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Essays in Macroeconomics and International Finance

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ABSTRACT OF THE DISSERTATION

Essays in Macroeconomics and International Finance

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This thesis consists of three chapters. In Chapter 1, we quantitatively analyze an interaction between the US fiscal debt and Net Foreign Asset (NFA) and examines the shock source of US NFA deterioration. We address two questions: 1. What is the impact of increased US government treasury as a safe asset on portfolio choices and welfare? 2. What shocks are appropriate to explain changes in individual asset classes since the financial crisis? To answer these questions, we construct a two-country heterogeneous agent general equilibrium model augmented with fiscal sector and portfolio choice calibrated to post the global financial crisis era. First, we see how the size of debt affects the impact of regime change from financial autarky to financial integration. We find that the benefit of financial integration is more substantial among wealthy households under a low level of debt. On the other hand, with the high level of debt, poor households would enjoy a convenience yield that emerges after financial integration. Second, we examine three alternative hypotheses for NFA deterioration advocated in the literature, the markup hypothesis, the fiscal deficit hypothesis, and the uncertainty increase in the Rest of the World. We investigate -5% NFA deterioration shock under the alternative hypothesis. To understand the dynamics of different asset classes which compose the US NFA position, the markup shock, and the uncertainty shock are more consistent with the data. All shocks improve US households' welfare by up to 0.1% in the

short run. On the other hand, in the long run, the welfare of US households deteriorates by 0.1%.

In Chapter 2, we provide a theoretical framework to solve the Fiscal Theory of Price Level (FTPL) puzzle, in which inflation remains low despite permanent budget deficits. According to the standard FTPL model, budget deficits lead to an increase in the price level. Nevertheless, some country is a counterexample of this theoretical prediction (Brunnermeier et al. (2020)). We solve this puzzle by adding a market segmentation assumption to the standard New Keynesian (NK)-FTPL model. Specifically, by adding the assumption that financial intermediaries directly hold all government bonds, we derived the conditions under which government spending leads to a decline in the price level. The condition is whether the government's stance toward a sound primary balance is so accommodative that it overrides the riskiness of long-term debt, calculated from the ratio of long-term debt to GDP and the risk premium. If the government's stance is stronger than the threshold, the fiscal deficit is deflationary; if it is weaker, the fiscal deficit is inflationary.

In Chapter 3, we revisit the interrelationship between exchange rates and stock prices and check the following stylized facts: 1) exchange rate volatility is lower than stock price volatility; 2) stock prices of the G7 countries are positively correlated; 3) countries with low-interest rates tend to see their stock prices rise when their exchange rates depreciate, while stock prices tend to fall for countries with high-interest rates as the exchange rate depreciates. We present a two-country Dynamic Stochastic General Equilibrium model with the market segmentation hypothesis (a combination of Itskhoki and Mukhin (2019a) and Hau and Rey (2006)) that can exhibit dynamic behavior consistent with the stylized facts. To reproduce the econometric moments, we propose a positively correlated risk appetite shock between the two countries in the financial market. The second-order moments, which determine the dynamics of exchange rate and stock price, are generated endogenously, given the exogenous shocks to the model.

The dissertation of Nobuhiro Abe is approved.

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To Chisa, Nagisa, and Yuri

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CHAPTER 1

Interaction of the US Twin Deficit: Source of US NFA Deterioration

1.1 Introduction

The increase in government debt and external liabilities in the US has become more pronounced in recent years following the Great Financial Crisis and the Covid-19 crisis (Figure 1.1.1). While government debt has continued to increase under the increased fiscal transfers from the government, the US debt has been a safe asset with high liquidity in the international financial markets. Thus, the amount of foreign holdings have increased, contributing to the deterioration of the Net Foreign Asset (NFA). As noted in Atkeson et al. (2020), one major factor in the worsening of NFA has been the increase in foreign holdings of US equities. When looking at the individual asset classes that constitute the NFA since

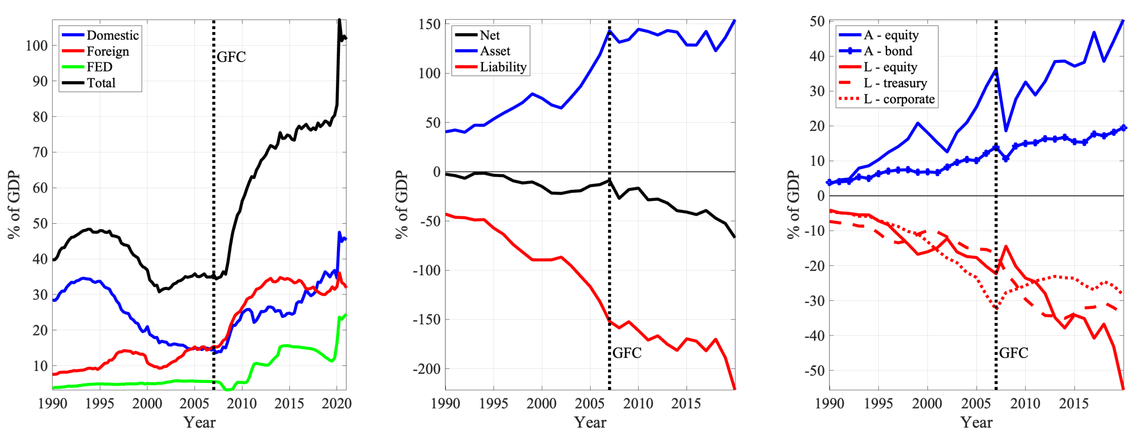

Figure 1.1.1: The holder of Treasury security, NFA position and its decomposition

the Great Financial Crisis, there has also been a simultaneous increase in foreign equity holdings by the US, which help halt the deterioration of the NFA. How do these cross-border demands for US government bonds and global equity assets relate to each other? What are the primary mechanisms that shape the US NFA position? Furthermore, what are their implications for the welfare of US and foreign households? These are essential questions when examining how to address the twin deficits in the US in the years ahead.

This paper aims to answer the following three questions: 1. How should we assess the impact of increased US government treasury as a safe asset on portfolio choices in other asset classes and on the social wellbeing of US households? 2. What shocks are appropriate to explain changes in individual asset classes since the financial crisis? 3. How would the social welfare of US households change under those shocks?

We construct a two-country general equilibrium model populated by heterogeneous households to answer these questions, incorporating the government sector. When considering the impact on social welfare, the heterogeneity of households (i.e., the difference between the poor and the rich) is a crucial aspect to take into account in the current era, when the financial assets that households can hold vary widely. Households constitute an economy where labor and capital income each have idiosyncratic risks and face net worth constraints. Households' portfolios consist of US Treasuries, US stocks, and foreign stocks, each with different returns and volatilities. The graph on the right side of Figure 1.1.1 depicts the trends in assets and liabilities of the individual asset classes over the past 30 years. As can be seen here, US Treasuries, US equities, and foreign equities have been significant factors in the trends of NFAs since the Great Financial Crisis. Thus, although foreign bonds appear in the model, they are not included in the portfolio selection for simplicity. The government collects a lump-sum tax from households and determines government transfers to households according to the path of government debt to satisfy the government's budget constraint. Two countries are the US and the rest of the world (RW), and households trade a single consumption good in a world of dollarization. In the baseline, we consider an environment in which the US NFA is negative, but RW stocks are high-risk, high-return, and the US Net Factor Payment (NFP) is nearly zero.

Interaction of the US Twin Debts. First, to examine the impact of the size of government debt on the portfolio choice of other asset classes and social welfare in the presence of foreign demand for safe assets, we consider the closed economy (Financial Autarky) and the two countries' economy (Financial Integration). We analyzed the impact of the transition from a closed economy to a two-country economy and how it varies depending on the size of government debt.

In the case of a two-country economy, interest rates rise more slowly as government debt increases. This is due to the demand for US debt as a safe asset from RW households, who hold riskier assets relative to US households as their primary portfolio, thus demanding safe assets. As a result, the pace of interest rate increases will be less than in a closed economy. Through a similar channel, the decline in the price of US equities associated with increased government debt will also be slower than in a closed economy. However, since the increase in government debt monotonically worsens the NFA along with the rise in RW holdings of US Treasuries, income transfers abroad will increase along with the rise in government debt. As a result, consumption in the US declines, creating the need to produce more. Thus, when we consider social welfare and the increase in government debt, the increase in government debt monotonically worsens the welfare of US households through an increase in net factor payment.

In terms of changes in welfare at different levels of government debt, there is more room for welfare improvement at lower levels of government debt, and welfare is likely to worsen as the debt level increases. Based on a model calibrated to US government debt, NFP, and NFA positions since the financial crisis (2012-2017), if government debt is smaller than that, NFP will be positive. If government debt is significant, NFP will be negative. Thus, for example, financial integration improves social welfare in an environment with no government debt, but the opposite is true at higher government debt levels (100%), etc.

Household heterogeneity is another important aspect. Financial integration improves welfare when government debt is low, especially for the wealthy who can hold riskier assets. In contrast, when government debt is high, the interest rate lowering effect through the demand channel for safe assets is stronger, and welfare improvement is more remarkable for the poor.

US NFA deterioration by what? What shocks are appropriate for understanding recent US NFA trends? We follow the previous literature and consider the following three shocks: A markup increase shock in the US (Atkeson et al. (2020)), a fiscal stimulus shock (Aggarwal et al. (2022)), and a foreign sector uncertainty shock (Rey (2015)). We add to the model each shock that causes the NFA to change by -5%, the annual average since the financial crisis.

Our analysis of the model's behavior for these three shocks reveals the following. The rise in the markup has high explanatory power for the increase in the price of US equities and the increase in foreign holdings of US equities. However, it has little explanatory power for the increase in foreign holdings of US equities by the US. Fiscal stimulus shocks can explain the temporary increase in foreign equity holdings by stimulating the investment behavior of US households. Nevertheless, there have been some difficulties explaining trends since the financial crisis, such as the decline in foreign investment in US equities and the fall in US stock prices. Finally, uncertainty shocks in the foreign sector have the highest explanatory power for the recent behavior of each asset class, especially the increase in foreign equity holdings by US households. This results from increased demand for safe assets from abroad, which has encouraged lower interest rates and risk-taking by US households. Given the sizable fiscal stimulus itself since the Great Financial Crisis, the results suggest that both markup shocks and uncertainty shocks in the foreign sector are essential in explaining the behavior of individual asset classes in the US NFA.

A summary of the results of our analysis of the normative aspects follows. We find that all shocks produce welfare improvements for US households in the short run (up to 0.1%) but worsen welfare in the long run (roughly -0.1% for all shocks). Conversely, in the RW, the results show that welfare deteriorates in the short run but improves in the long run. In the long run, the US loses, and foreign countries gain because of the increase in NFP from the US to foreign countries. Regarding household heterogeneity, the implications of the two shocks, the markup shock and the fiscal stimulus shock, and the uncertainty shock in the foreign sector are quite different. The first two shocks are severe for the poor because they entail lower wages and reduced fiscal transfers. However, the foreign sector uncertainty shock implies that assets with stable returns held by US households will flow out of the country, implying that the wealthy, the main creditors, will lose out.

Literature review. First, we relate this paper to the literature on fiscal debt with a heterogeneous agent model setting. Aiyagari and McGrattan (1998) shows that there is an optimal level for debt within the framework of Aiyagari (1994) augmented with the fiscal sector. The optimality for issuing debt comes from a tradeoff between the benefit of liquidity provision for smoothing consumption and the cost of an increase in tax and capital crowding out. Azzimonti et al. (2014) considers the role of public debt in an international setting when households face idiosyncratic shock. They show that when the financial market is integrated, the government chooses higher debt than in the closed economy case. Our paper differs in that we consider the model, where capital income entails idiosyncratic shock and the main focus is cross-border portfolio choice (risk-sharing).

The large negative position on NFA of the US and global imbalance have been the central issue in international economics. A large number of works have documented the speciality of the US external balance sheet (see Gourinchas and Rey (2007a), Gourinchas and Rey (2007b), and Gourinchas and Rey (2014)). Atkeson et al. (2020) revisited the US external balance sheet along with other major financial variables. Also, several theoretical frameworks to understand the mechanism to generate the global imbalance have been provided (see e.g., Caballero et al. (2008), Chien and Naknoi (2015)). Among others, Mendoza et al. (2009) shows how financial deepening induces the global imbalance relying on a heterogeneous agent framework. The other notable work, Maggiori (2017), argues an asymmetric risk-sharing between the US and the rest of the world is a source of the US external debt. More concretely, the US is an insurance provider to the rest of the world that supplies the treasury as insurance. Then, the exorbitant privilege emerges as an insurance premium (see also Krishnamurthy and Vissing-Jorgensen (2012), Jiang et al. (2019), Du et al. (2018))[1].

[1] More recent work by Chen (2021) argue that the US is the 'service provider' by showing that there was no exorbitant duty during the Great Financial Crisis

Broadly speaking, our motivation for this paper comes from the dollar usage in the world as the dominant currency. Under this perspective, this paper also relates to literature on the Dominant Currency Paradigm (Gopinath et al. (2020), Gopinath and Itskhoki (2021)). Gopinath and Stein (2020) provides a theoretical model to explain the interplay between trade invoicing patterns and the price of the treasury in different currencies. Chahrour et al. (2021) proposes a model of persistent coordination on a dominant international medium of exchange, which can rationalize the history of the international monetary system over the past century.

Lastly, the model also contributes to heterogeneous agent literature (Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998)). In particular, our model allows each agent to optimally choose a financial asset portfolio given the net worth constraint as in Mendoza et al. (2009). We depart from the model in that our model is a general equilibrium model and considers the being of public debt. Other literature following Krusell and Smith (1997) also build the model with portfolio choice, but the literature is still infant stage under general equilibrium setting (see Auclert and Rognlie (2018), Kaplan et al. (2018), Auclert et al. (2021a)).

Structure. This paper is constructed with the following menu. In Section 1.2, we describe the model. We show the optimality of portfolio choice, calibration details, solution method, and policy function of agents as a basic property of the model. Section 1.3 provides an exercise on financial integration, where the US and the RW transit from the financial autarky to capital mobility economy under different debt levels. We show the results for static and dynamic analysis. In Section 1.4, we do the counterfactual experiments for NFA deterioration of the US under three alternative hypotheses. We briefly summarize the hypotheses and then show the results. Section 1.5 concludes the paper.

1.2 Model

In this section, we describe an augmented version of the model in MQR - augmented to include government sector. The model is two country general equilibrium model, one of which country is supposed to be the US and the others is the rest of the world[2]. Those two regions are heterogenous in terms of financial liability constraint and the volatility of return from capital.

1.2.1 Two country model

Households. Firstly we introduce infinitely-lived households' behavior, who face idiosyncratic but no aggregate risk. Each household, living in both countries, the US and the Rest of the World (hereinafter RW), in each period t, has an idiosyncratic state $s_t \in \mathcal{S}$. s_t follows a Markov process with transition matrix Λ. Households face a portfolio problem where they choose how much to invest in US treasury bond, home capital, and foreign capital. The problem of a household of type (s, a) in country i is to choose consumption $c_{i,t}$, labor supply $n_{i,t}$, US bond holding $b_{i,t}$, and share holding $\theta^j_{i,t}$, given an initial value of asset $a_{i,t}$. $\theta^j_{i,t}$ is portfolio choice for capital in country j by a household in country i at time t. Households also have claim for the RW government bond, but we assume that its portfolio is exogenously given and hence it is not the control variable for households[3]. The households in country i, then, solve the following problem:

$$V_{i,t}(s,a) = \max_{c,n,\theta,b^{us}} \left\{ \frac{c_{i,t}(s,a)^{1-\sigma}}{1-\sigma} - \psi \frac{n_{i,t}(s,a)^{1+\phi}}{1+\phi} + \beta \mathbb{E}_t \left[V_{i,t+1}\left(s', a'\right)\right] \right\}, \tag{1.2.1}$$

subject to the flow budget constraint:

$$\begin{aligned} c_{i,t}(s,a) + \sum_j^I P^j_t \theta^j_{i,t}(s,a) + \frac{b^{us}_{i,t}(s,a)}{1+r^{us}_t} + \frac{b^{rw}_{i,t}}{1+r^{rw}} &= \left(1-\tau^w_i\right) W^i_t \epsilon^{l,i}_t n_{i,t}(s,a) + a_{i,t} + TR^i_t, \\ a_{i,t} \equiv \sum_j^I \left[P^j_t \theta^j_{i,t-1} + \delta^j_i \epsilon^{k,j}_t \left(\theta^j_{i,t-1}\right)^\kappa \left(1-\tau^k_j\right) \Pi^j_t \right] &+ b^{us}_{i,t-1} + b^{rw}_{i,t-1}, \end{aligned} \tag{1.2.2}$$

[2] In this paper, the rest of the world is supposed to be emerging countries.

[3] As explained later, we set the total net supply of RW bond over RW GDP as constant and we assume some fixed fraction of RW bond is hold by households in each country.

and the net worth constraints:

$$a_{i,t} \geq \underline{a}_i, \tag{1.2.3}$$

where the parameter $\sigma > 0$ is the relative risk aversion or the inverse elasticity of intertemporal substitution, $\phi > 0$ the inverse Frisch elasticity of labor supply, and $\psi > 0$ is a normalization constant. W_t^i is the real wage in country i, P_t^j is the price of one unit of share, r_t is the real interest rate. TR_t^i is the transfer from the government sector, which is identical for all households in country i[4]. τ_i^w is the tax for labor income in country i and τ_j^k is the tax for capital income in country j. The individual labor productivity shock $\epsilon_t^{l,i}$ evolves in an idiosyncratic fashion. Π_t^j is the capital income produced by the final good producing firm in country j. For the detail on portfolio holding on RW bond, see the paragraph for the fiscal sector in the below.

We assume the capital investment technology, $\delta_i^j \epsilon_t^{k,j} \left(\theta_{i,t-1}^j\right)^\kappa$, has an idiosyncratic productivity shock $\epsilon_t^{k,j}$, home bias δ_i^j, and the multiplier κ, which is less than 1. Hence, the technology is the decreasing return to scale (as in MQR)[5][6]. We can interpret the investment device as follows. The domestic mutual fund issues one unit of claims to households with price, P_t^j. The individual household invest in capital by buying claims, $P_t^j \theta_{i,t}^j$, from the mutual fund. The mutual fund allocates corporate profits according to the outcome of each banker, but the banker faces idiosyncratic productivity shock, which differ across countries. The production of each banker is decreasing return to scale and this assumption is essential when incorporating the portfolio choice in heterogenous-agent model, because otherwise the only richest household holds all risk assets. It is worth noting that individual bankers' total outcome does not necessarily become 1 because of the decreasing return to scale assumption.

[4]Without any further notice, the variables with capital letter indicates aggregate one and identical across households, while that of small letter is for individual one.

[5]For the literature of the individual entrepreneurial shock or the idiosyncratic investment shock, see Moskowitz and Vissing-Jørgensen (2002), Campbell et al. (2001), Moll (2014), Benhabib et al. (2015), Gabaix et al. (2016), Buera et al. (2011), and Midrigan and Xu (2014), among others.

[6]Mendoza et al. (2009) assumes that households also can optimally allocate managerial resource across border, which are added to be 1. However, that assumption leads to the unrealistic outcome in the portfolio choice, in a sense that the risky capital, which is the capital of the RW in MQR, only hold by the wealthier households. Moreover, as a result of this, these rich households do not have the US equity claims, which seems to be unrealistic. For more detail see Fig. 5 in Mendoza et al. (2009).

The generated residual is compensated or absorbed by the government, which we show the detail in later. δ_i^j represents for home bias in capital choice across border (see, for example, French and Poterba (1991), Coeurdacier and Rey (2013)),

$$\begin{cases} \delta_i^j = 1 & (\text{if} \quad j = i)\,, \\ 0 \le \delta_i^j \le 1 & (\text{if} \quad j \ne i)\,. \end{cases} \tag{1.2.4}$$

Thus the US households discount the performance of the banker of mutual funds in RW with δ_{us}^{rw}, while they appreciate the domestic bankers fully. The home bias parameter is identical across households within country. The home bias assumption is also key element to replicate realistic portfolio balance across border within the model.

The net worth constraint imposes an exogenous borrowing limit: the lower the value of a_i, the higher the agents' ability to borrow. In the benchmark calibration, we assume that the both US and RW households face the same level of net worth constraint. However, we can make the environment as US households have higher ability to borrow in the dollarization of economy and RW households have the higher constraint, which is assumed to be the reserve constraint[7].

Finally, we assume that households have perfect foresight over price sequences.

Production function. In each country there is a unit mass of different intermediate varieties indexed by $x \in [0,1]$. Let y_{xt}^i denote total production of variety x at date t. Domestic output of the final good is given by

$$Y_t^i = \left(\int_0^1 \left(y_{xt}^i \right)^{\frac{\nu^i - 1}{\nu^i}} dx \right)^{\frac{\nu^i}{\nu^i - 1}}$$

[7]We can think of alternative setting for borrowing constraints. The borrowing constraint with the treasury debt as Krusell and Smith (1997) or the net worth constraint as Mendoza et al. (2009). We explore the cost and benefit of another approach is argued in Appendix 1.A.4. With this specification, as we will see in Section 1.2.4, we allow households to borrow with the rate of US treasury. As a result, poor households borrow and rich households hold US treasury as asset. This property is important to think about the finding of Mian et al. (2020); *'Quantitatively, the demand for U.S. dollar-denominated safe assets comes almost as much from the rich within the United States as it does from the rest of the world'.*

where $\nu^i > 1$ is the elasticity of substitution in production between different varieties in country i. We consider the representative and competitive firm produces the final output in the economy in each country. The intermediate good producing firm has a Cobb-Douglas technology with labor input and we assume they are symmetric, $y^i_{xt} = Z^i l_{xt}$. They rent labor force from workers at rate W^i_t and faces the time-invariant total factor productivity Z^i. The firm's first-order conditions gives us the optimal markup:

$$\alpha^i = \frac{\nu^i}{\nu^i - 1}. \tag{1.2.5}$$

Then the equilibrium wage in country i is $W^i_t = \frac{Z^i}{\alpha^i}$, which is constant as long as the markup and the TFP is time-invariant in country i. The price of the final output is numeraire in the economy, then the total dividend produced from the final good producer is $\Pi^i_t = \frac{\alpha^i - 1}{\alpha^i} Y^i_t$.

Fiscal policy. The government flow budget constraint in country i at time t is:

$$\begin{aligned} TR^i_t &= \frac{B^i_t}{1 + r^i_t} - B^i_{t-1} - G^i_t + T^i_t + \Xi^i_t, \\ \text{where} \quad \Xi^i_t &\equiv \left[1 - \sum_j^I \Theta^i_{j,t}\right] \left(1 - \tau^k_i\right) \Pi^i_t \end{aligned} \tag{1.2.6}$$

where $\Theta^j_{i,t} \equiv \delta^j_i \int \epsilon^{k,j}_t \left(\theta^j_{i,t-1}\right)^\kappa dH^i_t\left(s_-, a_-, s, a\right)$ is the aggregate of country j's equity claims for corporate profit hold by households in country i. Since we assume the decreasing return to scale for the investment technology, $\sum_j^I \Theta^i_{j,t}$ deviates from 1. The extra profit forwards to the government or excess claims are financed by the government as subsidy. Dividing the both sides by the total output in country i, Y^i_t, we have:

$$tr^i_t = \frac{b^i_t}{1 + r^i_t} - \frac{b^i_{t-1}}{g^i_t} - gp^i_t + \tau^i_t + \xi^i_t, \tag{1.2.7}$$

where we define each fiscal variable as the fraction of GDP, $gp^i_t \equiv \frac{G^i_t}{Y^i_t}$, $tr^i_t \equiv \frac{TR^i_t}{Y^i_t}$, $\tau^i_t \equiv \frac{T^i_t}{Y^i_t}$, and $b^i_t \equiv \frac{B^i_t}{Y^i_t}$[8]. The domestic GDP growth is defined as $g^i_t \equiv \frac{Y^i_t}{Y^i_{t-1}}$. The tax is levied from

[8] gp represents for the government purchase.

labor and capital income, which is:

$$T_t^i = \tau_i^w \int W_t^i \epsilon_t^l n_{i,t} dH^i + \tau_i^k \Pi_t^i = \tau_i^w W_t^i L_t^i + \tau_i^k \Pi_t^i = \tau_i Y_t^i. \tag{1.2.8}$$

The aggregate tax rate is the weighted average of tax rate on labor and capital[9].

It should be noted that the debt to over GDP ratio of the RW, b^{rw}, is fixed. The fraction of holding of the RW government bond from each region is also time-invariant. We assume the fraction γ is hold by the households of the US proportionally and the remained RW bond is hold by foreigners:

$$\begin{cases} b_{us,t}^{rw} = \gamma \dfrac{B_t^{rw}}{\mu^{us}} = \gamma \dfrac{b^{rw}}{\mu^{us}} Y_t^{rw}, \\ b_{rw,t}^{rw} = (1-\gamma) \dfrac{B_t^{rw}}{1-\mu^{us}} = (1-\gamma) \dfrac{b^{rw}}{1-\mu^{us}} Y_t^{rw}, \end{cases} \tag{1.2.9}$$

where μ^{us} is the country size of the US. For the concrete value for b^{rw}, γ and μ^{us}, see Section 1.2.3.

Country budget constraint. We start with defining the net foreign asset position of the US as the sum of the net bond and equity positions,

$$NFA_t \equiv \int \left[\underbrace{P_t^{rw} \theta_{us,t}^{rw}}_{\equiv NFA^1} + \underbrace{\frac{b_{us,t}^{rw}}{1+r^{rw}}}_{\equiv NFA^2} \right] dH_t^{us} + \int \left[\underbrace{-P_t^{us} \theta_{rw,t}^{us}}_{\equiv NFA^3} - \underbrace{\frac{b_{rw,t}^{us}}{1+r_t^{us}}}_{\equiv NFA^4} \right] dH_t^{rw}, \tag{1.2.10}$$

where the first bracket is the US households' holding on the RW equity and government bond, and the second bracket is RW households' position in the US equity and treasury. Then the change in the net foreign asset position of the US is given by,

$$NFA_t - NFA_{t-1} = CA_t + VE_t, \tag{1.2.11}$$

[9]While in this paper we focus on proportional tax for labor and capital, but we can investigate how different tax structure change portfolio allocation and its consequence on aggregate variables and welfare as well within the same framework

where we divide the change in the NFA into two objects, the current account surplus and the valuation effects of capital, as follows[10],

$$\begin{aligned} CA_t =& P_t^{rw}\Delta\theta_{us,t}^{rw} + \left[\frac{\int b_{us,t}^{rw} dH_t^{us}}{1+r^{rw}} - \frac{\int b_{us,t-1}^{rw} dH_{t-1}^{us}}{1+r^{rw}}\right] - P_t^{us}\Delta\theta_{rw,t}^{us} - \left[\frac{\int b_{rw,t}^{us} dH_t^{rw}}{1+r_t^{us}} - \frac{\int b_{rw,t-1}^{us} dH_{t-1}^{rw}}{1+r_{t-1}^{us}}\right], \\ VE_t =& \left(P_t^{rw} - P_{t-1}^{rw}\right)\int \theta_{us,t-1}^{rw} dH_{t-1}^{us} - \left(P_t^{us} - P_{t-1}^{us}\right)\int \theta_{rw,t-1}^{us} dH_{t-1}^{rw}, \end{aligned} \tag{1.2.12}$$

where we should note that the current account is literally identical with the financial balance, $FB_t = CA_t$. Now, the current account is also decomposed to the two components, which are the net export and the net factor payment. On the one hand, the net export in the economy is:

$$NX_t = Y_t^{us} - \left(C_t^{us} + G_t^{us}\right), \tag{1.2.13}$$

where NX_t is the net-export from the US to RW. The other component of the current account is the net factor payment, which we define here as the net income flow from the RW to the US. Mathematically, we have:

$$\begin{aligned} NFP_t =& \Theta_{us,t}^{rw}\left(1-\tau_{rw}^k\right)\Pi_t^{rw} + \frac{r^{rw}}{1+r^{rw}}\int b_{us,t-1}^{rw} dH_{t-1}^{us} \\ & - \Theta_{rw,t}^{us}\left(1-\tau_{us}^k\right)\Pi_t^{us} - \frac{r_{t-1}^{us}}{1+r_{t-1}^{us}}\int b_{rw,t-1}^{us} dH_{t-1}^{rw}. \end{aligned} \tag{1.2.14}$$

Finally, we also have the current account of the US:

$$CA_t = NX_t + NFP_t. \tag{1.2.15}$$

If we combine this condition with the formula for financial balance, we get the country budget constraint of the US.

Equilibrium definition. Given a sequence of the aggregate supply of the government bond $\left\{B_t^i\right\}_{i=us,rw}$, a sequence of exogenous shocks $\left\{\epsilon_t^{l,i}, \epsilon_t^{k,i}\right\}_{i=us,rw}$, and an initial joint distribution $\left\{H_0^i\right\}_{i=us,rw}$ over idiosyncratic states, an equilibrium is a set of aggregate

[10] The whole derivation for the formula in this section is in Apeendix 1.A.1.

quantities for each country $\left\{C_t^i, Y_t^i, L_t^i, \Pi_t^i\right\}_{i=us,rw}$, another set of international aggregate quantities $\{CA_t, NX_t, NFP_t, VE_t, NFA_t\}$, prices $\{r_t, P_t^{us}, P_t^{rw}, W_t^{us}, W_t^{rw}\}$, government policy $\left\{G_t^i, T_t^i, TR_t^i, \Xi_t^i\right\}_{i=us,rw}$, individual decision rules $\left\{c_{i,t}(s,a), n_{i,t}(s,a), \theta_{i,t}^{us}(s,a), \theta_{i,t}^{rw}(s,a), b_{i,t}^{us}(s,a)\right\}_{i=us,rw}$ and joint distributions $\left\{H_t^i(s,a)\right\}_{i=us,rw}$, such that households maximize utility subject to their budget constraint, the final good producing firm and the intermediate good firms solve the optimality, the government satisfies its budget constraint, the distribution of households is consistent with the decision rules, and all markets clear,

$$\begin{aligned}
\text{Bond (US):} \quad B_t &= \sum_j^I \int b_{j,t}(s,a) dH_t^j(s,a) \\
\text{Capital claim:} \quad \mu^j &= \sum_i^I \int \theta_{i,t}^j(s,a) dH_t^i(s,a) \quad \forall j \\
\text{Labor:} \quad L_t^j &= \int \epsilon_t^{l,j} n_{j,t}(s,a) dH^j(s,a) \quad \forall j.
\end{aligned}$$

In the end, the Walras law holds.

1.2.2 Portfolio choice

This sub-section derives the optimality conditions of households in the asset market, which is a key component in the model. For whole derivation for the optimality conditions, see Appendix 1.A.3. If we define λ_t and ζ_t as the Lagrange multiplier for the households' budget constraint and the net worth constraint respectively, the optimality conditions for bond holding and equity claims are:

$$\begin{aligned}
\lambda_t &= \beta\left(1 + r_t\right) \mathbb{E}_t\left[\lambda_{t+1} + \zeta_{t+1}\right], \\
\lambda_t &= \beta \mathbb{E}_t\left[\left(\lambda_{t+1} + \zeta_{t+1}\right) R_{t+1}^j\right],
\end{aligned} \tag{1.2.16}$$

where $\lambda_t = \frac{\partial u_{i,t}}{\partial c_{i,t}} = \left(c_{i,t}\right)^{-\sigma}$ and we define that the expected return from capital in country j is:

$$\mathbb{E}_t\left[R_{t+1}^j\right] \equiv \mathbb{E}_t\left[\frac{P_{t+1}^j + \kappa \delta_i^j \epsilon_{t+1}^{k,j} \left(\theta_{i,t}^j\right)^{\kappa-1} \left(1 - \tau_j^k\right) \Pi_{t+1}^j}{P_t^j}\right]. \tag{1.2.17}$$

Then, if the net worth constraint does not bind (i.e., $\zeta_{t+1} = 0$), $(1 + r_t)\,\mathbb{E}_t[\lambda_{t+1}] = \mathbb{E}_t\left[\lambda_{t+1}R^j_{t+1}\right]$. Therefore we can have an usual expression for the risk premium:

$$\mathbb{E}_t\left[R^j_{t+1}\right] - (1 + r_t) = -\frac{\mathbb{C}ov_t\left[R^j_{t+1}\lambda_{t+1}\right]}{\mathbb{E}_t[\lambda_{t+1}]}, \tag{1.2.18}$$

which should be plus as long as λ_{t+1} and R^j_{t+1} are negatively correlated.

Next, we get the formula for portfolio choice on claims for risky assets by combining the optimality conditions in equation (1.2.16), in the following way:

$$\theta^j_{i,t} = \left[\frac{\beta\kappa\delta^j_i\left(1 - \tau^k_j\right)\frac{\Pi^j_{t+1}}{P^j_t}\mathbb{E}_t\left[\epsilon^{k,j}_{t+1}\left(\lambda_{t+1} + \zeta_{t+1}\right)\right]}{\lambda_t - \beta\frac{P^j_{t+1}}{P^j_t}\mathbb{E}_t\left[\lambda_{t+1} + \zeta_{t+1}\right]}\right]^{\frac{1}{1-\kappa}}. \tag{1.2.19}$$

From the formula, it is obvious that risky asset holding monotonically decrease with an increase capital tax and a decrease in home bias and price earning ratio. To understand heterogenous risk asset holding across households within the same country, we assume we are in the steady state. Then, we claim the following proposition.

Proposition 1. *At the steady state, the amount of equity claims in country j held by households in country i is:*

$$\theta^j_i = \left[\frac{\kappa\delta^j_i\left(1 - \tau^k_j\right)}{r}\frac{\Pi^j}{P^j}\frac{\mathbb{E}\left[\epsilon^{k,j'}\left(\lambda' + \zeta'\right)\right]}{\mathbb{E}\left[\lambda' + \zeta'\right]}\right]^{\frac{1}{1-\kappa}}.$$

If US treasury rate (r) increases or price earning ratio ($\frac{\Pi^j}{P^j}$) decreases, risky asset portfolio decreases. Also if asset households have is small, the shadow price of the net worth constraint is (ζ') large, then the amount of equity claims held declines.

Proof. See Appendix 1.A.3. ■

1.2.3 Calibration and solution method

Calibration. We calibrate the model to match the key moments during the periods post the Great Financial Crisis, from 2012 to 2017. Table 1.2.1 summarizes our benchmark calibration parameters.

The annual discount factor, β, is 0.965 to match the 10 year maturity rate of US treasury. The markup, α, is 1.15, which account for corporate profit over GDP in the US[11]. The multiplier of investment technology, κ, is 0.97, which is less than 1 to produce decreasing return to scale technology. This value is chosen to match the portfolio choice for risky assets of poor household observed in the individual data sets in Survey of Consumer Finance 2013 (SCF). The larger the value of κ, the smaller the portfolio weighting of poor households to risky assets. We discuss the reasoning for choosing this parameter in Appendix 1.A.3. The other parameters in the first set of rows are conventional except for the relative risk aversion, the coefficient of disutility from labor $\psi = 2$, and the Frisch elasticity of labor supply $1/\phi = 1$. We set the risk aversion parameter across countries, $\sigma^{us} = 2.0$ and $\sigma^{rw} = 1.3$. In the benchmark case, we consider the economy, in which the US earn higher return by investing in RW equity. Given home bias, the identical net worth constraint and higher risky premium in RW, household in the RW demand more safe assets as long as both US and RW households are equivalently risk averse because their main saving instrument is risky. To calibrate US treasury holding of the RW, we assume RW households are more risk appetite than US households.

For the net worth constraint, we assume the constraints in both countries are 0[12]. We assume that the labor income process is under the persistence probability of 0.95 and $\epsilon_t^{l,i} \in \{0.5, 1.5\}$ following MQR. These values are targeting an estimated value for log earnings,

[11] This setting is coincidentally close to the setting In MQR, where 85% of income comes from labor and 15% of income is generated from capital. Therefore, the following results in both static and dynamic exercise are comparable to MQR.

[12] In MQR, the heterogenous net worth constraint across border is considered. However, in this paper, we want to focus on heterogenous capital income risk and its implication for demand for safe asset, hence we set the same level of constraint, which is 0. If we think of the real world, governments in the rest of the world hold high volume of US treasury as foreign reserve partly due to the reserve requirement. Therefore, it might be natural to set the higher constraint in the RW. See Bianchi et al. (2021), Bianchi and Sosa-Padilla (2020), Amador et al. (2018), Bianchi and Lorenzoni (2021), among the literature.

which are autocorrelation coefficient of 0.9 and a standard deviation of 0.30, by Storesletten et al. (2004)[13]. As for the investment shock, we assume the correlation in investment shocks across the investment technologies is 0, as in MQR, for the RW. But, we assume that the investment shock for US equity is more persistent. Moreover, we assume the RW investment technologies are more volatile, twice as large as the US. The calibrated parameters for capital shock in the US is comparable with that of Angeletos (2007), which is alsoconsistent with the recent estimate for the idiosyncratic capital income volatility in Campbell et al. (2022). We need these settings to generate high risk premium on RW equity under the environment only with idiosyncratic shock. This is because the US NFA position is negative, but the net factor payment (NFP) is slightly positive. The net factor payment is an important target because this is one of 'Exorbitant privilege' of the US. We calibrate this moment computed in Chen (2021). We assume the home bias parameter, δ_i^j, are identical for two countries.

The country size of the U.S. is set to $\mu^{us} = 0.5$[14]. The Total Factor Productivity in each region is normalized to 1. The government sector variables for the US is calibrated to match the data from 2012 to 2017[15] and we assume the relative size of government purchase and tax to GDP in the RW is identical with the US. The size of debt in the RW is calibrated to match the US holding on RW bond given the same fraction is hold by the US and the RW. We assume the interest rate of RW bond is 3%. Finally, the benchmark calibration targets six moments, four decomposition entities for NFA in equation (1.2.10) adding to US treasury 10y yield and the net factor payment. Therefore six parameters $\{\beta, b^{rw}, \sigma^{rw}, \delta_i^j, \pi^{k,us}, \epsilon^{k,rw}\}$ are calibrated to match the six moments, which period spans from 2012 to 2017. Table 1.2.2 shows the target data sets and corresponding model generating moments, all of which are

[13]While the recent literature, as Kaplan et al. (2018), Auclert and Rognlie (2018) and others, consider the more sophisticated earning process than the setting in this paper, the main focus here is the portfolio choice of agents so that we simplify the labor income earning process.

[14]The assumption for the size of country is important because even if we set the volatility for the capital return, the liability constraint, and the size of debt are the same, the price of capital and wage are higher in the U.S. than the rest of the world. Bonds issued in the currencies of larger economies are expensive because they insure against shocks that affect a larger fraction of the world economy, see Hassan (2013) for more detail. MQR sets the country size of the US as 0.3 to reflect the ratio of US GDP to world GDP in the benchmark model, but we set symmetric size to each region to understand the underlying mechanism under our experiments.

[15]To see construction of government sector data, see Appendix 1.B.1.

reasonably matching.

Table 1.2.1: Benchmark calibration parameters

Parameters	Description	Main calibration	Target
β	Discount factor	0.965	US treasury 10y yield
σ^{us}	Relative risk aversion	2.0	Standard calibration
σ^{rw}	Relative risk aversion	1.3	NFA (NIIP 2012-2017)
ψ	Disutility from labor	2.0	Auclert et al. (2021a)
ϕ	Inverse Frisch elasticity	1.0	Standard calibration
α	Markup	1.15	Mendoza et al. (2009)
κ	Multiplier of investment tech.	0.97	SCF 2013
$\underline{a}_i \ \forall \ i$	Net worth const.	0.0	Simplifying assumption
δ^{rw}_{us}, δ^{us}_{rw}	Home bias	0.92	NFA (NIIP 2012-2017)
$\epsilon^{l,i}_t \ \forall \ i$	Idiosyncratic shock for labor	$\in \{0.5, 1.5\}$	Storesletten et al. (2004)
$\pi^{l,i} \ \forall \ i$	the persistence probability	0.95	Storesletten et al. (2004)
$\epsilon^{k,us}_t$	Idiosyncratic shock for capital in US	$\in \{0.4, 1.6\}$	NFA (NIIP 2012-2017)
$\pi^{k,us}$	the persistence probability in US	0.85	NFP (NIIP 2012-2017)
$\epsilon^{k,rw}_t$	Idiosyncratic shock for capital in RW	$\in \{-0.2, 2.2\}$	NFA (NIIP 2012-2017)
$\pi^{k,rw}$	the persistence probability in RW	0.50	Mendoza et al. (2009)
μ^{us}	US country size	0.5	Normalization
$Z^i \ \forall \ i$	TFP	1.0	Normalization
b^{us}	Government debt of US	74.9%	NIPA 2012-2017
b^{rw}	Government debt of RW	40.0%	NFA (NIIP 2012-2017)
r^{rw}	Return on RW debt	3.0%	Simplifying assumption
γ	Fraction of RW bond hold by US	0.5	NFA (NIIP 2012-2017)
$gp^i \ \forall \ i$	Government purchase	15.3%	NIPA 2012-2017
$\tau^i \ \forall \ i$	Government tax	27.2%	NIPA 2012-2017

Table 1.2.2: Matched steady state

Description	Data	Model
r	2.2%	2.3%
NFP/Y	1.0%	0.0%
NFA^1/Y	37.0%	35.0%
NFA^2/Y	16.0%	17.2%
NFA^3/Y	-34.0%	-32.2%
NFA^4/Y	-33.9%	-30.0%

Solution method. The model is solved out with the solution method developed in Auclert and Rognlie (2018) and Auclert et al. (2021a). To solve out the non-linear equilibrium system, we use the state space Jacobian in both static and dynamic exercise. See Appendix 1.C for computational detail.

1.2.4 Policy function

Figure 1.2.1 describes the policy function for the individuals both in the US (upper) and in the RW (bottom). Each line represents for the value of portfolios, $P^{us}\theta_i^{us}$ (blue), $P^{rw}\theta_i^{rw}$ (green), and $b_i^{us}/\left(1+r^{us}\right)$ (red). The solid thin lines are for the low wage type and the solid thick lines for the high wage type. Since the persistent idiosyncratic shock for investment technology in the US dramatically changes the policy function so we separate the policy function according to the type of US investment shock type. Since the idiosyncratic shock in the RW is assumed to be independently and identically distributed across type, the policy functions for each high and low type should be identical. All values are normalized by the population average of the net worth in each country.

It is natural to see the portfolio for risky assets of the high wage type is larger than that of the low type. This is because the higher wage you earn in the current period, you can consume more and marginal utility from consumption is small. As Proposition 1 states, the high wage type people less likely to hit the constraint and the shadow price, ζ, tends to be smaller. Thus, they can invest more in risky assets.

Moreover, the risk taking behavior is heterogenous even in the same wage type. If asset households have is small, the shadow price of the net worth constraint is large, the amount of equity claims held decline as Proposition 1 states. On the other hand, they strengthen their risk-taking behavior and increase their holdings of risky assets if they become richer. Finally, when their assets become large enough, they will no longer need to borrow and hold government bonds as assets. The same argument applies to the high wage type. But, the households can invest in risky assets by borrowing even if they have no net worth when wage type is high type.

Investment in US equity become largely different according to the type of investment shock for US equity as the US investment shock is persistent. Once if you hit the low type of US investment, you expect you are likely to hit the low type again. Thus, households facing the low type US investment shock are unwilling to hold US equity. On the other hand, if you enjoy the high type shock, you invest in US equity largely again. In terms of RW equity, however, the same argument does not apply as we assume i.i.d. shock for the investment technology in the RW[16].

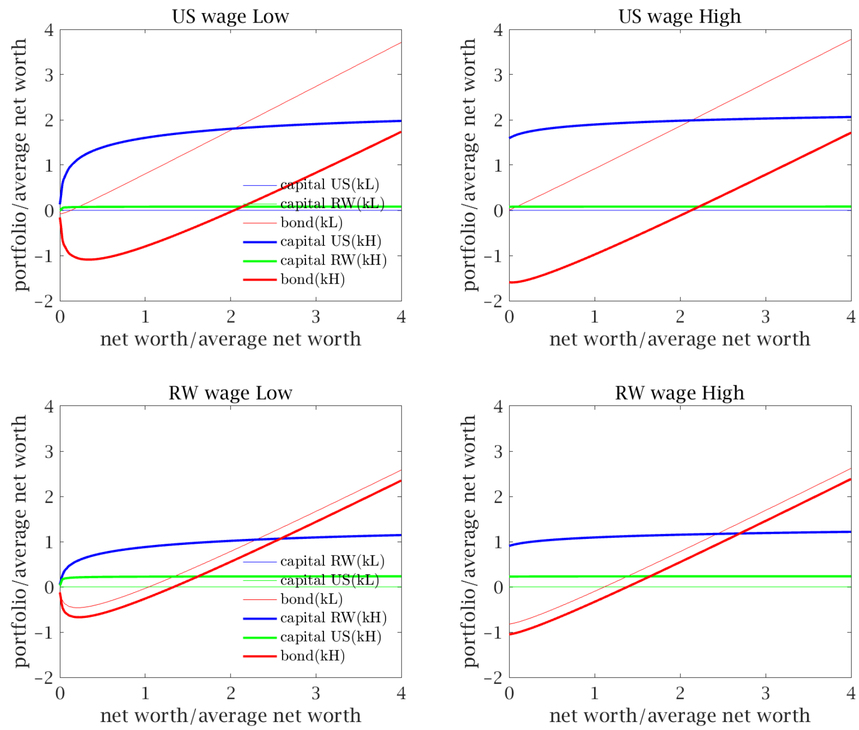

Figure 1.2.1: Policy functions as functions of net worth

NOTE.—The number of types in wage, US capital, and RW capital is two for each. Thus, we have eight idiosyncratic type combinations. We divide it into two according to the wage type. The left panels show policy functions for low-wage types, and the right panels show for high-wage types. In each panel, we have two different types according to capital type.

1.3 Interaction of US Twin Debts

In this section, we examine the impact of the size of government debt on the portfolio choice of other asset classes, the resulting NFA, and social welfare in the presence of foreign demand for safe assets. In order to do that, we consider separately a closed economy (Financial Autarky) and a two country economy (Financial Integration). We analyze how the transition from a

[16]The model prediction qualitatively match the portfolio data of each household in the US. In Appendix 1.B.3, we present the portfolios of each household found in SCF 2013. The data shows if households are high wage type, they borrow more and invest in financial and non-financial assets. However, the opposite thing are observed for low wage type households. It should be noted that the model cannot capture quantitative consistency in terms of borrowing limits and linear risk-taking behavior in the data. See Appendix 1.B.3 for more details.

closed economy to a bilateral economy changes and how it depends on the size of government debt. In an analysis for closed economy, we consider the situation of financial autarky, where the US households cannot access to RW equity and bond claims. We compare the outcome with the seminal work on welfare analysis under different debt size, Aiyagari and McGrattan (1998) (henceforth, AM)[17]. In two country case, we allow the households exchange equity claims and treasury bills in the financial market. So the special case that the net supply of US debt is 0 in our analysis is close to MQR, where net supply of international contingent claims is 0. Here we consider the case of zero net supply of government debt and the case of B=0.75Y government debt, which is the average of the five years since the financial crisis (2012-2017). We call the latter the benchmark.

1.3.1 Financial autarky

The left side of Table 1.3.1 shows the steady-state key moment results for the two cases of government debt. Figure 1.3.1 shows how the moments change as government debt increases. First, an increase in government debt leads to an increase in interest rates. Higher interest rates reduce the amount of government transfers through the government budget constraint equation. Stock prices fall as demand for savings shifts from equities to government bonds. With reduced government transfers, households between the poor and middle class will work more and consume less to avoid binding on the net worth constraint. The wealthy, on the other hand, are less affected by the decrease in government transfers. Because of our model setting, the wealthy are domestic creditors through government bonds and hold government bonds on the asset side, so higher interest rates are a tailwind for them.

When looking at individual households, welfare is deteriorating, but the population distribution shifts to the right because assets, including government bonds, are increasing in the economy as a whole. The combination of these two effects leaves overall welfare little changed. In the fine-grained view, these two opposing effects give the optimal size of

[17]In AM, the authors fix the rate of government consumption and government transfer and compute tax rate which satisfy the government budget constraint according to the debt size. In this paper, however, we fix tax rate and government consumption rate and consider how the size of transfer would change according to the debt size. But, these difference does not change the implication of our experiments.

government debt in a closed economy (B= 0.15Y). However, they are negligible in quantitative terms compared to the AM results discussed below. Note that since the overall economy's labor supply increases and wage, which is aggregate productivity divided by the markup, is constant, corporate profits increase. As a result, returns from equities increase.

Comparison with Aiyagari and McGrattan (1998). It is essential to discuss the difference between AM and the impact of the size of government debt on welfare in a closed economy. The AM model assumes capital accumulation in a form commonly found in classical economic growth models. Households save in government bonds or risk-free, depreciable capital. Thus, an increase in outstanding government debt would lead to a crowding out of capital, reducing household consumption expenditures; in AM, the optimal size of government debt is determined by a tradeoff between the supply of liquidity and the opposite forces of capital crowding out and higher taxes.

However, since the focus of this paper is on households' portfolio choices from differentially risky assets, total capital is fixed. Since the domestic output is determined by domestic labor supply, the crowding-in effect of capital is not observed in the model in this paper. Thus, it is a minor issue compared to the effects of income transfers, as seen in the two-country model below[18].

1.3.2 Financial integration

The right hand side of 1.3.1 provides the results for both zero net supply and benchmark size of the debt under financial integration.The effects on government debt welfare would be markedly different if capital flows between the U.S. and the rest of the world were allowed to move freely. First, in the case of a two-country economy, interest rates rise more slowly as government debt increases. This is due to demand from RW households for U.S. debt

[18] AM discusses two additional points regarding the impact of an increase in outstanding government debt on welfare when a lump-sum tax is used. The first is the distributional effect of a lump-sum tax. This is the point at which people with fewer assets and lower incomes bear a relatively greater burden than those with more assets and higher incomes. Since the marginal value of assets decreases with the amount of assets held, this tends to lower welfare. Second, using an ex ante interpretation of the welfare criterion, for a given household, due to earnings uncertainty, a lump-sum tax has the effect of worsening the volatility of after-tax earnings and increasing the volatility of consumption.

Table 1.3.1: Benchmark results

		Financial autarky		Financial integration			
		$B^{us}=0$	$B^{us}=0.75Y^{us}$	$B^{us}=0$		$B^{us}=0.75Y^{us}$	
Description	variable	US	US	US	RW	US	RW
Interest rate	r	2.21	2.29	2.23	-	2.28	-
Transfer rate	tr	11.90	10.22	11.90	10.58	10.23	10.58
Price of stock	P	4.43	4.28	4.38	2.80	4.31	2.73
Capital return	R	2.27	2.35	2.29	3.20	2.34	3.27
Std of capital return	$std\,(R)$	1.36	1.41	1.37	3.84	1.40	3.92
GDP	Y	0.770	0.772	0.767	0.687	0.772	0.685
Consumption	C	0.653	0.654	0.654	0.578	0.654	0.580
Welfare	V	-64.75	-64.75	-64.44	-119.63	-64.77	-119.34
Net Factor Payment	NFP	-	-	0.6	-	-0.0	-
Net Foreign Asset	NFA	-	-	15.9	-	-10.1	-
	NFA^1	-	-	36.2	-	35.0	-
	NFA^2	-	-	17.3	-	17.2	-
	NFA^3	-	-	-33.0	-	-32.2	-
	NFA^4	-	-	-4.7	-	-30.0	-

NOTE.—The definition of decomposition of the NFA is in equation (1.2.10).

as a safe asset, as RWs hold riskier assets as their primary portfolio relative to more U.S. households. As a result, the pace of interest rate increases will be less than in a closed economy. Through a similar channel, the decline in the price of U.S. equities associated with increased government debt will also be slower than in a closed economy. However, since the increase in government debt monotonically worsens the NFA along with the increase in RW holdings of U.S. Treasuries, income transfers abroad will increase along with the increase in government debt. This, in turn, results in less consumption in the U.S. and the need to produce more. Thus, the increase in government debt monotonically worsens the welfare of U.S. households through the channel of an increase in NFP.

Focusing on changes in welfare from financial autarky to financial integration at different government debt levels, there is more room for welfare improvement at lower levels of government debt, and welfare is likely to worsen as the debt level increases described above. While this outcome depends on what we set for the RW interest rate, if we consider the model based on a model calibrated to the level of U.S. government debt, NFP, and NFA positions since the financial crisis (2012-2017), if government debt is smaller than that, NFP will be positive. If government debt is larger, NFP will be negative. Thus, for example, financial integration improves social welfare in an environment with no government debt, but the opposite is true at higher levels of government debt (100%), etc. Finally, when observing welfare in other countries, an increase in U.S. debt works in the direction of improving welfare. This is due to the same effect as the improvement in welfare from the supply of liquidity in a closed economy.

Heterogeneity. Heterogeneity of households is also an important aspect to consider. As noted in MQR, the behavior of the interest rate after financial integration is important when considering the welfare of individual households. Figure 4 shows changes in the welfare of individual households and the population distribution for both cases between autarky and integration. When government debt is low, welfare improves mainly among the wealthy, who can hold riskier assets. In contrast, when government debt is high, the effect of lower interest rates through the demand channel for safe assets is more substantial, and the degree of improvement in welfare is remarkable for the poor. These are because the poor, who are initially net debtors, benefit from lower interest rates, while the wealthy, who are net creditors, lose from lower interest rates.

1.3.3 Transition path

This section analyzes the transition process from autarky to financial integration under the benchmark government debt size (B = 0.75Y). Note that in the RW under autarky, we assume that 0 net supply US Treasuries are included in the portfolio. First, as soon as the economy goes under financial integration, the interest rate on US Treasuries declines relative to that in the case of US autarky. The demand for safe assets from RW households with a high portfolio share of risky assets pushes interest rates lower than in the initial condition. With lower interest rates, US households can invest in riskier assets and increase their investments in stocks, and the price of US stocks rises. At this time, the price of RW stocks also rises. This is because, in the RW, interest rates rise from the initial state immediately after financial integration, which causes labor supply to increase, especially

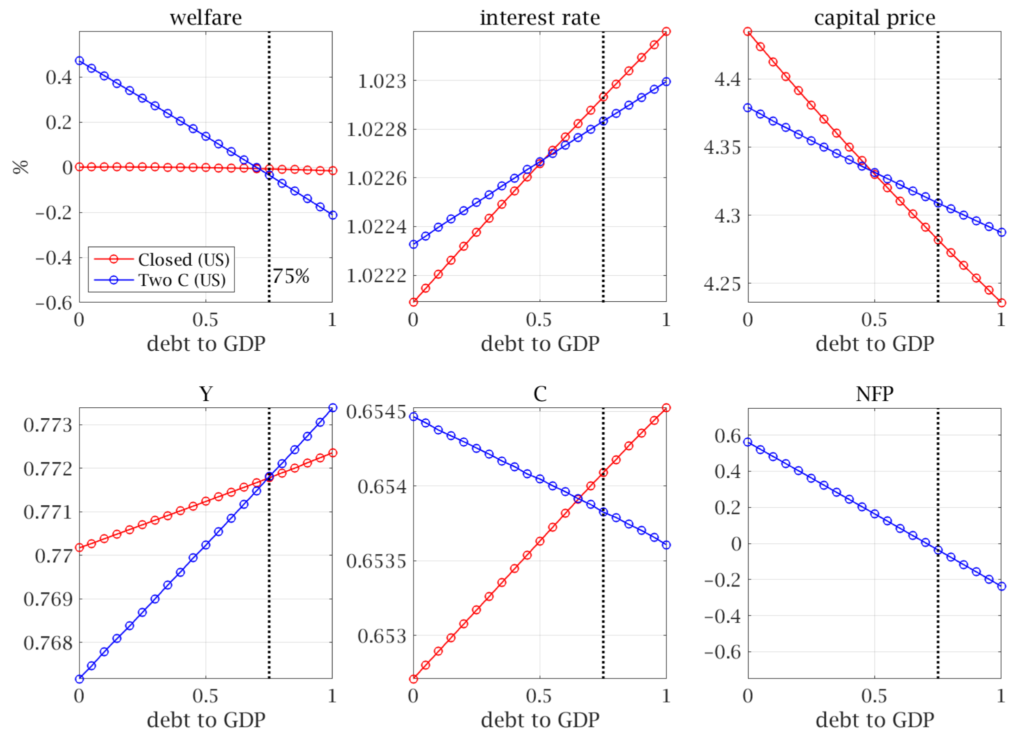

Figure 1.3.1: Price and GDP versus debt/GDP ratio for the benchmark economy in the US

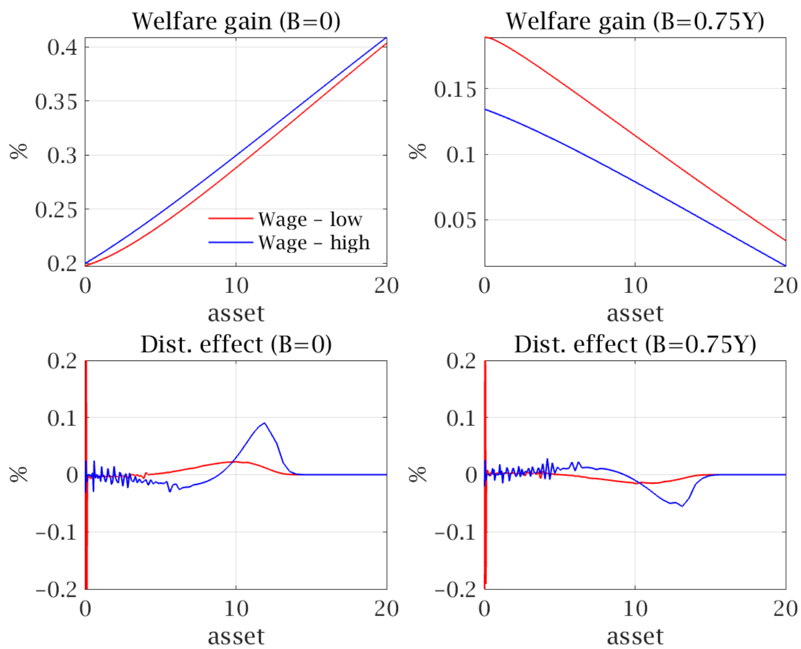

Figure 1.3.2: Welfare gain and distribution effect in the US under the two experiment with debt size of $B = 0$ and $B = 0.75Y$

among the poor, and corporate profits to increase (Proposition 1).

Looking at the composition of the NFA in the US, positions are quickly adjusted right after an integration. US households increase their holdings of foreign stocks, and foreign households increase their holdings of US stocks and US government bonds as safe assets under the risk-sharing motive. In this case, it is important to note that the US holds half of its foreign claims exogenously.

Financial integration causes interest rates to fall in the US and rise abroad. In the US, lower interest rates lead to higher consumption and lower labor supply, which improves welfare. On the other hand, in the foreign sector, the opposite occurs for all.

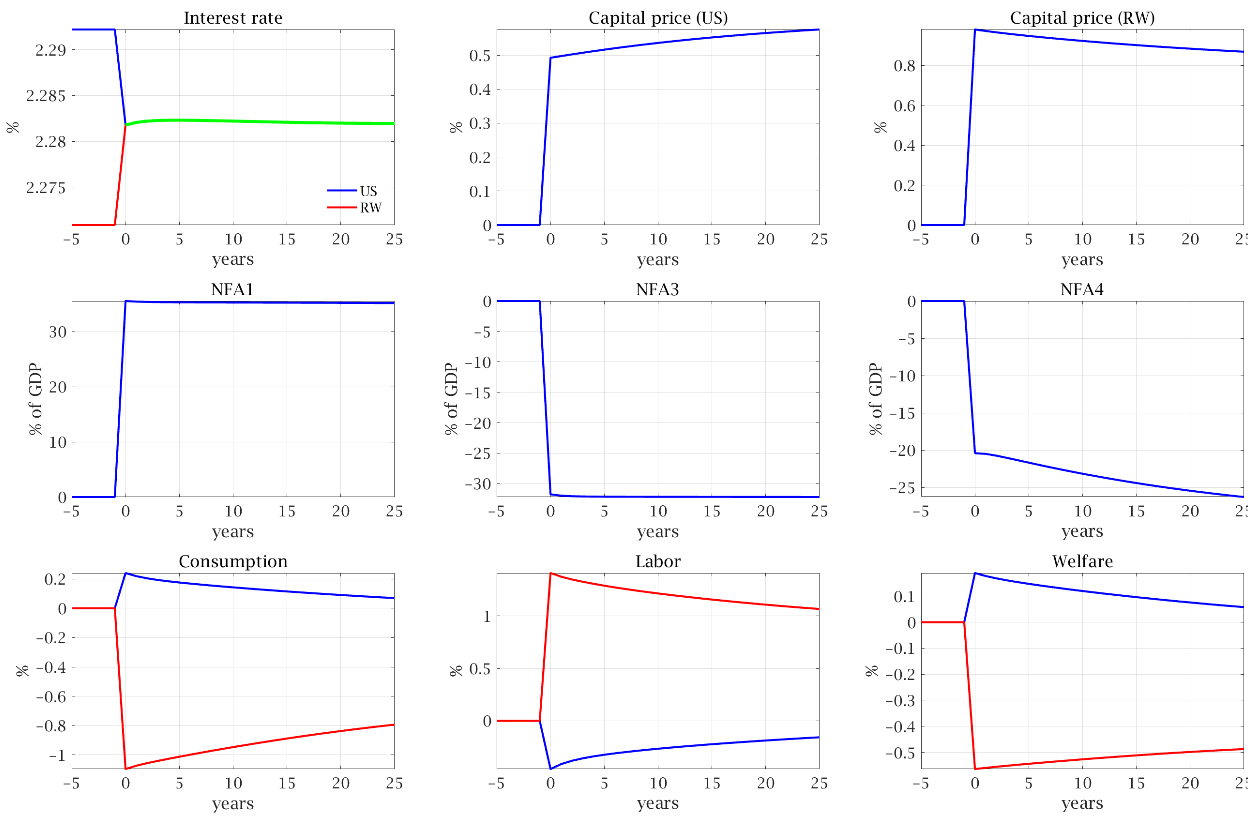

Figure 1.3.3: Transition path from financial autarky to financial integration economy with the benchmark case

Note.—If the description for measure unit in vertical axis shows '%,' it means the plot shows the change from the initial steady state, except for interest rate. If the unit of measure is '% of GDP,' it is straightforward.

1.4 NFA Deterioration by What?

In this section, we consider US NFA deterioration episode under different background mechanisms. We will simulate three forces, the markup increase hypothesis, the fiscal deficit hypothesis, and the global financial cycle hypothesis. We introduce the base notion for each hypothesis briefly, then examine the model outcome given -5% NFA position deterioration shock, which is corresponding to an average of yearly US NFA deterioration post the Great Financial Crisis (from 2008 to 2020).

- **Markup increase hypothesis** - Atkeson et al. (2020) argue the rise in the price of US equities (the Valuation Effect) as a major factor in the recent deterioration of the NFA, which they attribute to the rise in markups of US companies. They state that the markup hypothesis is the most likely explanation, based on the data for firm-side profit, dividend data, and Buffett ratios. An increase in markup of the US firms is pointed out in the literature (see, e.g., De Loecker et al. (2020), Edmond et al. (2018), and Boar and Midrigan (2019) among others).

- **Fiscal deficit hypothesis** - Aggarwal et al. (2022) point to three facts based on recent data. They are the increase in private savings in the U.S. and around the world, the widening of the current account deficit in the US, and the substantial expansion of budget deficits in countries around the world, especially in the US. The paper attributes the first two to the US government budget deficit. The paper develops a multinational heterogeneous agent model in which deficit fiscal transfers cause a substantial increase in private savings and persistent current account deficits.

- **Global financial cycle hypothesis** - The global financial cycle (GFC) refers to fluctuations in financial activity on a global scale (see Rey (2015)). Miranda-Agrippino and Rey (2020b) apply a medium-sized Bayesian VAR to data for the period 1980-2010 (or 1990-2010) to show that US monetary policy is a driver of the global financial cycle. In this context, Miranda-Agrippino and Rey (2020a) confirm that the U.S. exchange rate appreciates when prices of risky assets around the world fall and this trend has been more pronounced since the 2008 financial crisis. In other words, the results suggest that the volatility of stocks and other risky assets in other countries has increased when viewed in US dollar terms[19].

1.4.1 Steady state outcome

Table 1.4.1 summarizes the result of counter factual simulations for 5% US NFA position deterioration in each scenario. To deteriorate NFA position of the US by 5%, we need a positive shock to the markup in the US by 0.4% ($\alpha = 1.15 \Rightarrow 1.154$). In this scenario, the households in the US increase US equity holding because of its profitability. Thus, their demand for US treasury and RW equity from US households decreases. The households in the RW exploits this opportunity to have profitable US equity despite of its home bias. Moreover, they increase demand for US treasury as it becomes cheap as a result of less demand from the US investors. Consequently, the net factor payment to the RW increase and hence the welfare of the US households deteriorates.

Next, if an outstanding for the government debt increase by 15% ($B/Y = 0.75 \Rightarrow 0.90$), it also generates 5% US NFA position deterioration. In this case, as the supply of US treasury increase, the demand for it increases in both countries, Contrarily, the demand for equity claims become less so that mutual holdings of equity decrease but slightly. As a result, the US households become have to pay more interest payment for government bond, then, it worse off the US welfare.

Lastly, we think of the case that wage income and capital income in RW become more volatile ($\Delta\epsilon^{l,rw} = 0.50 \Rightarrow 0.505$, $\Delta\epsilon^{k,rw} = 1.2 \Rightarrow 1.4$), where RW households demand less volatile assets, and hence their portfolio shift to US equity or US treasury. However, due to home bias, they cannot diversify its own equity risk by investing in the US equity. Thus, most of their demand goes to US treasury. The US household enjoys the convenience yield generated due to higher demand for safe assets. Since their own stock is stable and they can

[19] Note that this result is also confirmed by the correlation between the first order difference between stock prices and exchange rates, even without using the VAR. Moreover this hypothesis is consistent with Maggiori et al. (2020). Maggiori et al. (2020) finds that the dollar becomes more dominant after the financial crisis. For more detail, see Appendix 1.B.4.

borrow more easily, they can leverage their position to hold the RW stock claim. While, the net factor payment to the US by holding RW equity increases, but the interest payment for US treasury to the RW also increases. Combining these effects, the US welfare deteriorates in total.

Heterogeneity. How would the impact of each shock on individual households differ? This subsection examines the impact of the three shocks on individual households. Figure 1.4.1 provides the welfare gains at the terminal steady state compared to the initial steady state and the change in distribution with respect to asset holding.

US: The implications of the US markup and fiscal shocks are similar. In a markup shock, wages decline, and in a fiscal shock, the ratio of fiscal transfers to GDP at the terminal steady state declines. Thus, the deterioration in social welfare is particularly large for poor households close to the net worth constraint. The overall distribution shifts to the right and the share of the wealthy households increases as the decline in wages and fiscal transfers stimulate households' saving motives.

When the shocks are attributed to foreign countries, the model's implication is the opposite, with the wealthier group experiencing a more significant deterioration in welfare and the overall distribution shifting to the left. This is because of the demand for US assets abroad and the outflow of US capital abroad due to increased uncertainty in the foreign sector. The wealthy households, in turn, hold more volatile foreign capital, making their savings less stable and worsening their welfare. Interestingly, welfare improves for households with lower wages and smaller assets. This is because the demand for safe assets from abroad has lowered interest rates and reduced borrowing costs for the poor.

RW: In the overseas sector, the implications of markup shocks and fiscal shocks are also similar. Welfare improves mainly for the wealthy, who can increase their holdings of assets due to increased holdings of US equities in the case of a markup shock and increased holdings of US treasury in the case of a fiscal shock. These increases in asset holdings shift the overall distribution to the right.

Finally, the increase in uncertainty in the foreign sector worsens the welfare of the low-wage type the most, while the welfare of the high-wage type does not deteriorate much. The distribution shifts to the right as declines in both labor and capital incomes combine to increase household asset holdings due to an increase in the precautionary saving motive. It is important to note that although the change in the welfare of individual households worsens, the overall welfare improves as the distribution shifts to the right.

Table 1.4.1: Counter factual simulation results

		Benchmark		Markup		Fiscal deficit		GFC	
Description	variable	US	RW	US	RW	US	RW	US	RW
Interest rate	r	2.28	-	2.29	-	2.29	-	2.28	-
Transfer rate	tr	10.23	10.58	10.22	10.58	9.89	10.58	10.23	10.58
Price of stock	P	4.31	2.73	4.38	2.72	4.30	2.72	4.32	2.73
Capital return	R	2.34	3.27	2.35	3.28	2.35	3.28	2.33	3.27
Std of capital return	$std(R)$	1.40	3.92	1.41	3.94	1.41	3.94	1.40	4.58
GDP	Y	0.772	0.685	0.772	0.684	0.773	0.684	0.772	0.684
Consumption	C	0.654	0.580	0.653	0.581	0.654	0.581	0.653	0.581
Welfare	V	0.00	0.00	-0.09	0.05	-0.10	0.05	-0.09	0.03
Net Factor Payment	NFP	-0.0	-	-0.2	-	-0.2	-	-0.1	-
Net Foreign Asset	NFA	-10.1	-	-15.1	-	-15.1	-	-15.1	-
	NFA^1	35.0	-	34.7	-	34.7	-	37.2	-
	NFA^2	17.2	-	17.1	-	17.1	-	17.1	-
	NFA^3	-32.2	-	-32.8	-	-32.1	-	-32.3	-
	NFA^4	-30.0	-	-34.2	-	-34.8	-	-37.2	-

NOTE.—The definition of decomposition of the NFA is in equation (1.2.10). The row for welfare shows the change in '%' from the benchmark on each simulation.

1.4.2 Transition path

We show the transition path from the benchmark equilibrium to the terminal steady state given US NFA deterioration shock in this subsection. We give the exogenous shock on debt and transfer schedule (Figure 1.4.2). We assume $AR(1)$ process with persistence 0.50 for the debt schedule, with which the level of debt approximately converges to the terminal level within 5 years[20]. In terms of transfer, as we assume the fiscal authority promptly dose not change the level of tax rate and government expenditure rate, transfer is determined to satisfy the government budget constraint along with the debt path. We provide the transition path

[20] The assumption for 5 years convergence comes from observations on the trajectory of the US debt after the Great Financial Crisis and the Covid-19 crisis. However, even if we change the assumption for convergence, the qualitative implications of transitory path across different parameters for persistence, 0 and 0.5, are identical.

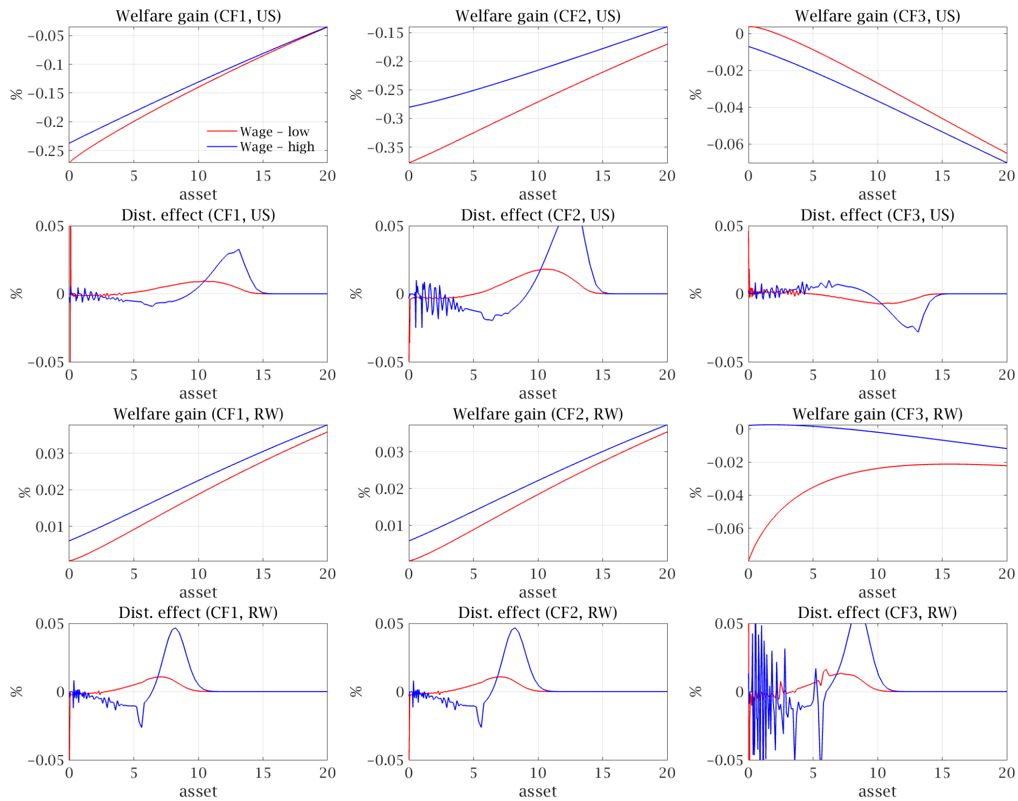

Figure 1.4.1: Welfare gain of individuals and the change in distribution after given the shock

after -5% NFA deterioration shock in each episode in Figure 1.4.3.

Markup hypothesis. When markup increase shock hits the final good producing firm in the US, the labor share decreases and the share of corporate profits over its production increases. Thus, demand for US equities increases as price to earning ratio, P^{us}/Π^{us}, increases (see Proposition 1). This results in a decrease in demand for US treasury and RW equity. Since there is no change in the supply of Treasuries but the demand decreases, interest rate rises. The price of RW equity also drops. Due to the presence of a home bias, a significant portion of the increase in demand for US equities comes from domestic households, while domestic demand for US government bonds will decline. Therefore, demand from foreign investors will also flow to US government bonds, resulting in both an increase in the acquisition of US equities and an increase in demand for US treasury from abroad. Note that the wealth effect from rising stock prices temporarily encourages an increase in US households' acquisition of foreign stocks.

In the US, the wealth effect from higher stock prices temporarily increases consumption and lower labor supply despite higher interest rates. As a result, social welfare improves. However, consumption declines in the long run because higher markups imply lower wages. In addition, corporate profits flow out through transfers to foreign investors, so consumption remains reduced and does not return to its original level.

The real side of the economy in the RW is forced to reduce consumption as interest rates rise and their own stock prices fall simultaneously. Needless to say, the labor supply will increase, and social welfare will temporarily deteriorate.

Fiscal deficit hypothesis. Next, we consider the increase in government debt due to fiscal spending. In this simulation, we assume that government transfers cause an increase in government debt[21]. We assume that government transfers are equal for each household. First, interest rates rise as the supply of US government bonds increases. Higher interest rates lower the aggregate demand for risky assets in the US and the RW combined. As a result, equity prices fall. However, investment attitudes toward risky assets differ between

[21] As checked in Appendix 1.B.1, government transfers were most significant in the government sector variables in the case of the Great Financial Crisis and the Covid-19 crisis. Therefore, in this paper, we consider that government transfers mainly drive the government debt schedule.

the two countries. In the US, households have more money to invest because of positive fiscal transfers. Therefore, they are more willing to invest in risky assets and increase their investments in foreign equities (NFA1). Overseas, however, investment in risky assets declines, and investment in US equities also declines (NFA3) because they are only affected by rising interest rates.

The response in the real economy depends on the level of assets held by households. First, the poor, who have a higher marginal propensity to consume, increase consumption and decrease labor supply in the US. It should be noted that a decline in labor supply leads to a decline in corporate profits, which in turn leads to a decline in US stock prices. In addition, wealthy households with a low marginal propensity to consume do not increase their consumption significantly when they receive transfers from the government. Instead, they save these transfers in government bonds and domestic and foreign stocks. Due to the combination of asset effects and increased consumption, social welfare in the US improves the most in the short run among the three simulations.

In the RW, consumption declines, and labor supply increases due to higher interest rates and lower RW stock prices.

Global Financial Cycle hypothesis. When a shock occurs that results in volatile wages and stock returns in the foreign country, households in the RW demand more US treasuries, interest rate falls. Since interest rate falls, US households can take higher leverage and hold more equity claims. While the volatility of RW equity increases, higher demand from US households pushes up RW equity claim price. NFA^1 and NFA^4, US holding on RW equity and RW holding on US treasury, are the most significant in magnitude among three simulations according to the mechanism.

In real side, interest rate decline allows US households to consume more and less supply labor force. As a consequence, US households' social welfare initially improves, but it goes to below 0 soon due to the same mechanism in the other two experiments. In the RW, notably, welfare deterioration is the hardest among due to exposure to the higher idiosyncratic risk in both labor and capital income.

The impact of higher labor and capital income volatility in the foreign sector is significant.

The investment in safe assets due to the precautionary saving motive reduces consumption and, at the same time, raises labor supply. As a result, social welfare in the foreign sector is the worst in the simulation.

NFA deterioration by what? We summarize the behavior of the NFA for the above three different shocks. The markup increase shock has high explanatory power for increasing US equity holdings from abroad. The uncertainty shock of wage and capital income in the foreign sector can explain the increase in RW equity holdings from US households, respectively. Given that fiscal stimulus shock alone cannot explain the increase in US stock holdings from abroad, the decomposition of the US NFA position implies that markup and RW volatility shocks coexisted after the financial crisis.

Wrap up. In this section, we examine the shocks that have been proposed in the existing literature which exacerbate the NFA position. Decomposing recent trends in the NFA position into its individual components, our analysis suggests that both markup and volatility shocks are necessary. Every shock improves US social welfare in the short run only immediately after the shock but worsens it by 0.1% in the long run. There is room for further study of exchange rate shocks and home bias shocks, which are not examined in this paper, but these will be the subject of future research.

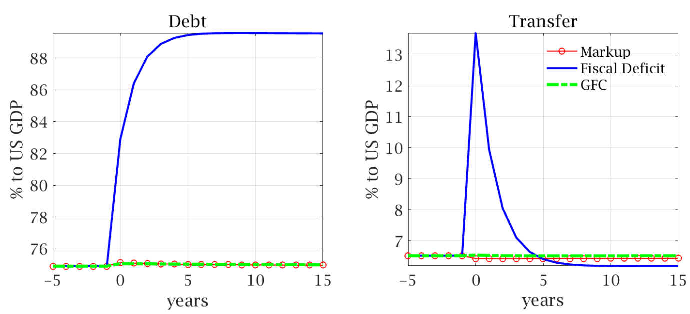

Figure 1.4.2: Exogenous path for fiscal variables for -5% US NFA deterioration simulation under alternative hypothesis

1.5 Conclusion

This paper analyzes the impact of US government debt and external debt on US households' welfare by constructing a two country heterogeneous agent general equilibrium model with

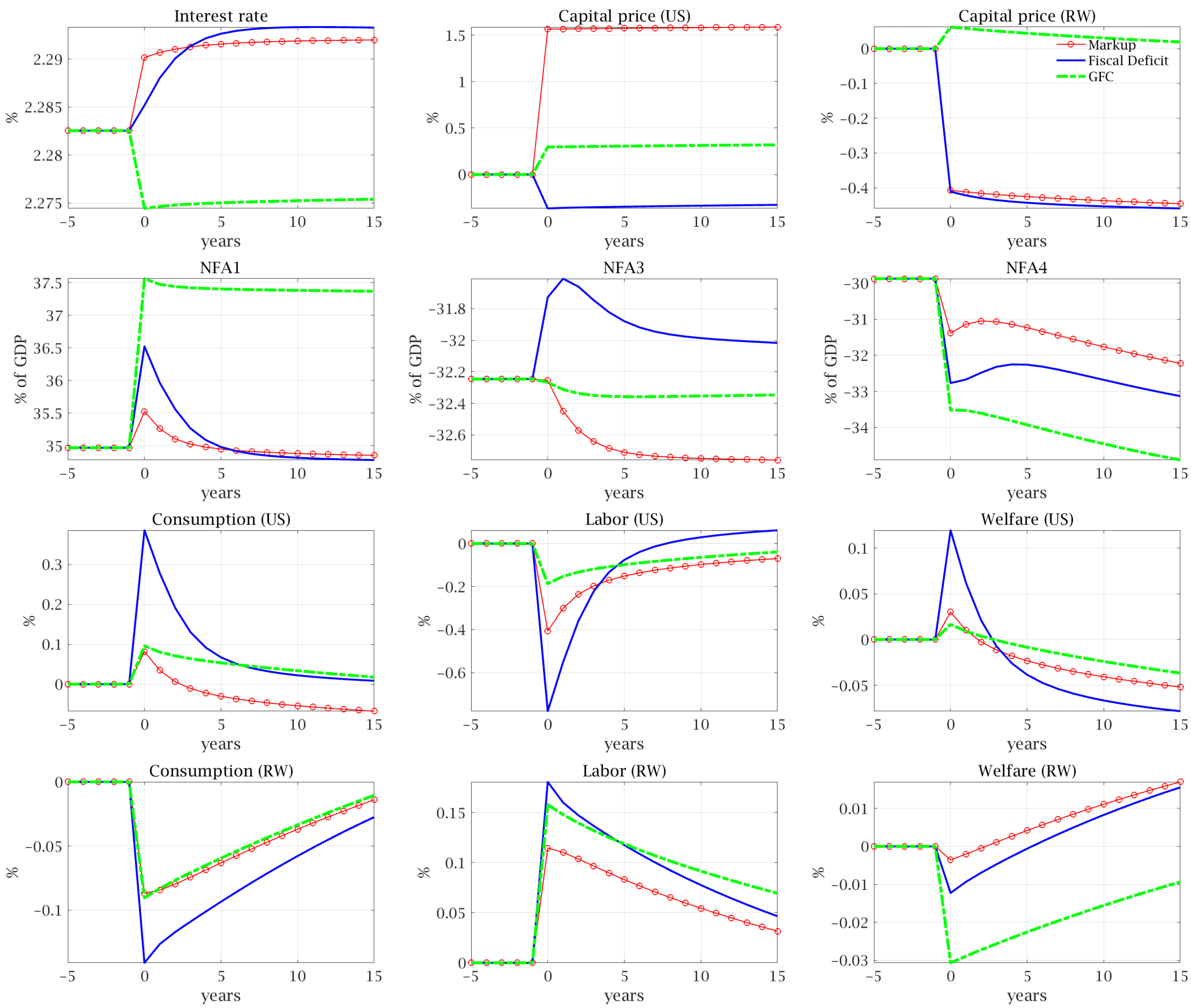

Figure 1.4.3: Transition path after -5% US NFA deterioration shock under alternative hypothesis

Note.—If the description for measure unit in vertical axis shows '%,' it means the plot shows the change from the initial steady state, except for interest rate. If the unit of measure is '% of GDP,' it is straightforward.

fiscal authority and cross-border portfolio choice. First, we identify the impact of increased government debt on household portfolios in the US and abroad and explore how and why households' welfare changes. Next, we exploit the model to investigate the impact of three types of shocks that worsen the NFA: a markup shock, a government stimulus shock, and a shock that causes foreign financial assets to become volatile[22].

The main findings are as follows. First, an increase in government debt increases the foreign sector's holdings of US treasury claims under the demand for safe assets from abroad. A decrease in the amount of liquid assets held within the US leads to a deterioration in rehabilitation. Compared to a closed economy, welfare improves when government debt is low because the effects of lower interest rates and risk-sharing are dominant. However, asset outflows abroad will outweigh the advantages as government debt increases. Next, when considering shocks that worsen NFA, it is essential to know the main underlying force when looking at the impact on welfare. A markup shock or a government stimulus shock will worsen household welfare because its consequence is to reduce household income. However, the implication is different when foreign financial assets become volatile. The reason is that, without any change in household incomes, demand for government debt from abroad will drive down interest rates, which in turn will facilitate risk-taking by US households and increase the net factor payment to the US from abroad.

The above analysis is not meant to identify the causes of the recent deterioration in the NFA's position. Knowing which of the above three factors dominates is a topic for future research, as it has significant policy implications. In addition, the following issues would matter when considering the welfare implication of the Dominant Currency Paradigm. While in the paper, we only consider the case with the idiosyncratic shock for labor income and capital income to feature each optimal portfolio choice and risk-sharing condition with a heterogeneous agent model. Seeing how the model outcome differs if we add an aggregate shock and the crucial price to consider a portfolio allocation, the exchange rate as in Auclert et al. (2021b) is fruitful.

[22]In this paper, we do not consider the fiscal risk of the treasury bond. For the fiscal risk of the US government bond with a safe asset view, see Liu et al. (2020). Liu et al. (2020) provide a general equilibrium asset pricing model with endogenous liquidity and corporate default premia subject to fiscal risk.

Next, although we assume exogenous markups in this paper, how portfolio choices affect firms' markups and profits by accumulating tangible or intangible assets will be a critical component in understanding the exorbitant privilege of the US dollar's currency hegemony (Atkeson and Burstein (2010), Itskhoki and Moll (2019)). The stock price increase allows US firms to invest in R&D and acquire intangible capital, including human resources.

Finally, although we have simplified fiscal policy in our analysis, it would be fruitful to further analyze progressive taxation, optimal capital and labor taxation, and taxation of assets under open economy framework (see e.g., Heathcote (2005), Conesa et al. (2009), Heathcote et al. (2017), Navarro and Ferriere (2016), Guvenen et al. (2022)).

1.A Model Details

1.A.1 Country budget constraint

The following two equations are the aggregated households' budget constraint (1.2.2) and the government flow budget constraint (1.2.6):

$$\begin{gathered}
C_t^i + \sum_j^I \left[P_t^j \int \theta_{i,t}^j dH_t^i + \frac{\int b_{i,t}^j dH_t^i}{1+r_t^j} \right] \\
= \left(1-\tau_i^w\right) W_t^i L_t^i + \sum_j^I \left[P_t^j \int \theta_{i,t-1}^j dH_{t-1}^i + \Theta_{i,t}^j \left(1-\tau_j^k\right) \Pi_t^j + \int b_{i,t-1}^j dH_{t-1}^i \right] + TR_t^i, \\
G_t^i + TR_t^i + B_{t-1}^i = \frac{B_t^i}{1+r_t^i} + T_t^i + \Xi_t^i, \\
\Xi_t^i = \left[1 - \sum_j^I \Theta_{j,t}^i \right] \left(1-\tau_i^k\right) \Pi_t^i,
\end{gathered} \tag{1.A.1}$$

where Ξ_t^i is the residual corporate profit in country i, which is not distributed to the investors. Since we assume the decreasing return to scale technology for the investment technology, the aggregate total output from investment is less than 1. The remained corporate profit after taxation become the resource of the government budget constraint. Combining these two equations gives the aggregate country budget constraint:

$$\begin{gathered}
C_t^i + G_t^i + \sum_j^I \left[P_t^j \int \theta_{i,t}^j dH_t^i + \frac{\int b_{i,t}^j dH_t^i}{1+r_t^j} \right] + \left[\sum_j^I \Theta_{j,t}^i - 1 \right] \left(1-\tau_i^k\right) \Pi_t^i + B_{t-1}^i \\
= \left(1-\tau_i^w\right) W_t^i L_t^i + \sum_j^I \left[P_t^j \int \theta_{i,t-1}^j dH_{t-1}^i + \Theta_{i,t}^j \left(1-\tau_j^k\right) \Pi_t^j + \int b_{i,t-1}^j dH_{t-1}^i \right] + \frac{B_t^i}{1+r_t^i} + T_t^i.
\end{gathered} \tag{1.A.2}$$

Now, if we only focus on the country budget constraint of the US, we have:

$$\begin{aligned} & C_t^{us} + G_t^{us} + \sum_j^I \left[P_t^j \Delta\theta_{us,t}^j + \frac{\int b_{us,t}^j dH_t^{us}}{1+r_t^j} \right] + \left[\sum_j^I \Theta_{j,t}^{us} - 1 \right] \left(1-\tau_{us}^k\right) \Pi_t^{us} + B_{t-1}^{us} \\ & = \left(1-\tau_{us}^w\right) W_t^{us} L_t^{us} + \sum_j^I \left[\Theta_{us,t}^j \left(1-\tau_j^k\right) \Pi_t^j + \int b_{us,t-1}^j \right] dH_{t-1}^{us} + \frac{B_t^{us}}{1+r_t^{us}} + T_t^{us}. \end{aligned} \tag{1.A.3}$$

where $\Delta\theta_{i,t}^j \equiv \int \theta_{i,t}^j dH_t^i - \int \theta_{i,t-1}^j dH_{t-1}^i$. The market clearing conditions for the US treasury gives us:

$$\begin{aligned} & C_t^{us} + G_t^{us} + \sum_j^I P_t^j \Delta\theta_{us,t}^j + \sum_j^I \Theta_{j,t}^{us} \left(1-\tau_{us}^k\right) \Pi_t^{us} + \frac{\int b_{us,t}^{rw} dH_t^{us}}{1+r_t^{rw}} + \int b_{rw,t-1}^{us} dH_{t-1}^{rw} \\ = & Y_t^{us} + \sum_j^I \Theta_{us,t}^j \left(1-\tau_j^k\right) \Pi_t^j + \frac{\int b_{rw,t}^{us} dH_t^{rw}}{1+r_t^{us}} + \int b_{us,t-1}^{rw} dH_{t-1}^{us}, \end{aligned} \tag{1.A.4}$$

where we use the relation for tax levied in the US, including the both labor income tax and capital income tax, which is:

$$T_t^{us} = \tau_{us}^w W_t^{us} L_t^{us} + \tau_{us}^k \Pi_t^{us}. \tag{1.A.5}$$

Decomposing the summation terms for corporation profits, then we have,

$$\begin{aligned} & C_t^{us} + G_t^{us} + \sum_j^I P_t^j \Delta\theta_{us,t}^j + \Theta_{rw,t}^{us} \left(1-\tau_{us}^k\right) \Pi_t^{us} + \frac{\int b_{us,t}^{rw} dH_t^{us}}{1+r_t^{rw}} + \int b_{rw,t-1}^{us} dH_{t-1}^{rw} \\ & \qquad = Y_t^{us} + \Theta_{us,t}^{rw} \left(1-\tau_{rw}^k\right) \Pi_t^{rw} + \frac{\int b_{rw,t}^{us} dH_t^{rw}}{1+r_t^{us}} + \int b_{us,t-1}^{rw} dH_{t-1}^{us}. \end{aligned} \tag{1.A.6}$$

We can get, in a similar way, the country budget constraint for the RW:

$$\begin{aligned} & C_t^{rw} + G_t^{rw} + \sum_j^I P_t^j \Delta\theta_{rw,t}^j + \Theta_{us,t}^{rw} \left(1-\tau_{rw}^k\right) \Pi_t^{rw} + \frac{\int b_{rw,t}^{us} dH_t^{rw}}{1+r_t^{us}} + \int b_{us,t-1}^{rw} dH_{t-1}^{us} \\ & \qquad = Y_t^{us} + \Theta_{rw,t}^{us} \left(1-\tau_{us}^k\right) \Pi_t^{us} + \frac{\int b_{us,t}^{rw} dH_t^{us}}{1+r_t^{rw}} + \int b_{rw,t-1}^{us} dH_{t-1}^{rw}. \end{aligned} \tag{1.A.7}$$

Combining the two country budget constraints, we have the world resource constraint as the following:

$$C_t^{us} + G_t^{us} + C_t^{rw} + G_t^{rw} = Y_t^{us} + Y_t^{rw}, \tag{1.A.8}$$

where we use the condition for total net supply of equity claims, $\Sigma_j \Delta\theta_{j,t}^i = 0$.

1.A.2 Current Account and Net Foreign Asset

Firstly, we define the net foreign asset position as the sum of the net bond and equity positions:

$$NFA_t \equiv \int \left[\underbrace{P_t^{rw}\theta_{us,t}^{rw}}_{\equiv NFA^1} + \underbrace{\frac{b_{us,t}^{rw}}{1+r_t^{rw}}}_{\equiv NFA^2} \right] dH_t^{us} + \int \left[\underbrace{-P_t^{us}\theta_{rw,t}^{us}}_{\equiv NFA^3} - \underbrace{\frac{b_{rw,t}^{us}}{1+r_t^{us}}}_{\equiv NFA^4} \right] dH_t^{rw} \tag{1.A.9}$$

The growth of the NFA position is sum of the current account and the valuation effect of each portfolio. The decomposition takes the form as the following:

$$\Delta NFA_t \equiv NFA_t - NFA_{t-1} = CA_t + VE_t. \tag{1.A.10}$$

We define the valuation effect comes from the valuation change in equity portfolio households choose in the last period:

$$VE_t \equiv \Delta P_t^{rw} \int \theta_{us,t-1}^{rw} dH_{t-1}^{us} - \Delta P_t^{us} \int \theta_{rw,t-1}^{us} dH_{t-1}^{rw}. \tag{1.A.11}$$

The remained component of the change in the NFA position is the financial balance which is identical with the current account in the model, so we formalize,

$$\begin{aligned} FB_t \equiv CA_t = P_t^{rw}\Delta\theta_{us,t}^{rw} &+ \left[\frac{\int b_{us,t}^{rw} dH_t^{us}}{1+r_t^{rw}} - \frac{\int b_{us,t-1}^{rw} dH_{t-1}^{us}}{1+r_{t-1}^{rw}} \right] \\ - P_t^{us}\Delta\theta_{rw,t}^{us} &- \left[\frac{\int b_{rw,t}^{us} dH_t^{rw}}{1+r_t^{us}} - \frac{\int b_{rw,t-1}^{us} dH_{t-1}^{rw}}{1+r_{t-1}^{us}} \right]. \end{aligned} \tag{1.A.12}$$

Furthermore, the current account can also be decomposed to the two elements, which are the next export and the net factor payment, as follows:

$$\begin{aligned} NX_t &\equiv Y_t^{us} - C_t^{us} - G_t^{us}, \\ NFP_t &\equiv \Theta_{us,t}^{rw}\left(1-\tau_{rw}^{k}\right)\Pi_t^{rw} + \frac{r_{t-1}^{rw}}{1+r_{t-1}^{rw}}\int b_{us,t-1}^{rw} dH_{t-1}^{us} \\ &\quad - \Theta_{rw,t}^{us}\left(1-\tau_{us}^{k}\right)\Pi_t^{us} - \frac{r_{t-1}^{us}}{1+r_{t-1}^{us}}\int b_{rw,t-1}^{us} dH_{t-1}^{rw}, \end{aligned} \tag{1.A.13}$$

where we define the net export is the export of goods from the US to the RW minus the import of the US from the RW and the net factor payment as the net income inflow from the RW to the US. Noting that total net supply of equity claims is 0, then the current account also has the following form:

$$\begin{aligned} CA_t &\equiv NX_t + NFP_t \\ &= Y_t^{us} - C_t^{us} - G_t^{us} \\ &+ \Theta_{us,t}^{rw}\left(1-\tau_{rw}^{k}\right)\Pi_t^{rw} + \frac{r_{t-1}^{rw}}{1+r_{t-1}^{rw}}\int b_{us,t-1}^{rw} dH_{t-1}^{us} - \Theta_{rw,t}^{us}\left(1-\tau_{us}^{k}\right)\Pi_t^{us} - \frac{r_{t-1}^{us}}{1+r_{t-1}^{us}}\int b_{rw,t-1}^{us} dH_{t-1}^{rw}. \end{aligned} \tag{1.A.14}$$

Combining (1.A.12) and (1.A.14), we have the budget constrain of the US households, which is (1.A.6).

1.A.3 Optimality condition

This sub-section derives the optimality conditions of firms and households in goods and asset markets. Substituting the net worth into the budget constraint and considering both the flow budget constraint and the net worth constraint, the Lagrangian for household utility maximization is:

$$
\begin{aligned}
\mathcal{L}_t =& \sum_{t,s^t} \beta^t \pi\left(s^t\right) \Bigg\{ \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \psi \frac{{n_{i,t}}^{1+\phi}}{1+\phi} \\
&+ \lambda_t \left[\left(1-\tau_i^w\right) W_t^i \epsilon_t^{l,i} n_{i,t} + \sum_j^I \left[P_t^j \theta_{i,t-1}^j + \epsilon_t^{k,j} \delta_i^j \left(\theta_{i,t-1}^j\right)^\kappa \left(1-\tau_j^k\right) \Pi_t^j \right] + b_{i,t-1} + TR_t^i \right] \\
&- \lambda_t \left[c_{i,t} + \sum_j^I P_t^j \theta_{i,t}^j + \frac{b_{i,t}}{1+r_t} \right] \\
&+ \zeta_t \left[\sum_j^I \left[P_t^j \theta_{i,t-1}^j + \epsilon_t^{k,j} \delta_i^j \left(\theta_{i,t-1}^j\right)^\kappa \left(1-\tau_j^k\right) \Pi_t^j \right] + b_{i,t-1} - \underline{a}_i \right] \Bigg\},
\end{aligned}
\tag{1.A.15}
$$

where $\pi\left(s^t\right)$ is probability of state $s^t = (s_0, s_1, \ldots, s_t)$ at time t. $\beta^t \pi\left(s^t\right) \lambda_t$ is the Lagrange multiplier on the flow budget constraint and $\beta^t \pi\left(s^t\right) \zeta_t$ is the Lagrange multiplier on the net worth constraint in state s^t at time t (we suppress the dependence of variables on s^t for the sake of brevity). The optimality conditions are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c_{i,t}} &= 0 \Leftrightarrow \lambda_t = \frac{\partial u_{i,t}}{\partial c_{i,t}} = \left(c_{i,t}\right)^{-\sigma}, \\
\frac{\partial \mathcal{L}_t}{\partial n_{i,t}} &= 0 \Leftrightarrow \left(1-\tau_i^w\right) W_t^i \epsilon_t^{l,i} \lambda_t = \frac{\partial u_{i,t}}{\partial n_{i,t}} = \psi n_{i,t}^\phi, \\
\frac{\partial \mathcal{L}_t}{\partial b_{i,t}} &= 0 \Leftrightarrow \lambda_t = \beta\left(1+r_t\right) \mathbb{E}_t\left[\lambda_{t+1} + \zeta_{t+1}\right], \\
\frac{\partial \mathcal{L}_t}{\partial \theta_{i,t}^j} &= 0 \Leftrightarrow \lambda_t = \beta \mathbb{E}_t\left[\left(\lambda_{t+1} + \zeta_{t+1}\right) R_{t+1}^j\right],
\end{aligned}
\tag{1.A.16}
$$

where we define that the expected return from capital in country j is:

$$
\mathbb{E}_t\left[R_{t+1}^j\right] \equiv \mathbb{E}_t\left[\frac{P_{t+1}^j + \kappa \delta_i^j \epsilon_{t+1}^{k,j} \left(\theta_{i,t}^j\right)^{\kappa-1} \left(1-\tau_j^k\right) \Pi_{t+1}^j}{P_t^j}\right]. \tag{1.A.17}
$$

Then, if the net worth constraint does not bind (i.e., $\zeta_{t+1} = 0$), $\left(1+r_t\right) \mathbb{E}_t\left[\lambda_{t+1}\right] = \mathbb{E}_t\left[\lambda_{t+1} R_{t+1}^j\right]$. Therefore we can have an usual expression for the risk premium:

$$
\mathbb{E}_t\left[R_{t+1}^j\right] - \left(1+r_t\right) = -\frac{\mathbb{C}ov_t\left[R_{t+1}^j \lambda_{t+1}\right]}{\mathbb{E}_t\left[\lambda_{t+1}\right]}, \tag{1.A.18}
$$

which should be plus as long as λ_{t+1} and R^j_{t+1} are negatively correlated.

Next, from forth optimality condition in equation (1.A.16), we write the optimal portfolio holding for the risky assets in the following way:

$$\begin{aligned}
\lambda_t &= \beta \mathbb{E}_t \left[(\lambda_{t+1} + \zeta_{t+1}) \left[\frac{P^j_{t+1} + \kappa \delta^j_i \epsilon^{k,j}_{t+1} \left(\theta^j_{i,t}\right)^{\kappa-1} \left(1 - \tau^k_j\right) \Pi^j_{t+1}}{P^j_t} \right] \right] \\
\Leftrightarrow \lambda_t - \beta \mathbb{E}_t \left[\frac{P^j_{t+1}}{P^j_t} (\lambda_{t+1} + \zeta_{t+1}) \right] &= \beta \mathbb{E}_t \left[\frac{\kappa \delta^j_i \epsilon^{k,j}_{t+1} \left(\theta^j_{i,t}\right)^{\kappa-1} \left(1 - \tau^k_j\right) \Pi^j_{t+1}}{P^j_t} (\lambda_{t+1} + \zeta_{t+1}) \right] \\
\Leftrightarrow \lambda_t - \beta \frac{P^j_{t+1}}{P^j_t} \mathbb{E}_t \left[\lambda_{t+1} + \zeta_{t+1} \right] &= \beta \kappa \delta^j_i \left(\theta^j_{i,t}\right)^{\kappa-1} \left(1 - \tau^k_j\right) \frac{\Pi^j_{t+1}}{P^j_t} \mathbb{E}_t \left[\epsilon^{k,j}_{t+1} (\lambda_{t+1} + \zeta_{t+1}) \right] \\
\Leftrightarrow \theta^j_{i,t} &= \left[\frac{\beta \kappa \delta^j_i \left(1 - \tau^k_j\right) \frac{\Pi^j_{t+1}}{P^j_t} \mathbb{E}_t \left[\epsilon^{k,j}_{t+1} (\lambda_{t+1} + \zeta_{t+1}) \right]}{\lambda_t - \beta \frac{P^j_{t+1}}{P^j_t} \mathbb{E}_t \left[\lambda_{t+1} + \zeta_{t+1} \right]} \right]^{\frac{1}{1-\kappa}},
\end{aligned} \tag{1.A.19}$$

where we extract aggregate terms from the expectation since we take the expectation over the idiosyncratic state.

Proof of Proposition 1: Now we assume we are in the steady state, thus we have:

$$\begin{aligned}
\theta^j_i &= \left[\frac{\beta \kappa \delta^j_i \left(1 - \tau^k_j\right) \frac{\Pi^j}{P^j} \mathbb{E} \left[\epsilon^{k,j'} (\lambda' + \zeta') \right]}{\lambda - \beta \mathbb{E} \left[\lambda' + \zeta' \right]} \right]^{\frac{1}{1-\kappa}} \\
\Leftrightarrow \theta^j_i &= \left[\frac{\kappa \delta^j_i \left(1 - \tau^k_j\right)}{r} \frac{\Pi^j}{P^j} \frac{\mathbb{E} \left[\epsilon^{k,j'} (\lambda' + \zeta') \right]}{\mathbb{E} \left[\lambda' + \zeta' \right]} \right]^{\frac{1}{1-\kappa}}.
\end{aligned} \tag{1.A.20}$$

This formulation completes the proof of Proposition 1. ■

Multiplier for the investment technology, κ, is a key parameter to govern individual portfolios. To understand the property of this parameter, we think of a following special case. Suppose that there is no home bias, no tax, and we are in the steady state, then we have the optimal portfolio holding as :

$$\theta^j_{i,t} = \left[\frac{\kappa \Pi^j}{r P^j} \frac{\mathbb{E}_t \left[\epsilon^{k,j}_{t+1} (\lambda_{t+1} + \zeta_{t+1}) \right]}{\mathbb{E}_t \left[\lambda_{t+1} + \zeta_{t+1} \right]} \right]^{\frac{1}{1-\kappa}}, \tag{1.A.21}$$

where we use the optimality condition for bond holding to derive the second equality. Noting that $\frac{\kappa\Pi^j}{rP^j}$ is an identical term across households, the remained expectation term works when generating heterogeneity in portfolio choice. To understand how the multiplier, κ, affect the curvature of portfolio choice, suppose that an agents would bind with the net worth constraint in the next period. This scenario happens only when the realized capital shock is low, i.e., $\epsilon_{t+1}^{k,j} < 1$. Noting that the capital shock is i.i.d. and normally distributed and the marginal utility from consumption, λ_{t+1}, does not depend on realization of capital shock type, we have the following inequality,

$$\frac{\mathbb{E}_t\left[\epsilon_{t+1}^{k,j}\left(\lambda_{t+1}+\zeta_{t+1}\right)\right]}{\mathbb{E}_t\left[\lambda_{t+1}+\zeta_{t+1}\right]} < 1. \tag{1.A.22}$$

Since $1/\left(1-\kappa\right)$ is an increasing function of κ, the bigger κ is, the less willing to hold risky assets households are. As you can see in Figure 1.A.1, where we show the portfolio choice under a different value for the multiplier, $\kappa = 0.5$, the portfolio for risky asset of both wage types are similar and linear except for the poorest households. Intuitively, in the case of $\kappa = 0.5$, the return for investment technology is smaller so that an individual holds risky assets and borrow more to smooth consumption by underestimating the risk of binding the constraint even if you are poorer. However, if return to scale increases, poor households do not want to take a risk, hence, portfolio for risky assets for the poorest is small and its position for risky assets gradually increases according to the size of net worth. In terms of borrowing position, the same argument applies. In the case of return to scale is large, i.e., $\kappa = 0.97$, high wage type agent can borrow more than low wage type, which is consistent with the date observed in SCF 2013 (see Appendix 1.B.3).

1.A.4 Alternative constraints

In the main text, we consider the net worth is the sum of values of three different assets, home equity claim, foreign equity claim, and US treasury as in equation (1.2.2). However, we can think of alternative specifications for this constraint. Here, we consider two other specifications and discuss the cost and benefit of these applications.

Firstly, Mendoza et al. (2009) considers the budget constraint and the net worth constraint

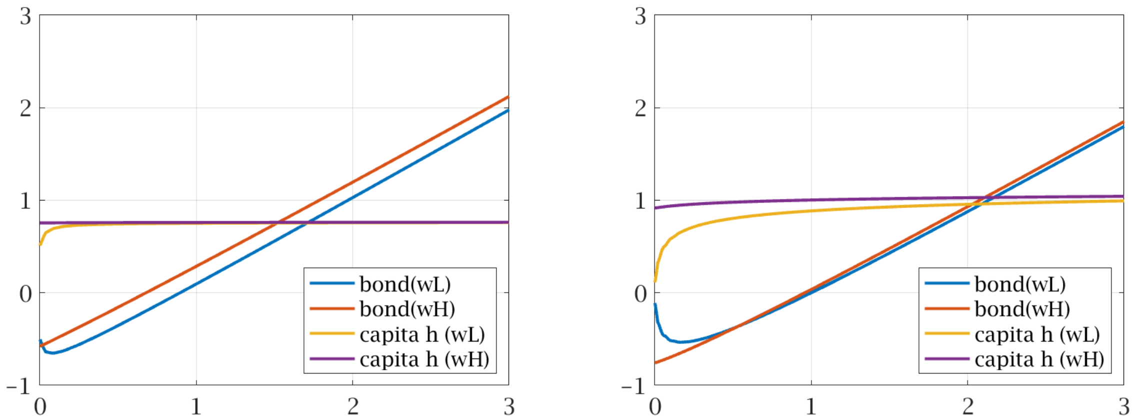

Figure 1.A.1: Policy functions for different values for the multiplier for investment technology ($P_t\theta_t$ and $b_t/\left(1+r_t\right)$ in the closed economy case). The unit of measure is an average of net worth. The left panel shows the policy function with $\kappa = 0.5$ and the right one shows with $\kappa = 0.97$ (benchmark) given the other settings than κ are the same for both experiments.

in the following way[23],

$$c_{i,t}(s,a) + \sum_{j}^{I} P_t^j \theta_{i,t}^j(s,a) + \frac{b_{i,t}(s,a)}{1+r_t} = a_{i,t} + TR_t^i$$

$$a_{i,t} \equiv \left(1-\tau_i^w\right) W_t^i \epsilon_t^{l,i} n_{i,t}(s,a) + \sum_{j}^{I} \left[P_t^j \theta_{i,t-1}^j + \epsilon_t^{k,j} \delta_i^j \left(\theta_{i,t-1}^j\right)^{\kappa} \left(1-\tau_j^k\right) \Pi_t^j\right] + b_{i,t-1}, \quad a_{i,t} \geq \underline{a}_i. \tag{1.A.23}$$

MQR includes labor income into net worth of each household. Obviously, this specification makes net worth more volatile as labor income is also exposed to the idiosyncratic shock. Thus the demand for US treasury would be higher compared to the benchmark specification given the rest of model settings are remained as the same. Furthermore, the incentive to diversify the risk also increases and the benefit from risk sharing also increases. These qualitative properties are acceptable.

However, one can look at the policy function for optimal portfolio as unpleasant. The left panel of Figure 1.A.2 describes the portfolio choice under MQR specification. The policy functions are mostly identical with that of the main text, but high wage type agents with lower level of assets hold less risky assets than low wage type agents with the same level of

[23] In MQR, precisely speaking, there is no government sector. Then there is no tax and transfer term, but these are minor points and dose not change the argument here. Also, while we only show the case for the closed economy version of the model, the argument dose not change again.

net worth. The reason is as follows. The individuals with high wage consume more than the low wage type agents. So if you had lower enough assets, you would like to save in safer way in order to smooth consumption. On the other hand, individuals with low wage and low assets do not have to tolerate and they can invest in risky capitals. As a result, portfolio positions between them is reversed, perhaps, in a surprising way and this result is inconsistent with the data, in which high wage type take riskier portfolio position (see Appendix 1.B.3).

The other specification follows other numerous heterogenous agent literature and think of pure borrowing constraint. Particularly, we consider the specification from Krusell and Smith (1997):

$$\begin{gathered} c_{i,t}(s,a) + \sum_{j}^{I} P_t^j \theta_{i,t}^j(s,a) + \frac{b_{i,t}(s,a)}{1+r_t} \\ = \left(1-\tau_i^w\right) W_t^i \epsilon_t^{l,i} n_{i,t}(s,a) + \sum_{j}^{I} \left[P_t^j \theta_{i,t-1}^j + \epsilon_t^{k,j} \delta_i^j \left(\theta_{i,t-1}^j\right)^{\kappa} \left(1-\tau_j^k\right) \Pi_t^j\right] + a_{i,t} + TR_t^i \\ a_{i,t} \equiv b_{i,t-1}, \quad a_{i,t} \geq \underline{a}_i. \end{gathered} \tag{1.A.24}$$

The right panel of Figure 1.A.2 shows the policy functions in this case. The portfolio for risky assets and treasury tracks US individuals' data (SCF 2013) (for details about data, see Appendix 1.B.3). However, we should face two crucial problems. First, since individuals want to borrow up to the borrowing constraint, both high type and low type agents borrow up to the threshold. However, as the data suggests, high wage type agents borrow more and low wage type agents borrow less. Taking a different limit to borrow according to individual state is a potential solution, but the model become complicated in another direction. Second one is about computational problem. As large number of populations are biding with the borrowing constraint so that we have to compute the shadow prices of the constraint, ζ, for many of agents. In this kind of portfolio choice model with binding constraints, we have to compute the shadow price in precise way, which needs formidable time. Therefore, it is hard to obtain an outcome under this specification with respect to accuracy and time.

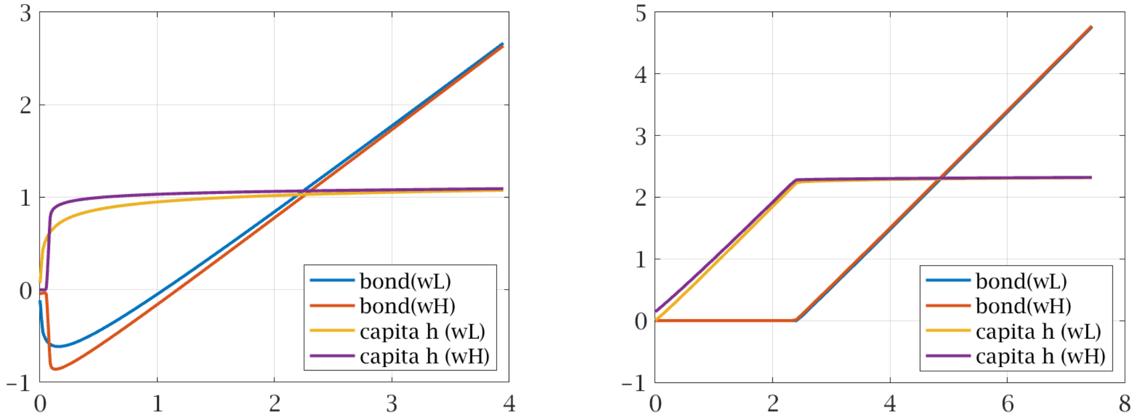

Figure 1.A.2: Policy functions for the alternative specifications. The unit of measure is an average of net worth. The left panel shows MQR version and the right one shows KS version.

1.B Data Sources and Definitions

1.B.1 Government sector

We construct the fiscal variables, government consumption (G), government transfer (TR), and tax (T), following Leeper et al. (2010) and using NIPA Table 3.2. "Federal Government Current Receipts and Expenditures." Government consumption is computed as the sum of consumption expenditures (L25), gross government investment (L45), net purchases of nonproduced assets (L47), less consumption of fixed capital (L48). Transfers are given by the sum of net current transfer payments (L26-L19), subsidies (L36), and net capital transfers (L46-L42). Then, total government expenditure is sum of government consumption and transfer. Tax revenues are given by the difference between current receipts (L41) and current transfer receipts (L19). All variables are then expressed as a fraction of GDP.

The left panel of Figure 1.B.1 shows government consumption (purchase), transfer, and tax income form 1990 to 2021. Government purchase and tax income have been relatively stable over the decades. However, as can be seen around the GFC and the Covid-19 crisis, government transfer is adjusted in more flexible way. Therefore, we fix the rate of government purchase and tax to GDP and use transfer as an adjustment tool to clear the government budget constraint.

The data for government bond supply, its holding, and its interest rate are downloaded from the St. Louis Fed web-site. The domestic holding is from "Federal Debt Held by Private Investors as Percent of Gross Domestic Product" (HBPIGDQ188S), the foreign investors' holding is from "Federal Debt Held by Foreign and International Investors as Percent of Gross Domestic Product" (HBFIGDQ188S), and Federal Reserve holding is from "Federal Debt Held by Federal Reserve Banks as Percent of Gross Domestic Product" (HBFRGDQ188S). GDP is obtained from the Bureau of Economic Analysis using NIPA Table 1.1.5. "Gross Domestic Product." The data sample is quarterly and spanning from 1Q:1990 to 1Q:2021.

The yield for 10 year treasury is "10-Year Treasury Constant Maturity Rate, Percent, Quarterly, Not Seasonally Adjusted" (DGS10). Table 1.B.1 shows the average of each series for the three periods before and after the GFC and around the Covid-19 crisis.

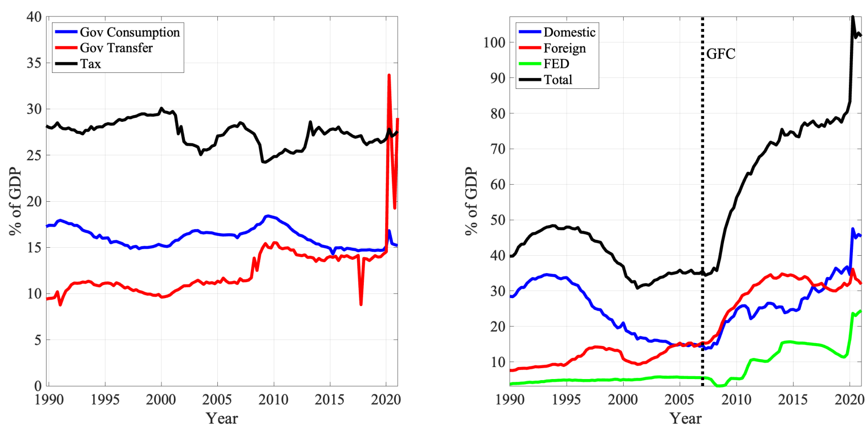

Figure 1.B.1: The time series for government sector's variables

Table 1.B.1: The government sector variables, owner of US treasury and 10 year maturity rate

Description	2002-2007	2012-2017	2020-2021
G/Y	16.5%	15.3%	15.6%
TR/Y	11.2%	13.7%	24.4%
T/Y	26.9%	27.2%	27.3%
B/Y	34.8%	74.9%	99.3%
$Domestic/Y$	14.9%	27.0%	43.7%
$Foreign/Y$	14.4%	33.4%	33.3%
FED/Y	5.6%	14.5%	22.3%
r	4.4%	2.2%	0.98%

1.B.2 Net Foreign Asset position of the US

For data on U.S. net foreign assets and their composition, we use 'Table 1.2. U.S. Net International Investment Position at the End of the Period, Expanded Detail,' in 'International Transactions, International Services, and International Investment Position Tables (BEA).' Specifically, we use 'U.S. net international investment position (L1)' for NFA data, 'U.S. assets (L4)' for external assets, 'Equity and investment fund shares (L11)' for assets held in equity, and 'Debt securities (L12)' for assets held in debt. On the liabilities side, we use 'U.S. liabilities (L36)' for overall, 'Equity and investment fund shares (L43)' for U.S. equity holdings from abroad, 'Treasury bills and certificates (L46)(L49)" for holdings in foreign U.S. government bonds, 'Other short-term securities (L47)(L50)' for holdings in other debt securities.

Figure 1.B.2 shows time-series data for each. Table 1.B.2 shows the average of each series for the three periods before and after the GFC and around the Covid-19 crisis.

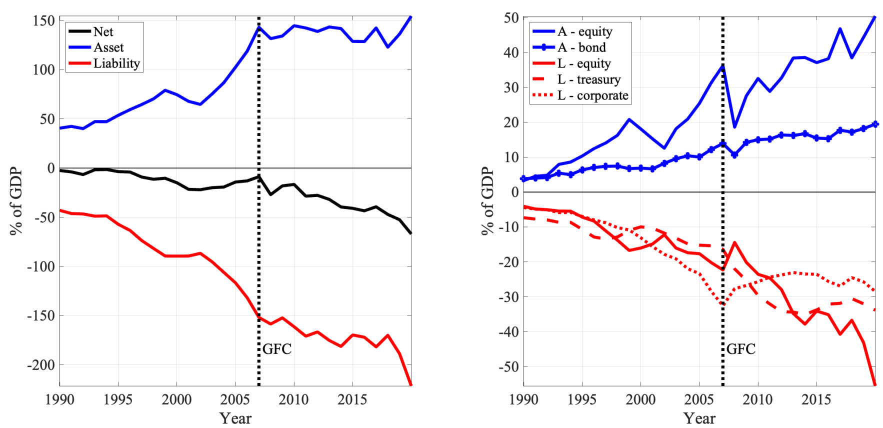

Figure 1.B.2: NFA position and its decomposition

1.B.3 Portfolio choice

How would households' asset holdings vary by net worth and wage type? We used data from the 2013 Survey of Consumer Finances (SCF) to check our data on households' borrowing and portfolio share of asset holdings. Specifically, we split the entire sample in two by the median wage. We then further categorize each group by the percentile of net worth i =

Table 1.B.2: Net Foreign Asset position of the US

Description	2002-2007	2012-2017	2020
NFA	−17.7%	−36.7%	−67.1%
Asset	89.6%	136.2%	154.4%
(A) Equity	21.7%	37.0%	50.4%
(A) Bond	10.1%	16.0%	19.5%
Liability	−107.3%	−173.0%	−221.4%
(L) Equity	−16.7%	−34.0%	−55.5%
(L) Treasury	−14.1%	−33.9%	−33.8%
(L) Corporate bond	−22.2%	−23.9%	−28.5%

NOTE.—(A) means that an entity is asset side of the US balance sheet, on the other hand, (L) means that an entity is liability side of the balance sheet.

1...100 to classify each group. For each group, assets and borrowing are plotted in Figure 1.B.3. The net worth of households is the sum of financial and nonfinancial assets minus debt. Thus, the asset side shows the sum of financial and nonfinancial assets.

Characteristically, the low-wage type borrows less and holds fewer assets than the high-wage type. We obtain this feature as a distinctive one in the policy function of the portfolio obtained from the model. It should be noted that while the only risky assets in our model are stocks, the risky assets in the data also include housing and other assets. In addition, bond holdings can be obtained from the data using savings bonds (SAVBND) and directly held bonds (BOND), but these do not tell us about households' borrowing status. Since borrowing by the poor through government bonds plays an essential role in the model, household borrowing was considered the corresponding debt in the data.

Auclert and Rognlie (2018), in their Appendix, examines three options for dividing households' net worth ai into the concepts of individual equity and bond holdings. In their explanation, the *narrow* definition for equity holdings shows a well-known pattern: the share of wealth invested in equities rises rapidly with wealth. This rapid rise is not captured in our model.

1.B.4 Equity performance of the RW after the Global Financial Crisis

To see how stock price in the RW has become volatile in US dollar term, we provide two data observations. The first one is correlations between the exchange rate and stock price in the

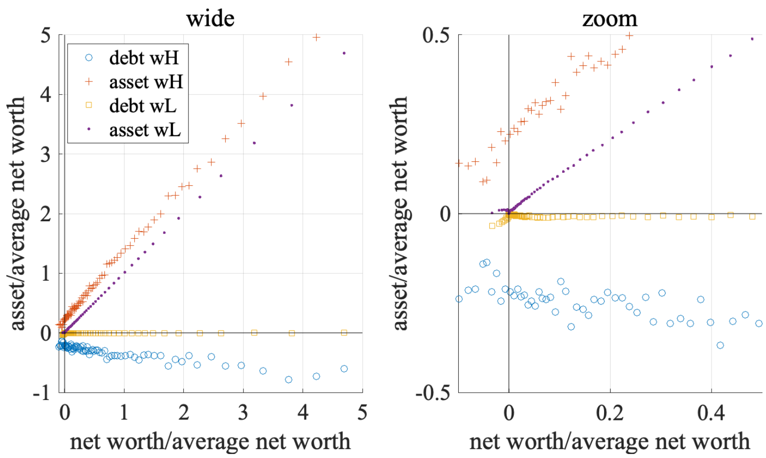

Figure 1.B.3: Individual holdings of assets and borrowings in the SCF 2013

RW. Here, we define the exchange rate, $\mathcal{E}_t^i$ is nominal and units of local currency in country i for one unit of dollar, then an increase in $\mathcal{E}_t^i$ corresponds to country i's currency devaluation. Figure 1.B.4 plots 10 quarter moving average of correlation between the change in stock price and the change in the exchange rate, $\mathbb{C}orr\left(\Delta P_{t+1}^i, \Delta\mathcal{E}_{t+1}^i\right)$, with quarterly data. We show the main countries in the European Union and some major emerging countries.

Before the Great Financial Crisis, the correlation had moved around 0. After 2008, however, the correlation went to negative, which implies when stock price drops in local currency unit, the corresponding country's currency devaluates. Further implication is that stock values in these countries had become volatile in US dollar unit after 2008, which is consistent with the finding in Miranda-Agrippino and Rey (2020a). To confirm the drop in equity performance of the RW in dollar unit, we see the sharpe ration during that era. Now we define stock price in country i in terms of US dollar unit as the following: $P_t^{i,\$} \equiv \frac{P_t^i}{\mathcal{E}_t^i}$. Then the Sharpe ratio is defined as:

$$SR_t^i \equiv \frac{\mathbb{E}_t\left[\Delta P_{t+1}^{i,\$}\right]}{std_t\left[\Delta P_{t+1}^{i,\$}\right]}. \tag{1.B.1}$$

Table 1.B.3 shows the *ex-post* sharpe ratio for the same sample of countries without countries whose part of data is unavailable at some point. We se the sharpe ration pre and post Great Financial Crisis period of 5 years and 10 years. The data shows performance of equity in

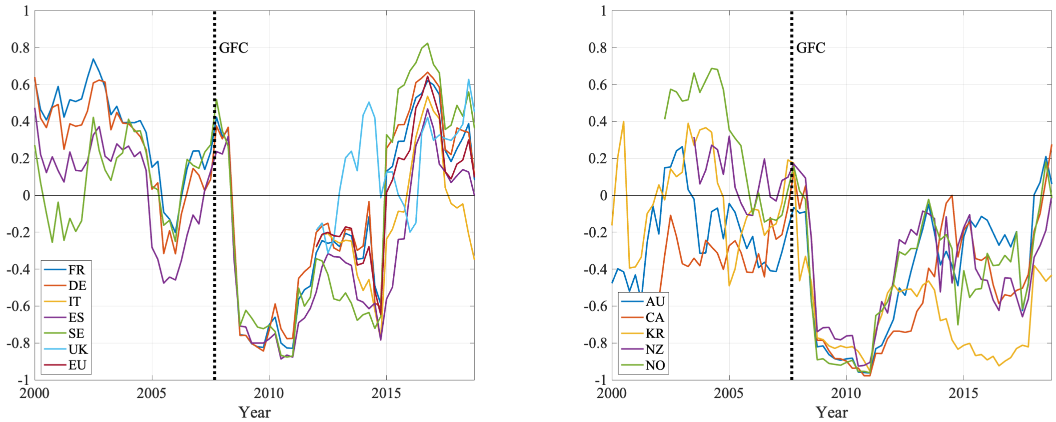

Figure 1.B.4: The 10 quarter moving average of correlation btw stock price in local currency and the exchange rate

the RW had become worsened compared to that of the US in both sets[24]. In the model, as we do not account for growth of the economy, we assume the decline in the sharpe ratio stems from an increase in the volatility of stock return. The bilateral nominal exchange rate is downloaded from BIS. The stock price data is downloaded from datastream (*Refiniv Workspace*).

Table 1.B.3: Sharpe ratio of stock pice in US dollar term

	(a) 5 years pre GFC	(b) 5 years post GFC	(b/a) ratio	(c) 10 years pre GFC	(d) 10 years post GFC	(d/c) ratio
France	7.8	-1.3	-0.2	5.3	-0.3	-0.0
Germany	11.5	1.2	0.1	4.9	4.0	0.8
Japan	4.9	2.1	0.4	0.9	7.8	8.4
Switzerland	10.3	2.6	0.3	4.8	4.2	0.9
Australia	8.7	0.5	0.1	14.5	0.4	0.0
South Korea	7.9	1.3	0.2	12.8	3.1	0.2
Spain	9.0	-2.8	-0.3	6.0	-3.8	-0.6
Sweden	7.1	2.2	0.3	3.9	2.6	0.7
US	4.7	3.0	0.6	2.6	11.3	4.4

[24] Japan is an outlier as an introducing Abenomics in 2013 induced an acute increase in stock price in Yen despite Japanese Yen's depreciation.

1.C Computation

In this paper, we employ the solution method developed in Auclert and Rognlie (2018) and Auclert et al. (2021a). To solve out the non-linear equilibrium system, we use the state space Jacobian. See Auclert and Rognlie (2018) and Auclert et al. (2021a) for a discussion of the superiority of the quasi-Newtonian method using the Jacobian to other methods.

1.C.1 Individual policy function

It is crucial to compute accurately the Lagrange multiplier for the borrowing constraint, the shadow price of the borrowing constraint, because it determines the portfolio of risky assets. During the iteration procedure, it should be noted that we cannot preserve the Lagrange multiplier since even if the constraint is binding at some point, this does not necessarily mean the constraint will be binding with the next candidate for unknowns. If we keep the multiplier in the next iteration, it would induce households precautionary saving motive and the policy function of them would be inaccurate.

Also, note that the Lagrange multiplier is identical when binding the constraint. To explain this, we give an example of binding cases. Suppose we were in state $\left(w_L, k_H^1, k_H^2\right)$, in which we get low type shock for labor and high types for both capital, then we consider the case we would bind the net worth constraint in the next period. Since the timing of realization of capital shock is earlier than wage shock, the possible states in which we hit the constraint in the next period are $\left(w_L, k_L^1, k_L^2\right)$ and $\left(w_H, k_L^1, k_L^2\right)$ out of 8 $\left(= 2^3\right)$ states. When realizing the capital shock, only the type of capital income and the net worth are *effective* state variable when households make a decision on policy function. Hence we think the Lagrange multiplier for the two states are identical.

1.C.2 Steady state

The unknown variables in this model are the interest rate, (r), the price of capital gain claims in the two countries, (P^{us}, P^{rw}), and labor demand, (L^{us}, L^{rw}). There is a market for each of the unknown variables, and we seek unknown variables with which all equilibrium conditions are satisfied.

- The demand for government bonds is determined by the policy function of households and their population distribution given the interest rate, each country wage, and the price of capital claims in the two countries. The supply of government debt is determined by US GDP, which is determined given US labor demand, and the exogenous debt-to-GDP ratio.

- The demand for capital claims is determined by the policy function of households and their population distribution, as above. Supply is fixed.

- The demand for labor in each country is unknown variable in this setting. The supply of labor is determined by the policy function of households and the population distribution, as above.

Once we define the unknowns and errors of the equilibrium conditions, $U \equiv (r, P^{us}, P^{rw}, L^{us}, L^{rw})$ and $\mathcal{E} = \left(\varepsilon^{B}, \varepsilon^{K^{us}}, \varepsilon^{K^{rw}}, \varepsilon^{L^{us}}, \varepsilon^{L^{rw}}\right)$, the above equilibrium condition can be summarized in the following functional form: $f : U \rightarrow \mathcal{E}$. We find the equilibrium unknowns in the following general iteration procedure.

- [*step* 1] Given a guess for the unknowns, apply f to obtain the errors for each equilibrium condition.

- [*step* 2] Find the approximate Jacobian J of f at the point evaluated in [*step* 1]. To obtain the approximate Jacobian, we use the direct method, in which we perturb the unknowns and compute the derivatives directly.

- [*step* 3] Use the errors in [*step* 1] and the approximate Jacobian in [*step* 2] to do a Newton iteration to find a new guess for the unknowns, then return to [*step* 1]. We loop out when the equilibrium condition errors become sufficiently small, which we set

10^{-6}.

$$\begin{pmatrix} r^{(n)} \\ P^{us,(n)} \\ P^{rw,(n)} \\ L^{us,(n)} \\ L^{rw,(n)} \end{pmatrix} = \begin{pmatrix} r^{(n-1)} \\ P^{us,(n-1)} \\ P^{rw,(n-1)} \\ L^{us,(n-1)} \\ L^{rw,(n-1)} \end{pmatrix} - J^{-1} \begin{pmatrix} \varepsilon^{B,(n-1)} \\ \varepsilon^{K^{us},(n-1)} \\ \varepsilon^{K^{rw},(n-1)} \\ \varepsilon^{L^{us},(n-1)} \\ \varepsilon^{L^{rw},(n-1)} \end{pmatrix} \tag{1.C.1}$$

While when obtaining the approximate Jacobian in [*step* 2], we use the direct method particularly when doing initial iterations. However, after the error of the equilibrium condition become relatively small, the Broyden method is useful to update the Jacobian utilizing the next form:

$$J^n = J^{n-1} + \frac{\Delta\mathcal{E}^n - J^{n-1}\Delta U^n}{\|\Delta U^n\|^2} (\Delta U^n)^T, \tag{1.C.2}$$

where $\Delta\mathcal{E}^n = \mathcal{E}^n - \mathcal{E}^{n-1}$ and $\Delta U^n = U^n - U^{n-1}$.

1.C.3 Transition

We obtain the transition path from the initial steady state to the terminal steady state in Section 1.3.3 using sequence-space Jacobians, $\mathbf{H_U}$, using the following the updating procedure:

$$\mathbf{U}^{j+1} = \mathbf{U}^j - [\mathbf{H_U}(\mathbf{U}_{ss}, \mathbf{Z})]^{-1} \mathbf{H}\left(\mathbf{U}^j, \mathbf{Z}\right), \tag{1.C.3}$$

where $\mathbf{H}$ is the equilibrium conditions which should be equalized to 0 in the equilibrium with given the vector of the exogenous variables, $\mathbf{Z}$, and the equilibrium path for the unknowns, $\mathbf{U}$. As have been discussed $U = (r, P^{us}, P^{rw}, L^{us}, L^{rw})$ and $\mathcal{E} = \left(\varepsilon^B, \varepsilon^{K^{us}}, \varepsilon^{K^{rw}}, \varepsilon^{L^{us}}, \varepsilon^{L^{rw}}\right)$, so $\mathbf{U}$ and $\mathbf{H}$ are the time series for these unknowns and errors. ss denotes the terminal steady state. To get Jacobians, we used the direct method.

It should be noted that when considering the dynamics path, we also have to compute the wealth effect term, $P_t^i \Delta\theta^i$, which changes the optimal decisions of the households through transfer from the government. We guess and verify these wealth effect terms in both countries with the following iteration algorithm.

- [*step* 1] Guess time series for the wealth effect terms, $P_t^i \Delta\theta^i \ \forall i$. We set all entities are 0 initially.

- [*step* 2] Given, the above the wealth effects, guess transition sequences for the unknowns. The value for unknowns in the last period are set to the terminal steady state value for each unknown.

- [*step* 3] Using the first order conditions of households, solve for the optimal decision rules backward starting from the terminal period T.

- [*step* 4] Using the optimal decision rules, we find the sequence of distributions over the all periods.

- [*step* 5] Compute the equilibrium error, $\mathbf{H}\left(\mathbf{U}^j, \mathbf{Z}\right)$, and use equation (1.C.3) to update the Jacobians. If the equilibrium error is sufficiently small we exit the inner loop, otherwise go back to [*step* 2].

- [*step* 6] Compute the wealth effect term and add this on transfer, then repeat the outer loop from [*step* 1]. If the updating error for the wealth effect term become substantially small, smaller than 10^{-4}, the outer loop finishes.

We set the truncated period to 300 years as suggested in Auclert et al. (2021a).

CHAPTER 2

FTPL Puzzle Redux with Market Segmentation

'A common story about the recent inflation surge in the United States–especially among members of Team Transitory–is that the surge is largely due to global supply shocks, such as rising energy prices, chip shortages, and various bottlenecks in the wake of the worldwide pandemic. Why then do we not see a similar inflation surge in Japan? There, inflation is running at about the same rate as it was pre-pandemic. See below. (Click on image to enlarge.) I am puzzled'.

– 'An Inflation Puzzle,' in GREG MANKIW'S BLOG, May 06, 2022

2.1 Introduction

The theory of inflation dynamics is an ongoing research topic among macroeconomists and policy circles, and many unresolved issues exist. Figure 2.1.1 shows the past 25 years of inflation and fiscal balances in developed countries. The US and European countries achieved inflation near 2% under chronic budget deficits, with large budget deficits after the Covid-19 crisis, and experienced high inflation in 2022. For Japan, however, inflation is still low, excluding energy and food, despite chronic budget deficits and fiscal spending under the Covid-19 crisis. In standard Fiscal Theory of Price Level (FTPL) theory, a negative budget surplus is compensated by an increase in the price level to satisfy the government's budget constraint equation. Real government debt, nominal government debt divided by the price level, is assumed to match the sum of future government budget surplus flows. However, this is an enigma in the context of the FTPL, unlike the phenomenon seen in Japan, for example (Brunnermeier et al. (2020)). It is also always problematic when the divergence between FTPL theory and reality is discussed (Cochrane (2022)).

This paper provides a theoretical framework for solving this FTPL puzzle. We define the FTPL puzzle as a situation in which inflation remains low despite permanent budget deficits. We solve this puzzle by adding long-term government debt and market segmentation assumptions to the standard FTPL model. By adding the assumption that all government bonds issued by the government are held by financial institutions and indirectly by households, we find that there is a region in which government deficits lead to a decline in the price level. The condition is whether the government's stance toward a sound primary balance is sufficient to exceed a threshold calculated from the government debt ratio to GDP and its risk premium. If the government's stance is more strongly fiscally responsible than the threshold, government spending is deflationary. Conversely, if its stance is weak enough so fiscal active, government spending is inflationary.

The baseline model is the New Keynesian-FTPL (NK-FTPL) model with nominal price rigidities, which is based on Leeper (1991) and Woodford (2003) in mind. The model has

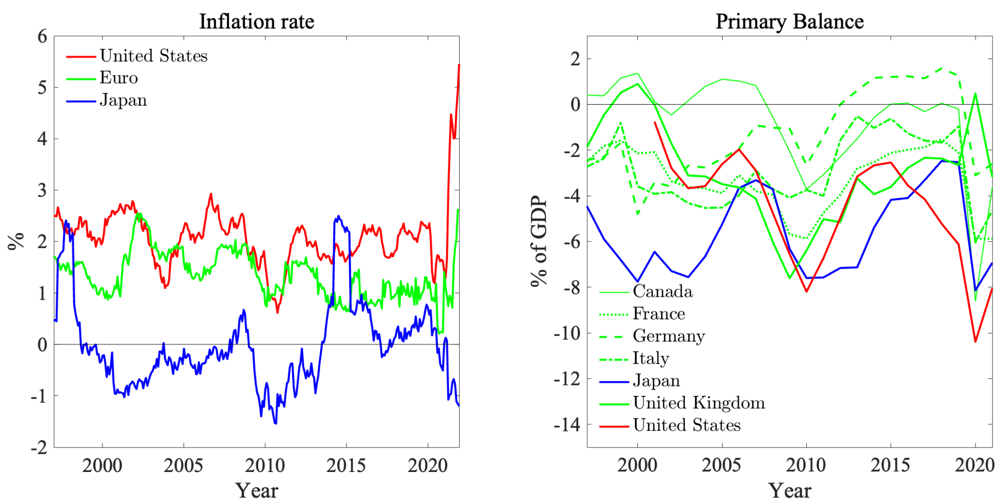

Figure 2.1.1: Core-core CPI and primary balance in three advanced regions

Note.—The data source for CPI is 'Federal Reserve Economic Data.' We use 'Consumer Price Index: All Items Excluding Food and Energy for Japan, Percent Change from Year Ago, Monthly, Not Seasonally Adjusted' for Japan, 'Consumer Price Index: All Items Excluding Food and Energy for the United States, Percent Change from Year Ago, Monthly, Not Seasonally Adjusted' for the U.S. and 'Consumer Price Index: Harmonized Prices: Total All Items Less Food, Energy, Tobacco, and Alcohol for the Euro Area, Percent Change from Year Ago, Monthly, Not Seasonally Adjusted' for the Euro area. The data source for the primary balance is 'Government Finance Statistics (IMF).' We use 'General government structural balance.'

a central bank that serves as the monetary policy authority and a government sector as the fiscal policy authority, each with its own separate policy rules (e.g., Taylor rule). To this standard NK-FTPL model, we introduce government bonds with different maturities (short-term and long-term) and add the assumption of market segmentation. There are two kinds of agents in the economy: households and financial intermediaries. Households make deposits with financial intermediaries, which hold all government debt as assets. Within the financial intermediaries' portfolio, the financial intermediaries profit from the interest rate differential between the short-term and long-term rates. The profits from this interest rate differential are returned to households.

The government deficit shocks in this model reveals two distinct regions, the standard inflationary and new deflationary regions. The inflationary region is identical to the behavior in the Fiscally-led (Passive Monetary and Active Fiscal, PM/AF) regime in the existing literature on NK-FTPL[1]. Since financial institutions bear the entire risk of long-term debt, when the government's fiscal discipline is insufficient, the financial sector reduces its portfolio share of long-term debt to anticipate a decline in the price of long-term debt. In this case, households pay the cost of meeting the government budget constraint equation in the form of inflation in response to government spending. Under nominal price rigidities, households' consumption behavior behaves non-Ricardian. That is, they increase consumption in response to fiscal lump-sum transfers.

A unique part of the model in this paper is the emergence of a deflationary region in the fiscal-led (PM/AF) regime. Suppose the government's stance toward fiscal discipline is sufficiently austere, and the current budget deficit is committed to being covered by future tax increases. In that case, financial institutions will predict that the value of long-term bonds will not fall and, conversely, increase the portfolio ratio of long-term bonds. In this case, households will face deflation rather than inflation because future tax increases will finance the government's budget constraint formula. When nominal price rigidity is

[1]We follow Leeper (1991) to define these terminologies. In general, the Monetary-led regime corresponds to Active Monetary and Passive Fiscal (AM/PF) regime, in which the Taylor principle is satisfied and fiscal backing is sufficient. The Fiscal-led regime is as in the main text, and it corresponds to the FTPL world, where the price level is determined to satisfy the government budget constraint.

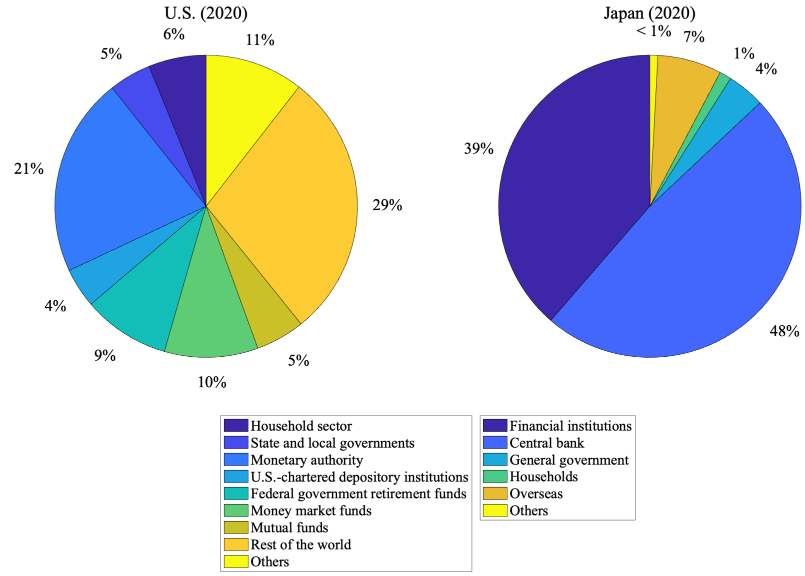

Figure 2.1.2: Owners of the government bonds

NOTE.—The data source for the US is 'Z.1 Financial Accounts of the United States' (Board of Governors of the Federal Reserve System). For Japan, we use 'Flow of Funds Accounts' (Bank of Japan).

introduced, households' consumption behavior is identical to Ricardian behavior. The threshold separating these two regions is characterized by the government debt to GDP ratio and the difference between long- and short-term interest rates. The larger the government debt ratio to GDP and the larger the difference between long- and short-term interest rates, the larger the threshold. Therefore, the parameters that characterize the stance toward fiscal discipline must be sufficient to avoid an inflationary regime.

Why market segmentation. The market segmentation hypothesis is this paper's most critical and key assumption. Before moving on to the main text, we would like to review the motivational facts. Figure 2.1.2 shows the fractions of ownership of the government bonds in both the U.S. and Japan. As well knows that the main owners of U.S. treasury are the Federal Reserve Board and the rest of the world. In Japan, the major holders of government bonds are financial institutions and the central bank. In both countries, the direct holding from households is small, 6% and 1% each. Put differently, households indirectly own their national bonds through financial institutions. Figure 2.1.3 describes financial assets held by households in the U.S., Euro region, and Japan. Households' investment in debt securities is

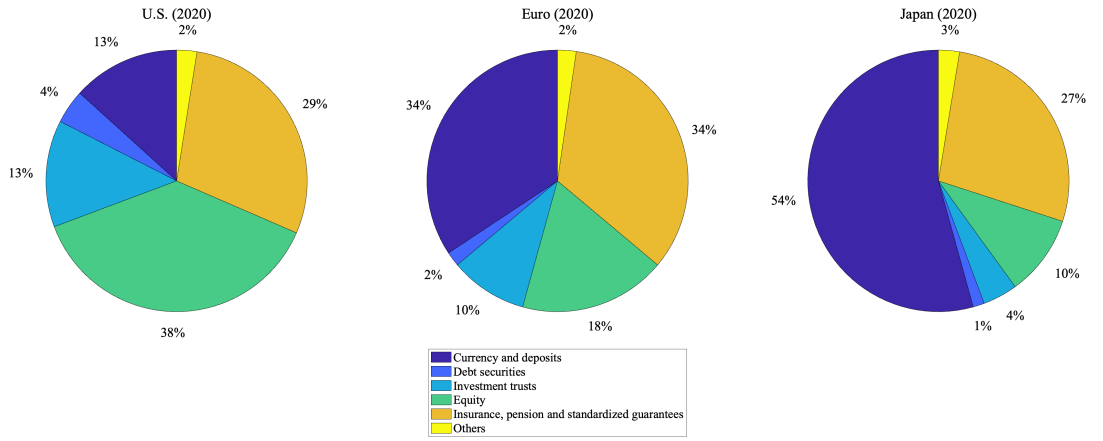

Figure 2.1.3: Financial assets held by households

NOTE.—The data source for the US is 'Z.1 Financial Accounts of the United States' (Board of Governors of the Federal Reserve System). For the Euro area, we use 'Euro Area Accounts' (European Central Bank). For Japan, we use 'Flow of Funds Accounts' (Bank of Japan).

relatively small, up to 4% even in the U.S. It should be noted that the fraction of cash and deposits is high in Japan and Euro region. We see from these two figures that households hold the government debt not directly but indirectly through financial institutions by lending or depositing their money[2].

Literature review. This paper is related to the NK and FTPL literature: the research on the interaction between fiscal and monetary policy and prices, which began with Sargent and Wallace (1981) as a precursor, followed by Leeper (1991), Woodford (2003), Sims (2011), Sims (2013), and other numerous literature. For a comprehensive survey of the FTPL, Leeper and Leith (2016) and Cochrane (2022) have conducted excellent surveys.

This paper contributes to the literature by attempting to solve the FTPL puzzle by introducing long-term bonds into the conventional NK-FTPL model. However, this paper is not the first to try to solve the puzzle. For example, recently, Cochrane (2021) discusses the possibility of deflation in response to positive fiscal surplus shock with the conventional FTPL

[2]Also, the recent micro evidence supports the idea of market segmentation. For instance, Bacchetta et al. (2020) shows that most agents do not participate in financial markets despite low transaction costs. It implies the market are effectively segmented between participants and non-participants in the short run. This slow portfolio adjustment is consistent with recent micro asset demand estimates by Koijen and Yogo (2019a) and Gabaix and Koijen (2021).

framework but with the long-term bond. Cochrane (2021) shows that the response of prices to government spending is negative even under the conventional government budget constraint formula by adding a positive shock on the discount rate to incentivize households to save more. Contemporaneously, Brunnermeier et al. (2020) constructed a Bewley (1983)-type model based on Brunnermeier and Sannikov (2016)[3]. They show that when economic agents can invest in capital and government bonds, a bubble term appears in the government's budget constraint equation due to the precautionary saving demand. Caramp and Silva (2022) provides a theoretical framework in which the inflation dynamics depend not only on monetary policy but also on the response of fiscal policy. They find that the major force in determining the initial response of inflation after the monetary policy shock is not the intertemporal substitution effect but the wealth effect determined by a simultaneous fiscal policy. They argue that a contractionary monetary policy reduces initial inflation if and only if a big contractionary fiscal policy follows because it make households poorer. Without the wealth effect, a Neo-Fisherian property emerges, and inflation rises after an interest rate hike shock (Uribe (2018)).

Compared to the above paper, this paper is unique due to its framework with long-term bond market segmentation. This setting is motivated by the fact that households are not the direct holders of long-term government bonds and the key friction to generating the deflationary pressure under the fiscal-led regime. The concept of market segmentation is often used in the international economics literature. In particular, in recent years, much of the literature has introduced this hypothesis to unravel the puzzle of exchange rates and capital flows (see e.g., Alvarez et al. (2002), Alvarez et al. (2009), Gabaix and Maggiori (2015), and Itskhoki and Mukhin (2021)). Although not much literature explicitly introduces the idea of market segmentation into closed economies, with notable exception Alvarez et al. (2001) and Bilbiie (2008), in which some fraction of agents are asset market participants,

[3]In addition to the Bewley type model, Brunnermeier et al. (2020) also analyzed extensions of the OLG model (Samuelson (1958)) and the perpetual youth (Blanchard (1985)) model. In both models, however, government debt can have value even in the absence of an underlying surplus because it has a bubble component. In the perpetual youth model, government debt becomes a store of value, allowing agents to exchange some of their current labor income for labor income claims of future generations. For more details, see Brunnermeier et al. (2020).

and others are non-asset market participants. However, suppose we regard the heterogeneity in the degree of market participants as separating all agents into two groups of households. In that case, several literatures considers the model with two types of agents with different objective functions and budget constraint equations. In the context of closed economies, the analysis has been conducted by dividing households such as borrower and saver or hand-to-mouth and saver (see e.g., Mankiw (2000), Bilbiie and Straub (2013), Bilbiie (2018), Debortoli and Galí (2017), and Cúrdia and Woodford (2016) among others).

Finally, this paper is also related to the recent accumulation of public debt since the Covid-19 crisis and the fiscal policy under it. In this regard, the literature such as Blanchard (2019), Hilscher et al. (2022), and Mian et al. (2021) that discusses $r - g$ is also relevant.

Structure. In the next section, Section 2.2, we develop the NK model nested FTPL model in which long-term bond is segmented from the households. Section 2.3 shows the model results, the conditions for Fiscally-led inflationary and deflationary regions, the main mechanism, and the equilibrium dynamics. Section 2.4 maps the theory to the data. Section 2.5 concludes and describes an avenue for future research.

2.2 Model

2.2.1 NK-FTPL model with long term bond market segmentation

This section describes the overview of the closed economy model. The model is the workhorse New Keynesian model with the government sector, but we incorporates the financial intermediary and it leads to market segmentation, following Leeper (1991) and Woodford (2003). The key ingredient of the model is the long-term bond issued by the government is hold only by the financial intermediary so that the risk of government budget constraint is held by the financial intermediary.

Households: The domestic representative household maximizes expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi} \right), \tag{2.2.1}$$

subject to the budget constraint:

$$P_t C_t + \frac{F_t^S}{R_t^S} + M_t \leq W_t L_t + F_{t-1}^S + M_{t-1} + \Pi_t^{PF} + \Pi_t^{FI} - T_t + TR_t, \tag{2.2.2}$$

where P_t is the consumer price index, C_t is consumption, W_t is the nominal wage, L_t is labor supply. T_t represents for lump-sum tax levied by the government and TR_t is transfer from the government to the households. Π_t^{PF} stands for dividends paid by domestic firms and Π_t^{FI} is the transfer from the financial intermediary. F_{t+1}^S is households' short-term bond holding, R_t^S is the short-term rate set by the monetary authority, and M_{t+1} is the money holding. When $R_t^S > 0$, the household demands no money. Since the short-term rate cannot be less than 0 in this model, households are forced to hold money M_t by the central bank. It should be noted that the short-term rate and the money supply is controlled by the monetary authority through the open market operation, so that M_{t+1} is *not* the control variable of the households. It is also worthwhile to note that we do not allow the households to hold the long-term government bond with the assumption of market segmentation which I will describe in the following.

Firm production: Each producer i produce $Y_t(i)$ with a standard constant return to scale production technology only with input of labor and the log total factor productivity, a_t:

$$\begin{aligned} Y_t(i) &= e^{a_t} L_t(i) \\ a_t &= \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \quad \varepsilon_t^a \sim iid(0,1). \end{aligned} \tag{2.2.3}$$

The log total factor productivity following an AR(1) process with persistence $\rho_a \in [0,1]$ and volatility of the innovation $\sigma_a \geq 0$. The marginal cost of the production is the wage rate discounted by the total factor productivity, which is identical across firms:

$$MC_t(i) = \frac{W_t(i) L_t(i)}{Y_t(i)} = e^{-a_t} W_t. \quad (2.2.4)$$

The profits of an individual firm $i \in [0, 1]$ is computed as:

$$\Pi_t(i)^{PF} = (P_t(i) - MC_t)\, Y_t(i). \quad (2.2.5)$$

We assume that firms set prices à la Calvo with a probability of changing price next period equal to $1 - \lambda_p$ (see e.g., Galí (2015)), and the each symmetric individual goods are aggregated with the constant elasticity of substitution aggregator which is specified by the CES parameter θ:

$$\max_{\widehat{P}_t(i), \{Y_s(i)\}} \mathbb{E}_t \sum_{s \geq t} \lambda_p^{s-t} \Theta_{t,s} \left[\widehat{P}_t(i) \left(\frac{\widehat{P}_t(i)}{P_s} \right)^{-\theta} C_s - MC_s Y_s(i) \right] \quad (2.2.6)$$

where $\widehat{P}_t(i)$ is a reset price and $\Theta_{t,s} = \prod_{j=1}^{s-t} \Theta_{t+j} = \prod_{j=1}^{s-t} \beta \left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+j}}$ is the stochastic discount factor of the households. We can derive the first-order conditions for reset prices in domestic goods market in log-linearization form:

$$\hat{p}_t = (1 - \beta\lambda_p)\, \mathbb{E}_t \textstyle\sum_{s \geq t} (\beta\lambda_p)^{s-t}\, mc_s. \quad (2.2.7)$$

As we know the law of motions for the home goods price are $\pi_t = (1 - \lambda_p)(\hat{p}_t - p_{t-1}) = \frac{1-\lambda_p}{\lambda_p}(\hat{p}_t - p_t)$, the New Keynesian Phillips Curve is:

$$\pi_t = k_p\, (mc_t - p_t) + \beta \mathbb{E}_t \pi_{t+1}, \quad (2.2.8)$$

where $k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}$.

Fiscal authority: We impose the restriction the short-term debt is zero-net supply and there is no government expenditure in the model to simplify the model analysis. The government levies the lump-sum tax from the households and deliver the lump-sum transfer to the households to satisfy the flow budget constraint. Now we define the primary surplus of the government is $S_t \equiv T_t - TR_t$., Note that while we remain the short-term bond, B_t^S, in the budget constraint, we assume those are zero-net supply in equilibrium:

$$\underbrace{B_{t-1}^S + (1 + \delta P_t^m) B_{t-1}^L}_{\text{Gov should pay at date } t} - \underbrace{(T_t - TR_t)}_{\equiv S_t} = \frac{B_t^S}{R_t^S} + P_t^m B_t^L, \tag{2.2.9}$$

where δ controls the maturity structure of the long-term bond. If $\delta = 0$, the long-term bond and short-term bond are equivalent in terms of risk and return. Now we define the debt-to-GDP ratio as $b_t \equiv \frac{P_t^m B_t^L}{P_t Y_t}$ and the fiscal surplus to GDP ratio as $s_t \equiv \frac{S_t}{P_t Y_t}$, and linearize with fiscal variables and log-linearize with the rest of variables. In the following, we define $\tilde{x}_t \equiv x_t - x_{ss}$ for fiscal variables and consider the shock to the primary surplus[4][5]:

$$\tilde{b}_t = \overline{R^L}\tilde{b}_{t-1} + \overline{R^L} b_{ss}\left(r_t^L - \pi_t - \Delta y_t\right) - \tilde{s}_t + \sigma_s \varepsilon_t^s, \quad \varepsilon_t^s \sim iid(0,1). \tag{2.2.10}$$

The rule of the fiscal authority is:

$$\tilde{s}_t = \phi_b \tilde{b}_{t-1}, \tag{2.2.11}$$

where ϕ_b governs the fiscal stance to stabilize its debt size relative to the total output.

Monetary authority: The monetary authority manipulate the interest rate of short-term bond following a standard Taylor rule:

$$\frac{R_t^S}{\overline{R}^S} = \left(\frac{R_{t-1}^S}{\overline{R}^S}\right)^{\rho_m} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\right]^{(1-\rho_m)} e^{\sigma_m \varepsilon_t^m} \tag{2.2.12}$$

$$r_t^S = \rho_m r_{t-1}^S + (1 - \rho_m)\phi_\pi \pi_t + \sigma_m \varepsilon_t^m, \quad \varepsilon_t^m \sim iid(0,1), \tag{2.2.13}$$

[4]For the derivation of the linearized form of the government budget constraint, see Appendix 2.A.

[5]This style of linearization is typical within NK-FTPL DSGE literature, see Leeper (1991) and Cochrane (2022), and the following literature, e.g., Bhattarai et al. (2012), Bhattarai et al. (2016), Bianchi and Melosi (2017), and Bianchi and Ilut (2017).

where $\overline{R}^S$ and $\overline{\Pi}$ are the steady state values of gross nominal interest rate and gross inflation rate. To control the short-term rate, the monetary authority adjusts the quantity of short-term bond in the financial market through the open market operations. When purchasing the short-term bond, the bank issues fiat money (narrow money), which is on the liability side of balance sheet of the central bank[6].

$$\frac{CB_t^S}{R_t^S} - CB_{t-1}^S = M_t - M_{t-1} \tag{2.2.14}$$

Financial intermediary: The investors are risk averse and maximize a mean-variance objective in local currency terms. Firstly, we introduce the mean-variance analysis with several risky assets in general setting following Campbell and Viceira (2002). Assuming that the investor trade off mean and variance in a linear fashion, the investor's maximizing problem is:

$$\max_{\boldsymbol{\alpha}_t} \boldsymbol{\alpha}_t' \left(\mathbb{E}_t \mathbf{R}_{t+1} - R_t^f \boldsymbol{\iota} \right) - \frac{\omega_t}{2} \boldsymbol{\alpha}_t' \boldsymbol{\Sigma}_t \boldsymbol{\alpha}_t, \tag{2.2.15}$$

where $\boldsymbol{\alpha}_t$ is a vector of allocations to the risky assets, $\boldsymbol{\iota}$ is a vector of ones, and $\boldsymbol{\Sigma}_t$ is a variance-covariance matrix of the return on the risky assets. $\mathbf{R}_t$ is the return for risky assets and R_t^f is the return for risk-free bond. ω_t is the risk aversion parameter. The solution the maximization problem is:

$$\boldsymbol{\alpha}_t = \frac{1}{\omega_t} \boldsymbol{\Sigma}_t^{-1} \left(\mathbb{E}_t \mathbf{R}_{t+1} - R_t^f \boldsymbol{\iota} \right). \tag{2.2.16}$$

In the closed economy, the financial intermediary face the portfolio choice problem between the government short-term bond and long-term bond, then the portfolio choice

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[6]Put differently, the central bank controls the deposit rate through the open market operation. If we combine the flow budget constraint of the households and the central bank balance sheet, we have:

$$P_t C_t + \frac{F_t^S + CB_t^S}{R_t^S} \leq W_t L_t + F_{t-1}^S + CB_{t-1}^S + \Pi_t^{PF} + \Pi_t^{FI} - T_t + TR_t.$$

If we redefine the deposit of the households as $D_t \equiv F_t^S + CB_t^S$, saving instruments for households is now only the deposit with the rate of R_t^S. This deposit is the liability of the financial intermediary since we assume the zero-net supply of the short-term bond. In this case, the market clearing condition for the short-term bond market become: $H_t^S + F_t^S + CB_t^S = B_t^S \Leftrightarrow H_t^S + D_t^S = B_t^S \ (= 0)$.

of the domestic financial intermediaries are:

$$P_t^m H_t^L = m P_t^m h_t^L = \frac{\mathbb{E}_t\left[R_{t+1}^L\right] - R_t^S}{\frac{\omega}{m}\Sigma}, \tag{2.2.17}$$

where P_t^m is the price of the long-term bond at time t, m is a measure of arbitrageur, H_t^L is the aggregate holding of the long-term bond, and h_t^L is that of a symmetric individual. ω is the risk aversion parameter, and we call $\frac{\omega}{m}$ as the effective risk aversion of the whole sector of intermediaries[7]. Σ is the squared volatility of the long-term bond return. Note that equation (2.2.17) has the implication on the relation between risk premium and market long-term bond holding, financial intermediaries' risk taking, and the volatility of long-term bond. If we convert the second equality in terms of long-term risk premium, we have:

$$\mathbb{E}_t\left[R_{t+1}^L\right] - R_t^S = \frac{\omega}{m}\Sigma P_t^m H_t^L. \tag{2.2.18}$$

Then if the market risk increases ($\Sigma \uparrow$) or financial intermediaries become risk averse ($\omega \uparrow$), financial intermediaries decreases their holding on the long-term bond ($h_t^L \downarrow$) given some risk premium and price of long-term bond. The expected return on the long-term bond is:

$$\mathbb{E}_t\left[R_{t+1}^L\right] \equiv \mathbb{E}_t \frac{1 + \delta P_{t+1}^m}{P_t^m}, \tag{2.2.19}$$

where δ is the parameter controlling the average maturity of the longer-term bond, which can be interpreted as a portfolio of infinitely many bonds[8].

The position of financial intermediary is net out to zero. In this paper, as we assume

[7]This kind of approach follows Hansen and Sargent (2011), which is applied in Itskhoki and Mukhin (2021) in open economy setting recently. Hansen and Sargent (2011) considers the continuous-time limit in their model of ambiguity aversion. The economic rationale for this asymptotic approach is that the risk premium term proportional to $\frac{\omega}{m}\Sigma$ is finite and nonzero. Our solution concept allows for a first-order component of the long-term and short-term rate differential, i.e., a nonzero expected return from yield curve carry trade.

[8]Usually, the price of longer-term increase as its maturity approach and its price become 1 when matured. Lustig et al. (2019) define the price of long-term zero-coupon bond of maturity k at time t as $P_t^{(k)} = \exp(-ky_t^{(k)})$, and define the return by holding long-term bond for one-period as $R_{t+1}^{(k)} \equiv P_{t+1}^{(k-1)}/P_t^{(k)}$. It leads to an approximation of the following definition of return on long-term bond. We can formalize the

not inverted yield but normal yield as a shape of yield curve, the risk-averse arbitrageurs take a positive position for the longer-maturity bond and borrow the short-term bond, at the steady state. The budget constraint of the intermediaries then become,

$$P_t^m H_t^L + \frac{H_t^S}{R_t^S} = 0. \tag{2.2.21}$$

The profit gain of the financial intermediaries is transferred to the households in lump-sum way:

$$\Pi_t^{FI} = \left(1 + \delta P_t^m\right) H_{t-1}^L + H_{t-1}^S. \tag{2.2.22}$$

Market clearing and the definition of equilibrium: The labor markets clear with the wage W_t. The both short-term and long-term bond market clear, but note that we assume supply of short-term bonds in the market are zero-net supply:

$$0 = F_t^S + H_t^S + CB_t^S \left(= B_t^S\right), \tag{2.2.23}$$

$$B_t^L = H_t^L. \tag{2.2.24}$$

Then, the Walras law holds so that the good market clear, $C_t = Y_t$.

Definition 1 (Equilibrium system)**.**

Given state variables at time t, $\{F_{t-1}^S, CB_{t-1}^S, B_{t-1}^L, R_{t-1}^S\}_t$ *and prices,*

---

return by holding one unit of longer-term bond as:

$$\begin{aligned}\mathbb{E}_t R_{t+1}^L &\equiv \mathbb{E}_t \frac{\sum_{k=1}^K P_{t+1}^{(k-1)}}{\sum_{k=1}^K P_t^{(k)}} = \mathbb{E}_t \frac{\sum_{k=1}^K \exp\left(-(k-1)y_{t+1}^{(k-1)}\right)}{\sum_{k=1}^K \exp\left(-ky_t^{(k)}\right)} = \mathbb{E}_t \frac{1 + \sum_{k=2}^K \exp\left(-(k-1)y_{t+1}^{(k-1)}\right)}{\sum_{k=1}^K \exp\left(-ky_t^{(k)}\right)} \\ &= \mathbb{E}_t \frac{1 + \sum_{k=1}^{K-1} \exp\left(-ky_{t+1}^{(k)}\right)}{\sum_{k=1}^K \exp\left(-ky_t^{(k)}\right)} \approx \mathbb{E}_t \frac{1 + (1 - 1/K)\, P_{t+1}^m}{P_t^m} = \mathbb{E}_t \frac{1 + \delta P_{t+1}^m}{P_t^m}.\end{aligned} \tag{2.2.20}$$

This notation for the return on the zero coupon bond is corresponding to $r_{t+1}^y = y_t - \frac{\delta}{1-\delta}\left(y_{t+1} - y_t\right)$ in Campbell (2017) and Greenwood et al. (2019), where y_t is the log yield-to-maturity on domestic bonds. We can check this equivalence by transforming the equation: $(1-\delta)r_{t+1}^y = (1-\delta)y_t - \delta\left(y_{t+1} - y_t\right) \Leftrightarrow (1-\delta)r_{t+1}^y = y_t - \delta y_{t+1}$. If we set $1 - \delta \equiv \frac{1}{k}$, this is equivalent to $\frac{1}{k}r_{t+1}^{(k)} = y_t^{(k)} - (1 - \frac{1}{k})(y_{t+1}^{(k-1)}) \Leftrightarrow r_{t+1}^{(k)} = -(k-1)y_{t+1}^{(k-1)} + ky_t^{(k)}$.

a competitive equilibrium consists of stochastic process

$\{C_t, L_t, Y_t, F_t^S, H_t^S, H_t^L, B_t^L, \Pi_t^P, \Pi_t^{FI}, P_t, W_t, R_t^S, R_t^L\}_t$ *such that:*

(i) $\{C_t, L_t, F_t^S\}$ *maximize the infinite horizon utility subject to the budget constraint;*

(ii) goods producing firms choose $\{P_t, Y_t, L_t\}$ *to maximize the profit;*

(iii) financial intermediaries's position solves mean-variance problem $\{H_t^S, H_t^L\}$*;*

(iv) government follows the tax rule to determine $\{S_t\}$*;*

(v) CB follows the Taylor rule through open market operation to choose $\{R_t^S\}$*; and*

(vi) market clearing conditions are satisfied.

2.2.2 Calibration

The calibration parameter for households and firms are conventional (Table 2.2.1). We set the risk aversion parameter, $\sigma = 2.0$, and the Frisch elasticity of labor supply, $1/\phi = 1$. The elasticity of substitution is $\theta = 11$. This implies a 10% steady-state mark up. As for the price nominal rigidity, we assume that prices adjust on average once a year, and thus set $\lambda_p = 0.75$. The discount factor is $\beta = 0.9975$, which implies the steady state quarterly short-term interest rate is 0.25%.

The quarterly long-term rate is 1.01, which corresponds to 4% annualized return to long-term holding. The maturity measure, δ, is 0.95 following Bianchi and Ilut (2017). This implies the average maturity of the long-term bond is 5 years (= 1/(1-0.95) quarters). The debt to GDP ration is 100% reflecting the US case after the Covid-19 crisis. The standard deviation of excess return from investing in the long-term bond is 1.5%[9].

Lastly, the policy parameter for both monetary and fiscal policy such as ϕ_π and ϕ_b

[9]Given these calibrating parameters, we can obtain implied effective risk aversion measure of financial arbitrageurs from equation (2.2.18), while we only need the risk premium on the long-term bond to solve the equilibrium system. In this economy, the markup over the marginal cost is $\frac{\theta}{\theta-1}$. Now, suppose the economy is at the steady state, we assume the general price level at the initial steady state is 1, i.e., $\overline{P} = 1$. Given this assumption, the nominal wage, which is equivalent to the marginal cost, is $\overline{W} = \frac{\theta-1}{\theta}$. From the optimal labor supply condition of the households, we have $W_t C_t^{-\sigma} = L_t^{\phi} \Rightarrow \overline{Y} = \overline{W}^{\frac{1}{\sigma+\phi}} = \frac{10}{11}^{\frac{1}{3}} \approx 0.97$. Given this, the effective aversion is:

$$\overline{R^L} - \overline{R^S} = \frac{\omega}{m}\Sigma\overline{P^m}\overline{H^L} \Leftrightarrow \frac{\omega}{m} = \frac{\overline{R^L} - \overline{R^S}}{\Sigma\overline{P^m}\overline{H^L}} = \frac{\overline{R^L} - 1/\beta}{\sigma_{r^L}^2 b_{ss}\overline{PY}} = \frac{1.010 - 1.0025}{(0.015)^2 \times 1.0 \times 0.97 \times 4} \approx 8.6.$$

This is in the range of "medium" risk aversion parameter in Gourinchas et al. (2020).

and the shock persistence depends on our exercise. We show the correspondence on each simulation.

Table 2.2.1: Baseline calibration parameters

Description	Parameter	Value
Households		
Discount factor	β	0.9975
Relative risk aversion	σ	2
Frisch elasticity of labor supply	ϕ	1
Good producing firms		
Elasticity of substitution	θ	11
Calvo probability for prices	λ_p	0.75
Financial market		
Return on long-term rate	$\overline{R^L}$	1.010
Maturity measure	δ	0.95
Debt to GDP ratio	b_{ss}	1.0
Std. of ex. return from long-term	σ_{r^L}	1.5%

2.3 Results

2.3.1 Deflationary fiscally-led region

In this subsection, we argue the main mechanism of the model. As is shown in Leeper (1991), the most of the Monetary-FTPL model have the four disjoint regions according to the policy parameter space (see e.g., Bhattarai et al. (2014)). The model in this paper, however, characterize the five regions by adding another region in the Fiscally-led region or the Passive Monetary and Active Fiscal region. Firstly, we see the parameter space in which we can find a unique saddle-path equilibrium.

Lemma 1. *i. Equilibrium is determinate when* $\{\phi_\pi > 1,\ \phi_b > \phi_b^{def}\}$ *or* $\{\phi_\pi < 1,\ \phi_b < \phi_b^{def}\}$*;*
ii. equilibrium is indeterminate when $\{\phi_\pi < 1,\ \phi_b > \phi_b^{def}\}$*;*
iii. equilibrium has no solution when $\{\phi_\pi > 1,\ \phi_b < \phi_b^{def}\}$*,*

$$\textit{where} \quad \phi_b^{def} \equiv \overline{R^L} - 1 + b_{ss}\left(\overline{R^L} - \overline{R^S}\right).$$

Proof. See Appendix 2.A. ■

This set of equilibrium region is consistent with the literature, however, the threshold for the fiscal parameter, ϕ_b^{def}, is differentiated. The presence of long-term debt and financial intermediary plays a crucial role for this difference (note: b_{ss} is the steady state value of the debt to GDP ratio). The combination of long-term bond risk premia and debt size is added to the threshold, which is a minimum tax requirement to escape from a unique equilibrium of the Fiscal-led regime. As the debt size or the interest rate differential between short-term and long-term bonds increases, the risk of being in the Fiscal-led regime increases. If we consider the economy in which government issue only short-term bond or there is no interest differential across short-term and long-term bond, the yield from both debts should be identical, i.e. $\overline{R^S} = \overline{R^L}$. Hence the fiscal threshold is $\phi_b^{def} = \overline{R^S} - 1 (= 1/\beta - 1)$, which is the threshold for the model with one class of government debt as in Leeper (1991).

For the sake of comparison, we should note that *indeterminate region* is corresponding to the Passive Monetary and Passive Fiscal (PM/PF) and *no solution region* is the Active Monetary and Active Fiscal (AM/AF) regime, if we use the terminology by Leeper (1991). The case of $\{\phi_\pi > 1,\, \phi_b > \phi_b^{def}\}$ is the Active Monetary and Passive Fiscal (AM/PF) regime, in which monetary policy can actively pursue price stability by reacting strongly to inflation. In this regime, fiscal policy obeys the tighter lump-sum tax rule to balance the budget. The fourth case of the Passive Monetary and Active Fiscal (PM/AF) regime, I will look into the detail in the following.

To pin down an argument, we now define the *deflationary region* and the *inflationary region* in the PM/AF regime to consider how the behavior of price level could differ due to the size and duration of the government debt.

Definition 2. *Consider the case of* $\{\phi_\pi < 1,\, \phi_b < \phi_b^{def}\}$. *If the price level increases, conditional on a positive government primary surplus shock, we define the regime is inflationary. If the price drops, the regime is deflationary.*

Given the definition for the inflationary region and the deflationary region, we can prove the following proposition under the fully nominal rigid case[10]:

Proposition 2. *In the case of full nominal rigidity,* $\lambda(=\kappa_p(\sigma+\phi)) \to 0$*:*

i. equilibrium is deflationary when $\{\phi_\pi < 1,\ \phi_b^{inf} < \phi_b < \phi_b^{def}\}$*;*

ii. equilibrium is inflationary when $\{\phi_\pi < 1,\ \phi_b < \phi_b^{inf}\}$*,*

$$\textit{where} \quad \phi_b^{inf} \equiv \frac{\overline{R^L}-\delta}{\overline{R^L}-\delta\frac{\overline{R^S}}{\overline{R^L}}}\phi_b^{def} - \frac{(1-\delta)\left(\overline{R^L}-\overline{R^S}\right)}{\overline{R^L}-\delta\frac{\overline{R^S}}{\overline{R^L}}}.$$

Proof. See Appendix 2.A. ∎

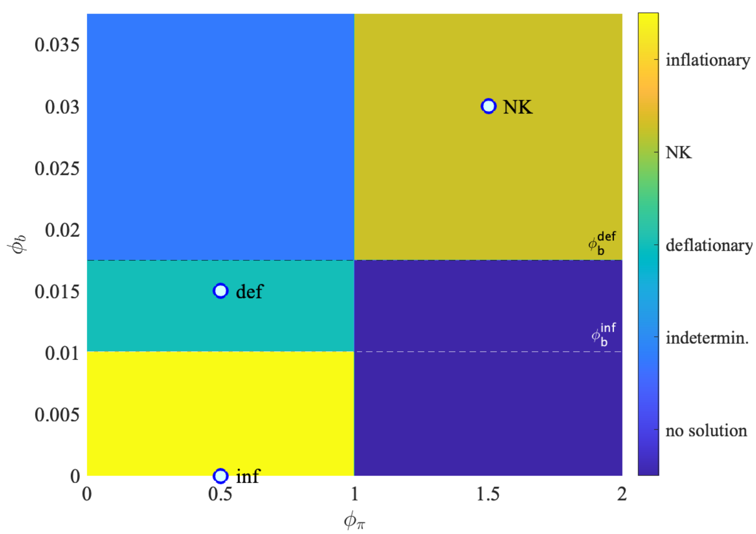

Figure 2.3.1: The determinacy region

Figure 2.3.1 shows the parameter space for each regime. Other than an emergence of the deflationary region in Fiscally-led (PM/AF) regime ($\phi_\pi < 1$ and $\phi_b < \phi_b^{def}$), the parameter space is identical with the literature following Leeper (1991). In each region, deflationary region and inflationary region, there are distinct mechanisms that are key results to understand the behavior of inflation responding to the fiscal shock[11].

[10]In Appendix 2.A, we derive the condition, which divides the inflationary region and the deflationary regions, in more general setting including flexible price case.

[11]In the main text, we only focus on the fully sticky price case as a limiting benchmark which simplifies the argument by avoiding a complex formula as is shown in Appendix 2.A. If the price stickiness is lessened,

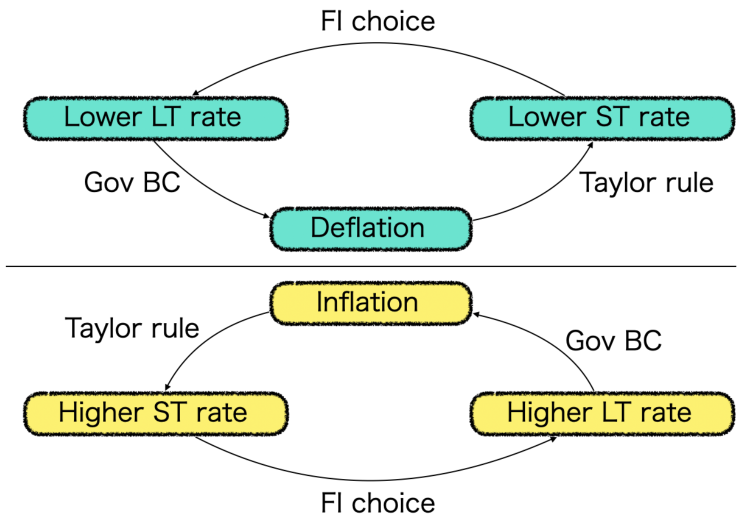

Figure 2.3.2: Main mechanism

- *Deflationary region*: The tax is sufficient, which is higher than ϕ_b^{inf}, so the government budget constraint is satisfied not by inflation but by deflation. In other words, the fiscal burden is financed by borrowing from the future surplus. The households decrease consumption because of deflation and subsequent real interest rate increase and save more in short-term bonds, which lower the nominal short-term interest rate[12]. Due to a drop in the price level and a negative surplus, the debt to GDP ratio rises, and its increase in the debt accelerates tax increase. The short-term rate drops, so the financial intermediaries' demand shifts to long-term bonds, and the nominal long-term rate is lowered.

- *Inflationary region*: In this region, the primary deficit is financed by inflation. Due to inflation, the households consume more and less save in short-term bonds, hence

the deflationary region expands, and the inflationary region shrinks. This is because the more flexible price is, the more critical the role of the Taylor rule is. The determination of whether deflationary or inflationary highly depends on the size of inflation and the nominal rate's response. The choice of whether consume or save by households relies on the behavior of the real interest rate so that if the monetary authority does not respond much toward deflation, the real interest rate tends to be higher. That is why the deflation region expands. For more detail, see Appendix 2.A.

[12]If the Taylor rule coefficient, ϕ_π, is greater than 0 and the price stickiness is there, the Taylor rule also plays a role to lower the nominal interest rate responding to deflation. Even in the case of $\phi_\pi = 0$ and the flexible price, however, the mechanism for lowering the short-term rate works without the central bank Taylor rule.

increasing the nominal short-term rate. Since the nominal short-term rate becomes higher than the initial, the financiers' demand shifts from long-term bonds. The price of long-term bonds drops, and its rate increases. Finally, the debt burden decreases because of inflation, and tax is reduced because of the smaller size of the debt, which supports the households' consumption.

The financial intermediaries are indifferent about the inflation because they carry the interest rate differential between short-term and long-term bond, which is independent of inflation. Hence, as long as the long-term bond carry higher yield than that of short-term bond conditional on the government transfer shock, they long long-term bond. Figure 2.3.2 describes the main mechanism argued in the above conceptually.

Given the calibrating parameters in Section 2.2.2 and the above argument, we characterize the three regions by using the following policy parameters in Table 2.3.1.

Table 2.3.1: Monetary and Fiscal policy rule coefficients under different regimes

Regime (region)	ϕ_b	ϕ_π
NK	0.03	1.5
FTPL (def)	0.015	0.5
FTPL (inf)	0.00	0.5

The condition for emergence of the deflationary region. When dose the deflationary region emerge? In the fully nominal rigidity case, we can see the condition for emergence in an analytical way. The size of deflationary regions is characterized by the difference between ϕ_b^{def} and ϕ_b^{inf}:

$$\phi_b^{def} - \phi_b^{inf} = \frac{\delta\left(1 - \frac{\overline{R^S}}{\overline{R^L}}\right)}{\overline{R^L} - \delta\frac{\overline{R^S}}{\overline{R^L}}}\phi_b^{def} + \frac{(1-\delta)\left(\overline{R^L} - \overline{R^S}\right)}{\overline{R^L} - \delta\frac{\overline{R^S}}{\overline{R^L}}} = \frac{1 - \frac{\overline{R^S}}{\overline{R^L}}}{\overline{R^L} - \delta\frac{\overline{R^S}}{\overline{R^L}}}\left[\overline{R^L} - \delta\left(1 - b_{ss}\left(\overline{R^L} - \overline{R^S}\right)\right)\right]. \tag{2.3.1}$$

Fist of all, the larger the interest rate differential is, the larger the deflationary region is. If the long-term bond rate is equivalent with the short-term bond rate, i.e., $\overline{R^S} = \overline{R^L}$, $\phi_b^{def} - \phi_b^{inf} = 0$. Thus the deflationary regions vanishes (Figure 2.3.3). To put it differently, as long as there is positive interest rate differential, the deflationary regions emerges. Given the maturity rate is the same, the larger the debt size is, the larger the deflationary region is[13].

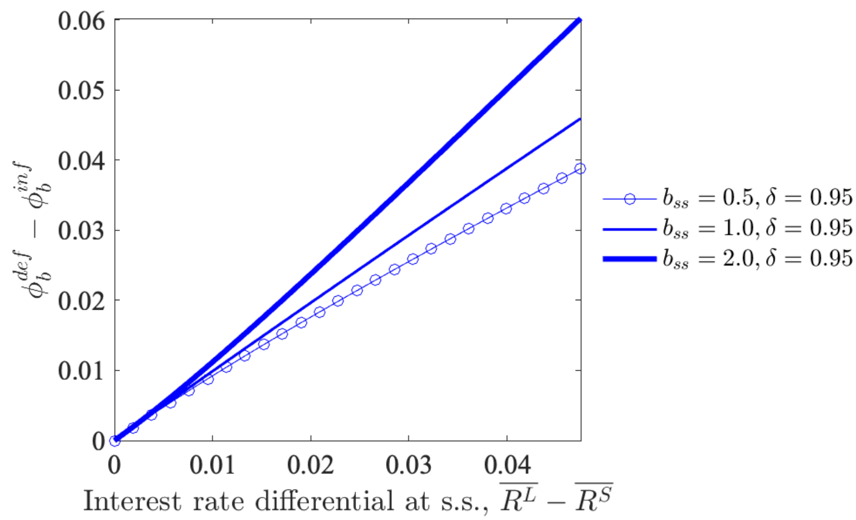

Figure 2.3.3: The measure of size of deflationary region with respect to the interest rate differential between long-term and short-term under different sets of the debt size and the maturity measure

2.3.2 Government time 0 budget constraint

How the deflationary equilibrium emerges as a result of the introduction of the long-term bond and the financial intermediaries? This section discusses how the linearized government budget constraint would change compared to the case of only with the short-term bonds. Solving the model in log-linearized form, we already have the government's flow budget constraint:

$$\frac{1}{\overline{R^L}}\tilde{b}_t = \tilde{b}_{t-1} + b_{ss}\left(r_t^L - \pi_t - \Delta y_t\right) - \frac{1}{\overline{R^L}}\tilde{s}_t, \tag{2.3.2}$$

[13]It should be noted that the model allows for a certain discontinuity between short-term and long-term bonds. Essentially, short-term and long-term bonds with one maturity period ($\delta = 0$) are perfect substitutes. However, in the model in this paper, short-term and long-term bonds are different in nature in terms of the risk each carries. That is, since the steady-state values of the returns of the short-term and long-term bonds are exogenously given, it is possible to assume that the maturity is one period, i.e., ($\delta = 0$), even if there is a difference in their returns. Then, if the maturity horizon of the long-term bond is quite short, i.e., $\delta \to 0$, the size of deflationary region can be characterized by $\phi_b^{def} - \phi_b^{inf} = 1 - \frac{\overline{R^S}}{\overline{R^L}}$, which dose not depend on the maturity rate but depends on the interest rate differential.

The households' inter-temporal Euler equation under the special case of $\sigma = 1$ is $\Delta y_{t+1} = r_t^S - \pi_{t+1}$. Putting together the government budget constraint and the households' Euler equation yields:

$$\frac{1}{\overline{R^L}}\tilde{b}_t = \tilde{b}_{t-1} + b_{ss}\left(r_t^L - r_{t-1}^S\right) - \frac{1}{\overline{R^L}}\tilde{s}_t. \tag{2.3.3}$$

Computing the time 0 budget constraint of the government by iterating the above forward, we have a present value identity:

$$\tilde{b}_0 = \sum_{t=1}^{T}\left(\frac{1}{\overline{R^L}}\right)^t \tilde{s}_t - \overline{R^L} b_{ss} \sum_{t=1}^{T}\left(\frac{1}{\overline{R^L}}\right)^t \left(r_t^L - r_{t-1}^S\right) + \left(\frac{1}{\overline{R^L}}\right)^T \tilde{b}_T. \tag{2.3.4}$$

Taking expectations on both sides and taking the limit as $T \to \infty$, and assuming that the limiting term goes to zero, we have:

$$\tilde{b}_0 = \mathbb{E}_0\left[\sum_{t=1}^{\infty}\left(\frac{1}{\overline{R^L}}\right)^t \tilde{s}_t\right] - \overline{R^L} b_{ss}\mathbb{E}_0\left[\sum_{t=1}^{\infty}\left(\frac{1}{\overline{R^L}}\right)^t \left(r_t^L - r_{t-1}^S\right)\right]. \tag{2.3.5}$$

The log value of real debt to GDP ratio equals the present value of future surplus-to-GDP ratios discounted by the nominal (real) return differential between the long-term and short-term government debt. If there is no additional term, the equation is similar one in the literature. The second bracket, however, has a crucial role to understand the price level dynamics with the model with the long-term debt. Suppose we were in the inflationary regime, the long-term rate response is larger than that of the short-term bond given the negative surplus shock and the second tern is positive. It means the real debt to GDP ratio is smaller than the case without the second term, hence, the price level increases more. On the other hand, if we were in the deflationary region, the effect of the second term is the opposite. The real value of debt is larger and the price level decreases.

2.3.3 Impulse response function

In this section, we add three alternative shocks to the economy, fiscal deficit shock, interest hike monetary policy shock, and quantitative easing monetary policy shock[14]. A goal of this section is to see how responses to each should would differ across monetary and fiscal regimes. It should be noted that the last shock, quantitative easing shock, is enabled by our setting with market segmentation and demand based asset pricing approach, which contributes to Cochrane (2021). In the first two exercises, we assume the persistence of monetary policy shock is $\rho_m = 0.96$, while we assume very persistent transition for asset purchase shock to quantitatively approximate the behavior of short-term rate binding with the Zero Lower Bound, $\rho_m \approx 1.0$.

2.3.3.1 Fiscal deficit shock

Figure 2.3.4 shows the IRFs of main variables under 1% fiscal deficit shock. Since we consider the economy without government purchase and lump-sum transfer and tax, consumption and price does not respond in the New Keynesian regime (henceforth NK, blue circle line). Due to the fiscal deficit, price of long-term bond drops and debt increases.

In the case of Fiscal Theory Inflationary regime (henceforth FTI, green solid line), price level increases to compensate the government budget constraint, which is typical with the literature of fiscal theory[15]. Remember that in FTI, the stance of fiscal authority is insufficient and fiscal authority does not increase tax or decrease transfer particularly in the case of $\phi_b = 0$. Since the price of long-term bond drops, the long-term rate increases[16]. Consumption increases due to a decrease in real rate and inter-temporal substitution.
Combined with price increase, the nominal GDP increases, thus the debt to GDP decreases.

The case of the Fiscal Theory Deflationary regime (henceforth FTD, dashed red line) is

[14]While in the main text, we only argue about the three shocks. In Appendix 2.B, we also consider the model dynamics under a negative productivity shock, which corresponds to the negative supply shock.

[15]If we do not introduce the nominal price rigidity ($\lambda_p \approx 0$), the real side of economy does not respond, i.e., consumption barely moves even though the price level increases. See Cochrane (2021).

[16]An initial decline in the long-term rate comes from a decline in price of that bond. After that, the long-term rate increases.

characteristic in the model. In this case, households expect future tax increase so that they do not increase consumption. The fiscal authority would increase tax (primary surplus), then price of the long-term bond does not drop as much as FTI. In this region, decrease in price and output as a result of this, are the equilibrium outcome.

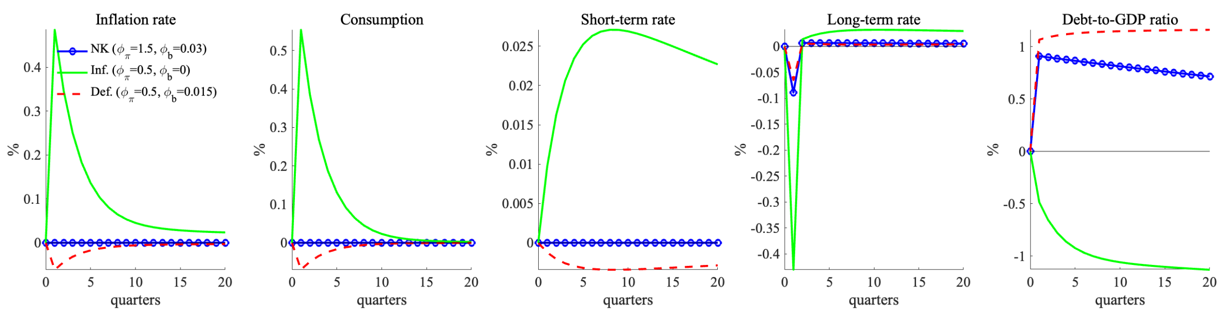

Figure 2.3.4: IRFs to 1% negative fiscal deficit shock

2.3.3.2 Conventional monetary policy shock

Figure 2.3.5 describes the IRFs given 1% interest hike monetary policy shock. The response of in NK is conventional that an increase in real rate decreases consumption. An increase in the short-term rate induce portfolio shift of financial intermediaries from the long-term bond. The long-term rate temporaly increases, but converges to the original steady state as well as the debt to GDP ratio, because the monetary policy is fiscally backed by the fiscal authority.

In FTI, the financial intermediaries shift portfolio toward the short-term bond as well as NK case. To satisfy the government budget constraint, the economy need inflation, since the response of fiscal surplus is 0 in FTI. Due to an increase in inflation, the monetary authority keeps the short-term rate high through the Taylor rule. Therefore, households reduce consumption because of high real rate. If we flip a sign of the shock, an economic behavior is entirely reversed. An increase in the nominal short-term rate increases inflation and consumption. The dynamics is similar that of Neo-Fischer ones (see e.g., Uribe (2018), Garín et al. (2018) among others).

Lastly, in FTD, we finds the IRFs are close to that of NK, while the magnitude of each response differs. The mechanism is also close to that of NK, but responses of inflation and consumption are amplified due to the weak commitment of monetary policy. Due to its weakness, the short-term rate remains higher and hence it leads to prolonging higher long-term rate. Higher long-term rate implies larger debt burden and it results in future surplus. It implies a larger debt over GDP Overall, these similarities are also pointed out in Cochrane (2021) as “observational equivalence.”

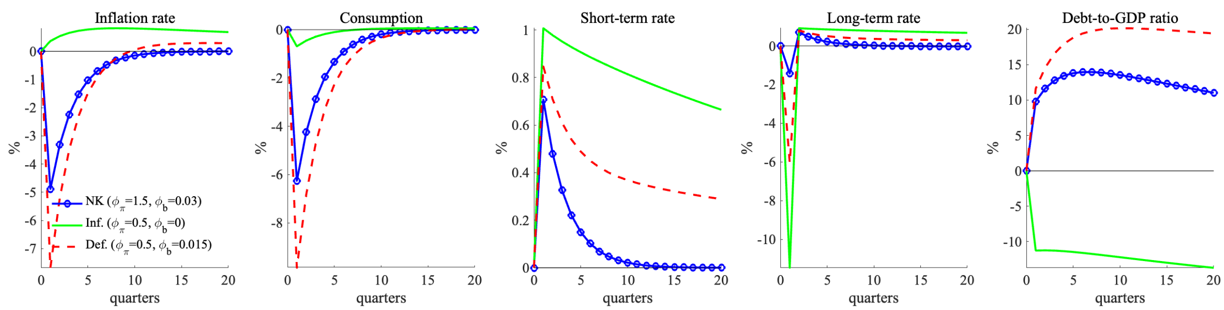

Figure 2.3.5: IRFs to 1% interest hike monetary policy shock

2.3.3.3 Unconventional monetary policy shock

Lastly, we show the IRFs to an asset purchase monetary policy shock in Figure 2.3.6, which is unique to the model setting compared to Cochrane (2021). The setting with market segmentation hypothesis allows us to examine the effect of the unconventional monetary policy easing straightforward. To do this, we have to modify two equations of the benchmark model, the balance sheet of the central bank (equation (2.2.14)) and the market cleating condition of the long-term bonds (equation (2.2.24)) as follows:

$$\begin{gathered} P_t^m CB_t^L - (1 + \delta P_t^m) CB_{t-1}^L + \frac{CB_t^S}{R_t^S} - CB_{t-1}^S = M_t - M_{t-1}, \\ B_t^L = H_t^L + CB_t^L. \end{gathered} \tag{2.3.6}$$

Now we assume the central bank holds 20% of debt in asset side of the balance sheet at the steady state, which is close to an average of the federal reserve holding of US treasury after the Great Financial Crisis. It means $\overline{CB^L} = 0.20\overline{B^L}$ at the steady state. We consider the following shock, which is quite persistent, $\rho_{cb} \approx 1.0$:

$$\frac{CB_t^L}{\overline{CB}^L} = \left(\frac{CB_{t-1}^L}{\overline{CB}^L}\right)^{\rho_{cb}} e^{\sigma_{cb}\varepsilon_t^{cb}}, \quad \varepsilon_t^{cb} \sim iid(0,1). \tag{2.3.7}$$

Then, we use the two equations log-linearized around the steady state:

$$\begin{aligned} b_t^L &= 0.8h_t^L + 0.2cb_t^L, \\ cb_t^L &= \rho_{cb} cb_{t-1}^L + \sigma_{cb}\varepsilon_t^{cb}. \end{aligned} \tag{2.3.8}$$

Figure 2.3.6 shows the IRFs for 1% long-term bond purchase monetary policy shock. Note that since we assume quite persistent process for the short-term rate, it barely moves for each simulation. In NK, an asset purchase has no effect on price even under the segmented market, because the monetary policy is fiscally backed with sufficient primary balance rule. Since price and short-term rate do not respond, thus real rate and consumption also are kept unchanged. On the fiscal side, an asset purchase by printing money increase price of long-term bond and lower rate of long-term bond. Therefore, in the long run, the fiscal burden decreases and debt-to GDP ratio also diminishes.

The response under FTI is, potentially, controversial. In general, a quantitative easing has an effect that decreases debt burden. Therefore, a decrease debt burden leads to lower inflation in FTI. Lowe inflation implies higher real rate, then consumption decrease and debt to GDP ratio increases.

Finally, the response under FTD suggests inflationary pressure and an increase in consumption. As like a previous argument, if the debt burden decreases, the fiscal authority dose not have to raise primary surplus, thus deflationary pressure is lessened and households can consume more. The debt burden results in small after the execution of quantitative easing.

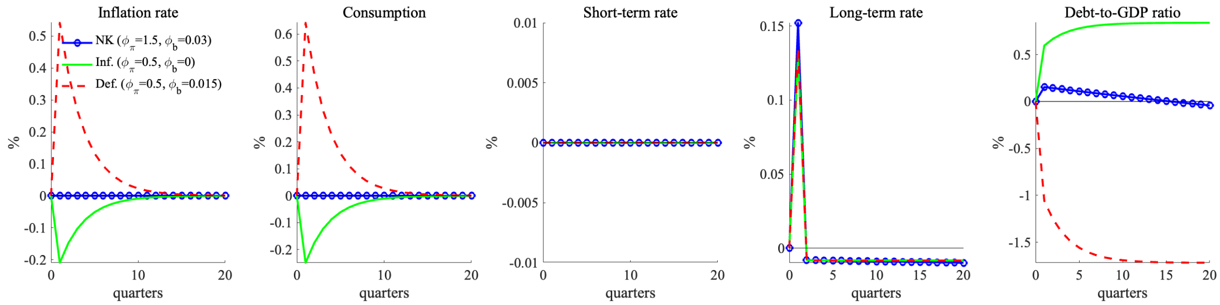

Figure 2.3.6: IRFs to 1% debt purchase shock

2.4 Mapping to the Data

The main point we have discussed in the paper is that there might be a deflationary region in Fiscal-led regime if we incorporate the long-term market segmentation element into the conventional NK-FTPL framework. Then, in a real world, what is the relation between inflation rate and fiscal policy stance? To see this, we estimate the fiscal policy stance parameter, $\hat{\phi}_b$, and compare it with the theoretical threshold that distinguish the inflationary region and the deflationary region for OECD countries using the data post the Great Financial Crisis[17][18].

Our data analysis reveals that there is remarkable difference between non-Euro countries and Euro countries. Thus we divide the whole sample into two groups, as non-Euro and Euro countries. Figure 2.4.1 plots $\hat{\phi}_b - \phi_b^{inf}$, which is a measure for the fiscal stance not to be inflationary, and π_t, average annual inflation rate. Given the most of OECD countries have been experienced chronic fiscal deficit, our theory predicts that the stronger fiscal austerity compared to the inflationary threshold implies that low inflation or deflationary pressure. In this regard, non-Euro countries match with this prediction.

However, the data for Euro countries shows the opposite to the model prediction. That is, in countries such as Greece, Italy, and Portugal, which have chronic fiscal deficits, inflation

[17] We describe the detail of data construction and data source in Appendix 2.C.

[18] The reason why we focus on post-era after the GFC is the fiscal stance had changed before and after the GFC. Moreover, 'missing inflation' episode has bee argued among some of the developed countries only after the GFC. See, for example, Constancio (2015) and Heise et al. (2022).

rates are low even though their fiscal stance is not adequate relative to the threshold. A potential explanation for this is that these three countries are within the EURO area and their currencies are stable, which means that the rise in interest rates does not occur in tandem with the rise in prices, i.e., the government debt cannot be inflated away. The another potential reason is the belief of agents matters. While the current fiscal stance is insufficient, the households in each country believe the future government would try to improve the primary balance of the country. Thus, they would decrease consumption in response to fiscal deficit shock in order to prepare future tax increase.

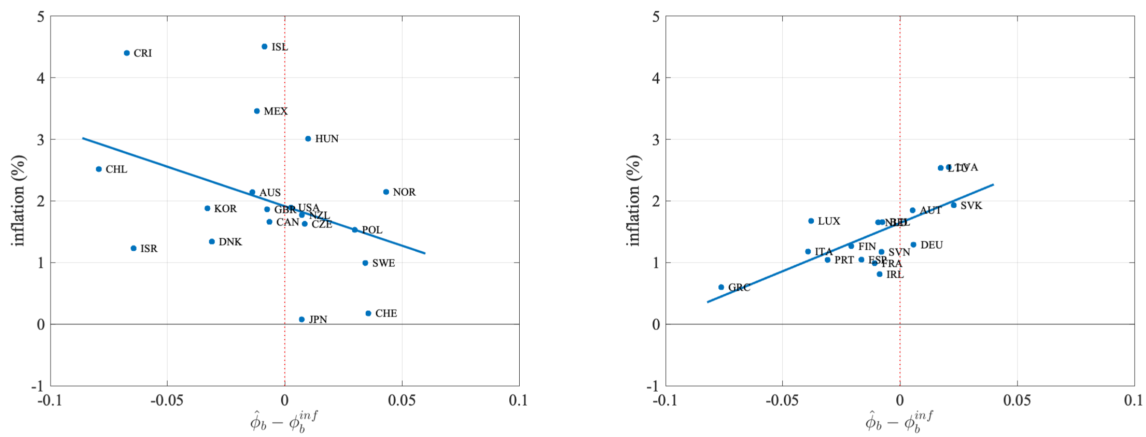

Figure 2.4.1: $\hat{\phi}_b - \phi_b^{inf}$ in non-EURO countries and EURO countries

NOTE.—The left panel shows a plot for $\hat{\phi}_b - \phi_b^{inf}$ and average inflation rates after the GFC for non-EURO countries. The right panel shows that for EURO-countries.

2.5 Conclusion

This paper provides a theoretical model that introduces the long-term debt and market segmentation hypotheses into the conventional NK-FTPL framework. Under this theory, we find that depending on the fiscal policy stance, there may be deflationary rather than the inflationary region in response to fiscal deficit shocks even under a fiscal-led regime. Under the market segmentation hypothesis, we confirm that QE, which has become the main instrument of monetary policy in recent years, works to boost GDP and inflation in deflationary regions. The future research agenda includes the following.

First, it is essential to deepen our discussion on the strategy of the policy planning side, applying a game-theoretic approach to coordinating monetary and fiscal policies. Given the findings of the deflationary part of the fiscal-led regime obtained in this paper, discussing the linkage between the monetary and fiscal policy is a very important perspective (e.g., Atkeson et al. (2010), Bassetto and Sargent (2020), and Christiano and Takahashi (2018)).

With the discovery of new regimes, research issues include estimating macroeconomic models that have regime transitions. However, since the responses in the NK and FTD regimes are qualitatively close (observatory equivalence), it is currently difficult to identify regimes using estimates such as those based on regime-switching (e.g., Bhattarai et al. (2016) and Bianchi and Ilut (2017) among others). To solve this problem, it is necessary to examine in more detail the response of each regime to different shocks, such as fiscal spending and additional taxation. This view of macro model estimation is discussed in more detail in Cochrane (2022).

The assumption of the market segmentation hypothesis also allows the model to be extended to open economies[19]. It is instructive to advance our understanding of how inflation and exchange rate dynamics behave under monetary and fiscal policy rules and parameters. When focusing this analysis on the U.S., it will also be essential to analyze what implications the demand for U.S. Treasuries as safe assets from abroad will have for monetary and fiscal policy, for example, by pushing down the neutral interest rate. Moreover, it is intriguing to investigate the implication of monetary and fiscal policy in the U.S. on the international monetary system and price system (Farhi and Maggiori (2018), Mukhin (2022)). These are also issues for future research.

[19] Jiang (2021) and Jiang (2022) discuss an implication of fiscal theory under open economies, including exchange rates.

APPENDIX

2.A Model Details

2.A.1 Walras law

In this subsection, we check if the Walras law holds in this economy with the case of asset purchase of the central bank. We starts with the households' budget constraint:

$$P_tC_t + \frac{F_t^S}{R_t^S} + M_t = W_tL_t + F_{t-1}^S + M_{t-1} + \Pi_t^{PF} + \Pi_t^{FI} - S_t. \tag{2.A.1}$$

Substituting the following government budget constraint, profits from the final goods producer, and profit from the financial intermediary,

$$\begin{aligned} B_{t-1}^S + \left(1 + \delta P_t^m\right) B_{t-1}^L - S_t &= \frac{B_t^S}{R_t^S} + P_t^m B_t^L, \\ \Pi_t^{PF} &= \left(P_t - MC_t\right) Y_t = P_tY_t - W_tL_t, \\ \Pi_t^{FI} &= \left(1 + \delta P_t^m\right) H_{t-1}^L + H_{t-1}^S, \end{aligned} \tag{2.A.2}$$

then we obtain:

$$\begin{aligned} &P_tC_t + \frac{F_t^S}{R_t^S} + M_t + B_{t-1}^S + \left(1 + \delta P_t^m\right) B_{t-1}^L \\ &= F_{t-1}^S + M_{t-1} + P_tY_t + \left(1 + \delta P_t^m\right) H_{t-1}^L + H_{t-1}^S + \frac{B_t^S}{R_t^S} + P_t^m B_t^L. \end{aligned} \tag{2.A.3}$$

The budget constraint of the financial intermediary, the market clearing conditions for short-term bond and long-term bond, and the balance sheet of the central bank are the last components to check the Walras law.

$$\begin{aligned} P_t^m H_t^L + \frac{H_t^S}{R_t^S} &= 0, \\ B_t^S &= F_t^S + H_t^S + CB_t^S, \\ B_t^L &= H_t^L + CB_t^L, \\ P_t^m CB_t^L - (1 + \delta P_t^m)\, CB_{t-1}^L + \frac{CB_t^S}{R_t^S} - CB_{t-1}^S &= M_t - M_{t-1}. \end{aligned} \tag{2.A.4}$$

Substituting the above 4 equations into equation (2.A.3), we finally get:

$$\begin{aligned} P_t C_t + \frac{F_t^S}{R_t^S} + \frac{H_t^S}{R_t^S} + \frac{CB_t^S}{R_t^S} + B_{t-1}^S &= P_t Y_t + \frac{B_t^S}{R_t^S} + F_{t-1}^S + H_{t-1}^S + CB_{t-1}^S \\ \Leftrightarrow \quad P_t C_t &= P_t Y_t. \end{aligned} \tag{2.A.5}$$

Therefore, the Walras law holds in this economy. The benchmark case economy without asset purchase of the central bank can be obtained by ignoring the term for the central bank holding of the long-term bond, CB^L.

2.A.2 Linearized government budget constraint

We set the following government budget constraint, which is composed of the short-term and long-term bond, and primary surplus.

$$B^S_{t-1} + \left(1+\delta P^m_t\right) B^L_{t-1} - S_t = \frac{B^S_t}{R^S_t} + P^m_t B^L_t, \tag{2.A.6}$$

Note that there is no government expenditure in the model to equate consumption and production, $C_t = Y_t$. Since we impose the restriction the short-term debt is zero-net supply, we delete the short-term bond from the equation, then we have:

$$\left(1+\delta P^m_t\right) B^L_{t-1} - S_t = P^m_t B^L_t. \tag{2.A.7}$$

Dividing the both sides by the nominal GDP, P_tY_t, and arranging price of the long-term bond with the definition of the return on long-term bond, $R^L_t \equiv \frac{1+\delta P^m_t}{P^m_{t-1}}$, we obtain the following,

$$\begin{aligned} \frac{P^m_t B^L_t}{P_tY_t} &= \frac{\left(1+\delta P^m_t\right) B^L_{t-1}}{P_tY_t} - \frac{S_t}{P_tY_t} \\ \Leftrightarrow \quad \frac{P^m_t B^L_t}{P_tY_t} &= \frac{1+\delta P^m_t}{P^m_{t-1}} \frac{P^m_{t-1}B^L_{t-1}}{P_{t-1}Y_{t-1}} \frac{P_{t-1}Y_{t-1}}{P_tY_t} - \frac{S_t}{P_tY_t} \\ \Leftrightarrow \quad b_t &= \frac{R^L_t}{\Pi_t}\frac{Y_{t-1}}{Y_t} b_{t-1} - s_t, \end{aligned} \tag{2.A.8}$$

where we use two definitions, the debt-to-GDP ratio, $b_t \equiv \frac{P^m_t B^L_t}{P_tY_t}$, and the fiscal surplus to GDP ratio, $s_t \equiv \frac{S_t}{P_tY_t}$. Π_t is the gross inflation rate. We linearize with fiscal variables and log-linearize with the rest of variables around zero inflation steady state[20]. We define $\tilde{x}_t \equiv x_t - x_{ss}$ for fiscal variables and consider the shock to the primary surplus:

$$\tilde{b}_t = \overline{R^L}\tilde{b}_{t-1} + \overline{R^L} b_{ss}\left(r^L_t - \pi_t - \Delta y_t\right) - \tilde{s}_t + \sigma_s \varepsilon^s_t, \quad \varepsilon^s_t \sim iid(0,1), \tag{2.A.9}$$

[20]This style of linearization is typical within NK-FTPL DSGE literature, see Leeper (1991) and Cochrane (2022), and the following literature, e.g., Bianchi and Melosi (2017), Bianchi and Ilut (2017), Bhattarai et al. (2016), and Bhattarai et al. (2012).

which is equation (2.2.10) in the main text.

2.A.3 Summary of equilibrium system in linearized form

In this subsection, we characterize the equilibrium system with 5 equations and 5 endogenous variables. We starts with 15 equations for 15 endogenous variables, $\{c_t, l_t, y_t, mc_t, w_t, p_t, \pi_t, r_t^S, r_t^L, \tilde{s}_t, \tilde{b}_t, p_t^m, b_t^L, h_t^L, a_t\}$, which characterize the whole equilibrium system of the benchmark economy without central banks' asset purchase. The model equations are summarized as the followings:

$$\begin{aligned}
\text{the households' intertemporal Euler equation:} \quad & \sigma\left(\mathbb{E}_t c_{t+1} - c_t\right) = r_t^S - \mathbb{E}_t \pi_{t+1}, \\
\text{the labor supply optimality:} \quad & \sigma c_t + \phi l_t = w_t - p_t, \\
\text{the production function:} \quad & y_t = l_t + a_t, \\
\text{the marginal cost:} \quad & mc_t = w_t - a_t, \\
\text{the TFP process:} \quad & a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \\
\text{the New Keynesian Phillips curve:} \quad & \pi_t = k_p\left(mc_t - p_t\right) + \beta \mathbb{E}_t \pi_{t+1}, \\
\text{the government budget constraint:} \quad & \tilde{b}_t = \overline{R^L}\tilde{b}_{t-1} + \overline{R^L} b_{ss}\left(r_t^L - \pi_t - \Delta y_t\right) - \tilde{s}_t + \sigma_s \varepsilon_t^s, \\
\text{the fiscal rule:} \quad & \tilde{s}_t = \phi_b \tilde{b}_{t-1}, \\
\text{the debt to GDP ratio:} \quad & \tilde{b}_t = p_t^m + b_t^L - p_t - y_t, \\
\text{the portfolio choice by financial intermediary:} \quad & \left(\overline{R^L} - \overline{R^S}\right)\left(p_t^m + h_t^L\right) = \overline{R^L}\mathbb{E}_t r_{t+1}^L - \overline{R^S} r_t^S, \\
\text{the return on the long-term rate:} \quad & r_t^L = \frac{\delta}{\overline{R^L}} p_t^m - p_{t-1}^m, \\
\text{the inflation rate:} \quad & \pi_t = p_t - p_{t-1}, \\
\text{the monetary policy rule:} \quad & r_t^S = \rho_m r_{t-1}^S + \left(1 - \rho_m\right)\phi_\pi \pi_t + \sigma_m \varepsilon_t^m, \\
\text{the goods market clearing:} \quad & y_t = c_t, \\
\text{the long-term bond market clearing:} \quad & b_t^L = h_t^L.
\end{aligned} \tag{2.A.10}$$

Since our main focus of the paper is the deflationary region under the fiscal-led regime and we define the deflationary region according to the impulse response to the primary surplus shock, we simplify the monetary policy rule by assuming $\rho_m = 0$ and $\sigma_m = 0$ and the process for the total factor productivity by setting $\sigma_a = 0$ and then $a_t = 0$. Then, we obtain 5 equations for 5 endogenous variables, $\{p_t, c_t, p_t^m, r_t^L, b_t\}$ by substituting the auxuary variables:

$$\begin{aligned}
\text{households' intertemporal Euler equation:} \quad & \sigma c_t = \sigma \mathbb{E}_t c_{t+1} - \phi_\pi \left(p_t - p_{t-1}\right) + \mathbb{E}_t \left(p_{t+1} - p_t\right), \\
\text{New Keynesian Phillips curve:} \quad & \left(p_t - p_{t-1}\right) = \lambda c_t + \beta \mathbb{E}_t \left(p_{t+1} - p_t\right), \\
\text{government budget constraint:} \quad & \tilde{b}_t = \left(\overline{R^L} - \phi_b\right) \tilde{b}_{t-1} + b_{ss} \overline{R^L} \left(r_t^L - \left(p_t - p_{t-1}\right) - \Delta c_t\right) + \sigma_s \varepsilon_t^s, \\
\text{debt to GDP ratio:} \quad & \tilde{b}_t = \kappa \mathbb{E}_t r_{t+1}^L + (1-\kappa) \phi_\pi \left(p_t - p_{t-1}\right) - p_t - c_t, \\
\text{return on the long-term rate:} \quad & r_t^L = \frac{\delta}{\overline{R^L}} p_t^m - p_{t-1}^m,
\end{aligned} \tag{2.A.11}$$

where $\lambda = \kappa_p \left(\sigma + \phi\right)$ and $\kappa = \frac{\overline{R^L}}{\overline{R^L} - \overline{R^S}}$.

2.A.4 Solving the system of equations and proofs of proposition

We solve a simplified equilibrium system with the workhorse method by Blanchard and Kahn (1980)[21]. We assume the size of the volatility of the primary surplus as $\sigma_s = 1$ for the sake of simplicity. In a matrix form, we have:

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\delta}{\overline{R^L}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\kappa \\ 0 & 0 & 0 & 0 & 1 & \sigma & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 \end{pmatrix}}_{\equiv A} \begin{pmatrix} p_t \\ c_t \\ p_t^m \\ \tilde{b}_t \\ \mathbb{E}_t p_{t+1} \\ \mathbb{E}_t c_{t+1} \\ \mathbb{E}_t r_{t+1}^L \end{pmatrix}$$
$$= \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ b_{ss}\overline{R^L} & b_{ss}\overline{R^L} & 0 & \overline{R^L} - \phi_b & -b_{ss}\overline{R^L} & -b_{ss}\overline{R^L} & b_{ss}\overline{R^L} \\ -\left(1-\kappa\right)\phi_\pi & 0 & 0 & 0 & \left(1-\kappa\right)\phi_\pi - 1 & -1 & 0 \\ -\phi_\pi & 0 & 0 & 0 & \phi_\pi + 1 & \sigma & 0 \\ -1 & 0 & 0 & 0 & 1+\beta & -\lambda & 0 \end{pmatrix}}_{\equiv B} \begin{pmatrix} p_{t-1} \\ c_{t-1} \\ p_{t-1}^m \\ \tilde{b}_{t-1} \\ p_t \\ c_t \\ r_t^L \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\equiv C} \varepsilon_t^s. \tag{2.A.12}$$

Note that $\tilde{b}$ is the state variable in this model. This corresponds the following usual matrix representation.

$$A \begin{bmatrix} x' \\ \mathbb{E}_t y' \end{bmatrix} = B \begin{bmatrix} x \\ y \end{bmatrix} + Cv', \tag{2.A.13}$$

[21] When proceeding this section, we follow the lecture note by Nakajima (2007).

where we define $x \equiv \left[p_{t-1}, c_{t-1}, p_{t-1}^m, \tilde{b}_{t-1}\right]$ and $y \equiv \left[p_t, c_t, r_t^L\right]$. Then p_t and c_t have one period lag-term. To proceed the following procedure, we have to assume A is non-singular. Multiplying A^{-1} from the left, we have:

$$
\begin{aligned}
&\begin{pmatrix} p_t \\ c_t \\ p_t^m \\ \tilde{b}_t \\ \mathbb{E}_t p_{t+1} \\ \mathbb{E}_t c_{t+1} \\ \mathbb{E}_t r_{t+1}^L \end{pmatrix} \\
&= \underbrace{\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & \frac{\overline{R^L}}{\delta} & 0 & 0 & 0 & \frac{\overline{R^L}}{\delta} \\
b_{ss}\overline{R^L} & b_{ss}\overline{R^L} & 0 & \overline{R^L} - \phi_b & -b_{ss}\overline{R^L} & -b_{ss}\overline{R^L} & b_{ss}\overline{R^L} \\
-\frac{1}{\beta} & 0 & 0 & 0 & 1 + \frac{1}{\beta} & -\frac{\lambda}{\beta} & 0 \\
\frac{1-\beta\phi_\pi}{\beta\sigma} & 0 & 0 & 0 & \frac{1+\phi_\pi}{\sigma} - \frac{1+\beta}{\beta\sigma} & 1 + \frac{\lambda}{\beta\sigma} & 0 \\
-\left(1 - \frac{1}{\kappa}\right)\phi_\pi + \frac{b_{ss}\overline{R^L}}{\kappa} & \frac{b_{ss}\overline{R^L}}{\kappa} & 0 & \frac{\overline{R^L} - \phi_b}{\kappa} & \left(1 - \frac{1}{\kappa}\right)\phi_\pi + \frac{1 - b_{ss}\overline{R^L}}{\kappa} & \frac{1 - b_{ss}\overline{R^L}}{\kappa} & \frac{b_{ss}\overline{R^L}}{\kappa}
\end{pmatrix}}_{A^{-1}B}
\begin{pmatrix} p_{t-1} \\ c_{t-1} \\ p_{t-1}^m \\ \tilde{b}_{t-1} \\ p_t \\ c_t \\ r_t^L \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \frac{1}{\kappa} \end{pmatrix} \varepsilon_t^s.
\end{aligned}
\tag{2.A.14}
$$

We want to have the following expression by applying Jordan decomposition to the matrix $A^{-1}B$.

$$
\begin{bmatrix} x' \\ \mathbb{E}_t y' \end{bmatrix} = H \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} H^{-1} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} v'. \tag{2.A.15}
$$

Firstly we compute the eigenvalues for $A^{-1}B$. After completing an algebra, we obtain the eigenvalues for $A^{-1}B$,

$$\begin{vmatrix} -x & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -x & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{\overline{R^L}}{\delta} - x & 0 & 0 & 0 & \frac{\overline{R^L}}{\delta} \\ b_{ss}\overline{R^L} & b_{ss}\overline{R^L} & 0 & \overline{R^L} - \phi_b - x & -b_{ss}\overline{R^L} & -b_{ss}\overline{R^L} & b_{ss}\overline{R^L} \\ -\frac{1}{\beta} & 0 & 0 & 0 & 1 + \frac{1}{\beta} - x & -\frac{\lambda}{\beta} & 0 \\ \frac{1-\beta\phi_\pi}{\beta\sigma} & 0 & 0 & 0 & \frac{1+\phi_\pi}{\sigma} - \frac{1+\beta}{\beta\sigma} & 1 + \frac{\lambda}{\beta\sigma} - x & 0 \\ -\left(1 - \frac{1}{\kappa}\right)\phi_\pi + \frac{b_{ss}\overline{R^L}}{\kappa} & \frac{b_{ss}\overline{R^L}}{\kappa} & 0 & \frac{\overline{R^L} - \phi_b}{\kappa} & \left(1 - \frac{1}{\kappa}\right)\phi_\pi + \frac{1 - b_{ss}\overline{R^L}}{\kappa} & \frac{1 - b_{ss}\overline{R^L}}{\kappa} & \frac{b_{ss}\overline{R^L}}{\kappa} - x \end{vmatrix}$$
$$= (x - e_1)(x - e_2)(x - e_3)(x - e_4)(x - e_5)(x - e_6)(x - e_7), \tag{2.A.16}$$

where

$$\begin{aligned} e_1 &= 0 \\ e_2 &= 0 \\ e_3 &= 1 \\ e_4 &= \frac{1}{2\beta}\left(\beta + \frac{\lambda}{\sigma} + 1 - \sqrt{\left(\beta + \frac{\lambda}{\sigma} + 1\right)^2 - 4\beta\left(1 + \frac{\lambda}{\sigma}\phi_\pi\right)}\right) \\ e_5 &= \overline{R^L} - \phi_b + \frac{b_{ss}\overline{R^L}}{\kappa} = \overline{R^L} - \phi_b + b_{ss}\left(\overline{R^L} - \overline{R^S}\right) \\ e_6 &= \frac{1}{2\beta}\left(\beta + \frac{\lambda}{\sigma} + 1 + \sqrt{\left(\beta + \frac{\lambda}{\sigma} + 1\right)^2 - 4\beta\left(1 + \frac{\lambda}{\sigma}\phi_\pi\right)}\right) \\ e_7 &= \frac{\overline{R^L}}{\delta}. \end{aligned}$$

Proof of Lemma 1. In the model economy equilibrium system, we have 3 jump variables, $\{p_t, c_t, r_t^L\}$. Let the number of eigenvalues outside the unit circle as h here. (i) If we have 3 eigenvalues outside the unit circle, i.e., $h = 3$, the solution of the system is unique. (ii) If $h < 3$, there is an infinity of solutions (indeterminacy). (iii) If $h > 3$, there is no solution to the system.

We assume the long-term rate is greater than 1, $\overline{R^L} > 1$, and the maturity rate is less than 1, $\delta < 1$, then we have $e_7 > 1$. We will see the conditions if e_4, e_5, and e_6 are greater or smaller than 1.

e_4 **and** e_6**:** In the following we assume that the inside of the root is positive, $(\beta + \frac{\lambda}{\sigma} + 1)^2 - 4\beta(1 + \frac{\lambda}{\sigma}\phi_\pi) > 0$. Given this assumption, we check the conditions, which $e_4 > 1$ and $e_6 > 1$.

$$\frac{1}{2\beta}\left(\beta+\frac{\lambda}{\sigma}+1\pm\sqrt{\left(\beta+\frac{\lambda}{\sigma}+1\right)^2-4\beta\left(1+\frac{\lambda}{\sigma}\phi_\pi\right)}\right)>1$$
$$\Leftrightarrow \quad 1-\beta+\frac{\lambda}{\sigma}>\mp\sqrt{\left(1-\beta+\frac{\lambda}{\sigma}\right)^2+4\beta\frac{\lambda}{\sigma}\left(1-\phi_\pi\right)} \tag{2.A.17}$$

Since the left hand side is positive, we have $e_6 > 1$ no matter what value of ϕ_π. As for e_4,

$$e_4 > 1$$
$$\Leftrightarrow \quad 1-\beta+\frac{\lambda}{\sigma}>\sqrt{\left(1-\beta+\frac{\lambda}{\sigma}\right)^2+4\beta\frac{\lambda}{\sigma}\left(1-\phi_\pi\right)} \tag{2.A.18}$$
$$\Leftrightarrow \quad \phi_\pi > 1.$$

e_5**:** The condition for e_5 to be greater than 1 is:

$$e_5 > 1$$
$$\Leftrightarrow \quad \overline{R^L}-\phi_b+b_{ss}\left(\overline{R^L}-\overline{R^S}\right)>1 \tag{2.A.19}$$
$$\Leftrightarrow \quad \phi_b<\overline{R^L}-1+b_{ss}\left(\overline{R^L}-\overline{R^S}\right).$$

The right hand side is defined as ϕ_b^{def} in the lemma. This completes the proof of the lemma. ■

Now we obtain the eigenvalues for $A^{-1}B\left(=HJH^{-1}\right)$. The eigenvectors, H^{-1} takes the form:

$$H^{-1}\equiv\begin{bmatrix}\hat{H}_{11}\,(4\times4) & \hat{H}_{12}\,(4\times3)\\ \hat{H}_{21}\,(3\times4) & \hat{H}_{22}\,(3\times3)\end{bmatrix}=\begin{pmatrix}1-q_{15} & q_{12} & 0 & 1 & q_{15} & q_{16} & -\kappa\\ 1-q_{25} & q_{22} & 0 & 1 & q_{25} & q_{26} & -\kappa\\ 1 & 0 & 0 & 0 & 0 & 0 & 0\\ -1 & 0 & 0 & 0 & 1 & \sigma\frac{1-\beta e_4}{1-\beta\phi_\pi} & 0\\ -\frac{1}{(1-e_5)\kappa}-q_{55} & 1 & 0 & \frac{e_5}{b_{ss}\overline{R^L}}-\frac{1}{\kappa} & q_{55} & q_{56} & 1\\ -1 & 0 & 0 & 0 & 1 & \sigma\frac{1-\beta e_6}{1-\beta\phi_\pi} & 0\\ q_{71} & q_{77}-1 & 1 & q_{74} & q_{75} & q_{76} & q_{77}\end{pmatrix}. \tag{2.A.20}$$

For later use, we note each component of $\hat{H}_{22}$. These are:

$$
\begin{aligned}
\begin{pmatrix} q_{55} \\ q_{56} \end{pmatrix} &= \frac{1}{(e_5 - e_6)(e_5 - e_4)} \begin{pmatrix} 1 + \frac{\lambda}{\beta\sigma} - e_5 & -\frac{\phi_\pi}{\sigma} + \frac{1}{\beta\sigma} \\ \frac{\lambda}{\beta} & \frac{1}{\beta} - e_5 \end{pmatrix} \begin{pmatrix} e_5 + \frac{e_5}{(1-e_5)}\frac{1}{\kappa} - \left(1 - \frac{1}{\kappa}\right)\phi_\pi \\ e_5 - \frac{1}{\kappa} - 1 \end{pmatrix}, \\
\begin{pmatrix} q_{75} \\ q_{76} \end{pmatrix} &= \frac{1}{(e_7 - e_4)(e_7 - e_6)} \begin{pmatrix} 1 + \frac{\lambda}{\beta\sigma} - e_7 & -\frac{\phi_\pi}{\sigma} + \frac{1}{\beta\sigma} \\ \frac{\lambda}{\beta} & \frac{1}{\beta} - e_7 \end{pmatrix} \left[\begin{pmatrix} e_7 + \frac{e_7}{(1-e_7)}\frac{1}{\kappa} - \left(1 - \frac{1}{\kappa}\right)\phi_\pi \\ e_7 - \frac{1}{\kappa} - 1 \end{pmatrix} q_{77} + \begin{pmatrix} -e_7 \\ -e_7 + 1 \end{pmatrix} \right], \\
\begin{pmatrix} q_{77} \\ q_{74} \end{pmatrix} &= \frac{-1}{e_7 - e_5} \begin{pmatrix} \overline{R^L} - \phi_b - e_7 \\ -\frac{\overline{R^L} - \phi_b}{\kappa} \end{pmatrix}.
\end{aligned}
\tag{2.A.21}
$$

Remembering equation (2.A.15), by multiplying H^{-1} from the left for both sides, we have:

$$
H^{-1} \begin{bmatrix} x' \\ \mathbb{E}y' \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} H^{-1} \begin{bmatrix} x \\ y \end{bmatrix} + H^{-1} \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} v'. \tag{2.A.22}
$$

Focusing on the unstable part of the system, we obtain the formula for y as the following:

$$
y = -\hat{H}_{22}^{-1}\hat{H}_{21}x - \hat{H}_{22}^{-1}J_2^{-1}\left[\hat{H}_{21}G_1 + \hat{H}_{22}G_2\right]v', \tag{2.A.23}
$$

where

$$
\begin{aligned}
\hat{H}_{22}^{-1}J_2^{-1} &= \begin{pmatrix} q_{55} & q_{56} & 1 \\ 1 & \sigma\frac{1-\beta e_6}{1-\beta\phi_\pi} & 0 \\ q_{75} & q_{76} & q_{77} \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{e_5} & 0 & 0 \\ 0 & \frac{1}{e_6} & 0 \\ 0 & 0 & \frac{1}{e_7} \end{pmatrix}, \\
\hat{H}_{21}G_1 + \hat{H}_{22}G_2 &= \begin{pmatrix} \frac{e_5}{b_{ss}\overline{R^L}} \\ 0 \\ \frac{1}{\kappa}\frac{e_7}{e_7 - e_5} \end{pmatrix}.
\end{aligned}
\tag{2.A.24}
$$

Now, as we define the inflationary region and deflationary region as 2, our interest is a response of y given the shock v. Ignoring the terms with x, we only see the terms pertaining to the shock:

$$
\begin{aligned}
&-\hat{H}_{22}^{-1}J_2^{-1}\left[\hat{H}_{21}G_1 + \hat{H}_{22}G_2\right] \\
&= -\frac{1}{q_{55}q_{66}q_{77} + q_{76} - q_{66}q_{75} - q_{56}q_{77}} \begin{bmatrix} q_{66}q_{77} & -(q_{56}q_{77} - q_{76}) & -q_{66} \\ -q_{77} & q_{55}q_{77} - q_{75} & 1 \\ q_{76} - q_{66}q_{75} & -(q_{55}q_{76} - q_{56}q_{75}) & q_{55}q_{66} - q_{56} \end{bmatrix} \begin{pmatrix} \frac{1}{b_{ss}\overline{R^L}} \\ 0 \\ \frac{1}{\kappa}\frac{1}{e_7 - e_5} \end{pmatrix}
\end{aligned}
\tag{2.A.25}
$$

The initial response of price level, p, is the 1st element of $-\hat{H}_{22}^{-1} J_2^{-1}\left[\hat{H}_{21} G_1+\hat{H}_{22} G_2\right]$. We define this as the function of ϕ_π and ϕ_b, $F\left(\phi_\pi, \phi_b\right)$. That is:

$$F\left(\phi_\pi, \phi_b\right) \equiv-\frac{\frac{q_{66} q_{77}}{b_{ss} \overline{R^L}}-\frac{q_{66}}{\kappa\left(e_7-e_5\right)}}{\left(q_{55} q_{66}-q_{56}\right) q_{77}-\left(q_{75} q_{66}-q_{76}\right)}. \tag{2.A.26}$$

The numerator is:

$$\frac{q_{66} q_{77}}{b_{ss} \overline{R^L}}-\frac{q_{66}}{\kappa\left(e_7-e_5\right)}=\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi} \frac{1}{b_{ss} \overline{R^L}}. \tag{2.A.27}$$

where

$$\begin{aligned}
q_{55} q_{66}-q_{56} &= \frac{1}{\left(e_5-e_6\right)\left(e_5-e_4\right)}\left[\left[\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi}\left(1-e_5\right)-\frac{\lambda}{\beta}\left(1-\frac{1-\beta e_6}{1-\beta \phi_\pi}\right)\right] A_5+\left[e_5-e_6\right] B_5\right], \\
q_{75} q_{66}-q_{76} &= \frac{1}{\left(e_7-e_6\right)\left(e_7-e_4\right)}\left[\left[\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi}\left(1-e_7\right)-\frac{\lambda}{\beta}\left(1-\frac{1-\beta e_6}{1-\beta \phi_\pi}\right)\right]\left(A_7 q_{77}-e_7\right)+\left[e_7-e_6\right]\left(B_7 q_{77}-e_7+1\right)\right].
\end{aligned} \tag{2.A.28}$$

This gives us the following proposition.

Proposition 3. *(i) Equilibrium is deflationary when* $\{\phi_\pi < 1,\ \phi_b < \phi_b^{def},\ F(\phi_\pi, \phi_b) < 0\}$*.* *(ii) Equilibrium is inflationary when* $\{\phi_\pi < 1,\ \phi_b < \phi_b^{def},\ F(\phi_\pi, \phi_b) > 0\}$*.*

$$
\begin{aligned}
F(\phi_\pi, \phi_b) &= -\frac{\sigma \frac{1-\beta e_6}{1-\beta \phi_\pi} \frac{1}{b_{ss}\overline{R^L}}}{(q_{55}q_{66} - q_{56})\, q_{77} - (q_{75}q_{66} - q_{76})}, \\
\textit{where} \quad q_{55}q_{66} - q_{56} &= \frac{\left[\sigma \frac{1-\beta e_6}{1-\beta\phi_\pi}(1-e_5) - \frac{\lambda}{\beta}\left(1 - \frac{1-\beta e_6}{1-\beta\phi_\pi}\right)\right] A_5 + (e_5 - e_6)\, B_5}{(e_5 - e_6)(e_5 - e_4)}, \\
q_{75}q_{66} - q_{76} &= \frac{\left[\sigma \frac{1-\beta e_6}{1-\beta\phi_\pi}(1-e_7) - \frac{\lambda}{\beta}\left(1 - \frac{1-\beta e_6}{1-\beta\phi_\pi}\right)\right] (A_7 q_{77} - e_7) + (e_7 - e_6)(B_7 q_{77} - e_7 + 1)}{(e_7 - e_6)(e_7 - e_4)}, \\
q_{77} &= -\frac{\overline{R^L} - \phi_b - e_7}{e_7 - e_5}, \\
e_{4,6} &= \frac{1}{2\beta}\left(\beta + \frac{\lambda}{\sigma} + 1 \mp \sqrt{\left(\beta + \frac{\lambda}{\sigma} + 1\right)^2 - 4\beta\left(1 + \frac{\lambda}{\sigma}\phi_\pi\right)}\right), \\
e_5 &= \overline{R^L} - \phi_b + b_{ss}\left(\overline{R^L} - \overline{R^S}\right), \quad e_7 = \frac{\overline{R^L}}{\delta}, \\
A_i &= e_i + \frac{e_i}{(1-e_i)}\frac{1}{\kappa} - \left(1 - \frac{1}{\kappa}\right)\phi_\pi, \quad B_i = e_i - \frac{1}{\kappa} - 1, \quad \textit{for } i = 5, 7, \\
\lambda &= \kappa_p(\sigma + \phi) \ \textit{and} \ \kappa = \frac{\overline{R^L}}{\overline{R^L} - \overline{R^S}}.
\end{aligned}
$$

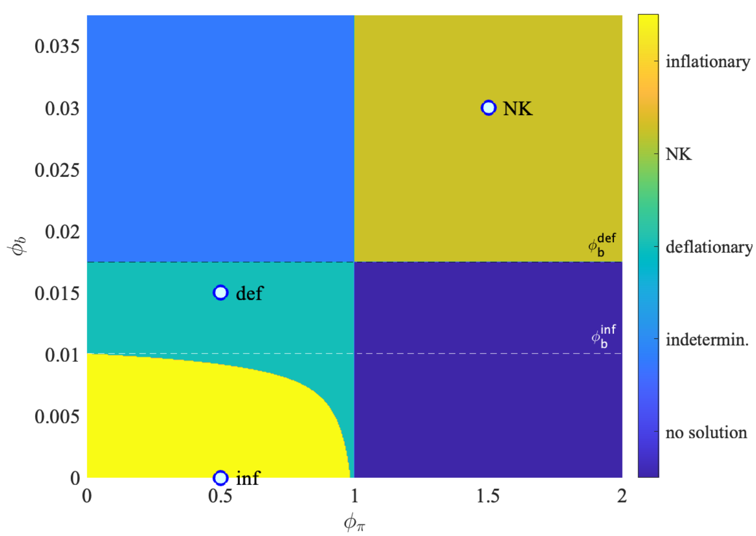

Figure 2.A.1: Determinacy regions with fiscal-led inflationary region and deflationary region under partial nominal price rigidity

Proof of Proposition 2 (fully rigid price case). In the following, we consider the special case, in which nominal price is rigid perfectly, $\lambda \to +0$. We consider the condition

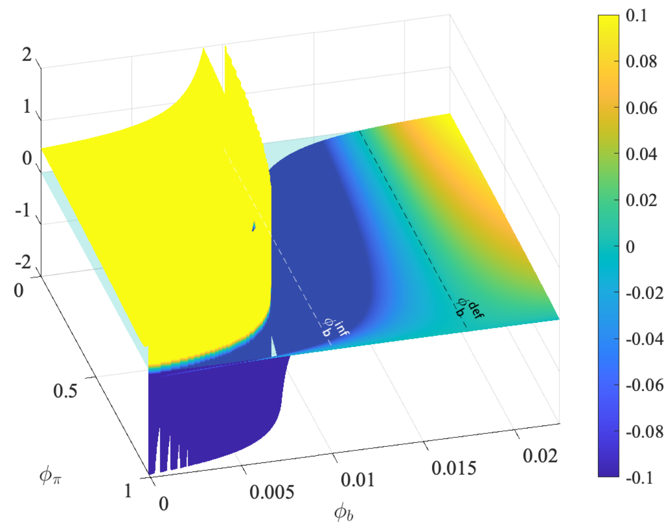

Figure 2.A.2: $F(\phi_\pi, \phi_b)$

for the deflationary region, in which the initial response of price given the positive primary surplus shock. From the general case, the corresponding condition is:

$$F(\phi_\pi, \phi_b) < 0 \quad \Leftrightarrow \quad -\frac{\sigma \frac{1-\beta e_6}{1-\beta\phi_\pi} \frac{1}{b_{ss}\overline{R^L}}}{(q_{55}q_{66} - q_{56})\, q_{77} - (q_{75}q_{66} - q_{76})} < 0. \tag{2.A.29}$$

When price rigidity is perfect, $\lim_{\lambda\to+0} e_4 = 1 - 0$ and $\lim_{\lambda\to+0} e_6 = \frac{1}{\beta} + 0$. Given these and $\phi_\pi < 1$, we want the condition,

$$(q_{55}q_{66} - q_{56})\, q_{77} - (q_{75}q_{66} - q_{76}) < 0. \tag{2.A.30}$$

Each component in the bracket can be simplifies as:

$$\begin{aligned} \lim_{\lambda\to+0} \{q_{55}q_{66} - q_{56}\} &= \frac{\left[\sigma \frac{1-\beta e_6}{1-\beta\phi_\pi}(1-e_5) - \frac{\lambda}{\beta}\left(1 - \frac{1-\beta e_6}{1-\beta\phi_\pi}\right)\right] A_5 + (e_5 - e_6) B_5}{(e_5 - e_6)(e_5 - e_4)} = \frac{B_5}{e_5 - e_4}, \\ \lim_{\lambda\to+0} \{q_{75}q_{66} - q_{76}\} &= \frac{\left[\sigma \frac{1-\beta e_6}{1-\beta\phi_\pi}(1-e_7) - \frac{\lambda}{\beta}\left(1 - \frac{1-\beta e_6}{1-\beta\phi_\pi}\right)\right](A_7 q_{77} - e_7) + (e_7 - e_6)(B_7 q_{77} - e_7 + 1)}{(e_7 - e_6)(e_7 - e_4)} = \frac{B_7 q_{77} - e_7 + 1}{e_7 - e_4}. \end{aligned} \tag{2.A.31}$$

Then, noting that $e_5 > e_4$, $e_7 > e_4$, $B_i = e_i - \frac{1}{\kappa} - 1$, $q_{77} = -\frac{\overline{R^L} - \phi_b - e_7}{e_7 - e_5}$, and $\kappa = \frac{\overline{R^L}}{\overline{R^L} - \overline{R^S}}$, we compute the requiring condition as:

$$\begin{aligned}
& (q_{55}q_{66} - q_{56})\, q_{77} - (q_{75}q_{66} - q_{76}) < 0 \\
\Leftrightarrow \quad & \frac{(e_7 - e_4)\, B_5 q_{77} - (e_5 - e_4)\,(B_7 q_{77} - e_7 + 1)}{(e_5 - e_4)\,(e_7 - e_4)} < 0 \\
\Leftrightarrow \quad & (e_7 - e_4)\, B_5 q_{77} - (e_5 - e_4)\,(B_7 q_{77} - e_7 + 1) < 0 \\
\Leftrightarrow \quad & \phi_b > \frac{\overline{R^L} - \delta}{\overline{R^L} - \delta \frac{\overline{R^S}}{\overline{R^L}}} \phi_b^{def} - \frac{(1-\delta)\left(\overline{R^L} - \overline{R^S}\right)}{\overline{R^L} - \delta \frac{\overline{R^S}}{\overline{R^L}}}.
\end{aligned} \tag{2.A.32}$$

Defining the right side as ϕ_b^{inf}, we complete the proof of Proposition 2. ■

2.B Negative Supply Shock

In this section, we consider the IRFs of the model for the remained shock, a negative productivity shock. Figure 2.B.1 shows the IRFs for -1% productivity shock, which corresponds to negative supply shock.

A negative supply shock causes the price of final goods to rise. Note that nominal GDP is rising at this time because an increase in the price level is greater than a decrease in the real GDP. The monetary policy authority responds to a price increase by raising short-term interest rates. Since an increase in short-term interest rates implies a reduction in financial intermediaries' portfolios to long-term bonds, the price of long-term bonds falls. The level of real debt (debt-to-GDP ratio) is lowered by an increase in nominal GDP and a fall in the price of long-term debt. This mechanism is common to all regimes, but since the NK regime is the most sensitive to price increases, the rise in short-term interest rates, the decline in the price of long-term bonds, and the decline in real debt are also greater.

2.C Mapping to the Data

In this section, we examine the relationship between fiscal stance and inflation using data from OECD member countries since the Great Financial Crisis (henceforth, GFC). The

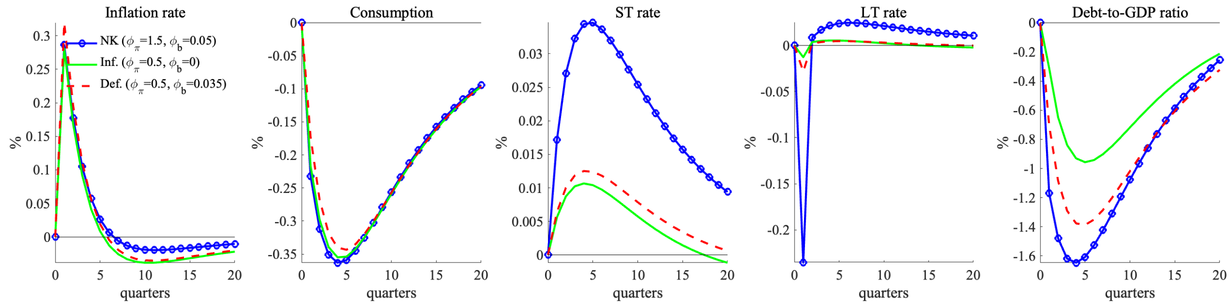

Figure 2.B.1: IRFs to -1% productivity shock

model in this paper points to the possibility that deflationary pressures may arise from fiscal deficit shocks even under a regime of fiscal dominance. Therefore, we will use data focused on the period after the financial crisis, when deflationary pressures began to be pointed out globally. In order to ascertain the extent to which the model in this paper is consistent with actual data on fiscal spending and inflation, it is necessary to estimate both monetary and fiscal policy parameters. However, as seen in the past literature, it is technically difficult to estimate a macroeconomic model through such a model with a combination of monetary and fiscal policy. This is due to the fact that in the real world, these policy parameters vary over time as regimes change.

The data are estimated for 35 OECD countries, excluding those with many missing data (Columbia, Estonia, Turkey), for the period 2007-2020[22]. In conducting our analysis, we also consider countries in the EURO currency area separately from the rest of the world. This is because the analysis that follows identifies clear differences between EURO-using and non-EURO-using countries.

First, the theoretical thresholds, ϕ_b^{def} and ϕ_b^{inf}, are measured based on the following equations, respectively.

$$\phi_b^{def} = \overline{R^L} - 1 + b_{ss}\left(\overline{R^L} - \overline{R^S}\right)$$

$$\phi_b^{inf} = \frac{\overline{R^L} - \delta}{\overline{R^L} - \delta \frac{\overline{R^S}}{\overline{R^L}}}\phi_b^{def} - \frac{(1-\delta)\left(\overline{R^L} - \overline{R^S}\right)}{\overline{R^L} - \delta \frac{\overline{R^S}}{\overline{R^L}}}$$

[22]The details of the data are summarized at the end of this section.

For the term of the long-term interest rate, we set $\delta = 0.95$ to match the 5-year maturity average of the U.S. and applied it to all countries. The results are shown in Table 2.C.1 and Figure 2.C.1. It is interesting to note that for countries other than EURO that issue their own currency, ϕ_b^{def} and ϕ_b^{inf} are positively correlated with the average of inflation, whereas for the EURO currency area, they are inversely negatively correlated. In non-EURO countries, inflation tends to be higher on average in countries with higher long-term interest rates and higher government debt. In the EURO currency area, however, inflation rates tend to be lower for countries with larger spreads between long- and short-term interest rates and higher government debt, as seen in Greece, Italy, and Portugal. These background considerations are summarized at the end of this section.

Next, we perform the estimation for $\hat{\phi}_b$; the results are shown in Figure 2.C.2. Although most of the results of each estimation are insignificant due to the small sample period of the data, the trends among individual countries show that for non-EURO countries, inflation rates are lower for countries with more fiscal austerity. On the other hand, in the EURO currency area, the more fiscally austere countries tended to have higher inflation rates.

$$\tilde{s}_t = \phi_b \tilde{b}_{t-1} + \epsilon$$

Finally, to ascertain whether budget deficits are inflationary or deflationary, we examine the relationship between inflation and $\hat{\phi}_b - \phi_b^{inf}$. The results are consistent with the model's prediction to some extent for all countries except for the EURO countries. That is, even in fiscally-led regimes, inflation rates are lower in countries with fiscal stances tighter than ϕ_b^{inf}, the threshold for not becoming inflationary, than in countries without such stances. This is consistent with the model's prediction, given that fiscal deficits are negative in most OECD countries. In the EURO currency area, on the other hand, the exact opposite argument applies. That is, in countries such as Greece, Italy, and Portugal, which have chronic fiscal deficits, inflation rates are low even though their fiscal stance is not adequate relative to the threshold. This is the exact opposite of the model's prediction. A potential explanation for this is that these three countries are within the EURO area and their currencies are stable, which means that the rise in interest rates does not occur in tandem with the rise in prices, i.e., the government debt cannot be inflated away. The another potential reason is the belief of agents matters. While the current fiscal stance is insufficient, the households in each country believe the future government would try to improve the primary balance of the country. Thus, they would decrease consumption in response to fiscal deficit shock in order to prepare future tax increase.

However, this point needs to be scrutinized more closely with what happens when the model is extended to an open economy. This point should be the subject of future research. Moreover, it should be noted that the above discussion focuses on the post-crisis period and does not estimate monetary policy parameters. We would like to leave as a topic for future research an attempt to estimate more precisely fiscal stance and monetary policy parameters for individual countries with long-horizon.

Data set construction. The sources of the data used in the above data analysis are listed below, respectively.

- Debt to GDP ratio - 'Gross debt (D4) at market value' (IMF.Government Finance Statistics). If the IMF data is unavailable we use 'General government gross debt as a percentage of GDP' (OECD.stat).
- Primary Balance - 'Revenue' (IMF.Government Finance Statistics) minus 'Expense' (IMF.Government Finance Statistics).
- Long-term rate - 'Long-term interest rates, Percent per annum' (OECD.stat).
- Short-term rate - 'Short-term interest rates, Percent per annum' (OECD.stat).

OECD member countries are classified in the following manner.

- non-EURO currency countries - Australia, Canada, Chile, Costa Rica, Czech Republic, Denmark, Hungary, Iceland, Israel, Japan, Korea, Mexico, New Zealand, Norway, Poland, Sweden, Switzerland, United Kingdom, and United States.

- EURO currency countries - Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Portugal, Slovak Republic, Slovenia, and Spain.

- Countries not included in the data - Colombia, Estonia, and Turkey.

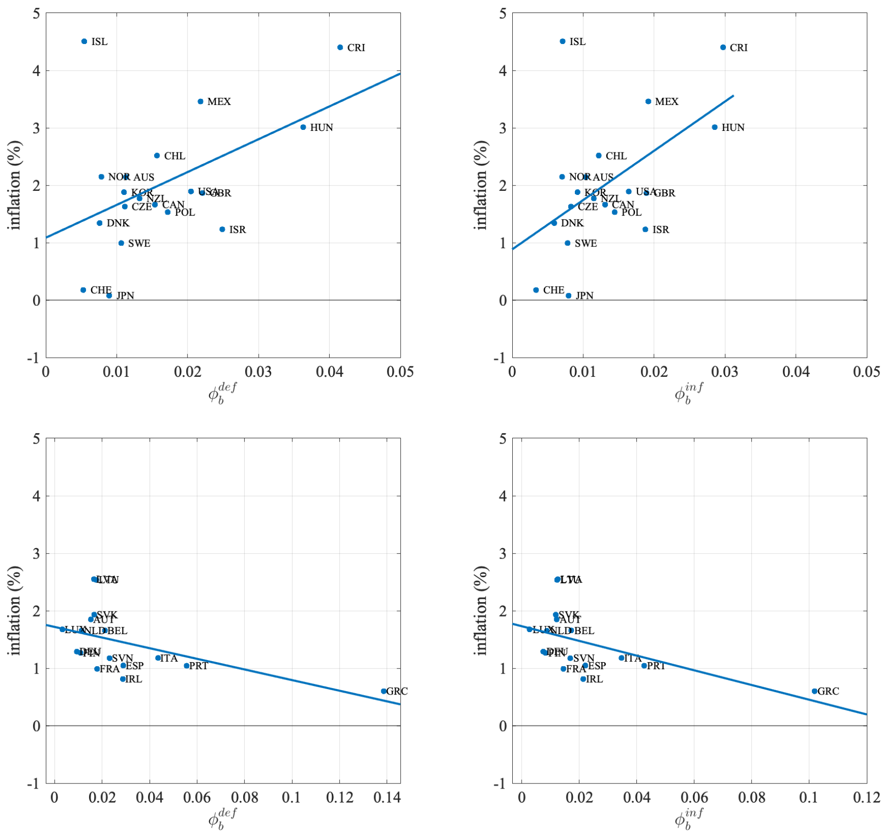

Figure 2.C.1: ϕ_b^{def} and ϕ_b^{inf} in non-EURO countries and EURO countries

Note.—The upper two panels show plots the theoretical thresholds, ϕ_b^{def} and ϕ_b^{inf}, and average inflation rate after the GFC for non-EURO countries. The lower two panels show that for EURO-countries.

Table 2.C.1: Estimation results and theoretical thresholds

Country	code	ϕ_b^{def}	ϕ_b^{inf}	$\hat{\phi}_b$	s.e.		s
Australia	AUS	0.011	0.010	-0.003	0.009		-0.011
Austria	AUT	0.015	0.012	0.018	0.015		-0.018
Belgium	BEL	0.021	0.017	0.010	0.015		-0.028
Canada	CAN	0.015	0.013	0.007	0.024		-0.009
Chile	CHL	0.016	0.012	-0.067	0.019	***	0.009
Costa Rica	CRI	0.042	0.030	-0.038	0.012	***	-0.021
Czech Republic	CZE	0.011	0.008	0.017	0.018		-0.017
Denmark	DNK	0.008	0.006	-0.025	0.022		0.006
Finland	FIN	0.011	0.008	-0.013	0.011		-0.006
France	FRA	0.018	0.015	0.004	0.008		-0.039
Germany	DEU	0.009	0.007	0.013	0.019		-0.004
Greece	GRC	0.139	0.102	0.026	0.008	***	-0.065
Hungary	HUN	0.036	0.029	0.039	0.014	**	-0.025
Iceland	ISL	0.005	0.007	-0.002	0.015		-0.009
Ireland	IRL	0.029	0.021	0.013	0.020		-0.055
Israel	ISR	0.025	0.019	-0.046	0.033		-0.031
Italy	ITA	0.044	0.035	-0.005	0.008		-0.034
Japan	JPN	0.009	0.008	0.015	0.007	**	-0.052
Korea	KOR	0.011	0.009	-0.024	0.011	*	0.050
Latvia	LVA	0.017	0.013	0.033	0.017	*	-0.020
Lithuania	LTU	0.018	0.012	0.030	0.018		-0.017
Luxembourg	LUX	0.003	0.003	-0.035	0.029		0.032
Mexico	MEX	0.022	0.019	0.007	0.007		-0.011
Netherlands	NLD	0.011	0.009	-0.001	0.019		-0.015
New Zealand	NZL	0.013	0.011	0.019	0.027		0.002
Norway	NOR	0.008	0.007	0.050	0.048		0.108
Poland	POL	0.017	0.014	0.044	0.019	**	-0.015
Portugal	PRT	0.056	0.043	0.012	0.007		-0.051
Slovak Republic	SVK	0.017	0.012	0.035	0.010	***	-0.028
Slovenia	SVN	0.023	0.017	0.009	0.011		-0.022
Spain	ESP	0.029	0.022	0.006	0.007		-0.055
Sweden	SWE	0.011	0.008	0.042	0.023	*	0.012
Switzerland	CHE	0.005	0.003	0.039	0.050		0.007
United Kingdom	GBR	0.022	0.019	0.011	0.012		-0.067
United States	USA	0.020	0.016	0.019	0.017		-0.070

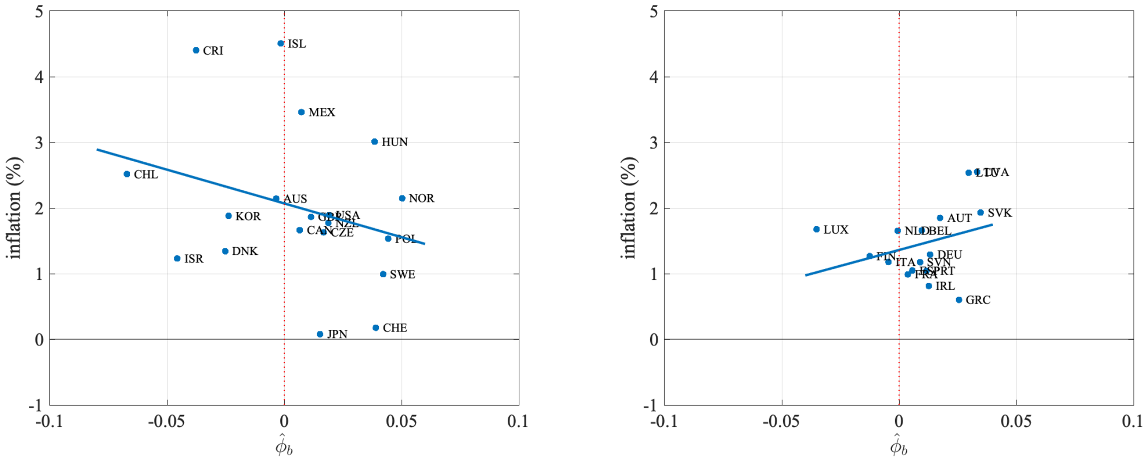

Figure 2.C.2: $\hat{\phi}_b$ in non-EURO countries and EURO countries

Note.—The left panel shows a plot for $\hat{\phi}_b$ and average inflation rates after the GFC for non-EURO countries. The right panel shows that for EURO-countries.

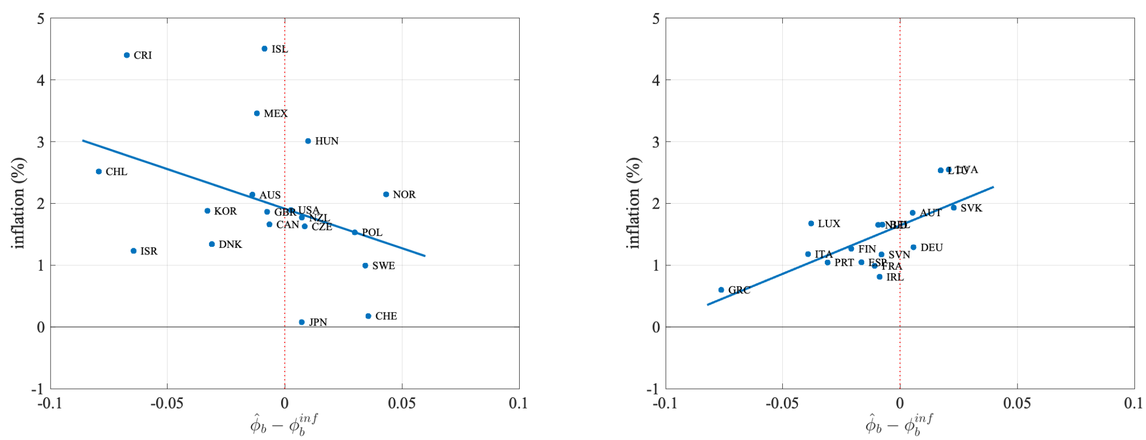

Figure 2.C.3: $\hat{\phi}_b - \phi_b^{inf}$ in non-EURO countries and EURO countries

Note.—The left panel shows a plot for $\hat{\phi}_b - \phi_b^{inf}$ and average inflation rates after the GFC for non-EURO countries. The right panel shows that for EURO-countries.

CHAPTER 3

Revisiting Exchange Rates and Equity Prices

3.1 Introduction

This paper revisits the interrelationship between exchange rates and stock prices. In particular, we review the existing literature on the difference between exchange rate volatility and stock price volatility, and the correlation of exchange rates with stock prices, and organize it in light of data from major countries since the beginning of the year 2000. This paper first presents three facts:

1. The market volatility of exchange rates is lower than the volatility of stock prices (Brandt et al. (2006)). Exchange rate volatility is about 15%, whereas stock return volatility is 40%.

2. Stock prices of G7 countries are positively correlated.

3. Exchange rates and stock prices are positively correlated in low-interest rate countries (stock prices rise as exchange rates depreciate). In high-interest rate countries, exchange rates and stock prices are negatively correlated (stock prices fall as exchange rates depreciate).

We present a two-country model that can show a mechanism consistent with the stylized facts. The model is a two-country model based on the market segmentation hypothesis of Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2019a). The main difference from these benchmark models is that it incorporates the market structure of Hau and Rey (2006), who studied equity capital flows and exchange rates. The model includes households that make deposits and financial intermediaries that use the deposits to invest in stocks of the two countries in the financial market. The financial intermediary adopts the Mean-Variance approach to maximize its profit per period (Campbell and Viceira (2002)). In the benchmark model, there are two risky assets, and the unique part is that the covariance between the two risky assets is determined endogenously given exogenous shocks.

To reproduce the econometric moments, including the above three facts and other macro variables, the model analysis proposes shocks to risk-taking in financial markets. We further assume that financial intermediaries' risk-taking is positively correlated between the two countries. This correlation is key in explaining the first and second facts. When financial intermediaries become risk averse, stock prices fall, and thus stock prices are positively correlated if risk-averse behavior occurs simultaneously. In addition, risk-averse behavior causes financial intermediaries to deleverage, which leads to an appreciation of the home currency and a decline in stock prices in both countries. Since this mechanism of deleveraging is common to both countries, if risk-averse behavior were to coincide, the exchange rates would cancel each other out, and stock prices would rise or fall more sharply.

Next, to express the third fact, we consider the different levels of safe interest rates at a steady state. The main intuition for the behavior of the model will be as follows. When there is an interest rate differential between one's own country and another, financial intermediaries in low-interest-rate countries go long the high-interest-rate currency and short the low-interest-rate currency in international financial markets (carry trade). On the other hand, in the domestic market, they invest in stocks. These are the positions of financial intermediaries in a steady state. Assuming that a risk-averse shock occurs here, the financial intermediary deleverages to unwind its carry position and stock position. As a result, an appreciation of the home currency and a decline in stock prices will co-occur. The behavior in high-interest-rate currency countries is in exactly the opposite direction. These behaviors suggest that not only the direction of the shock to interest rates but also the steady state level of interest rates is important when considering the spillover effects of monetary policy. The model in this paper has the potential to examine the differences in monetary policy shocks in a model with such an economic structure, i.e., the spillover effects of monetary policy on exchange rates and stock prices in low- and high-interest-rate countries.

The first of the specific quantitative findings, the enigma of exchange rate smoothness, has been variously examined by the literature beginning with Brandt et al. (2006). It finds that low exchange rate volatility and high stock return volatility are equivalent to marginal utility growth rate, implying a stochastic discount factor (SDF) correlation of at least 0.98 supposing a complete market condition. Lustig and Verdelhan (2019) relax the perfect span assumption and consider whether imperfect spans solve the core exchange rate puzzle, including low exchange rate volatility. However, another shortcoming remains: the puzzle of Backus and Smith (1993), the inability to solve the negative correlation between relative Stochastic Discount Factor (SDF) and exchange rate changes. Recently Chien et al. (2020) achieved highly correlated and volatile SDF and low correlation in consumption growth by partial entry into financial markets (infrequent portfolio choice). However, even their model still suffers from the Backus and Smith (1993) puzzle[1].

This paper also provides a comprehensive solution to the Backus-Smith puzzle, which is unresolved in the existing literature mentioned above. Our idea of risk-taking shocks mimics the role of financial liquidity shocks (Noise-trader shocks) in Itskhoki and Mukhin (2019a). Under the market segmentation hypothesis, the utility function of the financial intermediary is independent of the household's stochastic discount factor. Put differently, there are different stochastic discount factors held by households and financial intermediaries in the economy (incomplete market)[2]. Under this interpretation, when financial intermediaries' risk-taking is subject to an exogenous shock, the shock acts as a wedge of imperfect risk-sharing between households in the two countries.

[1] Along these lines, Fang and Liu (2021) develop a quantitative model of endogenous volatility of exchange rates and VaR constraints and provide related empirical support for a feedback loop between exchange rate volatility and the tightness of the leverage constraint. However, their model does not consider the volatility of stock prices or the correlation between stock prices and exchange rates.

[2] More strictly, financial intermediaries do not have a stochastic discount factor because they act to maximize their profit per period. However, following the recent macro-finance literature (see, for example, Greenwald et al. (2014)), we can interpret financial investors' risk aversion coefficient as their SDF. Under this, their SDF is supposed to increase when financial intermediaries become risk-averse.

Literature review. First, this paper contributes to the literature strand of cross-country portfolio selection in terms of analyzing the interconnection between exchange rates and stock prices (papers examining cross-border market stock portfolio selection include Hau and Rey (2006), Camanho et al. (2018), Bacchetta et al. (2020), among others). Also, Alvarez et al. (2009), Pavlova and Rigobon (2012), Gabaix and Maggiori (2015), and Maggiori (2017) are representative of the literature that explores portfolio flows in a setting that assumes market imperfection. Much of the literature consider models in which investors, who are also households, face an incomplete list of assets. However, this paper completely separates households from market investors. It considers an environment in which households rely on short-term rates determined by monetary policy to save but have no access to other assets[3]. Therefore, the financial intermediaries bear all the risk in the financial market since all portfolio choice in risky assets is done by the financial intermediary.

Shocks to the risk aversion parameter are an important feature of this paper. The recent decade has seen a growing literature examining the role of risk tolerance in financial markets; Greenwald et al. (2014) find that most of the short- and medium-term stock market volatility in historical data is driven by shocks to risk aversion. In other words, in the short run, shocks that affect the willingness to bear risk independent of macroeconomic fundamentals explain most of the market volatility. Gourinchas et al. (2010), in the context of international macro-finance, construct a model in which the U.S. is less risk averse than other countries, explaining the role of the U.S. as an insurer in bad times. In their model, the risk aversion parameter transitions by a Markov transition matrix, which calibrates the process of catastrophic events. In addition to them, several recent literatures investigate the role of market participants' risk sentiment in the context of financial and portfolio flows (e.g., Miranda-Agrippino and Rey (2015), Greenwald et al. (2019), Coeurdacier et al. (2019),

[3] Maggiori (2022) is excellent for a review of the market segmentation setting and international economics. Among the market segmentation hypothesis literature, Itskhoki and Mukhin (2019a) solves the major puzzles in international economics by applying a market segmentation setting. Gourinchas et al. (2020) and Greenwood et al. (2019) analyze the interrelationship between exchange rates and the term structure of government bonds. What this paper, Gabaix and Maggiori (2015), Itskhoki and Mukhin (2019a), and others have in common is that they use the demand asset pricing methodology, where the demand for each asset determines the price of the asset (Koijen and Yogo (2019b) and Koijen and Yogo (2019c)).

Giglio et al. (2019), and Berger et al. (2020))[4]. Also, some other literature, such as Dou and Verdelhan (2015), Pavlova and Rigobon (2012), and Fogli and Perri (2015), argue that the shock to risk aversion has a profound implication for the valuation effect.

Finally, the model must be solved around a stochastic steady state for portfolio choice in financial markets since it relies on mean-variance analysis. In this sense, this paper is also related to the literature on calculations around risky steady states following Coeurdacier et al. (2011) (see, for example, Gertler et al. (2012) and Basu and Bundick (2018)). It should be noted that the first order dynamics around the steady state are robust and unchanged even around the non-stochastic steady state, while we solve around the risk steady state for the sake of accuracy.

Structure. The rest of this paper is organized as follows. Section 3.2 reviews the data and the key moments. Section 3.3 develops the DSGE model which augment the financial sector model. Section 3.4 derives the quantitative implications of our model and thus the ability of incomplete spanning models to match the empirical moments of stock and exchange rate and their relation to the economic variables. Section 3.5 concludes and explore the future extensions.

3.2 Data on Exchange Rates and Stock Prices

In this section, we re-examine the relationship between stock prices and exchange rates. See Appendix 3.A for details about the data source and detailed decomposition of the data. While the main text emphasizes the relationship between exchange rate and stock price data, Appendix 3.A also confirms the second-order moments between macroeconomic variables

[4]In this paper, the risk premium occurs in the form of shocks to parameters of risk aversion. We cite Drechsler et al. (2018) and Fang and Liu (2021), which extends Drechsler et al. (2018) to the small open economy, as a model that endogenizes generating risk premium under monetary policy shock. The mechanism we discuss in this paper is similar to that of Fang and Liu (2021) but differs in that our model quantitatively matches Empirical moments, while Fang and Liu (2021) focuses on qualitative mechanisms. Models that examine larger disaster risk include Brunnermeier et al. (2008) (currency crash risk) and Farhi and Gabaix (2016) (rare disaster risk). Also, Cormun and De Leo (2020), in a form close to our model, read the noise trader shock in Itskhoki and Mukhin (2019a) as a shock to the risk-taking parameter and present evidence on the behavior of exchange rates and NFAs with VARs, their shocks have high explanatory power.

and stock prices. The first argument in this section is that exchange rate volatility is small compared to stock price volatility, and stock prices in each country are correlated with those in the United States. The second argument is that low interest rate countries experience an increase in stock prices along with a depreciation of exchange rates. In contrast, for high interest rate countries, a depreciation of exchange rates is accompanied by a decline in stock prices[5].

Volatility of exchange rate and stock price. The left and right graphs in Figure 3.2.1 show the quarterly stock price and exchange rate differences for each of the G7 countries, annualized, with standard errors for the ten quarters calculated as a moving average (Itskhoki and Mukhin (2019b)). The center graph shows the difference between U.S. stock prices and those of the rest of the G7 countries, and a moving average of the differences. First of all, for stock prices, the annualized volatility is around 40%, while the exchange rate volatility is roughly 15%, or about half. We note the following as known facts.

Stylized Fact 1. *Exchange rate volatility is lower than stock price volatility.*

This is not a new finding, as it has been material in previous studies, but we confirm it here because it is an important issue for this paper. See, for example, Brandt et al. (2006) and Chien et al. (2020), for previous studies on the relative smoothness of exchange rates.

Next, the middle graph of Figure 3.2.1 shows the differential volatility between the U.S. and the rest of the world for changes in stock prices. The standard deviations are averaged over ten quarters as in the other graphs. This volatility is small compared to the volatility of stock prices in a single country in the left panel. The implied fact is that the movements of the U.S. and other countries' stock prices are positively correlated. The same can be confirmed for combinations of non-U.S. countries.

Stylized Fact 2. *Stock prices of the G7 countries are positively correlated.*

The correlation of risk asset prices, not only among the G7 countries but also broader set of countries, is discussed in detail in Miranda-Agrippino and Rey (2021) in the context of the *Global Financial Cycle*. In particular, the following is one of the stylized facts discussed

[5]Note that the stock prices in this paper are based on the local currency of each country and are not converted to the U.S. dollar.

in Miranda-Agrippino and Rey (2021): *"One global factor in risky asset prices, correlated with global risk appetite, explains about a quarter of the variance of the data. quarter of the variance of the data".* In the models in the following sections, the fact that shocks to risk-taking are positively correlated across the two countries has the same implication as to the fact that risk appetites co-move across the world.

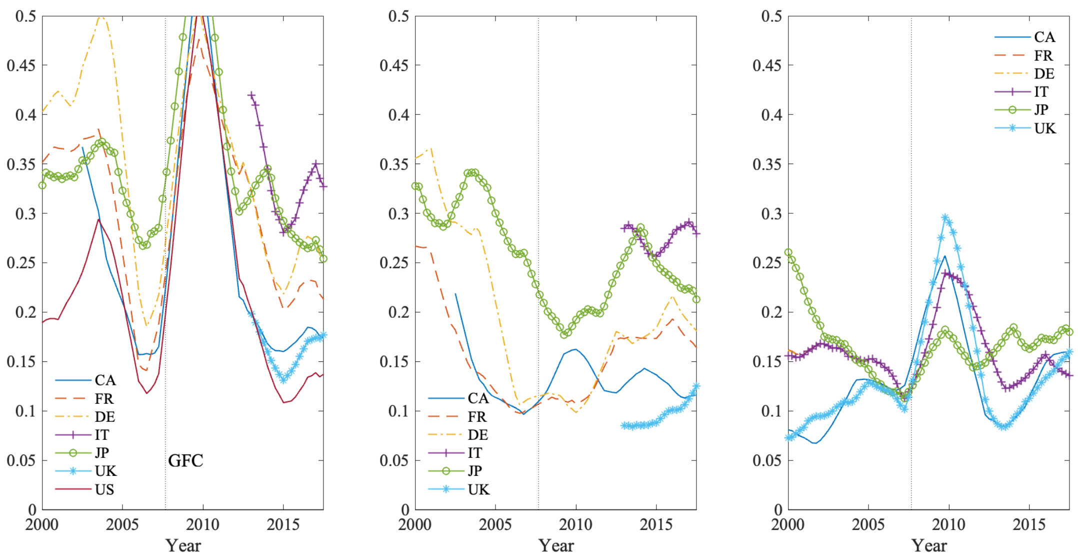

Figure 3.2.1: The left panel is a time series of $Std(\Delta p^{s}_{t+1})$ of G7 countries. The center panel is a time series of $Std(\Delta p^{s}_{t+1} - \Delta p^{s,us}_{t+1})$, the right panel is time series of $Std(\Delta e_{t+1})$

NOTE.—All time series are represented for moving average of std of 10 quarters window. The original data is monthly but annualized before computing the std and time period is from 2000:Q1 to 2018:Q4. The data for stock prices is basing on its local currency term.

Interest rate differentials and correlation between exchange rate and stock price. Next, we review the time series of correlations between exchange rates and stock prices for the G7 countries. Figure 3.2.2 plots the correlations between stock price differentials and exchange rate differentials over a 10-quarter Window from 2000 through 2018. These time series indicate that for EURO-using countries and Canada, the correlation between exchange rates and stock prices has become negatively correlated since the Great Financial Crisis. This means that in these countries, exchange rate depreciation and stock price declines coincide. It implies that when stock prices fall in EURO, which can be considered bad times for euro investors, the US dollar is highly demanded in the financial market. The rush to

US dollar phenomenon during bad times is well documented in the literature in *US dollar as safe currency* (see. e.g., Jiang et al. (2018), Maggiori et al. (2019), Bianchi et al. (2021) among others). In contrast, the correlation between exchange rates and stock prices has been positive in Japan. This indicates that the rise in stock prices rises along with the depreciation of the exchange rate. What explains these differences in the correlation between exchange rates and stock prices across countries? Below we present an analysis focusing on the interest rate differential with the US

The analysis is expanded from non-US G7 countries to 13 countries, including emerging economies, and the correlation between the interest rate differential with the US and the correlation between exchange rates and stock prices is plotted (Figure 3.2.3)[6]. The leftmost graph shows the plots for the entire period covered (2000-2018). The horizontal axis shows the correlation between the exchange rate and stock prices over the ten quarters as in Figure 3.2.2. The vertical axis represents the average interest rate differential with the US over the last ten quarters. Thus, countries in the upper part of the plot are those with high interest rates, such as emerging countries, while countries in the lower part of the plot are those with low interest rates, such as Switzerland and Japan. This confirms that the correlation between exchange rates and stock prices are negatively correlated with interest rate differentials. In other words, in high-interest-rate countries, the exchange rate depreciates and stock prices fall simultaneously, while in low-interest-rate countries, the exchange rate depreciates and stock prices rise simultaneously. To the best of the author's knowledge, this is not a well-known and stylized fact in the literature. And this is the last stylized fact:

Stylized Fact 3. *Countries with low interest rates tend to see their stock prices rise when their exchange rates depreciate. On the other hand, stock prices tend to fall for countries with high interest rates as the exchange rate depreciates.*

Note that the remaining three graphs in Figure 3.2.3 show scatter plots for subsamples with three different sample periods, but the trend still remains the same. Also, this negative

[6] Other countries than G7 countries are Australia, South Korea, New Zealand, Norway, Spain, Sweden, and Switzerland. In the Appendix, we present time series data on the correlation between exchange rates and stock prices for these additional countries. Also, the 13 countries are grouped into three groups according to the characteristics of their currencies. We show the scatter plots for each by dividing the whole period into sub-sample periods. For more detail, see Appendix 3.A.

correlation between the interest rate differential and the correlation between the exchange rate and stock prices would be observed unchanged even if the interest rate differential were changed from a 3-month maturity to a 10-year maturity. See Appendix 3.A for this result.

In the model part below, we present a model capable of representing these three stylized facts.

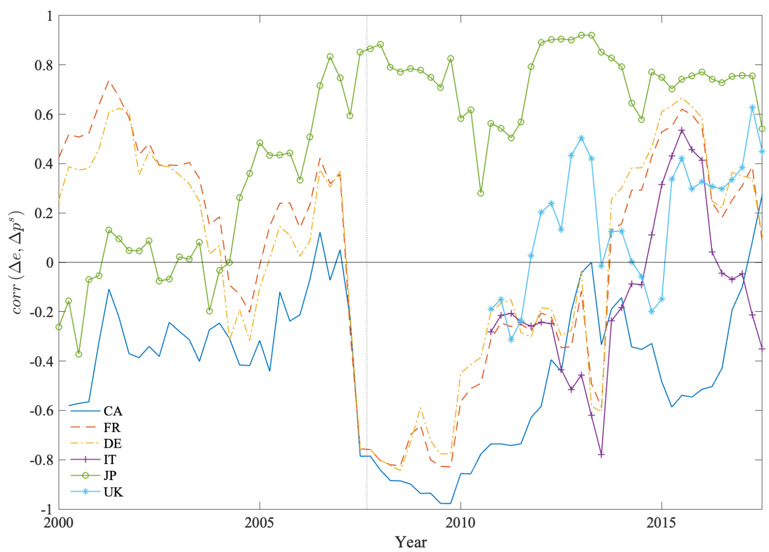

Figure 3.2.2: The time series of corr $(\Delta e, \Delta p^s)$

NOTE.—The data sample period is from 2000:Q1 to 2018:Q4. All time series are represented for moving average of std of 10 quarters back-ward window.

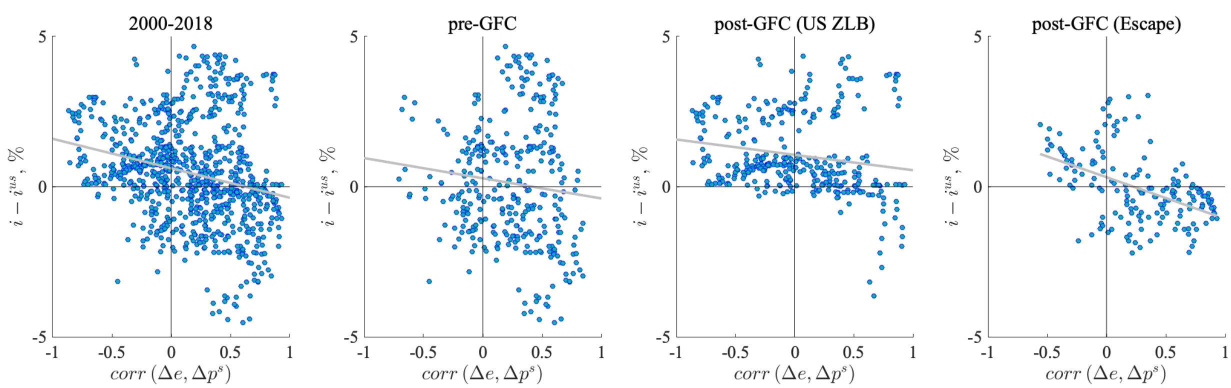

Figure 3.2.3: The scatter plots for corr $(\Delta e, \Delta p^s)$ and $i - i^{us}$ (3-month maturity)

NOTE.—The maturity for interest rate is three months. The correlation between the growth of exchange rate and the growth of stock price is computed with ten quarters with a backward window. We take ten quarters moving average for the interest rate differential. The sample countries are all thirteen countries. The data sample period is from 2000:Q1 to 2018:Q4. 'pre-GFC' period corresponds to the period from 2000:Q1 to 2008:Q3, and 'post-GFC (US ZLB)' period is from 2008:Q4 to 2015:Q3, and 'post-GFC (Escape)' period is from 2015:Q4 to 2018:Q4.

3.3 Two-Country DSGE Framework

In this section we introduce an international two-country business cycle model augmented with the financial intermediaries following Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2019a). The real side of the model is the simplest case of Itskhoki and Mukhin (2019a), where the demand aggregator of domestic goods and imported goods is the CES aggregator and the only input for the production function is labor with the TFP shock. But, we extend the financial sector side by adding the extra asset, equity, as in Hau and Rey (2006). The model consists of two countries, home and foreign country. We regard home country as a low interest rate country with higher discount rate, say Japan, and foreign as a high interest rate country like the US (denoted with a $*$). In the benchmark model, financial intermediaries trade domestic risk free bond and stocks in home and foreign country. The latter one allows hedging by allowing investing in foreign bond adding to cross-border stock trading. For the sake of brevity, we describe only the former case since it is simpler structure[7]. The nominal exchange rate, $\mathcal{E}_t$, denotes the nominal price of one unit of the US dollar in terms of the yen, hence an increase in $\mathcal{E}_t$ means depreciation of the yen. The financial intermediary are risk-averse and trade financial assets mean-variance analysis with net-zero budget constraint.

3.3.1 Real side of the economy

Households. The domestic representative household maximizes expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\phi}}{1+\phi} \right) \tag{3.3.1}$$

subject to the budget constraint:

$$\begin{aligned} P_t C_t + \frac{S_t}{R_t} &\leq W_t L_t + S_{t-1} + \left(1 - \overline{H}^S\right) D_t + \Pi_t^{FI}, \\ \Pi_t^{FI} &= \mathcal{E}_t \left(P_t^{S*} + D_t^*\right) H_{Ht-1}^{S*} + \left(P_t^S + D_t\right) H_{Ht-1}^S + H_{Ht-1}^B, \end{aligned} \tag{3.3.2}$$

[7] As for the latter case, we summarize key equations and log-linearization around the stochastic steady state in Appendix 3.C.

where C_t is consumption, P_t is the consumer price index, W_t is the nominal wage, L_t is labor supply. S_t is households' choice of bond holding. P_t^S is stock prices for one unit of the home firms' equity at time t. D_t stands for dividends paid by domestic firms. The total supply of equity claims is $\overline{H}^S$, which is less than or equal to 1. The remained corporate profits distributes toward the households directly. H_{Ht-1}^{S*}, H_{Ht-1}^{S}, and H_{Ht-1}^{B} are portfolio choices of financial intermediary in the previous period: H_{Ht-1}^{S*} is stock holding in the foreign, H_{Ht-1}^{S} is portfolio in domestic stock, and H_{Ht-1}^{B} is domestic bond. $\mathcal{E}_t$ is the nominal exchange rate and an increase in $\mathcal{E}_t$ (denominated in yen per dollar) corresponds to a dollar appreciation against the yen. The return from the financial intermediaries' choice, Π_t^{FI}, is income gain or loss from investors's trading. We assume that households trade only local-currency bonds, which is the part of market segmentation assumption. With this regard, it should be noted that stock holding is not choice variable of household and only chosen by the financial intermediaries. The foreign households are symmetric in terms of the expected utility, the budget constraint, the access to the financial assets.

Demand aggregator. The domestic households divide their consumption into home and foreign variety of the final products. We use a simple CES demand aggregator[8]:

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft} = \int_0^1 \left[P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \right] \mathrm{d}i, \tag{3.3.3}$$

$$C_t = \int_0^1 \left[(1-\gamma)^{\frac{1}{\theta}} C_{Ht}(i)^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} C_{Ft}(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \mathrm{d}i. \tag{3.3.4}$$

where C_{Ht} is the mass of consumption of home goods, and C_{Ft} is a counterpart of foreign. θ is elasticity of substitution between home and foreign goods and $\gamma \in [0,1]$ reflects home bias in preferences. The solution to the optimal expenditure allocation results in the following

[8] If we adopt the a Kimball aggregator, we can allow for pricing to market. But for the sake of simplicity, we begin with the conventional CES demand structure. We remain extending to nest in a Kimball aggregator as the future work, since it potentially could change the dynamics of corporate profits and hence that of equity prices.

homothetic demand schedules:

$$C_{Ht}(i) = (1-\gamma)\left(\frac{P_{Ht}(i)}{P_t}\right)^{-\theta} C_t \quad \text{and} \quad C_{Ft}(j) = \gamma\left(\frac{P_{Ft}(j)}{P_t}\right)^{-\theta} C_t, \tag{3.3.5}$$

and the price indexes in home is,

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\theta} + \gamma P_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{3.3.6}$$

Since we consider the case where home and foreign are asymmetry, the expenditure allocation of the foreign households is characterized by a similar but different demand system. Specifically, we allow the net export at the steady state departs from 0, so the home bias on consumption bundle of final products would be different as γ^*. The consumer price index in the foreign is also characterized in a similar manner.

Now, we can define the real exchange rate as the relative consumer price level in the two countries:

$$\mathcal{Q}_t \equiv \frac{P_t^*}{P_t}\mathcal{E}_t. \tag{3.3.7}$$

An increase in $\mathcal{Q}_t$ corresponds to a real depreciation, that is a decrease in the relative price of the home consumption basket.

Production function. Each producer i produce $Y_t(i)$ with a Cobb-Douglas production technology.

$$Y_t(i) = e^{a_t}L_t(i). \tag{3.3.8}$$

The aggregate productivity follows an AR(1) process:

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \quad \varepsilon_t^a \sim iid(0,1), \tag{3.3.9}$$

where $\rho_a \in [0,1]$ is the persistence parameter and $\sigma_a \geq 0$ is the volatility of the innovation. Then the marginal cost to produce one unit of home produced goods in characterized by:

$$MC_t = \frac{W_t L_t(i)}{Y_t(i)} = e^{-a_t} W_t. \tag{3.3.10}$$

Profits and price setting. The firm serves both home and foreign markets. The profits of an individual firm $i \in [0,1]$ is computed as:

$$D_t(i) = (P_{Ht}(i) - MC_t)\, Y_{Ht}(i) + (\mathcal{E}_t P^*_{Ht}(i) - MC_t)\, Y^*_{Ht}(i). \tag{3.3.11}$$

The firm maximizes its profits by setting the optimal home market price $P_{Ht}(i)$, and the optimal foreign market price $P_{Ft}(i)$:

$$\begin{aligned} P_{Ht}(i) &= \arg\max_{P_{Ht}(i)} \left\{ (P_{Ht}(i) - MC_t)\,(1-\gamma) \left(\frac{P_{Ht}(i)}{P_t}\right)^{-\theta} C_t \right\} = \frac{\theta}{\theta-1} MC_t, \\ P^*_{Ht}(i) &= \arg\max_{P^*_{Ht}(i)} \left\{ (\mathcal{E}_t P^*_{Ht}(i) - MC_t)\,(1-\gamma) \left(\frac{P^*_{Ht}(i)}{P^*_t}\right)^{-\theta} C^*_t \right\} = \frac{\theta}{\theta-1} \frac{MC_t}{\mathcal{E}_t}, \end{aligned} \tag{3.3.12}$$

where we should note that the marginal cost, MC_t, is identical for all i, then the optimal prices are also identical across firms from the firm symmetric assumption. Now we know P_{Ht} and P^*_{Ht} are corresponding to the price indexes of the home good in the home and foreign markets. Lastly, we aggregate the expenditure in the home and foreign markets:

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft} \quad \text{and} \quad P^*_t C^*_t = P^*_{Ht} C^*_{Ht} + P^*_{Ft} C^*_{Ft}. \tag{3.3.13}$$

Noting that the final goods producing firms are symmetry and we have the optimal price for both domestic and foreign markets, we can compute firm profits across all domestic firms as follows:

$$\begin{aligned} D_t &= (P_{Ht} - MC_t)\, Y_{Ht} + (\mathcal{E}_t P^*_{Ht} - MC_t)\, Y^*_{Ht} \\ &= P_{Ht} Y_{Ht} + \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - MC_t Y_t \\ &= P_{Ht} Y_{Ht} + \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - W_t L_t. \end{aligned} \tag{3.3.14}$$

Good market clearing. There is no capital and intermediate goods in this economy, the good market clearing conditions requires the total production by the home firms should be divided into the home and foreign market,

$$Y_t = Y_{Ht} + Y_{Ht}^* \quad \text{and} \quad Y_t^* = Y_{Ft}^* + Y_{Ft}. \tag{3.3.15}$$

We know all firms i set the same price in each market. Thus, the local demand in the home and foreign market for the home produced final goods are described as:

$$\begin{aligned} Y_{Ht} &= C_{Ht} = (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\theta} C_t, \\ Y_{Ht}^* &= C_{Ht}^* = \gamma^* \left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\theta} C_t^*. \end{aligned} \tag{3.3.16}$$

A symmetric demand schedules hold for the foreign-produced output supplied to the both foreign and home markets.

Finally, we remark the net export, export minus import of the home, and the the term of trade, the relative price at which home exchanges its exports for imports. We define,

$$NX_t \equiv \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - P_{Ft} Y_{Ft}, \tag{3.3.17}$$

$$\mathcal{S}_t \equiv \frac{P_{Ft}}{\mathcal{E}_t P_{Ht}^*}. \tag{3.3.18}$$

We argue the country budget constraint in the following subsection.

3.3.2 Financial market

The model of financial sector follows the spirit of Hau and Rey (2006), but different from them with the following perspective. Since we are interested in the dynamics of the stock return and the exchange rate given the shock to the risk aversion parameter, we assume the risk aversion parameter is not time-invariant but exposed to AR(1) process shock. The dividend is determined in general equilibrium, while in Hau and Rey (2006) the dividends follow Ornstein-Uhlenbeck processes.

The investors are risk averse and maximize a mean-variance objective in local currency terms. Firstly we introduce the mean-variance analysis with several risky assets in general setting following Campbell and Viceira (2002). Assuming that the investor trade off mean and variance in a linear fashion, the investor's maximizing problem is:

$$\max_{\boldsymbol{\alpha}_t} \boldsymbol{\alpha}_t' \left(\mathbb{E}_t \mathbf{R}_{t+1} - R_t^f \boldsymbol{\iota} \right) - \frac{\omega_t}{2} \boldsymbol{\alpha}_t' \boldsymbol{\Sigma}_t \boldsymbol{\alpha}_t, \tag{3.3.19}$$

where $\boldsymbol{\alpha}_t$ is a vector of allocations to the risky assets, $\boldsymbol{\iota}$ is a vector of ones, and $\boldsymbol{\Sigma}_t$ is a variance-covariance matrix of the return on the risky assets. $\mathbf{R}_t$ is the return for risky assets and R_t^f is the return for risk-free bond. ω_t is the risk aversion parameter. The solution the maximization problem is:

$$\boldsymbol{\alpha}_t = \frac{1}{\omega_t} \boldsymbol{\Sigma}_t^{-1} \left(\mathbb{E}_t \mathbf{R}_{t+1} - R_t^f \boldsymbol{\iota} \right). \tag{3.3.20}$$

Cross-border stock trading. We now consider the portfolio problem in which agents can access to two risk assets, home stock and foreign risk-free bond, and one safe asset, domestic risk-free bond. The portfolio choice of domestic and foreign financial intermediaries are:

$$\begin{pmatrix} P_t^S H_{Ht}^S \\ \mathcal{E}_t P_t^{S*} H_{Ht}^{S*} \end{pmatrix} = m \begin{pmatrix} P_t^S h_{Ht}^S \\ \mathcal{E}_t P_t^{S*} h_{Ht}^{S*} \end{pmatrix} = \frac{\Sigma^{-1}}{\frac{\omega}{m} e^{\xi_t}} \begin{pmatrix} \mathbb{E}_t \frac{R_{t+1}^S}{R_t} - 1 \\ \mathbb{E}_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \frac{R_{t+1}^{S*}}{R_t} - 1 \end{pmatrix} \tag{3.3.21}$$
$$\xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_t^\xi, \quad \varepsilon_t^\xi \sim iid(0,1)$$

$$\begin{pmatrix} P_t^{S*} H_{Ft}^{S*} \\ \frac{1}{\mathcal{E}_t} P_t^S H_{Ft}^S \end{pmatrix} = m^* \begin{pmatrix} P_t^{S*} h_{Ft}^{S*} \\ \frac{1}{\mathcal{E}_t} P_t^S h_{Ft}^S \end{pmatrix} = \frac{\Sigma^{*-1}}{\frac{\omega^*}{m^*} e^{\xi_t^*}} \begin{pmatrix} \mathbb{E}_t \frac{R_{t+1}^{S*}}{R_t^*} - 1 \\ \mathbb{E}_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \frac{R_{t+1}^S}{R_t^*} - 1 \end{pmatrix} \tag{3.3.22}$$
$$\xi_t^* = \rho_\xi \xi_{t-1}^* + \sigma_\xi \varepsilon_t^{\xi^*}, \quad \varepsilon_t^{\xi^*} \sim iid(0,1)$$

where $\mathcal{E}_t$ is the nominal exchange rate[9]. P_t^S is a price of home stock, P_t^{S*} is a price of foreign stock, h_{Ht}^S is a position of home equity, and h_{Ht}^{S*} is foreign equity position of each home investor. m is a measure of arbitrageur, ω is the risk aversion parameter, and we call $\frac{\omega}{m}$ as the effective risk aversion of the whole sector of intermediaries. $\boldsymbol{\Sigma}$ is the variance covariance

[9]Note that the nominal exchange rate is equivalent to the real exchange rate, since we do not consider inflation process. In other words, we consider the economy with full flexible-price case.

matrix of the excess return from investment[10]. The returns on stocks one period ahead are denoted by R^S_{t+1} and R^{S*}_{t+1}, and defined as:

$$\begin{aligned} \mathbb{E}_t R^S_{t+1} &\equiv \mathbb{E}_t \frac{P^S_{t+1} + D_{t+1}}{P^S_t} \\ \mathbb{E}_t R^{S*}_{t+1} &\equiv \mathbb{E}_t \frac{P^{S*}_{t+1} + D^*_{t+1}}{P^{S*}_t} \end{aligned} \tag{3.3.24}$$

where D_t is the dividend of one unit of equity.

The position of financial intermediary is net out to zero. The risk-averse arbitrageurs take a positive position for stock and borrow or lend the short-term bond in home and foreign. The budget constraint of the intermediaries then become,

$$\begin{aligned} P^S_t H^S_{Ht} + \mathcal{E}_t P^{S*}_t H^{S*}_{Ht} + \frac{H^B_{Ht}}{R_t} &= 0, \\ P^{S*}_t H^{S*}_{Ft} + \frac{1}{\mathcal{E}_t} P^S_t H^S_{Ft} + \frac{H^{B*}_{Ft}}{R^*_t} &= 0. \end{aligned} \tag{3.3.25}$$

Monetary policy rule. The central bank follows a standard Taylor rule:

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\rho_m} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\right]^{(1-\rho_m)} e^{\sigma_m \varepsilon^m_t} \tag{3.3.26}$$

where $\overline{R}$ and $\overline{\Pi}$ are the steady state values of gross nominal interest rate and gross inflation rate.

[10]The structure of the variance-covariance matrix is:

$$\Sigma \equiv \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma^2_s & \rho_{ss^*}\sigma_s\sigma_{s^*} + \rho_{se}\sigma_s\sigma_e \\ \rho_{ss^*}\sigma_s\sigma_{s^*} + \rho_{se}\sigma_s\sigma_e & \sigma^2_{s^*} + \sigma^2_e + 2\rho_{s^*e}\sigma_{s^*}\sigma_e \end{pmatrix}, \tag{3.3.23}$$

where ρ_{ss^*} is the correlation of home stock return and foreign stock return, whereas ρ_{se} is the correlation of home stock return and the exchange rate. The variance and covariance matrix for the foreign country intermediaries is given in a symmetric way.

Market clearing in financial markets. We assume supplies of short-term bonds of both countries are zero-net supply:

$$S_t + H^B_{Ht} = 0 \quad \text{and} \quad S^*_t + H^{B*}_{Ft} = 0. \tag{3.3.27}$$

The aggregate supply of stock is also constant and we consider the case of cross-border stock trading as the benchmark case, then the stock market clearing is:

$$H^S_{Ht} + H^S_{Ft} = \overline{H}^S \quad \text{and} \quad H^{S*}_{Ft} + H^{S*}_{Ht} = \overline{H}^{S*}. \tag{3.3.28}$$

Country budget constraint. Firstly, we define the net foreign asset position of home country as:

$$NFA_t \equiv \mathcal{E}_t P^{S*}_t H^{S*}_{Ht} - P^S_t H^S_{Ft}. \tag{3.3.29}$$

The growth of the NFA position is computed and decomposed to the current account and the valuation effect term as:

$$\begin{aligned} NFA_t - NFA_{t-1} &= \left[\mathcal{E}_t P^{S*}_t H^{S*}_{Ht} - P^S_t H^S_{Ft}\right] - \left[\mathcal{E}_{t-1} P^{S*}_{t-1} H^{S*}_{Ht-1} - P^S_{t-1} H^S_{Ft-1}\right] \\ &= \underbrace{\left[\mathcal{E}_t P^{S*}_t \left(H^{S*}_{Ht} - H^{S*}_{Ht-1}\right) - P^S_t \left(H^S_{Ft} - H^S_{Ft-1}\right)\right]}_{\text{current account}} \\ &+ \underbrace{\left[\left(\mathcal{E}_t P^{S*}_t - \mathcal{E}_{t-1} P^{S*}_{t-1}\right) H^{S*}_{Ht-1} - \left(P^S_t - P^S_{t-1}\right) H^S_{Ft-1}\right]}_{\text{valuation effect}} = CA_t + VE_t. \end{aligned} \tag{3.3.30}$$

Notice that we have the net export and the net factor payment as:

$$\begin{aligned} NX_t \equiv& \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft} = W_t L_t + D_t - P_t C_t, \\ NFP_t =& \underbrace{\mathcal{E}_t D^*_t H^{S*}_{Ht-1}}_{\text{payment from abroad}} \underbrace{- D_t H^S_{Ft-1}}_{\text{payment toward abroad}}, \end{aligned} \tag{3.3.31}$$

where we use $D_t = P_{Ht}Y_{Ht} + \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - W_t L_t$ and $P_t C_t = P_{Ht} Y_{Ht} + P_{Ft} Y_{Ft}$ for first equality[11]. Finally, we can derive current account for the home country as the sum of NX_t and NFP_t by rearranging the budget constraint of households and the profit earned by the financial intermediaries:

$$\begin{aligned} & P_t C_t + \frac{S_t}{R_t} = W_t L_t + S_{t-1} + \left(1 - \overline{H}^S\right) D_t + \Pi_t^{FI} \\ \Leftrightarrow \quad & CA_t = NX_t + NFP_t. \end{aligned} \tag{3.3.32}$$

As for the foreign country, we can apply the same argument to derive the current account. We have, $CA^*_t = NX^*_t + NFP^*_t$, where $NX^*_t = -\frac{NX_t}{\mathcal{E}_t}$ and $NFP^*_t = -\frac{NFP_t}{\mathcal{E}_t}$. Finally, we have the condition of Walras Law in this economy by adding current account of both countries, $CA_t + \mathcal{E}_t CA^*_t = 0$. Therefore, we do not need the foreign budget constraint in the equilibrium system.

Definition 3 (Equilibrium system)**.** *Given state variables at time t,* $\{S_{t-1}, H^S_{Ht-1}, H^{S*}_{Ht-1}, H^B_{Ht-1}, R_{t-1}\}_t$, $\{S^*_{t-1}, H^{S*}_{Ft-1}, H^S_{Ft-1}, H^B_{Ft-1}, R^*_{t-1}\}_t$*, stochastic processes,* $\{a_t, a^*_t, \xi_t, \xi^*_t\}_t$*, and prices,* $\{W_t, W^*_t, P_t, P^*_t, R_t, R^*_t, P^S_t, P^{S*}_t, \mathcal{E}_t\}_t$*, a competitive equilibrium consists of stochastic process,* $\{C_t, L_t, P_{Ht}, P^*_{Ht}, Y_t, Y_{Ht}, Y^*_{Ht}, D_t, S_t, H^S_{Ht}, H^{S*}_{Ht}, H^B_{Ht}, \Pi^{FI}_t\}_t$, $\{C^*_t, L^*_t, P^*_{Ft}, P_{Ft}, Y^*_t, Y^*_{Ft}, Y_{Ft}, D^*_t, S^*_t, H^{S*}_{Ft}, H^S_{Ft}, H^B_{Ft}, \Pi^{FI*}_t\}_t$*, and auxiliary variables,* $\{NX_t, NFP_t, CA_t, VE_t, NFA_t\}_t$, $\{NX^*_t, NFP^*_t, CA^*_t, VE^*_t, NFA^*_t\}_t$*, such that:*
(i) $\{C_t, L_t, S_t\}$ *and* $\{C^*_t, L^*_t, S^*_t\}$ *maximize the infinite horizon utility subject to the budget constraint; (ii) goods producing firms choose* $\{P_{Ht}, P^*_{Ht}, Y_{Ht}, Y^*_{Ht}, L_t\}$ *and* $\{P^*_{Ft}, P_{Ft}, Y^*_{Ft}, Y_{Ft}, L^*_t\}$ *to maximize the profits; (iii) financial intermediaries's position solves mean-variance problem* $\{H^S_{Ht}, H^{S*}_{Ht}, H^B_{Ht}\}_t$ *and* $\{H^{S*}_{Ft}, H^S_{Ft}, H^B_{Ft}\}_t$*; and (iv) market clearing conditions are satisfied.*

[11] See Appendix 3.C for the derivation of the country budget constraint in more general setting with cross-border stock trading and equity risk hedging, which nests the benchmark model.

3.3.3 Solution method

The model has the finance sector, which choose an optimal portfolio with mean-variance analysis. The risk perceptions are matter in this economy and then we should deal with the second order moment. We construct the risky steady state which notion provided in Coeurdacier et al. (2011). The risk-adjusted steady state differs from the deterministic steady state in due to being of second order terms. The solution method has been proposed in several literature (e.g., Juillard (2011), Lopez et al. (2017)), and exploited for the model which entails portfolio choice (e.g., Gertler et al. (2012)).

We describe the model has a form of:

$$\mathbb{E}_t \left[f \left(Y_{t+1}, Y_t, Y_{t-1}, \epsilon_t \right) \right] = 0, \tag{3.3.33}$$

where $\mathbb{V}_t \equiv [Y_{t+1}, Y_t, Y_{t-1}]$ is all the endogenous variables in the model. Coeurdacier et al. (2011) defines the model function around the risk-adjusted steady state by taking a second-order approximation of f around $\mathbb{E}_t \mathbb{V}_{t+1}$:

$$\Phi \left(\mathbb{E}_t \mathbb{V}_{t+1} \right) = f \left(\mathbb{E}_t \mathbb{V}_{t+1} \right) + \mathbb{E}_t \left[f'' \cdot \left[\mathbb{V}_{t+1} - \mathbb{E}_t \mathbb{V}_{t+1} \right]^2 \right] \tag{3.3.34}$$

where f'' is evaluated at $\mathbb{E}_t \mathbb{V}_{t+1}$. Without the last term of second-order, it is equivalent to usual expression around the deterministic steady state. The equilibrium system is defined by $\Phi \left(\mathbb{E}_t \mathbb{V}_{t+1} \right) = 0$ and the risk-adjusted steady state is characterized by $\Phi(\overline{\mathbb{V}}) = 0$. The solution algorithm is the following, while we describe the more details on computation to solve for $\Phi \left(\mathbb{E}_t \mathbb{V}_{t+1} \right)$ in Appendix 3.B.

Algorithm. We should find the risky-steady state and conditional second moment simultaneously. Since the conditional second moments should be computed by the log-linear dynamics around the risky-steady state, we need an iterative procedure. Denote M as the vector of conditional second moments. Given a set of conditional second moments, the risky steady state is a function of the conditional second moments: $X' = g_X(M)$. Next, given a new temporal risky steady state, we can compute a new conditional second moments around it, $M' = g_M(X')$. Therefore computing the risky steady state is therefore a fixed point problem of finding a M^* such that $M^* = g_M(g_X(M^*))$. We iterate over this procedure until we achieve convergence.

3.4 Quantitative Analysis

We calibrate the empirical moment using the model built in the previous section, which allows financial intermediaries to trade equities across the border. In this section, we provide our results under two different alternative settings, in which two countries are symmetric or asymmetric. In both cases, we consider how model-generated moments would differ according to the correlation of the risk sentiment shock to the financial intermediaries. In an asymmetric case, two countries are asymmetric in the discount rate, hence interest rates on risk-free bonds.

3.4.1 Calibration

We choose calibrating parameters following the IRBC model in Itskhoki and Mukhin (2019a). Table 3.4.1 summarizes the calibrated parameters for our calibration. It should be noted that the trade openness parameter for the foreign is also 0.07 in a symmetric case, but the value would be different in an asymmetric case. Conventional parameters for the elasticity of substitution between home and foreign goods are from 1.5 to 2.0. Nevertheless, the size of corporate profits to aggregate GDP is important in this calibration. Thus, we set this parameter as 3.0 so that the fraction of capital income to aggregate income is one-third. The Taylor rule determines the interest rate, but the Taylor rule coefficient is infinitely large since we impose zero inflation on this economy. The shock persistence of risk aversion shock, ρ_ξ, is 0.97 to match the high persistence of interest rate differential across countries. The correlation of productivity shock, $\text{corr}\,(a, a^*)$, is 0.3, which is suggested by the estimation results of Itskhoki and Mukhin (2019a). Finally, we assume the correlation of productivity and risk aversion shock, $\text{corr}\,(a, \xi)$, is 0.

Table 3.4.1: Calibrated parameters

Conventional parameters			**Shock persistence**		
Relative risk aversion	σ	2	Productivity	ρ_a	0.97
Frisch elasticity of labor supply	ϕ	1	Risk aversion	ρ_ξ	0.97
Trade parameters			**Correlation of shocks**		
Trade openness in home	γ	0.07	Corr. productivity shocks	$\text{corr}\,(a, a^*)$	0.3
Elasticity of substitution	θ	3.0	Corr. prod. and risk aver.	$\text{corr}\,(a, \xi)$	0
Monetary parameters					
Taylor rule coefficient	ϕ_π	∞			
Interest rate smoothing	ρ_m	0.95			

Parameter estimates. Unlike standard calibration, as we solve the model around the stochastic steady state, it is not straightforward to attain the exact parameter values that match the target moment. Thus, we conduct a parametric experiment to estimate the parameter to match the target moment in a symmetric model and apply those estimated parameters to other specifications. Given parameter estimates, we see how a change in the correlation of the risk aversion parameter affects the moments. We consider two cases that the risk-taking behavior are correlated and non-correlated, which are represented by the correlation of risk aversion, $\text{corr}\,(\xi, \xi^*)$, 0.5 and 0,

For the discount rate, we consider the quarterly discount rate as $\beta = 0.99$ in a symmetric case. In an asymmetric case, we set a rate of a high-interest currency as $\beta = 0.985$, which implies the annual interest rate is 6%. For a low-interest rate country, we set $\beta = 0.995$.

Table 3.4.2 summarizes the estimated parameters to match the key moments. The size of risk aversion shock, σ_ξ, the effective risk aversion parameter, ω/m, and the size of equity claims, $\overline{H}^S$, are critical for the size of the volatility of both exchange rate and stock price (return). Our initial moment matching is done with a symmetric case under a positive correlation of risk-averse shocks. We implement the same parameter for the other three specifications. To match the volatility of the exchange rate and stock price, we choose 1 for ω/m and 0.45 for σ_ξ. The size of the firm which issues equity, $\overline{H}^S$ is estimated as 0.2 to match the ratio of the market value of the total stock to GDP equals 1 in the symmetric model[12]. Finally, we estimate the size of the shock on productivity to match the consumption volatility with data.

Table 3.4.2: Estimated parameters

Effective risk aversion	ω/m	1
Fraction of stock issued	$\overline{H}^S$	0.20
Std. productivity shocks	σ_a	0.005
Std. risk aversion shocks	σ_ξ	0.450

3.4.2 Discussion

Exchange rate disconnect and Backus-Smith correlation. The results of parametric experiments are summarized in Table 3.4.3. In this model, we mimic the financial shock in Itskhoki and Mukhin (2019a)'s model (Noise-trader shock) in the form of a risk-taking shock for the financial intermediary. We adjust the risk-taking shocks' size to match the exchange rate volatility and stock prices. As a result, it is possible to obtain results where exchange rate volatility is higher than the volatility of consumption and output in the real economy. The risk-taking shocks of financial intermediaries also create a wedge between the stochastic discount factors of households in the home country and the foreign. As a result, risk-sharing is incomplete, and the Backus-Smith correlation is negative.

Exchange rate volatility and correlation of risk-taking of financial intermediaries. Next, we check how the size of volatility in exchange rates and stock prices varies with the correlation of risk-taking shocks. First, we use the Impulse Response Functions to see how the exchange rate changes with risk-taking shocks. Figure 3.4.1 shows the changes in key variables when a positive shock to risk-taking by financial intermediaries (they become risk-averse) is applied in an asymmetric setting, where the home is a low-interest rate country and the foreign is a high-interest rate country. For exchange rates, when each country becomes risk-averse, the respective country's currency appreciates (a decrease in $\mathcal{E}_t$ is an appreciation of the home currency, and an increase in $\mathcal{E}_t$ is an appreciation of the foreign

[12] According to the FRED data base, 'Stock Market Capitalization to GDP for United States (DDDM01USA156NWDB),' stock market capitalization to GDP in the U.S. has been around 100% (from 80 to 160%).

currency). In other words, the risk-averse behavior of the financial intermediaries in each country generates opposite forces on the exchange rate. Thus, when risk-taking behavior is positively correlated, the net exchange rate behavior is smaller than when it is not. When risk-averse behavior is correlated, exchange rate volatility becomes less. Next, looking at stock price behavior, risk-averse shocks in both home and foreign cause stock prices in both countries to fall. Thus, stock price swings are more significant when risk-taking behavior is positively correlated. In sum, when risk-taking behavior is positively correlated, exchange rate volatility is relatively small relative to stock price volatility.

Correlation of stock return and exchange rate. What would be the correlation between exchange rates and stock prices? Revisiting the response of exchange rates and stock prices to risk-averse shocks in an asymmetric setting where the home is a low-interest rate country and the foreign is a high-interest rate country, the size of an appreciation of the home currency and a decrease in stock price is larger than that of the foreign currency and stock price (Figure 3.4.1). This is because the home has lower interest rates and therefore holds more risky assets on its own and foreign equities at a steady state. Nevertheless, the exchange rate and stock price responses are also perfectly symmetric when the two countries are symmetric. Therefore, if the two countries are symmetric, $\text{corr}\left(\Delta e, \Delta p^{S*}\right)$ will be negative. However, when there is a difference in the safe interest rates of the respective countries, the effect of risk-averse shocks in the home will dominate, so the appreciation of the home currency will be associated with a fall in the stock prices of both countries and $\text{corr}\left(\Delta e, \Delta p^{S*}\right)$ will be positive. In other words, the foreign stock price increases when the foreign exchange rate increases.

Stock price and consumption. Finally, we discuss the negative correlation between consumption and stock prices. Intuitively, a stock price decline would imply a consumption decline, but here consumption is temporarily increasing. This is because the risk-averse behavior of financial intermediaries increases demand for risk-free assets, which causes interest rates to fall. The decline in the interest rate here exalts consumption through the inter-temporal substitution channel of consumption across different points in time. It is important to note that the risk-averse behavior of financial intermediaries also reduces transfers to households (wealth effect) because it causes stock prices to fall. However, the occurrence of this asset effect is accompanied by some lags. It takes about ten quarters for the asset effect to outweigh the substitution channel effect of consumption across different time points. Although this point is counterintuitive, the following two accommodations allow us to derive an intuitive response. The first is to add a similarly risk-averse shock to the household discount rate. This would reduce the initial increase in consumption. Second, we would place a gap between the savings rate faced by households and the market policy rate. For simplicity in the model, this paper assumes that the household deposit rate and the financial intermediary's borrowing rate are identical. However, this effect can be attenuated by assuming that the household deposit rate is less responsive to the interest rate on short-term bonds.

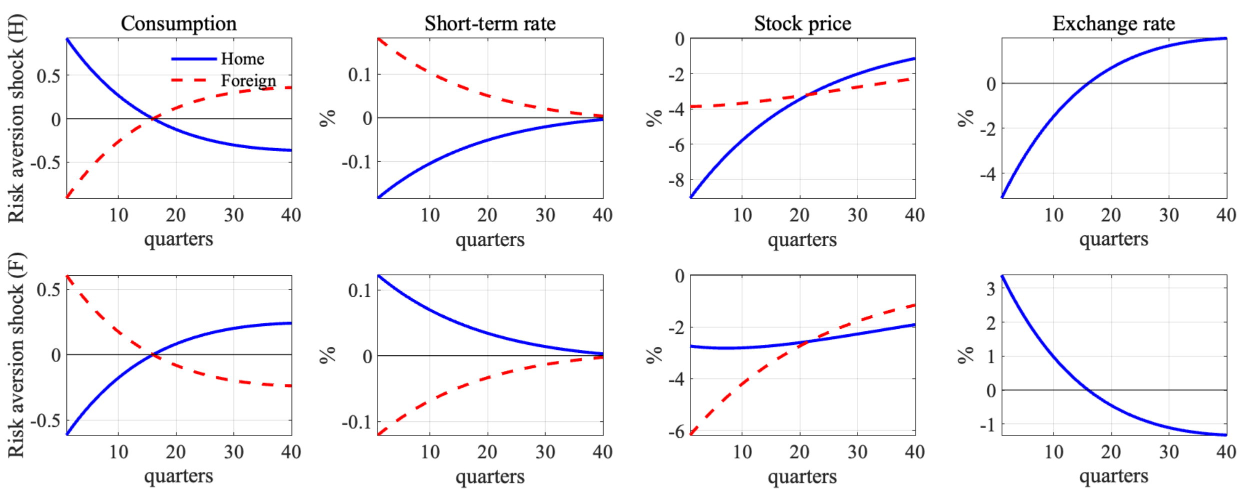

Figure 3.4.1: The impulse responses for 1% positive shock on the risk aversion parameter of investors under an asymmetric case

NOTE.—The unit of measure is percent. The top line represents shock for the home investors, and the bottom line is for the foreign investors. The home country is a low-interest rate country, and the foreign is a high-interest rate country. We show the case of the correlation of the risk aversion shock, corr(ξ, ξ^*), is 0.5.

3.5 Conclusion

In this paper, we revisit the interrelationship between exchange rates and stock prices and confirm the following facts: 1) exchange rate volatility is lower than stock price volatility;

Table 3.4.3: Moments generated in the model

Moments	Data	Model			
		Symmetric		Asymmetric	
A. Exchange rate disconnect					
$\rho(\Delta e)$	≈ 0	-0.03	-0.05	-0.05	-0.05
$\sigma(\Delta e)/\sigma(\Delta gdp)$	5.20	3.29	3.29	3.28	3.44
$\sigma(\Delta e)/\sigma(\Delta c)$	6.30	5.01	4.97	5.00	5.16
B. Backus-Smith correlation					
corr $(\Delta q, \Delta c - \Delta c^*)$	-0.40	-0.97	-0.97	-0.97	-0.97
D. Exchange rates and Stock prices					
$\sigma(\Delta e)$	0.18	0.19	0.17	0.21	0.19
$\sigma(\Delta p^S)$	0.34	0.25	0.40	0.31	0.43
$\sigma(\Delta p^{S*})$	0.34	0.24	0.40	0.16	0.35
$\sigma(\Delta(p^S - p^{S*}))$	0.23	0.20	0.17	0.21	0.19
$\text{corr}(\Delta p^S, \Delta p^{S*})$	0.74	0.70	0.90	0.77	0.90
$\text{corr}(\Delta e, \Delta p^S)$	>0	0.38	0.21	0.85	0.60
$\text{corr}(\Delta e, \Delta p^{S*})$		-0.39	-0.22	0.33	0.19
$\sigma(\Delta c)$	0.03	0.04	0.03	0.04	0.04
$\text{corr}(\Delta c, \Delta p^S)$		-0.33	-0.20	-0.78	-0.56
$\text{corr}(\Delta c^*, \Delta p^{S*})$		-0.34	-0.20	0.34	0.20
Calibration parameter					
$\text{corr}(\xi, \xi^*)$		0.00	0.50	0.00	0.50
β_1		0.990	0.990	0.995	0.995
β_2		0.990	0.990	0.985	0.985
γ^*		0.070	0.070	0.071	0.066

Note.—This panel reports the simulation results of median moments across 1,000 simulations of 120 quarters. The trade openness parameter for the foreign γ^* is an equilibrium outcome. If this value is higher than γ, the home country is the excess export country. $\text{corr}(\Delta e, \Delta p^{S*})$ supposed to be positive if there is an interest differential between safe rates in two countries.

2) stock prices of the G7 countries are positively correlated; 3) countries with low interest rates tend to see their stock prices rise when their exchange rates depreciate, while stock prices tend to fall for countries with high interest rates as the exchange rate depreciates. In the latter part of the paper, we present a two-country DSGE model with the market segmentation hypothesis that can exhibit behavior consistent with the facts obtained in the first part of the paper. To reproduce the econometric moments, including the two facts and the quadratic moments of the other macro variables and the exchange rate, we proposed a positively correlated risk appetite shock between the two countries in the financial market.

Possible directions for future research include the following. In this paper, we have only focused on reproducing the second-order moments of the exchange rate and stock price data. However, exchange rates and stock prices are themselves important economic variables for monetary and fiscal policies. Stock prices may widen the gap between wealthy and poor households, accompanied by an increase in asset prices. A spillover effect of monetary policy, how monetary policy shocks affect the value of assets, and the consideration of optimal economic policy under such shocks are recent hot topics that have been actively studied (Caballero and Simsek (2022), Kekre and Lenel (2021b), and Caramp and Silva (2021)). However, these studies were conducted in closed economies, and none of them have yet examined the distribution of dividends and asset effects of cross-border stock transactions in an open economy with floating exchange rates[13]. In this regard, to know how stock dividends affect household consumption behavior under portfolio choice, we should consider the case in which financiers trade equity cross-border as Hau and Rey (2006). While we show the part of the results in Appendix 3.C, we should explore how hedging demand affects the exchange rate dynamics and the distributional effect of cross-border portfolio choice. Although we considered two different entities in the form of households and financial intermediaries in this paper, it is essential from the perspective of overall economic welfare to consider heterogeneity between Hand to mouth households and Wealthier households, for example. We want to leave these issues for future research.

[13]An exception is Kekre and Lenel (2021a). However, their model does not take into account intra-country heterogeneity, and their analysis focuses on the U.S. without generalizing to other countries.

APPENDIX

3.A Data details

Data set construction. We construct the data sets for major quantity variables, e.g., GDP, Consumption, and CPI, which are a quarterly base from the OECD database. The bilateral nominal exchange rate is downloaded from BIS. The stock price data is downloaded from datastream (*Refiniv Workspace*). Also, it should be noted that the stock return we calculate here is just the growth of stock price and does not include dividends. All quantity variables (GDP, consumption) are real and seasonally adjusted, while nominal exchange rates and consumer price indexes are not. All data are annualized to make volatilities (standard deviations) comparable across series. The sample countries in the main paper are G7 countries plus Spain. In the model part, when we consider the two symmetric countries model, we have a basic idea that those two are the US and the four euro countries, given that those two can be considered equivalent with respect to size and the currency significance. Therefore we include Spain adding to three other European countries. The sample period is from 1990:1Q to 2018:4Q[14].

The followings summarizes the data sources for all data sets.

- Variables for real economy - 'Gross domestic product - expenditure approach,' 'Private final consumption expenditure,' 'Consumer price indices (CPIs) - Complete database, CPI: 01-12 - All items' (OECD.stat).
- Nominal exchange rate - 'US dollar exchange rates' (BIS Statistics).
- Stock price - 'S&P ASX 200' for Australia, 'S&P TSX 60' for Canada, 'CAC 40' for France, 'DAX' for Germany, 'FTSE MIB' for Italy, 'nikkei 225' for Japan, 'KOSPI 200' for South Korea, 'NZX 50' for New Zealand, 'Oslo OBX' for Norway, 'IBEX 35' for Spain, 'OMX Stockholm 30' for Sweden, 'SMI' for Switzerland, 'FTSE 100' for the

[14]We set the initial period as 1990:1Q because the acute appreciation among developed countries, Japan, France, Germany, and Italy after the Plaza accord.

Table 3.A.1: Empirical Moments

Moments	Canada	France	Germany	Italy	Japan	Spain	U.K.	U.S.	EURO
A. Exchange rate disconnect									
$\sigma(\Delta e)/\sigma(\Delta gdp)$	4.95	9.49	5.09	6.71	4.88	5.95	7.10	-	6.81
$\sigma(\Delta e)/\sigma(\Delta c)$	5.54	8.20	5.57	7.24	5.04	5.20	5.83	-	6.55
B. Backus-Smith correlation									
corr $(\Delta q, \Delta c - \Delta c^{us})$	-0.24	-0.23	-0.08	-0.21	0.13	-0.23	-0.06	-	-0.19
C. Business cycle moments									
$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.89	1.16	0.92	0.93	0.97	1.15	1.22	0.84	1.04
corr$(\Delta c, \Delta gdp)$	0.51	0.55	0.37	0.59	0.64	0.61	0.56	0.68	0.53
corr $(\Delta gdp, \Delta gdp^{us})$	0.56	0.53	0.22	0.36	0.23	0.26	0.53	-	0.34
corr $(\Delta c, \Delta c^{us})$	0.42	0.31	0.02	0.28	0.04	0.43	0.51	-	0.26
D. Exchange rates and Stock prices									
$\sigma(\Delta e)$	0.13	0.17	0.17	0.18	0.19	0.18	0.17	-	0.18
$\sigma(\Delta p^s)$	0.30	0.32	0.36	0.34	0.36	0.36	0.18	0.25	0.34
$\sigma(\Delta p^s - \Delta p^{s,us})$	0.17	0.19	0.23	0.26	0.30	0.25	0.11	-	0.23
corr $(\Delta p^s, \Delta p^{s,us})$	0.84	0.81	0.78	0.66	0.57	0.72	0.80	-	0.74
corr $(\Delta e, \Delta p^s)$	-0.53	0.18	0.19	-0.19	0.35	0.00	0.09	0.05	0.05
corr $(\Delta e, \Delta p^s - \Delta p^{s,us})$	0.11	0.36	-0.12	0.03	0.09	0.12	0.37	-	0.10
corr $(\Delta c, \Delta p^s)$	0.54	0.11	-0.09	0.12	0.06	0.09	-0.05	0.36	0.06

NOTE.—The time window is from 1990:Q1 to 2018:Q4. "EURO" is computed by taking arithmetic mean of four major euro countries: France, Germany, Italy and Spain. We compute corr $(\Delta e, \Delta p^s)$ for the US using a bilateral exchange rate between US and EURO as the exchange rate. As for a pre-period of the union of EURO, we use Germany Mark.

U.K., 'S&P 500' for the U.S. (Datastream).

- Short-term rate (3-month) - 'Short-term interest rates, Percent per annum' (OECD.stat).

- Long-term rate (10-year) - 'Long-term interest rates, Percent per annum' (OECD.stat).

Exchange rates, stock prices, and macro economic variables. Here, we check statistics about stock return and exchange rate, and economic variables. Table 3.A.1 shows the results for G7 countries and first three sets of rows repeat an exercise on Meese and Rogoff (1983) puzzle and Backus and Smith (1993) puzzle as in Itskhoki and Mukhin (2019a).

In the last set of rows, we have the second-order moments among exchange rates and stock prices. The exchange rate volatility is lower than that of stock price, and the stock prices among G7 countries are positively correlated. The correlation between exchange rates and stock prices is ambiguous, but this would be discussed in the following paragraphs with time series and other countries. Lastly, the correlation between consumption and stock prices, which is supposed to be positive, is also ambiguous in other countries that the U.S. and Canada. For the U.S., several literature has explored how stock market outcome resulting in consumption behavior (see e.g., Campbell and Cochrane (1999), Ludvigson et al. (1998), and Poterba (2000)). Nevertheless, these arguments, perhaps, cannot be applied directly to the countries with the lower positive or negative correlation.

Correlation between exchange rate and stock price. We present the time series data on the correlation between exchange rates and stock prices for other countries than G7 countries in Figure 3.A.1. The seven countries adding to non-US G7 countries are grouped into three categories according to the characteristics of their currencies, as the following:

- EURO currency countries - France, Germany, Italy, and Spain.
- Emerging countries and the U.K. - Australia, Canada, South Korea, New Zealand, Norway, Sweden, and the United Kingdom.
- Low interest rate countries - Japan and Switzerland.

Two major points need to be mentioned here. First, looking at the correlation between exchange rates and stock prices for emerging economies, before the GFC, there was no clear positive or negative distinction. However, after the GFC, the correlation between exchange rates and stock prices turned negative, as in the case of countries using the EURO currency. Furthermore, that trend has generally continued through 2018, with the exception of Sweden and the U.K.

Next, we turn to Switzerland, which we have classified as a safe-haven currency. While a positive correlation between the exchange rate and stock prices can be observed for the Swiss before the GFC, this trend has been lost after the GFC. This is because Switzerland's main trading partner is the eurozone and its trade invoicing currency is the euro, so the stock price in Swiss has been influenced by the euro currency area[15].

[15]For more information on the trade invoicing currencies used over the whole world see Boz et al. (2020).

Interest rate differentials and correlation between exchange rate and stock price. The leftmost panel in Figure 3.A.2 shows that a negative correlation between the interest rate differential and the correlation between the exchange rate and stock prices would be observed unchanged even if the interest rate differential were changed from a 3-month maturity to a 10-year maturity. Moreover, it should be noted that the remaining three graphs on the right in Figure 3.A.2 show that the same tendency has been observed even if we change the sub-sample period.

Figure 3.A.3 and Figure 3.A.4 present scatter plots in the three currency groups, segregated by currency characteristics, for all sample periods and the three subsample periods. Figure 3.A.3 shows the interest rate differentials calculated using interest rates with 3-month maturities, while Figure 3.A.4 shows those using interest rates with 10-year maturities. The first is the EURO currency group, which is unique: prior to the GFC, exchange rates and stock prices were positively correlated (i.e., exchange rate depreciation and stock price appreciation occurred simultaneously) under low policy rates relative to the U.S. interest rate. After the GFC, however, quantitative easing was taken in the U.S., policy rates were bound to the ZLB, and interest rates in Euro countries were relatively high. The correlation between exchange rates and stock prices was negative during that period. Finally, under the normalization of quantitative easing in the U.S., when the policy rate was raised and the interest rate in the euro area was lower, the correlation between exchange rates and stock prices returned to a positive correlation.

Turning now to the second group of emerging market economies and the U.K., we find that in this currency group, where interest rates are higher than in the U.S., exchange rates and stock prices should be negatively correlated, in line with our argument. In this regard, the relationship was not clear before the GFC, but exchange rates and stock prices are negatively correlated in many countries after the GFC. The disagreement comes from Sweden and the U.K.; see Figure 3.A.1 too.

Finally, for Japan and Switzerland, which are considered safe-haven currencies, we observe a positive correlation between exchange rates and stock prices over the sample period, when the policy interest rate is kept consistently lower than the U.S. interest rate.

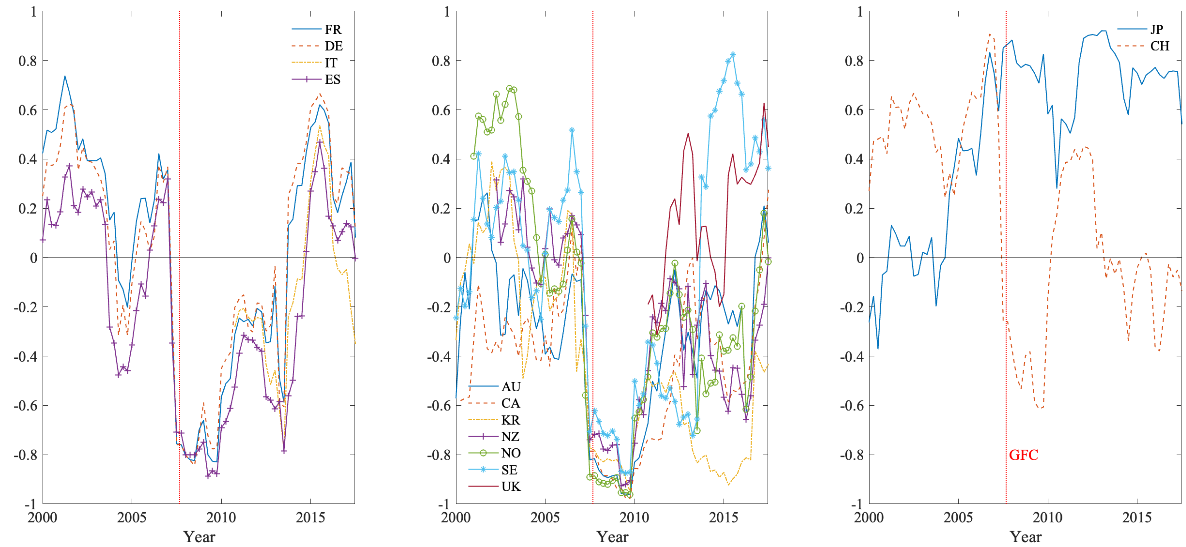

Figure 3.A.1: The time series of corr $(\Delta e, \Delta p^s)$

NOTE.—The data sample period is from 2000:Q1 to 2018:Q4. All time series are represented for moving average of std of 10 quarters back-ward window.

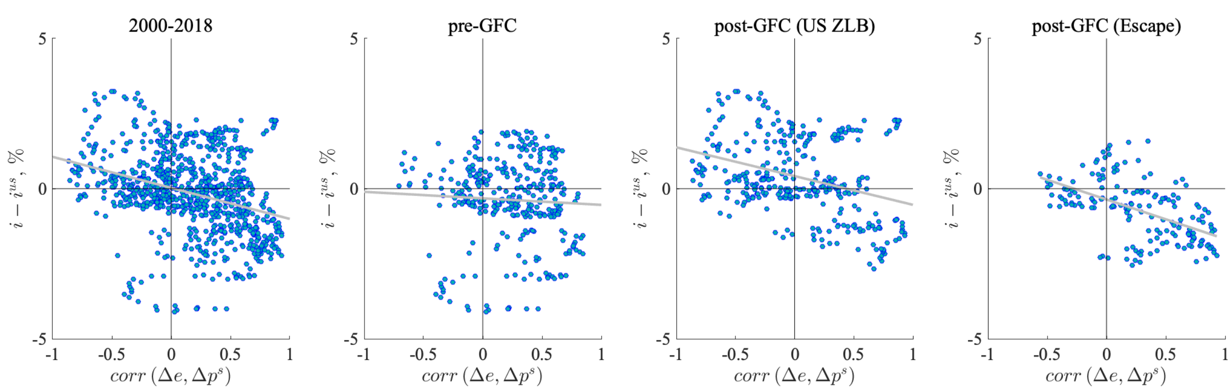

Figure 3.A.2: The scatter plots for corr $(\Delta e, \Delta p^s)$ and $i - i^{us}$ (10-year maturity)

NOTE.—The maturity for interest rate is ten years. The correlation between the growth of exchange rate and the growth of stock price is computed with ten quarters with a backward window. We take ten quarters moving average for the interest rate differential. The sample countries are all thirteen countries. The data sample period is from 2000:Q1 to 2018:Q4. 'pre-GFC' period corresponds to the period from 2000:Q1 to 2008:Q3, and 'post-GFC (US ZLB)' period is from 2008:Q4 to 2015:Q3, and 'post-GFC (Escape)' period is from 2015:Q4 to 2018:Q4.

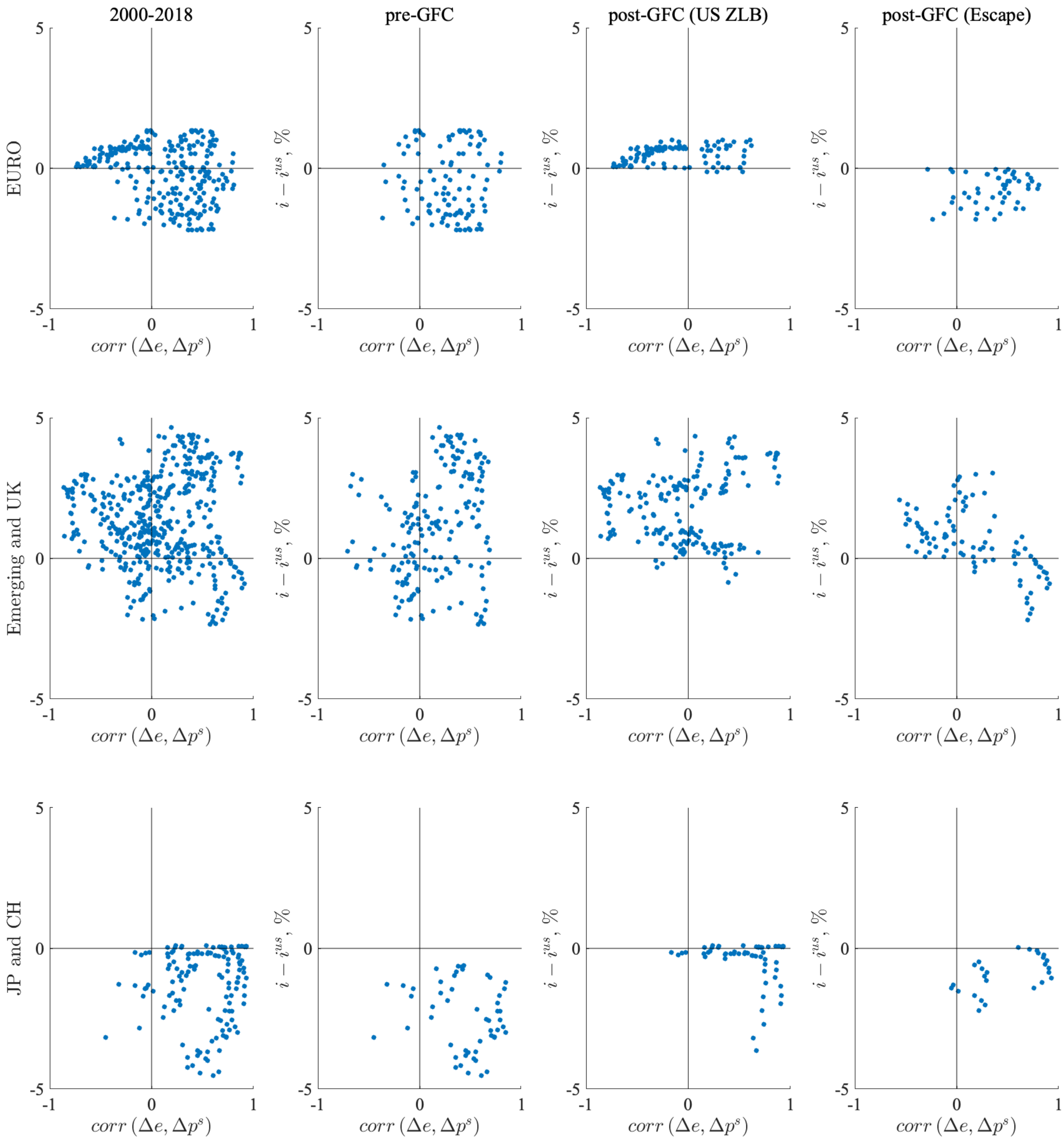

Figure 3.A.3: The scatter plots for corr $(\Delta e, \Delta p^s)$ and $i - i^{us}$ (3-month maturity) for sub group of countries

Note.—The maturity for interest rate is three months. The correlation between the growth of exchange rate and the growth of stock price is computed with ten quarters with a backward window. We take ten quarters moving average for the interest rate differential. The data sample period is from 2000:Q1 to 2018:Q4. 'pre-GFC' period corresponds to the period from 2000:Q1 to 2008:Q3, and 'post-GFC (US ZLB)' period is from 2008:Q4 to 2015:Q3, and 'post-GFC (Escape)' period is from 2015:Q4 to 2018:Q4.

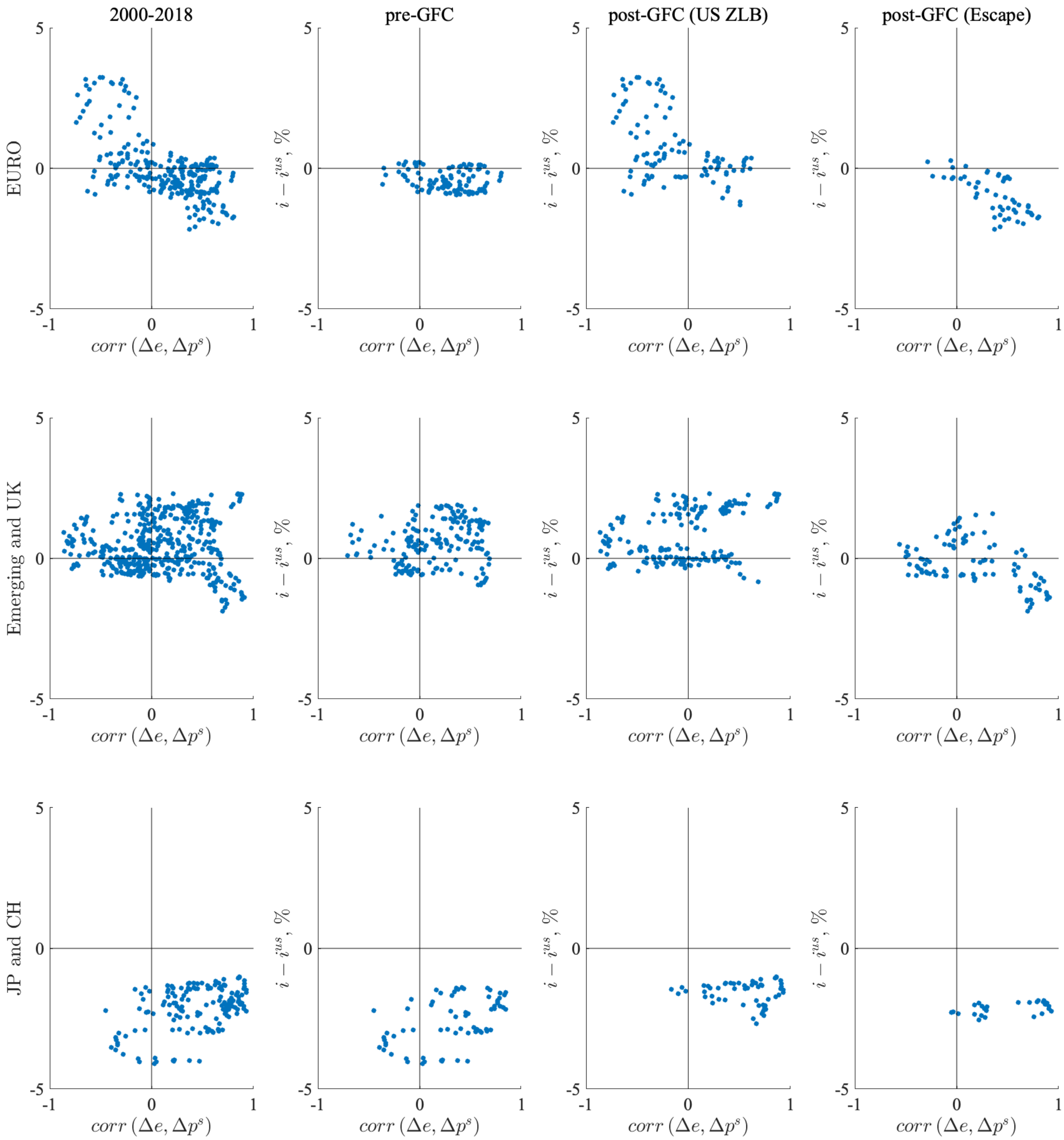

Figure 3.A.4: The scatter plots for $\text{corr}\,(\Delta e, \Delta p^s)$ and $i - i^{us}$ (10-year maturity) for sub group of countries

NOTE.—The maturity for interest rate is ten years. The correlation between the growth of exchange rate and the growth of stock price is computed with ten quarters with a backward window. We take ten quarters moving average for the interest rate differential. The data sample period is from 2000:Q1 to 2018:Q4. 'pre-GFC' period corresponds to the period from 2000:Q1 to 2008:Q3, and 'post-GFC (US ZLB)' period is from 2008:Q4 to 2015:Q3, and 'post-GFC (Escape)' period is from 2015:Q4 to 2018:Q4.

3.B Risky steady state argument

We describe the detail for calculation to obtain $\Phi\left(\mathbb{E}_t \mathbb{V}_{t+1}\right)$ following a way of Coeurdacier et al. (2011). In this economy, only the Euler equation is affected by an uncertainty, which is:

$$\text{Euler equations:} \quad \beta R_t \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} = 1. \tag{3.B.1}$$

We think about the second order expansion Φ around the expected future variables.

$$\Phi\left(\mathbb{E}_t \mathbb{V}_{t+1}\right) = f\left(\mathbb{E}_t\left[\mathbb{V}_{t+1}\right]\right) + \mathbb{E}_t\left[f''\left[\mathbb{V}_{t+1} - \mathbb{E}_t \mathbb{V}_{t+1}\right]^2\right] \tag{3.B.2}$$

In the case of the Euler equation in this economy,

$$f\left(\mathbb{V}_{t+1}\right) = \beta R_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} - 1 = \beta R_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} - 1 \tag{3.B.3}$$

$$\begin{aligned}
\frac{\partial f\left(\mathbb{V}_{t+1}\right)}{\partial C_{t+1}} &= \beta R_t \frac{-\sigma}{C_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma-1} \frac{1}{\Pi_{t+1}} \\
\frac{\partial f\left(\mathbb{V}_{t+1}\right)}{\partial \Pi_{t+1}} &= \beta R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{-1}{\Pi_{t+1}^2} \\
\frac{\partial^2 f\left(\mathbb{V}_{t+1}\right)}{\partial C_{t+1}^2} &= \beta R_t \frac{\sigma(\sigma+1)}{C_t^2} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma-2} \frac{1}{\Pi_{t+1}} \\
\frac{\partial^2 f\left(\mathbb{V}_{t+1}\right)}{\partial C_{t+1} \partial \Pi_{t+1}} &= \beta R_t \frac{-\sigma}{C_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma-1} \frac{-1}{\Pi_{t+1}^2} \\
\frac{\partial^2 f\left(\mathbb{V}_{t+1}\right)}{\partial \Pi_{t+1}^2} &= \beta R_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{2}{\Pi_{t+1}^3}
\end{aligned} \tag{3.B.4}$$

To solve the optimality conditions, we have,

$$\begin{aligned}
0 &= \beta R_t \left\{ \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma} \frac{1}{\mathbb{E}_t\left[\Pi_{t+1}\right]} \right\} - 1 + \beta R_t \frac{\sigma(\sigma+1)}{C_t^2} \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma-2} \frac{1}{\mathbb{E}_t\left[\Pi_{t+1}\right]} Var_t(C_{t+1}) \\
&+ \beta R_t \frac{-\sigma}{C_t} \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma-1} \frac{-1}{\mathbb{E}_t\left[\Pi_{t+1}\right]^2} Cov_t(C_{t+1}, \Pi_{t+1}) + \beta R_t \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma} \frac{2}{\mathbb{E}_t\left[\Pi_{t+1}\right]^3} Var_t(\Pi_{t+1}) \\
\Leftrightarrow \frac{1}{\beta R_t} &= \left\{ \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma} \frac{1}{\mathbb{E}_t\left[\Pi_{t+1}\right]} \right\} + \frac{\sigma(\sigma+1)}{C_t^2} \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma-2} \frac{1}{\mathbb{E}_t\left[\Pi_{t+1}\right]} Var_t(C_{t+1}) \\
&+ \frac{-\sigma}{C_t} \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma-1} \frac{-1}{\mathbb{E}_t\left[\Pi_{t+1}\right]^2} Cov_t(C_{t+1}, \Pi_{t+1}) + \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma} \frac{2}{\mathbb{E}_t\left[\Pi_{t+1}\right]^3} Var_t(\Pi_{t+1}) \\
&= \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma} \frac{1}{\mathbb{E}_t\left[\Pi_{t+1}\right]} \left[1 + \sigma(\sigma+1) \frac{Var_t(C_{t+1})}{\mathbb{E}_t\left[C_{t+1}\right]^2} + \sigma \frac{Cov_t(C_{t+1}, \Pi_{t+1})}{\mathbb{E}_t\left[C_{t+1}\right] \mathbb{E}_t\left[\Pi_{t+1}\right]} + 2 \frac{Var_t(\Pi_{t+1})}{\mathbb{E}_t\left[\Pi_{t+1}\right]^2} \right]
\end{aligned} \tag{3.B.5}$$

Since we are thinking of no inflation economy with full flexible-price model, this setting simplifies the optimality condition as:

$$\frac{1}{\beta R_t} = \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t} \right)^{-\sigma} \left[1 + \sigma(\sigma+1) \frac{Var_t(C_{t+1})}{\mathbb{E}_t\left[C_{t+1}\right]^2} \right]. \tag{3.B.6}$$

At the risky steady state, the following equation should hold,

$$\begin{aligned}
\frac{1}{\beta \overline{R}} &= 1 + \sigma(\sigma+1) \frac{\overline{Var}_t(C_{t+1})}{\overline{C}^2} \\
\Leftrightarrow \overline{R} &= \frac{1}{\beta \left[1 + \sigma(\sigma+1) \overline{Var}_t(c_{t+1}) \right]}.
\end{aligned} \tag{3.B.7}$$

It implies, as the variance of consumption or the risk aversion parameter increase, the interest rate at the steady state decline. Log-linearizing around the risky steady state derives,

$$\begin{aligned}
\frac{1}{\beta \overline{R}} \{ \sigma(c_{t+1} - c_t) - r_t \} &= -2\sigma(\sigma+1) \overline{Var}_t(c_{t+1}) c_{t+1} \\
\Leftrightarrow \left[1 + \sigma(\sigma+1) \overline{\sigma}_c^2 \right] \{ \sigma(c_{t+1} - c_t) - r_t \} &= -2\sigma(\sigma+1) \overline{\sigma}_c^2 c_{t+1},
\end{aligned} \tag{3.B.8}$$

where $\overline{\sigma}_c^2$ is the conditional variance of consumption, corresponding to $\overline{Var}_t(c_{t+1})$.

3.C Cross-border Stock Trading and Stock Risk Hedging Model

In this section, we show the model with equity risk hedging adding to cross-border stock trading, where the financial intermediaries can invest in foreign bonds.

3.C.1 Real side of the economy

The real side of the economy is identical with the benchmark model as the following. The model equations for the foreign country are descried in a similar way.

$$\begin{aligned}
\text{Euler equation:}\quad & 1 = \beta R_t \left(\frac{\mathbb{E}_t\left[C_{t+1}\right]}{C_t}\right)^{-\sigma}\left[1+\sigma(\sigma+1)\frac{\mathrm{Var}_t\left(C_{t+1}\right)}{\mathbb{E}_t\left[C_{t+1}\right]^2}\right] \\
\text{Labor supply:}\quad & C_t^{\sigma} L_t^{\varphi} = \frac{W_t}{P_t} \\
\text{Production function:}\quad & Y_t = e^{a_t} L_t \\
\text{Marginal cost:}\quad & MC_t = e^{-a_t} W_t \\
\text{Price setting in the domestic:}\quad & P_{Ht} = \frac{\theta}{\theta-1} MC_t \\
\text{Price setting in the foreign:}\quad & P_{Ht}^{*} = \frac{\theta}{\theta-1}\frac{MC_t}{\mathcal{E}_t} \\
\text{Price index:}\quad & P_t = \left[(1-\gamma)P_{Ht}^{1-\theta} + \gamma P_{Ft}^{1-\theta}\right]^{1/(1-\theta)} \\
\text{Local demand in the home:}\quad & Y_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\theta} C_t \\
\text{Local demand in the foreign:}\quad & Y_{Ht}^{*} = \gamma\left(\frac{P_H^{*}}{P_t^{*}}\right)^{-\theta} C_t^{*} \\
\text{Goods market clearing:}\quad & Y_t = Y_{Ht} + Y_{Ht}^{*} \\
\text{Firm profits:}\quad & D_t = P_{Ht} Y_{Ht} + \mathcal{E}_t P_{Ht}^{*} Y_{Ht}^{*} - MC_t Y_t \\
\text{Stock return:}\quad & \mathbb{E}_t R_{t+1}^{S} \equiv \mathbb{E}_t \frac{P_{t+1}^{S} + D_{t+1}}{P_t^{S}} \\
\text{Monetary policy:}\quad & \frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\rho_m}\left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\right]^{(1-\rho_m)} e^{\sigma_m \varepsilon_t^m}, \quad \varepsilon_t^m \sim iid(0,1) \\
\text{Productivity shock process:}\quad & a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \quad \varepsilon_t^a \sim iid(0,1) \\
\text{Risk aversion shock process:}\quad & \xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_t^\xi \quad \varepsilon_t^\xi \sim iid(0,1)
\end{aligned} \tag{3.C.1}$$

3.C.2 Country budget constraint and portfolio choice

In this subsection, we briefly explains the model of trading all assets by reviewing the equations which differs from the model of cross-border stock trading benchmark model. Since we aim to clarify the difference, we only describe the equations which are distinct from the main text. Firstly, the budget constraint of households is:

$$\begin{aligned} P_t C_t + \frac{S_t}{R_t} &= W_t L_t + S_{t-1} + \left(1 - \overline{H}^S\right) D_t + \Pi_t^{FI} + \left(\frac{\overline{H}^B}{R_t} - \overline{H}^B\right) \\ \Pi_t^{FI} &= \mathcal{E}_t \left(P_t^{S*} + D_t^*\right) H_{Ht-1}^{S*} + \mathcal{E}_t H_{Ht-1}^{B*} + \left(P_t^S + D_t\right) H_{Ht-1}^S + H_{Ht-1}^B \end{aligned} \tag{3.C.2}$$

where H_{Ht-1}^S, H_{Ht-1}^{S*}, H_{Ht-1}^B, and H_{Ht-1}^{B*} are portfolio choices of the home financial intermediaries in the previous period: H_{Ht-1}^S is portfolio in the home stock, H_{Ht-1}^{S*} is the foreign stock, and H_{Ht-1}^B and H_{Ht-1}^{B*} are holding of risk-free bond in the home and foreign. H_{Ft-1}^S and H_{Ft-1}^B are the portfolio invested for the home stock and bond by the foreign investors. D_t stands for dividends paid by domestic firms and D_t^* is the dividends by foreign firms. Note that while we restrict the net supply of the risk-free bond in each country is 0 in the main text, we generalize this setting by allowing the positive net-supply of risk-free bond with $\overline{H}^B$ in this appendix.

The portfolio choice of domestic and foreign financial intermediaries are:

$$\begin{pmatrix} P_t^S H_{Ht}^S \\ \mathcal{E}_t P_t^{S*} H_{Ht}^{S*} \\ \mathcal{E}_t \frac{H_{Ht}^{B*}}{R_t^*} \end{pmatrix} = m \begin{pmatrix} P_t^S h_{Ht}^S \\ \mathcal{E}_t P_t^{S*} h_{Ht}^{S*} \\ \mathcal{E}_t \frac{h_{Ht}^{B*}}{R_t^*} \end{pmatrix} = \frac{\Sigma^{-1}}{\frac{\omega}{m} e^{\xi_t}} \begin{pmatrix} \mathbb{E}_t \frac{R_{t+1}^S}{R_t} - 1 \\ \mathbb{E}_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \frac{R_{t+1}^{S*}}{R_t} - 1 \\ \mathbb{E}_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \frac{R_t^*}{R_t} - 1 \end{pmatrix}, \tag{3.C.3}$$

$$\begin{pmatrix} P_t^{S*} H_{Ft}^{S*} \\ \frac{1}{\mathcal{E}_t} P_t^S H_{Ft}^S \\ \frac{1}{\mathcal{E}_t} \frac{H_{Ft}^B}{R_t} \end{pmatrix} = m^* \begin{pmatrix} P_t^{S*} h_{Ft}^{S*} \\ \frac{1}{\mathcal{E}_t} P_t^S h_{Ft}^S \\ \frac{1}{\mathcal{E}_t} \frac{h_{Ft}^B}{R_t} \end{pmatrix} = \frac{\Sigma^{*-1}}{\frac{\omega^*}{m^*} e^{\xi_t^*}} \begin{pmatrix} \mathbb{E}_t \frac{R_{t+1}^{S*}}{R_t^*} - 1 \\ \mathbb{E}_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \frac{R_{t+1}^S}{R_t^*} - 1 \\ \mathbb{E}_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \frac{R_t}{R_t^*} - 1 \end{pmatrix}, \tag{3.C.4}$$

where the investors can take a position in the counterpart country bond, so they are able to hedge the foreign equity risk by short-selling the foreign bond. The structure of the variance-covariance matrix of excess return is:

$$\Sigma \equiv \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_s^2 & \rho_{ss^*}\sigma_s\sigma_{s^*} + \rho_{se}\sigma_s\sigma_e & \rho_{se}\sigma_s\sigma_e \\ \rho_{ss^*}\sigma_s\sigma_{s^*} + \rho_{se}\sigma_s\sigma_e & \sigma_{s^*}^2 + \sigma_e^2 + 2\rho_{s^*e}\sigma_{s^*}\sigma_e & \sigma_e^2 + \rho_{s^*e}\sigma_{s^*}\sigma_e \\ \rho_{se}\sigma_s\sigma_e & \sigma_e^2 + \rho_{s^*e}\sigma_{s^*}\sigma_e & \sigma_e^2 \end{pmatrix}, \tag{3.C.5}$$

$$\Sigma^* \equiv \begin{pmatrix} \sigma_{11}^* & \sigma_{12}^* & \sigma_{13}^* \\ \sigma_{12}^* & \sigma_{22}^* & \sigma_{23}^* \\ \sigma_{13}^* & \sigma_{23}^* & \sigma_{33}^* \end{pmatrix} = \begin{pmatrix} \sigma_{s^*}^2 & \rho_{ss^*}\sigma_s\sigma_{s^*} - \rho_{s^*e}\sigma_{s^*}\sigma_e & -\rho_{s^*e}\sigma_{s^*}\sigma_e \\ \rho_{ss^*}\sigma_s\sigma_{s^*} - \rho_{s^*e}\sigma_{s^*}\sigma_e & \sigma_s^2 + \sigma_e^2 - 2\rho_{se}\sigma_s\sigma_e & \sigma_e^2 - \rho_{se}\sigma_s\sigma_e \\ -\rho_{s^*e}\sigma_{s^*}\sigma_e & \sigma_e^2 - \rho_{se}\sigma_s\sigma_e & \sigma_e^2 \end{pmatrix}, \tag{3.C.6}$$

where ρ_{ss^*} is the correlation of home stock return and foreign stock return, whereas ρ_{se} is the correlation of home stock return and the exchange rate.

The budget constraint of financial intermediaries are:

$$\begin{aligned} P_t^S H_{Ht}^S + \frac{H_{Ht}^B}{R_t} + \mathcal{E}_t P_t^{S*} H_{Ht}^{S*} + \mathcal{E}_t \frac{H_{Ht}^{B*}}{R_t^*} = 0, \\ P_t^{S*} H_{Ft}^{S*} + \frac{H_{Ft}^{B*}}{R_t^*} + \frac{1}{\mathcal{E}_t} P_t^S H_{Ft}^S + \frac{1}{\mathcal{E}_t} \frac{H_{Ft}^B}{R_t} = 0. \end{aligned} \tag{3.C.7}$$

The market clearing conditions for bond market and stock market are as follows. Since we assume supplies of short-term bonds in the market are zero-net.

$$\begin{aligned} S_t + H_{Ht}^B + H_{Ft}^B = \overline{H}^B \\ S_t^* + H_{Ft}^{B*} + H_{Ht}^{B*} = \overline{H}^{B*}. \end{aligned} \tag{3.C.8}$$

The stock market clearing is:

$$H_{Ht}^S + H_{Ft}^S = \overline{H}^S \quad \text{and} \quad H_{Ft}^{S*} + H_{Ht}^{S*} = \overline{H}^{S*}. \tag{3.C.9}$$

By definition, the NFA position of the home country is:

$$NFA_t \equiv \mathcal{E}_t P_t^{S*} H_{Ht}^{S*} + \mathcal{E}_t \frac{H_{Ht}^{B*}}{R_t^*} - P_t^S H_{Ft}^S - \frac{H_{Ft}^B}{R_t}. \tag{3.C.10}$$

Therefore, the growth of the NFA position is computed and decomposed to the current account and the valuation effect term as:

$$\begin{aligned}
NFA_t - NFA_{t-1} &= \left[\mathcal{E}_t P_t^{S*} H_{Ht}^{S*} + \mathcal{E}_t \frac{H_{Ht}^{B*}}{R_t^*} - P_t^S H_{Ft}^S - \frac{H_{Ft}^B}{R_t}\right] - \left[\mathcal{E}_{t-1} P_{t-1}^{S*} H_{Ht-1}^{S*} + \mathcal{E}_{t-1} \frac{H_{Ht-1}^{B*}}{R_{t-1}^*} - P_{t-1}^S H_{Ft-1}^S - \frac{H_{Ft-1}^B}{R_{t-1}}\right] \\
&= \underbrace{\left[\mathcal{E}_t P_t^{S*}\left(H_{Ht}^{S*} - H_{Ht-1}^{S*}\right) + \mathcal{E}_t\left(\frac{H_{Ht}^{B*}}{R_t^*} - \frac{H_{Ht-1}^{B*}}{R_{t-1}^*}\right) - P_t^S\left(H_{Ft}^S - H_{Ft-1}^S\right) - \left(\frac{H_{Ft}^B}{R_t} - \frac{H_{Ft-1}^B}{R_{t-1}}\right)\right]}_{\text{current account}} \\
&+ \underbrace{\left[\left(\mathcal{E}_t P_t^{S*} - \mathcal{E}_{t-1} P_{t-1}^{S*}\right) H_{Ht-1}^{S*} + \left(\mathcal{E}_t - \mathcal{E}_{t-1}\right)\frac{H_{Ht-1}^{B*}}{R_{t-1}^*} - \left(P_t^S - P_{t-1}^S\right) H_{Ft-1}^S\right]}_{\text{valuation effect}} = CA_t + VE_t.
\end{aligned} \tag{3.C.11}$$

Notice that we have the net export and the net factor payment as:

$$\begin{aligned}
NX_t &= W_t L_t + D_t - P_t C_t, \\
NFP_t &= \underbrace{\mathcal{E}_t D_t^* H_{Ht-1}^{S*} + \mathcal{E}_t\left[1 - \frac{1}{R_{t-1}^*}\right] H_{Ht-1}^{B*}}_{\text{payment from abroad}} \underbrace{- D_t H_{Ft-1}^S - \left[1 - \frac{1}{R_{t-1}}\right] H_{Ft-1}^B}_{\text{payment toward abroad}},
\end{aligned} \tag{3.C.12}$$

where, we use $D_t = P_{Ht}Y_{Ht} + \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - W_t L_t$, $NX_t = \mathcal{E}_t P_{Ht}^* Y_{Ht}^* - P_{Ft}Y_{Ft}$, and $P_t C_t = P_{Ht}Y_{Ht} + P_{Ft}Y_{Ft}$ for first equality. Finally, we derive current account for the home country by rearranging the budget constraint of households and the profit earned by the financial intermediaries:

$$\begin{aligned}
& P_t C_t + \frac{S_t}{R_t} = W_t L_t + S_{t-1} + \left(1 - \overline{H}^S\right) D_t + \Pi_t^{FI} + \left(\frac{\overline{H}^B}{R_t} - \overline{H}^B\right) \\
\Leftrightarrow & P_t C_t + \left[\mathcal{E}_t P_t^{S*} H_{Ht}^{S*} + \mathcal{E}_t \frac{H_{Ht}^{B*}}{R_t^*} + P_t^S H_{Ht}^S + \frac{H_{Ht}^B}{R_t}\right] \\
= & W_t L_t + \left(1 - \overline{H}^S\right) D_t + \left[\mathcal{E}_t\left(P_t^{S*} + D_t^*\right) H_{Ht-1}^{S*} + \mathcal{E}_t H_{Ht-1}^{B*} + \left(P_t^S + D_t\right) H_{Ht-1}^S + H_{Ht-1}^B\right] + \left(\frac{\overline{H}^B - S_t}{R_t} - \left(\overline{H}^B - S_{t-1}\right)\right) \\
\Leftrightarrow & \left[\mathcal{E}_t P_t^{S*}\left(H_{Ht}^{S*} - H_{Ht-1}^{S*}\right) + \mathcal{E}_t\left(\frac{H_{Ht}^{B*}}{R_t^*} - \frac{H_{Ht-1}^{B*}}{R_{t-1}^*}\right) - P_t^S\left(H_{Ft}^S - H_{Ft-1}^S\right) - \left(\frac{H_{Ft}^B}{R_t} - \frac{H_{Ft-1}^B}{R_{t-1}}\right)\right] \\
= & - P_t C_t + W_t L_t + D_t + \mathcal{E}_t D_t^* H_{Ht-1}^{S*} + \mathcal{E}_t\left[1 - \frac{1}{R_{t-1}^*}\right] H_{Ht-1}^{B*} - D_t H_{Ft-1}^S - \left[1 - \frac{1}{R_{t-1}}\right] H_{Ft-1}^B \\
\Leftrightarrow & CA_t = NX_t + NFP_t.
\end{aligned} \tag{3.C.13}$$

Next, we have current account for the foreign in a similar way as, $CA_t^* = NX_t^* + NFP_t^*$, where

$$
\begin{aligned}
NX_t^* &= \frac{1}{\mathcal{E}_t} P_{Ft} Y_{Ft} - P_{Ht}^* Y_{Ht}^* = -\frac{NX_t}{\mathcal{E}_t} \\
NFP_t^* &= \underbrace{\frac{1}{\mathcal{E}_t} D_t H_{Ft-1}^S + \frac{1}{\mathcal{E}_t}\left[1 - \frac{1}{R_{t-1}}\right] H_{Ft-1}^B}_{\text{payment from home}} \underbrace{- D_t^* H_{Ht-1}^{S*} - \left[1 - \frac{1}{R_{t-1}^*}\right] H_{Ht-1}^{B*}}_{\text{payment toward home}} = -\frac{NFP_t}{\mathcal{E}_t}
\end{aligned}
\tag{3.C.14}
$$

We have the condition of Walras Law in this economy by adding current account of both countries, $CA_t + \mathcal{E}_t CA_t^* = 0$. Therefore, we do not need the foreign budget constraint in the equilibrium system.

Finally, it is worthwhile to check the relation between the households' saving and NFA position. By rearranging the households' budget constraint with the definition of NFA, we have:

$$
\begin{aligned}
& \frac{S_t}{R_t} - S_{t-1} = -P_t C_t + W_t L_t + D_t - \overline{H}^S D_t + \Pi_t^{FI} + \left(\frac{\overline{H}^B}{R_t} - \overline{H}^B\right) \\
\Leftrightarrow \quad & \frac{S_t}{R_t} - S_{t-1} = NX_t - \overline{H}^S D_t + \Pi_t^{FI} + \left(\frac{\overline{H}^B}{R_t} - \overline{H}^B\right) \\
\Leftrightarrow \quad & \frac{S_t}{R_t} = \frac{\overline{H}^B}{R_t} + P_t^S \overline{H}^S + NFA_t.
\end{aligned}
\tag{3.C.15}
$$

Therefore, the total saving in the home country is sum of the total value of assets, bonds and stocks, and value of NFA.

3.C.3 Steady state

Here, we derive the steady state and log-linearize the model around the stochastic steady state. It should be noted that the argument here for cross-border stock and bond trading model nests the case of the benchmark model. Also, it should be noted that we dispense the Euler equation without the price term as we consider the IRBC model in which a monetary policy completely stabilizes price and ensures zero inflation. Therefore, the Euler equation around the stochastic steady state, described in the previous section, include the price term, we ignore those terms.

The variables governing the trade between the home and foreign in financial markets and good markets and the auxiliary variable, the term of trade and the current account are:

$$\begin{aligned}
\text{Net export:} \quad & NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft}, \\
\text{Net factor payment:} \quad & NFP_t = \mathcal{E}_t D^*_t H^{S*}_{Ht-1} + \mathcal{E}_t \left[1 - \frac{1}{R^*_{t-1}}\right] H^{B*}_{Ht-1} - D_t H^S_{Ft-1} - \left[1 - \frac{1}{R_{t-1}}\right] H^B_{Ft-1}, \\
\text{Term of trade:} \quad & \mathcal{S}_t \equiv \frac{P_{Ft}}{P^*_{Ht}\mathcal{E}_t}, \\
\text{Curent account:} \quad & CA_t \equiv NX_t + NFP_t.
\end{aligned} \tag{3.C.16}$$

At the steady state, the growth of the NFA position is 0. Therefore, the sum of the current account and the valuation effect term is also 0 at the steady state. Remember the formula for the valuation effect is:

$$VE_t \equiv \left(\mathcal{E}_t P^{S*}_t - \mathcal{E}_{t-1} P^{S*}_{t-1}\right) H^{S*}_{Ht-1} + \left(\mathcal{E}_t - \mathcal{E}_{t-1}\right) \frac{H^{B*}_{Ht-1}}{R^*_{t-1}} - \left(P^S_t - P^S_{t-1}\right) H^S_{Ft-1}, \tag{3.C.17}$$

which also should be 0 at the steady state. Then, the net export and the net factor payment are balanced out at the steady state,

$$\overline{NX} + \overline{NFP} = 0. \tag{3.C.18}$$

Note that the net export and the net factor payment are all balanced in the case of symmetric two country, $\overline{NFP} = 0$ and $\overline{NX} = 0$. But this is not the case here. We are considering the economy in which two countries are asymmetry, in one of which the short-term rate is low and in the other that rate is high. As for the prices, we normalize $\overline{P} = \overline{P}^* = 1$. We also assume the economy starts with $\overline{\mathcal{E}} = \overline{\mathcal{S}} = 1$ by normalization. Given those, we have the marginal cost in symmetric steady state as, $\overline{MC} = \overline{MC}^* = (\theta - 1)/\theta$. The nominal wage follows the same value, $\overline{W} = \overline{W}^* = (\theta - 1)/\theta$. Referring the risky steady state argument, the interest rate at the steady state follows:

$$\begin{aligned}
\overline{R} &= \frac{1}{\beta\left[1 + \sigma(\sigma+1)\overline{\mathrm{Var}_t}\left(c_{t+1}\right)\right]}, \\
\overline{R}^* &= \frac{1}{\beta^*\left[1 + \sigma(\sigma+1)\overline{\mathrm{Var}_t}\left(c^*_{t+1}\right)\right]},
\end{aligned} \tag{3.C.19}$$

where $\overline{\mathrm{Var}_t}\left(c_{t+1}\right)$ is the conditional variance of consumption deviation from the steady state. Next, product and factor market clearing in an asymmetric steady state requires:

$$\begin{aligned} \overline{Y} &= \overline{C} + \overline{NX}, \quad \overline{C}^{\sigma}\overline{Y}^{\phi} = \overline{W} = \frac{\theta - 1}{\theta}, \\ \overline{Y}^{*} &= \overline{C}^{*} - \frac{\overline{NX}}{\overline{\mathcal{E}}}, \quad \overline{C}^{*\sigma}\overline{Y}^{*\phi} = \overline{W}^{*} = \frac{\theta - 1}{\theta}. \end{aligned} \tag{3.C.20}$$

From the local demands in domestic and foreign, the net export at the steady state is

$$\overline{NX} = \overline{Y}_H^{*} - \overline{Y}_F = \gamma^{*}\overline{C}^{*} - \gamma\overline{C}. \tag{3.C.21}$$

The dividend is determined by the economic outcome,

$$\begin{aligned} \overline{D} &= \overline{PY} - \overline{WL} = \left(1 - \frac{1}{\mu}\right)\overline{Y}, \\ \overline{D}^{*} &= \overline{P}^{*}\overline{Y}^{*} - \overline{W}^{*}\overline{L}^{*} = \left(1 - \frac{1}{\mu}\right)\overline{Y}^{*}, \end{aligned} \tag{3.C.22}$$

where we define the markup, $\mu \equiv \theta / \left(\theta - 1\right)$.

Let move on to the financial sector, now we should recall we give the following exogenous parameters, ω/m, ω^{*}/m^{*}, $\overline{H}^{S}$ and $\overline{H}^{S*}$ into the model. The stock return in a steady state is $\overline{R}^{S} = \left(\overline{P}^{S} + \overline{D}\right)/\overline{P}^{S}$ and a similar equity return applies to the foreign. Using those conditions, we rewrite the portfolio choice equation and the market clearing for stock as:

$$\begin{aligned} \begin{pmatrix} \overline{P}^{S}\overline{H}_H^{S} \\ \overline{\mathcal{E}}\overline{P}^{S*}\overline{H}_H^{S*} \\ \overline{\mathcal{E}}\frac{\overline{H}_H^{B*}}{\overline{R}^{*}} \end{pmatrix} &= \frac{\Sigma^{-1}}{\frac{\omega}{m}} \begin{pmatrix} \frac{\left(\overline{P}^{S}+\overline{D}\right)/\overline{P}^{S}}{\overline{R}} - 1 \\ \frac{\left(\overline{P}^{S*}+\overline{D}^{*}\right)/\overline{P}^{S*}}{\overline{R}} - 1 \\ \frac{\overline{R}^{*}}{\overline{R}} - 1 \end{pmatrix}, \\ \begin{pmatrix} \overline{P}^{S*}\overline{H}_F^{S*} \\ \frac{1}{\overline{\mathcal{E}}}\overline{P}^{S}\overline{H}_F^{S} \\ \frac{1}{\overline{\mathcal{E}}}\frac{\overline{H}_F^{B}}{\overline{R}} \end{pmatrix} &= \frac{\Sigma^{*-1}}{\frac{\omega^{*}}{m^{*}}} \begin{pmatrix} \frac{\left(\overline{P}^{S*}+\overline{D}^{*}\right)/\overline{P}^{S*}}{\overline{R}^{*}} - 1 \\ \frac{\left(\overline{P}^{S}+\overline{D}\right)/\overline{P}^{S}}{\overline{R}^{*}} - 1 \\ \frac{\overline{R}}{\overline{R}^{*}} - 1 \end{pmatrix}, \end{aligned} \tag{3.C.23}$$

$$\overline{H}_H^S + \overline{H}_F^S = \overline{H}^S,$$
$$\overline{H}_F^{S*} + \overline{H}_H^{S*} = \overline{H}^{S*}. \tag{3.C.24}$$

Notice that the net factor payment at the steady state is:

$$\overline{NFP} = \overline{\mathcal{E}}\overline{D}^*\overline{H}_H^{S*} + \overline{\mathcal{E}}\left[1 - \frac{1}{\overline{R}^*}\right]\overline{H}_H^{B*} - \overline{D}\overline{H}_F^S - \left[1 - \frac{1}{\overline{R}}\right]\overline{H}_F^B. \tag{3.C.25}$$

We have the following 16 unknowns at the steady state, $\overline{P}^S, \overline{P}^{S*}, \overline{H}_H^S, \overline{H}_F^S, \overline{H}_F^{S*}, \overline{H}_H^{S*}, \overline{H}_H^{B*}, \overline{H}_F^B$, $\overline{NFP}, \overline{NX}, \overline{C}, \overline{Y}, \overline{D}, \overline{C}^*, \overline{Y}^*, \overline{D}^*$. We use the equations (3.C.18), (3.C.20), (3.C.22), (3.C.23), (3.C.24), (3.C.25), which are in total 16 equations, to solve out the steady state values of unknowns numerically. Note that values for $\boldsymbol{\Sigma}$, $\boldsymbol{\Sigma}^*$, $\overline{R}$ and $\overline{R}^*$ are attained after solving the model with some specific shock process. Once we solve out the steady state values, we would get the second order moments again with given shock process. We iterate this procedure until the second order moments and the steady state values are converged.

3.C.4 Log-linearized system

We consider here the log-linearized system around the stochastic steady state which we describe in the previous section. We describe the log-linear expression under the which a monetary policy ensure zero inflation rate. In the following, we define the log-linearized term, $x_t \equiv \mathrm{d}\log X_t$. We only describe the log-linearized expression for every equation that we need for home country for the sake of brevity.

$$
\begin{aligned}
&\text{Euler equation:} && \left[1+\sigma(\sigma+1)\overline{\sigma}_c^2\right]\{\sigma\left(c_{t+1}-c_t\right)-r_t\} = -2\sigma(\sigma+1)\overline{\sigma}_c^2 c_{t+1} \\
&\text{Labor supply:} && \sigma c_t + \phi \ell_t = w_t - p_t \\
&\text{Production function:} && y_t = a_t + \ell_t \\
&\text{Marginal cost:} && mc_t = -a_t + w_t \\
&\text{Price setting in the domestic:} && p_{Ht} = mc_t \\
&\text{Price setting in the foreign:} && p_{Ht}^* = mc_t - e_t \\
&\text{Price index:} && p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft} \\
&\text{Local demand in the home:} && y_{Ht} = -\theta\left(p_{Ht}-p_t\right) + c_t \\
&\text{Local demand in the foreign:} && y_{Ht}^* = -\theta\left(p_H^* - p_t^*\right) + c_t^* \\
&\text{Goods market clearing:} && (1-\gamma)\,\overline{C}y_{Ht} + \gamma^*\overline{C}^* y_{Ht}^* \\
&\text{Firm profits:} && \left(1-\frac{1}{\mu}\right)d_t = \frac{(1-\gamma)\overline{C}}{\overline{Y}}\left[\left(p_{Ht}+y_{Ht}\right) - \frac{1}{\mu}\left(mc_t + y_{Ht}\right)\right] + \frac{\gamma^*\overline{C}^*}{\overline{Y}}\left[\left(p_{Ht}^* + e_t + y_{Ht}^*\right) - \frac{1}{\mu}\left(mc_t + y_{Ht}^*\right)\right] \\
&\text{Stock return:} && \overline{R}^S r_{t+1}^S = \left(p_{t+1}^S - p_t^S\right) + \left(\overline{R}^S - 1\right)\left(d_{t+1} - p_t^S\right) \\
&\text{Monetary policy:} && r_t = \rho_m r_{t-1} + (1-\rho_m)\,\phi_\pi \pi_t + \sigma_m \varepsilon_t^m, \quad \varepsilon_t^m \sim iid(0,1) \\
&\text{Productivity shock process:} && a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \quad \varepsilon_t^a \sim iid(0,1) \\
&\text{Risk aversion shock process:} && \xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_t^\xi, \quad \varepsilon_t^\xi \sim iid(0,1) \\
&\text{Portfolio choice:} && \begin{pmatrix} \overline{P}^S\overline{H}_H^S\left(p_t^S + h_{Ht}^S + \xi_t\right) \\ \overline{P}^{S*}\overline{H}_H^{S*}\left(e_t + p_t^{S*} + h_{Ht}^{S*} + \xi_t\right) \\ \frac{\overline{H}_H^{B*}}{\overline{R}^*}\left(e_t + h_{Ht}^{B*} - r_t^* + \xi_t\right) \end{pmatrix} = \frac{\Sigma^{-1}}{\frac{\omega}{m}} \begin{pmatrix} \frac{\overline{R}^S}{\overline{R}}\left(r_{t+1}^S - r_t\right) \\ \frac{\overline{R}^{S*}}{\overline{R}}\left(e_{t+1} - e_t + r_{t+1}^{S*} - r_t\right) \\ \frac{\overline{R}^*}{\overline{R}}\left(e_{t+1} - e_t + r_t^* - r_t\right) \end{pmatrix} \\
&\text{Net-zero position of arbitrageurs:} && \overline{P}^S\overline{H}_H^S\left(p_t^S + h_{Ht}^S\right) + \frac{\overline{H}_H^B}{\overline{R}}\left(h_{Ht}^B - r_t\right) + \overline{\mathcal{E}P}^{S*}\overline{H}_H^{S*}\left(e_t + p_t^{S*} + h_{Ht}^{S*}\right) + \overline{\mathcal{E}}\frac{\overline{H}_H^{B*}}{\overline{R}^*}\left(e_t + h_{Ht}^{B*} - r_t^*\right) = 0 \\
&\text{Stock market clearing:} && \overline{H}_H^S h_{Ht}^S + \overline{H}_F^S h_{Ft}^S = 0 \\
&\text{Bond market clearing:} && \overline{S}s_t + \overline{H}_H^B h_{Ht}^B + \overline{H}_F^B h_{Ft}^B = 0 \\
&\text{Net export:} && \overline{NX}nx_t = \gamma^*\overline{C}^*\left(e_t + p_{Ht}^* + y_{Ht}^*\right) - \gamma\overline{C}\left(p_{Ft} + y_{Ft}\right) \\
&\text{Net factor payment:} && \overline{NFP}nfp_t = \left(e_t + d_t^* + h_{Ht-1}^{S*}\right)\overline{D}^*\overline{H}_H^{S*} + \left(e_t + h_{Ht-1}^{B*}\right)\overline{H}_H^{B*} - \left(e_t - r_{t-1}^* + h_{Ht-1}^{B*}\right)\frac{\overline{H}_H^{B*}}{\overline{R}^*} \\
& && - \left(d_t + h_{Ft-1}^S\right)\overline{DH}_F^S - h_{Ft-1}^B\overline{H}_F^B + \left(h_{Ft-1}^B - r_{t-1}\right)\frac{\overline{H}_F^B}{\overline{R}} \\
&\text{Current account:} && ca_t = \frac{\overline{NFP}}{\overline{PY}}nfp_t + \frac{\overline{NX}}{\overline{PY}}nx_t
\end{aligned}
\tag{3.C.26}
$$

The equations for foreign country is obtained in the similar way.

3.C.5 Simulation results

Finally, we present the results of simulations in an environment where financial intermediaries in both countries have access to all assets. Calibrations here are the same parameters used in the benchmark model without hedging in the main text.

Figure 3.C.1 shows the IRFs with shocks that make financial intermediaries in each country risk averse. The home country currency appreciates when financial intermediaries in either country become risk averse. This is identical to the results of the benchmark model.

Table 3.C.1 shows the simulation results under the full asset trading model in this section. A unique feature is that the investment in equities is the same between the two countries. This is because the inclusion of foreign bonds in the portfolio hedges the foreign exchange risk associated with holding foreign stocks. Since the risk inherent in equities (Productivity shock) is the same in both countries, the amount of equities held will be the same in both countries. In addition, the model is similar to the benchmark model in that when the risk-averse behavior of financial intermediaries in both countries is positively correlated, the relative volatility of the exchange rate relative to equities is lower.

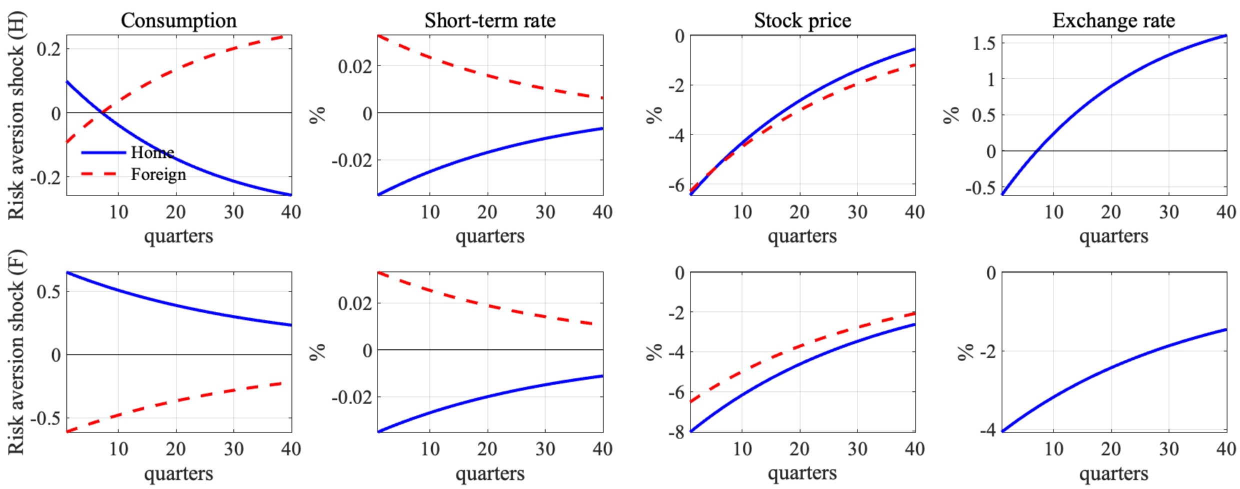

Figure 3.C.1: The impulse responses for 1% positive shock on the risk aversion parameter of investors under an asymmetric case

NOTE.—The unit of measure is percent. The top line represents shock for the home investors, and the bottom line is for the foreign investors. The home country is a low-interest rate country, and the foreign is a high-interest rate country. We show the case of the correlation of the risk aversion shock, corr(ξ, ξ^*), is 0.5.

Table 3.C.1: Moments generated in the model

Moments	Data	Model			
		Symmetric		Asymmetric	
A. Exchange rate disconnect					
$\rho(\Delta e)$	≈ 0	-0.04	-0.04	-0.03	-0.02
$\sigma(\Delta e)/\sigma(\Delta gdp)$	5.20	1.92	1.68	4.24	3.92
$\sigma(\Delta e)/\sigma(\Delta c)$	6.30	2.67	2.30	6.15	5.74
B. Backus-Smith correlation					
$\text{corr}\left(\Delta q, \Delta c - \Delta c^*\right)$	-0.40	-0.38	-0.20	-0.95	-0.96
C. Business cycle moments					
$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.82	0.72	0.74	0.69	0.68
$\text{corr}(\Delta c, \Delta gdp)$	0.68	0.50	0.64	-0.72	-0.74
$\text{corr}\left(\Delta gdp, \Delta gdp^*\right)$	0.34	-0.33	-0.26	-0.88	-0.89
$\text{corr}\left(\Delta c, \Delta c^*\right)$	0.26	0.30	0.40	-0.73	-0.75
D. Exchange rates and Stock prices					
$\sigma(\Delta e)$	0.18	0.04	0.03	0.19	0.18
$\sigma(\Delta p^S)$	0.34	0.25	0.36	0.33	0.51
$\sigma(\Delta p^{S*})$	0.34	0.25	0.36	0.31	0.45
$\sigma(\Delta(p^S - p^{S*}))$	0.23	0.03	0.02	0.05	0.07
$\text{corr}(\Delta p^S, \Delta p^{S*})$	0.74	1.00	1.00	0.99	1.00
$\text{corr}(\Delta e, \Delta p^S)$	>0	0.05	0.02	0.88	0.94
$\text{corr}(\Delta e, \Delta p^{S*})$		-0.04	-0.03	0.81	0.92
$\sigma(\Delta c)$	0.03	0.01	0.01	0.03	0.03
$\text{corr}(\Delta c, \Delta p^S)$		0.04	0.03	-0.79	-0.86
$\text{corr}(\Delta c^*, \Delta p^{S*})$		0.07	0.04	0.75	0.86
E. Steady state					
$\overline{R}$		1.01	1.01	1.00	1.00
$\overline{R}^S$		1.02	1.03	1.02	1.03
$\overline{R}^*$		1.01	1.01	1.01	1.01
$\overline{R}^{S*}$		1.02	1.03	1.03	1.04
$\overline{H}_H^S$		0.10	0.10	0.10	0.10
$\overline{H}_H^{S*}$		0.10	0.10	0.10	0.10
Calibration parameter					
$\text{corr}(\xi, \xi^*)$		0.00	0.50	0.00	0.50
β_1		0.990	0.990	0.995	0.995
β_2		0.990	0.990	0.985	0.985
γ^*		0.070	0.070	0.040	0.049

NOTE.—This panel reports the simulation results of median moments across 1,000 simulations of 120 quarters. The trade openness parameter for the foreign γ^* is an equilibrium outcome. If this value is higher than γ, the home country is the excess export country. $\text{corr}(\Delta e, \Delta p^{S*})$ supposed to be positive if there is an interest differential between safe rates in two countries.

Bibliography

Aggarwal, R., Auclert, A., Rognlie, M., Straub, L., et al. (2022). Excess savings and twin deficits: The transmission of fiscal stimulus in open economies. *NBER Chapters*.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684.

Aiyagari, S. R. and McGrattan, E. R. (1998). The optimum quantity of debt. *Journal of Monetary Economics*, 42(3):447–469.

Alvarez, F., Atkeson, A., and Kehoe, P. J. (2002). Money, interest rates, and exchange rates with endogenously segmented markets. *Journal of political Economy*, 110(1):73–112.

Alvarez, F., Atkeson, A., and Kehoe, P. J. (2009). Time-varying risk, interest rates, and exchange rates in general equilibrium. 76(3):851–878.

Alvarez, F., Lucas, R. E., and Weber, W. E. (2001). Interest rates and inflation. *American Economic Review*, 91(2):219–225.

Amador, M., Bianchi, J., Bocola, L., and Perri, F. (2018). Foreign reserve management. *Unpublished Manuscript, Federal Reserve Bank of Minneapolis*.

Angeletos, G.-M. (2007). Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic dynamics*, 10(1):1–30.

Atkeson, A. and Burstein, A. T. (2010). Innovation, firm dynamics, and international trade. *Journal of political economy*, 118(3):433–484.

Atkeson, A., Chari, V. V., and Kehoe, P. J. (2010). Sophisticated monetary policies. *The Quarterly Journal of Economics*, 125(1):47–89.

Atkeson, A., Heathcote, J., and Perri, F. (2020). The end of privilege. mimeo.

Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021a). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408.

Auclert, A. and Rognlie, M. (2018). Inequality and aggregate demand. Technical report, National Bureau of Economic Research.

Auclert, A., Rognlie, M., Souchier, M., and Straub, L. (2021b). Exchange rates and monetary policy with heterogeneous agents: Sizing up the real income channel. Technical report, National Bureau of Economic Research.

Azzimonti, M., De Francisco, E., and Quadrini, V. (2014). Financial globalization, inequality, and the rising public debt. *American Economic Review*, 104(8):2267–2302.

Bacchetta, P., Tieche, S., and Van Wincoop, E. (2020). International portfolio choice with frictions: Evidence from mutual funds. *Swiss Finance Institute Research Paper*, (20-46).

Backus, D. K. and Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35(3-4):297–316.

Bassetto, M. and Sargent, T. J. (2020). Shotgun wedding: Fiscal and monetary policy. Technical report, National Bureau of Economic Research.

Basu, S. and Bundick, B. (2018). Uncertainty shocks in a model of effective demand: Reply. *Econometrica*, 86(4):1527–1531.

Benhabib, J., Bisin, A., and Zhu, S. (2015). The wealth distribution in bewley economies with capital income risk. *Journal of Economic Theory*, 159:489–515.

Berger, D., Dew-Becker, I., and Giglio, S. (2020). Uncertainty shocks as second-moment news shocks. *The Review of Economic Studies*, 87(1):40–76.

Bewley, T. (1983). A difficulty with the optimum quantity of money. *Econometrica: Journal of the Econometric Society*, pages 1485–1504.

Bhattarai, S., Lee, J. W., and Park, W. Y. (2012). Monetary-fiscal policy interactions and indeterminacy in postwar us data. *American Economic Review*, 102(3):173–78.

Bhattarai, S., Lee, J. W., and Park, W. Y. (2014). Inflation dynamics: The role of public debt and policy regimes. *Journal of Monetary Economics*, 67:93–108.

Bhattarai, S., Lee, J. W., and Park, W. Y. (2016). Policy regimes, policy shifts, and us business cycles. *Review of Economics and Statistics*, 98(5):968–983.

Bianchi, F. and Ilut, C. (2017). Monetary/fiscal policy mix and agents' beliefs. *Review of Economic Dynamics*, 26:113–139.

Bianchi, F. and Melosi, L. (2017). Escaping the great recession. *American Economic Review*, 107(4):1030–58.

Bianchi, J., Bigio, S., and Engel, C. (2021). Scrambling for dollars: International liquidity, banks and exchange rates. Technical report, National Bureau of Economic Research.

Bianchi, J. and Lorenzoni, G. (2021). The prudential use of capital controls and foreign currency reserves. Technical report, National Bureau of Economic Research.

Bianchi, J. and Sosa-Padilla, C. (2020). Reserve accumulation, macroeconomic stabilization, and sovereign risk. Technical report, National Bureau of Economic Research.

Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of economic theory*, 140(1):162–196.

Bilbiie, F. O. (2018). Monetary policy and heterogeneity: An analytical framework. *Available at SSRN 3106805*.

Bilbiie, F. O. and Straub, R. (2013). Asset market participation, monetary policy rules, and the great inflation. *Review of Economics and Statistics*, 95(2):377–392.

Blanchard, O. (2019). Public debt and low interest rates. *American Economic Review*, 109(4):1197–1229.

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of political economy*, 93(2):223–247.

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311.

Boar, C. and Midrigan, V. (2019). Markups and inequality. Technical report, National Bureau of Economic Research.

Boz, E., Casas, C., Georgiadis, G., Gopinath, G., Le Mezo, H., Mehl, A., and Nguyen, T. (2020). Patterns in invoicing currency in global trade.

Brandt, M. W., Cochrane, J. H., and Santa-Clara, P. (2006). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics*, 53(4):671–698.

Brunnermeier, M. K., Merkel, S. A., and Sannikov, Y. (2020). The fiscal theory of price level with a bubble. Technical report, National Bureau of Economic Research.

Brunnermeier, M. K., Nagel, S., and Pedersen, L. H. (2008). Carry trades and currency crashes. *NBER macroeconomics annual*, 23(1):313–348.

Brunnermeier, M. K. and Sannikov, Y. (2016). The i theory of money. Technical report, National Bureau of Economic Research.

Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and development: A tale of two sectors. *American economic review*, 101(5):1964–2002.

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2008). An equilibrium model of" global imbalances" and low interest rates. *American economic review*, 98(1):358–93.

Caballero, R. J. and Simsek, A. (2022). A monetary policy asset pricing model. *Available at SSRN 4113332*.

Camanho, N., Hau, H., and Rey, H. (2018). Global portfolio rebalancing and exchange rates. Technical report, National Bureau of Economic Research.

Campbell, J. Y. (2017). *Financial decisions and markets: a course in asset pricing*. Princeton University Press.

Campbell, J. Y. and Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of political Economy*, 107(2):205–251.

Campbell, J. Y., Lettau, M., Malkiel, B. G., and Xu, Y. (2001). Have individual stocks become more volatile? an empirical exploration of idiosyncratic risk. *The journal of finance*, 56(1):1–43.

Campbell, J. Y., Lettau, M., Malkiel, B. G., and Xu, Y. (2022). Idiosyncratic equity risk two decades later. *Available at SSRN*.

Campbell, J. Y. and Viceira, L. M. (2002). *Strategic asset allocation: portfolio choice for long-term investors*. Clarendon Lectures in Economic.

Caramp, N. and Silva, D. H. (2021). Monetary policy and wealth effects: The role of risk and heterogeneity. Technical report, Working Paper.

Caramp, N. and Silva, D. H. (2022). Fiscal policy and the monetary transmission mechanism. Mimeo.

Chahrour, R., Valchev, R., et al. (2021). Trade finance and the durability of the dollar. *Review of Economic Studies*.

Chen, Z. (2021). Global safe assets and the us exorbitant privilege. *Available at SSRN 3963553*.

Chien, Y., Lustig, H., and Naknoi, K. (2020). Why are exchange rates so smooth? a household finance explanation. *Journal of Monetary Economics*, 112:129–144.

Chien, Y. and Naknoi, K. (2015). The risk premium and long-run global imbalances. *Journal of Monetary Economics*, 76:299–315.

Christiano, L. J. and Takahashi, Y. (2018). Discouraging deviant behavior in monetary economics. Mimeo.

Cochrane, J. H. (2021). A fiscal theory of monetary policy with partially-repaid long-term debt. *Review of Economic Dynamics*.

Cochrane, J. H. (2022). *The Fiscal Theory of the Price Level*. Princeton University Press.

Coeurdacier, N. and Rey, H. (2013). Home bias in open economy financial macroeconomics. *Journal of Economic Literature*, 51(1):63–115.

Coeurdacier, N., Rey, H., and Winant, P. (2011). The risky steady state. *American Economic Review*, 101(3):398–401.

Coeurdacier, N., Rey, H., and Winant, P. (2019). Financial integration and growth in a risky world. *Journal of Monetary Economics*.

Conesa, J. C., Kitao, S., and Krueger, D. (2009). Taxing capital? not a bad idea after all! *American Economic Review*, 99(1):25–48.

Constancio, V. (2015). Understanding inflation dynamics and monetary policy. In *Speech at the Jackson Hole Economic Policy Symposium*, volume 29.

Cormun, V. and De Leo, P. (2020). Shocks and exchange rates in small open economies.

Cúrdia, V. and Woodford, M. (2016). Credit frictions and optimal monetary policy. *Journal of Monetary Economics*, 84:30–65.

De Loecker, J., Eeckhout, J., and Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, 135(2):561–644.

Debortoli, D. and Galí, J. (2017). Monetary policy with heterogeneous agents: Insights from tank models.

Dou, W. W. and Verdelhan, A. (2015). The volatility of international capital flows and foreign assets. Technical report, Working Paper, MIT.

Drechsler, I., Savov, A., and Schnabl, P. (2018). A model of monetary policy and risk premia. *The Journal of Finance*, 73(1):317–373.

Du, W., Im, J., and Schreger, J. (2018). The us treasury premium. *Journal of International Economics*, 112:167–181.

Edmond, C., Midrigan, V., and Xu, D. Y. (2018). How costly are markups? Technical report, National Bureau of Economic Research.

Fang, X. and Liu, Y. (2021). Volatility, intermediaries, and exchange rates. *Journal of Financial Economics*, 141(1):217–233.

Farhi, E. and Gabaix, X. (2016). Rare disasters and exchange rates. *The Quarterly Journal of Economics*, 131(1):1–52.

Farhi, E. and Maggiori, M. (2018). A model of the international monetary system. *The Quarterly Journal of Economics*, 133(1):295–355.

Fogli, A. and Perri, F. (2015). Macroeconomic volatility and external imbalances. *Journal of Monetary Economics*, 69:1–15.

French, K. R. and Poterba, J. M. (1991). Investor diversification and international equity markets. *The American Economic Review*, 81(2):222–226.

Gabaix, X. and Koijen, R. S. (2021). In search of the origins of financial fluctuations: The inelastic markets hypothesis. Technical report, National Bureau of Economic Research.

Gabaix, X., Lasry, J.-M., Lions, P.-L., and Moll, B. (2016). The dynamics of inequality. *Econometrica*, 84(6):2071–2111.

Gabaix, X. and Maggiori, M. (2015). International liquidity and exchange rate dynamics. *The Quarterly Journal of Economics*, 130(3):1369–1420.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Garín, J., Lester, R., and Sims, E. (2018). Raise rates to raise inflation? neo-fisherianism in the new keynesian model. *Journal of Money, Credit and Banking*, 50(1):243–259.

Gertler, M., Kiyotaki, N., and Queralto, A. (2012). Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics*, 59:S17–S34.

Giglio, S., Maggiori, M., Stroebel, J., and Utkus, S. (2019). Five facts about beliefs and portfolios. Technical report, National Bureau of Economic Research.

Gopinath, G., Boz, E., Casas, C., Díez, F. J., Gourinchas, P.-O., and Plagborg-Møller, M. (2020). Dominant currency paradigm. *American Economic Review*, 110(3):677–719.

Gopinath, G. and Itskhoki, O. (2021). Dominant currency paradigm: a review.

Gopinath, G. and Stein, J. C. (2020). Banking, Trade, and the Making of a Dominant Currency*. *The Quarterly Journal of Economics*, 136(2):783–830.

Gourinchas, P.-O., Ray, W., and Vayanos, D. (2020). A preferred-habitat model of term premia and currency risk. Technical report, National Bureau of Economic Research.

Gourinchas, P.-O. and Rey, H. (2007a). From world banker to world venture capitalist: Us external adjustment and the exorbitant privilege. In *G7 current account imbalances: sustainability and adjustment*, pages 11–66. University of Chicago Press.

Gourinchas, P.-O. and Rey, H. (2007b). International financial adjustment. *Journal of political economy*, 115(4):665–703.

Gourinchas, P.-O. and Rey, H. (2014). External adjustment, global imbalances, valuation effects. In *Handbook of international economics*, volume 4, pages 585–645. Elsevier.

Gourinchas, P.-O., Rey, H., Govillot, N., et al. (2010). Exorbitant privilege and exorbitant duty. Technical report, Institute for Monetary and Economic Studies, Bank of Japan Tokyo.

Greenwald, D. L., Lettau, M., and Ludvigson, S. C. (2014). Origins of stock market fluctuations. Technical report, National Bureau of Economic Research.

Greenwald, D. L., Lettau, M., and Ludvigson, S. C. (2019). How the wealth was won: Factors shares as market fundamentals. Technical report, National Bureau of Economic Research.

Greenwood, R., Hanson, S. G., Stein, J. C., and Sunderam, A. (2019). A quantity-driven theory of term premiums and exchange rates. *mimeo*.

Guvenen, F., Kambourov, G., Kuruscu, B., and Ocampo, S. (2022). Taxing wealth and capital income with heterogeneous returns. Mimeo.

Hansen, L. P. and Sargent, T. J. (2011). Robustness and ambiguity in continuous time. *Journal of Economic Theory*, 146(3):1195–1223.

Hassan, T. A. (2013). Country size, currency unions, and international asset returns. *The Journal of Finance*, 68(6):2269–2308.

Hau, H. and Rey, H. (2006). Exchange rates, equity prices, and capital flows. *The Review of Financial Studies*, 19(1):273–317.

Heathcote, J. (2005). Fiscal policy with heterogeneous agents and incomplete markets. *The Review of Economic Studies*, 72(1):161–188.

Heathcote, J., Storesletten, K., and Violante, G. L. (2017). Optimal tax progressivity: An analytical framework. *The Quarterly Journal of Economics*, 132(4):1693–1754.

Heise, S., Karahan, F., and Şahin, A. (2022). The missing inflation puzzle: The role of the wage-price pass-through. *Journal of Money, Credit and Banking*, 54(S1):7–51.

Hilscher, J., Raviv, A., and Reis, R. (2022). Inflating away the public debt? an empirical assessment. *The Review of Financial Studies*, 35(3):1553–1595.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of economic Dynamics and Control*, 17(5-6):953–969.

Itskhoki, O. and Moll, B. (2019). Optimal development policies with financial frictions. *Econometrica*, 87(1):139–173.

Itskhoki, O. and Mukhin, D. (2019a). Exchange rate disconnect in general eqilibrium. Technical report, Princeton University.

Itskhoki, O. and Mukhin, D. (2019b). Mussa puzzle redux. Technical report, Princeton University.

Itskhoki, O. and Mukhin, D. (2021). Exchange rate disconnect in general equilibrium. *Journal of Political Economy*, 129(8):2183–2232.

Jiang, Z. (2021). Us fiscal cycle and the dollar. *Journal of Monetary Economics*, 124:91–106.

Jiang, Z. (2022). Fiscal cyclicality and currency risk premia. *The Review of Financial Studies*, 35(3):1527–1552.

Jiang, Z., Krishnamurthy, A., and Lustig, H. (2018). Foreign safe asset demand for us treasurys and the dollar. In *AEA Papers and Proceedings*, volume 108, pages 537–41.

Jiang, Z., Krishnamurthy, A., and Lustig, H. (2019). Dollar safety and the global financial cycle. mimeo.

Juillard, M. (2011). Local approximation of dsge models around the risky steady state. *Wp. comunite*, 87:21–42.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.

Kekre, R. and Lenel, M. (2021a). The flight to safety and international risk sharing. Technical report, National Bureau of Economic Research.

Kekre, R. and Lenel, M. (2021b). Monetary policy, redistribution, and risk premia. Technical report, National Bureau of Economic Research.

Koijen, R. S. and Yogo, M. (2019a). A demand system approach to asset pricing. *Journal of Political Economy*, 127(4):1475–1515.

Koijen, R. S. J. and Yogo, M. (2019b). A demand system approach to asset pricing. *Journal Political Economy*, 127(4):1475–1515.

Koijen, R. S. J. and Yogo, M. (2019c). Exchange rates and asset prices in a global demand system. mimeo.

Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267.

Krusell, P. and Smith, A. A. (1997). Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns. *Macroeconomic dynamics*, 1(2):387–422.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896.

Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27:129–147.

Leeper, E. M. and Leith, C. (2016). Understanding inflation as a joint monetary–fiscal phenomenon. In *Handbook of Macroeconomics*, volume 2, pages 2305–2415. Elsevier.

Leeper, E. M., Plante, M., and Traum, N. (2010). Dynamics of fiscal financing in the united states. *Journal of Econometrics*, 156(2):304–321.

Liu, Y., Schmid, L., and Yaron, A. (2020). The risks of safe assets. *Available at SSRN 3699618*.

Lopez, P., Lopez-Salido, D., and Vazquez-Grande, F. (2017). Risk-adjusted linearizations of dynamic equilibrium models. *Available at SSRN 2626644*.

Ludvigson, S. C., Steindel, C., et al. (1998). *How important is the stock market effect on consumption?*, volume 9821. Federal Reserve Bank of New York New York.

Lustig, H., Stathopoulos, A., and Verdelhan, A. (2019). The term structure of currency carry trade risk premia. *American Economic Review*, 109(12):4142–77.

Lustig, H. and Verdelhan, A. (2019). Does incomplete spanning in international financial markets help to explain exchange rates? *American Economic Review*, 109(6):2208–44.

Maggiori, M. (2017). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(10):3038–71.

Maggiori, M. (2022). International macroeconomics with imperfect financial markets.

Maggiori, M., Neiman, B., and Schreger, J. (2019). International currencies and capital allocation. mimeo.

Maggiori, M., Neiman, B., and Schreger, J. (2020). International currencies and capital allocation. *Journal of Political Economy*, 128(6):2019–2066.

Mankiw, N. G. (2000). The savers-spenders theory of fiscal policy. *American economic review*, 90(2):120–125.

Meese, R. A. and Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of international economics*, 14(1-2):3–24.

Mendoza, E. G., Quadrini, V., and Rios-Rull, J.-V. (2009). Financial integration, financial development, and global imbalances. *Journal of Political economy*, 117(3):371–416.

Mian, A., Straub, L., Sufi, A., et al. (2021). A goldilocks theory of fiscal policy. *NBER Working Paper*, 29351.

Mian, A. R., Straub, L., and Sufi, A. (2020). The saving glut of the rich. Technical report, National Bureau of Economic Research.

Midrigan, V. and Xu, D. Y. (2014). Finance and misallocation: Evidence from plant-level data. *American economic review*, 104(2):422–58.

Miranda-Agrippino, S. and Rey, H. (2015). Us monetary policy and the global financial cycle. Technical report, National Bureau of Economic Research.

Miranda-Agrippino, S. and Rey, H. (2020a). The global financial cycle after lehman. 110:523–28.

Miranda-Agrippino, S. and Rey, H. (2020b). Us monetary policy and the global financial cycle. *The Review of Economic Studies*, 87(6):2754–2776.

Miranda-Agrippino, S. and Rey, H. (2021). The global financial cycle. Technical report, National Bureau of Economic Research.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, 104(10):3186–3221.

Moskowitz, T. J. and Vissing-Jørgensen, A. (2002). The returns to entrepreneurial investment: A private equity premium puzzle? *American Economic Review*, 92(4):745–778.

Mukhin, D. (2022). An equilibrium model of the international price system. *American Economic Review*, 112(2):650–88.

Nakajima, M. (2007). Solving rbc models with linearized euler equations: Blanchard-kahn method. mimeo.

Navarro, G. and Ferriere, A. (2016). The heterogeneous effects of government spending: It's all about taxes. In *2016 Meeting Papers*, number 1286. Society for Economic Dynamics.

Pavlova, A. and Rigobon, R. (2012). Equilibrium portfolios and external adjustment under incomplete markets. In *AFA 2009 San Francisco Meetings Paper*.

Poterba, J. M. (2000). Stock market wealth and consumption. *Journal of economic perspectives*, 14(2):99–118.

Rey, H. (2015). Dilemma not trilemma: the global financial cycle and monetary policy independence. Technical report, National Bureau of Economic Research.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy*, 66(6):467–482.

Sargent, T. J. and Wallace, N. (1981). Some unpleasant monetarist arithmetic. *Federal reserve bank of minneapolis quarterly review*, 5(3):1–17.

Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56.

Sims, C. A. (2013). Paper money. *American Economic Review*, 103(2):563–84.

Storesletten, K., Telmer, C. I., and Yaron, A. (2004). Cyclical dynamics in idiosyncratic labor market risk. *Journal of political Economy*, 112(3):695–717.

Uribe, M. (2018). The neo-fisher effect: Econometric evidence from empirical and optimizing models. Technical report, National Bureau of Economic Research.

Woodford, M. (2003). Interest and prices.