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Time Domain Green's Function for a Phased Semi-Infinite Periodic Line Array of Dipoles

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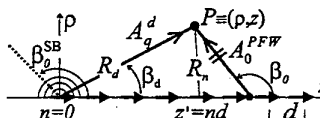1) Dept. Inform. Eng., Universitá di Siena, via Roma 56, Siena, Italy
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I. INTRODUCTION

We have begun a systematic investigation of the time domain (TD) behavior of Green's functions which are relevant for the characterization of truncated planar periodic arrays, with emphasis on the TD Floquet waves (FW) in the propagating *and* evanescent parameter regimes. Our principal aim is to understand the TD wave physics and phenomenologies on simple prototypes, invoking various methods that synthesize the solution from different perspectives. The first prototype, an infinite phased periodic line array of axial dipole radiators arranged along the z-axis of a cylindrical (ρ, z) coordinate system (Fig.1) has been studied in [1], and is reviewed in Sec. III. This prototype is a basic building block for *transversely* truncated plane arrays formed by a finite number of infinite line sources, which has so far been studied only in the frequency domain (FD)[2]. Truncation *along* the axis of a phased line array of dipoles is explored in this paper for the FD *and* TD. This modification of the infinite line dipole array leads to a new set of FD and TD truncation phenomena.

II. STATEMENT OF THE PROBLEM

The geometry of the truncated semi-infinite linear array of dipoles with period d, oriented along the z direction and excited by transient currents in free space, is shown in Fig.1. The E field is related to the z-directed magnetic scalar potential A which shall be used throughout. The FD $(\exp(j\omega t))$ and TD currents $J(\omega)$ and $\hat{J}(t)$, respectively, due to the dipoles, are related via a Fourier transform pair,

$$\left. \begin{array}{c} J(\omega) \\ \hat{J}(t) \end{array} \right\} = \sum_{n=0}^{\infty} \delta(z' - nd) \left\{ \begin{array}{c} \exp(-j\omega\gamma_z nd) \\ \delta(t - \gamma_z nd) \end{array} \right\} \tag{1}$$

Fig. 1

with the caret $\wedge$ denoting time-dependent functions. In the n-tagged element current amplitudes multiplying the delta function in (1), the FD portion $\omega\gamma_z d$ accounts for an assumed (linear) phase difference between adjacent elements, with γ_z denoting the interelement phase gradient normalized with respect to ω. The TD portion identifies sequentially pulsed dipoles, with the element at $z' = nd$ turned on at time $t_n = \gamma_z nd$. In this paper, we only consider the radiating case $v_z^{(p)} = \gamma_z^{-1} > c$, where c is the ambient propagation wave speed.

The *truncated* Poisson summation formula plays an essential role in converting the collective radiation from the truncated periodic array of *individual* phased *dipole* radiators into an infinite superposition of truncated linearly *smoothly* phased equivalent *line source* radiators. For the series of spherical wave fields A_n due to the n-indexed individual dipole radiators, the formula is

$$\sum_{n=0}^{\infty} A_n = \frac{A_0}{2} + \sum_{q=-\infty}^{\infty} \tilde{A}_q , \quad \tilde{A}_q = \int_0^{\infty} A(\nu) e^{j2\pi q\nu} d\nu, \tag{2}$$

which converts the n-series fields into a q-series whose summands $\tilde{A}_q$ are the truncated Fourier transforms of the smoothed-out spherical wave function $A(\nu)$, sampled at integer multiples of 2π. Since n and therefore ν in (2) are tied only to a spatial coordinate (see(1)), A_n and $A(\nu)$ can be either functions of ω or t.

III. FD- AND TD- FLOQUET WAVES FOR THE INFINITE ARRAY

Results pertaining to the *infinite* prototype phased (sequentially excited) periodic line array of dipoles in [1] are now summarized. In this case the n-sums in (1), (2), and Fig.1 extend from $-\infty$ to $+\infty$, the truncation term $A_0/2$ in (2) is omitted, and the $\tilde{A}_q$ integral, now from $\nu = -\infty$ to $+\infty$, becomes the complete Fourier transform (FT). The "infinite version" of (2) is written (2∞).

Frequency domain FW. Applying (2∞) to the radiated spherical FD fields from each dipole [1] yields for $\tilde{A}_q$ at $\mathbf{r} = (\rho, z)$ a tabulated FT which is the FD-FW$_q$ smoothly phased line source field

$$A_q^{FW}(\mathbf{r}, \omega) = \frac{e^{-jk_{zq}z}}{4jd} H_0^{(2)}(k_{\rho q}\rho) \sim \frac{e^{-j\pi/4}}{2d\sqrt{2\pi k_{\rho q}\rho}} e^{-j(k_{\rho q}\rho + k_{zq}z)}, \tag{3}$$

with

$$k_{zq} = \omega\gamma_z + \alpha_q, \qquad \alpha_q = 2\pi q/d, \qquad k_{\rho q} = \sqrt{k^2 - k_{zq}^2}. \tag{4}$$

The FW$_q$ wavenumber along z, $k_{zq}(\omega)$, represents the Floquet-type dispersion relation. The square root defining the radial wavenumber $k_{\rho q}$ is defined so that $\Im m\, k_{\rho q} \leq 0$ on the top Riemann sheet, consistent with the radiation condition at $\rho = \infty$. In (3), FWs with z-domain propagation constants $|k_{zq}| < |k|$ characterize radially propagating FWs (PFW) while those with $|k_{zq}| > |k|$ characterize radially evanescent FWs (EFW). Each PFW contributes at $\mathbf{r}$ a ray asymptotic field lying on a ray cone with semiangle $\beta_q(\omega) = \cos^{-1}(k_{zq}/k)$ (for both positive and negative frequencies)(see Fig.1 for $q = 0$). When $|k_{zq}| > |k|$, the cone angle becomes complex, with evanescent field along ρ; the EFW portion of $\sum_q A_q^{FW}$ converges rapidly away from the array axis and may require only a few terms.

Time domain FW. Applying (2∞) to the TD impulsive individually radiated fields $\hat{A}_n(\mathbf{r}, t) = \delta(t - \gamma_z nd - R_n/c)/(4\pi R_n)$, see Fig.1, the ν-integral for $\tilde{\hat{A}}_q(\mathbf{r}, t)$ is again evaluated in closed form. Since the δ function in $\hat{A}(\mathbf{r}, \nu, t)$ contributes to the integral only for those ν-values in $-\infty < \nu < \infty$ that satisfy $(t - \gamma_z \nu d - R_\nu/c) = 0$, one finds for the propagating TD-FW$_q$

$$\hat{A}_q^{PFW}(\mathbf{r}, t) = \sum_{i=1}^2 \hat{A}_{q,i}^{PFW}(\mathbf{r}, t), \qquad \hat{A}_{q,i}^{PFW} = \frac{e^{-j\alpha_q z_i'(t)}}{4\pi d\sqrt{\tau^2 - \tau_0^2}} U(\tau - \tau_0). \tag{5}$$

In this causal *fundamental solution* for the (q, t) indexed TD-FW, with $i = 1, 2$,

$$z_i'(t) = z - \gamma_\rho^{-2}\left(\tau\gamma_z + (-1)^i c^{-1}\sqrt{\tau^2 - \tau_0^2}\right), \quad \tau = t - \gamma_z z, \quad \tau_0 = \gamma_\rho \rho, \tag{6}$$

$\gamma_\rho = (c^{-2} - \gamma_z^2)^{1/2}$, $\Im m\, \gamma_\rho \leq 0$, and $U(\tau) = 1$ or 0 if $\tau > 0$ or $\tau < 0$, respectively. Note that $z_1'(t)$ and $z_2'(t)$ do *not depend on* q, are real for $\tau > \tau_0$ and coincide at $\tau = \tau_0$, which represents the causal turn-on time $t = t_0 = \gamma_z z + \tau_0$. For a stationary observer at P, each of the $i = 1$ or $i = 2$ smooth PFW contributions in (5) are shown in Figs. 2a,b when extended from $z = -\infty$ to $+\infty$. At the turn-on time t_0 (i.e. $\tau = \tau_0$), when the conical wavefront reaches P in Fig. 2a, the current impulse is located at $z' = t_0/\gamma_z = t_0 v_z^{(p)}$, where the wavefront originates. At the earlier time $t' = t - R(z_0')/c$, the current impulse was located at the point $z_0' = z_1'(t_0) = z_2'(t_0)$, with $R(z_0')/c$ as the time required for the field launched from z_0' to reach the observer. The angle β_0 is here defined as $\cos\beta_0 = (z - z_0')/R(z_0')$ and coincides with that in the FD case for $q = 0$ (i.e. $\alpha_q = 0$). This implies that at the turn-on time t_0 pertaining to the fixed observation point P, the wavefront arrives from a direction coincident with that of the FD-FW$_0$ wave vector $\mathbf{k}_0^{FW} = (k_{z0}, k_{\rho 0})$. At times $t > t_0$ (i.e. $\tau > \tau_0$), the wavefront moves beyond P to the location shown in Fig. 2b, which is tagged on the z

axis by $z' = t/\gamma_z = tv_z^{(p)}$. The observer now receives two distinct contributions that arrive simultaneously but are launched from points $z_i'(t)$, $i = 1,2$ at times $t_i' = t - R(z_i'(t))/c$, respectively. The two angles $\beta_i(t)$ depicted in Fig. 2b are defined as $\cos\beta_i = (z - z_i'(t))/R(z_i'(t))$. The results in (5) are complex. To obtain a physical (i.e., real causal) field from (5), we take the real part.

Fig. 2
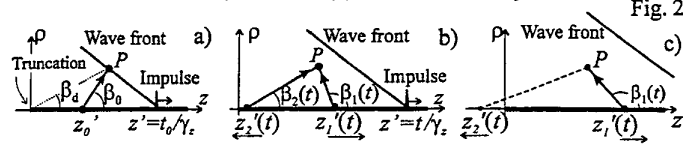
IV. FD- AND TD-FW FOR THE SEMI-INFINITE ARRAY

With the infinite array results in mind, we procede to the semi-infinite case.

Frequency domain truncated FW. Applying (2) to the FD radiated fields $A_n(\mathbf{r},\omega)$ leads to the truncated integral representation $\widetilde{A}_q(\mathbf{r},\omega)$ which is asymptotically evaluated via the following steps. First, the spherical wave function $A(\mathbf{r},\nu,\omega)$ is expressed as a wavenumber spectral integral over axially phased cylindrical waves. Interchanging the order of integration, the ν-integral is calculated exactly as a wavenumber spectral pole. The remaining spectral integral is deformed via the saddle point method into the steepest descent path (SDP) and evaluated asymptotically

$$\widetilde{A}_q(\mathbf{r},\omega) = A_q^{FW} U(\beta_q^{SB} - \beta_d) + A_q^d, \qquad A_q^d \sim \frac{e^{-jkR_d}}{j4\pi dk R_d} \frac{F(\delta_q^2)}{\cos\beta_q - \cos\beta_d} \qquad (7)$$

where the FW A_q^{FW} arises from the residue of the pole encountered in the deformation, and the q-th spherical diffracted fields A_q^d due to the truncation are obtained from a uniform saddle point evaluation along the SDP. U is the Heaviside unit function; β_q^{SB} is the shadow boundary of the truncated FW$_q$ which, for propagating FWs, coincides with the FW$_q$ propagation angle $\beta_q = \cos^{-1}(k_{zq}/k)$ (see Fig.1 for $q = 0$); $F(x)$ is the transition function of the Uniform Theory of Diffraction (UTD)[2], with argument $\delta_q = \sqrt{2kR_d}\sin((\beta_q - \beta_d)/2)$; $\beta_d = \cos^{-1}(z/R_d)$ is the observation angle mesured from the truncation of the array, see Fig.1. Every FW$_q$ in (7) is the same as that for the infinite array, except that its domain of existence is the region $\beta_d < \beta_q^{SB}$. The discontinuity of the PFW$_q$ at the shadow boundary $\beta_d = \beta_q^{SB}$ is compensated by the diffracted field A_q^d.

Time domain truncated FW. The TD-FW field $\widetilde{\widetilde{A}}_q$ is merely a truncated version of the infinite array field in (5) since it is generated by the $\nu \geq 0$ contributions in the ν-integral of (2) instead of the $-\infty < \nu < \infty$ range in (5). Accordingly,

$$\widetilde{\widetilde{A}}_q(\mathbf{r},t) = \sum_{i=i}^{2} \hat{A}_{q,i}^{PFW}(\mathbf{r},t) U(z_i'(t)), \qquad (8)$$

where the *fundamental* TD-FW $\hat{A}_{q,i}^{PFW}(\mathbf{r},t)$ and $z_i'(t)$ are the same as in (5),(6) and the U function expresses the truncation at the $\nu = 0$ end point. The $\hat{A}_{q,i}^{PFW}$ space-time phenomenology has already been explained in connection with (5), except that due to the truncation, at later times (Fig.2c) than those in Figs.2a,b, the observer receives only the contribution arriving from $z_1'(t)$. The (dashed) contribution from $z_2'(t)$ is off the array and is therefore not excited. Since $z_i'(t) = 0$ corresponds to $\beta_i(t) = \beta_d$, the FW existence condition $U(z_i'(t)) = U(\beta_i(t) - \beta_d)$, $i = 1, 2$, can be parameterized in terms of instantaneous propagation angles.

Asymptotic inversion. The TD field $\widetilde{A}_q(\mathbf{r}, t)$ can also be found through Fourier inversion of the high-frequency FD field in (7). Due to their nondispersive behavior, the $q = 0$ FW and diffracted field in (7) can be inverted exactly, while the dispersive case $|q| \neq 0$ is evaluated asymptotically. The FW field $A_q^{FW}(\mathbf{r}, \omega) \exp(j\omega t)$ has a high frequency asymptotic phase $\hat{\psi}(\omega) = k_{zq}z + k_{pq}\rho - \omega t$ (see (3),(4)). The uniform diffracted field $A_q^d(\mathbf{r}, \omega) \exp(j\omega t)$ in (7) has a more complicated phase which simplifies, however, close to the SB at $\beta_d = \beta_q$ where it can be shown to have the same phase as the FW [2]. This common phase $\hat{\psi}$ contributes to the inverse FT through the two asymptotic instantaneous frequencies $\omega_{q,i}(\mathbf{r}, t)$, $i = 1, 2$, which satisfy the saddle point condition $\frac{d}{d\omega}\hat{\psi}(\omega) = 0$ and are real in the radiating domain $\tau > \tau_0$ ($t > t_0$). Thus the asymptotics yields $\widetilde{A}_q \sim \sum_{i=1}^{2}[\hat{A}_{q,i}^{PFW}$ $U(\beta_i(t) - \beta_d) + \hat{A}_{q,i}^d]$, in which $\hat{A}_{q,i}^{PFW}$ is the same as in (5) and the TD diffracted field $\hat{A}_{q,i}^d$ is a spherical contribution centered at the array truncation. The instantaneous shadow boundaries $\beta_{q,i}^{SB}(t) = \cos^{-1}(k_{zq}/k)_{\omega_{q,i}(t)}$ (see (7)) and the FW propagation angle $\beta_{q,i} = \beta_{q,i}^{SB}$ are found to be *independent* of q and equal to the above $\beta_i(t)$. Thus, *all* $\hat{A}_{q,i}^{FW}(\mathbf{r}, t)$ fields at a given instant t reach an observer at $\mathbf{r}$ from the *same* direction $\beta_i(t)$. The asymptotic diffracted field $\hat{A}_{q,i}^d$ provides the required continuity across the instantaneous FW shadow boundaries $\beta_i(t)$ and is negligible elsewhere (see also (8)). When each dipole in (1) radiates a band-limited pulse $\hat{G}(t - \gamma_z nd)$, the band-limited TD field $\widetilde{A}_q{}^{BL}$ is found by multiplying each $i = 1, 2$ constituent by the pulse spectrum $G(\omega)$ evaluated at the instantaneous frequency $\omega_{q,i}$. Thus, only those TD-FW$_q$ fields with $\omega_{q,i}$ in the $\hat{G}(t)$ signal bandwidth contribute to the actual field. In Fig. 3, this TD asymptotic solution is compared with a reference solution obtained by an element by element summation over the pulsed radiation from all dipoles, for an array of 10 elements with interelement phasing $\gamma_z = 0.2/c$ ($\beta_0 = 78°$). Both left and right truncation effects have been considered, treating the actual array as the difference between two semi-infinite arrays. In Fig.3, we have chosen a Rayleigh pulse $\hat{G}(t) = \Re e[j/(j + \omega_M t/4)^5]$, with $\omega_M = 4\pi c/d$, d being the interelement spacing. The field is plotted vs. normalized time t/T, with $T = d/c$. The observer is located at two points $z = 0$ (a), and $z = 5d$ (b) with the same radial distance $\rho = 10d$. In both cases, the included asymptotic terms are $|q| \leq 3$, thereby demonstrating the good convergence property of the TD-FW field representation.

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Fig.3