Title
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Publication Date
1997

CALIFORNIA PATH PROGRAM
INSTITUTE OF TRANSPORTATION STUDIES
UNIVERSITY OF CALIFORNIA, BERKELEY

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California PATH Research Report
UCB-ITS-PRR-97-37

This work was performed as part of the California PATH Program of the University of California, in cooperation with the State of California Business, Transportation, and Housing Agency, Department of Transportation; and the United States Department of Transportation, Federal Highway Administration.

Final Report for MOU 219

October 1997

ISSN 1055-1425

CALIFORNIA PARTNERS FOR ADVANCED TRANSIT AND HIGHWAYS

EVALUATION OF ITS TECHNOLOGY FOR BUS TIMED TRANSFERS

October 10, 1997

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ABSTRACT

This paper evaluates dispatching rules at timed transfer transit terminals with respect to total passenger delay and number of passengers missing their connections. We consider dispatching rules that make use of Intelligent Transportation Systems (ITS), such as Global Positioning Systems (GPS) for bus tracking, as well as those that do not use ITS (i.e., they do not rely on communication or tracking). Both analytical and simulation models are developed to evaluate the benefit of ITS. Empirical data collected by the Los Angeles County/Metropolitan Transit Agency on bus stop arrival times are used to develop a model for delay over bus line segments that is conditioned to lateness at the start of the segment. We conclude that ITS provides modest benefits in terms of reduction in passenger delay -- on the order of 20 seconds per transferring passenger on average. Until the technology can be implemented at a relatively low cost (about $2000 per vehicle), the benefit of ITS for timed transfer alone does not justify the implementation of tracking technology.

Keywords: Public Transit, Simulation, Scheduling, and Lateness

EXECUTIVE SUMMARY

A timed transfer exists when multiple bus routes are scheduled to arrive on or about the same time at a transit terminal, with the goal of enabling short waiting times for passengers transferring between buses. Timed transfers have become increasingly common in bus systems, because they provide greater connectivity between origins and destinations, especially for bus lines with large headways.

In this report, we augment the prior work on timed transfer terminals by evaluating the performance of dispatching rules with ITS (Intelligent Transportation Systems) versus those without ITS. Two levels of ITS are considered: (1) system with centralized tracking, and (2) system with information on connecting passengers, as well as centralized tracking. Performance metrics that are studied include total passenger delay and number of passengers missing their connections. Both analytical and simulation models are developed to evaluate the benefit of using ITS for bus dispatching at timed transfer terminals.

Empirical data collected by the Los Angeles County/Metropolitan Transit Agency on bus stop arrival times are used to develop a model for delay over bus line segments that is conditioned to lateness at the start of the segment. The lateness model is used to forecast the arrival time of buses at timed transfer terminals. These forecasts are the basis for control rules which can decide whether to hold or release a connecting bus, depending on the forecasted delay.

We conclude that ITS provides modest benefits in terms of reduction in passenger delay -- on the order of 20 seconds per transferring passenger on average. Based on historical data on ITS implementations in transit, this benefit alone would not justify installation costs. However, in the future it should be possible to purchase off-the-shelf systems at significantly lower cost than the past. If implementation costs can be brought down to about $2000/bus (a realistic objective), then tracking could well be justified for this application. Nevertheless, tracking systems can provide many other benefits, such as improved traveler information and better planning data. These additional benefits could further justify implementation of tracking technology.

1. INTRODUCTION

A timed transfer exists when multiple bus routes are scheduled to arrive on or about the same time at a transit terminal, with the goal of enabling short waiting times for passengers transferring between buses. Timed transfers have become increasingly common in bus systems, because they provide greater connectivity between origins and destinations, especially for bus lines with large headways. Clever (1997) outlines the increased integration of timed transfer terminals with other transit services such as rail in several European Countries, including Germany and Austria.

Most major airlines have established “hub-and-spoke” type networks in which flights are funneled through a limited number of airports. Flights are scheduled in and out of these airports in flight "banks", a period on the order of one hour in which up to 100 flights may arrive and depart. Like bus systems, the objective is to provide greater connectivity between origins and destinations, while minimizing waits. When an incoming flight is delayed, airlines have the option of holding other flights to allow passengers to make connections, or alternatively allowing them to depart on schedule. The decision on whether to hold a flight or not depends on a variety of factors, including: (1) magnitude of delay, (2) number of connecting passengers, and (3) frequency of service on the connecting flight. Furthermore, real-time information regarding these factors enables controllers and pilots to make decisions regarding whether to increase or reduce the travel speed in order to provide greater connectivity.

These same considerations are also important to bus systems. However, bus systems have historically lacked real-time information regarding vehicle location and number of passengers intending to transfer to make similar dispatching decisions. Recently, bus transit service providers have begun to adopt Intelligent Transportation Systems (ITS) technologies such as Global Positioning Systems (GPS), Mobile Data Terminals, and Electronic Fare boxes (Khattak et al., 1993; Hansen, 1994; Hickman, 1996). GPS systems are particularly useful for vehicle tracking and mobile data terminals may be used for passenger counting.

There exists some literature on scheduling timed transfers in the presence of random delays. Based on the probability distribution for schedule "lateness", arrival and departure times are optimized, accounting for the likelihood of making a transfer connection and the expected waiting time for the transfer connection. Hall (1985), for instance, examines transfers to and from a rail line, and develops formulas for optimal "safety margins" (i.e., the expected time between arrival of an inbound bus and an outbound train). U.S. UMTA (1983) and Vuchic et al (1981) provide fairly comprehensive manuals for design of timed transfer systems. Abkowitz et al. (1987) simulate a variety of dispatching strategies at a timed transfer hub. Their simulation results on two bus lines show that a “no holding” strategy is best when the bus lines have unequal headways, and a “double holding” strategy is best when the bus lines have equal headways. Lee and Schonfeld (1992) simultaneously optimize headways and safety margins at a timed transfer terminal. Shih et al. (1997) develop a trip assignment model for timed transit terminals. Application of their model to an example network shows that demand tends to be assigned to higher frequency paths in uncoordinated cases.

In this report, we augment the prior work on timed transfer terminals by evaluating the performance of dispatching rules with ITS versus those without ITS. Two levels of ITS are considered: (1) system with centralized tracking, and (2) system with information on connecting passengers, as well as centralized tracking. Performance metrics that are studied include total passenger delay and number of passengers missing their connections. Both analytical and simulation models are developed to evaluate the benefit of using ITS for bus dispatching at timed transfer terminals.

An input to both the analytical and simulation models is the probability distribution of delay at each stop. Prior studies on lateness or delay distributions focused on bus lines with frequent service (i.e., small headways). For example, several authors (e.g., Turnquist, 1978; Andersson and Scalia-Tomba, 1981) argue that for this environment an initial delay in service causes deteriorating service farther down the line due to the increasing accumulation of boarding passengers as the headway becomes larger than scheduled. Our interest is on bus lines with large headways, since there is little benefit to coordinating bus transfers if the headways are small. For bus lines with large headways the opposite effect may result. That is, service may not deteriorate as the bus moves farther down the line, because there may be sufficient slack in the schedule to enable the bus to get back on schedule.

The remainder of this paper is divided into five sections. First, technologies for automatic vehicle location (AVL) (and related hardware and software) are reviewed. Next, we develop a model for bus delay over segments and lateness arriving at check points based on empirical data collected from the LA County/Metropolitan Transit Agency (LAC/MTA). Third, analytical models are derived to optimize the times buses are dispatched from a timed transfer terminal. Fourth, a simulation model is created and used to compare different strategies for controlling bus dispatch times. The final section provides conclusions and an economic analysis.

2. TECHNOLOGY REVIEW

This section reviews advances that have taken place in Advanced Public Transit Systems (APTS) as they apply to timed transfer systems, based on reviews by Casey and Labell (1996), Cassey et al. (1996), Goeddel (1996), and Schweiger (1994). To place timed transfer in context, Table 1 summarizes applications of APTS in the United States.

Table 1. Summary of APTS Applications in the United States

TECHNOLOGY	Operational	Implementation	Planning
Advanced Communication System	77	11	16
Automated Vehicle Location	22	47	30
Automated Passenger Counters	11	6	13
Vehicle Component Monitoring	5	16	9
Travel Signal Priority	9	6	7
Transit Operations Software	25	35	20
Traveler Information	49	25	21
Electronic Fare Payment	25	19	23

The basic components of a timed transfer system are:

1. Automatic Vehicle Location System
2. Mobile Data Terminal for Driver Interface
3. Advanced Wireless Communication System
4. Transit Operations Software and Hardware
5. Passenger Interface, Passenger Counter and/or Electronic Fare Payment (optional)

All of these products are readily available as commercial products, with the exception that base station systems typically are not programmed to provide timed transfer functionality. However, this amounts to a simple modification of existing software, and should not add appreciably to the cost. A timed transfer system does not require vehicle component monitoring, traffic signal priority, or traveler information.

The method of operation entails tracking the location of vehicles, and then executing control actions if a vehicle is determined to be behind schedule. The control action can either to be hold or release a connecting bus at the transfer terminal. The action is communicated from the base station to the connecting bus through either voice communication (e.g., conventional radio) or through transmitting a digital message to the driver's mobile data terminal. The control action can be conditioned to information on the number of passengers on board the late bus or the number of passengers transferring from the late bus. Automated passenger counters and electronic fair boxes can be used to track the number of passengers on board. However, transferring passengers can only be counted if passengers register their destination, either directly (e.g., a mobile data terminal accessible to passengers) or indirectly (the driver asks passengers where they are going, and registers this information for them).

2.1 AVL Systems

Automatic Vehicle Location (AVL) is the core system component. The system works by measuring the real-time location of each vehicle and transmitting the information to the base station. At the control center, transmitted information is processed and the location of the bus is graphically displayed on an electronic map. There are several advantages of an AVL system:

- Increased dispatching and operating efficiency
- More reliable service, encouraging ridership
- Quicker response to service disruption
- Inputs to passenger information system
- Increased driver and passenger safety
- Quicker notice of mechanical problems
- Inputs to traffic signal preferential treatment actuators
- Extensive planning information

Three different technologies are currently used: (1) Signpost and odometer (SO), (2) Global Positioning System (GPS), and (3) Radio Navigation, as described in the following.

Signpost and Odometer A few years back, SO was the most widely-used vehicle location technology. The system uses radio beacons mounted on top of utility poles. Each beacon has a unique I.D. Beacons send a low-powered signal, which is detected by vehicles fitted with receivers. When it is time to report, vehicles send the I.D of the last beacon crossed and the distance traveled since. An alternative to the above approach is to associate a unique I.D with each bus. When a bus passes a signpost, its position is relayed back to the control center. Implementing the alternative approach has the advantage of eliminating the need for wireless transfer of data from the vehicle; however, it suffers from the drawbacks that stray vehicles fail to send information to the control center. SO suffers from the following drawbacks:

- Signposts can be placed at only a small number of locations. If vehicles stray into locations that do not have signposts, their positions cannot be detected.
- It is costly to install, as it involves mounting radio beacons in the field
- It requires a high degree of maintenance.

Global Positioning System In recent years GPS has been the most widely-used vehicle location technology. It utilizes the signal transmitted from a constellation of 24 satellites along with receivers mounted on the roof of each bus. The bus reads the signal and transmits its latitude and longitude to the base station. The advantages of GPS are:

- It works anywhere the satellite signals reach
- It requires no field infrastructure. Since the satellites are already in orbit, the application of GPS technology only involves installing the receivers on buses.

The disadvantages of GPS are:

- The satellite signals do not reach underground and can be obstructed by tall buildings. Hence, GPS has to be supplemented with signposts or with compass/odometer to determine the location of vehicles in some locations.
- To obtain sufficient accuracy, "differential GPS" (DGPS) must be used. Under DGPS, a receiver is placed at a known position. The difference between the exact location and the GPS measured location is used to improve the accuracy of the position determination of vehicles.
- A wireless system is required to communicate between vehicles and the base station.

Radio Navigation Ground-based radio navigation (Loran-C) uses low-frequency waves to provide coverage. It determines location based on the reception of transmissions and the associated timing. Ground-based radio navigation is susceptible to interference, and proximity to overhead power lines causes significant error. However, Teletrac, a company in Los Angeles, has been successful in utilizing radio navigation for accurate location of vehicles by mounting transmitting and receiving towers all over the city.

A variety of communication media, including commercial radio, cellular, and satellite, are available for transmitting messages. Economic factors typically drive the choice. No matter which medium is selected, two methodologies are available for data transmission from the bus to the control center:

∑ Polling: each bus is polled in sequence. The accuracy of information transferred depends on how fast the buses are polled. Since different channels can be used for this purpose, the polling frequency of each bus could be quite high.

∑ Exception Reporting: buses report to the base station only when they run off the schedule beyond a specified tolerance. Exception reporting requires that buses not only to know their present location but also their scheduled position. This increases the hardware and software requirements, making it costlier to install.

Signpost and Odometer was the choice of most transit agencies until just recently. However, the reduced cost of GPS receivers has resulted in a shift in that direction. Due to generally lower accuracy for radio navigation, it is rarely used these days. A recent survey (Casey and Lebel, 1996) found that 82 of 102 AVL installations in the US use GPS (Table 2).

Table 2. Application of AVL Technologies

Technology	Applications
GPS	82
Signpost/Odometer	15
Loran-C (Radio Navigation)	2
Other	3

Several companies provide vehicle location devices, as shown by category in Table 3.

Table 3. AVL Vendors

GPS	S O	Radio Navigation
Rockwell	Gen. Roadway Signal	Teletrac
3M Corp	Motorola	II Morrow
UMA	Glenayre	
Trimble	F&M Global	
TMS	Siemens	
Harris Corp.	Amtech	
Ericson	Bell Radio	
Orbital Science	Fishback and Mare	
Electrocom		
AutoTrac		

2.2 Automatic Passenger Counters

Automatic passenger counters (APC) are a well-established automated means of collecting data on passengers boarding and alighting by time and location. The data may be used in real time or later for different applications. Some of the uses of data collected are:

- Input to dispatcher decision for immediate corrective action
- Input to real-time passenger information system
- Future scheduling
- Positioning new shelters
- Fleet Planning

Automatic Passenger Counters have the following advantages: (1) Decreased data collection, (2) Decreased time and effort for data processing, (3) Increased operating efficiency, and (4) Provided data to passenger information system. Infrared beams and treadle mats are the two most common technologies:

- *Infrared Beams*: Two infrared beams are placed across a passenger's path. As passengers board and alight the bus, they interrupt the beam in a particular sequence, thus activating the APC device. This is the most common technology.
- *Treadle Mats*: Two treadle mats are placed on the steps of doorway. The pressure of passengers stepping on them activates the APC.

APC is usually used as part of an AVL system, for the purpose of determining boardings by stop. Table 4 lists some of the vendors.

Table 4. Automatic Passenger Counter Vendors

Infrared Beams	Treadle Mats
Urban Transportation Associates	Microtronics
Red Pine	Wardup
London Mat (Vertical IR beams)	London Mat

Over the last couple of years transit agencies have also introduced the concept of electronic fare payment to eliminate cash/coin handling, automate accounting, eliminate moving parts in fare boxes, and to make fare schedules more sophisticated. Electronic fare boxes also provide a capability for counting boardings.

Electronic fare payment systems employ electronic communication, data processing, and data storage techniques. A number of technological advances in devices for electronic fare collection have been available over the last couple of years. Some of the most popular ones are:

- *Magnetic Strip card*: Magnetic strips are imprinted on cards. The magnetic strip stores the cash content of the card. On use, the card gets charged, reducing its cash balance.
- *Contact-Type Integrated Circuit Smart Cards*: Each card contains a microcomputer, along with an EEPROM and ROM. While the ROM is used to store the cash content on the card, EEPROM is used for storing the operating system, for performing the transaction and identification. Since the card allows for user identification, it provides a safe guard against theft.
- *Proximity Cards*: The proximity cards do not require the card and reader to be in contact, thus making the payment process faster and more comfortable for users. The system uses a radio

frequency (RF) magnetic field generated from an inductive coil on the read/write unit of the bus to power the card's circuitry.

2.3 Transit Operation Software

Transit operation software has the capability to perform many transit operations functions such as schedule control (adjusting dwell time, inserting vehicles to a route, signal preemption, etc.), computer-aided-dispatch, and service monitoring. While initially transit software focused on run scheduling activities, it has changed with time to provide for more efficient real-time management.

Most fleet management functions are performed at the base station. The hardware usually involves a pair of PCs. One computer serves as a mapping station, utilizing software to display geographic areas and locations of the buses. Most commercial software displays buses running late in a different color, thereby making it easier for the operators to visualize the problematic routes. The other computer functions as a communication controller and uses sophisticated software to manage a host of data transmissions and messages to and from the vehicles. Table 5 lists some of the available software.

Table 5. Transit Operation Software Vendors

Product	Vendor
CADMOS Pro+	Micro Dynamics Corporation
DISPATCH-A-RIDE	Multisystems
Dispatch Manager	Innovative Software Designers, Inc.
Easy Lift	Easy Lift Solutions
FAS (Fully Automated Scheduler)	Automated Business Solution
GIRO/ACCES	GIRO
MIDAS	Multisystems
PASS	On-Line Data Products, Inc.
TRIPS	Micro Dynamics Corporation
TRAPEZE-QV	Trapeze Software Inc.

2.4 Cost Estimates Of Advanced Public Transportation System

Many factors affect the cost of an APTS installation, including in-vehicle hardware, field hardware, control-center hardware, software, training, and support. Several steps were taken to develop an approximate cost estimate:

∑ Different vendors were contacted

∑ Transit Agencies with APTS were interviewed

∑ Publications in related fields were reviewed

Companies were reluctant to reveal the price of their product, probably because the actual price varied from one implementation to another, as projects are usually bid out on a competitive basis. Further, transit agencies could not provide a breakdown of costs by system component, and had only a rough estimate of how much it cost to implement the whole project. Only Phoenix, AZ was able to provide a detailed breakdown for purchase cost. No one had data on operation and maintenance cost.

The primary information source proved to be a publication by the U.S. Department of Transportation that reviews the state-of-the-art in APTS as of 1996 (Casey, 1996). We summarize in Table 6 data on the larger installations. The cost figures in Table 6 are in millions and 1's and 0's indicate the presence or absence of the respective technology. The data set was sub-divided into two categories, systems with GPS as their vehicle location technology and those with SO as their vehicle location technology. A separate linear regression was performed on each category to predict cost as a function of fleet size. Results follow:

GPS: Cost = \$1.30 million + \$13,000 x (fleet size)

SO: Cost = \$1.65 million + \$6,500 x (fleet size)

In both cases, the linear model explained most of the variation in cost data, with R^2 values of .85 and .855. R^2 indicates the fraction of the variance explained by the model. Further analysis of the data based on presence or absence of accessories like APC failed to significantly explain any further variation between the predicted and actual costs. This was perhaps because of the small data set.

Table 6. Costs for Installing APTS Systems

City	Cost	Vehicles	GPS	SO	CAD	APC	SP	EP	PI	SA	SC
Atlanta	7	250	1	0	1	1	1	0	1	1	0
Baltimore	8.9	935	1	0	1	1	0	0	1	1	0
Buffalo	9.6	415	1	0	1	1	0	1	1	1	0
Dallas	16.4	1200	1	0	1	0	0	1	0	1	0
Denver	11	900	1	0	1	0	0	0	1	1	1
Houston	28	1750	1	0	1	1	1	1	1	1	0
LA	12	2085	0	1	1	0	1	1	1	1	1
Louisville	2.5	257	0	1	1	1	0	1	0	1	1
Miami	14.5	610	1	0	1	0	0	1	1	1	0
Milwaukee	7.8	600	1	0	1	1	0	0	1	1	0
Norfolk	2	151	0	1	0	0	0	1	0	1	0
Phoenix	0.4	65	1	0	0	0	0	0	0	0	0
Portland	5.2	770	1	0	1	1	1	0	1	1	0
San Antonio	3.7	531	0	1	0	0	0	0	0	0	0
Seattle	15	1250	0	1	1	1	1	0	0	1	0
Tucson	3.5	200	1	0	1	1	1	1	0	1	0

GPS(Global Positioning Satellite), SO(Signpost/Odometer), CAD(Computer aided dispatch), APC(Automatic Passenger Counters, SP(Signal Preemption), EP(Engine Probe), PI(Passenger Information), SA(Silent Alarm), SC(Smart Cards)

By contrast, Table 7 provides cost data from smaller AVL installations (Casey and Labell, 1996). Here, the costs per vehicle (fixed plus variable) ranges from under $1,000 for a Teletrac installation in Santa Monica, CA to $22,000 per vehicle in Palatka. All of these costs are significantly lower than the larger installations. Data obtained from Phoenix was more in line with the smaller installations, with a cost per vehicle of about $6000 and a base station cost of $24,000 for its 65 vehicle fleet. The marked difference in costs at different installations is likely the result of the amount of customization required, the year of installation, the number of options, and the added overhead of working with larger government agencies.

Table 7. Cost of Installing Smaller Systems

City	Installation Cost	Fleet Size	Per Vehicle Cost
Bremerton, WA	$600,000	155	$7,200
Gary, IN	$140,000	32	$900
London, CT	$2,000,000	160	$22,000
Napa, CA	$130,000	18	$4,400
Palatka, FL	$440,000	20	$3,800
Rochester, NY	$50,000	13	$3,900
Santa Monica, CA	$130,000	135	$5,000
Sheboygen, WI	$100,000	20	$12,500

3. PROBABILITY DISTRIBUTION MODEL FOR DELAY

This section develops a probability models that are used to evaluate the benefits of an ITS based timed transfer system. The probability models are used to represent the lateness distribution for buses arrived at bus stops. We define *lateness* as the deviation from the scheduled arrival time at a check point or stop and *delay* as the deviation from the scheduled travel time over a segment. The relationship between lateness and delay can be expressed as follows:

$$L_k = A_k - S_k \quad (1)$$

$$D_k = L_k - L_{k-1} \quad (2)$$

where:

A_k	=	the actual arrival time of a bus at stop k
S_k	=	the scheduled arrival time of a bus at stop k
L_k	=	the lateness of a bus at stop k
D_k	=	the delay on the bus segment preceding stop k

Based on the above definitions, the actual travel time over the bus segment preceding stop k, $A_k - A_{k-1}$, equals the sum of the scheduled travel time, $S_k - S_{k-1}$, and a random delay, D_k. The lateness at stop k, L_k, can be interpreted as the cumulative delay of all bus segments preceding stop k; $L_k = \sum_{j=1}^{k} D_j$. Negative lateness means that a bus arrived early at a stop and negative delay means that a bus traveled faster than scheduled on a particular segment.

Table 8 summarizes probability distributions for the various random variables used in past studies. The work by Abkowitz et al. (1987), Lee and Shonfeld (1991), Hall (1985), and Talley and Becker (1987) can be classified as either delay (travel) or lateness (arrival), since they modeled a single bus stop and under this case the two variables are the same.

Table 8. Summary of Previous Studies

Authors	Random Variable	Probability Distribution
Turnquist (1978)	Arrival Time	Lognormal
Bookbinder and Desilets (1992)	Arrival Time	Truncated Exponential
Strathman and Hopper (1993)	Arrival Time	Lognormal
Guenthner and Hamat (1988)	Arrival Time	Gamma
Abkowitz et al. (1987)	Arrival Time, Travel Time	Normal
Lee and Shonfeld (1991)	Arrival Time, Travel Time	Normal, Gumbel
Hall (1985)	Lateness, Delay	Exponential
Talley and Becker (1987)	Lateness, Delay	Exponential
Jenkins (1976)	Travel Time	Normal
Andersson et al. (1979)	Travel Time	Lognormal
Turnquist and Bowman (1980)	Travel Time	Gamma
Seneviratne (1990)	Travel Time	Normal, Gamma
Wirasinghe and Liu (1995)	Travel Time	Gamma

Most studies prefer to use a skewed distribution such as lognormal or gamma since a bus is more than likely to be behind schedule than ahead of it. (A lognormal random variable is the log of a random variable which comes from a normal probability distribution and the exponential distribution is a special case of the gamma distribution). Some authors selected the probability distribution based on empirical studies (e.g., Turnquist, 1978; Talley and Becker, 1987; Guenthner and Hamat, 1988; Seneviratne, 1990; Strathman and Hooper, 1993) while others based their selection on model simplification (e.g., Andersson and Scalia-Tomba, 1981; Hall, 1985).

These past studies are not directly applicable to our situation since (1) they are intended for frequent-service bus lines (i.e., small headways), or (2) they do not consider the relationship between delay and lateness for adjacent bus stops. The correlation between delay and lateness is especially important in modeling bus lines. Turnquist (1978) observed that an initial delay in service on a short headway line causes deteriorating service downstream due to increased boardings. Hence, the random variables L_{k-1} and D_k are positively correlated on short headway lines.

For bus lines with large headways, service would not necessarily deteriorate downstream, as passengers are likely to plan their arrivals at bus stops according to the bus schedule. Hence, if a bus falls behind schedule, it would not carry additional boardings forcing it to fall further behind. In fact, because most bus schedules contain built-in slack, it may be that buses have a tendency to catch up when they fall behind. This is reinforced by operator policies that penalize drivers when they operate as little as 30 seconds ahead of schedule, or when they fall significantly behind schedule. Therefore, we hypothesize that L_{k-1} and D_k are negatively correlated on long headway lines. We test this hypothesis and develop a new delay probability distribution function for infrequent service bus lines based on data generated by LAC/MTA.

3.1 Description of LAC/MTA

The Los Angeles County/Metropolitan Transportation Agency (LAC/MTA) operates almost 200 bus routes and has more than 2,500 buses in the fleet. Bus service is provided 24 hours a day, with limited service during late night hours. Currently, MTA coordinates timed transfer stations at three locations with the biggest being at 7th and Broadway operating at night. Once a year, a person is assigned by the MTA to collect data on each scheduled trip of each bus line. The data consist of actual arrival times, the number of passengers boarding and alighting, and fare types used at all stops. For this analysis, we selected two bus lines, #26 and #30, that converge near the timed transfer center at 7th and Broadway. Bus line #26 has 17 major stops and bus line #30 has 11 major stops. All the stops contain data from at least 30 trips. Both lines operate with a headway of 60 minutes at night.

To demonstrate the relationship between the delay and lateness for adjacent stops, Table 9 shows the correlation results of the data collected from bus lines 26 and 30. We also list the

Table 9. Summary of Correlation Results

Bus Line	L_k vs. L_{k-1}		D_k vs. D_{k-1}		D_k vs. L_{k-1}	
Number	Correlation	t-value	Correlation	t-value	Correlation	t-value
26	.70	28.9	-.19	5.8	-.33	10.3
30	.66	27.4	-.26	8.3	-.34	10.6

t-statistic for each correlation coefficient for the hypothesis that it is equal to zero. Based on the t-statistic, all the correlation coefficients listed in the table are significant at a 99.5% confidence level, since $t_{.995} = 2.57$. As expected, the latenesses at adjacent stops (L_{k-1} and L_k) are positively correlated, meaning that if a bus is late at a stop it is also likely to be late at the next stop. This result implies that buses do not recover immediately from disturbances that place them behind (or ahead) of schedule. However, buses can at least partially recover from their lateness, as reflected in the negative correlation between D_k and D_{k-1} and between D_k and L_{k-1}.

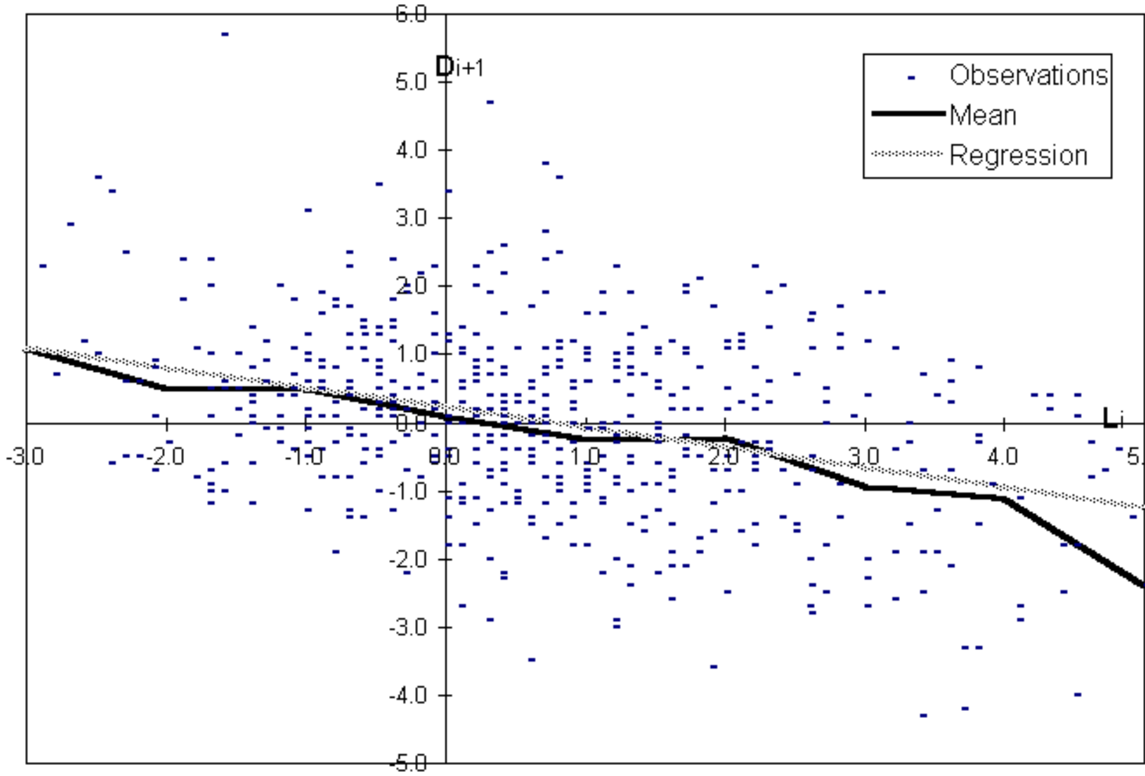
3.2 A Conditional Probability Model for Delay

To further demonstrate the effect of lateness on delay, Figure 1 plots the delay on segment k+1 versus the lateness at stop k for bus line #30. (The mean delays, shown on the solid line, are not centered on the actual values because each tick may represent more than one observation.) As one might expect, the plot shows that buses travel slower when ahead of schedule and faster when behind schedule. The plot suggests the following linear relationship for the expected delay, $E(D_k \mid L_{k-1})$, on segment k.

$$E(D_k \mid L_{k-1}) = a + b\, L_{k-1} \qquad (3)$$

The constant value, a, can be interpreted as the expected delay on segment k when the bus is ontime at the previous stop and the slope, b, is the correction factor when the bus is not on time. Figure 1 also shows the results of the regression analysis; the dashed line shows the regression line. For bus line #30, a = .23 (min) and b = -.29. We note that we found that Eq. (3)

also holds for bus line #26 with a = .06 (min) and b = -.28.



Regression Statistics	
Multiple R	0.3422914
R Square	0.1171634
Adjusted R Square	0.115662
Standard Error	1.2257286
Observations	590

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	117.240506	117.24	78.03	1.1644E-17
Residual	588	883.417477	1.5024		
Total	589	1000.65798			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept (a)	**0.2336**	0.05500825	4.2466	3E-05	0.12555922	0.3416326	0.12555922	0.34163255
X Variable 1 (b)	**-0.29247**	0.03310823	-8.834	1E-17	-0.3574943	-0.227445	-0.3574943	-0.2274446

Figure 1. Relationship Between Delay and Lateness from Previous Segment for Bus Line #30

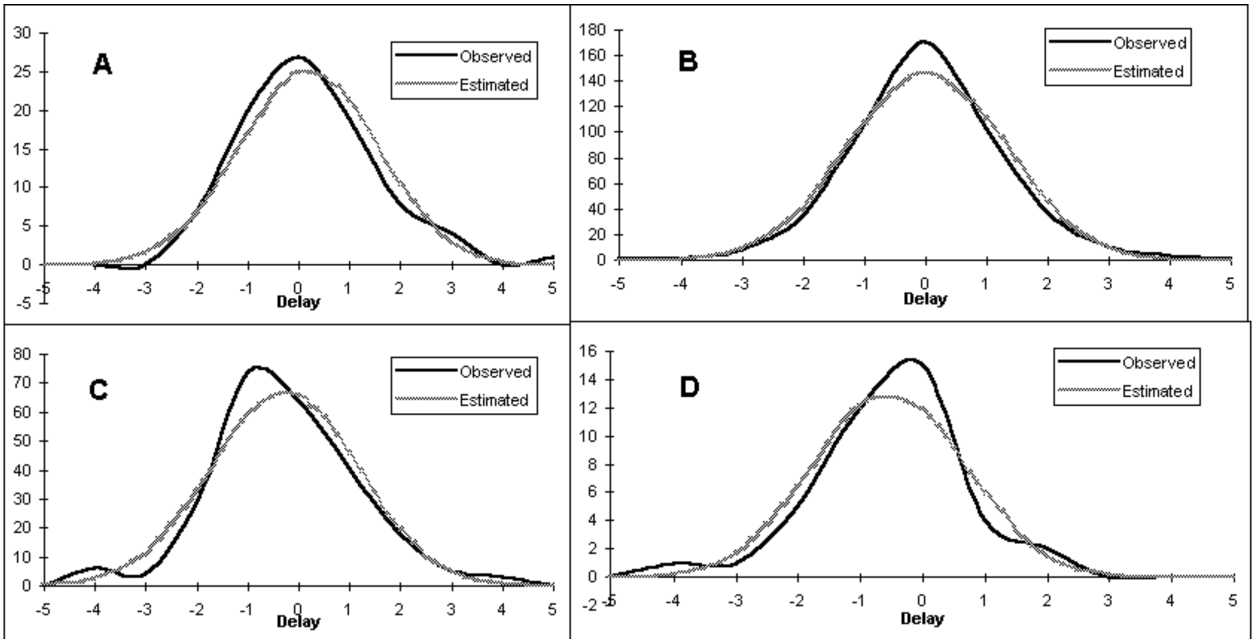
To derive the conditional probability density for delay given the lateness, $f(D_k \mid L_{k-1})$, the frequency distribution of the observed delay was evaluated over given ranges of lateness. We divided the lateness into five intervals: (-4, -2], (-2, 0], (0, 2], (2,4], and (4,4]. Figure 2 shows the resulting frequency distribution of delay over each lateness interval superimposed on a normal probability density function for bus line #26. The figure also lists the mean and standard deviation of delay for each lateness interval. Although the expected delay clearly depends on the lateness, there does not appear to be any relationship between the standard deviation of delay and lateness. The figure also shows the result of the Kolmogorov-Smirnov. Test for each lateness interval. At a 95% confidence level, the hypothesis that the conditional probability density of delay given the lateness, $f(D_k \mid L_{k-1})$, is a normal distribution cannot be rejected. We note that this analysis was performed for several other distributions such as lognormal and gamma and the normal distribution gave the best fit for the given data set

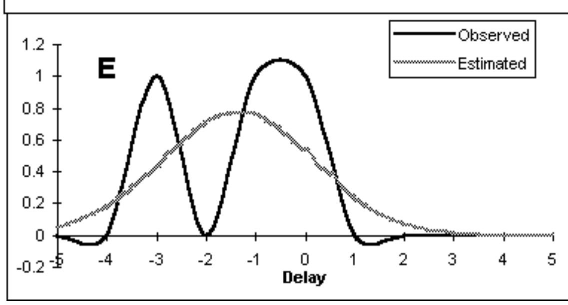
Some of the simulations that appear in Section IV are based on major schedule disturbances that place buses as much as 30 minutes behind schedule. In such instances, the model of Eq. (3) is unlikely to apply due to insufficient slack in the schedule. Therefore, for our simulations, the catch-up capability of a bus was truncated according to our estimate for the slack imbedded in the schedule. This is represented in the following relationship:

$$D_k = \max\left(g\,(S_k - S_{k-1}),\ \text{Normal}\,(E(D_k \mid L_{k-1}), s) \right) \qquad (4)$$

where:

g = the slack as a proportion of the schedule segment travel time





Interval	A	B	C	D	E
Mean	0.20930	0.03983	-0.26230	-0.52500	-1.33333
StDev	1.35574	1.28685	1.45043	1.21924	1.52753
K-s Test (α=0.05)	0.1467	0.0623	0.0871	0.2150	0.7080
Maximum |Xo-Xe|	0.0381	0.0289	0.0339	0.0410	0.1223

Figure 2. Distribution of Delay for Given Lateness Intervals for Bus Line #26.
(Range: A, (-•, -2]; B, (-2, 0]; C, (0, 2]; D, (2, 4]; E, (4, •])

The expected delay, $E(D_k | L_{k-1})$, is derived from Eq. (3) and s is the standard error. From Figure 1, the standard error of delay for bus line #30 is 1.22 minutes. Analysis of data from LAC/MTA suggest that g is on the order of .25, meaning that actual travel time over any segment can be no less than 75% of the scheduled travel time, even when far behind schedule. g is estimated by calculating the ratio of the maximum earliness over the scheduled travel time.

3.3 Forecasts Utilizing Tracking

In the presence of a bus tracking system, arrival times can be forecast for a timed transfer terminal using the conditional distribution model. We assume that the forecast is updated each time the bus passes a schedule check-point. The forecast arrival time is the sum of the scheduled arrival time at the transfer terminal and the forecast lateness time at the terminal. The forecast lateness is the sum of the forecasted delays over the segments between the current check point and the transfer terminal and the current lateness. We adopt the non-truncated model in this section, which should be accurate in the absence of unusually large schedule disturbances: $E(D_k | L_{k-1}) = bL_{k-1} + a$. We assume that the conditioned distribution for D_k is normal, with variance independent of L_{k-1}. Finally, for the sake of simplicity, we assume that the delay distribution is identical for all line segments.

Let L_0 be the lateness at the current stop. Then, the expected lateness at the next stop can be calculated as:

$$E(L_1) = (1+b)L_0 + a \quad (5)$$

The expected lateness at all subsequent stops (including the transfer terminal) is iteratively determined:

$$E(L_k) = (1+b)\, E(L_{k-1}) + a, \quad k > 1 \quad (6)$$

The variance of the lateness can similarly be derived from Eq. (6) by solving for the expectation of L_k^2 conditioned to L_{k-1}, and subtracting $E^2(L_k)$, yielding the following result:

$$V(L_k) = V(L_{k-1})\,(1+b)^2 + V(D) \quad (7)$$

where V(D) is the variance of the delay on an individual segment.

As k becomes large, $E(L_k)$ approaches a limiting value, which is found by solving for $E(L_k) = E(L_{k-1})$, resulting in:

$$E(L) = -(a/b), \quad \text{large } k \quad (8)$$

$V(L_k)$ also approaches a limiting value:

$$V(L) = -V(D)/(2b + b^2), \quad -2 < b < 0, \text{ large } k \quad (9)$$

For the lines studied at LAC/MTA, the limiting value of E(L) is on the order of .5 minutes, and the limiting value of V(L) is on the order of 3 min^2, indicating a good level of schedule adherence. For values of b smaller than -2 or greater than 0, lateness is naturally unstable, with the variance growing without bound as k becomes large. The variance is minimized when

b = -1, for which V(L) = V(D).

4. ANALYTICAL MODEL FOR SCHEDULING TIMED TRANSFERS

We consider dispatching strategies associated with a "bank" of buses, scheduled to arrive and depart at a terminal within a narrow time window. Each bank is defined by a set of bus lines, along with their scheduled arrival and departure times. We assume a one-to-one correspondence between arriving and departing buses.

Ideally, the terminal would operate in a manner that minimizes passenger and bus waiting time, and minimizes the number of passengers who miss connections (i.e., buses that arrive later than connecting buses depart). These objectives naturally conflict, in that holding a bus for a late connection may cause more delay for the passengers already on the bus than the amount saved for the connecting passengers. To simplify the analysis, the models that follow have a single attribute objective function, which counts all passenger waiting time (either for successful or missed connections) identically. However, the simulation model presented in the following section will provide separate performance statistics for bus waiting time, passenger waiting time, and missed connections to assess the validity of the single attribute objective.

We utilize the following objective, which counts the total waiting time among passengers on J lines:

$$\min_{t_{dj}} \sum_{j=1}^{J} W(t_{dj}) = \text{total waiting time for all passengers, with dispatch times } t_{dj}$$

$$= \sum_{j=1}^{J} (t_{dj}-t_j)B_j + \sum_{t_i \le t_{dj}} (t_{dj}-t_i)M_{ij} + \sum_{t_{dj}<t_i} (t_j-t_i)M_{ij} \qquad (10)$$

$$\text{s.t.} \quad t_{dj} \ge t_j, \qquad j = 1,2,\ldots J$$

where:

M_{ij} = number of passengers transferring from bus i to bus j

t_i = the maximum of the actual arrival time and scheduled arrival time ($\max(A_i, S_i)$) for bus line i

B_j = number of passengers originating at the stop, continuing on the bus, or delayed at later stops for line j

t_j = expected time of next departure for line j

t_{dj} = time that bus is dispatched on line j

$W(t_{dj})$ = the total passenger waiting if bus line j is dispatched at time t_{dj}

Note that the objective function is decomposable into J separate subproblems, where the objective of each subproblem is to optimize the dispatch time for one of the lines. Therefore, hereafter we drop the subscript j, recognizing that each parameter represents a specific line.

To keep the model easily tractable, we assume that any delays imposed on passengers at downstream stops are reflected in the parameter B, which could account for passengers originating at the stop along with passengers that board at later stops.

To create a lower bound on the objective function, we begin in the following sub-section with a holding strategy based on deterministic data. This is followed by a real-time dispatching strategy based on a fixed holding time. The appendix provides an additional model for optimizing holding time that follows a periodic review policy.

4.1 Deterministic Holding Strategy

In this section, we assume that all parameters in Eq. (10) are known with certainty, and t_d is the single decision variable, which is optimized to minimize $W(t_d)$. A bus is considered *available* when all passengers that are present at the terminal have boarded the bus. A bus is *ready* for dispatch if it is both available and the current time equals or exceeds its scheduled departure time. Suppose that at time 0 a bus is ready for dispatch but that another set of buses (numbered i = 1,2,...,N in order of arrival) are late and have not yet arrived. B represents the number of passengers currently on board, and M_1, M_2, ... represents the number of transferring passengers that will arrive from the late buses, with arrival times t_1, t_2, ... ($t \geq t_i$, for all i).

It is easily shown that $W(t_d)$ is minimized either at one of the t_i, or at 0. The derivative of $W(t_d)$ with respect to t_d equals:

$$W'_j(t_d) = \quad B + \sum_{t_i \leq t_d} M_i > 0, \qquad t_d \neq t_i \tag{11}$$

$W'_j(t_d)$ is strictly positive for all values of t_d other than t_i, i=1,2,...,N. $W(t_d)$ exhibits a discontinuity at t_i, when the function declines by the value $(t-t_i)M_i$. The function as a whole exhibits a sawtooth shape and the optimal solution cannot occur at any time other than at t_i or 0.

An optimal solution can be found through enumeration of t_i. A necessary condition for optimality for any value t_i is:

$$W(t_i) \quad \leq \quad W(t_{i-1}) \tag{12}$$

which can be expressed as:

$$(t_i - t_{i-1})(B + \sum_{j=1}^{i-1} M_j) \leq (t - t_i)M_i \qquad (13)$$

or, alternatively:

$$(B + \sum_{j=1}^{i-1} M_j) / (t - t_i) \leq M_i / (t_i - t_{i-1}) \qquad (14)$$

The left-hand side of the equation increases monotonically with i, and equals the ratio of the number of passengers on board at time t_{i-1} to the time remaining between t_i and the next dispatch. The right-hand side can vary up or down as i increases, depending on how long the bus must be held (t_i-t_{i-1}) or how many passengers will transfer (M_i).

4.2 Real-time Dispatching Strategies

In actual operation, most of the variables in Eq. (10) are unknown and therefore can only be forecast when the decision is made whether to hold or dispatch a bus. The probability distributions for these variables depend on the information available and the quality of travel time forecasts. It may be that these values vary significantly from run to run on a bus line, but they can nevertheless be forecast accurately due to availability of tracking and communication devices. Instead of making explicit use of these forecasts, this section considers a class of strategies in which buses can be held until a pre-determined dispatch time. Such a strategy is likely to approximate an optimal holding strategy. In the first model, buses must be held until this pre-determined time whether or not connecting buses have arrived. Effectively, this amounts to inserting a slack time into the schedule. This assumption is later relaxed. Appendix A provides a model that explicitly utilizes forecasts in its holding policy, which will be the subject of future research.

The holding time is determined from an optimal dispatching time. If the bus arrives prior to the optimal dispatch time, then it is held until this dispatch time. If it arrives after the optimal dispatch time, then it is dispatched immediately. The hold time equals the difference between the optimal dispatch time and the scheduled departure time. The objective function is expressed as follows.

$\min_{t_d}$ $W(t_d)$ = total waiting time among all passengers, for dispatch time t_d on line j

$$= t_d E(B) + \sum_{i=1}^{N} E(M_i) \int_{m_i}^{t_d} (t_d - t) f_i(t)dt + \sum_{i=1}^{N} E(M_i) \int_{t_d}^{\infty} (t-t) f_i(t)dt \qquad (15)$$

where $E(B)$ represents the expected number of continuing passengers on the bus line, $f_i(t)$ is the probability density function for the arrival time of bus i, and m_i is the minimum possible arrival time for line i. We assume that the waiting time for passengers who originate at the terminal is independent of t_d, as they can adjust their arrival time at the stop to the scheduled departure time. Taking the derivative of Eq. (15) with respect to t_d yields:

$$dW(t_d)/dt_d = E(B) + \sum_{i=1}^{N} E(M_i) F_i(t_d) - \sum_{i=1}^{N} E(M_i)(t-t_d) f_i(t_d) \qquad (16)$$

In the special case where bus arrival times are identically distributed, Eq (16) is minimized by solving for:

$$(t-t_d)f(t_d) \quad = \quad F(t_d) + E(B)/E(M)N \qquad (17)$$

where E(M) is the expected number of connecting passengers per bus.

In our simulations, we will also provide the option of dispatching a bus prior to t_d in the event that all connecting buses have already arrived. Eq. (15) can be modified to reflect the reduced waiting time as follows:

$$W(t_d) = [t_d E(B) + \sum_{i=1}^{N} E(M_i) \int_{m_i}^{t_d} (t_d-t)f_i(t)dt + \sum_{i=1}^{N} E(M_i) \int_{t_d}^{\infty} (t-t)\, f_i(t)dt] - [(t_d - g) - \int_{g}^{t_d} 1- \prod_{i=1}^{N} F_i(t)\, dt](\sum_{i=1}^{N} E(M_i) + E(B))] \qquad (18)$$

where g is the minimum arrival time of any approaching bus, $g = \min (m_1, m_2, \ldots, m_N)$. The added bracketed term gives the expectation of the minimum of $\{t_d$, arrival of last bus$\}$. Taking the derivative of Eq. (18) with respect to t_d yields:

$$dW(t_d)/dt_d = E(B) + \sum_{i=1}^{N} E(M_i)F_i(t_d) - \sum_{i=1}^{N} E(M_i)(t-t_d)\, f_i(t_d) - [\prod_{i=1}^{N} F_i(t_d)]\, (\sum_{i=1}^{N} E(M_i) + E(B)) \qquad (19)$$

To optimize for t_d , solve for Eq. (19) = 0. Because buses are allowed to depart prior to t_d, the optimal value of t_d is somewhat larger than provided by Eq. (16).

5. SIMULATION MODEL FOR SCHEDULING TIMED TRANSFERS

A simulation model of a timed transfer terminal is developed to evaluate the performance of dispatching rules with ITS versus those without ITS. The model is developed using a general-purpose simulation language, SLAMSYSTEM (Pritsker, 1986). The advantage of using a process-oriented language to model bus operations is that a small generic network, which has the flexibility to test many different dispatching strategies, can be used to represent detailed bus movement.

The scheduled arrival times at each major stop and at the timed transfer terminal for each bus line are input to the model. The scheduled travel time between major stops defines a particular segment along a bus line. The model simulates the movement of a bus on each segment on the line until it reaches the timed transfer terminal. The actual travel time on each segment, $A_k - A_{k-1}$, is the scheduled travel time plus the delay: $S_k - S_{k-1} + D_k$. We use Eq. (4) to sample the delay on the segment, D_k . Consistent with the simulation model of Seneviratne (1990) we assume that the distribution of the number of boarding passengers at each stop is Poisson distributed. Since we are only concerned with predicting the behavior of the system at the terminal, the model simulates only the passengers that stay on the bus until they reach the terminal (i.e., passengers that exit prior to the terminal are not simulated). Since we are only concerned with long headway lines, the model does not attempt to correlate travel time to the number of passengers boarding and exiting.

We model the detailed movement of a bus along its line instead of sampling a single arrival time at the terminal, which was the approach used by Abkowitz (1987), to allow for bus tracking capabilities and the subsequent mechanism to forecast its arrival time to the timed transfer terminal. The forecast arrival time of a bus to the terminal at any instant of time is the scheduled arrival time plus the expected arrival lateness to the terminal, which is based on its current location, and is iteratively calculated using Eq. (6).

Two levels of ITS are considered: (1) system with centralized tracking, and (2) system with information on connecting passengers, as well as centralized tracking. The tested strategies are:

1) Hold until all scheduled buses have arrived;
2) Do not hold and dispatch bus at the maximum of its scheduled departure time and actual arrival time;
3) Hold the bus for a maximum period of 1.5 minutes if all scheduled buses have not arrived;
4) Forecast bus arrivals and hold if another bus is expected to arrive within 1.5 minutes;
5) Forecast bus arrivals and hold if another bus is expected to arrive within 1.5 minutes and has at least one passenger who plans to transfer to the holding bus; and

6-8) Same as Strategies 3-5, except hold time is changed to 3 minutes.

Strategies 4 and 7 require bus tracking only while Strategies 5 and 8 require bus tracking as well as passenger counting capabilities. The holding times used in the simulation are smaller than the values that would minimize the average total delay for transferring passengers. Based on Eq. 15-

19, the optimal pre-determined hold times would be larger, due to the high penalty of missing a bus with a large headway. Somewhat smaller values are used in the analysis because we believed that larger hold times would be too disruptive to the downstream schedule. We revisit this issue later by studying the sensitivity of the results as a function of the holding time.

The parameters used for the simulation model are as follows. Each bus line is scheduled to depart from the terminal at the same time. We assume 24 major stops which are uniformly spaced with a duration of 2.5 minutes, giving a scheduled trip time to the terminal of 60 minutes. The expected number of passengers boarding at each stop is the same, with a mean of .42, which gives an expectation of 10 passengers on the bus when arriving at the terminal. Passengers are equally likely to transfer to any one bus. Based on the data analysis in Section II, we initially set a = .20 min., b = -.30, and g = .25 for each bus segment. We later discuss the sensitivity of the results to different values of all the above mentioned parameters.

A summary of the simulation results is provided in Figure 3. We show the results for three different cases representing 2, 5, or 10 bus lines connecting at the terminal (N=2, 5, and 10). Included in the figure are the average departure lateness and the fraction of passengers missing their connection of 500 simulation runs for each scenario.

As expected, the "all hold" strategy (#1) has the highest average departure lateness but has no passengers missing their connections. Alternatively, the "do not hold" strategy (#2) has the most passengers missing their connection and the smallest departure lateness. Comparing strategy 3 with 4 and strategy 6 with 7 shows the impact of bus tracking, as the former lack a forecasting ability. On average, bus tracking reduces the departure lateness by about 20 seconds without a subsequent increase in the number of passengers missing their connections.

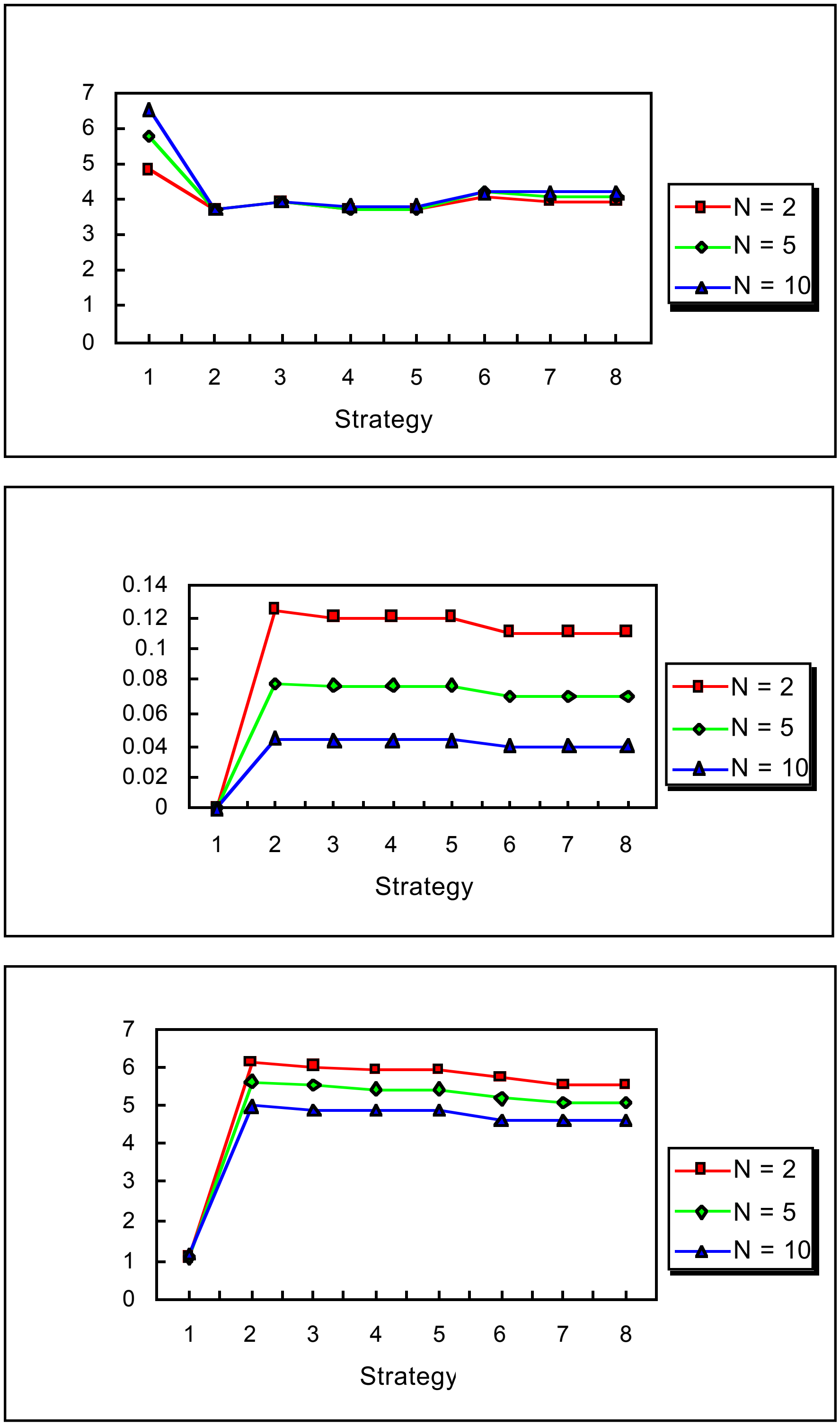


Figure 3. Summary of Results for the Different Dispatching Strategies.

Comparing strategy 4 with 5 and strategy 7 with 8 shows the impact of passenger counting. In our simulations there is a high likelihood that a passenger will be transferring to a holding bus. Therefore, passenger counting provides little added value and the two strategies are almost identical.

Figure 3 also plots the ratio of the total passenger delay for the given scenario over a lower bound, as defined by Eq. (10). As shown in Section III, with deterministic data the optimal dispatching time point for a bus at the terminal will either be at its scheduled departure time or the arrival time of a bus not already at the terminal. Hence, the optimal dispatching time point can be found by enumerating all possible combinations and selecting the time point which minimizes Eq. (10). Treating the actual arrival times and passenger information at the end of each simulation run as deterministic data in evaluating Eq. (10) provides a lower bound for the stochastic case.

As Figure 3 shows the "all hold" strategy provides the best ratio to the lower bound. However, this strategy is not a realistic one to implement since it yields unusually high bus departure lateness at the terminal, which will have detrimental effects on subsequent stops. Due to the high penalty of missing a bus (i.e., large bus headway), the table shows that it is preferable to use a holding time of 3 minutes as opposed to 1.5 minutes. In this case, holding for a fixed amount of time (Strategy 6) and the forecasting strategies (Strategies 7 and 8) are the best performing strategies, based on the total passenger delay criterion.

Figure 4 shows the sensitivity of the results for the forecasting strategy as a function of the maximum holding time. As the maximum holding time increases, the rate of increase in the average departure lateness is the greatest when the number of transferring buses is large (e.g., N=10). For the given data set, a maximum holding time greater than 3 minutes gives an average departure lateness of more than 4 minutes, and the lateness starts to significantly increase as the holding time increases, especially when N=10. In terms of the fraction of passengers missing their connection, the rate of decrease is the greatest when N is small. Based on the objective function defined in Eq. (18), the graph shows that the optimal holding time is over 8 minutes, although it gives a high average departure lateness at this level. The optimal holding time is much larger than the expected arrival lateness due to a high penalty associated with missing a bus with a large headway in the objective function. We do not recommend using a holding time this large because it would too disruptive to the downstream schedule. These findings suggest that for large headways the single attribute objective function defined in Eq. (18) is relevant only over a certain range of the bus dispatching time.

We next study the impact of ITS when one of the connecting buses experiences a breakdown or major delay. For these scenarios, we randomly selected one of the buses to be delayed for an extra 30 minutes at a random point on the line. Figure 5 shows the results for this case. We do not plot the departure lateness for the "all hold" strategy since its value (over 22 minutes on average) is significantly larger than the other strategies. Comparing strategy 3 with 4 and strategy 6 with 7 shows that the impact of bus tracking is more significant when one of the buses experiences a major delay, especially when there is a small number of connecting buses (N=2). Due to the high departure lateness associated with the "all hold" strategy, it provides the worst ratio to the lower bound. In this case, "not holding" (Strategy 2) and the forecasting strategies (Strategies 4, 5, 7 and 8) perform the best based on the total passenger delay criterion.

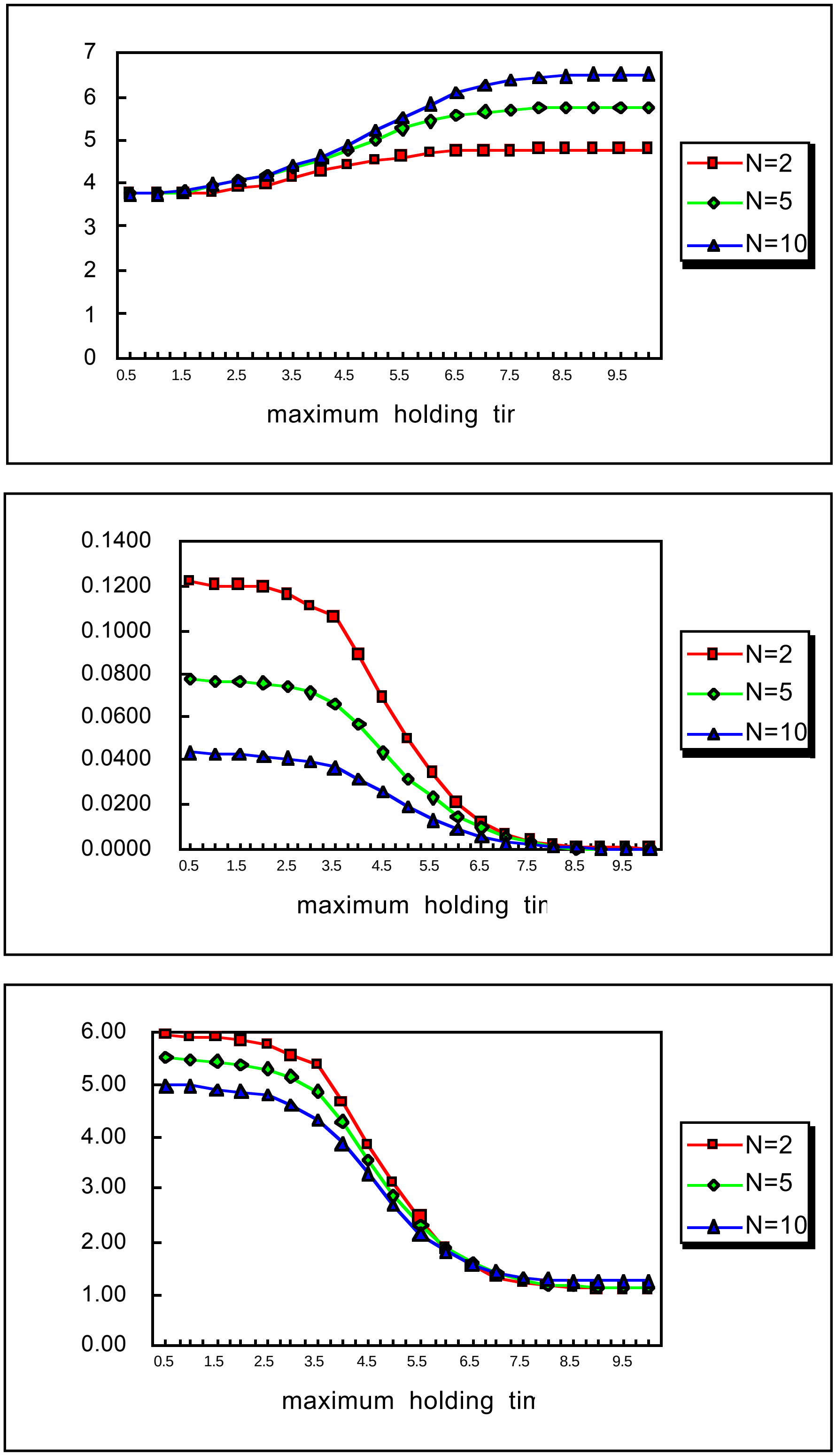


Figure 4. Summary of Results for Forecasting and Hold Strategy as the Maximum Holding Time Varies.

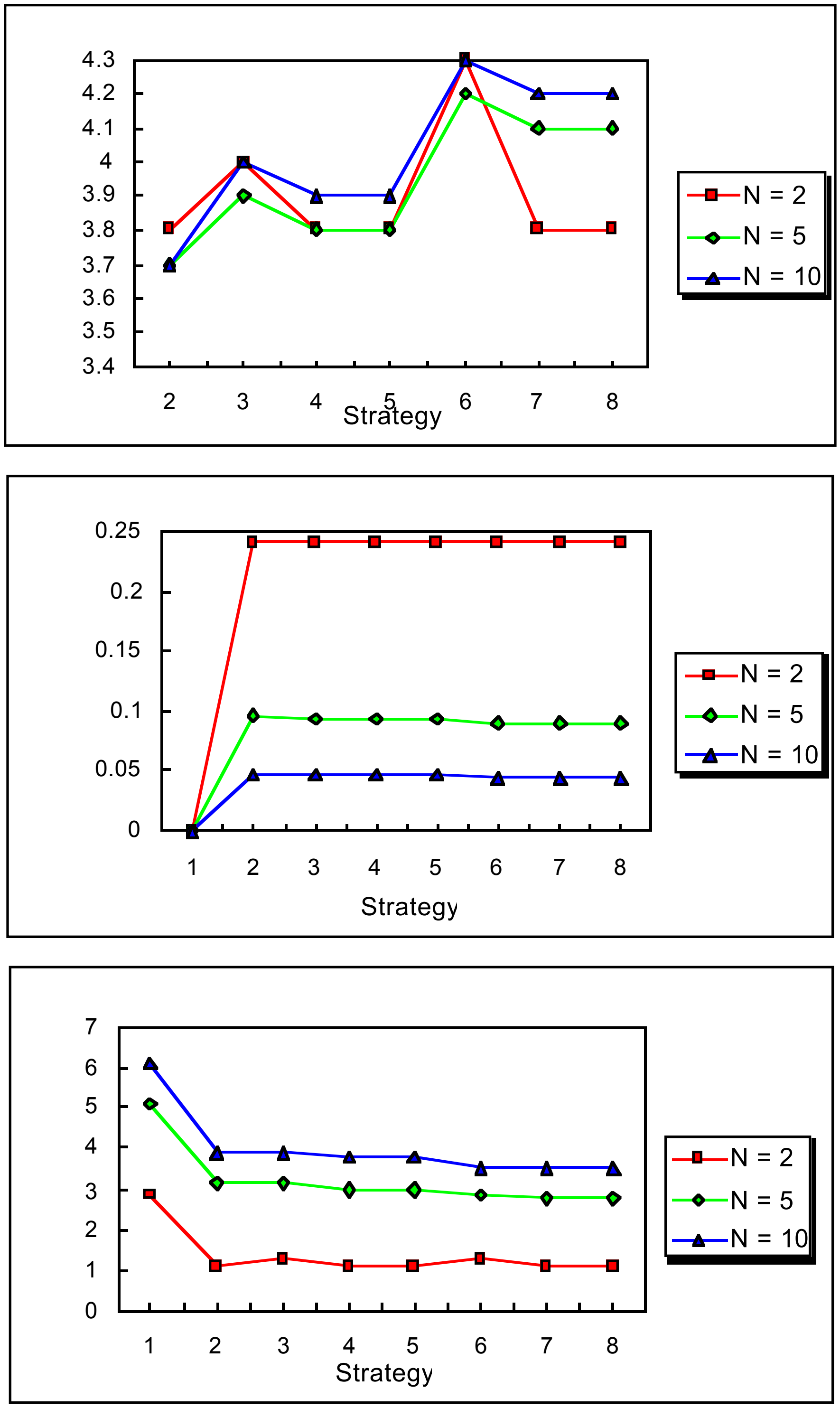


Figure 5. Summary of Results When There is a 30 Minute Breakdown.

We tested the sensitivity of the above findings for numerous other parameter values and found no difference in the results. The other values tested include a total scheduled trip length of 30 minutes, expected number of passengers on the bus when arriving to the terminal of 25 and 50 passengers, distance between major stops of 5 minutes, a = .05 and a = 1.0.

6. CONCLUSIONS

Intelligent transportation systems have the potential for improving the performance of bus systems by providing improved connectivity between lines at timed transfer terminals. By relaying a bus location in real time, and relaying information regarding whether or not transferring passengers are on board, an informed decision can be made as to whether to hold or release connecting buses.

The potential benefits of such a system were estimated through simulation of timed transfer terminals with 2 to 10 connecting bus lines. Under the ITS strategies investigated, a bus is only held when a late-arriving bus is forecast to arrive within a pre-set holding time. Without ITS, connecting buses are held up to the pre-set holding time automatically when other buses are late. As a consequence, ITS has the potential to reduce waiting time for passengers on the connecting buses without greatly increasing the number of missed connections. This hypothesis was borne out in our simulations, which resulted in about a 20 second reduction in delay per passenger through use of ITS. To evaluate whether this time saving justifies investment in AVL technology and associated systems, an economic analysis was completed in Table 10 for a range of scenarios. In the first part of the table, the net annual benefit (value of time savings minus system cost) was calculated as a function of fleet size, based on the historical cost of larger AVL installation. Costs are amortized over a five-year lifetime at a 10% discount rate, and passenger time is valued at $10/hour. Based on these costs, the benefits of improved timed transfer alone do not justify installation cost, except for very large installations with an unusually large number of transfers per vehicle.

Table 10. Economic Benefit of Timed Transfer System

Net Benefit For Timed Transfer System: Historical Costs

	GPS			Sign Post		
Transfer/Bus/Year	10 Veh	100 Veh	1000 Veh	10 Veh	100 Veh	1000 Veh
1000	-37667.5	-6803.18	-3716.75	-45133	-5959.03	-2041.63
10000	-37167.5	-6303.18	-3216.75	-44633	-5459.03	-1541.63
100000	-32167.5	-1303.18	1783.252	-39633	-459.027	3458.366

Net Benefit for Timed Transfer System: Varying Costs

	Fixed Cost			Monthly Cost Only		
Transfer/Bus/Year	2000	5000	10000	$ 20/mth	$ 50/mth	$ 100/mth
1000	-472.039	-1263.43	-2582.42	-184.444	-544.444	-1144.44
10000	27.96059	-763.432	-2082.42	315.5556	-44.4444	-644.444
100000	5027.961	4236.568	2917.581	5315.556	4955.556	4355.556

The second part of the table evaluates the net benefit for varying system costs. As can be seen, ITS technology appears to be economically justified if costs can be brought down to about 2000/vehicle (or $20/month service charge) and each bus handles about 10,000 transfers per year (about 40 per day of operation). Given the general direction of costs for GPS systems, such a scenario is plausible, assuming that transit operators purchase off-the-shelf systems. This analysis does not include operation and maintenance costs; however, it also does not include ancillary benefits from tracking, such as improved planning data.

A coordinated timed transfer might be achieved through use of radio communication instead of AVL, if the bus operator is able to report to the dispatcher whenever behind schedule by a critical amount of time. However, the system is less fail-safe than tracking technology.

In most of our simulations, average passenger delay is minimized when following the policy of holding until all buses arrive. This is obviously a simple and attractive policy that guarantees that every passenger makes his or her connection, and it does not require ITS. The drawback is that an "all hold" policy performs very poorly when a major delay occurs on one of the connecting lines. Another drawback is that downstream slack may not be sufficient to recover the schedule. It may be that a high level of performance can be achieved by asking drivers to radio in to their dispatcher when falling more than a set number of minutes behind schedule. This could then provide an exception to the all-hold policy, which would otherwise be employed.

As a side issue, we discovered in our empirical analysis that delays over segments of bus lines with large headways are negatively correlated with lateness at the start of the segment. This indicates that buses that are behind schedule tend to catch up and buses that are ahead of schedule tend to slow down, counter to the phenomenon observed on short headway lines. It also appears that bus lines contain considerable slack. It may be that ITS would provide greater benefits if slack times were reduced. Moreover, the greatest benefit of ITS could come from enabling bus systems to meet their scheduled headway with fewer buses, which would be achieved through tighter schedules. This is the subject of ongoing research.

ACKNOWLEDGEMENT

The research reported in this paper was partially supported by Partners for Advanced Transit and Highways (PATH) and the California Department of Transportation (CALTRANS).

Our appreciation goes to the Los Angeles County Metropolitan Transportation Authority for their cooperation in providing data.

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APPENDIX A

A PERIODIC REVIEW HOLDING POLICY

This section considers real-time dispatching strategies with periodic updates, allowing a bus to be dispatched at any update time, provided that the relevant dispatch criterion is satisfied. As an approximation, the deterministic dispatch rule can be operationalized by periodically reviewing Eq. (11) and determining whether $W(t_d)$ is minimized at the current time or at some future time (substituting expectations for random variables). If $W(t_d)$ is minimized at the current time, then the bus is dispatched immediately; otherwise, it is held until the next review time. The bus can be dispatched at any review time, whether or not a bus has just arrived. However, once a bus is dispatched, the decision cannot be reversed. This solution is only approximate in that the decision rule does not reflect future updates or possible dependencies between M_i and t_i. However, if the relationship between $W(t_d)$ and t_d is flat then the errors produced should be negligible.

An optimal decision rule can be developed through use of decision trees. However, it is difficult to formulate and solve an exact decision tree for the following reasons:

1) Probability distributions are inherently continuous.
2) Forecasts may be updated frequently.
3) Solution requires convoluting distributions representing minima or maxima of sets of random variables.

The following is suggested as a simplified decision tree with an approximate objective function. Although the decision can be reviewed periodically, the decision tree is collapsed to a relatively small number of future events. Suppose that time 0 represents the current time. Let the capitalized T_i represent the value of a random variable representing arrival time on bus line i.

Suppose that the late buses are numbered according to expected arrival time ($E(T_1) < E(T_2) < ... < E(T_L)$). The decision is restricted to the following set of N+1 alternatives:

0) No hold (dispatch immediately)

1) Wait for bus 1 only

2) Wait for both bus 1 and bus 2

3) Wait for all of bus 1,2, and 3, ...

In each option, the decision is to wait for the entire set of buses to arrive, independent of their actual order of arrival (which can differ from the order of expectations).

For each alternative, the expected waiting time is calculated exactly. The bus is dispatched immediately if alternative 0 produces the smallest expectation. Otherwise, the bus is held to the next update, at which time the expectations are re-evaluated. Note that the decision at time 0 does not reflect possible changes that occur at future updates, and is therefore approximate.

The expected waiting times are calculated as in the following examples, where $E(W_i)$ is the expected waiting time when holding for buses 1 to i:

No Hold

$$E(W_0) = \sum_{i=1}^{N} E[(t - T_i)M_i] \qquad \text{(A1a)}$$

Hold for Bus 1 Only

$$E(W_1) = E(BT_1) + \sum_{i=2}^{N} P(T_i \leq T_1)E[(T_1 - T_i)M_i|T_i \leq T_1] + \sum_{i=2}^{N} P(T_i > T_1)E[(t - T_i)M_i|T_i > T_1] \qquad \text{(A1b)}$$

Hold for Buses 1 and 2 Only

$$E(W_2) = E(B \max\{T_1,T_2\}) + \sum_{i=1}^{2} E[(\max\{T_1,T_2\} - T_i)M_i] \quad (A1c)$$

$$+ \sum_{i=3}^{N} P(T_i \leq \max\{T_1,T_2\})E[(\max\{T_1,T_2\} - T_i)M_i | T_i \leq \max\{T_1,T_2\}]$$

$$+ \sum_{i=3}^{N} P(T_i > \max\{T_1,T_2\})E[(t - T_i)M_i | T_i > \max\{T_1,T_2\}]$$

Even this simplified model is not easily tractable for large N. As a further simplification, cases where $T_i < T_j$ for $i > j$ might be ignored. Furthermore, random variables could be assumed to be independent, resulting in the following:

No Hold

$$E(W_0) = \sum_{i=1}^{N} [E(t) - E(T_i)]E(M_i) \quad (A2a)$$

Hold for Bus 1 Only

$$E(W_1) = E(B)E(T_1) + \sum_{i=2}^{N} [E(t) - E(T_i)]E(M_i) \quad (A2b)$$

Hold for Buses 1 and 2 Only

$$E(W_2) = E(B)E(\max\{T_1,T_2\}) + \sum_{i=1}^{2} [E(\max\{T_1,T_2\}) - E(T_i)]E(M_i) + \sum_{i=3}^{N} [E(t) - E(T_i)]E(M_i) \quad (A2c)$$

These expressions only require calculations for the expectation of the maximum of a set of random variables. If independent, this is defined by:

$$E[\max\{ T_1,T_2,\ldots, T_N\}] = \int_0^{\infty} 1 - \prod_{i=1}^{N} F_i(t)\, dt \qquad \text{(A3)}$$

where $F_i(t)$ is the distribution function for the arrival time on bus line i.

These policies, in theory, could out-perform the fixed holding time policies studied in the paper and will be the subject of future research.

APPENDIX B

SIMULATION MODEL DOCUMENTATION

Documentation

This appendix documents the simulation model for dispatching buses at a timed transfer terminal. The following dispatching strategies are included in the simulation model.

1- Hold buses at timed transfer terminal until all buses arrive.

2- Do not hold any bus at the timed transfer terminal.

3- Hold buses at timed transfer terminal for a fixed period.

4- Forecast bus arrivals and hold the bus at timed transfer terminal if forecast arrival of any approaching bus is within a fixed period.

5- Forecast bus arrivals and hold the bus at timed transfer terminal if <u>both</u> of the followings conditions are true for any approaching bus:

a) Forecast arrival of the bus is within a fixed period

b) More than a fixed number of passengers on all the approaching buses which are forecasted to arrive within the holding time of the current bus at timed transfer terminal

6- Forecast bus arrivals and hold the bus at timed transfer terminal if <u>both</u> of the followings conditions are true for any approaching bus:

a) Forecast arrival of the bus is within a fixed period

b) More than a fixed number of passengers are currently on all the approaching buses, arriving within the holding time of the current bus at the terminal, who want to transfer to the holding bus

The SLAMSYSTEM software package (Pritker, 1986) is used to simulate the above dispatching strategies. SLAMSYSTEM is a general-purpose simulation language distributed by Pritsker Corporation. Statistical output of the simulation is linked to an EXCEL spreadsheet for statistical analysis. An animation model is also developed for each strategy which dynamically shows bus behavior at the transfer terminal as well as approaching buses. A sample animation screen is shown in Figure B1. We next present in detail the simulation model.

Simulation Model

The simulation models the detailed movement of a bus along its route by dividing the line into segments. At the end of each segment is a major stop or check point where passengers can board the bus. The scheduled travel time on each segment is the difference of the scheduled arrival time of the bus to the stop at the end point of the segment from the stop at the beginning of the segment.

The entities in the system are the buses. Each entity in the simulation model has its unique set of attributes which are collected in the ATRIB array. The attributes are:

ATRIB#	*Description*
1	*Line number of the bus*
2	*Last stop number visited for the bus at the current time (TNOW).*
3	*Total number of passengers on the bus at the current time (TNOW) that will continue on to the timed transfer terminal.*
4	*Marker for current bus location (0=not at terminal, 1=at terminal)*
5	*Actual travel time on bus segment*

The SLAM network is displayed in Figure B2. The simulation model is a mixture of SLAM network code and user-written routines representing simulation events. They are:

EVENT #	*Description*
1	*Read data*
2	*Update bus information upon arrival to a bus stop*

3	*Arrival to the timed transfer terminal*
4	*Departure from timed transfer terminal*
5	*Update animation screen*
6	*Collect statistics*
7	*Hold buses based on forecast*

The first node in the slam network is a create node where a dummy entity is generated at time 0 to start the simulation. The dummy entity is first routed to EVENT Node #1 (INPT) where the data is read from file "BUS.DAT". Then, the dummy entity goes into a cycle and the bus entities are generated in this cycle. Each bus is sent to its respective first stop (at node STOP) and its attributes are updated for the first stop, upon arrival. This bus will travel to the next stop with duration ATRIB(5). We later discuss how the duration ATRIB(5) is calculated. Afterward each bus goes into a cycle which represents the movement of the bus to the next stop and updates its attributes, variables and statistics for that stop. This cycle is represented by EVENT Node #2. When the bus arrives at the timed transfer terminal (i.e., ATRIB(4)=1), it exits the cycle and dispatches to EVENT Node #3 (TT). All of the statistics on the bus and on its passengers will be calculated at node TT, and the bus then exits the network simulation model.

As Figure B2 shows, most of the simulation logic is conducted using user-written routines. Figure B3 shows the overall logic for the user-written routines. The major steps of the model are as follows.

Step 1. Select strategy:

The model is prepared for 6 strategies, regarding control of the buses at a timed transfer station.

1- ALL HOLDING: As each bus arrives to the timed transfer station, hold the bus until all other buses arrive. When all buses have arrived, release each bus at the maximum of the current time and its own scheduled departure time from the timed transfer terminal.

2- NO HOLDING: Bus leaves the timed transfer station either on schedule, or when it arrives to the timed transfer terminal if later than the scheduled departure time.

3- HOLD FOR A FIXED PERIOD : Similar to strategy # 1 but do not hold any bus more than a fixed period, inputted by a scheduler. No forecasting is involved in this strategy.

4- FORECAST & HOLD IF WITHIN A FIXED PERIOD: Each time a bus arrives at a stop, forecast its arrival to the timed transfer terminal. If the forecast arrival of at least ONE of the approaching buses is within the holding time of a particular bus (j) at the timed transfer terminal, then hold bus (j). Otherwise release the bus at the maximum of the current time and its own scheduled departure time from the timed transfer terminal.

5- Forecast & hold if within a fixed time and also if more than a fixed number of passengers are on board on all the arriving buses, which are forecasted to arrive within the holding time.

6- Forecast & hold if within a fixed time and also if more than a fixed number of passengers who will transfer to the holding bus and arriving within the holding time.

Step 2. Input data and initialize variables and attributes:

Let the variable i be the stop index and the variable j be the bus line index. The variable i is also the index for the bus segment preceding stop i. The following data is required for the simulation model:

NLINE: Number of bus lines requiring transfer at the timed transfer terminal.

$NSTOP_j$: Number of stops for each bus line for j = 1,2, ... , NLINE where $NSTOP_j + 1$ represents timed transfer terminal.

$A_{j,i}$: Scheduled arrival time for each bus at each stop (j,i) (and transfer terminal) for j = 1,2,...,NLINE and i = 1,2,...,NSTOP $_j$+1

$MEAN_{j,i}$: The mean delay on segment (j,i) for j = 1,2,...,NLINE and i = 1,2,...,NSTOP $_j$+1.

$VAR_{j,i}$: The variance of delay on segment (j,i) for j = 1,2,...,NLINE and i = 1,2,...,NSTOP $_j$+1.

$N_{j,i}$: Mean number of passengers boarding at stop (j,i) for j = 1,2,..., NLINE and i = 1,2,..., NSTOP $_j$ +1, which will continue onto the timed transfer terminal.

D_j: Scheduled departure time of each bus line from the timed transfer terminal, for j = 1,2,..., NLINE.

NO_j: The expected number of passengers originating at timed transfer terminal, for j = 1,2,..., NLINE.

$NT_{j1,j2}$: "From / To" matrix which represents cumulative probability of passengers transferring from one bus-line to another for j1 = 1,2,..., NLINE and j2 = 1,2,..., NLINE . $NT_{j1,j1}$ represents the probability that a passenger will continue on the same bus.

H_j: Maximum allowable holding time for bus line j at timed transfer terminal for j = 1,2,..., NLINE.

M_{j1}: Minimum total number of passengers required to be on board on all approaching buses to hold bus j1 at timed transfer terminal

MM_{j1}: Minimum total number of passengers transferring from all approaching buses to bus j1 in order to hold it

The following user defined variables are also initialized:

$DLY_{j,i}$: The actual delay on segment (j,i) for bus lines j = 1,2,..., NLINE at stop i = 1,2, ... , NSTOP $_{j}$+1, generated from Normal distribution using $MEAN_{j,i}$ and $VAR_{j,i}$.

$N'_{j,i}$: The actual number of passengers boarding at stop (j,i) for bus lines j = 1,2,...,NLINE at stops i = 1,2, ... , NSTOP $_{j}$, generated from a Poisson distribution using mean $N_{j,i}$.

$NT'_{j2,j1}$: The actual number of passengers transferring from bus j2 to bus j1 for bus lines j1 = 1,2,...,NLINE and j2 = 1,2,...,NLINE, generated from a Random distribution using "From / To Matrix" ($NT_{j2,j1}$).

NO'_{j}: The actual number of passengers originating at timed transfer terminal for bus lines j = 1,2,...,NLINE, generated from a Poisson distribution using mean NO_{j}.

F_{j}: The current forecast arrival time at timed transfer terminal for bus lines j = 1,2,...,NLINE, initially equal to the scheduled arrival time at the timed transfer terminal ($A_{j,NSTOP_j+1}$).

$AA_{j,i}$: Actual arrival of bus j to stop i for j = 1,2, ... ,NLINE and i = 1,2, ... , NSTOP $_{j}$+1

AD_{j}: Actual departure of bus j from timed transfer terminal for j = 1,2, ... , NLINE.

Step 3. Generate bus lines & send them to STOP #1:

At this step, the buses are generated and sent to the first stop.

```
For j = 1 TO NLINE => Update Attributes & Variables:
           ATRIB(1) = j
           ATRIB(2) = 1
           ATRIB(3) = N'j,1     where N'j,1 ~ POISSON (Nj,1)
```

Step 4. Generate number of passengers who transfer

Number of passengers expected to transfer at timed transfer terminal from the current bus to all other busses is randomly generated at this stop, based on the total number of passengers on board [ATRIB(3)].

```
j1=ATRIB(1)
For j2 = 1 TO NLINE
```

$NT'_{j1,j2}$ ~ RANDOM number incremented based on ($NT_{j1,j2}$)

Step 5. Update variables & attributes for next stop

Actual arrival at each stop equals scheduled arrival at that stop plus current accumulated delays on the previous segments. Forecast arrival to timed transfer terminal is updated by adding the current lateness plus expected delay at consequent segments to the scheduled arrival to timed transfer terminal

```
j=ATRIB(1)
ATRIB(2)=ATRIB(2)+1
i=ATRIB(2)
```

$DLY_{j,i}$ ~ NORMAL($MEAN_{j,i}$, $VAR_{j,i}$)

Actual arrival ($AA_{j,i}$) $= A_{j,i} + \sum_{l=1}^{i} DLY_{j,l}$

ATRIB(5) = $AA_{j,i}$ - TNOW

Step 6. Check hold status for strategies 4, 5, and 6

If the strategy includes forecasting (4, 5, or 6), a check to release a holding bus j1 at the timed transfer terminal needs to be made whenever another bus j2 arrives at any stop. The event that performs this check is EVENT #7 which is scheduled to occur at the maximum of the current simulation time and the scheduled departure time of bus j1. The following rules are used to determine whether or not to hold a bus in EVENT #7.

If the forecast arrival for any bus j2 approaching the timed transfer terminal is less than the holding time for some bus j1, at the timed transfer terminal, then :

i) For strategy #4 which does not use passengers' information, continue holding bus j1 at timed transfer terminal.

ii) For strategy #5, **only** if more than M_{j1} are on board on all the approaching buses, continue holding bus j1 at timed transfer terminal.

iii) For strategy #6, **only** if more than MM_{j1} will transfer from all the approaching buses, to bus j1, continue holding bus j1 at timed transfer terminal.

Otherwise, schedule departure (EVENT #4) for bus j1 at the maximum of the current simulation time and the scheduled departure time of bus j1.

Step 7 . Current bus at timed transfer terminal

If the current bus j has arrived at the timed transfer terminal then go to step 7. Otherwise, increment the number of passengers on the bus by the number originating at this stop, and go back to step 4 (next stop)

```
j=ATRIB(1)
i=ATRIB(2)
IF (i=NSTOP j+1) then GOTO step 8
ELSE
N' j,i ~ POISSON(N j,i )
ATRIB(3) = ATRIB(3)+N' j,i
Go back to step 4
```

Step 8. Schedule bus departure from timed transfer terminal

Figure B4 shows the detailed logic for scheduling bus departure at the timed transfer terminal for bus j. The EVENT that handles the departure logic is EVENT #4 and is described later.

For strategy # 2:

Schedule bus departure (EVENT #4) at the maximum of scheduled departure and current simulation time

```
Schedule EVENT #4 at MAX(TNOW,D j)
```

For all other strategies :

If the current bus (j1) is the last bus to arrive to the timed transfer terminal, schedule the actual departure of all buses at their scheduled departure or current time, whichever is later (EVENT #4):

```
IF all buses have already arrived when j1 arrives at
terminal =>
For j2 = 1 TO NLINE
Schedule EVENT #4 (departure) for j2 at MAX (TNOW , D j2)
```

Also for strategy #3: if current bus (j1) is not the last bus to arrive at the timed transfer terminal schedule EVENT #4 to occur at maximum of the current simulation time and the scheduled departure time plus holding time of bus j1.

```
Schedule EVENT #4 (departure) for j1 at
MAX (TNOW , D j1 +H j1 )
```

Also for strategies #4, #5, and #6: if current bus (j1) is not the last bus to arrive at the timed transfer terminal schedule EVENT #7 to occur at maximum of the current simulation time and the scheduled departure time of bus j1.

```
Schedule EVENT #7 (forecasting hold strategy) for j1 at
MAX (TNOW , D j1 )
```

Step 9. Passengers' logic when bus arrives at timed transfer terminal

A detailed description of the passengers' logic when a bus arrives to the timed transfer terminal is shown on Figure B5. For each passenger on the current bus j1 (note that the total number of passengers is in ATRIB(3)), reinitialize and update array $NT_{j1,j2}$ to determine if the passenger transfers to another bus j2 or continues on bus j1 (Repeat step #4). Increment the number on bus j1 [ATRIB(3)] by the number of originating passengers at timed transfer terminal, and the number of passengers waiting in the queue and place bus j1 in File #1. If another bus j2 is at the timed transfer terminal and it has not departed yet, board the passengers transferring from bus j1 to bus j2 by incrementing ATRIB(3) for bus j2 and decrement ATRIB(3) for bus j1. If bus j2 has departed, increment the number of passengers missing the connection to bus j2 by the number transferring from bus j1 to bus j2. If bus j2 has not arrived to the time transfer terminal, place the transferring passengers in a queue waiting for bus j2. We next describe the departing logic at the timed transfer terminal.

EVENT #4: Bus departs from timed transfer terminal

The current time is the departure time of the entity (Bus j) arriving at this node. Therefore remove the entity from file #1 (if it is NOT already removed). Calculate lateness statistics for the bus and its passengers and link them to EXCEL to conduct statistical analysis:

$LATEB_{j1}$: Lateness of bus j1 (Difference between actual departure and scheduled departure of bus j1)

$$LATEB_{j1} = AD_{j1} - D_{j1}$$

$TRNSLT_{j2,j1}$: Lateness for passengers transferring from bus j2 to the current bus (j1)

$$TRNSLT_{j2,j1} = (NT_{j2,j1}) \times [AD_{j1} - MAX(D_{j1}, AA_{j2,NSTOP_{j2}+1})] \quad \text{For} \quad j2 = 1, 2, ..., NLINE$$

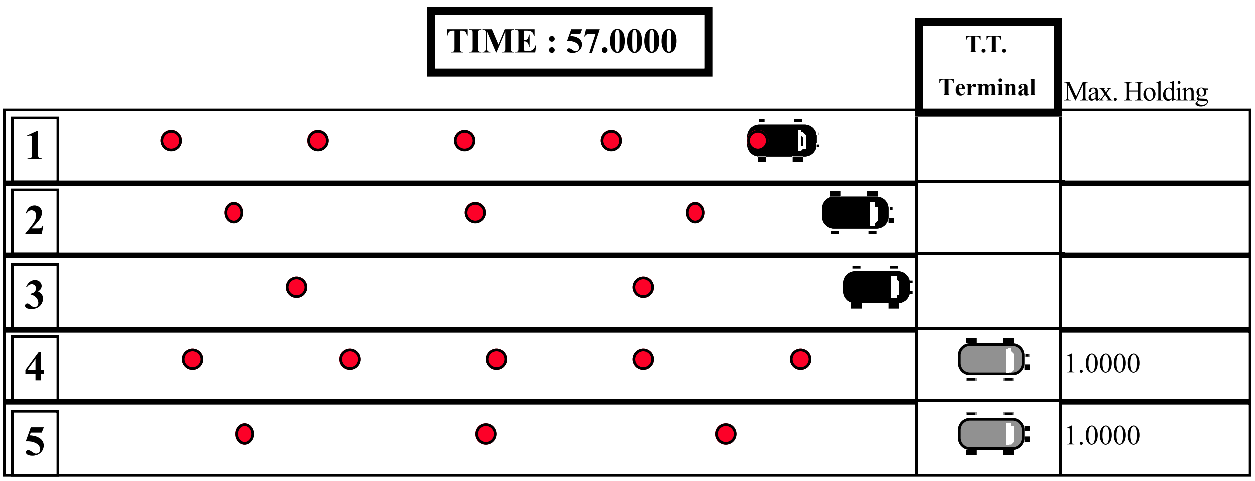
NOTE : The passengers who miss the connection must be excluded from this calculation to prevent negative statistics.

$TTLT_{j1}$: Total lateness of passengers in current bus (j1) equals total lateness for transferring passengers plus total lateness for originating passengers

$$TTLT_{j1} = \sum_{j2=1}^{NLINE} (TRNSLT_{j2,j1}) + (LATEB_{j1} \; x \; NO'_{j1})$$

$LTPP_{j1}$: Lateness per passenger for bus j1 equals total lateness of passengers in bus j1 divided by total number of passengers on board for bus j1 at departure time

$$LTPP_{j1} = (TTLT_{j1}) \; / \; ATRIB(3)$$



	AT STOP				At Timed Transfer Terminal			
Bus #	Sch. Arv	Lateness	# on Bus	Forecast	Sch. Dep.	Act. Dep.	# Waiting	# Missing
1	55.0000	7.61431	19	62.6143	60.0000		5	
2	53.0000	3.87546	18	56.8754	60.0000		11	
3	57.0000	1.54534	5	58.5453	60.0000		5	
4	56.0000	0.62921	22	56.6292	60.0000		0	
5	55.0000	0.79728	14	55.7972	60.0000		0	

Figure B1. Sample Animation Screen

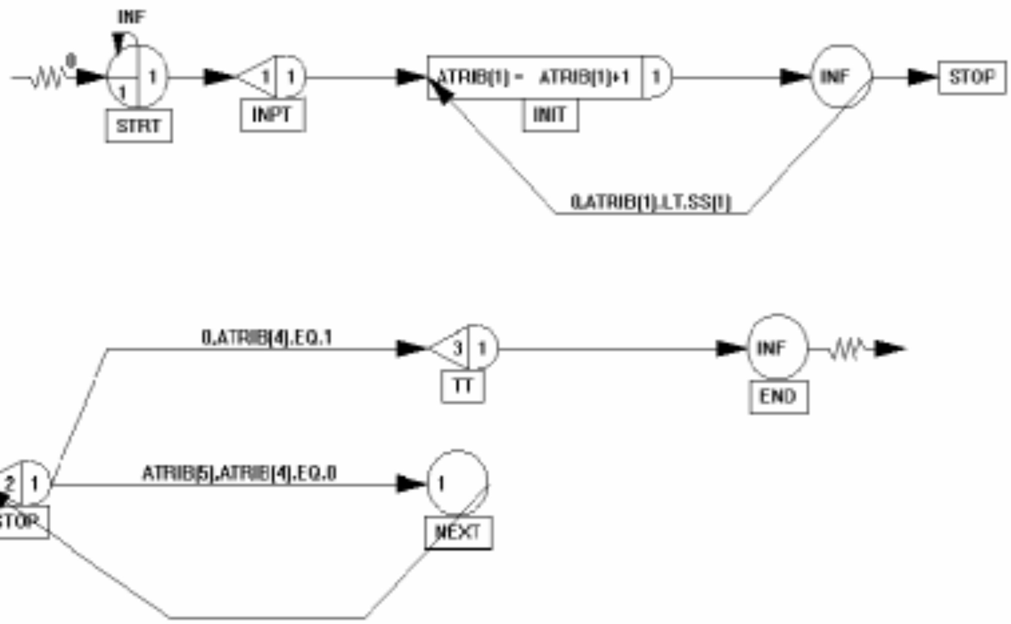


Figure B2. Simulation Network

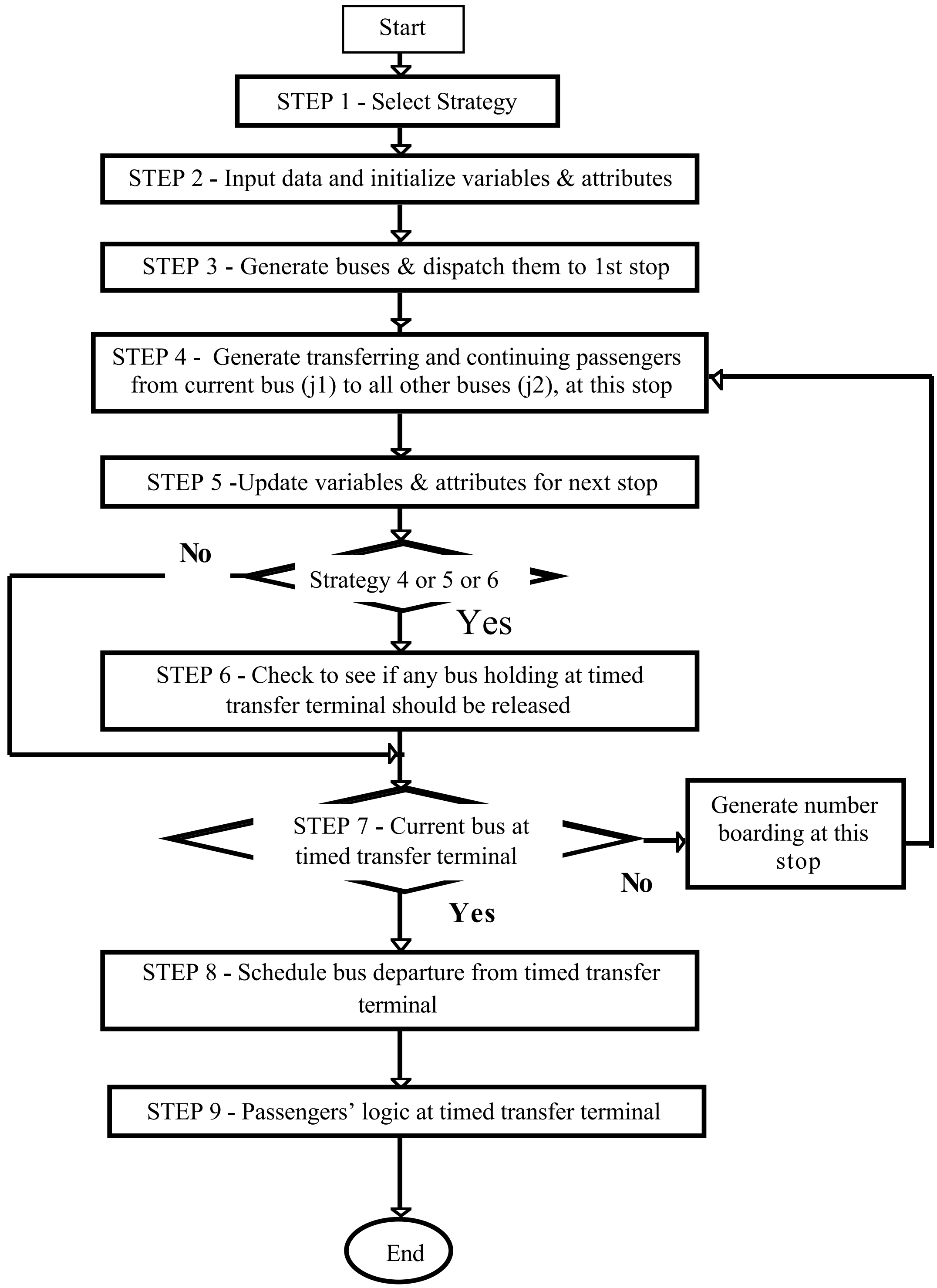


Figure B3. Overall Flowchart of Simulation Model

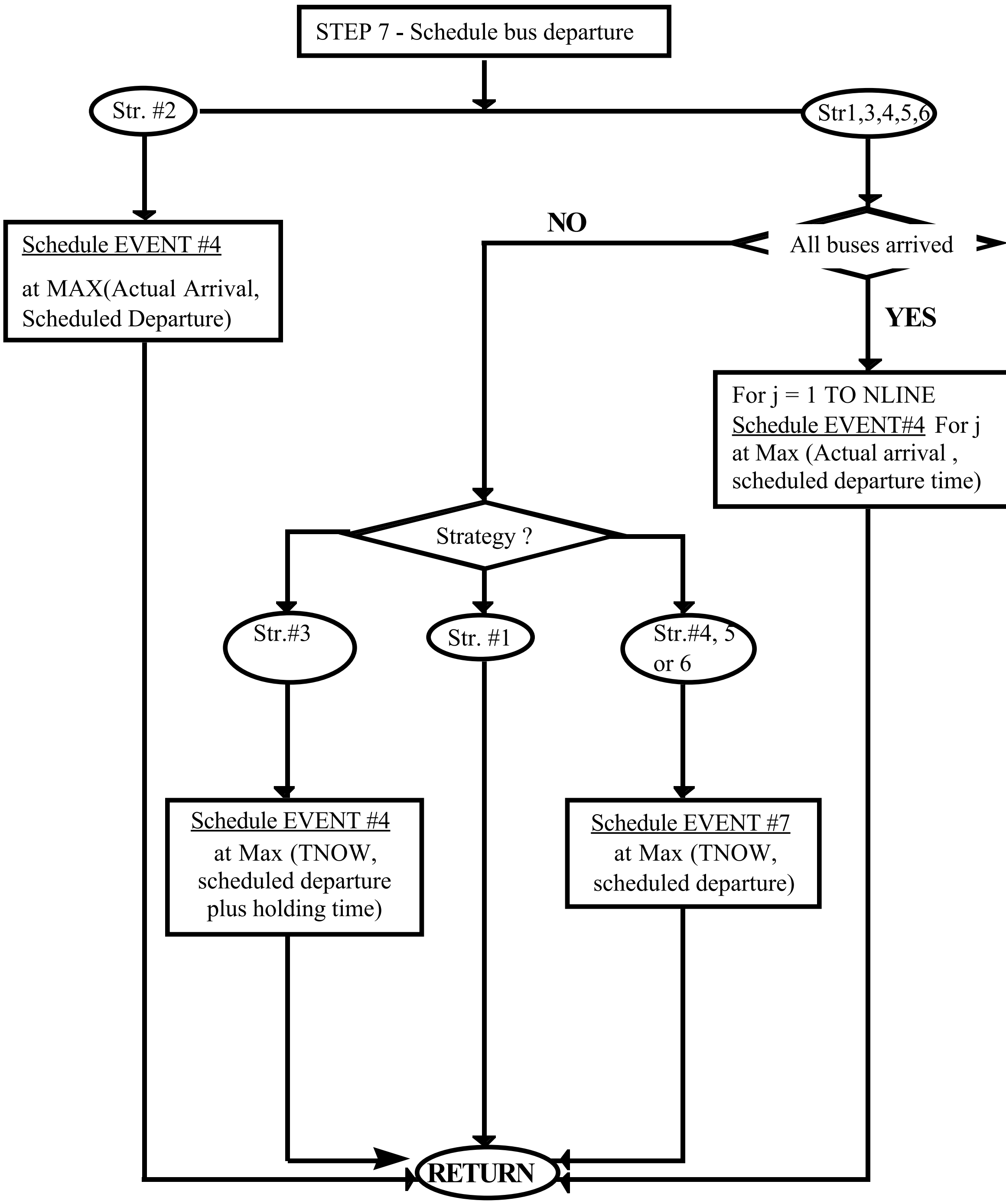


Figure B4. Flowchart of Departure Strategies

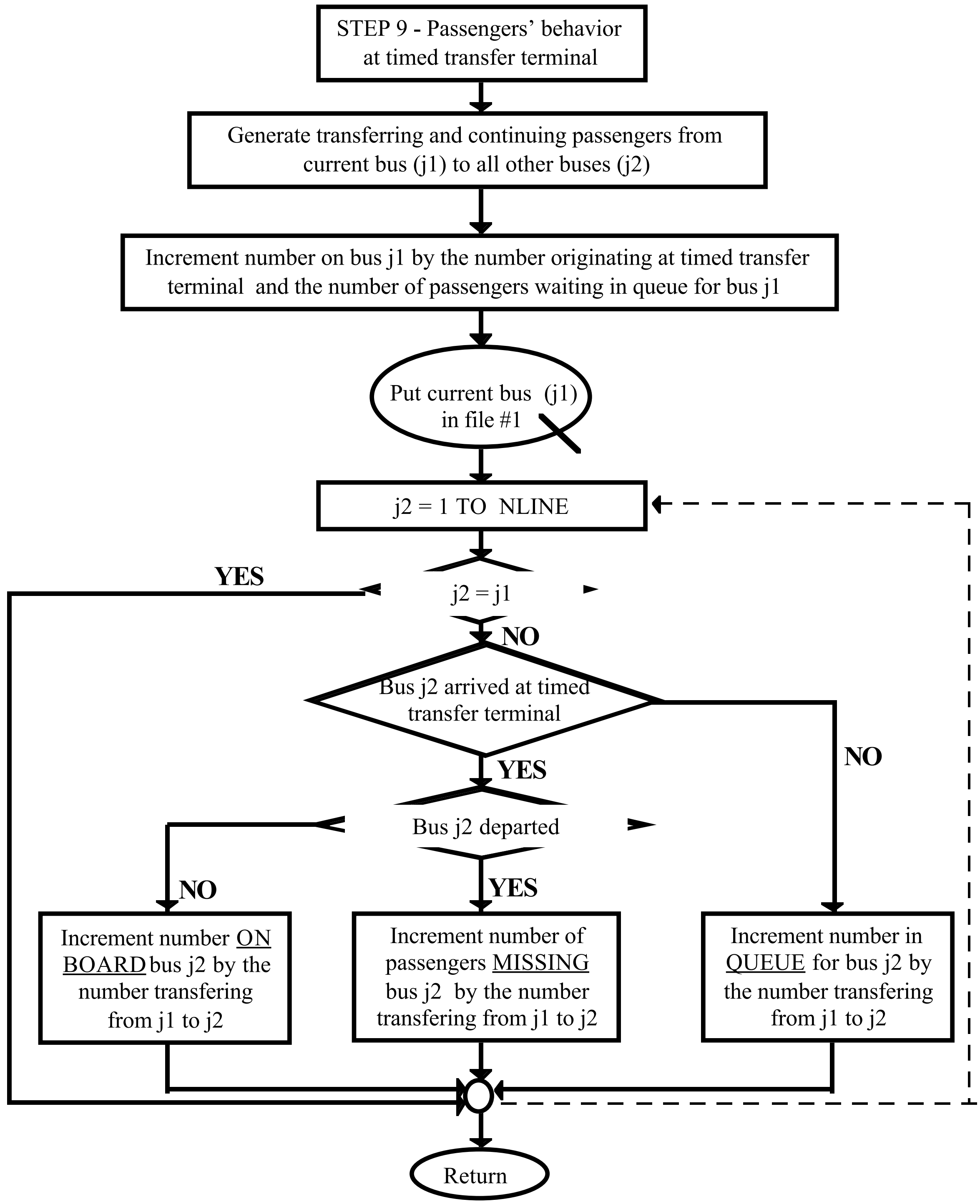


Figure B5. Passenger Logic at Timed Transfer Terminal