G \sim 10^{-5} M_p^{-2} . \tag{1.5}$$

The coupling of J_λ^{ℓ} with itself gives also rise to other reactions, like

$$\nu_e + e \rightarrow \nu_e + e$$

$$e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e .$$

These are very difficult to observe in laboratory experiments, but could have some importance in cosmological problems.

The Weak Current J_λ^s

It is not possible to give an expression for J_λ^s in terms of fields,[3] as we did for the lepton current J_λ^{ℓ}, since we lack a field-theoretical description of strongly interacting particles. Different approaches have been used to gain information on J_λ^s; these are the principals:

1. Study of selection rules of J_λ^s in respect to quantities which are conserved by strong interactions. If some quantity (e.g. strangeness) is not conserved by J_λ^s, we can inquire whether the violation assumes some simple form. All this will be discussed in Section II.

2. Dispersion relations. The main success of this approach is the discovery of Goldberger-Treiman relations. Other applications are to the study of "weak" form factors, in close analogy to the analysis of electromagnetic form factors. The knowledge of weak form factors is of special importance in the study of elastic neutrino processes, like, e.g.,

$$\nu_\mu + n \to p + \mu^-$$

3. Conserved currents. Strong interactions provide conserved quantities, like charge, strangeness, third components of the isotopic spin, baryon number. These are additive quantities: the charge of a system of particles is the arithmetic sum of the individual charges. To each of these quantities there corresponds a current

$$Q \to j_\lambda(x) \; ; \qquad B \to j_\lambda^{B}(x)$$

$$S \to j_\lambda^{S}(x) \; ; \qquad I_3 \to j_\lambda^{3}(x)$$

The currents are four vectors; the fourth component gives the local density of the corresponding quantity, which is then obtained by integrating this over all space:

$$iQ = \int j_4(x)\, d^3x \quad , \text{etc.}$$

Currents which correspond to conserved quantities (briefly: conserved currents) obey a continuity equation:

$$\partial_\lambda j_\lambda = \frac{\partial}{\partial(i\ t)} j_4(x) + \vec{\nabla}\vec{j}(x) = 0$$

$$\partial_\lambda j_\lambda^{S} = \partial_\lambda j_\lambda^{B} = \partial_\lambda j_\lambda^{3} = 0 .$$

We have other quantities which are conserved by strong interactions: the charge symmetry requires conservation not only of I^3, but also of the raising operators

$$I^{\pm} = I^1 \pm i I^2$$

to which correspond two conserved currents $j_\mu^{\pm}$:

$$\partial_\mu j_\mu^{+} = \partial_\mu j_\mu^{-} = 0 .$$

The physical meaning of these currents is not as transparent as that of currents which correspond to diagonal quantities, but is clear that charge symmetry requires them to exist and to obey continuity equations, as $j_\lambda^{3}(x)$ does. We come to the conclusion that there is an elite of current operators which are singled out as being important in the description of strong interaction symmetries.

It is interesting to ask whether J_λ^{s}, or at least some part of it, belongs to this elite. This brings--as we will see--to straightforward predictions which have been found in good agreement with experiment. The first attempt along this line has been the hypothesis that the $\Delta S = 0$ vector part of J_λ^{s} coincides with j_λ^{+} (conserved vector current of Feynman and Gell-Mann, Gherstein and Zheldovich).

More recent developments involve the use of vector currents whose existence is implied by the approximate SU_3 invariance of strong interactions.

This approach can also be used --in a more limited sense--for the axial part of $J_\lambda^{\,s}$, and has brought to a deeper understanding of Goldberger and Treiman relations.

Vector Bosons

The V-A Hamiltonian for leptonic decays of strong interacting particles

$$\frac{G}{\sqrt{2}} \left(J_\lambda^{\,s} J_\lambda^{\,\ell *} + J_\lambda^{\,s*} J_\lambda^{\,\ell} \right) \tag{1.6}$$

is very similar to that Hamiltonian which describes photon interactions:

$$e \, j_\lambda A_\lambda \quad . \tag{1.7}$$

The correspondence being

$$j_\lambda \rightarrow J_\lambda^{\,s}$$

$$A_\lambda \rightarrow J_\lambda^{\,\ell *} \text{ (photon } \rightarrow \text{ lepton pair) .}$$

The similarity of the emission of a lepton pair with the emission of a photon was the basis in fact of Fermi's theory of beta decay.

It has been proposed that the similarity could be even deeper, and that weak interactions are mediated by the exchange of a charged vector meson $W^{\pm}$; beta decay would then be a two-step process

$$n \rightarrow p + W^-$$
$$\qquad\qquad\quad \hookrightarrow e^- + \bar{\nu}_e$$

and the true Hamiltonian would not be (1.1) but

$$H = g W_\lambda J_\lambda + h.c. \tag{1.7}$$

which is completely similar to the e.m. Hamiltonian 1.7. The current-current Hamiltonian (1.1) would still give an approximate phenomenological description of weak (2-step) processes, if we put:

$$G = \sqrt{2} \; \frac{g^2}{M_W^2} . \tag{1.8}$$

The effects which would allow an experimental distinction between (1.1) and (1.7) are generally small and (or) difficult to identify. A clear-cut proof of the existence of $W^\pm$ can probably only come from the direct observation of its production and decay. A very promising reaction is neutrino production:

$$\nu_\mu + Z \to \mu^- + W^+ + Z$$
$$\hookrightarrow e^+ + \nu_e, \; \mu^+ + \nu_\mu . \tag{1.9}$$

The lifetime of $W^\pm$ would be small ($\sim 10^{-17}$sec) so that the overall process looks like the production of a lepton pair, $\mu^-\mu^+$ or μ^-e^+. The neutrino experiment at CERN has given indication for the existence of such processes, but the evidence for the existence of W-meson is not yet compelling.

Universality

A very interesting hypothesis on weak interactions regards their "universal" character. We see that muon and electron appear in a symmetrical way in the lepton current (Eq. 1.3), each with its own neutrino. This property is called "muon-electron universality" and is part of the general identity of behavior of muons and electrons (apart from the mass difference).

It is very appealing to speculate on a possible extension of universality to J_λ^s, the weak current of strongly interacting particles. To do so we would like to say something like:

"J_λ^s has the same strength as J_λ^ℓ".

This statement is rather vague, as it stands, and the problem is to attach some meaning to it. A naive way would be to compare matrix elements of J_λ^s with those of J_λ^ℓ; for example, the matrix element of J_λ^s in beta decay is (experimentally)

$$\langle\, p \mid J_\lambda^s \mid n \,\rangle \approx \left[\bar{u}(p)\ \gamma_\lambda(1 + 1.25\ \gamma_5)\ u(n)\right] \quad . \tag{1.10}$$

This compares satisfactorily with the matrix elements of J_λ^ℓ, for example

$$\langle\, e^- \mid J_\lambda^\ell \mid \nu_e \,\rangle = \left[\bar{u}(e)\ \gamma_\lambda(1 + \gamma_5)\ u(\nu_e)\right] . \tag{1.11}$$

The nearly perfect agreement in the vector part gave rise to the conserved vector current hypothesis. If we extend this comparison to the beta decay of the hyperons, the result is completely unsatisfactory; the observed rates are consistently 1/20 - 1/60 too small. The search for a better way of comparing different matrix elements of J_λ^s has recently brought to the use of SU_3 symmetry, which allows connections between $\Delta S = 0$ and $\Delta S = 1$ processes, and suggests a new interpretation of universality, which allows $\Delta S = 1$ leptonic processes to be less intense than $\Delta S = 0$ ones.

Non-leptonic decays

In the V-A theory the problem of leptonic decays is reduced, as we have seen, to the study of the current J_λ^s , and much progress has been made. The reason for this success is twofold: on one side any experimental result on leptonic decays illuminates some property of J_λ^s , and on the other side the introduction of a new hypothesis on the current--like that of conserved currents--can in general be tested by some experiment.

In the case of non-leptonic decays things are much worse. Even if the basic interaction is current-current, as in Eq. (1.1), it is not possible to prove so, since strong interactions among all the particles involved in any such process tend to cover any underlying structure. The main lines of approach is the study of selection rules which can be very informative on the possible current-current schemes.

FOOTNOTES

1. What is written here is actually a "density of Hamiltonian". The "real" Hamiltonian is obtained by integrating this over all space

$$\mathcal{H} = \int d^3x \; H(x) .$$

2. Here μ, e indicate the fields of μ^- and e^-, which are taken to be particles. The different fields can be expressed in terms of creation and distruction operators. For example

$$e(x) = \frac{1}{(2\pi)^{3/2}} \sum_i \int d^3k (u_i(k) \; a_i(k) \; e^{i(kx)} + v_i(k) \; b_i^*(k) \; e^{-ikx}).$$

The sum is over the two states of helicity $(i = \pm 1)$ which are available for each value of $\vec{k}$, $u_i(k)$ and $v_i(k)$ are Dirac spinors, and

$a_i(k)$ annihilates e^-

$b_i^*(k)$ creates e^+.

Altogether J_λ^ℓ: creates $e^+ + \nu_e$, $\mu^+ + \nu_\mu$ pairs

annihilates $\bar{\nu}_e$, $\bar{\nu}_\mu$ and creates e^+, μ^+, etc.

3. Field theory is however used in a restricted sense in the building of so-called "phenomenological" expressions. A phenomenological Hamiltonian is by definition an Hamiltonian which gives the correct value of a certain group of transiton matrix elements when used as

3. Cont.

a first-order perturbation, and the fields which it contains are considered <u>free</u> fields (and therefore expressed in terms of creation and annihilation operators). For example beta decay could be described by

$$H \approx \frac{G}{\sqrt{2}} \left(\bar{p}\, \gamma_\lambda (1 + 1.25\, \gamma_5) n\right)\left(\bar{e}\ \ \gamma_\lambda (1 + \gamma_5) \nu_e\right) + \text{h.c.}$$

Phenomenological expressions are a useful tool because they describe more than one process, taking automatically into account the consequencies of such general requirements as crossing and CPT theorems. The above expression, for example, describes not only beta decay, but also

$e^- + p \rightarrow n + \bar{\nu}_e$ - K capture

$\nu_e + p \rightarrow n + e^+$ - neutrino absorption (at low energy),

etc.

LECTURE II: LEPTONIC DECAYS - SELECTION RULES

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March 24, 1964

1. The matrix element

In this section we continue the discussions of the week current J_λ^s and of its selection rules.

Let us consider a leptonic decay

$$A \to B + \ell^- + \bar{\nu}_\ell \tag{2.1}$$

where ℓ^- could be e^- or μ^-, A and B are strongly interacting particles. Examples:

$$n \to p + e^- + \bar{\nu}_e$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

$$K^- \to \pi^+ + \pi^- + e^- + \bar{\nu}_e \quad . \tag{2.2}$$

If we are interested in the lowest order in the weak interactions and neglect electromagnetic corrections, the S-matrix element is given by the perturbation theory result .

$$S = \frac{G}{\sqrt{2}} d^4 x \ \langle B + \ell^- + \bar{\nu}_\ell \mid J_\lambda^{s}(x) \, J_\lambda^{\ell}(x) \mid A \rangle \qquad (2.3)$$

We can further reduce this (see Appendix A) to

$$S = \frac{G}{\sqrt{2}} (2\pi) \ \langle B | J_\lambda^{s} | A \rangle \ [\bar{u}(\ell^-) \, \gamma_\lambda \ (1 + \gamma_5) \ u(\bar{\nu}_\ell)]$$

$$\delta^4 (p^\ell + p^{\bar{\nu}} + p^B - p^A) \qquad (2.4)$$

The relevant unknown factor is therefore the matrix element of the current:

$$F_\lambda = \langle B | J_\lambda^{s} | A \rangle \qquad (2.5)$$

Lorentz invariance requires F_λ

to transform like a four vector. If A or B (or both) contain particles with spin $\neq 0$, F_λ should be <u>linear</u> in their spinors (if spin $\frac{1}{2}, \frac{3}{2} \cdots$) or polarization vectors (if spin 1) or polarization tensors (for integer spin > 1). This follows from Lorentz invariance, but could be seen as a consequence of the superposition principle.

In the case of spin $\frac{1}{2}$ particles (p, n, Λ, etc.) it is customary to use four component (Dirac) spinors, instead of two component (Pauli) spinors, because this greatly simplifies the discussion of Lorentz invariance.

Therefore, certain factors appear in F_λ for each incoming and outgoing spin $\frac{1}{2}$ particle, according to the following table:

fermion	incoming	$u(\vec{p})$
	outgoing	$\bar{u}(\vec{p})$
antifermion	incoming	$\bar{v}(\vec{p})$
	outgoing	$v(\vec{p})$

TABLE 2, I

These rules are the same as in perturbation theory, and this could be useful as a mnemmonic aid.

2. Parity

The weak current J_λ^s is, like J_λ^ℓ, a mixture of a vector and of an axial vector part:

$$J_\lambda^s = J_\lambda^V + J_\lambda^A \quad .$$

Their contribution to F_λ, F_λ^V and F_λ^A will be vector and axial-vector expressions or vice versa, according to the intrinsic parities of the particles which constitute states A and B; the rule is the following: Define P (A) and P(B) as the products of intrinsic parities of particles in A and B, then

$P(A) = P(B)$	$\langle B\|J_\lambda^V\|A\rangle$ is a vector expression
	$\langle B\|J_\lambda^A\|A\rangle$ is an axial expression
$P(A) = -P(B)$	$\langle B\|J_\lambda^V\|A\rangle$ is an axial expression
	$\langle B\|J_\lambda^A\|A\rangle$ is a vector expression

TABLE 2, II

The proof of this is given in Appendix B, where P,C,CP, CPT are briefly discussed. As an example, consider the pion decay; the relevant matrix element is

$$\langle 0|J_\lambda^{\,s}|\pi\rangle = \langle 0|\, J_\lambda^{\,V} + J_\lambda^{\,A}\,|\pi\rangle = F_\lambda^{\,V} + F_\lambda^{\,A} \ .$$

There is a parity change (the vacuum has even parity) so that we expect $F_\lambda^{\,V}$ to be an axial expression, $F_\lambda^{\,A}$ to be a vector expression. However, we have only one vector at our disposal, namely the pion four-momentum p_λ, and we are not able to build axial expressions, so that:

$$\langle 0|J_\lambda^{\,V}|\pi\rangle = 0$$

$$\langle 0|J_\lambda^{\,A}|\pi\rangle = f\,p_\lambda$$

Other examples are proposed as problems (problems 1 to 3). In solving these one should remember that it is possible to build an axial vector out of three independent vectors by means of the completely antisymmetric Ricci symbol:

$\epsilon^{\mu\nu\rho\sigma}$ $p_\nu q_\rho k_\sigma$ is an axial vector.

$$\epsilon^{1234} = 1 \;, \quad \epsilon^{2134} = -1 \;, \text{ etc.}$$

This is not zero only if p, q, k are independent (try to put $k_\lambda = a\,p_\lambda + b\,q_\lambda$ and use the antisymmetry of $\epsilon^{\mu\nu\rho\sigma}$)

3. Charge, Strangeness and Isotopic Spin

First of all it is clear(charge conservation):

$$\Delta Q = Q(B) - Q(A) = 1$$

We can classify different parts of $J_\lambda{}^S$ according to the strangness charge they cause: a priori we can have

$$\Delta S = S(B) - S(A) = 0 \;, \pm 1 \;, \cdots$$

We have abundant examples of $\Delta S = 0$ (beta decay, $\pi \to \mu\nu$, etc.) and $\Delta S = \Delta Q = 1$ ($K \to \mu\nu$, $K \to \mu 3$, $\Lambda \to p e^- \bar{\nu}$, etc...) but the existence of $\Delta S = -\Delta Q$ transitions has not been firmly established, while there is no evidence for (and some evidence against) $|\Delta S| \geq 2$

The change in the third component of isotopic spin is related to the previous two: we have

$$Q = I_3 + \frac{Y}{2} = I_3 + \frac{S + B}{2} .$$

Since the barionic number B is conserved,

$$\Delta I_3 = I_3(B) - I_3(A) = \Delta Q - \frac{1}{2}\Delta S = 1 - \frac{1}{2}\Delta S .$$

Therefore

$$\Delta I_3 = 1 \quad \text{for} \quad \Delta S = 0$$

$$\Delta I_3 = \frac{1}{2} \quad \text{for} \quad \Delta S = \Delta Q = 1$$

$$\Delta I_3 = \frac{3}{2} \quad \text{for} \quad \Delta S = -\Delta Q = -1$$

We want now to define a <u>total</u> isotopic spin charge, ΔI. Let us consider again the pion decay; the matrix element is:

$$\langle 0 | J_\lambda^{\,s} | \pi \rangle$$

It is clear that only that part of $J_\lambda^{\,s}$ can contribute, which transforms as a $I = 1$, $I_3 = 1$ object: it is the only part whose I-spin can add to the I-spin of the pion to give the I-spin zero of the vacuum. The same is true for beta decay: again only the $I = 1$, $I_3 = 1$ part of J^s can couple to $I = \frac{1}{2}$, $I_3 = -\frac{1}{2}$ (the neutron) to give $I = \frac{1}{2}$, $I_3 = \frac{1}{2}$(proton), in

$$\langle p | J_\lambda^{\,s} | n \rangle \quad .$$

One can advance the hypothesis that this portion of $J_\lambda^{\,s}$ which behaves as an $I = 1$, $I_3 = 1$ object in respect to isotopic spin is the <u>only</u> part with $\Delta S = 0$.

This assymption is usually referred to as the $\Delta I = 1$ selection rule for $\Delta S = 0$ transitions. The rule is not empty, as would appear from the cases of pion and beta decay: consider for example the following neutrino reactions

$$\nu_\mu + n \to N^{*+} + \mu^-$$

$$\nu_\mu + p \to N^{*++} + \mu^-$$

in which a $\left(\frac{3}{2}, \frac{3}{2}\right)$ resonant state is produced. Only the $\Delta S = 0$ part of J^s contributes; if it behaves as $I = 1, I_3 = 1$, i) and ii) are related by Clebsch-Gordan coefficients:

$$\langle N^{*+}|J_\lambda{}^s|n\rangle = \left(\frac{3}{2}, \frac{1}{2} / 1, 1; \frac{1}{2}, -\frac{1}{2}\right) M^0 = \sqrt{\frac{1}{3}}\, M^0$$

$$\langle N^{*++}|J_\lambda{}^s|p\rangle = \left(\frac{3}{2}, \frac{3}{2} / 1, 1; \frac{1}{2}, +\frac{1}{2}\right) M^0 = M^0$$

M^0 is a "reduced matrix element"; the cross sections are therefore expected to be in the ratio 1:3. We can repeat the argument for $\Delta S = 1$ transitions: the existence of $K^- \to \mu^- + \bar{\nu}_\mu$ requires that some part of $J_\lambda{}^s$ behaves as an $I = \frac{1}{2}, I_3 = \frac{1}{2}$ object, and we can make the hypothesis that the whole $\Delta S = 1$ part of $J_\lambda{}^s$ behaves in this way (no $I = \frac{3}{2}$ or larger). This is the $\Delta I = \frac{1}{2}$ rule for $\Delta S = 1$ leptonic decays. From this one gets a straightforward prediction on the $K \to \ell 3$ decays:

$$\langle \pi^0 | J_\lambda^{\ s} | K^- \rangle = \frac{1}{2} \langle \pi^+ | J_\lambda^{\ s} | \bar{K}^0 \rangle$$

the decay rates of $K^- \to \pi^0 + \mu^- + \bar{\nu}_\mu$ and $\bar{K}^0 \to \pi + \mu^- + \bar{\nu}_\mu$ should be in the ration 1:2, while all correleations (including, e.g. muon and pion spectrum, muon polarization, etc..) should be the same for the two decays. The same should be true for the corresponding electron modes. Similar relations, one between Ξ^- and Ξ^0 leptonic decays, the other between cross sections for neutrino production of I = 1 hyperons, are given in problems 4 and 5.

To conclude this section, the only two parts of $J_\lambda^{\ s}$ whose existence is firmly established are

i) $\Delta S = 0$, $\Delta I = 1$

ii) $\Delta S = \Delta Q = 1$, $\Delta I = \frac{1}{2}$

The theory of leptonic processes based on SU_3 invariance, which we will discuss later, requires $J_\lambda^{\ s}$ to contain only these two parts, and it is therefore very interesting to check these selection

Problems:

1) The most general expression for

$$\langle \pi^0 | J_\lambda^{\,s} | K^- \rangle$$

can be expressed in terms of the functions of the only scalar in the problem, namely $(p_\lambda^K - p_\lambda^\pi)^2$. Does the axial part of $J_\lambda^{\,s}$ contribute here?

2) Write the most general expression for the matrix element of $K^- \to \pi^+ + \pi^- + e^- + \bar{\nu}_e$

$$\langle \pi^+ + \pi^- | J_\lambda^{\,S} | K^- \rangle$$

and identify the axial and vector contributions.

3) In these matrix elements:

$$\langle \pi^0 + \pi^0 | J_\lambda^{\,S} | K^- \rangle$$

$$\langle \pi^+ + \pi^+ | J_\lambda^{\,S} | K^+ \rangle$$

We have two identical bosons in the final state; this leads to a simplification in respect to the previous case--discuss this.

4) The $\Delta I = \frac{1}{2}$ rule for $\Delta S = 1$ leptonic decays implies a relation among

$$\Xi^- \rightarrow \Sigma^0 + e^- + \bar{\nu}; \quad \Xi^0 \rightarrow \Sigma^+ + e^- + \bar{\nu}_e .$$

5) The $\Delta I = \frac{1}{2}$ rule ($\Delta S = 1$) implies a relation between the following reactions for the reutrino production of Σ hyperons: (and also among the corresponding reactions for the production of Y_1 resonances):

$$\bar{\nu}_\mu + n \rightarrow \Sigma^- + \mu^+$$

$$\bar{\nu}_\mu + p \rightarrow \Sigma^0 + \mu^+$$

(These involve emission of a positive lepton--see footnote 1).

---

(1) The Hamiltonian for leptonic decays is:

$$H = \frac{G}{2} J_\lambda^s J_\lambda^{e*} + J_\lambda^{s*} J_\lambda^{\ell} . \qquad (1.6)$$

The first part enters in reactions with emission of a negative lepton (Eq. 2.1) or others related by crossing to such a reaction like

(cont.)

$$\nu_\ell + A \to B + \ell^-$$

$$\ell^+ + A \to B + \bar{\nu}_\ell \quad \text{etc.}$$

The second in reactions with emission of a positive lepton:

$$A' \to B' = \ell^+ + \nu_\ell$$

$$\bar{\nu}_\ell + A' \to B' + \ell^+ \quad \text{etc.}$$

The matrix element for the first of these would then be (see Eq. 2.4)

$$\frac{G}{2} (2\pi) \langle B' | J_\lambda^{s*} | A' \rangle \left\{ \bar{u}(\nu_\ell)\, \gamma_\lambda (1 + \gamma_5)\, v(\ell^+) \right\} \delta^4 (p^\ell + p^{\bar{\nu}} + p^{B'} - p^{A'})$$

We are therefore interested in the matrix element:

$$\langle B' | J_\lambda^{s*} | A' \rangle = \langle A' | J_\lambda^{s} | B' \rangle .$$

LECTURES ON WEAK INTERACTIONS – III

N. Cabibbo

APPENDIX I

This appendix and other to follow will contain "basic material" which I will not cover in the lectures, but can nevertheless be useful to understand them. Many steps in the proofs are not explicitly given, but they are simple and short, and can be worked out by the reader.

In this section I review the properties of γ– matrices, the Dirac equation, the four fermion interactions, and discuss the experimental basis of the A–V theory.

The discussion of the Dirac equation is rather short and the readers are referred to standard textbooks for more details. In doing so, however, remember that different books use different metric (and therefore different sets of conventions on the γ matrices). The notations used here are in concord, e.g., with those in Pauli's article on quantum mechanics in the "Handbuch der Physik", band V, but not with those in Messiah's book.

Covariant Notation

I will use the metric with an imaginary fourth component. A vector has components:

$$V_\lambda = (\vec{V}, V_4 = iV_o)$$

The scalar product is defined

$$(V, T) = \vec{V}\cdot\vec{T} + V_4 T_4 = \vec{V}\cdot\vec{T} - V_o T_o$$

There are two invariant tensors (i.e., which have the same components in all Lorentz frames), the Kronecker symbol $\delta_{\lambda\mu}$ and the Ricci tensor $\mathcal{E}^{\mu\nu\sigma\lambda}$.

Dirac Equation

The Dirac equation can be written

$$(\partial\!\!\!/ + m)\psi = 0 \qquad (1)$$

where $\partial_\lambda = \partial/\partial x_\lambda$ $\quad \partial\!\!\!/ = \partial_\lambda \gamma_\lambda$, and ψ is a four component spinor. Since describes particles of mass m, it has to satisfy the Klein Gordon equation:

$$(\Box^2 + m^2)\psi = -(p^2 + m^2)\psi = 0 \qquad (2)$$

This requires the different γ_λ to anticommute: $\gamma_1\gamma_2 = -\gamma_2\gamma_1$, etc.; $\gamma_1^2 = \gamma_2^2 = \ldots = 1$, or in compact notation:

$$\{\gamma_\mu, \gamma_\lambda\} = \gamma_\mu\gamma_\lambda + \gamma_\lambda\gamma_\mu = 2\delta_{\mu\lambda} \tag{3}$$

It is <u>not</u> necessary to give explicit expressions for the γ_λ, since eq. (3) is all that we need to know. In fact many choices are possible. We restrict this by asking γ_λ to be hermitian matrices: (there still are many choices)

$$\gamma_\lambda^+ = \gamma_\lambda \tag{4}$$

Along with a spinor, it is convenient to introduce it's adjoint,

$$\bar{\psi} = \psi^* \gamma_4 \tag{5}$$

If ψ is considered as a <u>column</u> matrix, $\bar{\psi}$ will be a <u>row</u> matrix (operated by the right). It satisfies the equation

$$\bar{\psi}(-\not{\partial} + m) = 0 \tag{6}$$

Under a Lorentz transformation $a_{\mu\nu}$

$$\begin{aligned} x' &= a_{\mu\nu}x_\nu \\ a_{\mu\nu}a_{\mu\lambda} &= \delta_{\nu\lambda} \end{aligned} \tag{7}$$

The four component spinors transform in the following way

$$\psi'(x') = S(a)\psi(x) \tag{8}$$

where $S(a)$ is a 4 by 4 matrix. $S(a)$ must be chosen in such a way that the Dirac equation maintains its form in the new reference frame:

$$\left(\gamma_\mu \frac{\partial}{\partial x'_\mu} + m\right)\psi'(x') = 0 \tag{9}$$

but

$$\frac{\partial}{\partial x'_\mu} = a_{\mu\nu}\frac{\partial}{\partial x_\nu} \tag{10}$$

so that from (9) we get:

$$(\gamma_\mu a_{\mu\nu} \frac{\partial}{\partial x_\nu} + m) S \psi(x) = 0$$

Multiply on the left by S^{-1}:

$$(S^{-1} \gamma_\mu S a_{\mu\nu} \frac{\partial}{\partial x_\nu} + m) \psi(x) = 0$$

This is equivalent to the Dirac equation if

$$S^{-1} \gamma_\mu S = a_{\mu\lambda} \gamma_\lambda \qquad (11)$$

(remember the orthogonality condition $a_{\mu\nu} a_{\mu\lambda} = \delta_{\mu\lambda}$). The adjoint equation is also invariant if

$$\bar{\psi}'(x') = \bar{\psi}(x) S^{-1} \qquad (12)$$

If we want the relation between ψ and its adjoint (Eq. 5) to be valid in all frames of reference, we must therefore have

$$S^{-1} = \gamma_4 S^{+} \gamma_4 \qquad (13)$$

Bilinear Covariants – γ Matrices

From two spinors ψ and Φ, we can build quantities which behave in definite ways under Lorentz transformation, namely a scalar (S), a vector (V) an anti-symmetric tensor (T), an axial vector (A) and a pseudoscalar (P). It is easy to see that $(\bar{\psi}\Phi)$ is a scalar:

$$(\bar{\psi}'(x')\Phi'(x')) = (\bar{\psi}(x) S^{-1} S \Phi(x)) = (\bar{\psi}(x)\Phi(x))$$

also

$(\bar{\psi} \gamma_\mu \Phi)$ is a vector:

$$(\bar{\psi}' \gamma_\mu \Phi') = (\bar{\psi} S^{-1} \gamma_\mu S \psi) = a_{\mu\nu} (\bar{\psi} \gamma_\mu \Phi);$$

$$(\bar{\psi} \gamma_{\mu_1} \gamma_{\mu_2} \cdots \gamma_{\mu_n} \psi)$$

is a tensor of rank n.

In fact, we can only build sixteen matrices by multiplying γ's: these are called γ^A, and are divided into five types:

Type	γ^A	Notes
S(scalar)	1	The unit matrix
V(vector)	γ_λ	
T(tensor)	$\sigma_{\mu\nu} = \frac{i}{2}\,(\gamma_\lambda\gamma_\mu = \gamma_\mu\gamma_\lambda)$	Six different matrices: $i\gamma_1\gamma_2\ldots$
A(axial)	$i\gamma_\lambda\gamma_5$	
P(pseudoscalar)	$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$	

The γ^A have the following properties:

$$(\gamma^A)^2 = 1 \tag{14}$$

$$\gamma^{A+} = \gamma^A \tag{15}$$

$$\text{Trace }(\gamma^A) = 0\,,\ \text{if } \gamma^A \neq 1 \tag{16}$$

<u>Problem</u>: Prove these properties by verifying (14) and (15); (16) requires the following trick: for any γ^A find a γ^B which anticommutes with it. Then

$$\gamma^A = \gamma^A\gamma^B\gamma^B = -\,\gamma^B\gamma^A\gamma^B$$

But $\text{tr}\left[(\gamma^A\gamma^B)\gamma^B\right] = \text{tr}\left[\gamma^B(\gamma^A\gamma^B)\right]$, so $\text{tr}\,(\gamma^A) = 0$. You can also verify that $\gamma^A\gamma^B = \pm\, i\gamma^C$ with $\gamma^C \neq 1$ if $A \neq B$, so that

$$\text{tr}\,\gamma^A\gamma^B = 4\delta_{AB} \tag{17}$$

The sixteen γ^A are <u>linearly</u> <u>independent</u>: suppose that

$$\sum_A C^A\gamma^A = 0$$

then, for any γ^B

$$\text{tr}\,\sum_A C^A\gamma^A\gamma^B = 4\sum_A C^A\,\delta_{AB} = 4\,C^B = 0$$

Since there cannot be more than 16 independent 4 by 4 matrices, the γ^A give a complete set. Incidentally, this construction (from Eq.3) of 16 independent matrices proves that the dimensionality of the γ's must at least be 4 x 4.

Coming back to our "bilinear covariants", we can build five of them.

Scalar	S	$\bar{\psi}\ \Phi$	
Pseudoscalar	P	$\bar{\psi}\,\gamma_5\ \Phi$	
Antisymmetric Tensor	T	$\bar{\psi}\sigma_{\mu\nu}\ \Phi$	(18)
Vector	V	$\bar{\psi}\,\gamma_\mu\ \Phi$	
Axial Vector	A	$\bar{\psi}\ i\gamma_\mu\gamma_5\ \Phi$	

Problem: Prove that $\bar{\psi}\,\gamma_5\Phi$ is a pseudoscalar. Hint:

$$\gamma_5 = \frac{1}{4!}\,\varepsilon^{\mu\nu\rho\sigma}\ \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$$

and $\varepsilon^{\mu\nu\rho\sigma}$ is a pseudotensor. Note that γ_5 commutes with $1, \sigma_{\mu\nu}, \gamma_5$, but anticommutes with γ_μ, $i\gamma_\mu\gamma_5$

Plane Wave Solutions of Dirac Equation

Put

$$\psi\,(x) = u(k)\ {}^{i\,(kx)} \tag{19}$$

where $u(k)$ is a constant spinor, $k_\lambda \equiv (\vec{k}, iE)$. Then the equation becomes

$$(i\,\not{k} + m)\ u(k) = 0 \tag{20}$$

$\psi(x)$ is the wave function of a particle of momentum k and energy E. We have another kind of solution, apparently corresponding to negative energy $-E$ and momentum $-k$:

$$\psi(x) = v(k)\ e^{-i\,(kx)}$$

$v(k)$ satisfies the equation

$$(-i\,\not{k} + m)\ v(k) = 0 \tag{21}$$

As you know, this possibility brought Dirac to the discovery of the positron: he argued that all negative energy levels should be filled. Then an unfilled level with negative energy $-E$ behaves as a particle of energy E, momentum k. If $u(k)$ represents the absorption of a normal particle, $v(k)$ represents the absorption of a particle of negative energy, which is equivalent to the emission of an antiparticle. Thus, if we proceed to a quantization of the (free) field $\psi(x)$, we get:

$$\psi(X) = (2\pi)^{-\frac{3}{2}} \sum_i \int d^3k \, \left(u_i(\vec{k})\, a_i(\vec{k})\, e^{ikx} + v_i(\vec{k})\, b_i^*(\vec{k})\, e^{-ikx}\right) \tag{22}$$

The index i refers to the two different polarization states; the $a_i(k)$ are destruction operators for particles, $b_i^*(k)$ creation operators for antiparticles. The $u_i(k)$ and $v_i(k)$ are normalized according to

$$\left(u_i^*(\vec{k})\; u_j(\vec{k})\right) = \left(v_i^*(\vec{k})\; v_j(\vec{k})\right) = \delta_{ij} \tag{23}$$

<u>Problem:</u> Prove that we can put

$$v_i(k) = \gamma_5\, u_i(k), \text{ and satisfy Eq. (21)}$$

The adjoint of u and v obey the following equations (derive from Eqs. 20 and 21)

$$\bar{u}_j(k)\,(i\not{k} + m) = 0 \tag{24}$$

$$\bar{v}_j(k)\,(-i\not{k} + m) = 0 \tag{25}$$

If we multiply (24) by $\gamma_4 u_i(k)$ from the right, (20) by $u_j^*(k)$ on the left and add, we get

$$0 = \left(\bar{u}_j(\vec{k})\,(i\not{k} + m)\,\gamma_4\, u_i(\vec{k})\right) + \left(u_j^*(k)\,(i\not{k} + m)\, u_i(\vec{k})\right) =$$

$$2\left(u_j^*(\vec{k})\;(-\gamma_4 E + m)\, u_i(\vec{k})\right)$$

from which

$$\left(\bar{u}_j(\vec{k})\; u_i(\vec{k})\right) = \frac{m}{E}\left(u_j^*(\vec{K})\; u_i(\vec{k})\right) = \frac{m}{E}\,\delta_{ji} \tag{26}$$

Problem: Verify the steps in the above proof, and prove that

$$\bar{v}_j(\vec{k})\, v_i(\vec{k}) = -\frac{m}{E}\,\delta_{ji} \tag{27}$$

We have not yet specified how to chose the two polarization states indicated by the index i. To do so we should discuss a little the definition of spin and polarization.

Polarization

To discuss the problem of polarization, we must take into account the transformation properties of the spin. This is a delicate matter, but we can sidestep the difficulty by taking the convention that by polarization of a particle, we mean the polarization in its rest system. In the rest system the Dirac equation becomes

$$(-\gamma_4 + i)\, u_i(k) = 0 \tag{28}$$

To describe the spin of the particle in this system, we can use the following three matrices:

$$\begin{aligned} \sigma_1 &= \sigma_{23} = i\gamma_2\gamma_3 \\ \sigma_2 &= \sigma_{31} = i\gamma_3\gamma_1 \\ \sigma_3 &= \sigma_{12} = i\gamma_1\gamma_2 \end{aligned} \tag{29}$$

which obey (verify this) the usual commutation relations

$$\left[\sigma_1, \sigma_2\right] = 2i\sigma_3$$

So, if $u_i(0)$ is to represent a particle with spin component $\pm\frac{1}{2}$ along a direction $\hat{s}$, we must have

$$(\vec{\sigma}\cdot\hat{s})\, u_\pm(0) = \pm u_\pm(0) \tag{30}$$

The definition of polarization as "spin direction in the rest system" is obviously invariant, so that we must be able to put Eq. (30) in a form which is more obviously covariant. To do so, note that

$$\sigma_3 = i\gamma_1\gamma_2 = -i\gamma_5\gamma_3\gamma_4$$

And in general

$$\sigma_i = -i\,\gamma_5\gamma_i\gamma_4 \qquad (31)$$

or $$(\vec{\sigma}\cdot\hat{s}) = -i\,\gamma_5(\vec{\gamma}\cdot\hat{s})\,\gamma_4$$

where s_μ is a four vector, which in the rest system is:

$$s_\mu \equiv (\vec{s},0)$$

So we can rewrite (30) as:

$$-i\,\gamma_5\,(\gamma\cdot s)\,\gamma_4\,u_\pm(0) = \pm\,u_\pm(0)$$

And, using the Dirac equation (28):

$$\left(1 \pm i\,\gamma_5 \not{s}\right)u_\pm(0) = 0$$

This is obviously covariant, if we transform s_μ as a four vector. So the spinor of a particle of momentum k, which <u>in its rest system</u> has spinor $\pm\frac{1}{2}$ along $\hat{s}$ will obey the following set of equations:

$$\left[i\not{k} + m\right]\;u_\pm(\vec{k}) = 0$$

$$\left[1 \pm i\gamma_5\not{s}\right]u_\pm(\vec{k}) = 0 \qquad (32)$$

s_λ is obtained through a Lorentz transformation:

$$s_\lambda = \left\{\hat{s} + \hat{k}\,(\hat{s}\,.\,\hat{k})\,\frac{E-m}{m}\;,\; i\,\frac{|\vec{k}|}{m}\,(\hat{s}.\hat{k})\right\} \qquad (33)$$

Note the properties

$$(sp) = 0$$
$$s^2 = 1 \qquad (34)$$

<u>Helicity</u>

Helicity is defined as the component of the spin along the momentum. It can be treated by the above formalism, taking $\hat{s} = \hat{k}$, from which

$$s \equiv \left(\hat{k}\,\frac{E}{m}\;,\; i\,\frac{|\vec{k}|}{m}\right) \qquad (35)$$

We can however use a simpler form: by use of the Dirac equation

$$(i\gamma_5 \not{s}) \; u_\pm(\vec{k}) = \frac{1}{m}\gamma_5 \not{s} \not{k} \, u_\pm(\vec{k}) = -(\vec{\sigma}.\hat{k}) \, u_\pm(\vec{k})$$

The set of equation for the helicity states is then

$$\left[i\not{k} + m\right] u_\pm(\vec{k}) = 0 \tag{36}$$

$$\left[1 \pm (\vec{\sigma}.\hat{k})\right] u_\pm(\vec{k}) = 0 \tag{37}$$

Zero Mass Particles

In the case of zero mass particles there is no rest system so that our definition of polarization fails. We can try to define it by taking the limit from the case with $m \neq 0$. It is easy to see that the description in terms of s_λ becomes meaningless in this limit (the components of s_λ go to infinity). We can however still define helicity states through eqs. (36) and (37) which have a well defined limit:

$$i\not{k} \; u_\pm(\vec{k}) = 0 \tag{38}$$

$$\left[1 \pm (\vec{\sigma}.\hat{k})\right] u_\pm(\vec{k}) = 0 \tag{39}$$

There is an important simplification of (39): We can rewrite (38) as:

$$(i\vec{k}.\vec{\gamma} - E\gamma_4) \; u_\pm(\vec{k}) = 0 \qquad E = |\vec{k}|$$

but, from eq. (31):

$$i\vec{\gamma} = \gamma_4\gamma_5\vec{\sigma}$$

so that

$$(\gamma_5 \vec{\sigma}.\vec{k} - E) \; u_\pm(\vec{k}) = 0$$

$$\vec{\sigma}.\hat{k} \; u_\pm = \gamma_5 \, u_\pm$$

And we can rewrite (39) in the form

$$\left[1 \mp \gamma_5\right] u_\pm \tag{40}$$

This is the well known result that γ_5 is a helicity operator for zero ~~spin~~ mass particles. This can happen because in the limit $m = 0$ the helicity becomes a Lorentz invariant: you cannot change the helicity of a $m = 0$ particle by applying Lorentz transformations.

Problem: Describe a chain of Lorentz transformations by which you can invert the helicity of a particle with $m \neq 0$. Discuss why it does not work for $m = 0$. What about the v – solutions, which are connected with antiparticles? The $v_{\pm}(0)$ which corresponds to an antiparticle of zero momentum with spin $\pm 1/2$ along the $\hat{s}$ direction can be thought as describing a particle of negative energy (–m) and spin $= \mp 1/2$ along the $\hat{s}$ direction (note the inversion) and should therefore obey the set of equations (see 28 and 30).

$$(\gamma_4 + 1) \; v_{\pm}(0) = 0$$

$$(1 \pm \vec{\sigma} . \vec{s}) \; v_{\pm}(0) = 0$$

We can repeat the above arguments and obtain for the case of momentum $\vec{k}$

$$(1 \pm i\gamma_5 \not{s}) \; v_{\pm}(\vec{k}) = 0 \text{ (general case , } m \neq 0) \qquad (41)$$

$$(1 \pm \vec{\sigma} . \hat{k}) \; v_{\pm}(\vec{k}) = 0 \text{ (helicity states, any } m) \qquad (42)$$

$$(1 \pm \gamma_5) \; v_{\pm}(\vec{k}) = 0 \text{ (helicity states for } m = 0) \qquad (43)$$

Problem: Derive these equations, and discuss why in (41) the signs are the same as in (32), while those in (42) and (43) are opposite to those in (37) and (40).

LECTURES ON WEAK INTERACTIONS - IV

Third Lecture

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Experimental Basis of V-A Theory

In this section we briefly consider beta decay, the $\pi \to \mu\nu$ decay, and muon decay.

1. Beta Decay

The Fermi Hamiltonian for beta decay had the form

$$H = g(\overline{\psi}_p \gamma_\lambda \psi_n)(\overline{e} \gamma_\lambda \nu) + \text{h.c.} \tag{3.1}$$

Following Fermi's work, this has been generalized in different ways: first of all, to the most general form which, like 1, is local and does not contain field derivatives.

$$H = \sum g_i(\overline{\psi}_p O^i \psi_n)(\overline{e} O^i \nu) + \text{h.c.} \tag{3.2}$$

The sum is here over the five basic forms S, V, T, A, P (see Appendix). This contains (1) as a particular case. Other possible generalizations include derivative couplings. If the derivatives of the lepton fields appear in the Hamiltonian, this gives extra powers of the lepton momenta and energies in the transition matrix elements (remember $\psi \propto u\, e^{ikx}$). The ensuing modifications of the spectra are excluded from experimental data. A more important generalization became necessary after the discovery of parity non-conservation. The Hamiltonian should then be a mixture of a scalar and a pseudoscalar part:

$$H = \sum_i (\overline{\psi}_p O^i \psi_n)(\overline{e}(g_i + g_i' \gamma_5) O^i \nu) . \tag{3.3}$$

Parity non-violating effects (like spectra, angular correlations) will be described by bilinear forms like $g_i\, g_j$ and $g_i{}'\, g_j{}'$. Parity violating effects arise from the interference of the two parts, and are proportional to mixed terms like $g_i\, g_j{}'$. Note that if $g_i = 0$, $g_i{}' \neq 0$:

$$H = \sum g_i{}'(\overline{\psi}_p \;\; O^i \;\; \psi_n)(\overline{e} \;\; O^i \;\; \gamma_5\, \nu)$$

parity would be conserved (only we would say that the neutrino has a negative parity in respect to the electron). The analysis of beta decay is simplified by the circumstance that the velocity of nucleons which take part in it--both as free nucleons, as in the decay of a neutron, or within a nucleus--is a small quantity. We can therefore develop the matrix element in powers of the velocity.

<u>Problem:</u> What is the maximum velocity of the proton recoil in neutron decay? And in $\Lambda \rightarrow p + e^- + \overline{\nu}$?

As a first approximation, we can take ψ_p and ψ_n to obey the Dirac equation for a zero energy spinor:

$$\gamma_4\, \psi_n \simeq \psi_n$$

$$\overline{\psi}_p\, \gamma_4 \simeq \overline{\psi}_p\, .$$

The big (allowed) contributions to the matrix element will come from those parts of each O^i which commute with γ_4 :

$$\overline{\psi}_p\, O^i\, \psi_n \simeq \overline{\psi}_p\, O^i\, \gamma_4\, \psi_n = \pm\, \overline{\psi}_p\, \gamma_4\, O^i\, \psi_n$$

$$\approx \pm\, \psi_p\, O^i\, \psi_n \begin{cases} \neq 0 & \text{if} \quad [\, O^i, \gamma_4] = 0 \\ \approx 0 & \text{if} \quad O^i\, \gamma_4 = -\gamma_4\, O^i\, . \end{cases}$$

Furthermore terms which differ by a γ_4 factor give equal contributions. The allowed contributions are identified in Table 1, according to these rules.

Kind	Form of O^i	Commutes with γ_4 ?	Allowed contribution	Comments
S	1	Yes	1	
V	$\gamma_\lambda \Rightarrow \gamma_i$,	No	--	
	γ_4	Yes	1	
T	$\sigma_{\mu\lambda} \Rightarrow i\,\gamma_i\,\gamma_j$,	Yes	σ_ℓ	$(\sigma_1 = i\,\gamma_2\,\gamma_3$, etc.)
	$i\,\gamma_i\,\gamma_4$	No	--	
A	$i\,\gamma_\lambda\,\gamma_5 \Rightarrow i\,\gamma_i\,\gamma_5$,	Yes	σ_i	$(\sigma_3 = i\gamma_1\gamma_2 = i\gamma_5\gamma_4\gamma_3 = i\gamma_3\gamma_5\gamma_4 \approx i\gamma_3\gamma_5)$
	$i\,\gamma_4\,\gamma_5$	No	--	
P	γ_5	No	--	

Table 1. The allowed contributions to beta decay. Latin indices, i and j, range from 1 to 3.

Fermi and Gamow-Teller Selection Rules.

The allowed contributions arise from two kinds of "nuclear" operators: $\overline{\psi}_p \ \psi_n$ and $\overline{\psi}_p \ \vec{\sigma} \ \psi_n$. The first is a scalar, the second an axial vector, and they give rise to the following selection rules.

Term	Name	Selection rules
$\overline{\psi} \ \psi$	F. (Fermi)	$\Delta J = 0$, no parity change
$\overline{\psi} \ \vec{\sigma} \ \psi$	G.T. (Gamow-Teller)	$\Delta J = 0, \pm 1$, no parity change (no $0 \to 0$)

The Hamiltonian (4.3) is reduced to:

$$H = (\overline{\psi}_p \ \psi_n)(\overline{e} \left\{ (g_S + \gamma_4 \ g_V) + \gamma_5 (g_S{'} + \gamma_4 \ g_V{'}) \right\} \nu) \ + $$

$$(\overline{\psi}_p \vec{\sigma} \ \psi_n)(\overline{e} \ \left\{ (g_T + \gamma_4 \ g_A) + \gamma_5 (g_T{'} + \gamma \ \ g_A{'}) \right\} \ \vec{\sigma} \ \ \nu). \qquad (3.4)$$

Beta decays which are forbidden by these rules can still take place thanks to terms proportional to the nucleon velocity, and are called first forbidden, second forbidden, etc. How can we obtain the values of $g_S \cdots g_A$ from the experimental data on allowed transitions?

A. Parity Conserving Effects

In discussing these, we can neglect the parity violating coefficient $g_i{'}$. It can be shown that in parity conserving effects g_i and $g_i{'}$ always appear in the combination $g_i \ g_j + g_i{'} \ g_j{'}$. This is a consequence of the zero mass of the neutrino. The electron spectrum in any allowed decay is the same for each of the pure couplings:

$$W_0(E_e)dE_e = F(E_e)\, p_e\, E_e\, E_\nu^{\,2}\, dE_e \tag{3.5}$$

where $F(E_e)$ is the coulomb correction, which depends on the charge of the final nucleus, and is equal to 1 when coulomb effects can be neglected (e.g., in neutron beta decay). $W_0(E_e)$ is called the allowed spectrum.

For a general coupling including all four types, the specturm becomes

$$W(E_e) = W_0(E_e)(1 + b\ \frac{m}{E_e}) \ . \tag{3.6}$$

The new term (Fierz term) arises from $g_V\, g_S$ and $g_T\, g_A$ interference. Experimentally this term is not there, so that we can draw our first conclusion:

The Fermi coupling is either <u>only</u> S or <u>only</u> V .

The G-T coupling is either <u>only</u> A or <u>only</u> T .

Another parity non-violating effect is the electron neutrino correlation. This can be measured through the spectrum of recoils of the final nucleus. If electron and neutrino tend to go together, the nucleus will tend to have a large recoil, and inverse. The angular correlation for pure couplings is (β_e, the electron velocity)

$$\begin{array}{ll} 1 + \beta_e \cos\theta & V \\ 1 - \beta_e \cos\theta & S \\ 1 + \frac{1}{3}\beta_e \cos\theta & T \\ 1 - \frac{1}{3}\beta_e \cos\theta & A \ . \end{array} \tag{3.7}$$

The angular correlation has been extensively studied in the decay of He^6, a G.T. transition, and of A^{35}, a pure Fermi ($J = 0 \rightarrow J = 0$) transition. These experiments bring uniquely to the choice of V and A as the leading couplings, among the four choices allowed by the absence of Fierz terms. For the interesting story of these experiments see J. Allen, Revs. Modern Phys. 31, 791 (1959).

Parity Non-Conserving Effects

The two classical examples of parity violating effects are:

1. the longitudinal polarization of the emitted electron and neutrino,

2. "up-down" asymmetries.

The maximum longitudinal polarization for beta decay electrons of a given velocity v_e is $\pm\ v_e/c$, and is attained for maximum parity violation, i.e.,

$g_i' = g_i$ gives electrons with helicity $+v/c$

" positrons with helicity $-v/c$

$g_i' = -g_i$ gives electrons with helicity $-v/c$

" positrons with helicity $+v/c$.

Experimental data indicate that the second possibility is realized. Together with the results previously discussed, this suggests an Hamiltonian:

$$g_V(\overline{\psi}_p\ \gamma_\lambda\ \psi_n)(\overline{e}\ (1 - \gamma_5)\gamma_\lambda\ \nu) + g_A(\overline{\psi}_p\ i\ \gamma_\lambda\ \gamma_5\ \psi_n)(\overline{e}(1 - \gamma_5) i\ \gamma_\lambda\ \gamma_5\ \nu) + h.c.$$

which can be rewritten (verify this passage)

$$(\overline{\psi}_p\ \gamma_\lambda(g_V - \gamma_5\ g_A)\psi_n)(\overline{e}(1 - \gamma_5)\ \gamma_\lambda\ \nu) + h.c. \tag{3.8}$$

or

$$\left(\bar{\psi}_p\, \gamma_\lambda(g_V - \gamma_5\, g_A)\psi_n\right)\left(\bar{e}\;\; \gamma_\lambda(1 + \gamma_5)\nu\right) \;+\; \text{h.c.} \tag{3.9}$$

We can easily understand why $(1 - \gamma_5)$ gives rise to full polarization when $v/c \to 1$. In this limit we can neglect the electron mass, and we know that γ_5 is the helicity operator for a zero mass particle. A factor $(1 - \gamma_5)$ plays the role of a projection operator: It picks out of $\bar{\psi}_e$ that part for which $\gamma_5 \approx -1$, which describes creation of electrons with negative helicity and destruction of positrons of positive helicity (all this in the limit $m_e \to 0$).

The Two-Component Neutrino

Lepton conservation

The situation with the neutrino is rather similar. The neutrino field appears (see e.g. 3.9) in the combination

$$(1 + \gamma_5)\psi_\nu \;.$$

This causes emission of antineutrinos with positive helicity and the absorption of neutrinos with negative helicity.

The nomenclature of the neutrino states is rather arbitrary. It is only important to realize that there exist two kind of neutrinos which are connected with electrons:

Kind	Emitted together with	Helicity
$\bar{\nu}$ (antineutrino)	negative electron	positive
ν (neutrino)	positive electron	negative

The last two columns of this table embody a basic law of nature, which is usually called "lepton number conservation." We can attribute a lepton number to electrons and neutrinos

$$\ell = +1 \qquad \text{for } e^- \text{ and } \nu \text{ (negative helicity)}$$

$$\ell = -1 \qquad \text{for } e^+ \text{ and } \bar{\nu} \text{ (positive helicity).}$$

This is an additive quantum number, conserved like the charge and the baryonic number.

<u>Connection between Electron Polarization and Neutrino Helicity</u>

As we have seen e^- and $\bar{\nu}$ have opposite longitudinal polarization. This is peculiar of the A-V theory, and follows (compare 3.8 and 3.9) from the fact that

$$(1 - \gamma_5)\gamma_\lambda = \gamma_\lambda(1 + \gamma_5) .$$

The opposite happens with S, T, P terms; for example, we have

$$(1 - \gamma_5)\sigma_{\lambda\mu} = \sigma_{\lambda\mu}(1 - \gamma_5) .$$

S,T,P give e^- and neutrino with the <u>same</u> helicity. An independent measurement of the neutrino polarization can distinguish between S, T, P and A, V. Such a measurement was performed by a very clever indirect method on a G-T transition (Goldhaber, Grodzins and Sunyar, Phys. Rev. <u>109</u>, 1015 (1958)).

Up-Down Asymmetry

The best measurement of the ratio g_A/g_V (see, e. g. 3.9) in beta decay comes from a measurement of the up down asymmetry in angular distribution of electrons and neutrinos emitted from polarized neutrons. These distributions are:

$$1 + c \frac{\vec{p}_e}{E_e} \cdot \vec{P}_n \qquad \text{for the electron} \tag{3.10}$$

$$1 + d\, p_\nu \cdot \vec{P}_n \qquad \text{for the neutrino .} \tag{3.11}$$

Where $\vec{P}_n$ is the neutron polarization, and

$$c = (-2|g_A|^2 - 2 \,\mathrm{Re}\, g_V\, g_A^{*})/(|g_V|^2 + 3|g_A|^2) \;, \tag{3.12}$$

$$d = (2\,|g_A|^2 - 2 \,\mathrm{Re}\, g_V\, g_A^{*})/(|g_V|^2 + 3|g_A|^2) \;. \tag{3.13}$$

From a measurement of these parameters it has been obtained (assuming g_A and g_V to be both real)

$$g_A = -(1.25 \pm 0.04)g_V \;. \tag{3.14}$$

Parity, Charge Conjugation, CP.

It is interesting to see how these operate on neutrino states. Their operations on a particle are

P	Particle → particle	$\vec{\sigma} \to \vec{\sigma}$	$\vec{p} \to -\vec{p}$	hel → -hel
C	Particle → antiparticle	$\vec{\sigma} \to \vec{\sigma}$	$\vec{p} \to \vec{p}$	hel → hel
CP	Particle → antiparticle	$\vec{\sigma} \to \vec{\sigma}$	$\vec{p} \to -\vec{p}$	hel → -hel

P and C transform a neutrino into another which does not exist (or, if you wish, is never emitted or absorbed). On the other hand, CP symmetry is consistent with the two component neutrino. CP symmetry is very appealing, because it preserves--to a certain extent--the undistinguishibility of left and right.

For any experiement which gives a certain result in a left-handed coordinate system, there is another one (performed with anti-matter) which gives the same result when described in a right-handed coordinate system.

Problem. Suppose we contact by radio a physicist in another galaxy. Can we instruct him on how to build a left-handed coordinate system?

CP conservation in beta decay requires g_V and g_A to be relatively real (to have the same phase):

$$\mathrm{Im}(g_V\, g_A^*) = 0 . \tag{3.15}$$

This condition excludes the presence of a third kind of angular correlation, namely:

$$1 + e\, P_n \cdot \frac{\vec{p}_e}{E_e} \times p_\nu \tag{3.16}$$

$$e = \frac{2\, \mathrm{Im}(g_V\, g_A^*)}{|g_V|^2 + 3|g_A|^2} . \tag{3.17}$$

In the experiments on neutron decay, this has been found to be small and consistent with zero $(e = -0.04 \pm 0.07)$.

We can of course reach the same conclusion also from time reversal invariance, since the CPT theorem insures that CP conservation implies T conservation, and inverse. Time reversal invariance can be defined in this way:

The matrix element for the two processes

$$
\begin{aligned}
&A \rightarrow B \\
&B^T \rightarrow A^T
\end{aligned}
\tag{3.18}
$$

are equal. (Up to an unimportant phase factor.) Here B^T indicates the state obtained from B by reversing spin and momenta

$$
\begin{aligned}
&\vec{p} \rightarrow -\vec{p} \\
&\vec{\sigma} \rightarrow -\vec{\sigma}.
\end{aligned}
\tag{3.19}
$$

We obtain therefore a relation

$$\langle B_{out} \mid A_{in} \rangle = \langle A^T_{out} \mid B^T_{in} \rangle . \tag{3.20}$$

The subscripts "out" and "in" are extremely important if the particles which make up states A or B are strongly interacting: The physical definition of states is generally given in terms of free particles; we say "pion plus nucleon" or "electron plus neutrino", and so on. In fact this description is good only in an asymptotic sense; we can define a state

1. by requiring that before a certain process it is made of certain initial particles with given moments, spins, etc. (<u>in</u> state)
2. by requiring that after the process is over, it contains given particles of given momenta, etc. ··· (<u>out</u> state).

For a state which contains a single particle there is no distinction between <u>in</u> and <u>out</u>. In the case of beta decay we need not distinguish between in and out states, since the final state ($e^- + p + \nu$) is made of essentially non-interacting particles. (We neglect here coulomb corrections, or higher order weak interactions.)

Whenever this is possible, time reversal invariance reduces to a simple relation:

$$\langle B \mid A \rangle = \langle A^T \mid B^T \rangle = \langle B^T \mid A^T \rangle^* . \qquad (3.21)$$

This is a sort of reality condition, and can be worked out for beta decay to give (3.15). The consequences of T can be seen even more directly in this case in which we neglect the initial and final state interactions. Equation (3.21) implies that the processes

$$\begin{array}{l} A \rightarrow B \\ A^T \rightarrow B^T \end{array} \qquad (3.22)$$

have the same probability. These two are obtained the one from the other by reversing all spins and momenta. A correlation like (3.16)--which involves a triple product--is therefore forbidden.

Note that the use of (3.22), which is a sort of "naive" time reversal, is erroneous when the initial or final state interaction cannot be neglected. The coulomb correction to beta decay does indeed give rise to a correlation of kind 3.16, but with a small coefficient ($e \approx 1/137$).

Pion Decay

The matrix element for pion decay is given by

$$\frac{G}{\sqrt{2}} \langle\, 0 \mid J_\lambda{}^s \mid \pi^+ \,\rangle\; \bar{u}(\nu)(1 - \gamma_5)\gamma_\lambda\; v(\ell^+) \;; \tag{3.23}$$

here we consider $\pi^+ \to \ell^+ + \nu$, and $\ell^+ = e^+$ or μ^+ . As we have seen

$$\frac{G}{\sqrt{2}} \langle\, 0 \mid J_\lambda{}^s \mid \pi^+ \,\rangle \;=\; f\, p_\lambda{}^\pi \,. \tag{3.24}$$

But, thanks to energy momentum conservation

$$p_\lambda{}^\pi = p_\lambda{}^\ell + p_\lambda{}^\nu$$

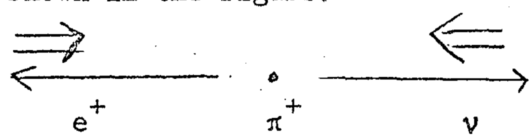
so that the matrix element becomes

$$f\; \bar{u}(\nu)(1 - \gamma_5)(\not{p}^\ell + \not{p}^\nu)\; v(\ell^+) = -i m_\ell\, f\; \bar{u}(\nu)(1 - \gamma_5)\; v(\ell^+) \;.$$

The matrix element results proportional to the lepton mass. The ratio between the two decay results

$$\frac{\pi \to e\nu}{\pi \to \mu\nu} = (m_e/m_\mu)^2 \left[(1 - m_e{}^2/m_\pi{}^2)/(1 - m_\mu{}^2/m_\pi{}^2) \right]^2$$
$$\approx 1.28 \times 10^{-4} \;.$$

This prediction, which has been checked to a few percent, is characteristic of the V-A theory. The depression of the electron in respect to the muon can be easily understood. Conservation of angular momentum insures that the electron and neutrino will have the <u>same</u> helicity in the pion rest system, as shown in the figure:



,

but the A-V Lagrangian favors the emission of $\ell^+\nu$ pair of opposite helicity; the emission of pairs of <u>like</u> helicity, forced by angular momentum conservation, is depressed. If the interaction were of the form S, P, T, which favors emission of pairs with like helicity, the ratio would ~1, and in fact the electron mode would be favored by phase space.

Problems

Many of the unproved assertions in this chapter can be checked without a complete calculation.

1. In Eq. (3.7) we see that the e-ν correlation is opposite for V and S, and for T and A. You can prove this directly from Eq. (3.4). The square of the matrix element for pure S and pure V results

$$\cdots\ g_S\ v(\bar{\nu})\ \bar{v}(\bar{\nu})\, g_S^{\;*}\ \cdots\ \approx\ \cdots\ |g_S|^2\ \not{p}_\nu\ \cdots$$

and

$$\cdots\ g_V\ \gamma_4\ \not{p}_\nu\ \gamma_4\ g_V^{\;*}\ \cdots\ .$$

The two differ only by $g_S \to g_V$, $\vec{p}_\nu \to -\vec{p}_\nu$ which gives opposite $\vec{p}_e \cdot \vec{p}_\nu$ terms.

Complete the argument for T and A. Are you able to actually derive 3.7?

2. Using the same kind of partial computation as in problem 1, show that the parity conserving effects are proportional to $g_i\, g_j + g_i{}'\, g_j{}'$. (For example, $g_S\, g_V + g_S{}'\, g_V{}'$). To do this, use Eq. (3.4), and separate in the square of the matrix element those terms which do not contain a γ_5. (Those which contain two can be reduced!) Remember that the complex conjugate of

$$\bar{e}\left\{O_1 + \gamma_5\, O_2\right\}\nu$$

is

$$\nu^*\left\{O_1^{\,+} + O_2^{\,+}\,\gamma_5\right\}\gamma_4\, e$$

which you want to rewrite as $(\bar{\nu} = \nu^* \gamma_5)$

$$\bar{\nu} \quad \cdots \quad e$$

since

$$v(\nu)\ \bar{v}(\nu) \Longrightarrow \not{p}_\nu \ .$$

3. One of the methods for measuring the helicity of positrons is by "in-flight" annihilation on polorized electrons. This is based on the difference in annihilation cross section of pairs with the same or opposite helicities. Electromagnetic interactions are of the form γ_μ, so that they favor at relativistic energy $(m_e/E_e \approx 0)$ a definite relation among the helicities. Which one?

For very low energy the annihilation should mainly proceed from S waves, and for the annihilation into 2γ one obtains again a simple result (apply the selection rules for the S states of positronium). From this qualitative argument, sketch the behavior of the quantity

$$\frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}$$

as a function of the energy.

Compare with the theoretical results in I. A. Page, Ann. Rev. Nuclear Sci. <u>12</u>, 43, 1962.

WEAK INTERACTIONS V

Lecture IV

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1. Muon Decay

Muon decay was around 1957 in a state very similar to that of beta decay: the Hamiltonian was also supposed to be of the four Fermion non-derivative kind, like:

$$H = \sum_i (\bar{\mu}\, O^i\, \nu)(\bar{\nu}\, O^i (f_i + f_i'\, \gamma_5)\, e) + h.\, c. \tag{4.1}$$

Even more complicated forms were possible if one released the condition of lepton conservation, so that, together with

$$\mu^+ \to e^+ + \nu + \bar{\nu} \tag{4.2}$$

one would have

$$\mu^+ \to e^+ + \nu + \nu \tag{4.3}$$

$$\mu^+ \to e^+ + \bar{\nu} + \bar{\nu}$$

If we assume some form of universality in the weak interactions of leptons, we can make the following guesses.

i. Leptonic number is conserved, so that muon decay is described by (4.2).

ii. The neutrinos are the same two component neutrinos which enter in beta decay and pion decay.

iii. The e^+ will have positive helicity, (the electron is ultra relativistic, so that $\beta_e \approx 1$).

This determines the form of H up to a constant. From ii. and iii. we can rewrite

$$H = \Sigma f_i (\bar{\mu}\, O^i (1 + \gamma_5)\, \nu)(\bar{\nu}\, (1 - \gamma_5)\, O^i (1 + \gamma_5)\, e)$$

Only A and V can enter, since the operators for SPT commute with γ_5, and we have, e.g.

$$(1 - \gamma_5)\, \sigma_{\mu\nu}\, (1 + \gamma_5) = \sigma_{\mu\nu}\, (1 - \gamma_5)(1 + \gamma_5) = 0$$

furthermore $O^i = \gamma_\mu$ and $O^i = i\gamma_\mu \gamma_5$ reduce to the same form, so that we can write

$$H = \frac{G}{\sqrt{2}}\, (\bar{\mu}\, \gamma_\lambda\, (1 + \gamma_5)\, \nu)(\bar{\nu}\, \gamma_\lambda\, (1 + \gamma_5)\, e) + \text{h. c.} \qquad (4.4)$$

Note that $(\bar{\mu}\, \nu)$ and $(\bar{\nu}\, e)$ appear in the same combinations that enter in beta decay.

By definition we will call G the Fermi coupling constants. G has the dimension of ℓ^2 or M^{-2} (in units $\hbar = c = 1$,. In fact H is an energy density, $M\ell^{-3}$ or M^4 (see footnote 1 to 1st Lecture), while $(\bar{\mu}\, \nu), (\bar{\nu}\, e)$ have the dimensions of a density, ℓ^{-3} or M^3. G can be obtained from the muon lifetime: without taking into account radiative corrections (and putting $m_\ell = 0$). This is given by

$$\tau_\mu^{-1} = \frac{G^2\, m_\mu^5}{24 (2\pi)^3} \qquad (4.5)$$

From the experimental $(\tau_\mu \sim 2.2 \times 10^{-6}$ s) we get

$$G \sim 10^{-5} \, m_p^{-2} \tag{4.6}$$

The use of m_p^{-2} units is useful because of this rather neat result (good to about 2%).

Other measurable quantities in muon decay are:

1. The electron spectrum.
2. The up-down asymmetry of the electron in respect to the muon polarization, as a function of the electron energy.
3. The electron polarization.

A general expression for 1. and 2. (valid for the Hamiltonian (4.1), and also in the case of non conservation of the lepton number) has been given by Bouchiat and Michel Phys. Rev. 106, 170 (1957):

$$dW = x^2 \left\{ 3(1-x) + 2\,\rho\left(\frac{4}{3}x - 1\right) + \xi(\vec{P}\cdot\hat{p}_e)\left[(1-x)+2\delta\left(\frac{4}{3}x-1\right)\right]\right\} dx\, d\Omega_e \tag{4.7}$$

Here $x = 2E_e/m_\mu$ is the electron energy in units of its maximum value; ρ, ξ, δ are parameters which depend on the different coupling constants. The V - A Hamiltonian (4.4) predicts (for μ^-)

$$\rho = 3/4$$

$$\xi = -1 \tag{4.8}$$

$$\delta = 3/4$$

for μ^+, $\xi = +1$.

Furthermore the $e^-(e^+)$ should have negative (positive) helicity. All these predictions have been experimentally checked within some percent.

Problems

1. The spectrum for $\rho = 3/4$ is peaked at $x = 1$. Find a simple argument (based on the conservation of angular momentum) to connect the value of ξ to the helicities of electron and neutrino.

2. If the two neutrinos which are emitted are identical, the spectrum cannot be peaked at $x = 1$, so that $\rho \neq 3/4$. Hint: what kinematical configuration corresponds to $x = 1$? By a qualitative argument show that in this case you expect $\rho = 0$.

2. The Conserved Vector Current Hypothesis

We have seen that an analysis of beta decay brought to the V - A Theory, embodied in a lagrangian of the form discussed in the first lecture

$$\frac{G}{\sqrt{2}}(J^s{}_\lambda J^\ell{}_\lambda + J^{s^*}{}_\lambda J^{\ell^*}{}_\lambda + J_\lambda{}^\ell J^{\ell^*}{}_\lambda) \tag{5.1}$$

This form has also been found to be very successful in explaining the details of muon decay and the ratio $\pi \to e\nu/\pi \to \mu\nu$. From the study of beta decay we know that a phenomenological form for the J^s is given by

$$\frac{G}{\sqrt{2}} J_\lambda{}^s \approx \bar{\psi}_p(g_v + g_a\gamma_5)\gamma_\lambda \psi_n \tag{5.2}$$

The numerical value of g_v is obtained from the lifetime of pure Fermi beta decays, ($J = 0 \to J = 0$, no parity change) where only the vector part can contribute, like

$$O^{14} \to N^{14} + e^+ + \nu \tag{5.3}$$

Problem

The axial current cannot contribute to a "pure Fermi" transition, independently of the approximation of allowed transition. Show this by the same technique you used to solve problem # 1 in the second lecture, relating to $K^- \to e^- + \bar{\nu} + \pi^0$.

Although equation (2) refers to free nucleons, it can be used to evaluate nuclear beta decay if

a) nuclei are considered as made up of essentially free nucleons (i.e. whose physical properties like mass, charge, magnetic moment, etc. are the same as those of free nucleons).

b) We have some knowledge of the wave functions.

In the case of O^{14} beta decay, the fact that O^{14} and N^{14} belong to the same I - spin triplet gives sufficient information to evaluate the lifetime from (1) and (2) and obtain g_V from a comparison with experiment. The result is that, within a few per cent,

$$g_v = \frac{G}{\sqrt{2}} \tag{5.4}$$

We can now compare beta decay with muon decay; the Hamiltonians are respectively (within some percent)

$$\frac{G}{\sqrt{2}} \left(\bar{\psi}_p \gamma_\lambda (1 + 1.25\, \gamma_5)\, \psi_n \right) \left(\bar{e}\, \gamma_\lambda (1 + \gamma_5) \nu \right) \tag{5.5}$$

$$\frac{G'}{\sqrt{2}} \left(\bar{\nu}\, \gamma_\lambda (1 + \gamma_5) \mu \right) \left(\bar{e}\, \gamma_\lambda\, (1 + \gamma_5)\, \nu \right) \tag{5.6}$$

The vector part in beta decay is the same as in μ-decay, to a good accuracy, too good in fact to be a mere coincidence. The solution which was independently proposed by Gershtein and Zeldovich, and Feynman and Gell-Mann, is based on two assumptions:

1. The weak interactions are universal. In the absence of strong interactions the coupling constants of beta decay and muon decay would be equal.
2. The vector coupling constant is not changed by the presence of strong interactions. (In the theoretist's language, it is not renormalized) Both assumptions are necessary for a satisfactory explanation.

Non renormalization is a well known effect; for example the charge of a proton is equal to that of an electron, in spite of the strong interactions of the proton. This can be explained in two steps similar to 1 and 2. First we assume that all strongly interacting particles have charge which is a multiple of e, the electron charge, and then we show that this situation is stable against any effects of the strong interactions, if strong interactions conserve charge. The charge of a proton, e. g., will not be changed by the fact that it can go virtually into a neutron plus a positive pion.

In a similar way it is possible to show that non renormalization of the vector coupling in beta decay can be assured if the $\Delta S = 0$ vector part of J^S_λ is a conserved current.

$$\partial_\lambda J_\lambda^{V,0} = 0 \qquad (5.7)$$

(The subscript 0 indicates the strangeness change.) From Eq. (7) it follows that

$$U = \int J_0^{V,0} d^3x$$

is conserved by strong interactions.

This could not have been passed unobserved, so let us see whether we can identify U with a known quantity. The only possible candidate is the I^+ component of the isotopic spin. We will therefore set

$$j_\lambda^{v,0} \propto j_\lambda^{(+)} = j_\lambda^1 + ij^2_\lambda \tag{5.8}$$

Equation (8) is rich of physical consequences. The current $j_\lambda^{(+)}$ is a close relative of the electromagnetic current. From the relation

$$Q = I^3 + \frac{1}{2} Y$$

we have

$$j_\lambda = j^3 + \frac{1}{2} j_\lambda^Y \tag{5.9}$$

j^3 is the third component of the J-spin triplet of currents to which $j^{(+)}$ belongs. This means that we can find relations among matrix elements of j^3 and of $j^{(+)}$.

Let us obtain these from the commutation relations

$$[I_1\, j^2_\lambda] = i\, j_\lambda^3 \qquad \text{and cyclic} \tag{5.10}$$

(These are a direct consequence of j_λ^i being a triplet, i. e., an I-spin vector. Remember the similar commutation relations among a vector and the total angular momentum.) From these we can get .

$$j_\lambda^{(+)} = [j^3 , I^+] \tag{5.11}$$

Furthermore the hypercharge current j^Y commutes with the isotopic spin (it is an iso-scalar), so that

$0 = \frac{1}{2} [j_\lambda^Y , I^+]$ and adding this to Eq.(11),

$$j^{(+)}_\lambda = [j_\lambda , I^+] \tag{5.12}$$

To see how this relation works, take it among a proton and a neutron state:

$$\langle p|\, j_\lambda^{(+)}\, |n\rangle = \langle p|\, j_\lambda\, I^+\, |n\rangle - \langle p|\, I^+\, j_\lambda\, |n\rangle$$

but, since proton and neutron form a doublet

$$I^+|n\rangle = |p\rangle$$

$$I^-|p\rangle = |n\rangle \qquad \text{or} \qquad \langle p|I^+ = \langle n| \tag{5.13}$$

and

$$\langle p|j^{(+)}{}_\lambda|n\rangle = \langle p|j_\lambda|p\rangle - \langle n|\, j_\lambda\, |n\rangle \tag{5.14}$$

In the limit of small velocity and momentum transfer, we have

$$\langle p|j|p\rangle \approx \bar{u}\, \gamma_\mu\, u$$

$$\langle n|j|n\rangle \approx 0$$

so that

$$\langle p|j^{(+)}\, |n\rangle = \bar{u}_p\, \gamma_\mu\, u_n \tag{5.15}$$

It is easy to see that equality of vector parts in μ-decay and beta decay (Eq. 4) follows, if we assume that Eq. (8) holds, with an equal sign. In general Eq. (8), (compare Eqs. (2) and (15)) can be rewritten as

$$J_\lambda^{V,0} = \frac{g_V \sqrt{2}}{G}\, j^{(+)} \tag{5.16}$$

Now we assume that the vector current is conserved and we ask: is Eq. (4) strictly valid? The best test is to compare muon decay and the O^{14} decay (or another Fermi transition). If we assume the validity of Eq. (16) we are in a much better situation than before in predicting the O^{14} lifetime.

we can evaluate this decay like that of any other decay of elementary particles, as discussed in the second lecture. The matrix element is given by (see footnote 1 to second lecture.)

$$\frac{G}{\sqrt{2}} \left\langle O^{14} \right| J_\lambda^{\,s} \left| N^{14} \right\rangle F \,\bar{u}(\nu)\, \gamma_\lambda \,(1 + \gamma_5)\, v(e) \tag{5.17}$$

Where F is the Coulomb correction to the electron wave-function, not negligible here. Now, from Eq. (16) we get (the A part does not contribute)

$$\frac{G}{\sqrt{2}} \left\langle O^{14} \right| J_\lambda^{\,s} \left| N^{14} \right\rangle = g_V \left\langle O^{14} \right| j_\lambda^{(+)} \left| N^{14} \right\rangle \tag{5.18}$$

but (Eq. 12)

$$\left\langle O^{14} \right| j_\lambda^{(+)} \left| N^{14} \right\rangle = \left\langle O^{14} \right| j_\lambda \, I^+ \left| N^{14} \right\rangle - \left\langle O^{14} \right| I^+ \, j \left| N^{14} \right\rangle$$

Since N^{14} and O^{14} belong to an I-spin triplet: $|N^{14}\rangle = |1, 0\rangle$, $O^{14} = |1, 1\rangle$, we have

$$I^+ |N^{14}\rangle = |O^{14}\rangle \sqrt{2}$$

$$\langle O^{14}| I^+ + \langle N^{14}| \sqrt{2}$$

and

$$\left\langle O^{14} \right| j_\lambda^{(+)} \left| N^{14} \right\rangle = \sqrt{2}\Big(\left\langle O^{14} \right| j_\lambda \left| O^{14} \right\rangle - \left\langle N^{14} \right| j_\lambda \left| N^{14} \right\rangle \Big)$$

The right member of this equation is well known: the matrix element of e.m. current among spin zero particles:

$$\left\langle O^{14} \right| j_\lambda \left| O^{14} \right\rangle = \frac{8\, F(q^2)}{2(2\pi)^3 \sqrt{E\,E'}} \quad (p_\lambda + p_\lambda{}')$$

Where $F(q^2)$ is the electric form factor, function of the momentum transfer $q^2 = (p^\nu + p^e)^2$. p_λ and p_λ' are the initial and final four momenta of the nucleus, E and E' the initial and final <u>total</u> energies. In our conditions only the fourth component of the current is important:

$$\langle O^{14}|j_0|O^{14}\rangle \approx \delta(2\pi)^{-3}$$

but we can use the exact expression directly. The final result is:

$$\frac{G}{\sqrt{2}} \langle O^{14}|J_\lambda^{\,s}|N^{14}\rangle = g_V \frac{F(g^2)}{2(2\pi)^3\sqrt{E\,E'}} (p_\lambda + p_\lambda') \qquad (5.19)$$

Note that this result depends only on I-spin considerations, not on any hypothesis on the nucleus structure. Even if N^{14} is not made of $7p + 7n$, but, say, $14n + 7\pi^+$ the result would be the same, once Eq. 16 is accepted.

Pushing this analysis to the end, one finds a small discrepancy: g_V^2 is about 2% smaller than $G^2/2$. The situation seems not to be bettered by radiative corrections to both muon decay and O^{14}. These are somewhat controversial, but the most widely accepted computations, due to Berman, increase the discrepancy to about 4.5% (± 1.5%). A clear discussion of this problem has been given by R. P. Feynman in the Rochester Conference 1960 (Reports, page 501). The discrepancy, which is there of 4.0% has slightly increased due to more recent measurements of the muon lifetime.

What is the meaning of the discrepancy? As we will discuss later, the best solution is probably in a revision of the concept of universality. In fact the CVC hypothesis has now received independent confirmation from other experiments, as we will now discuss.

Other Consequences of CVC

Another process which is assimilable to a pure Fermi transition is the "beta decay" of the pion

$$\pi^+ \to \pi^0 + e^+ + \nu$$

This is very similar to O^{14} beta decay (a transition among two members of a 0-spin triplet) so that we can derive the matrix element in close parallel with the previous use, and we will get the same result for the matrix element:

$$\frac{G}{\sqrt{2}} \left\langle \pi^+ \middle| J_\lambda^{\,s} \middle| \pi^0 \right\rangle = \frac{\sqrt{2}\ \mathcal{E}_V}{2\sqrt{E\,E'}\ (2\pi)^3}\ (p_\lambda + p_\lambda') \tag{5.20}$$

The rate for this mode is therefore uniquely determined and gives a branching ratio

$$\frac{\pi^+ \to \pi^0\ e\ \nu}{\pi^+ \to \mu^+ + \nu} = 10^{-8} \tag{5.21}$$

Problem

Prove Eq. (20) by the same procedure used in the O^{14} case. An experiment at CERN has checked this prediction within the experimental errors which were of some tens %. It seems difficult to do much better, but this is already a very good result, since the rate of this process could a priori (without CVC) vary in a wide range of values, perhaps from 10 times less to 10 times more than predicted by CVC.

The Weak Magnetism

Eq. should be valid in general, not only in the limit of slow nucleons and small momentum transfer. In the general case we get

$$g_V \langle p | j_\lambda^{(+)} | n \rangle = g_V \bar{u}(p) \left\{ F_1^V(q^2)\gamma_\lambda + \frac{1}{2M} F_2^V(q^2)\sigma_{\lambda\mu} q_\mu \right\} u(n) \qquad (5.22)$$

Where $F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$ are the isovector form factor of the nucleon, as measured in electron scattering experiments. Beyond the usual term γ_λ we find a new term, whose magnitude is exactly predicted. This is called the weak magnetism, since the corresponding terms in the e.m. current are associated with the anomolous magnetic moments of proton and neutron. In the limit of $q^2 = 0$ we have

$$F_1^V \to 1$$

$$F_2^V \to \mu_p - \mu_n \sim 3.7 \qquad (5.23)$$

Weak magnetism gives at most a first forbidden contribution to beta decay, since the term is of order compared to the γ_μ term. This first forbidden contribution is

$$g_V \frac{1}{2M} F_2 \sigma_{\lambda\mu} g_\mu \Longrightarrow g_V \frac{\mu_p - \mu_n}{2M} \vec{q} \times \vec{\sigma} \qquad (5.24)$$

A similar contribution comes from the γ_μ term, and, as can be expected, is obtained by substituting the anomolous magnetic moment of the proton with the total magnetic moment:

$$g_V \frac{1 + \mu_p - \mu_n}{2M} \vec{q} \times \vec{\sigma} \qquad (5.25)$$

In a beta decay this term would give the same selection rules as $\vec{\sigma}$, the allowed part of the axial contribution, so that it will appear as a correction to Gamow-Teller transitions. (Note to this end that $\vec{q} = \vec{P}_e + \vec{P}_\nu$ does not depend on the nucleon coordinates.)

The main effect of the weak magnetism term on an allowed beta decay would be to modify the spectrum from the allowed form to

$$W_0(E_e) \Longrightarrow W_0(E_e) \quad (1 + \frac{8}{3} \; a \; E_e) \tag{5.26}$$

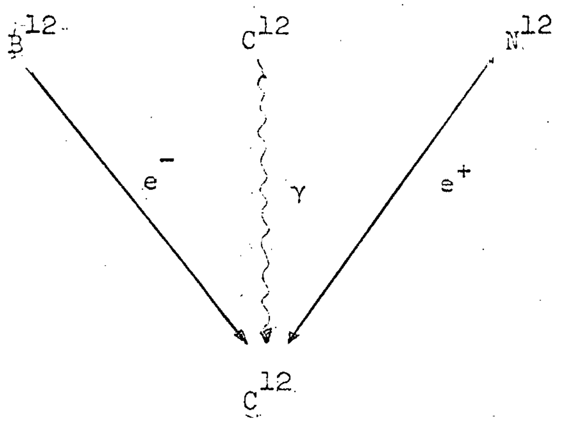
where

$$a \approx \frac{1 + \mu_p - \mu_n}{M \sqrt{2}} \quad \frac{g_V}{g_A}$$

Problem

Derive the non relativistic form of the weak magnetism term according to the methods given in the third lecture.

The existence of this eviation has been found, with a magnitude consistent with the predictions, following a suggestion of Gell-Mann, by comparing the spectra of two mirror processes, namely the β^+ decay of N^{12}



and the β^- decay of B^{12}, both going to the ground state of C^{12}. B^{12} and N^{12} are members of an I-spin triplet with $J = 1$, positive parity, together with an excited state of C^{12} which decays to the ground state $(I = 0)$ by emission of a γ (via magnetic dipole). The interest of comparing the two spectra comes from the fact that the correction term, arising from an A - V interference, has the opposite sign in the two. Note that

a. the effect is expected also without CVC, from the normal contribution. It is the magnitude which is important, $1+\mu_p-\mu_n \approx 4.7$, to be compared with ≈ 1.

b. In principle it is possible to obtain the correction directly from the knowledge of the transition rate for $C^{12*} \rightarrow C^{12}$, but in practice the experiments are compared with a prediction based on nuclear physics. This prediction is expected to be rather accurate, since weak magnetism and axial vector contributions are respectively proportional to the matrix elements of $\bar{\psi}_p \vec{\sigma} \psi_n$ and $\bar{\psi}_p \gamma_4 \vec{\sigma} \psi_n$ which, according to our discussion in the last lecture, are expected to be essentially equal.

WEAK INTERACTIONS VI

Lecture V

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Any Other Conserved Current?

The $\Delta S = 0$ vector part of $J_\lambda^{\,s}$ has been found to be conserved. What about other parts? Neglecting for the moment the possibility of $\Delta S = -\Delta Q$ transitions, we can write

$$J_\lambda^{\,s} = J_\lambda^{\,V,0} + J_\lambda^{\,V,1} + J_\lambda^{\,A,0} + J_\lambda^{\,A,1} \tag{6.1}$$

Let us see whether $J_\lambda^{\,A,0}$ can be conserved. To do so, consider pion decay (see Eq. 3.24)

$$\frac{G}{\sqrt{2}} \left\langle \pi^+ \middle| J_\lambda^{\,A,0} \middle| 0 \right\rangle = f\, p_\lambda^{\,\pi} \tag{6.2}$$

In order to calculate the matrix element of the divergence of the current (expected to be zero if $J_\lambda^{\,A,0}$ is conserved), we must know that

$$\left\langle \pi^+ \middle| J_\lambda^{\,A,0}(x) \middle| 0 \right\rangle = \exp\,[ip^\pi x]\, \left\langle \pi^+ \middle| J_\lambda^{\,A,0}(0) \middle| 0 \right\rangle . \tag{6.3}$$

This follows from translational invariance: the matrix element of $J(x)$ among certain states is equal to the matrix element of $J(0)$ among the same states "translated" by $-x$. States which have a definite total momentum simply pick up a factor under translation

$$T(x)|p\rangle = e^{i(px)}|p\rangle$$

$$\langle p|T^{*}(x) = \langle p|e^{-i(px)}$$

Incidentally, note that we write $\langle B|J_\lambda{}^s|A\rangle$ forgetting that $J_\lambda{}^s$ depends on x. The reason is that (use momentum conservation)

$$\langle B + \ell^- + \bar{\nu}\ |J_\lambda{}^s(x)\ J_\lambda{}^\ell(x)|A\rangle = \exp\ [i(p^B + p^\ell + p^\nu - p^A)x]$$

$$\cdot \langle B + \ell^- + \bar{\nu}\ |J_\lambda{}^s(0)J_\lambda{}^\ell(0)|A\rangle = \langle B + \ell^- + \bar{\nu}\ |J_\lambda{}^s(0)J_\lambda{}^\ell(0)|A\rangle$$

the argument is conventionally taken as $x = 0$ and dropped. Now from 3 and 2 we get:

$$\frac{G}{\sqrt{2}}\langle\pi^+|\partial_\lambda J_\lambda{}^{A,0}|0\rangle = \frac{G}{\sqrt{2}}\, ip_\lambda{}^\pi\ \langle\pi^+|J_\lambda{}^{A,0}|0\rangle\ = -i\ f\ m_\pi{}^2 \qquad (6.4)$$

This means that $J_\lambda{}^{A,0}$ <u>cannot</u> be conserved, since the pion decays ($f \neq 0$), and its mass is different from zero.

Another way of proving that $J_\lambda{}^{A,0}(x)$ cannot obey a continuity equation is to explore the properties of the pseudoscalar quantity which would be as a consequence conserved: (this argument is due to Okubo)

$$U' = \int J_0{}^{A,0}(x)\ d^3x \qquad (6.5)$$

If U' is conserved, it commutes with the total Hamiltonian: $[U',H] = 0$. This means that, if $|A\rangle$ is a state of energy E

$$H|A\rangle = E|A\rangle$$

then

$$H\ U'|A\rangle = U'H|A\rangle = E\ U'|A\rangle$$

so that (unless $U'|A\rangle = 0$) the state $U'|A\rangle$ would have the same energy as $|A\rangle$. $U'|A\rangle$ should have the same strangeness and baryon number as $|A\rangle$, but opposite parity. We know that this is not so: there is <u>no</u> state with the same baryon number, strangeness, and energy as a neutron at rest, and having the opposite parity. A possible state would be a $p + \pi^0$, in an S state, but the energy is at least one pion mass too large.

The second argument can be applied to the other two parts of the current, $J_\lambda^{V,1}$ and $J_\lambda^{A,1}$. In this case we run into the difficulty of not having states of the same baryon number as the neutron, but strangeness ± 1 . (One would consider the action of both $\int J_0^{V,1}\, d^3x$ and it is Hermitian conjugate). If we start from neutron, we reach either Σ^- or $n + K^-$ which have higher energy. If we assume that, according to the SU_3 scheme, the eight previous N, Σ, Λ, Ξ have the same parity we can see that

a. $J_\lambda^{V,1}$ cannot be conserved because of the mass differences among baryons of different strangeness.

b. $J_\lambda^{A,0}$ and $J_\lambda^{A,1}$ cannot be conserved because starting from the eight baryons we cannot find other states having the same baryonic number but opposite parity. This could be blamed on the fact that m_K and m_π are $\neq 0$.

<u>Problems</u>

1. These last arguments would be only slightly modified if parity assignments within the baryon octet were different.

2. Prove that $J_\lambda^{A,1}$ is not conserved directly from the existence of $K \to \mu\nu$ decay.

2. The Structure of Axial Currents

In this and the following subsections, we discuss the possible structure of the axial currents, especially the $\Delta S = 0$ part $J_\lambda^{A,0}$. We have seen that the allowed contribution of $J_\lambda^{A,0}$ to beta decay is

$$\frac{G}{\sqrt{2}} J_\lambda^{A,0} \approx g_A \bar{\psi}_p \gamma_5 \gamma_\lambda \psi_n \tag{6.6}$$

If we extend this to include all possible contributions, even if they do not give allowed contributions, we get three possible terms

$$\frac{G}{\sqrt{2}} J_\lambda^{A,0} \approx \bar{\psi}_p \gamma_5 \left\{ g_A \gamma_\lambda + g_A' q_\lambda + h_A \frac{1}{2\pi} \sigma_{\mu\lambda} \quad \lambda \right\} \psi_n \tag{6.7}$$

q_λ being the momentum transfer to the leptons. Any other possible term is reduced to one of these by use of the Dirac Equation (remember that fields in a phenomenological expression are to be considered as free fields). The third term is assumed to be zero. The reasons for this are connected with the G-parity behavior of the different terms: G parity means $n \leftrightarrow \bar{p}$ so that the current 6.7 (which gives processes like $n + \bar{p} \to \mu^- + \bar{\nu}$) is transformed into itself. Weinberg (P.R. 112, 1375, 1958) proved that the h_A term has positive G parity, while the other two have negative one. Since G-parity is a good symmetry for strong interactions, we can classify currents according to it, as an extension of the process discussed in the second lecture:

$$J_\lambda^{A,0} = J_\lambda^{A,0,-1} + J_\lambda^{A,0,+1} \tag{6.8}$$

Since the guiding principle is to assume that a new piece of the current is not accepted until it is proved to exist, it is now generally assumed that $h_A = 0$.

The same situation appears in the case of the vector current. The most general form of it's matrix element among nucleon states contains a third term, beyond the two discussed in the last lectur (see Eq. 5.22); we can write

$$\frac{G}{\sqrt{2}} J_\lambda^{V,O} = \bar{\psi}_p \left\{ g_V \gamma_\lambda + \frac{g_V'}{2M} \sigma_{\lambda\mu} q_\mu + h_V q_\lambda \right\} \psi_n$$

The "new" term, h_V , comes from a part of the current which has a negative G-parity, while the other two have a positive G-parity. Note that according to the conserved vector current hypothesis $J_\lambda^{V,O} \propto j_\lambda^{(+)}$, which has positive G parity, so that h_V cannot arise from ~~it~~. Note that the h_V term, even if present would not substantially alter any of the extant proofs of the CVC hypothesis:

i. A vector current with $G = -1$ cannot contribute to $\pi^+ \to \pi^0 e^+ \nu$

ii. By use of the Dirac equation for electron and neutrino we can operate the following reduction:

$$h_V (\bar{\psi}_p \psi_n) \left(\bar{e} (\not{p}_e + \not{p}_\nu)(1 + \gamma_5) \nu \right) = i\, h_V m_e (\bar{\psi}_p \psi_n) \left(\bar{e} (1 + \gamma_5) \nu \right) \quad (6.9)$$

The h_V contribution is therefore equivalent to a <u>scalar</u> contribution with $g_s = m_e h_V$. Given the smallness of m_e this would be a <u>small</u> contribution to pure Fermi decays. (O^{14})

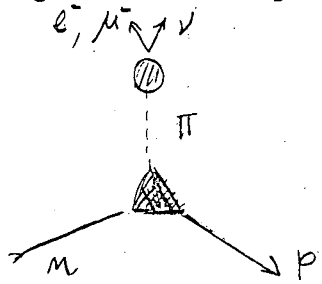
iii. The selection rules of the h_V term would not allow contributions to the B^{12}, $N^{12} \to C^{12}$ transitions.

We will for the moment assume that $h_V = h_A = 0$. The best way to search for these terms is presumably through muon capture. The assumed absence of $J_\lambda^{A,O,+1}$ and $J_\lambda^{A,O,-1}$ can also be tested in some hyperon decay, like, e.g. $\Sigma^\pm \to \Lambda + e^\pm + \nu$.

Problem

Check points i. to iii. in particular Eq. (9).

Let us come back to the axial part. Some insight is gained into it's structure through the use of dispersion relations. The idea is to investigate what systems can be exchanged between the nucleon and the lepton pairs (in analogy with the analysis of electromagnetic form factors). Only one contribution - that of a pion exchange - can be completely evaluated in terms of known quantities:



$$= i\, \gamma_5\, q_\mu \frac{g_{\pi np}\, f}{q^2 + m_\pi^2}$$

One recognizes here the π decay current ($f\, q_\mu$), the pion propagator $\frac{1}{q^2+m_\pi^2}$, the pion-nucleon coupling $\gamma_5\, g_{\pi np}$.

The pion pole contributes to the g_A' term: we assume in fact that g_A' is dominated by this contribution, at least for the low momentum transfers which are relevant in beta decay or muon capture. We are not able to evaluate other contributions explicitly, so that g_A remains totally unaccounted for. We will write:

$$\frac{G}{\sqrt{2}} \left\langle p \middle| J_\lambda^{A,0} \middle| n \right\rangle \approx \bar{u}_p \quad \gamma_5 \left(g_A\, \gamma_\lambda + i\, \frac{g_{\pi np}\, f}{q^2 + m_\pi^2}\, q_\lambda \right) u_n \qquad (6.11)$$

Note that we have only learned the <u>magnitude</u> of g_A' , not its sign, since from the pion lifetime, we can only determine $|f|^2$. The sign is however determined by the Goldberger-Treiman relations.

<u>Problem</u>

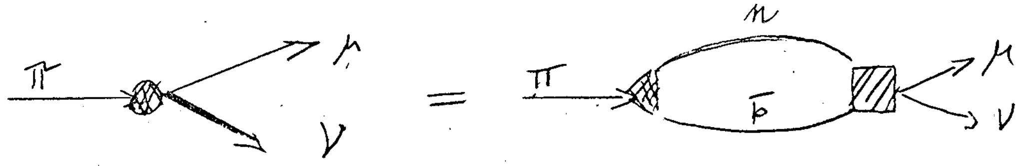
Show that a pole term which contributes to g_A should come form the exchange of a boson with $J^P = 1^+$, $I = 1$, $G = -1$.

The Goldberger and Trieman Relation

This was obtained in an attempt to evaluate the pion lifetime by dispersion techniques, and accounted for it with a remarkable accuracy.

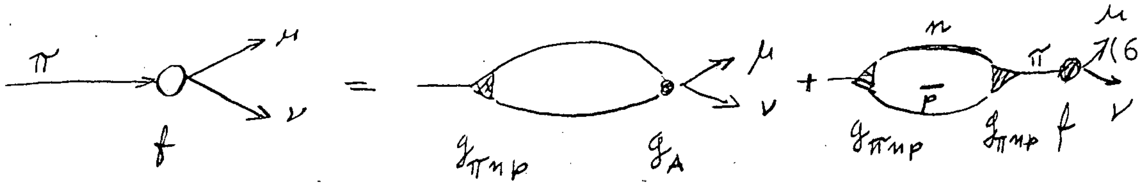
I will only give a sketch of the original derivation and will rather emphasize later derivations of the G-T relation, due to Nambu, Gell-Mann and Levy.

The original derivation started from the hypothesis that f is dominated by the virtual decomposition of the pion into an $N\bar{N}$ pair:



(6.12)

The square box indicates here the process-connected with beta decay, $n + \bar{p} \to \mu^- + \bar{\nu}$. This is assumed to have the form discussed in the preceding subsection, so that the second memeber of the graphical equation above can be split in two terms:



(6.13)

where we have noted the coupling constant at each vertex. The loops on the two sides correspond to integration over all possible momenta of $\bar{p}$ n pairs. The two loops result in essentially the same integral, J . Since f appears on the two sides, we can solve for f. The result is:

$$f = 2m_N \frac{g_{\pi np}\, g_A\, J/4\pi^2}{1 + g^2_{\pi np}\, J/4\pi^2} \tag{6.14}$$

Goldberger and Treiman found that J is probably large enough, so that we can neglect the 1 in the denominator, and have:

$$f = \frac{2m_V\, g_A}{g_{\pi np}} \tag{6.15}$$

This equation is nearly satisfied if we insert experimental numbers.

Problem

Show that up to factors 2π , Eq. 14 follows from Eq. 13. The factor m_N will be explained later.

G-T Relations - other interpretations

The success of the G-T approach remains somewhat mysterious, in view of the many drastic approximations involved: neglect any contribution apart from $\overline{p}n$, use of Eq. 11 also far from the region of low momentum transfer, etc.

To understand this let us note that in obtaining the G-T relation, Eq. 15, the first member of the symbolic equation 13 has completely dropped out. We can interpret Eq. 15 as a condition which insures that the second member of the symbolic equation is not too large, and large it tends to be, since it involves an integration over all possible $\bar{p}\, n$ states of any energy. In this sense the G-T condition can be seen as a sort of self-consistency condition, which should remain valid even if we include other contributions (e.g. that of $\Sigma^- \bar{\Lambda}$. and other hyperon-antihyperon pairs.)

A very simple derivation of Eq. 15 was given by Nambu. He noted that--as we have discussed at the beginning of this lecture--all impediments to the conservation of $J_\lambda^{A,0}$. are connected with the fact that the pion has a non-zero mass. He advanced the hypothesis that $J_\lambda^{A,0}$ is nearly conserved, and that it is actually a conserved current in the limit in which we put $m_\pi = 0$. Consider the matrix element of $\partial_\lambda J_\lambda^{A,0}$ among nucleon states. We get (see Eq. 11)

$$\frac{G}{\sqrt{2}} \langle p | \partial_\lambda J_\lambda^{A,0} | n \rangle = \frac{G}{\sqrt{2}} i\, q_\lambda \langle p | J_\lambda^{A,0} | n \rangle$$

$$= \bar{u}(p)\, \gamma_5 \left\{ g_A\, i(\not{p} - \not{n}) - \frac{g_{\pi np}\, f}{q^2 + m_\pi^2}\, q^2 \right\} u(n)$$

$$= \bar{u}(p)\, \gamma_5 \left\{ 2m_N\, g_A - \frac{g_{\pi np}\, f}{q^2 + m_\pi^2}\, q^2 \right\} u(n)$$

where we have used the Dirac equations for $\bar{u}(p)$ and $u(n)_1 (i\not{n} + m_N) u(n) = \bar{u}(p)(i\not{p} + m_n) = 0$. It is easy to see that in the limit $m_\pi \to 0$:

$$\langle p| \partial_\lambda J_\lambda^{A,0} |n\rangle \xRightarrow[m_\Lambda \to 0]{} 0 \qquad (6.16)$$

if Eq. 15 is satisfied.

The procedure of passing to the limit $m_\pi \to 0$ is not rigorously justified. We could also obtain the G-T relation by requiring that the axial current is conserved in the limit of large momentum transfers:

$$\langle p| \partial_\lambda J_\lambda^{A,0} |n\rangle \xRightarrow[q^2 \to \infty]{} 0 \qquad (6.17)$$

Here one could question the validity of Eq. 11 at large momentum transfers.

Still another approach to the G-T relation has been proposed by Gell-Mann and Levy, who derive them from the hypothesis that the divergence of the axial current is proportional to the pion field:

$$\partial_\lambda \; J_\lambda^{A,0}(x) \;\alpha\; \phi_\pi(x) \qquad (6.18)$$

(If we introduce a phenomenological form for the current, ad hoc to explain $\pi \to \mu + \nu$

$$\frac{G}{\sqrt{2}} J_\lambda^{A,0}(x) = \partial_\lambda \phi_\pi(x) \tag{6.19}$$

Eq. 18 is satisfied.)

In conclusion, the Goldberger-Treiman relation (Eq. 15) is probably true to some accuracy, and can be derived from suggestive principles (Eqs. 16, 17, 18). The G-T relation is not, however, on an equally firm ground as those derived from C V C for the vector part. The understanding of the G-T relation and of its limits will improve with the improving of our knowledge of strong interactions.

Generalization of G-T Relations

We can repeat the above arguments for other beta decays of baryons.

A particularly interesting case is the $\Delta S \equiv 0$ transition

$$\Sigma^- \to \Lambda^0 + e^- + \bar{\nu} \quad . \tag{6.20}$$

In analogy with Eq. 11 we expect:

$$\frac{G}{\sqrt{2}} \langle \Lambda | J_\lambda^{A,0} | \Sigma^- \rangle = \bar{u}(\Lambda)\gamma_5 \left(g_A^{\Sigma\Lambda} \gamma_\lambda + \frac{g_{\pi^+\Sigma^-\Lambda} f}{q^2+m_\pi^2} q_\lambda \right) u(\Sigma^-).$$

if we apply to this the Nambu argument (Eq. 16, 17), we obtain an equation similar to 15:

$$f = (m_\Lambda + m_\Sigma) \frac{g_{\pi^+\Sigma^-\Lambda}}{g_A^{\Sigma\Lambda}} . \tag{6.21}$$

If we compare 21 with 15 and (within the expected accuracy of both) put $m_\Lambda + m_\Sigma \approx 2m_N$ we get:

$$\frac{g_A^{\Sigma\Lambda}}{g_A} = \frac{g_{\pi n p}}{g_{\pi\Sigma\Lambda}} . \tag{6.22}$$

We can apply the same arguments to $\Delta S = 1$ transitions, only the pole term is here provided by an intermediate K, instead of a pion. In this case the dominance of the pole term, as well as the process of sending $m_K \to 0$ is however less justified.

WEAK INTERACTIONS VII

Lecture VI

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May 12, 1964

Neutrino Processes - elastic interactions

Neutrino experiments give an interesting possibility to study weak interactions. Some examples of neutrino processes are

$$\nu + n \to p + \ell^-$$

$$\bar{\nu} + p \to u + \ell^+ \tag{7.1}$$

$$\bar{\nu} + p \to p + \pi^- + \ell^+ \quad \text{, etc.} \qquad \ell^+ \text{ being } \mu^+ \text{ or } e^+$$

The first two are conventionally called "elastic" processes. One obvious reason of interest in these processes is the study of the matrix elements of the weak current J_λ^s in regions of momentum transfers which are not available in decay processes. In this sense the neutrino reactions study the "weak structure" of the nucleon in a way very similar to that in which electron scattering studies their electromagnetic structure.

Let us consider, for example, the first process in our list. The matrix element for such a process is given, in the $V - A$ theory, by

$$\frac{G}{\sqrt{2}} \langle p | J_\lambda^s | n \rangle \; [\bar{u}(\ell^-) \; \gamma_\lambda (1+\gamma_5) \; u(\nu)] \tag{7.2}$$

This is very similar to that for beta decay, the only noticeable difference being that the v spinor for an outgoing antineutrino has been substituted by an u spinor for an ingoing neutrino.

We can write the matrix element of $J_\lambda^{\,s}$ in terms of four "form factors":

$$\langle p|J_\lambda^{\,s}|n\rangle = \bar{u}(p)\ \{F_1(q^2)\ \gamma_\lambda + \frac{F_2(q^2)}{2M_{\mathcal{N}}}\sigma_{\lambda\nu}q_\nu + F_A(q^2)\gamma_\lambda\gamma_5 + F_A'(q^2)q_\lambda\gamma_5\}\ u(n) \tag{7.3}$$

Where $q_\lambda = p_\lambda - n_\lambda$ is the momentum transfer to the nucleon. For simplicity we have assumed that terms with "wrong" G parity are absent (see discussion in previous lecture). For $q^2 \sim 0$ this expression should reproduce the expression used earlier in beta decay, so that

$$\begin{aligned} &F_1(0) \approx 1 \\ &F_A(0) \approx 1.25 \end{aligned} \tag{7.4}$$

The CVC hypothesis gives a straigthforward prediction on the vector part: the two vector form factors F_1 and F_2 should be equal to the "isovector" combination of the nucleon electromagnetic form factors:

$$F_{1,2}(q^2) = F_{1,2}^{\,p}(q^2) - F_{1,2}^{\,n}(q^2) \tag{7.5}$$

(and in particular $F_2(0) = \mu_p - \mu_n$, as we have already seen). The axial form factors cannot be related to other quantities. They are however, expected to fall down with increasing q^2 as the electromagnetic (and vector) ones. F_A' can be connected again to a pion exchange, but its contribution is always rather small.

The predictions on elastic processes can be summarized in the following way: At low energy the cross section increases with the square of the neutrino energy. This is due to the increase in available phase space. When the energy goes to high values, the phase space increases, but parts of it correspond to larger momentum transfers q^2, so that the high energy behaviour of the cross section is essentially determined by the value of the form factors for high momentum transfer.

If the form factors tend to zero fast enough, the cross section tends to a constant value at high energy:

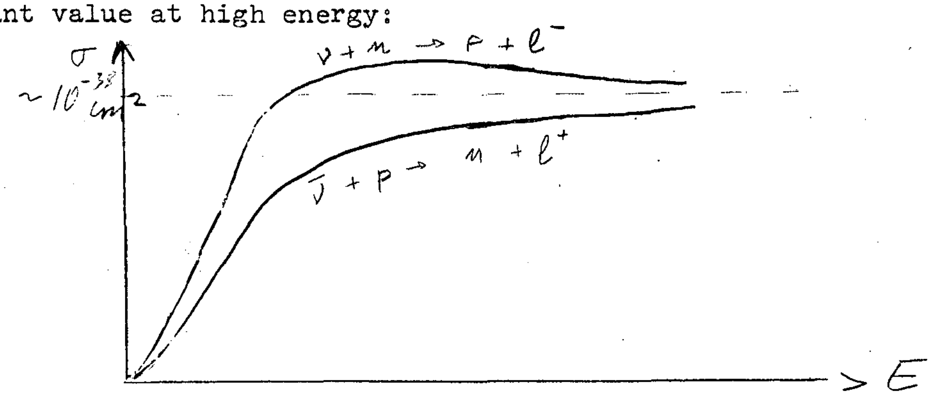


Fig. 7.1

This situation is shown in the figure. The difference between the two curves is due to a change of sign in the interference term among V and A. The order of magnitude of the cross sections at high energy (~ 1 GeV) is typically $\sim 10^{-38}$ cm, at low energy (few MeV) typically $\sim 10^{-44}$ cm.

A detailed experiment in which angular distributions (and perhaps polorizations of the final nucleons) are measured for the two elastic processes could allow a determination of the relevant form factors.

Such a program cannot be pushed very far with present techniques, where the actual target is not an isolated nucleon, but a composite nucleus. An analysis of the data of the CERN experiment indicates that the picture described here, with vector form factors equal to the electromagnetic ones (as determined in e - p scattering experiments) and axial vector form factors having a similiar behaviour, is essentially correct.

More detailed information on the form factors (in particular the axial ones) will have to wait for the use of hydrogen (bubble chamber) targets.

Nature of the Leptons - Lepton Conservation

The first results of extreme interest from neuterino experiments are related to the nature of the leptons and their interactions. This is particularly interesting as the leptons are among the few particles (together and with the photon and the vector mesons which mediate weak interactions, if they exist) which have some claim to the title of "elementary."

The first neutrino experiment was performed by Cowan and Reines with pile neutrinos. These are $\bar{\nu}$, being produced in the beta decay of neutron rich fission fragments, according to the scheme

$$(Z,A) \rightarrow (Z+1,\ A) + e^- + \bar{\nu} \qquad (7.6)$$

They are expected to produce positrons, and in fact were found to give rise to inverse reactions like

$$\bar{\nu} + (Z+1,\ A) \rightarrow (Z,A) + e^+ \qquad (7.7)$$

The main results of the low energy neutrino experiments are

1. Verification of the existence of neutrinos.
2. The measured cross sections were in good agreement with the computed ones. (for the elementary process $\bar{\nu} + p \rightarrow n + e^+$, see Nature, 178, 446 (1956)).
3. Lepton conservation.

Let's discuss briefly this last point: It has been checked that pile antineutrinos don't give rise to reactions like

$$\bar{\nu} + Cl^{37} \rightarrow A^{37} + e^- \qquad (7.8)$$

(see R. Davis: Bull. Am. Phys. Soc. 1, 219, 1956 and C. O. Muelhouse and S. Oleska: Phys. Rev. 105, 1333, 1957).

At present times we would not be surprised at all of this result, since we know that ν and $\bar{\nu}$ (defined as the neutrinos emitted with e^+ and e^-) have always opposite helicity; this is enough to predict the result of the chlorine experiments. (see the discussion in the third lecture).

<u>The Two Neutrinos</u> - <u>Muon Conservation</u> - <u>Neutral Currents</u>

The main result of the recent Brookhaven and Cern experiments concerns the existence of two different neutrinos, one of them associated with electrons, the other with muons, ν_e and ν_μ.

The existence of two neutrinos was already suspected by using theoretists, as the only means fo forbidding transitions of muons into electrons, namely processes like

$$\begin{aligned} &\text{a. } \mu^{\pm} \to e^{\pm} + \gamma \\ &\text{b. } \mu^- + p \to e^- + p \qquad (7.9) \\ &\text{c. } \mu^{\pm} \to 3e \end{aligned}$$

for each of these processes we have now very good higher limits, if compared with the normal known decay or muon capture. The first idea in looking for these processes was to see whether one could detect the coupling of neutral lepton currents. All the known leptonic processes involve the interaction of a charged current, $(\bar{e}\nu)$ and $(\bar{\mu}\nu)$. There is apparently no a priori reason why neutral lepton currents like, $(\bar{\nu}\nu)$, $(\bar{e}e)$, $(\bar{\mu}e)$and $(\bar{\mu}\mu)$ are not coupled among themselves and with corresponding currents of strongly interacting particles, like $(\bar{p}p)$, $(\bar{\Lambda}n)$, etc.

The coupling of neutral lepton currents with themselves would give rise (if the $(\overline{\mu}e)$ current is among them) to the processes 7.9, and also given contributions to processes which already happen via the charged current J_{λ}^{e}. Muon decay, e.g., would have two contributions:

$$(\overline{\mu}\,\nu)\,(\overline{\nu}\,e) + (\overline{\mu}\,e)\,(\overline{\nu}\,\nu) \qquad (7.10)$$

Other lepton processes induced by neutral currents, like $\nu - \nu$ scattering are clearly undetectable.

The coupling of lepton neutral currents to their strong interacting particle counterparts could give rise to

$$K^{+} \to \mu^{+} + e^{-} + \pi^{+}$$

$$K^{+} \to e^{+} + e^{-} + \pi^{+}$$

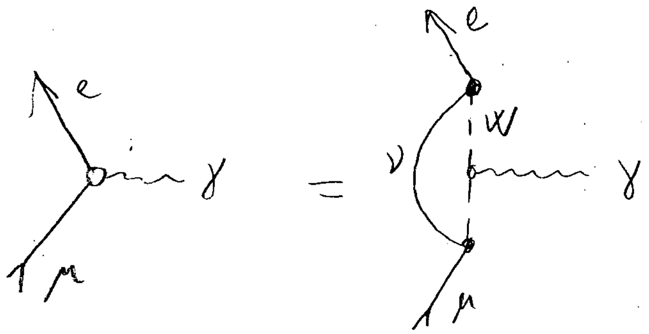
$$K^{+} \to \mu^{+} + \mu^{-} + \pi^{+} \qquad (7.11)$$

$$\nu + p \to \nu + p \quad \text{etc.}$$

All these are known to be less frequent by at least $\sim 10^{2} \leftrightarrow 10^{3}$ than corresponding "charged current" modes.

As we will see later when we discuss non leptonic decays, neutral currents of strong interacting particles are probably coupled among themselves, so that the non coupling of lepton currents is rather puzzling, and has always been a serious obstacle to a really universal theory of weak interactions.

It was soon realized that the "unwanted processes" 7.9 would also happen without the intervention of neutral currents. If we assume that the interaction between the charged currents $(\overline{\mu}\,\nu)$ and $(\overline{\nu}\,e)$, responsible for muon decay, is mediated by a charged vector meson $W^{\pm}$, $\mu \to e + \gamma$ could proceed through the following mechanism:



(Fig. 7.2)

Once $\mu \to e + \gamma$ is allowed, the other two could also be produced through the exchange of virtual photons:

(Fig. 7.3)

Even if W^{+} does not exist, it is possible to find other mechanisms to produce $\mu \to e + \gamma$.

A selection rule forbidding 7.9 can be established if there are two different neutrinos, one connected with the muon, the other with the electron. The lepton current is then written as

$$J_\lambda^{\ell} = \left(\bar{\mu} \;\gamma_\lambda (1 + \gamma_5) \nu_\mu\right) + \left(\bar{e} \;\gamma_\lambda (1 + \gamma_5)\; \nu_e\right) \tag{7.12}$$

In this case the unwanted processes 7.9 are forbidden; for example the mechanism in Fig. 2 would not work because at the lower vertex a $\bar{\nu}_\mu$ is emitted which cannot be subsequently reabsorbed at the upper vertex. In fact, if the leptons couple through the current 7.12 we have two conserved quantum numbers relating to leptons, instead of only one (the lepton number). There are many different but equivalent ways in which these numbers are defined. The table below shows a possible choice.

	N_ℓ	N_μ
e^+, $\bar{\nu}_e$	-1	0
e^-, ν_e	+1	0
μ^+, $\bar{\nu}_\mu$	-1	-1
μ^-, ν_μ	+1	+1

N_ℓ is here the old lepton number, N_μ is the "muon" number. In $\mu \to e + \gamma$ and other processes 7.9 the muon number N_μ changes by one unit, so that they are forbidden.

The neutrinos used in high energy neutrino experiments at Brookhaven and CERN come mainly from $\pi \to \mu\nu$ decay. Under the two-neutrino hypothesis they should give rise to muons, according to

$$\begin{aligned} &\nu_\mu + n \to p + \mu^- \\ &\bar{\nu}_\mu + p \to n + \mu^+ \qquad \text{etc,} \end{aligned} \tag{7.13}$$

and not to electrons. If there is only one kind of neutrino, electrons and muons should be produced in essentially equal numbers. In both experiments the observed events were mainly muons, the few electrons being compatible with a small admisture of ν_e from $K \to e + \pi + \nu_e$ or muon decay.

"Neutrino Flip"

Another theoretical possibility which has been discarded after the recent neutrino experiments is that of a "neutrino flip". The idea was based on a possible similarity between muon number and strangeness: it was proposed that in $\Delta S = 1$ processes the neutrino - charged lepton pairing could be opposite to the normal one: for example

Normal	"Neutrino Flip"
$\pi^+ \to \mu^+ + \nu_\mu$	$\pi^+ \to \mu^+ + \nu_\mu$
$K^+ \to \mu^+ + \nu_\mu$	$K^+ \to \mu^+ + \nu_\mu$
$K^+ \to \pi^0 + e^+ + \nu_e$	$K^+ \to \pi^0 + e^+ + \nu_\mu$
	etc.

(7.14)

The high energy neutrino beams, apart from $\pi \to \mu\nu$ neutrinos, contain also a sizeable number of $K \to \mu\nu$ neutrinos which according to the neutrino flip hypothesis should then interact and give rise to electrons instead of muons (in $\Delta S = 0$ interactions like those in Eq. 7.1). This possibility has been experimentally excluded; and one must conclude that the same current J_λ^{ℓ} (Eq. 7.12) acts in both $\Delta S = 0$ and $\Delta S = 1$ leptonic processes.

WEAK INTERACTIONS VIII

Lecture VII

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Schemes for Weak Interactions

In the preceding lectures we have mainly discussed leptonic decays and leptonic interactions in general. Our knowledge of these can be reviewed in the following:

a. Leptonic weak interactions are of the A - V type, namely described by

$$\frac{G}{\sqrt{2}} \{ J_\lambda^{\ell} J_\lambda^{\ell *} + J_\lambda^{s} J_\lambda^{\ell} + J_\lambda^{s*} J_\lambda^{\ell *} \} \qquad (8.1)$$

b. The structure of J_λ^{ℓ} is well known. This structure has been tested in many experiments, among which we can recall those on beta decay, the measurement of different parameters in muon decay (the ρ-value, the longitudinal polarization of $e^{\pm}$ in $\mu^{\pm}$ decay, etc.), the ratio of $\pi \to e\nu/\pi \to \mu\nu$, the helicity of muons in $\pi \to \mu\nu$, and finally the neutrino experiments (for the existence of two neutrinos with no neutrino flip).

c. The structure of J_λ^{s}, the weak current of strongly interacting particles, is not known in comparable detail. The accent here is more on general properties of J_λ^{s} than on a single piece of information about one of its matrix elements.

The most remarkable facts about J_λ^s are its selection rules (see second lecture), the validity of the CVC hypothesis, which connects the $\Delta S = 0$ vector part of J_λ^s with the current of isospin, and the success of Goldberger-Treiman relations, which connect the $\Delta S = 0$ axial part of J_λ^s with pion physics. All these aspects can be extended and unified if the larger (SU_3) symmetry of strong interactions is taken into account.

Non leptonic decays

Before going into a discussion of this let us discuss briefly the non-leptonic decays. The problem here is that the strong interactions among all particles involved make it difficult to retrieve the structure of the weak interactions which cause such decays.

The only experimental feature of non leptonic decay, which has any relevance to their general structure is the validity - with good approximation- of two selection rules:

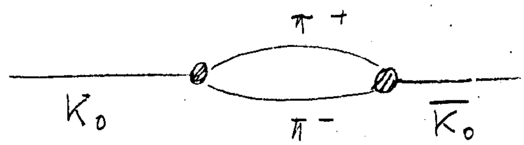
$$|\Delta S| < 2 \tag{8.2}$$

$$\Delta I = 1/2 \quad \text{for} \quad \Delta S = \pm 1$$

A direct evidence against the existence of $\Delta S = 2$ transitions is the absence of the decay

$$\Xi^- \rightarrow n + \pi^- \tag{8.3}$$

which is at least ~ 100 times less frequent than the normal mode $\Xi^- \rightarrow \lambda + \pi^-$. An indirect but more stringent evidence comes from the $K_0^1 - K_0^2$ mass difference. If a $K_0 \rightarrow \overline{K}_0$ transition were directly permitted in the lowest order of weak interactions, this would bring to a mass difference of the order of $1 - 10$ eV. The observed mass difference is however much smaller, of the order of 10^{-5} eV and is understood in terms of second order $K_0 \to \bar{K}_0$ transitions, e. g.:



The $\Delta I = 1/2$ selection rule has only "direct" checks, based on predictions of the kind

$$\frac{K_0^1 \to \pi^0 + \pi^0}{K_0^1 \quad \pi^+ + \pi^-} = \frac{1}{2} \qquad \text{etc.} \tag{8.4}$$

A discussion of the experimental checks of $\Delta I = 1/2$ has been given by R. H. Dalitz (Reports of the Int. Conf. on Fundamental Aspects of Weak Interactions, Brookhaven, Sept. 1963). The evidence for the rule, which was already found convincing at that time, has improved recently. The first hint for the validity of $\Delta I = 1/2$ came from the very slow rate of $K^+ \to \pi^+ + \pi^0$ which is ~ 700 time less than the rate for $K_0^1 \to 2\pi$. The two pions in K^+ decay are emitted in a $I = 2$ state, so that the process would be forbidden by $\Delta I = 1/2$.

<u>Problem</u> Prove the above, and also that the two pions in $K_0^1 \to \pi + \pi$ can be in $I = 0$, which leads to Eq. 8.4.

It is amusing to note that this ratio 1:700 is now one of the main difficulties to the unrestricted validity of $\Delta I = 1/2$. Violation of these rules are expected to arise from electromagnetic corrections (virtual emission and reabsorption of a photon) and to be therefore of order $\approx \alpha^2 \approx 10^{-4}$, instead of $\approx 10^{-3}$. A possible explanation of this descrepancy arises from SU_3, and will be discussed later.

Theoretical Interpretations

It would seem that the simplest possibility to explain non leptonic decays on a par with leptonic ones is to add to Eq. 8.1 a term with couples J_λ^s with itself:

$$\frac{G}{\sqrt{2}} J_\lambda^s J_\lambda^{s*} \tag{8.5}$$

this approach has the nice feature that Eq. 8.5 and 8.1 can be rewritten together as the coupling of a single current with

$$\frac{G}{\sqrt{2}} (J_\lambda^s + J_\lambda^\ell)(J_\lambda^s + J_\lambda^\ell)^* \tag{8.6}$$

itself. This scheme is unfortunately not very satisfactory because it does not give rise to the $\Delta I = 1/2$ rule.

In fact we know that the $\Delta S = 0$ part of J_λ^s has $\Delta I = 1$, the $\Delta S = 1$ part has $\Delta I = 1/2$. (Both selection rules are assumed to be valid, as discussed in the second lecture, and are supported at the moment by some experimental evidence. The following argument would not be weakened if J_λ^s had also, e. g., some $\Delta S = 0$, $\Delta I = 2$, but it would be some what different). The two parts have therefore the same I spin behavior of π^+ and K^+. It follows that the $\Delta S = 1$ part of Eq. 8.5 has the same I spin behavior as $K^+ \pi^-$, which is a mixture of $I = 1/2$ and $I = 3/2$.

In order to get pure $\Delta I = 1/2$ we should add to Eq. 8.5 and 8.6 a term which contains the coupling of neutral currents. This can be seen immediately through the analogy with pions and Kaons ; a neutral system with $I = 1/2$ and zero charge is

$$\sqrt{\frac{1}{3}}\, K^+ \pi^- + \sqrt{\frac{2}{3}}\, K^0 \pi^0$$

Similarly with our currents: to get an $I = 1/2$ object from a triplet ($\Delta S = 0$, $I = 1$) of currents coupled to a doublet ($\Delta S = 1$, $I = 1/2$) we must also couple the neutral components. As we have seen it is difficult to include this into a really elegant scheme in view of the fact that neutral lepton currents do not seem to be coupled at all. Even if they have a very small coupling (which is still possible) the situation would not be symmetric.

If the term 8.5 enters (even if not alone) in the description of non leptonic decays, there is a connection of the $|\Delta S| < 2$ selection rule for non leptonic decays with the $\Delta S = \Delta Q$ selection rule for leptonic decays. If both $\Delta S = \Delta Q = 1$ and $\Delta S = -\Delta Q = -1$ terms enter in $J_\lambda{}^s$:

$$J_\lambda{}^s = J_\lambda{}^{(\Delta S=0)} + J_\lambda{}^{(\Delta S=1)} + J_\lambda{}^{(\Delta S=-1)}$$

Then Eq. 8.5 would also give rise to $\Delta S = 2$ transitions, through terms like

$$J_\lambda{}^{(\Delta S=1)} \; J_\lambda{}^{(\Delta S=-1)^*} + \text{h. c.}$$

I don't want to discuss here the many theoretical schemes based on current-current interactions to include leptonic and non leptonic interactions. Many of these were strongly influenced by the uncertain experimental situation of the $\Delta I = 1/2$ rule for non leptonic decays and the $\Delta S = \Delta Q$ rule (and $\Delta I = 1/2$) for leptonic decays.

WEAK INTERACTIONS VIII

Lecture VII
(Second Part)

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Universality of Weak Interactions; A Simple Model

I. Universality

In the following discussion of universality we will limit ourselves to the case of Leptonic processes, where the influence of strong interactions can be to a certain extent taken into account.

The necessity of understanding weak interactions as a "universal" force stems from a comparison with our conceptions about the strength of the other three basic forces in nature. Electromagnetic and gravitational forces are described in terms of two universal parameters: the gravitational constant, and the elementary unit of charge. Strong interactions are not characterized by a well defined number, but we think now that their strength is dynamically determined by conditions of analyticity and unitarity of the S matrix.

Weak interactions being akin to electromagnetic and gravitational ones in their less than "maximal" strength, it is desirable to describe them in terms of a single parameter, for example the Fermi coupling constant G, as measured in muon decay.

To see whether this is possible, and in what sense, we should compare the strength of different weak processes. In this we are to a certain extent helped by the conserved vector current hypothesis. Assuming this to be valid, and taking into account radiative corrections to both μ-decay and beta decay one obtains (see the lecture on CVC).

$$g_V{}^2 = \frac{G^2}{2} \; [1 - (4.5 \pm 1.5)\%] \tag{9.1}$$

A more serious discrepancy arises in the case of $\Delta S = 1$ decays of hyperons. In the case of

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e \tag{9.2}$$

we find experimentally a branching ratio which about 15 times smaller than one would expect assuming that the vector and axial coupling constants are of the same order of magnitude as in beta decay. The same happens in all $\Delta S = 1$ leptonic decays which are consistently less intense than corresponding $\Delta S = 0$ decays. Since the $\Delta S = 1$ current cannot be conserved, this latter discrepancy could be attributed to large renormalization effect, but this is not a very satisfactory solution.

A better one seems to accept these discrepancies as basic "facts of life". In the next lecture I will describe a theory of leptonic interactions which is based on a modification of the concept of universality and on the approximate unitarity symmetry of strong interactions (SU_3). Here I would like to discuss a simple but unrealistic model, in order to illustrate this new conception of universality without the machinery of SU_3.

II. A Simple Model

Let us consider the use in which P, n, and Λ are the only existing baryons. We will have two kinds of leptonic decays: the beta decay of the neutron and the beta decay of the Λ (9.2). Assuming the two interactions to be of the $\gamma_\lambda(1 + \gamma_5)$ kind we can write the effective Hamiltonian in the form

$$\frac{G}{\sqrt{2}}\left[a(\bar{p}\,\gamma_\lambda(1+\gamma_5)n) + b(\bar{p}\,\gamma_\lambda(1+\gamma_5)\Lambda)\right]\,(\bar{e}\,\gamma_\lambda(1+\gamma_5)\nu_e) \tag{9.3}$$

Where a and b are the strengths of the two interactions. Note that both the $\Delta S = 0$ and $\Delta S = 1$ parts of the current of strong interacting particles multiply the same lepton current. It is convenient to regroup the factors in Eq. 9.3 and rewrite it as:

$$(a^2+b^2)^{1/2}\;\frac{G}{\sqrt{2}}\left[\bar{p}\,\gamma_\lambda(1+\gamma_5)\frac{(an + b\Lambda)}{(a^2+b^2)^{1/2}}\right](\bar{e}\,\gamma_\lambda(1+\gamma_5)\;\nu_e) \tag{9.4}$$

Equation 9.4 has very interesting consequences in the case in which Λ and n are supposed to have equal masses (but still higher than the proton to allow their leptonic decays). In this case one would observe particle mixture effects not unlike those which are actually observed for K^o and $\bar{K}^o$.

The decays of N and Λ will be better described in terms of two mixtures:

$$n' = (an + b\Lambda)\,(a^2 + b^2)^{-\frac{1}{2}}$$

$$\Lambda' = (bn - a\Lambda)(a^2 + b^2)^{-\frac{1}{2}} \tag{9.5}$$

of these only the first could beta decay, the second being stable.

the values

So we see that, independently/of a and b we cannot in the limit of mass degeneracy have all decays of equal strength. We end up with one particle, n', which has a beta decay with strength $G(a^2 + b^2)^{1/2}$, and another, Λ', which is stable. We can still have universality if we require

$$a^2 + b^2 = 1 \quad \text{or} \quad \begin{array}{l} a = \cos\theta \\ b = \sin\theta \end{array} \tag{9.6}$$

This condition insures that the amplitude for the β decay of n' is equal to that of muon decay. This situation is similar to that in electromagnetic interactions: not all the particles have an electric charge, but those which have one, have a multiple of the electron charge. In this model, universality does not say anything on the ratio $a:b$, but only concerns the overall coupling strength.

If we reinstate the $n - \Lambda$ mass difference we find their two beta decays to be (apart from phasespace factors) in the ratio $a^2:b^2$, and if we choose

$$\sin^2\theta \approx 0.06 \qquad\qquad \cos^2\theta \approx 0.94 , \qquad\qquad (9.7)$$

we can roughly understand both the low rate of Λ beta decay and the small discrepancy in normal beta decay (Eq. 9.1). This unrealistic model, where we neglect the existence of Σ and Ξ hyperons, contains the interesting idea of the sharing of a universal weak interaction among $\Delta S = 0$ and $\Delta S = 1$ processes.

The sharing depends on the particular way in which the degeneracy of and n is removed by strong interactions. This is done in such a way that Λ and n have well defined mass, while from the point of view of weak interactions n' and Λ' would be more significant. In fact if strong interactions did not provide a mass split between n and Λ, weak interactions would provide a small mass split between n' and Λ', as is the case in $K^o\,\overline{K}^o$ system, where K^1 and K^2 have definite masses.