Submitted to Physics Letters B

LBL-371
Preprint

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AEC Contract No. W-7405-eng-48

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ANALYSIS OF THE OBSERVED ANOMALY IN $\pi\pi$ S-WAVE SCATTERING NEAR $K\bar{K}$ THRESHOLD*

S. M. Flatté,** M. Alston-Garnjost, A. Barbaro-Galtieri, J. H. Friedman,† G. R. Lynch, S. D. Protopopescu, M. S. Rabin, and F. T. Solmitz

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

December 1971

ABSTRACT

The character of the $\pi\pi$ s-wave scattering amplitude near 980 MeV is determined from the reactions $\pi^+p \rightarrow \Delta^{++}\pi^+\pi^-$ and $\Delta^{++}K^+K^-$ at 7 GeV/c.

* Work done under the auspices of the U.S. Atomic Energy Commission

** Present Address: Division of Natural Sciences, University of California, Santa Cruz, California 95060.

† Present address: Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305.

Our present understanding of strong interactions comes in large measure from information on the $\pi\pi$ system; the ρ and f^0 mesons play a central role in most strong interaction schemes. However, use of the s-wave $\pi\pi$ interaction has been hampered in the past by persistent ambiguity in determination of the phase shift below 900 MeV, and by lack of precise data above 900 MeV.[1,2] We have recently reported our observation of a strong anomaly in the $\pi\pi$ s-wave amplitude at 980 MeV in the reaction $\pi^+p \rightarrow \Delta^{++}\pi^+\pi^-$ at 7 GeV/c.[3] We pointed out that the anomaly is most easily understood as a rapid variation in the s-wave amplitude associated with $K\bar{K}$ threshold.

In the present paper, we present a qualitative picture of the $\pi\pi$ elastic s-wave scattering amplitude near $K\bar{K}$ threshold (i.e., between 900 MeV and 1,050 MeV), using the reactions $\pi^+p \rightarrow \Delta^{++}\pi^+\pi^-$ and $\Delta^{++}K^+K^-$. We find that the s-wave phase shift is close to 90° at 900 MeV and moves rapidly to 180° near 990 MeV. We conclude that the "up" solution in the 900 MeV region is eliminated, leaving the "down" solution as the unique result. Our data near $K\bar{K}$ threshold can be parametrized by an S^* resonance ($I^G J^{PC} = 0^+ 0^{++}$) at 990 MeV which couples to both $\pi\pi$ and $K\bar{K}$, but more strongly to $K\bar{K}$.

In our analysis we assume that pion exchange is the dominant mechanism for producing the $\pi^+\pi^-$ and K^+K^- systems in our reactions. Our data as a function of momentum transfer are consistent with previous analyses that have demonstrated pion-exchange dominance.[4] We use a simple form of the one-pion exchange (OPE) mechanism in order to present the qualitative features of the scattering amplitude; later we list a number of complications which must be treated if any truly quantitative analysis is desired.

It has been known for some time that a strong s-wave interaction

exists in the $K\bar{K}$ system at threshold.[5] This enhancement [S^*] has been described as either a resonance or as a large scattering length in an effective-range approximation.[6] Thus we must treat both reactions which can take place via the S^* enhancement:

$$\pi^+\pi^- \rightarrow \pi^+\pi^- \qquad (1)\ ;$$

$$\pi^+\pi^- \rightarrow K^+K^- \qquad (2)\ ;$$

and a proper treatment will involve coupling these two channels through the unitarity condition.

Our simple treatment of the OPE formula is

$$\frac{d^4\sigma_i}{dm\,d\Omega\,dt} = \frac{C\,m^2}{q(\mu^2 - t)^2}\,|\,T_i\,|^2$$

where we have ignored the slight kinematic dependence on the Δ^{++} mass. T_i is the T-matrix element for reaction i (i = π or K). The solid angle Ω represents the direction of the outgoing π^+ in the $\pi^+\pi^-$ rest frame, where the Z-axis is the incoming π^+; m is the mass of the $\pi^+\pi^-$ or K^+K^- system; t is the 4-momentum transfer squared from proton to Δ^{++}, q is the momentum of the π^- in the $\pi^+\pi^-$ rest frame, and C is a normalization constant.

Integrating over the physical region of t gives

$$\frac{d^3\sigma}{dm\,d\Omega} = \frac{C\,m^2}{q}\,|\,T_i\,|^2\left(\frac{1}{\mu^2 - t_{min}} - \frac{1}{\mu^2 - t_{max}}\right) .$$

We made the approximation that T_i is independent of t. Since we have chosen events with $|\,t'\,| < 0.1\ \text{GeV}^2$ ($t' = t - t_{min}$), we set $t_{max} = t_{min} - 0.1\ \text{GeV}^2$. The effect of t_{min} is substantial in our reactions; at the ρ-mass, $t_{min} = -1.5\,\mu^2$; at the f^0-mass, $t_{min} = -5\mu^2$. We have

$$T_\pi = \frac{2}{3}\,T_\pi^s + \sqrt{3}\,T_\pi^p + \frac{2\sqrt{5}}{3}\,T_\pi^d$$

$$T_k = \sqrt{\frac{1}{3}}\,T_k^s$$

where the superscripts indicate the angular momentum. We are assuming that only s, p and d waves are important*, and that the s-wave has substantial inelasticity coupled to the $K\bar{K}$ channel.

Two very important qualitative features of the s-wave amplitude may be determined simply by looking at the data (see Fig. 1):

a) The drop in the cross section between 950 MeV and 980 MeV was previously shown to be an s-wave effect.[3] Its size, compared to the p-wave maximum at the ρ peak, implies that the s-wave must have begun near its maximum value (a phase shift of 90°) and ended near its minimum. Thus the s-wave amplitude must start at about 900 MeV with $\delta_s \sim 90°$ and reach $\delta_s \sim 180°$ at about 990 MeV.

b) The drastic drop in $\langle Y_1 \rangle$ at 980 MeV requires that the s-p interference term pass through zero near 980 MeV. Because at this mass the p-wave phase shift is near 150° and the s-wave is elastic,[3] one finds that the s-wave phase must be either 60° or 180°. Only the 180° possibility is compatible with the cross section behaviour.

To gain a more quantitative understanding of these effects, we use as our model of $\pi\pi$ scattering (which must be incorporated into T_i) the following:

d-wave the f^0 meson $M_f = 1270$ MeV, $\Gamma_{f_0} = 180$ MeV

p-wave the ρ-meson $M_\rho = 770$ MeV, $\Gamma_{\rho_0} = 140$ MeV

s-wave a) The S^*-meson with coupling to both $K\bar{K}$ and $\pi\pi$, but no coupling to other channels

b) An elastic s-wave background represented by a constant phase shift, δ.

Even though we include the f^0 in our model, we are <u>not</u> attempting to explain all of our data in the f^0 region; a region we find to be very complicated. Also, we do not expect nor require this model of the

* Limitations of these assumptions will be indicated later in this letter.

s-wave to be meaningful below 850 MeV. We are only interested here in the $K\bar{K}$ threshold region.

We have

$$T_\pi^p = \frac{r_\rho m_\rho \Gamma_\rho Y_1^0(\Omega)}{m_\rho^2 - m^2 - i m_\rho \Gamma_\rho} \quad ; \quad \Gamma_\rho = \Gamma_{\rho_0}\left(\frac{q}{q_\rho}\right)^3\left(\frac{x^2+q_\rho^2}{x^2+q^2}\right)$$

$$T_\pi^d = \frac{r_f m_f \Gamma_f Y_2^0(\Omega)}{m_f^2 - m^2 - i m_f \Gamma_f} \quad ; \quad \Gamma_f = \Gamma_{f_0}\left(\frac{q}{q_f}\right)^5$$

$$T_\pi^s = \frac{1}{\sqrt{4\pi}}\left[\frac{e^{2i\delta}-1}{2i} + \frac{e^{2i\delta} m_s \Gamma_\pi}{m_s^2 - m^2 - i m_s(\Gamma_\pi + \Gamma_k)}\right] ;$$

$$T_k^s = \frac{1}{\sqrt{4\pi}}\left[\frac{m_s\sqrt{\Gamma_\pi \Gamma_k}}{m_s^2 - m^2 - i m_s(\Gamma_\pi + \Gamma_k)}\right] \text{ (above threshold),}$$

where q_ρ and q_f are the values of q at the ρ and f peaks, respectively. The value of x was set at 0.8 GeV/c, which was required for a reasonable fit to the ρ meson. The constants r_ρ and r_f would be unity if we were dealing with on-shell $\pi\pi$ scattering. However, because of off-shell effects and absorption, they can differ from unity in the physical region. The partial widths of the S^* into $\pi^+\pi^-$ and K^+K^- are given by Γ_π and Γ_k respectively; we have $\Gamma_\pi = g_\pi q$ and $\Gamma_k = g_k q_k$, where q_k is the momentum of the K^- in the K^+K^- rest frame. g_π and g_k are the squares of the coupling constants for the $S^*\pi\pi$ and $S^*K\bar{K}$ vertices.

Our forms for the s-wave matrix elements T_π^s and T_k^s satisfy all the requirements of unitarity. However, there are additional requirements of analyticity across the $K\bar{K}$ threshold. These requirements are properly handled by treating Γ_k correctly below threshold. We have

$$\Gamma_k = g_k q_k = \begin{cases} g_k\sqrt{\dfrac{m^2}{4} - m_k^2} & \text{above threshold} \\ i g_k\sqrt{m_k^2 - m^2/4} & \text{below threshold} \end{cases}$$

One result of the unitarity constraints is that the shape of the K^+K^- cross section comes mainly from the product of $\Gamma_\pi \Gamma_k$ and not from the Breit-Wigner (BW) denominator of the S^*. This is a consequence of the S^* mass being close to $K\bar{K}$ threshold. To see this, let us ignore $m_s^2 - m^2$ in the BW denominator. Then

$$\frac{d\sigma_{k\bar{k}}}{dm} \propto \frac{\Gamma_\pi \Gamma_k}{(\Gamma_\pi + \Gamma_k)^2} = \frac{\Gamma_\pi\, g_k\, q_k}{(\Gamma_\pi + g_k\, q_k)^2}$$

where we have treated Γ_π as constant over the mass region near threshold. This form has a maximum as a function of q_k that occurs when $\Gamma_\pi = \Gamma_k$, and the maximum value is 0.25, independent of Γ_π or g_k — in other words, independent of the S^* coupling to $\pi\pi$ or $K\bar{K}$. The width of the peak is proportional to Γ_π/g_k so that we have the following peculiar fact: the larger the S^* coupling to $K\bar{K}$, the narrower (but not higher) the peak in the KK cross section becomes.

Because of the many effects that we are ignoring in our simple analysis (and which we list later), an actual fit to the data is not appropriate. However, the qualitative arguments stated earlier lead to a rather narrow range of possibilities. The values we have found that best represent the qualitative features of the data in the 900-1050 MeV region are:

$\delta = 90°$

$M_s = 990$ MeV

$g_\pi = 0.2$

$g_k = 0.5$

$r_\rho = 0.9$

$r_f = 0.7$

The Argand diagram associated with these parameters, as well as the predictions to be compared with the data, are shown in

Fig. 1 (labelled Nominal). We note that the drops in the cross section and in $\langle Y_1 \rangle$, as well as the rise in $\langle Y_2 \rangle$ are well produced at the correct masses, and the cross section for K^+K^- is reproduced within 20% by the model. (We have chosen the overall normalization to fit the ρ peak in the $\pi^+\pi^-$ mass plot.)

We do see significant discrepancies in shape; for example, in $\langle Y_2 \rangle$ above 1050 MeV the curve shows a drop and then a slow rise, while the data seem to rise continually. Also $\langle Y_4 \rangle$ is not explained by the f^0 meson (see Fig. 3). However, the $\langle Y_4 \rangle$ moment is small below 1050 MeV and varies slowly, so that our conclusions, which depend on the rapid variations in the data near 1 GeV, should not be affected by these complications.

With the above parameters, the model gives higher values of $\langle Y_1 \rangle$ below 980 MeV than are seen in the data. However, we have found that extrapolations of the data below 980 MeV to the pion pole can easily increase $\langle Y_1 \rangle$ by 20%.

It is difficult to assign errors to our determination of the above parameters because of the qualitative nature of our analysis. In order to give some feeling for what changes in each parameter can be tolerated, we have shown the predictions of several variations in Fig. 2. We comment briefly on each case:

$\delta = 135°$ — The shape of the predicted $\langle Y_1 \rangle$ does not describe the effect near 980 MeV.

$g_k = 0$ — This requires that none of the K^+K^- cross section come from π exchange. The model does not predict the shoulder in the $\pi^+\pi^-$ mass plot at 970 MeV, and the calculated $\langle Y_1 \rangle$ is much too negative above 980 MeV.

m_s = 1040 MeV The theoretical $\langle Y_1 \rangle$ does not drop fast enough; (it reaches zero 50 MeV beyond threshold, whereas the data do so within 10 MeV).

Our nominal picture of the s-wave Argand diagram has accounted for all of the important qualitative features of the effect in our data between 900 MeV and 1050 MeV. To be more quantitative we would have to treat the following problems:

1) How to extrapolate to the pion pole. Our preliminary work on extrapolation shows no qualitative changes. However, quantitative changes do occur. For example, our extrapolations of $\langle Y_1 \rangle$ below 980 MeV lie near 0.2, whereas the values in the physical region are near 0.15 (see Fig. 1).

2) Inelasticity in the p and d waves (e.g. $\rho\omega\pi$ coupling and $fK\bar{K}$ coupling).

3) How to allow for variation in the s-wave background (now represented by one number, δ).

4) The presence in the data of strong $\langle Y_4 \rangle$ below the f^0.

5) Non-zero $\langle Y_5 \rangle$ above 1 GeV, indicating the necessity of an f-wave.[3]

6) How to treat I = 2 contributions in both s and d waves.

We also note that our S^* parametrization of the s-wave is only one possibility. Since we are restricting ourselves to a narrow region of mass (900-1050 MeV), an effective range parametrization would serve equally well.

In conclusion, the following picture of the s-wave $\pi\pi$ interaction near 1 GeV qualitatively describes the effect seen in our data. The s-wave phase shift at 900 MeV is close to 90°, and it moves very

rapidly to 180° at 990 MeV. (This rapid behavior can be parametrized in terms of an S^* resonance at 990 MeV which couples to both $\pi\pi$ and $K\bar{K}$ channels, $K\bar{K}$ more strongly than $\pi\pi$). Our value for the phase shift near 900 MeV eliminates the up-down ambiguity in the s-wave phase shift, since the "up" solution requires a phase shift of 160° at 900 MeV whereas the "down" solution is compatible with 90°.[7]

We gratefully acknowledge helpful conversations with Dr. W. Beusch, Professor J. D. Jackson, and Dr. G. Smadja.

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7. For the various solutions found previously, see reference 1.

FIGURE CAPTIONS

1. Number of events and spherical harmonic moments as a function of mass in the reactions $\pi^+ p \rightarrow \Delta^{++}\pi^+\pi^-$ and $\pi^+ p \rightarrow \Delta^{++}K^+K^-$ at 7.1 GeV/c. See Ref. 3 for selection criteria. The curves are derived from an OPE approximation which treats the $\pi^+\pi^-$ and K^+K^- systems as final states in the $\pi^+\pi^-$ scattering process. The amplitude includes a ρ-meson and f-meson, and s-wave interaction which includes a constant-phase-shift background and an S^* resonance which couples to both ππ and $K\overline{K}$. The s-wave amplitude is shown on the Argand plot. Since our model fails above 1050 MeV, this amplitude is probably very inaccurate in that region (see text for definitions and values of parameters).

2. The effect of changing the s-wave parameters from the nominal values used in Fig. 1.

3. Spherical harmonic moments $\langle Y_3 \rangle$ and $\langle Y_4 \rangle$ for the reaction $\pi^+ p \rightarrow \Delta^{++}\pi^+\pi^-$ at 7.1 GeV/c. The curves are derived from the nominal parameters.

Note the failure of the f-meson to explain $\langle Y_4 \rangle$ below 1200 MeV. For data on $\langle Y_5 \rangle$ and $\langle Y_6 \rangle$ see Ref. 3.

0 0 0 0 3 7 0 1 8 0 1

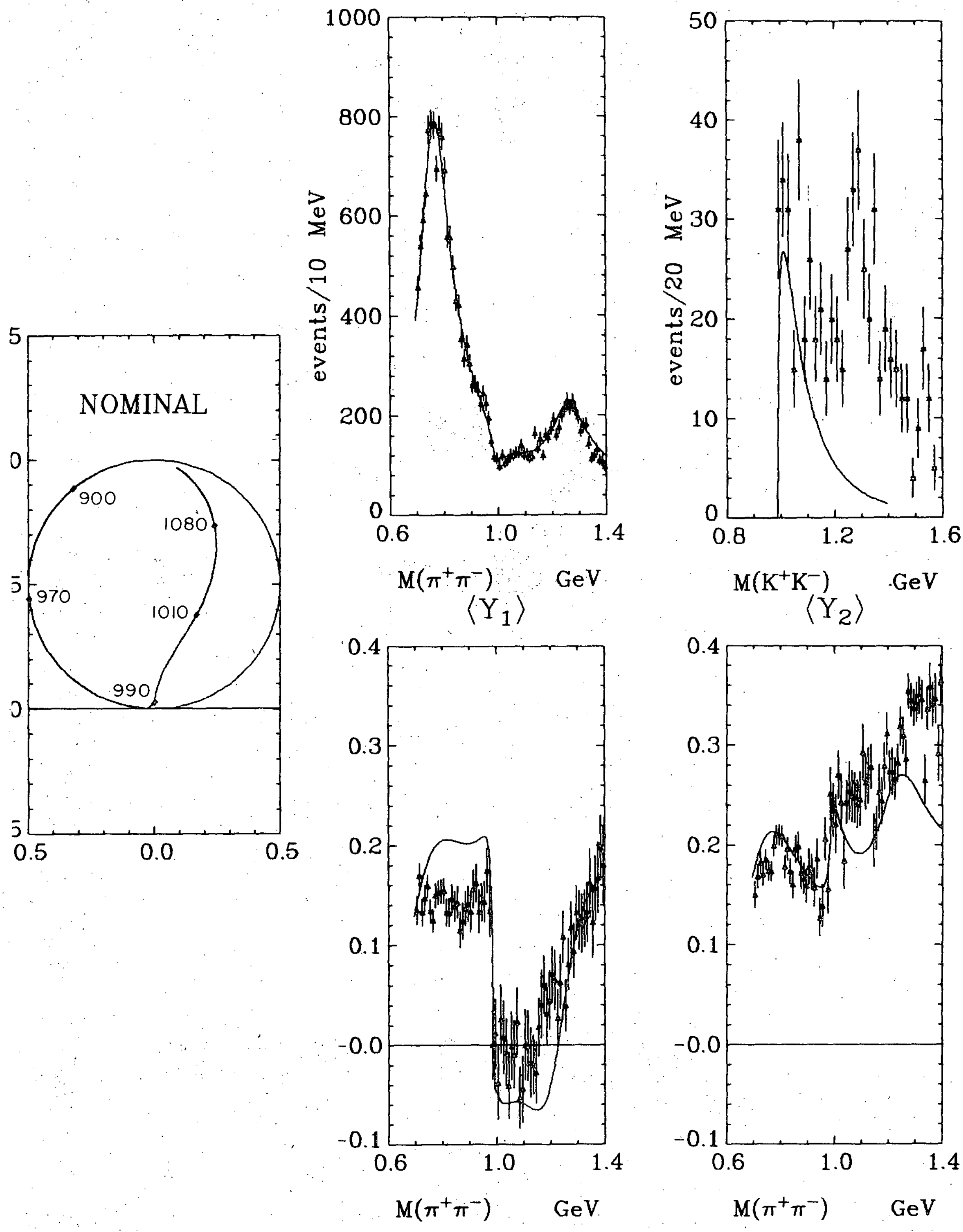

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Fig. 1

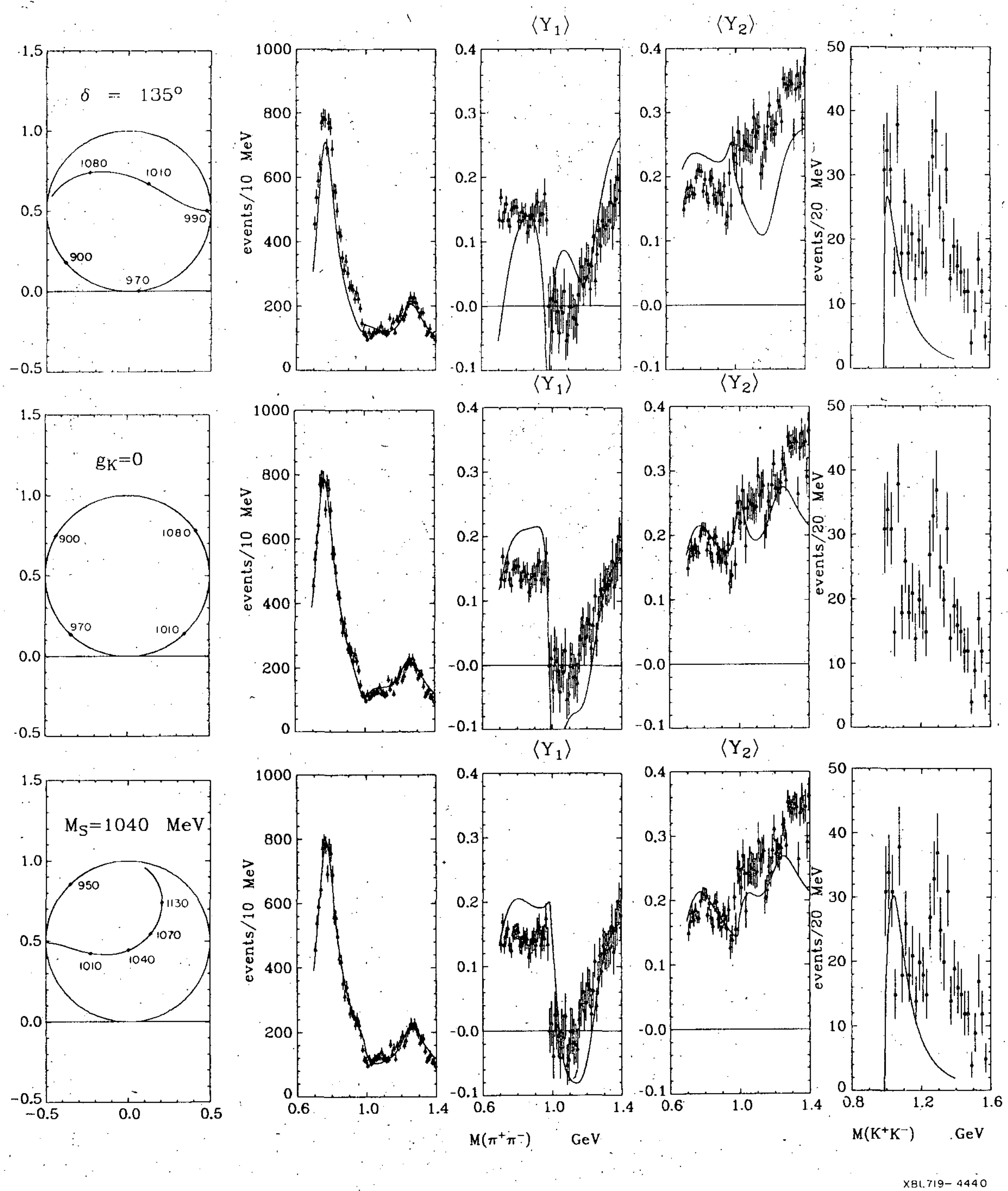
δ = 135°
1080
1010
990
900
970
events/10 MeV
⟨Y1⟩
⟨Y2⟩
events/20 MeV
gK=0
900
1080
970
1010
MS=1040 MeV
950
1130
1070
1010
1040
M(π+π−) GeV
M(K+K−) GeV
XBL719- 4440

Fig. 2

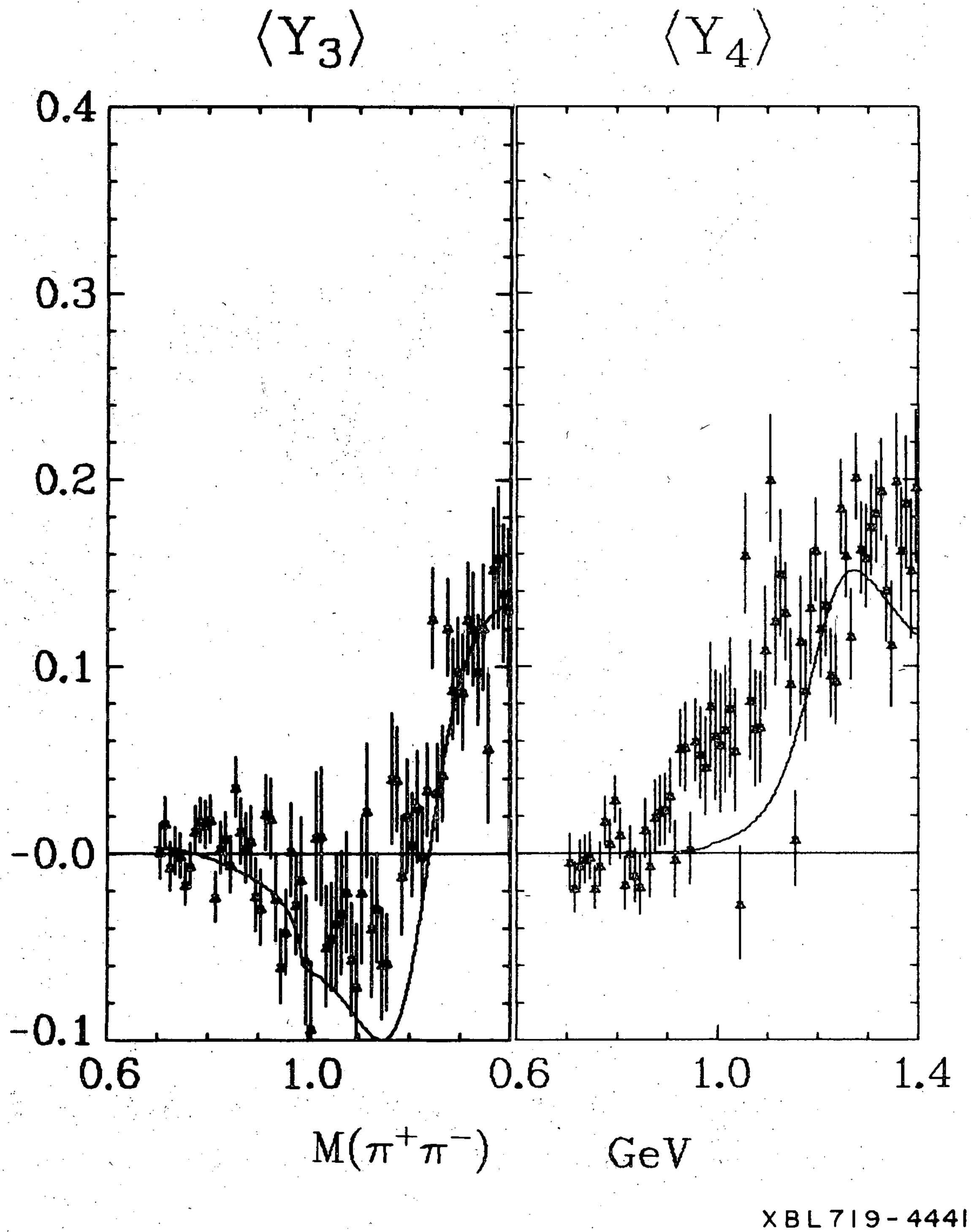

Fig. 3

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720