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From Strange Stars to Strange Dwarfs[†]

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Abstract

We determine all possible equilibrium sequences of compact strange-matter stars with nuclear crusts, which range from massive strange stars to strange white-dwarf-like objects (strange dwarfs). The properties of such stars are compared with those of their non-strange counterparts, – neutron stars and ordinary white dwarfs. The main emphasize of this paper is on strange dwarfs, which we divide into two distinct categories. The first one consists of a core of strange matter enveloped within ordinary white dwarf matter. Such stars are hydrostatically stable with or without the strange core and are therefore referred to as 'trivial' strange dwarfs. This is different for the second category which forms an entirely new class of dwarf stars that contain nuclear material up to $\sim 4 \times 10^4$ times denser than in ordinary white dwarfs of average mass, $M \sim 0.6\, M_\odot$, and still about 400 times denser that in the densest white dwarfs. The entire family of such dwarfs, denoted dense strange dwarfs, owes its hydrostatic stability to the strange core. One of the striking features of strange dwarfs is that the entire sequence from the maximum-mass strange star to the maximum-mass strange dwarf is stable to radial oscillations. The minimum-mass star is only conditionally stable, and the sequences on both sides are stable. Such a stable, continuous connection does not exist between ordinary white dwarfs and neutron stars, which are known to be separated by a broad range of unstable stars. As a result, we find an expansive range of very-low-mass (planetary-like) strange-matter stars (masses even below $\sim 10^{-4}\, M_\odot$ are possible) that arise as natural dark-matter candidates, which, if abundant enough in our Galaxy, should be seen in the gravitational microlensing searches that are presently being performed.

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Contents

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1 Introduction

The hypothesis that strange quark matter may be the absolute ground state of the strong interaction rather than ^{56}Fe was proposed independently by Bodmer [1] and Witten [2]. There is still no sound scientific basis on which one can either confirm or reject this hypothesis, so it remains a serious possibility of fundamental significance for rare but exotic phenomena. For a review of recent work, and a complete bibliography up to 1991, see Refs. [3, 4, 5, 6].

If the hypothesis is true, then there could exist compact stars containing such matter. The purpose of this paper is to explore, in greater detail than heretofore, strange-star sequences and their continuous connection to a previously unexplored section of the sequences, strange white dwarfs.

A bare strange star [2, 7] may acquire a nuclear crust [8], either by accretion from the interstellar medium onto an initially and possibly primordial strange star, or during its creation in a supernova, whose progenitor has already accreted strange matter nuggets. The crust is suspended out of contact with the quark core due to the existence of an electric dipole layer on the core's surface [8], which insulates the crust from conversion to quark matter. Even so, the maximum density of the crust is strictly limited by the neutron drip density ($\epsilon_{\rm drip} = 4 \times 10^{11}$ g/cm^3), at which neutrons begin to drip out of the nuclei and would gravitate into the core where they would be dissolved into strange matter. In principle both strange and neutron stars could co-exist. However if strange stars exist, the Galaxy is likely to be contaminated by a sufficient number of strange quark nuggets as would convert all neutron stars to strange stars [2, 4, 6, 9]. This in turn would mean that the objects known to astronomers as pulsars would be strange-matter stars, not neutron stars!

Strange white dwarfs comprise a hitherto unexplored consequence of the strange matter hypothesis, and our main emphasis is on them. It is well known that the maximum density attained in the limiting-mass white dwarf is about $\epsilon_{\rm wd} = 10^9$ g/cm^3 [10, 11]. Above this density, the electron pressure is insufficient to support the star, and there are no stable equilibrium configurations until densities of the order of nuclear density ($\gtrsim 10^{14}$ g/cm^3) are reached, which are neutron or strange stars. So we envision one class of strange dwarfs as consisting of a core of strange matter enveloped within what would otherwise be an ordinary white dwarf. They would be practically indistinguishable from white dwarfs. Of greater interest is the possible existence of a class of white dwarfs that contain nuclear material up to the neutron drip density, which would not exist without the stabilizing influence of the strange-quark core [12]. These strange dwarfs could have densities of the nuclear envelope or crust, at its inner edge, that fall in the range $\epsilon_{\rm wd} < \epsilon_{\rm crust} < \epsilon_{\rm drip}$. The maximum inner crust density therefore can be 400 times the central density of the limiting mass white dwarf and 4×10^4 times that of the typical 0.6 $M_\odot$ white dwarf. We investigate the stability of such very dense dwarf configurations to acoustical vibrations and find an expansive range of stability from masses $\sim 10^{-3}\, M_\odot$ to slightly more than $M_\odot$. This is the same range as ordinary white dwarfs except that the lower mass limit is smaller by a factor of $\sim 1/100$. This is because of the influence of the strange core, to which the entire class owes its stability.

2 Description of Strange Stars with Crust

Beta equilibrated strange quark-star matter consists of an approximately equal mixture of up, down and strange quarks, with a slight deficit of the latter. This leads to a net positive charge of the quarks. Since stars in their lowest energy state are charge neutral, the net positive quark charge in strange-matter stars must be balanced by electrons [8]. Being bound by the Coulomb force, rather than the strong force as is the case for the quarks, the electrons extend several hundred fermis beyond the surface of the strange star [8]. Associated with this electron displacement is a electric dipole layer which can support, out of contact with the surface of the strange star, a crust of nuclear material, which it polarizes [8, 13]. The crust is *gravitationally* bound to the strange star. By hypothesis, strange matter is absolutely stable, so neutrons will be dissolved into quark matter as they gravitate from the nuclear crust into the star's strange core. Consequently the maximum density of the crust is strictly limited by neutron drip, $\epsilon_{\rm drip} = 4 \times 10^{11}$ g/cm^3. We shall be interested in strange stars that have a crust that has reached the final state of stellar evolution, namely cold catalyzed matter appropriate to the range of pressures found in the crust.

As shown elsewhere [14], the somewhat complicated situation just described can be very simply represented by an appropriate choice of equation of state. It consists of two parts. At densities below the inner crust density, which as noted must be less than neutron drip, it is represented by the low-density equation of state of charge neutral nuclear matter. For this we use the equation of state of Baym, Pethick, and Sutherland [11]. At pressures above that of the inner crust the equation of state corresponds to strange quark matter, which we describe in the framework of the bag model. The chosen values for the bag constant are $B^{1/4}$=145 and 160 MeV. For massless strange quarks, these bag constants correspond to an equilibrium energy per baryon number of strange quark matter of about 830 and 915 MeV, respectively. For strange quarks of 100 MeV mass, they correspond to about 855 and 930 MeV [15]. Hence these B values describe strongly ($\sim$ 100 MeV) and weakly bound strange matter, which in every case is absolutely stable with respect to ^{56}Fe, as required by the hypothesis.

The graphical representation of the equation of state of a strange star with nuclear

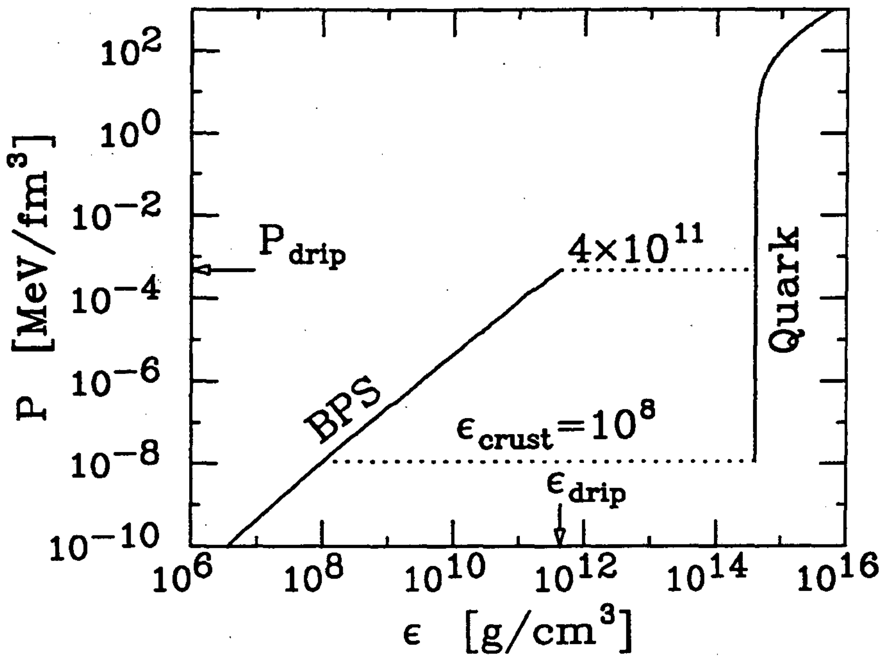

Figure 1: Equation of state of a strange star with nuclear crust. The density at the base of the crust (inner crust density), ϵ_{crust}, is limited by neutron drip, $\epsilon_{\text{drip}} = 4 \times 10^{11}$ g/cm^3. Any inner crust value smaller than that is possible. As an example, we show the equation of state for $\epsilon_{\text{crust}} = 10^8$ g/cm^3.

crust is represented in Fig. 1. The pressure in a star is a continuous and monotonically decreasing function of the Schwarzschild radial coordinate, but naturally there is a discontinuity in energy density between strange quark matter and hadronic matter across the electric dipole gap inside the star where the pressure at the base of the hadronic crust equals the pressure of the strange matter core at its surface. Although the pressure at the crust's base is limited by neutron drip, any value $P \leq P_{\text{drip}}$ is possible.

3 Complete Sequences of Strange-Matter Stars with Nuclear Crusts

3.1 Dense strange stars compared with neutron stars

The star sequences with which we shall be concerned are two-parameter sequences, defined by the inner crust density, and for a fixed value of that, the central density of the quark core. This leads to a whole variety of strange-matter sequences that are considerably more complex than the one-parameter sequence consisting of white dwarfs at sub-nuclear densities and neutron stars at super-nuclear densities. To give an initial impression we compare in Fig. 2 the mass-radius relationship for ordinary

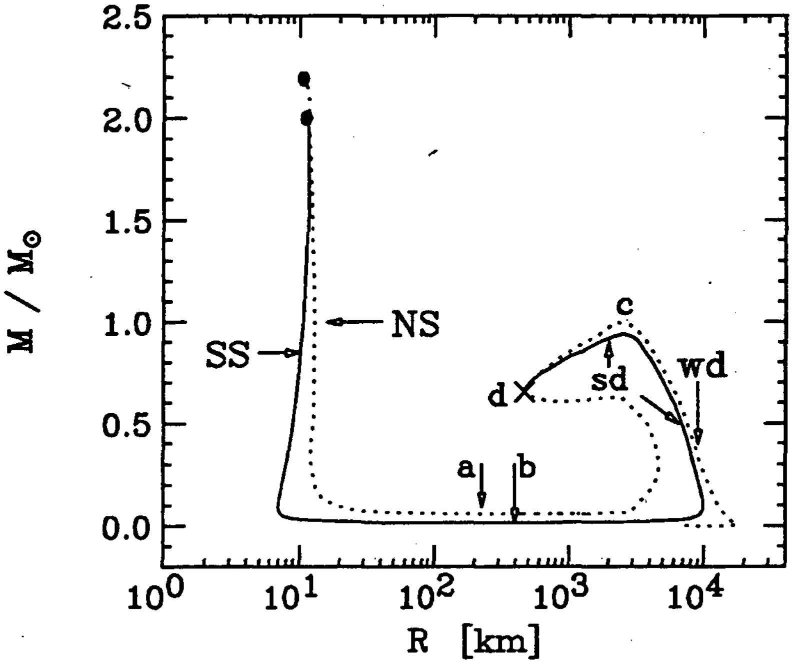

Figure 2: Mass versus radius of strange-star configurations with nuclear crust ($\epsilon_{\rm crust} = \epsilon_{\rm drip}$) and gravitationally bound stars (NS=neutron star, SS=strange star, wd=white dwarf, sd=strange dwarf). The dots and arrows at 'a' and 'b' denoted the heaviest and lightest star of each sequence. Label 'c' refers to the most massive dwarf stars. The strange-dwarf sequence terminates at 'd'.

neutron stars (constructed for the relativistic Hartree-Fock equation of state HFV of Ref. [16]) and white dwarfs and the *complete* sequence of strange-matter stars, from dense strange stars, the counterparts of neutron stars, to strange dwarfs, the strange counterparts of ordinary white dwarfs. The central density decreases monotonically throughout both sequences being maximum in the compact configurations. Here we have selected, for illustration, an inner nuclear crust density for the strange sequence that is equal to the neutron drip. Other sequences with $\epsilon_{\rm crust} < \epsilon_{\rm drip}$ will be discussed below.

The two sequences of Fig. 2 have certain resemblances, including the mass and radius of the limiting compact star of each sequence (see Table 1). However several differences will be noticed. The white dwarfs terminate at low mass in planetary objects. The strange stars terminate when the strange core has shrunk to zero radius at a white dwarf. Secondly, in the dwarf region, the direction of decreasing central density is in different senses on the strange dwarf and white dwarf sequences. We will discuss this later in an analysis of stability to radial oscillations.

Next we compare in Fig. 3 two strange sequences corresponding to different values of the inner crust density, (1) the maximum value which is the neutron drip, (2) a substantially lower value. An enlargement of the left portion of this figure is shown in Fig. 4. There is some residue of the $R \propto M^{1/3}$ law that holds for low-mass bare

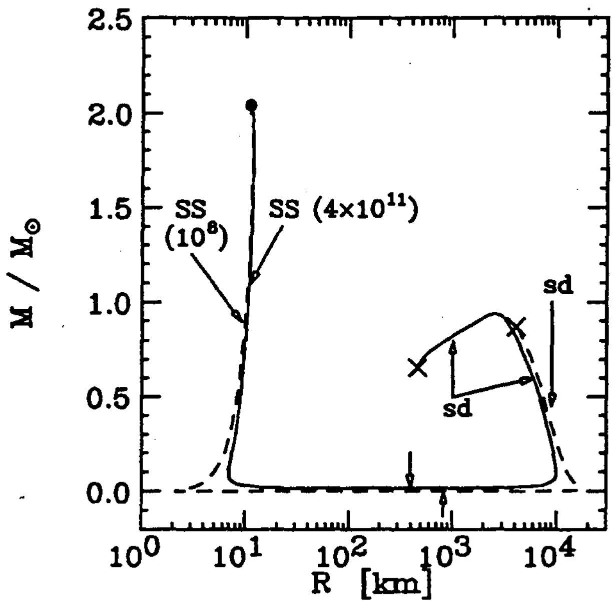

Figure 3: Mass versus radius of two complete sequences of strange-matter stars, computed for inner crust densities $\epsilon_{\rm crust} = 4 \times 10^{11}$ g/cm^3 (neutron drip) and 10^8 g/cm^3. The labeling is the same as in Fig. 2.

strange stars, especially for low crust densities. However, since the nuclear crust on a strange star is bound by gravity and not by confinement as for the strange core or a bare strange star, the mass radius relationship ultimately becomes characteristic of gravitationally bound stars, – the radius becomes large as the mass approaches a small value – about $1/10\ M_\odot$ or less.

3.1.1 Limiting rotational periods of strange stars

We note the small radii of the compact strange stars as compared to neutron stars from Fig. 4. This suggests that compact strange stars may have especially large mass-shedding (or Kepler) frequencies, $\Omega_{\rm K}$, which set an *absolute* upper limit on stable rotation [17, 18, 19, 20]. Classically, the balance of the gravitational and centrifugal forces acting on a particle that rotates at the star's equator is given by $\Omega_{\rm K} = \sqrt{M/R^3}$. The general relativistic expression, which is needed to find $\Omega_{\rm K}$ for massive rotating neutron and strange stars, is given by [17, 19, 21]

$$\Omega_{\rm K} = \omega + \frac{\omega'}{2\psi'} + e^{\nu-\psi}\sqrt{\frac{\nu'}{\psi'} + \left(\frac{\omega'}{2\psi'}e^{\psi-\nu}\right)^2} \,. \tag{1}$$

It can only be evaluated in a self-consistent algorithm for solving Einstein's equations for a rotating stellar configuration because all quantities on the right (the metric functions ν and ψ, the frame dragging frequency $\omega(r)$, and their radial derivatives denoted by primes, which are evaluated at the equator) depend on $\Omega_{\rm K}$. However the

Table 1: Properties of the maximum-mass star configurations. The properties listed are: star's radius, R; radius of strange core, $R_{\rm core}$; total mass, M; crust mass, $M_{\rm crust}$; central density, ϵ_c (in units of the density of normal nuclear matter, $\epsilon_0 = 2.5 \times 10^{14}$ g/cm^3); total number of baryons in the star, A; number of baryons in star's crust (core), $A_{\rm crust}$ ($A_{\rm core}$); gravitational redshift, z. The symbol '–' indicates that this quantity has no meaning for neutron stars.

Quantity	Neutron Star HFV†	Strange Stars $\epsilon_{\rm crust} = 4 \times 10^{11}$ g/cm^3	Strange Stars $\epsilon_{\rm crust} = 10^9$ g/cm^3
R [km]	10.70	11.143	11.052
$R_{\rm core}$ [km]	–	10.969	11.039
$M/M_\odot$	2.198	2.005	2.046
$M_{\rm crust}/M_\odot$	–	0.13×10^{-4}	0.34×10^{-3}
ϵ_c/ϵ_0	9.64	7.81	7.86
$\log A$	57.5253	57.5178	57.5202
$\log A_{\rm crust}$	–	53.7724	53.8369
$\log A_{\rm core}$	–	57.5177	57.5201
z	0.6143	0.4611	0.4857

† Taken from Ref. [16].

qualitative dependence of $\Omega_{\rm K}$ on mass and radius is approximately represented by the classical expression as has been found by comparing it with numerical solutions to Einstein's equations [20, 22].

From the mass-radius relationships exhibited in Figs. 2–4 it is clear that strange stars with crusts can have limiting rotational frequencies that are *larger* than those of the gravitationally bound neutron stars, due to their much smaller radii [3, 4, 23]. Furthermore, not only the strange star at the termination point can sustain rapid rotation but *all* stars down to rather small masses. In fact, as can be seen from Fig. 5, we find that even a certain part of the sequence of the low-mass strange stars can have rotational frequencies that rival even the maximum possible frequency of heavy neutron stars! The neutron star sequence is constructed for the same particular choice of equation of state as in Sect. 3.1. However from our detailed investigation performed elsewhere [18, 24] it is known that this trend seems to hold for any realistic nuclear equation of state and thus is not a specific feature of the model used here. Of course bare strange stars, possessing no nuclear crusts, can rotate faster than those with gravitationally bound crusts. This is particularly evident for the lighter strange stars of the sequences shown in Fig. 5. Strange-matter stars with typical pulsars masses, $\sim 1.45\, M_\odot$, can rotate at Kepler periods in the range $0.55 \lesssim P_{\rm K}/{\rm msec} \lesssim 0.8$, depending on the value of the bag constant and the existence of a crust on their surfaces. The situation is different for neutron stars of the same mass, for which a limiting rotational period of about 1 msec has been established [18, 24, 25, 26, 27].

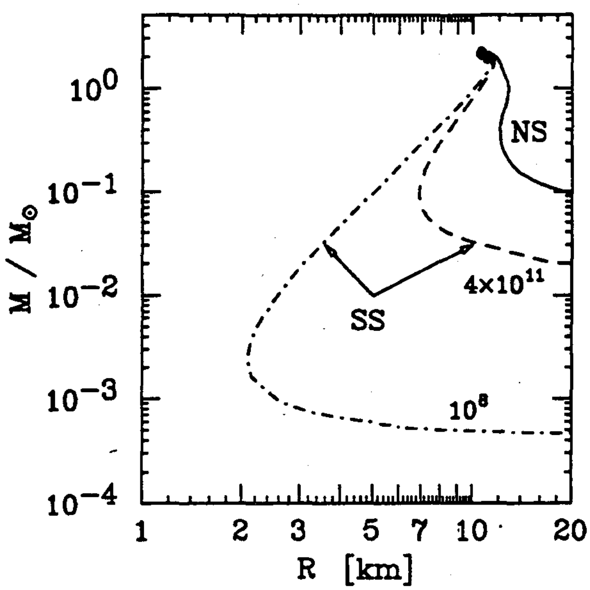

Figure 4: Mass versus radius of strange (SS) and neutron stars (NS) for inner crust densities 4×10^{11} g/cm^3 and 10^8 g/cm^3.

3.1.2 Properties of light strange stars

As the core mass decreases sufficiently, the mass-radius relationship of strange stars with crust abruptly becomes dominated by the gravitationally bound crust, and the radius increases very strongly with decreasing mass. The masses of the lightest strange stars of the sequence are considerably smaller than those of the neutron-star sequence, even less than 1/100'th, as can be seen from Fig. 4. We are able to identify the minimum, even in so flat a curve as shown in Fig. 2 as is demonstrated in Fig. 6. As two-parameters sequences, the mass of these light strange stars depends on central density as well as the crust's inner density. The latter dependence is relatively strong. For example, the mass of the lightest strange star of the sequence with maximal inner crust density, $\epsilon_{\rm crust} = 4 \times 10^{11}$ g/cm^3, is $\sim 0.017\, M_\odot$, or about 17 Jupiter masses, as illustrated in Fig. 6. It drops by two additional orders of magnitude if $\epsilon_{\rm crust}$ is reduced to 10^8 g/cm^3, as shown in Table 2. So, if the hypothesis is correct there could exist stellar strange-matter configurations with masses similar to those of brown dwarfs or ordinary planets but radii that can be dramatically different. These can be as small as those of neutron stars! If abundant, this class of strange-matter stars would constitute possible candidates for gravitational microlensing searches [28, 29, 30, 31, 32].[1]

[1]Microlensing searches can give definitive results on dark objects in the mass range 10^{-5} to $10^2\, M_\odot$ [28, 29]. In fact the presently performed searches may eventually be sensitive to the entire theoretically possible range of baryonic dark matter, 10^{-9} to $10^6\, M_\odot$ [33]. If the dark matter consists of these objects, these experiments should find it! [31]. The observation of the effects of an invisible object at least as massive as the planet Jupiter has been reported independently by the MACHO and French collaboration (see Refs. [34] and [35], respectively). 27 microlensing events in the Galactic Bulge have been reported in [36].

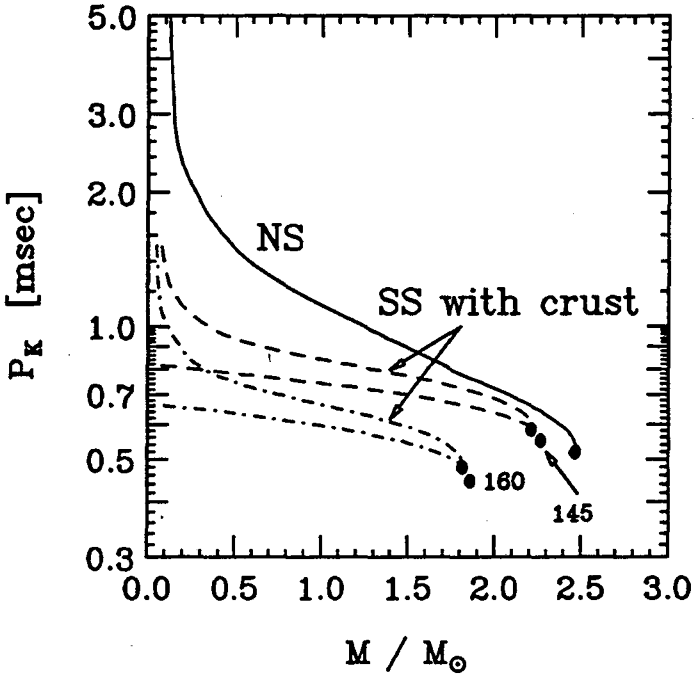

Figure 5: General relativistic Kepler period, $P_K \equiv 2\pi/\Omega_K$, versus rotational star mass for neutron stars (NS) and strange stars (SS) with ($\epsilon_{crust} = \epsilon_{drip}$) and without nuclear crust. The numbers denote the value of the bag constant (in MeV).

3.2 Strange dwarfs versus white dwarfs

3.2.1 Strange cores in white dwarfs

As illustrated in Fig. 2, strange-matter stars with central densities smaller than the lightest strange star of the sequence (at 'b') possess masses and radii similar to those of ordinary white dwarfs. We recall that the strange-matter cores are monotonically shrinking in the direction from 'b' to 'd', since this is the direction in which the central star density decreases. It is striking that the central density varies from 'b' to 'd' by less than 2%. The cross at 'd' denotes that particular strange star whose core has shrunk to zero. What is left is a star made up of the crust matter of the former strange dwarf, that is – a white dwarf. Its central density is determined by the inner crust density of the former strange dwarf, which in the present case is the neutron drip density, 4×10^{11} g/cm^3. This is the central density of the white dwarf at 'd' in Fig. 2. A comparison with Fig. 7 (solid curve) reveals that the interiors of the strange-matter stars between 'b' and 'd' consist of very small strange cores relative to the equatorial radii. The nuclear envelopes of these dwarfs are up to a few thousand kilometers thick. As the strange matter core shrinks along any sequence corresponding to a chosen inner crust density, the final configuration will be a white dwarf. Thus each of the strange-matter sequences shown in Figs. 2, 3 and 8 terminates on the white dwarf sequence. The crosses mark the termination points.

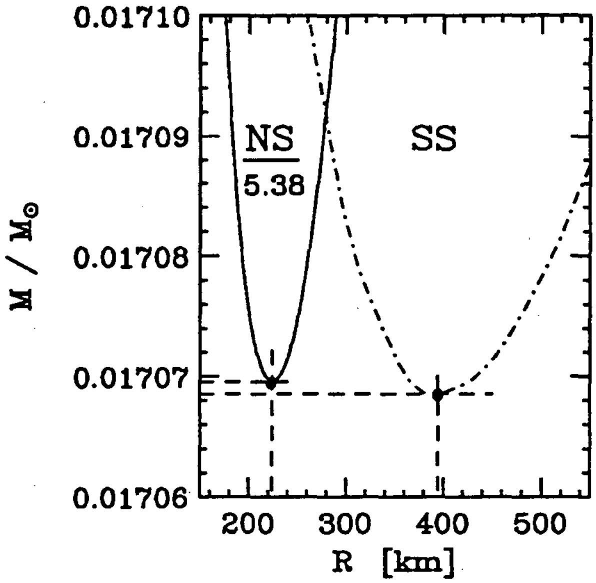

Figure 6: Minimum-mass stars of neutron and strange-star sequences. For convenience, the masses of the former has been rescaled. They are by a factor of 5.38 larger than shown here. The strange-star sequence is computed for inner nuclear crust density $\epsilon_{\rm crust} = \epsilon_{\rm drip}$.

3.2.2 Mass-density profiles

We now compare the mass-energy profiles of ordinary white dwarfs with those of dwarfs possessing strange matter cores. A first impression is given by Figs. 9 and 10. In the latter figure, the inner crust density of nuclear matter that overlays the strange core is equal to the neutron drip density. Due to the fact that the central density in stable dwarfs with strange matter cores changes only very little with mass (see, for example, the stars in Fig. 7 along sequence '1' between 'b' and 'c'), the radii of their strange cores are nearly independent of star mass, as can be seen in Fig. 10. Such cores, because of their high mass-density, still exert a non-negligible influence on the hydrostatic equilibrium of strange dwarfs, as follows from the profiles shown in Fig. 10. Indeed, as will be pointed out below (Sect. 3.2.3), strange cores with $R_{\rm core} \sim 3$ km are sufficient to stabilize a new type of white-dwarf-like stars that are much denser than ordinary white dwarfs [12].

The mass-energy profiles of strange dwarfs constructed for several different inner crust densities *smaller* than the limiting drip density are compared to a white dwarf profile in Fig. 11. All profiles correspond to stars with $M = 0.6\, M_\odot$, the average mass value of observed white dwarfs [37]. The radii of the strange cores in these models decrease from about 3 km, obtained for $\epsilon_{\rm crust} = \epsilon_{\rm drip}$, to about 10^{-1} km if $\epsilon_{\rm crust} = 10^7$ g/cm^3 (Fig. 7). Due to the core's much weaker gravitational impact in the latter case, the density profile of this strange dwarf resembles that of an ordinary white dwarf of the same mass.

Table 2: Properties of minimum-mass star configurations. The properties listed are explained in Table 1. The symbol '–' indicates that this quantity has no meaning for neutron stars.

Quantity	Neutron Star	Strange Stars	
		$\epsilon_{crust} = 4 \times 10^{11}$ g/cm^3	$\epsilon_{crust} = 10^8$ g/cm^3
R [km]	224	395	820
R_{core} [km]	–	2.69	0.77
$M/M_\odot$	0.092	0.017	4×10^{-4}
$M_{crust}/M_\odot$	–	0.18×10^{-3}	0.22×10^{-5}
ϵ_c/ϵ_0	0.575	1.667	1.646
$\log A$	56.0395	55.3637	53.7298
$\log A_{crust}$	–	53.3149	51.4246
$\log A_{core}$	–	55.3598	53.7276
z	0.6220×10^{-3}	0.6729×10^{-4}	0.7153×10^{-6}

3.2.3 Possible new class of dense white dwarfs

We can now make an important distinction among what we have heretofore referred to as strange dwarfs. If the inner density of white dwarf material (the crust) lies below $\sim 10^9$ g/cm^3, which is the central density of the maximum-mass white dwarf [11], the star that would remain, were the strange core shrunk to zero, would be a stable ordinary white dwarf. We refer to such stars as 'trivial' strange dwarfs. The presence of the strange core does not disturb the configuration of the star appreciably, as can be verified by comparison of two of the strange dwarf configurations that lie close to the white dwarf in Fig. 11. Since there are no stable white dwarfs with central densities higher than $\sim 10^9$ g/cm^3, the strange dwarfs with inner crust densities of the nuclear material that lie in the range $10^9 < \epsilon_{crust}/(\text{g/cm}^3) \leq 4 \times 10^{11}$ are entirely new objects that owe their stability to the presence of the strange core and would be unstable without it [12]. They would fall in the unstable region between the maximum-mass white dwarf and minimum-mass compact star ('c' and 'a' respectively in Fig. 2). Such dwarfs have nuclear material that is 400 times more dense than the central density of the maximum-mass white dwarf, and 4×10^4 that of the typical $0.6\,M_\odot$ white dwarf. As such, provided that they are stable to acoustical vibrations, they would constitute a new class of white dwarfs, under the strange matter hypothesis. To make a definitive statement as to whether such strange dwarfs can indeed exist in nature, we need to go beyond the construction of hydrostatic equilibrium configurations and investigate their stability against radial oscillations (acoustical vibrations), which will be performed in Sect. 4.

The baryon numbers of the strange cores in stable strange dwarfs are shown in Fig. 12. We recall that the trivial dwarf sequences ($\epsilon_{crust} \leq 10^9$ g/cm^3, curves '4' to '7') are stable down to the termination points, where the strange cores have shrunk to zero (cf. Fig. 8). Therefore the core baryon numbers in the trivial dwarfs can range from

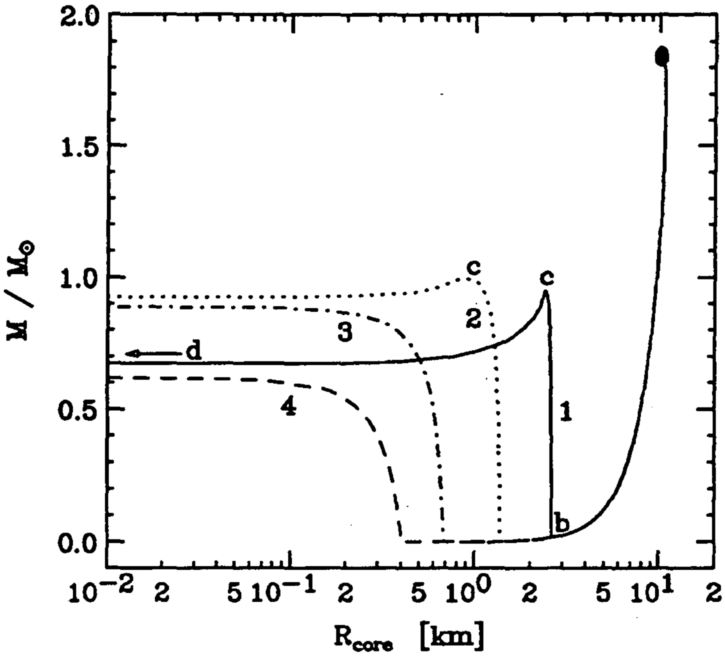

Figure 7: Mass versus strange-core radius for selected strange star sequences with inner crust densities (1) 4×10^{11} g/cm^3, (2) 5×10^9, (3) 10^8, and (4) 10^7. The mass-radius diagram of sequences '1' and '3' are shown in Figs. 2 and 3. The label 'c' denotes maximum-mass strange dwarfs (cf. Fig. 2), which do not exist for sequences with $\epsilon_{\rm crust} \lesssim 10^9$ g/cm^3 ('3' and '4'). Stars to the left of 'c' are unstable against compressional modes, which will be demonstrated in Sect. 4. The minimum-mass star models possess core radii $0.4 \lesssim R_{\rm core}/{\rm km} \lesssim 3$.

at most $\sim 10^{54}$, determined by the hydrostatic equilibrium equations, down to zero. This is different for the dense strange-dwarf sequences ($\epsilon_{\rm crust} > 10^9$ g/cm^3, curves '1' to '3') because these terminate at the unstable dwarf region, beyond the mass peaks in Fig. 8 where all dense sequences become unstable against radial oscillations (Sect. 4). Hence the first stable strange dwarfs (in the direction of growing strange cores) of the dense category, located right at the mass peaks, already possess strange cores of certain finite sizes, which depends on $\epsilon_{\rm crust}$ and star mass. Three examples are shown in Fig. 12. The solid dots refer to the cores' baryon number at the mass peaks of each sequence. For all dwarfs, $A_{\rm core}$ increases with decreasing star mass since this is the direction in which the central star density increases.

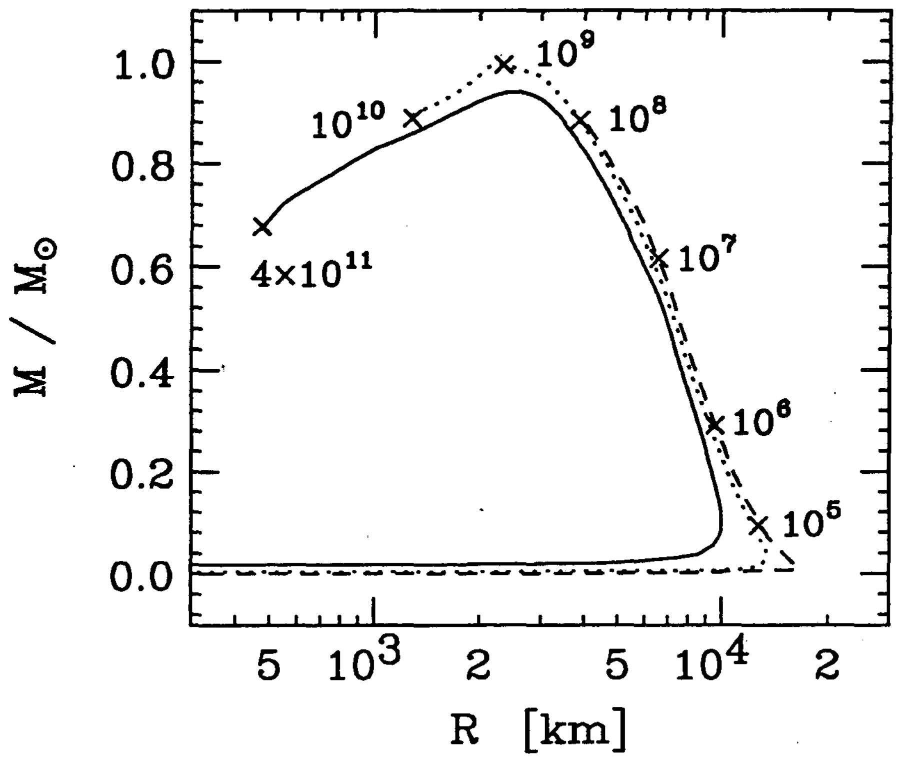

Figure 8: Impact of the value of inner crust density, $\epsilon_{\rm crust}$ (numbers ranging from 4×10^{11} g/cm^3 to 10^5 g/cm^3), on the termination point (crossed) of a strange-dwarf sequence. Since the sequences with $\epsilon_{\rm crust} \lesssim 10^9$ g/cm^3 nearly overlap to the resolution of the figure only their termination points are shown.

4 Stability of Strange Stars against Radial Oscillations

4.1 Mathematical framework

The equations that are to be solved to obtain the eigenfrequencies and eigenfunctions of radial normal modes of a star are taken, with slight modifications, from the catalogue of methods for studying the normal modes of radial pulsations provided by Bardeen, Thorne, and Meltzer [38]. The analysis is carried out for solutions of the Oppenheimer-Volkoff equations which are subjected to small Lagrangian perturbations, which for the nth normal mode ($n = 0$ is the fundamental mode) are expressed in terms of amplitudes $u_n(r)$ by

$$\delta r(r,t) \;=\; e^{\nu}\, u_n(r)\, e^{i\,\omega_n\, t} \,/\, r^2 \;\;, \tag{2}$$

where $\delta r(r,t)$ denotes small Lagrangian perturbations in r. The quantity $\omega_n(t)$ is the star's oscillation frequency, which we want to compute. The eigenequation for $u_n(r)$ (first derived by Chandrasekhar [39]), which governs the normal modes, has the

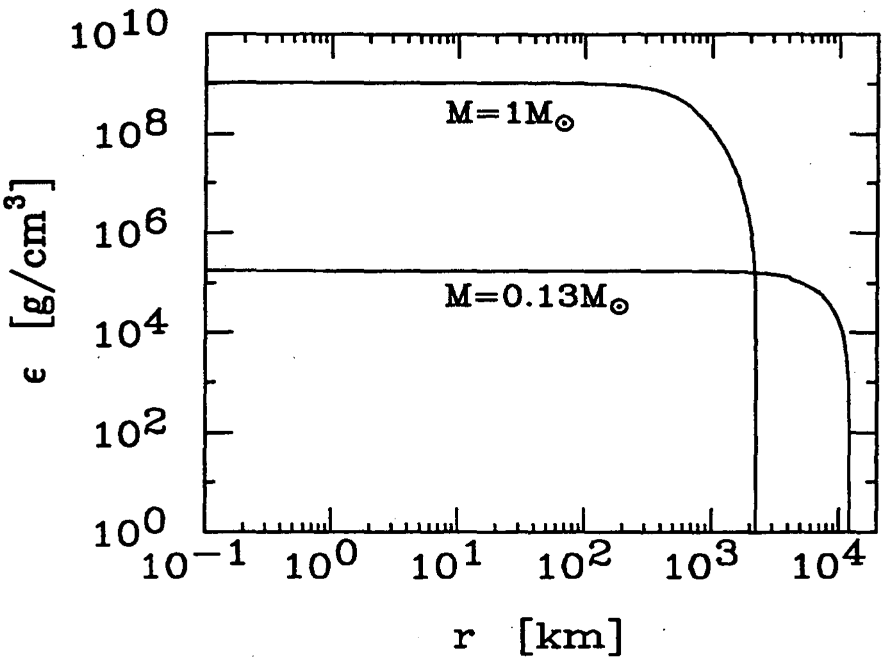

Figure 9: Mass-density versus radius of a heavy and a light stable white dwarf.

Sturm-Liouville form,

$$\frac{d}{dr}\left(\Pi\,\frac{du_n}{dr}\right) \;+\; \left(Q \,+\, \omega_n^2\, W\right)\, u_n \;=\; 0\;. \tag{3}$$

The functions $\Pi(r)$, $Q(r)$, and $W(r)$ are expressed in terms of the equilibrium configurations of the star [12, 13, 38]. Solving this equation subject to the appropriate boundary conditions yields the frequency spectrum ω_n^2 ($n = 0, 1, 2, \ldots$) of the normal radial modes. As a characteristic feature, the eigenfrequencies ω_n^2 form an infinite discrete sequence, i.e. $\omega_0^2 < \omega_1^2 < \omega_2^2 < \ldots$. If any of these is negative for a particular star, the frequency is imaginary to which there corresponds an exponentially growing amplitude of oscillation. Such a star would be unstable.

4.2 Eigenfrequencies of strange stars and strange dwarfs

The four lowest-lying eigenfrequencies of the complete sequence of strange-matter stars, from strange stars to strange dwarfs (Figs. 2 and 3), are shown in Fig. 13. An enlargement of the low-density portion of this figure is given in Fig. 14. One sees that the higher-lying modes form a discrete sequence, i.e., $\omega_1^2 < \omega_2^2 < \omega_3^2$, as enforced by the mathematical structure of the eigenequation (3). Specifically, the eigenfrequencies of strange stars in the vicinity of the minimum-mass configuration at 'b' of Fig. 14, which exhibit a dip-like behavior in that density region, fulfill this condition. This can barely be seen in Fig. 14 but has been confirmed numerically. These figures also show that strange stars possess a characteristic mode of vibration of zero frequency

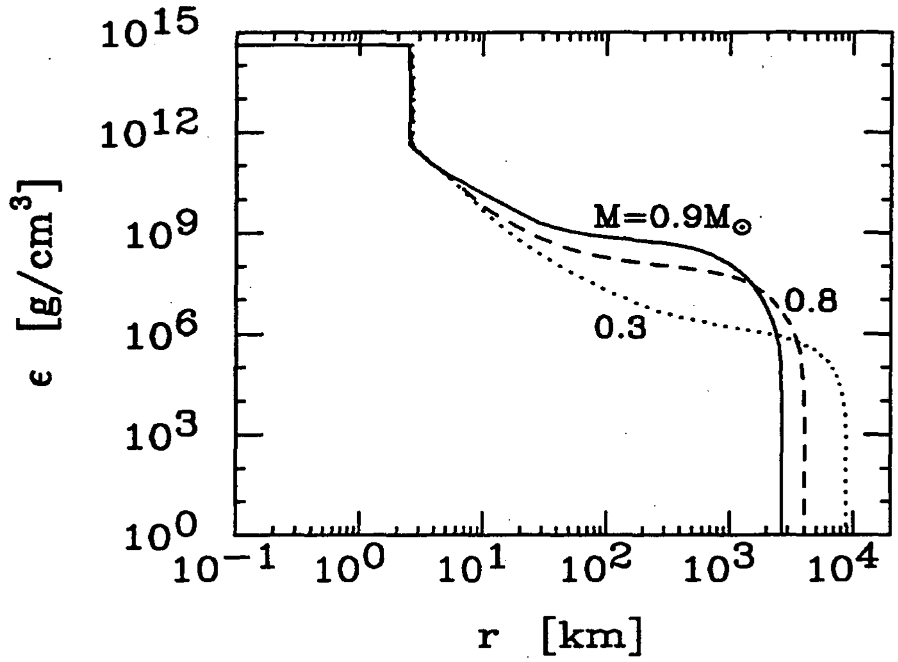

Figure 10: Mass-density versus radius of a few selected stable strange dwarf models ($\epsilon_{\rm crust} = \epsilon_{\rm drip}$).

($\omega_n^2 = 0$) when and only when $dM/d\epsilon_c = 0$, that is, only when the star's mass attains an extremum. Such a behavior is consistent with what has been established for neutron stars and ordinary white dwarfs [10]. Most interestingly, the lowest-lying eigenmode ($n = 0$) passes through zero at 'c' of Fig. 14 and *remains* negative for all densities down to the central density of the strange dwarf at the termination point 'd'. Therefore all members along this particular section are *unstable!* Those between 'b' and 'c' are stable against radial oscillations because ω_0^2 (and thus all higher-lying ones) is positive. So we conclude that the strange-dwarf sequence with $\epsilon_{\rm crust} = \epsilon_{\rm drip}$ has a segment whose stars are stable against vibrational modes, which is of decisive importance for their stable existence! These stars range in mass from exceedingly light ones (masses typical of ordinary planets) to about one solar mass.

The eigenfrequencies of strange-dwarf sequences show a behavior qualitatively similar to those computed for the $\epsilon_{\rm crust} = \epsilon_{\rm drip}$ sequence as long as the inner crust density lies in the range $10^9 < \epsilon_{\rm crust}/({\rm g/cm}^3) \leq \epsilon_{\rm drip}$, which is illustrated in Fig. 15.

This is different for sequences with $\epsilon_{\rm crust} \lesssim 10^9$ g/cm^3 because these do not possess a maximum-mass dwarf star (see Fig. 8) where ω_0^2 turns negative. Therefore the *complete* sequences of strange-matter stars with $\epsilon_{\rm crust} < 10^9$ g/cm^3, from the dense strange stars to the white dwarf at the termination point, are stable. However since they are essentially white dwarfs which are only mildly perturbed by the strange core, they are not as interesting as the *stable* strange dwarfs with $\epsilon_{\rm crust} > 10^9$ g/cm^3.

Nevertheless there is one aspect in which they important. Their stability demonstrates that the strange matter hypothesis is not inconsistent with observation of

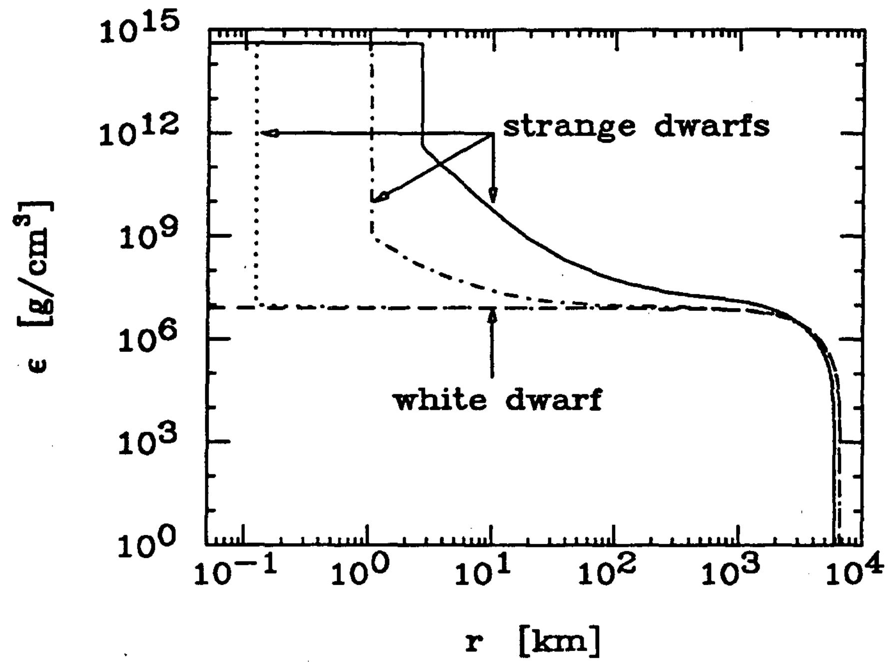

Figure 11: Mass-density profiles of a white dwarf and several strange dwarfs of the same mass, $M = 0.6\, M_{\odot}$. The inner crust densities of the latter are $\epsilon_{\rm crust} = 4 \times 10^{11}$ g/cm^3 (solid curve), 10^9 g/cm^3 (dot-dashed), and 10^7 g/cm^3 (dotted).

white dwarfs. For if the hypothesis is correct, then almost certainly all stars contain some strange matter.

For the rest of this section, we turn our interest to the eigenfrequencies and locations of the zero-frequency vibrations of the denser strange stars with crust, possessing central densities up to those at which even charm-quark states become populated in their dense interiors (charm stars). The second zero point of the oscillation frequencies ω_n^2 in Fig. 13 in the direction of increasing density is located at that density at which the strange stars attain their maximum mass (solid dots in Fig. 2). Since ω_0^2 remains negative at all densities larger than this one, it follows that no quark-matter stars can exist in nature that are more compact than the hypothetical strange stars [13, 40]. Specifically this rules out the possible existence of charm stars. In fact, as one sees from Fig. 13, going to higher and higher central star densities leads to the successive excitation of more and more unstable modes ($\omega_n^2 < 0$, $n = 2, 3, \ldots$). This situation is analogous to that of hydrostatic equilibrium configurations in the neutron star sequence with central densities above that of the maximum-mass neutron star [38].

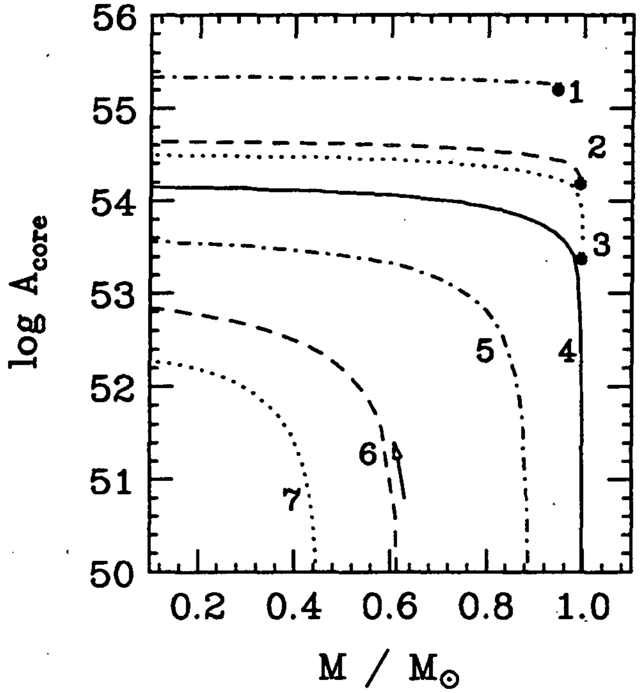

Figure 12: Baryon number of strange cores, $A_{\rm core}$, in stable strange dwarfs versus mass. The inner crust densities (in units of g/cm^3) are: (1) neutron drip, 4×10^{11}, (2) 10^{10}, (3) 4×10^9, (4) 10^9, (5) 10^8, (6) 10^7, and (7) 10^6. The hydrostatically stable parts of sequences '1' to '3' terminate at the solid dots, which denote maximum-mass strange dwarfs labeled 'c' in Fig. 7. The arrow indicates the direction of increasing central density.

4.3 Schematic arguments for the stability of strange dwarfs against compressional modes

The strange dwarfs between 'b' and 'c' in Figs. 2, 14 and 15 obey $dM/d\epsilon_c < 0$, since the mass of these stars increases with decreasing central density. In the case of neutron stars and ordinary white dwarfs, $dM/d\epsilon_c < 0$ implies instability against (at least) the lowest-lying eigenmode of oscillation. This however is not the case for these strange dwarfs since their eigenfrequencies were found above to be all positive. So these stars are stable against vibrational modes. The physical reason for this is that the acoustic modes in them propagate mostly in the compressible nuclear (white dwarf) material which comprises most of these stars and not in the much higher-density incompressible quark core of at most a few kilometers in radius for which the local adiabatic index, $\Gamma(\epsilon) \equiv [(\epsilon + P)/P] dP/d\epsilon$, is very large, as can be understood in reference to Fig. 1.

Due to the strict ordering of the eigenfrequencies, a star is stable against radial oscillations if it is stable against the $n = 0$ eigenmode. This mode corresponds to an oscillation with no nodes inside the star (pure compressional mode). Thus any

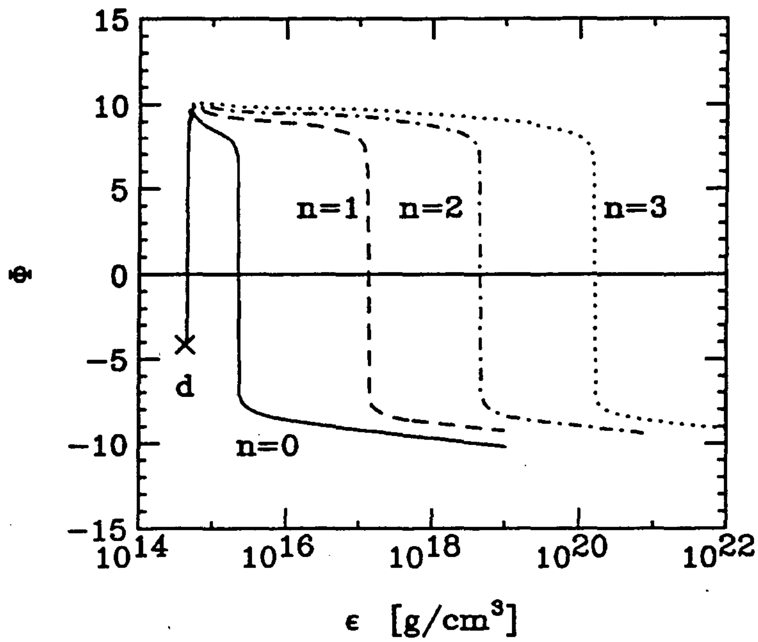

Figure 13: Oscillation frequencies of the lowest four ($n = 0, 1, 2$, and 3) normal radial modes of strange-matter stars ($\epsilon_{\rm crust} = \epsilon_{\rm drip}$) measured by $\Phi(x) \equiv {\rm sign}(x) \log[1 + |x|]$ with $x \equiv (\omega_n/{\rm sec}^{-1})^2$ as a function of central star density. The cross at 'd' refers to the termination point of the strange dwarf sequence (cf. Fig. 2). For $\Phi < 0$ the mode is unstable.

star oscillating in this mode shrinks and expands as a whole. A strange dwarf with its highly incompressible strange core in its center can only oscillate in such a mode by means of varying its inner crust density, rather than the central density as is the case for neutron stars and ordinary white dwarfs. This leads one to reason that $dM/d\epsilon_{\rm crust} > 0$, rather than $dM/d\epsilon_c > 0$, constitutes the relevant condition necessary for stability against compressional vibrations. That this is indeed the case can be verified in close analogy to gravitationally bound stars [10]. As an example, any oscillating strange dwarf of the segment 'b'–'c' with a given fixed strange core in its center contracts ($\epsilon_{\rm crust}$ increases) toward hydrostatic neighboring configurations that possess larger strange cores (which can be inferred from Fig. 8 by remembering that the strange cores are monotonically growing in the clockwise direction) and thus a stronger gravitational binding force. So the oscillating strange dwarf possessing too small a strange core expands back to its initial state. Conversely, any strange dwarfs of the segment expands ($\epsilon_{\rm crust}$ decreases) during its oscillatory motion toward neighboring equilibrium configurations with cores that are smaller, and thus gravity pulls the star back. So any oscillating strange dwarf between 'b' and 'c' is stable against compressional oscillations. The strict ordering of the eigenmodes implies stability against all higher-lying oscillation modes too.

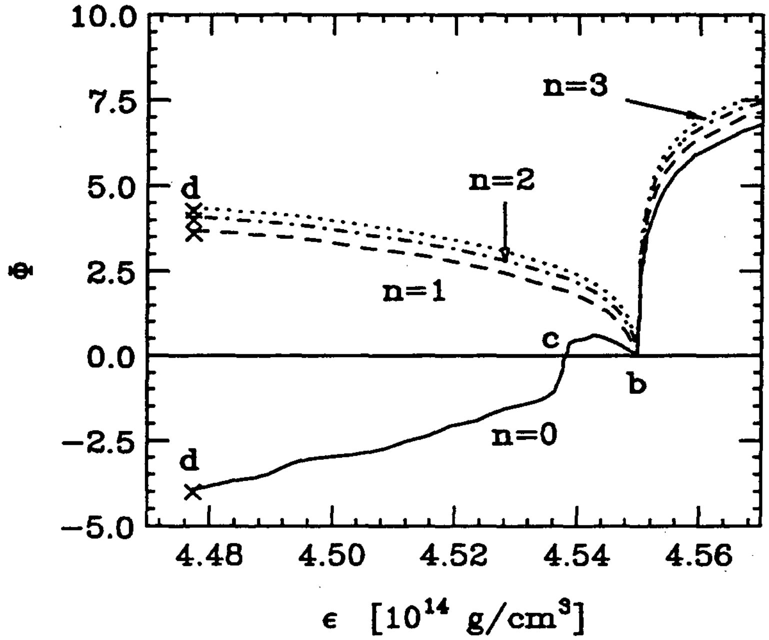

Figure 14: Same as Fig. 13, but for strange dwarfs in the vicinity of the termination point, 'd', of the sequence. The labels 'b' and 'c' refer to the minimum- and maximum-mass stars respectively marked in Fig. 2.

5 Summary

The theoretical possibility that strange matter may be absolutely stable, that is, the energy per baryon is $\lesssim 930$ MeV, is one of the most startling speculations of modern physics, which, if correct, would have implications of fundamental importance for our understanding of the early universe, its evolution to the present day, astrophysical compact objects, as well as laboratory physics [5, 6, 8]. Unfortunately it seems unlikely that QCD calculations will be accurate enough in the foreseeable future to give a definitive prediction on the absolute stability of strange matter, and one is thus left with experiment [41, 42, 43, 44, 45, 46, 47] and astrophysical tests [14] to either confirm or reject the strange-matter hypothesis.

In this investigation we construct complete sequences of strange-matter stars with nuclear crusts, which range from compact strange stars to strange dwarfs, and explore their properties (e.g., mass-radius relationship, limiting rotational periods, stability against radial oscillations) in great detail. Because such stars form two-parameter sequences (central star density and inner crust density), rather than one-parameter families as is the case for stars bound by gravity only, one arrives at a variety of hydrostatically stable strange-star sequences whose properties are much more complex than those of the conventional sequence, which extends from neutron stars to white dwarfs.

One of the most striking differences between sequences of strange-matter stars

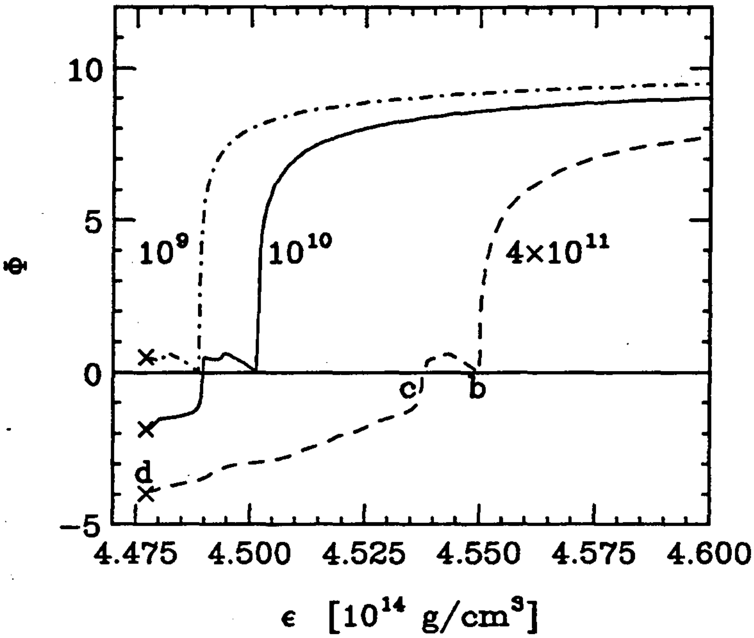

Figure 15: Eigenfrequencies (for the notation, see Fig. 13) of strange-dwarf sequences with $\epsilon_{\rm crust} < \epsilon_{\rm drip}$. The labels 'b', 'c', and 'd' are the same as in Figs. 2 and 14. The numbers refer to the value of $\epsilon_{\rm crust}$. Note that strange dwarfs along sequences with $\epsilon_{\rm crust} \lesssim 10^9$ g/cm^3 are all stable against radial oscillations.

with crust and the conventional sequence consists in the continuous, hydrostatically stable connection of massive strange stars with strange dwarfs. This is in sharp contrast to their non-strange counterparts which are separated by a broad range of unstable stellar configurations. Of possible importance for the dark matter problem in our Galaxy, one obtains an expansive range of extremely light strange-matter stars with masses that rival even those of ordinary planets (e.g., $\sim 10^{-3}\,M_\odot$, which is the mass of Jupiter), that is completely excluded for neutron stars and white dwarfs. Such light stars arise as natural dark matter candidates which, if abundant enough, should be seen in the gravitational microlensing searches that are being performed presently. The thickness of the nuclear crusts on such stars lies in the enormous range between ~ 10 and 10^4 kilometers, depending on inner crust density and central star density. Such crusts are likely to alter the cooling behavior of such stars, a subject which is presently being studied [48].

Besides the very light planetary-like strange-matter stars, we also find an expansive range of stable white-dwarf-like strange stars, the strange dwarfs, which we divide into two generically different categories. The dwarfs of the first category possess strange cores with radii in the range $R_{\rm core} \sim 1-3$ km. Such cores compress the surrounding nuclear material up to densities that are about 400 times higher than those in the most massive ordinary white dwarfs ($\epsilon_c \sim 10^9$ g/cm^3) and $\sim 4\times10^4$ times that of the typical $0.6\,M_\odot$ white dwarf ($\epsilon_c \sim 10^7$ g/cm^3). The entire sequences of such strange dwarfs owe their stability solely to the strange-matter cores, which possess baryon numbers of $\log A \lesssim 54 - 55.2$, compared to $\log A \sim 56 - 57$ for the baryon number of neutron stars or ordinary white dwarfs in their observed mass range. The second, trivial, category of strange dwarfs consists of ordinary white dwarfs which envelope small quark cores, $R_{\rm core} \lesssim 0.7$ km ($\log A_{\rm core} < 55$). Being essentially ordinary white dwarfs, such configurations would be stable with or without the strange cores. This statement proves the compatibility of the strange matter hypothesis with the existence of white dwarfs.

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