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Diversity in Mathematical Insight Experiences in the Wild: Evidence of Opportunistic Assimilation

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Abstract

The opportunistic assimilation hypothesis posits that struggling and failing to solve a problem creates failure indexes, or long-term memory traces of the problem, that creates sensitivity to environment hints that trigger insight experiences. Past laboratory research has cast doubt on the usefulness of such hints during incubation breaks, but laboratory work is limited in its ability to recreate the diversity of stimuli in everyday life the opportunistic assimilation hypothesis requires. The current paper evaluates the insight experiences of over 150 participants who solved an insight math puzzle outside the lab for the presence of incidental hints that aided with problem solving. Across two studies, participants reported that chance hints in the wild had helped them to solve the puzzle and triggered insight moments. This suggests that opportunistic assimilation may play a role in everyday insight experiences and should not be discounted in future research.

Keywords: insight; incubation; mathematics; problem-solving; education

Introduction

Insight moments represent a powerful cognitive experience and have capture the interest of cognitive scientists for decades. Also known as aha! moments, insight moments are described as, "the sudden experience of comprehending something that you didn't understand before, thinking about a familiar thing in a novel way, or combining familiar things to for something new." (Kounios & Beeman, 2015). Insights are unique from other experiences of problem solving, as they involve a sense of suddenness (Gick & Lockhart, 1995; Metcalfe & Wiebe, 1987), a new way of looking at a problem (Csikszentmihalyi & Sawyer, 2014) and often produce a feeling of elation or happiness (Shen et al., 2016).

Researchers frequently point to insight moments in mathematics as powerful examples of these experiences. For example, two of the most cited insight examples are of Poincaré, who reported making a major breakthrough in his Fuchsian function problem while stepping off a bus during a geological excavation (e.g. Benedek & Jauk, 2018; Gilhooly, 2016; Sadler-Smith, 2015), as well as the famed story of Archimedes who had a moment of sudden realization regarding how to calculate the volume of a crown using displaced water in the bathtub (e.g. Lawson, 2001; Simonton, 2018; Ward et al., 1999). For mathematicians, insight has been described as finding a remote connection, switching on a light, and suddenly developing a greater understanding for how concepts relate together (Burton, 1998). But it's not just expert mathematicians who experience insight in math. Students also experience insights when learning math (Barnes, 2000; Liljedahl, 2005), and that these insight moments mark important cognitive shifts in understanding and thinking within STEM education.

One model that remains popular today to explain how people reach insight moments is the four-stage model of creativity first described by Wallas (1926). According to this model creative problem-solving begins by first, immersing oneself in the problem to better understand it and exhaust conventional ideas. After a period of immersion, one reaches an impasse, or a mental block. Once an impasse has been reached, people enter a period of incubation, where they temporarily shift their attention away from the problem and do something else. During this period of incubation, or upon return to the problem, people can experience an aha! moment of insight, where an idea surfaces with "brevity, suddenness, and immediate certainty" (Hardy, 1946). After experiencing insight, the potential solution or breakthrough is evaluated.

The traditional paradigm in incubation and insight research involves bringing participants into the lab and providing them with a problem or set of problems to solve. After an initial work period, participants either receive an incubation break from the problem, or continue to work. Then, students in the incubation condition return to the problem, and their performance is compared to a continuous work condition. There is widespread evidence for the effects of incubation (Sio & Ormerod, 2009), and researchers have spent much time trying to pinpoint the exact mechanism that sparks insight moments.

Mechanisms of Incubation and Insight

Theories tested and developed to explain incubation and insight generally point to unconscious mechanisms (Ritter & Dijksterhuis, 2014). For example, Smith and Blankenship (1991) found that incubation periods allow for *selected forgetting,* whereby false cues that stymie the participant from accessing more relevant information during solving are inhibited during breaks, allowing one to see other relevant information more easily. Another proposed mechanism proposed is the *spreading activation account* (Sio & Rudowicz, 2007) in which incubation allows for greater semantic activation, facilitating connections to more relevant information necessary for problem solving.

A third hypothesis termed *opportunistic assimilation* was posited by Seifert et al. (1994). According to this hypothesis, immersion periods of struggling to solve a problem creates failure indexes, or long-term memory traces of the problem. When the individual enters an incubation period, the diversity

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in their environment can provide them with incidental cues relevant to the problem that their memory traces are sensitive to. To put simply, opportunistic assimilation is a hypothesis about becoming sensitive to environmental hints that may aid with problem solving, but it has not received widespread support in the literature in the past few decades. For example, Smith et al. (2012) used incubation breaks to provide participants with hints to solution words for remote associate test (RAT) problems. During the break, participants would complete a lexical decision task that contained hints(e.g. seeing the word "Flake" to help participants think of the solution word "Snow") and found that it does not help solution rates, unless participants are told ahead of time that they may encounter hints that are helpful for solving. Other research has also found a similar lack of evidence for the effect of hints during incubation breaks (e.g. Dodds et al., 2002).

Limitation of Previous Research

There are a number of factors that might not have allowed past laboratory research to fully test the effects of incidental hints on insight problem solving. First, some research may not have provided participants with enough time to fully immerse themselves in a problem and reach an impasse. For example, Smith et al., (2012) only allowed participants 10 seconds to immerse themselves in the problem. This is likely not enough time for immersion, in fact, this time has been used in research to ensure participants do not have enough time to reach an impasse (Moss et al., 2011). Even research that uses 15 seconds (Kohn & Smith, 2009; Smith et al. 1998) or 30 seconds (Dodds et al., 2002) for immersion may still be on the lower end of immersion and may not be enough to create the failure indexes required for environmental hints to have any effect.

Second, although researchers go to great lengths to create useful and semantically-related hints to test opportunistic assimilation, there is still an assumption that all hints will work for all participants. Opportunistic assimilation posits that it's the diversity of stimuli in the environment that aids problem solving, allowing for problem solvers to attend to any hint that is most helpful to them. Using semantically related words as hints not only makes an assumption that a single hint should be effective for everyone, but by presenting these hints, there is the additional confound of activating semantically related words, which may inadvertently have negative effects on problem solving. For instance, Storm et al., (2011) found that when participants are asked to think of semantically-related words, they show retrieval-induced forgetting, which can make it even more difficult to retrieve the target word.

Third, classic incubation studies give upwards of 20 RAT items to solve (60 words in total), provide a break, and then assess how many more can be solved after a break (e.g. Dodds et al., 2002; Smith, & Blankenship, 1991) but some work suggests that students have difficulty remembering so many different sets of problems (Moss, et al., 2007), possibly overpowering the very processes involved in opportunistic assimilation (e.g. memory traces).

Lastly, the incubation periods in laboratory studies are traditionally not periods of rest, but other forms of quick work such as lexical decision tasks, other insight puzzles, or sorting tasks. Research on incubation in natural settings suggests that incubation periods often happen during the "five b's"– buses (driving), bedrooms (falling asleep/waking up), bathrooms (showering), boring meetings (presentations, meetings, lectures), and booze (intoxication) (Benedek & Jauk, 2018). These incubation breaks also take longer than an hour which is the standard time of a lab experiment (Savic, 2012).

Taken together, opportunistic assimilation is difficult to assess in laboratory work, and with this hypothesis depending on the diversity of stimuli and chance incidental hints one would experience in everyday life, within the conditions of everyday life, it is not surprising that laboratory work has not found strong evidence for it.

The Case for Mathematics

Traditional tasks used to study insight, such as anagrams or RAT items, provide a useful tool to explore mechanisms inside the lab and have contributed much to cognitive science. However, like any other task they come with limitations, such as being confounded with verbal fluency and lacking direct ties to problems one might face in everyday life. Shifting to other tasks to study insight, such as those that occur within the context of mathematics, adds not just important diversity to the tasks used to study insight, but provides important ecologically validity. Specifically, *aha* moments can be critical moments in contexts like learning mathematics, in which we want people to feel relief of the value of hard work, even when it feels like they're not getting anywhere. In fact, from the limited research of insight moments in math classrooms, we know that insight moments constitute some of the only positive experiences students have in mathematics (Liljedahl, 2005), signaling incredible value to STEM educators.

Even more beneficial would be if educators could identify ways that they could help students reach and experience insight moments on their own, such as the use of cues and hints. Sure, educators can always provide strong-handed hints or solutions, but there is more value in allowing students to experience the moment of insight for themselves. Indeed, some math students report that before reaching an insight moment, they never realized that they could struggle for long periods of time, feeling like they were not making any progress, and then to experience the sudden burst of insight, making them appreciate persistence in math (Liljedahl, 2005).

The question for many educators is simply, how can I help students make connections and create these moments of insight? Opportunistic assimilation may have something to offer, such as the use of subtle cues in math textbooks, the classroom, or even homework. However, with much research published arguing that humans are quite bad at picking up hints, a revisiting of the opportunistic assimilation hypothesis is needed, especially in the wild environment that it requires. Preliminary evidence of the opportunistic assimilation in mathematics and in everyday life would provide a first step toward rethinking the role of this unfavored hypothesis as research moves away from traditional insight paradigms in the lab and into more modern methods of capturing these moments in the wild.

The Current Study

In the current study, open-ended responses of mathematical insight experiences that occurred outside the lab were evaluated for evidence of the opportunistic assimilation hypothesis—namely that participants' reports of insight experiences were triggered by environmental hints in their environment. Data come from two larger studies (one study and subsequent replication study) that assessed the effect of an incubation manipulation in the lab on an insight math puzzle [Shaw, 2020].

Participants who were unable to solve during the lab portion of the experiment were instructed to live their lives as normal, and if they happened to think of the solution to the problem, immediately report the details of their solve experience through an online survey. Of particular interest, participants were asked to describe exactly what they were doing before they solved, and in the replication study, asked if anything in their environment had helped them think of the solution. It is important to note that evidence of the opportunistic assimilation hypothesis in this study does not argue that other mechanisms are not at play as well, but rather would argue that hints can and do play a role in insight moments.

Methods

Participants

Participants were comprised of undergraduate psychology students who participated for course credit at a large public university.

Procedure

Study 1 recruited 231 students who came into the lab, were presented with the math insight puzzle and then given 6 minutes to try to solve it. Participants were informed that if they did not solve the problem during the lab session, they'd be enrolled in a second phase of the study where they'd get an opportunity to solve it in their everyday lives up to three days later. In the lab portion of the study, students were randomized to receive no incubation break, a low demand incubation break half-way through, or a high demand incubation break. There were no differences in solution rates between conditions (p = .282 - .574). Subsequently, n = 166 students who did not solve the insight puzzle in the lab were enrolled in and completed the second phase of the study. These participants were not instructed to try find the solution, but rather live their lives as normal and, if they happened to think of the solution, they were asked to immediately report their problem-solving experiences through an online survey that they received before leaving the lab.

Study 2, an expanded replication of study 1, recruited 257 students and followed a similar procedure, except only two conditions were administered in the lab (a low demand incubation break vs. continuous control, with no significant difference on solution rate p = .28), and for students who were enrolled in, and completed the follow-up phase of the study (n = 147), they were asked more specific questions about their problem-solving experiences. Data for the current study focus on students who solved the insight puzzle outside the lab across both studies (n = 157). The two studies were approved by the participating university's institutional review board.

Materials

Math Insight Puzzle

The math puzzle used in the current study presents participants with four digits (2 3 4 5) and two symbols (+ =) and asks them to create a balanced equation using each digit and symbol once and only once, without adding any digits or symbols (Miller, n.d.). Participants were encouraged to use any mathematical procedure they could think of, as long as it satisfied the rules. The solution to this problem is the equation $3^2 = 4 + 5$, requiring students to think beyond addition and make a connection to more advanced procedures of mathematics (exponents). Participants who believed that this would require adding an additional symbol of "^" and thus, violated the rules, were screened for at the end of both studies and removed from analyses.

Solve Experience Questions

Participants who did not solve in the lab but later solved in the wild were asked to "*...describe in at least two sentences what were you doing right before you solved the problem. Please provide as much detail as possible!".* In study 2, these instructions were expanded to "*…describe in at least four sentences what were you doing right before you solved the problem. Please provide as much detail as possible (even if it does not seem relevant)."*

In addition to their problem-solving experiences, study 2 participants were asked if there was there anything in their environment that helped them solve the problem (e.g. someone having a conversation about math, seeing a formula written on a whiteboard, etc.) through a yes/no response item. One difficulty in capturing the influence of environmental hints with this method is that students are not always aware of environmental cues that might help them solve. Therefore, students were also asked to describe the environment around them when they solved in at least four sentences.

Results

In study 1, 52% of participants reached the solution outside the lab, resulting in a sample size of n = 87 of problem solvers, and four students had open-ended responses that identified incidental cues in their environments as helping them reach their insight moment, signaling evidence for the opportunistic assimilation hypothesis. As a reminder, in study 1 there were no instructions to report potential hints or environmental information that helped students reach insight moments, but rather these cues were reported without solicitation. One student recalled their insight moment, stating:

> I was in my dorm's floor lounge playing a game on my laptop where I had to avoid obstacles and my floormates were working out a math problem on the board, where at one point they talked about squaring both sides of an equation and I thought of the problem.

This example illustrates how incidental cues in the environment can redirect attention back to stubborn problems, and even offer solutions to problem solvers. But not all hints were necessarily mathematical in nature. For instance, one student shared their insight moment, stating:

> I was doing my homework for English 4W and the main topics for this class are 'Form and Power.' That's when it hit me, that taking the power of a number doesn't require another symbol and I smacked myself in the face.

In this example, the student was presented with an incidental cue in the form of the word "power" through their homework, and was able to make the connection between the word in the context of their English class and the word in relation to exponents (taking the power of a number). These two examples show how hints were able to help some students reach insight moments. Unfortunately, many of the participants who solved did not provide detailed descriptions of their solve experience in study 1 (e.g. "I was eating cereal", "I thought about it"), so a subsequent replication was conducted to capture more detailed information.

In study 2, 48% of the participants solved the puzzle outside the lab, resulting in n = 70 problem solvers. These participants were asked if they had reached an impasse on the problem to ensure they received enough time to immerse themselves in the problem. Across participants who solved, 90% said they reached an impasse on the puzzle. Additionally, in study 2 participants were explicitly asked about the role of hints in the environment when solving. A total of 21 students (24% of solvers) reported that, yes, a hint in their environment had helped them to solve. Another two students who did not reported the presence of a hint did report the presence of mathematics or statistics context in their environment when they solved (e.g. using statistical software). Student accounts are quite telling about variability in the environment hints as well as the strength of the association. For example, one student stated:

> I went outside and sat [on] the benches… I listened to music and stared at the tour group where my attention was caught when the tour guide pointed at the [building name]. I looked up and realized the equation written on the mural...

The building the student is referring to contains a small section of a larger mural that has the equation $e = mc^2$ (see Figure 1). In this happenstance moment, the student followed the attention of a tour guide to look up and see an exponent in the wild. They then made the connection back to the insight problem, and quickly found the solution.

Figure 1: A picture of the mural a participant reported seeing that helped them to solve.

Other students, however, made connections to cues in their environment that were much more abstract. For example, one participant said:

> I was watching Pewdiepie play a 12 hour minecraft stream. I went to heat up some ramen and as I was eating, my boyfriend called. I told him about the study… and I started staring at this corner on my desk. Then, I thought about the Pythagorean theorem and wrote down the 4 digits and 2 symbols to solve it.

In this case, the participant had seen a right angle, which had brought up the idea of Pythagorean's theorem, which uses exponents ($a^2 + b^2 = c^2$). The environmental hint of a desk corner may be considered much weaker than seeing an exponent in the wild, but it still helped this student reach an insight moment. This suggests that students do not necessarily need to see the solution to solve but can make remote connections given the *right* hint.

In total, 26% percent of people who solved outside the lab in study 2 reported either hints had helped them to solve or reported mathematical content in their environment. Certainly, there are likely cases where students were unaware of such environment hints that may have helped them solve, and this went undetected through the current studies' self-report methodology. However, the presence of any student experiences solving through the use of hints in the environment suggests that opportunistic assimilation may help students reach insight in mathematics. This one account likely cannot explain *all* insight experience, as there was great diversity of experiences, but rather the current results suggest it is one of multiple mechanisms at play in the wild.

Discussion

Across two studies, insight experiences from 157 participants who solved a mathematics insight puzzle in their everyday lives were analyzed for evidence of the opportunistic assimilation hypothesis. This account of incubation and insight suggests that problem solvers who reach impasse become sensitive to environmental cues relevant to problem solving, and from exposure to the rich diversity of the environment, chance encounters with relevant cues trigger insight. Past research on incubation and insight has cast doubt on the utility of hints to help problem solvers. But many of these studies cannot properly test the opportunistic assimilation hypothesis, as participants were not always ensured enough time to immerse themselves in the problem and reach impasse, were not given the context of a diverse environment to encounter relevant cues during actual rest periods, and some research gave participants many problems to try to reach insight on at once, weakening memory traces that the opportunistic assimilation argues helps the problem solver attend to relevant information.

To set more optimal parameters to test this hypothesis, the current set of studies provided participants with a single math insight puzzle to try to solve that required students to think past addition and think of exponents. Participants were initially given an opportunity to try to solve in the lab, and those participants who were unable to solve were released from the lab and instructed to live their lives as normal. However, if a participant reached the solution within three days, they were instructed to immediately report their problem-solving experiences through an online survey.

Results from the first study showed that several students, without being prompted, identified and described hints in their environment that helped them find the solution. This provided evidence that, at least for some students, opportunistic assimilation may have been at work. To more thoroughly investigate this hypothesis, in study 2, participants were asked if they had reached an impasse, and if there were any hints in their environment that had helped them solve (yes/no). In addition to describing their insight moment, participants were asked to describe the environment around them when they solved the problem in attempt to capture hints that may have been present, but the problem solver was unaware of. Exactly 90% of participants reported reaching impasse, suggesting that if opportunistic assimilation does in fact play a role in insight, most of these students would become sensitive to environment hints. Indeed, it was found that 24% of participants who solved reported a hint had helped them to solve the problem, and another two participants did not report hints, but did provide an environmental description in which there was math present in their immediate surroundings.

Taken together, these two studies offer preliminary evidence that opportunistic assimilation can play a role in insight experiences, and suggests that under certain conditions, environment hints can help problem solvers reach insights. However, the magnitude of its effect, interaction with individual differences, and pairings with other potential mechanisms (e.g. selective forgetting) are still unknown. It is important to note that the present results do not argue that opportunistic assimilation is the *only* mechanism at play, but rather cognitive and environment factors likely work together in various ways to trigger these moments for the individual. For example, the insight task used in the current study creates a large amount of fixation on combining numbers (e.g. 5 + 4 = 23) strengthening activation of addition. Might students have picked up on exponents if there without inhibition of false cues (*selective forgetting*)? Or more relevant semantic activation coming online over time (*spreading activation*)? It is an open question if students would still be open and sensitive to hints without these cognitive mechanisms, but insight moments may be complex enough to involve multiple mechanisms.

Another important consideration is that the mathematical nature of the problem may be particularly well-suited to benefit from opportunistic assimilation. For studies that use college students as participants, such as the current set of studies, math may be more likely to exist in the environment compared to cues related to other solutions (e.g. the solution word of "snowflake" for a RAT item). On college campuses, many students are enrolled in math courses or around others who are engaged in math (such as the roommate doing math homework). Thus, insight tasks with greater relevance and ties to everyday life may also be positioned well to benefit from hints in the environment.

Further, mathematics may also be particularly ripe for opportunistic assimilation because student participants have extensive experience in math classrooms making connections between mathematical concepts, which may have led to a more complex interweaving of mathematical knowledge—creating greater reception to any and all hints. For instance, one student looked at the right angle of a desk, made a connection to Pythagorean's theorem, to the formula, to exponents, and back to the insight puzzle. This example illustrates how prior knowledge and existing complex semantic networks surrounding exponents may had led to even abstract hints, such as the angles of furniture, aiding in insight moments.

Limitations and Future Directions

One limitation of the current set of studies is that data were collected through self-report measures, which are subject to bias and are only able to capture what the participant is consciously aware of. For example, some students may have had hints in their environment help them solve the problem but were not consciously aware of them to note this in their self-reports. Another potential limitation is the nature of the insight task used. As previously noted, math tasks may be especially well-suited to benefit from environmental hints, especially for student participants. Future work would do well to expand on the findings in this study by diversifying insight tasks and studying how participants reach insights in everyday life.

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