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Variability in Math Achievement Growth Among Students With Early Math Learning Difficulties and the Role of School Supports

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Math is a challenging subject, with many students demonstrating math learning difficulties (MD) as early as kindergarten. However, our knowledge of how these students progress in their math achievement and ways in which individual skill differences and supports facilitate math achievement for this group is limited. We used the Early Childhood Longitudinal Study - Kindergarten data set ($N \sim 18{,}000$) to examine the patterns of math achievement growth displayed by students with MD between kindergarten and fourth grade. Using latent class growth analysis, we revealed three distinct growth classes, characterized by levels of math performance that fell very low, moderately low, and slightly low below the MD cutoff range across time. Those who displayed the lowest math performance in kindergarten also had substantially slower growth than the other classes and their non-MD peers. Students with higher initial reading achievement, cognitive flexibility, working memory, higher math skill ratings, and without an individualized education program (IEP) in kindergarten, were more likely to display more favorable growth trajectories, ending with relatively higher levels of math achievement. Students with higher growth were also more likely to demonstrate higher math skill ratings and were less likely to have an IEP and receive math instructional support in fifth grade. However, among the three growth classes, fewer underrepresented minority (URM) than non-URM students received IEP supports in fifth grade, indicating potential disproportionality in schools' provisions of supports. Collectively, our findings offer theoretical insight on math achievement growth among students with MD, highlighting areas of need and leverage points to reduce extensive math difficulties.

Educational Impact and Implications Statement

Many children begin formal schooling with underlying math learning difficulties (MD). We demonstrate using latent class growth analysis that students with early MD continue to demonstrate persistent difficulties with great room for improvement. However, most students display levels of math achievement that consistently fall just below the MD range (i.e., 25th percentile), suggesting that their math difficulties are more mild and show potential promise for overcoming their math difficulties in the future. In addition, by identifying predictors of growth, we provide insight on students with early MD who are most at risk for extensive math difficulties and highlight the need for greater allocation of early academic supports in kindergarten. Through our examination of distal outcomes, we also reveal areas where schools can improve in their continued provision of academic supports for struggling students, especially those from racially and ethnically minoritized groups. Furthermore, our findings provide evidence that students with the lowest math performance in kindergarten continue to lose ground relative to their peers, supporting the Matthew effect that "the rich get richer and the poor get poorer." To help reduce this effect, results from our study suggest that these students would likely benefit from early math instruction and interventions that take into account and help bolster their executive function and reading skills.

Keywords: math difficulties, latent growth class analysis, academic support, math achievement

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Mathematical skills are crucial for daily life and future educational advancement. However, a recent 2022 National Assessment for Educational Progress (U.S. Department of Education et al., 2022) indicated that only 36% of fourth graders in the United States were proficient in math, with a notable decline in the lowest performing students previously reported in 2019. Failure to develop foundational math skills yields a host of negative effects, including decreased college entry and success (National Mathematics Advisory Panel, 2008) and the pursuit of science, technology, engineering, and mathematics careers (Shapka et al., 2006). This problem is especially troubling for children who demonstrate math learning difficulties (MD).

Struggling students often exhibit signs of MD early in their education. Currently, students enter kindergarten with great variability in their math skills and this is well documented at the start of formal schooling (e.g., G. J. Duncan et al., 2007; Wirt et al., 2004). However, it is unclear what happens to students who exhibit early MD in terms of their future math achievement. That is, do students with MD in kindergarten demonstrate subsequent growth or are their struggles with mathematics sustained? Furthermore, is there additional heterogeneity in the longitudinal patterns of math achievement they display?

Elementary school is a pivotal time for shaping students' math achievement, as this is when many foundational skills are built (e.g., Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). It is also the time when students with MD are initially identified by their school as having learning difficulties and provided necessary academic supports; however, these identification processes are not always accurate (e.g., Ruffini et al., 2016). In this study, we investigated: (a) the types of growth students with early MD display in their math achievement from kindergarten through fourth grade, (b) the role background characteristics, and (c) school supports play in students' growth and related academic outcomes to gain a better theoretical understanding of MD in early childhood.

MD

Students with MD are considered those who demonstrate low math performance based percentile ranks on standardized diagnostic tests (e.g., Connolly, 2007). However, there is a lack of consensus of the specific definition of MD due to variation in cutoff criterion used by researchers to identify these students. For example, some scholars use scores as high as the 46th percentile and others use a more stringent cutoff of the 10th percentile (e.g., Cirino et al., 2015; Lewis & Fisher, 2016). Despite this variability, systematic reviews indicate ≤25th percentile as the most commonly used cutoff criteria used for identifying individuals with MD (e.g., Lewis & Fisher, 2016; Nelson & Powell, 2018). In contrast, students with a math learning disability (LD), a more severe form of math difficulty, are typically identified using cutoff scores of <11th percentile or two or more *SD*s below the mean on standardized measures of math performance (e.g., Geary, 2013; Lewis & Fisher, 2016).

Considerable overlap exists between students with MD and a math LD in terms of number-specific and cognitive difficulties (e.g., Andersson & Östergren, 2012; Vukovic & Siegel 2010). However, those with a math LD tend to also display large discrepancies between their levels of academic achievement in math and overall level of cognitive skills (e.g., intelligence quotient [IQ]), commonly referred to as the discrepancy model (e.g., Mazzocco & Myers, 2003). Over recent decades, the use of this model to identify and differentiate students with learning difficulties versus learning disabilities has been highly criticized as a "wait to fail" model (e.g., D. Fuchs & Fuchs, 1998) because it requires a student to exhibit exceedingly low levels of performance for a prolonged period of time before they are identified as needing academic supports. As a result, both researchers and practitioners have moved away from the overreliance on the presence of an IQ-achievement discrepancy and have instead focused on other methods such as achievement cutoff scores and students' improvement when provided additional tailored instruction and interventions (Individuals with Disabilities Education Improvement Act [IDEA], 2004; Taylor, 2014). In the current study, we use the term MD to refer to students who perform at or below the 25th percentile, as our focus is to examine differential growth among students who perform on the lower end of the math continuum, rather than differentially identifying those with a math learning difficulty versus a math LD. However, in using this definition, we recognize that our sample will include a mixture of students with both MD and math learning disabilities.

Risk Factors for MD

Children with MD demonstrate challenges in various cognitive and academic skills. In the current study, we consider how these and other known risk factors of MD serve as potential predictors of MD growth. One well-documented area of difficulty for students with MD is executive function. Executive function refers to an interrelated set of cognitive processes, which includes ignoring distractions and inhibiting inappropriate responses (inhibitory control), holding and manipulating information in memory (working memory), and shifting between tasks (cognitive flexibility) (e.g., Best et al., 2011). Executive function is correlated with various academic outcomes and learning disabilities (Peng & Fuchs, 2016; Spiegel et al., 2021).

Executive function is also predictive of learning mathematics and solving math problems (e.g., Ribner, 2020). Children with MD frequently struggle in one or more areas of executive function (e.g., L. S. Fuchs, et al., 2014). In particular, prior work has highlighted the role of inhibition in math calculation tasks. For example, children with MD may demonstrate difficulty suppressing inappropriate calculation strategies, like using addition when subtraction is more appropriate or subtracting the larger number from the smaller one on multidigit written calculations regardless of its position (Bull & Lee, 2014). Young children with MD may recruit working memory, a component of executive function, to execute calculation strategies because their basic calculation fluency skills (i.e., fact knowledge) are not yet well automated (Zhang et al., 2023). This is further demonstrated through studies showing that fact retrieval delays and difficulties are a characteristic trait of students with both MD and math learning disabilities and coincide with difficulties on working memory assessments (e.g., Geary et al., 2012). For example, Geary et al. (2012) compared fact retrieval skills across four groups of primary grade students (i.e., typically developing children, children with moderate MD (performing between the 10th and 25th percentile on a math achievement test) with mild fact retrieval delays, children with moderate MD and severe fact retrieval delays, and children with a severe math LD (performing at or below the 10th percentile on a math achievement test) and severe fact retrieval delays) and found that students with varying levels of MD and math learning disabilities all displayed to some extent fact retrieval difficulties and students with MD demonstrated the weakest performance on working memory tasks. Furthermore, other work indicates that children with poor counting skills also demonstrate lower working memory task performance (Allen et al., 2020; Siegel & Ryan, 1989). These findings suggest that executive function skills may explain some of the difficulties displayed by children with MD and math learning disabilities.

Children with MD not only demonstrate difficulties in math and executive function, but in reading as well. Children's reading and math performance are highly correlated (Ünal et al., 2023), with this relationship extending to a wide range of skills in both academic areas (e.g., Ding & Homer, 2020). For example, students' reading fluency skills are correlated with their calculation fluency skills in elementary school (e.g., Koponen et al., 2007). A positive relationship between reading comprehension and math problem solving has also been demonstrated among third- and fourth-grade children. Furthermore, students' early reading skills in elementary school are shown to predict their growth in their general math achievement over the course of several years well into secondary school (Grimm, 2008). Thus, a lack of strong reading skills can have a lasting impact on students' math proficiency. Given the strong relationship between reading and math, it is not surprising that there is a high prevalence (i.e., 17%–20%) of students with comorbid reading and math difficulties/disabilities (Landerl & Moll, 2010). Neuropsychological studies indicate that this co-occurrence between MD and reading difficulties (RD) is likely due to the shared brain regions and underlying cognitive processes (e.g., executive functioning, representation of symbolic information) required to learn skills in each area (e.g., Wang et al., 2020). These findings align with educational research, which shows students with MD and RD tend to experience greater academic difficulties and lower levels of math achievement growth than those with only MD (e.g., Cirino et al., 2015; Jordon et al., 2002), evidencing the impact difficulties in other academic areas can have on students' math proficiency.

Differences in Students' Math Achievement by Socioeconomic Status (SES) and Race/Ethnicity

Differences in children's math achievement are salient when observing performance variability between students from high- and low-income families (see García & Weiss, 2017 for a review). This variability reflects external factors beyond individual abilities. For instance, students from low-income homes have fewer opportunities to practice math, stemming from limited access to such resources (for a review, see Banerjee, 2016). Similar trends exist for ethnic/racial disparities. Students from racially and ethnically minoritized groups such as Black or Hispanic/Latine children who are also underrepresented minority (URM) students in science, technology, engineering, and mathematics demonstrate lower levels of math performance upon school entry (e.g., Sonnenschein & Galindo, 2015). As income, race, and ethnicity are generally intertwined in the United States, due to the long-standing inequitable social stratification of groups based on race, it is not surprising but still disheartening to know that students from low-income homes and URM students are at a greater risk for showing early and persistent MD. Due to this profound problem, we aimed to examine how the growth in children's math achievement trajectories differ by ethnicity/race and income. A deficit-based approach that focuses on individual student-level factors without consideration of structural ones narrows our understanding of how inequities are formed and maintained and can lead researchers to view certain kinds of inequalities as justified, resulting from the existence of a (false) meritocracy rather than the reality that inequities are undeserved and can be temporary realities if addressed (Hetey & Eberhardt, 2018). Marginalization, by definition, comes with negative experiences such as discrimination and lack of resources and opportunities. Thus, in the current article we consider the instructional supports offered to children to consider whether there are structural inequities that we may be able to document that may explain some of the variation we see in growth trajectories of students with MD for students from minoritized and marginalized groups (e.g., race, ethnicity, SES).

The Role of Schools in the Identification and Support of Students With MD

Response to Intervention (RTI)

Schools play a critical role in helping prevent persistent academic difficulties and promoting academic achievement among struggling students through the early identification of children with learning difficulties (e.g., MD) and the provision of necessary academic supports (e.g., Mellard et al., 2010). Many U.S. schools use an RTI model, also known as multitiered systems of support, as a framework for this process (IDEA, 2004). The RTI model was developed as an alternative to the "wait to fail" approach (i.e., IQ-achievement discrepancy model) to help combat the delay of initial supports to struggling students and reduce the over-identification of students in special education for learning disabilities (e.g., Grigorenko, 2009).

The RTI model consists of three tiers of increasing supports based on academic needs and instructional response, with initial tier placement based on students' performance on school-wide screening assessments. The three tiers of the RTI include: Tier-1 traditional large group classroom instruction for students demonstrating average academic performance; Tier-2 small group instruction for students demonstrating mild-to-moderate difficulties; and Tier-3 intensive individual instruction for students who do not make adequate progress at the Tier-2 level. Students who do not demonstrate improvement through Tier-3 instruction are referred for special education services. Those who are found eligible for special education services receive more personalized and intensive supports through an individualized education program (IEP), such as extended time, preferential seating, assistive materials, and so on. Although IEPs are reserved for students in special education, children can still receive supports in math as part of their IEP regardless of their specific education classification. For example, a student with an LD in reading can receive academic supports in math as part of their IEP if it is determined to be an additional area of need. In the current study, we include small group and/or individualized math instruction (i.e., math instructional support) and IEP status as proxy of academic supports and potential predictors of growth for students with early MD.

Disproportionality of Academic Supports and Services

Although RTI is designed to make the provision of academic supports and referral to special education more equitable (e.g., Kramarczuk Voulgarides et al., 2017), many schools struggle to implement this intervention model with fidelity (Balu et al., 2015; Ruffini et al., 2016), resulting in either too many or too few students receiving instructional support. In the latter situation, students who exhibit prolonged academic difficulties continue to go undetected for MD with a delay or lack of academic supports which can hinder their future academic achievement (e.g., Flett & Hewitt, 2013). This issue is particularly common for URM students (Delgado & Scott, 2006). However, disparities in the literature exist regarding URM students' overall identification rates in special education, with some studies demonstrating that URM students are underidentified (e.g., Morgan et al., 2017), while others reporting that they are overrepresented (Skiba et al., 2016; Sullivan & Bal, 2013). Yet, with respect to high-incidence (i.e., more prevalent) disabilities such as specific learning disabilities (e.g., math learning disabilities), URM students tend to be overidentified (Brandenburg et al., 2016; Fletcher & Navarrete, 2003). Regardless of the direction of these identification rates, URM students are disproportionately represented in special education and in their provision of supports. Students from low-income backgrounds also face issues of disproportionality in these areas (e.g., O'Connor & Fernandez, 2006). Thus, URM and low-income students who display early learning (e.g., math) difficulties may be at a disadvantage for future academic growth relative to their non-URM and higher-income peers due to the extent of supports they receive.

Teachers' Views and Ratings of Students' Academic Skills

Teachers' ratings (i.e., judgments or perceptions) of students' academic achievement also serve as a method for identifying students with learning difficulties. However, disparate findings are noted in the literature with regard to how accurate teachers' judgments of students' academic skills are in relation to their actual levels of achievement. For example, a previous meta-analysis across multiple grades and subject areas found a positive and moderately high correlation ($r = .63$) between teachers' ratings of students' academic skills (Südkamp et al., 2012). Moderately high levels of correlation ($r = .70$) were also found for elementary school teachers' judgments of students' academic skills and students' levels of academic performance (Demaray & Elliot, 1998). However, levels of agreement were higher for students with above-average academic skills compared to their below-average peers, indicating a potential gradient in the utility of teacher's ratings of students' academic skills. In addition, teachers' rating of students' early academic skills (i.e., in kindergarten) in math are shown to predict children's later academic performance in elementary school (Teisl et al., 2001). In contrast, other research shows teachers' ratings of students' academic achievement are influenced based on local, rather than national levels, of student performance and can thus lead to inaccuracies when trying to judge achievement on a nationally normed assessment (Mowrey & Farran, 2016). Student characteristics such as race and ethnicity as well as social skills can also influence teachers' ratings of student achievement (Furnari et al., 2017).

There is some recent evidence to suggest that teachers may display biases in their ratings of students' competencies. For example, Copur-Gencturk et al. (2020) found that when assessing mathematical ability, teachers displayed large biases against Black and Hispanic/Latine girls. This varied by teacher race. White teachers tended to have higher (inaccurate) estimates of boys' math ability than girls. Non-White teachers tended to have higher (inaccurate) estimates of White students' math ability over students of color. Relatedly, Ouazad (2014) found that White teachers rated their Hispanic students' mathematics proficiency lower than their White students, even when accounting for differences in students' performance. Papageorge et al. (2020) found that White high school teachers were generally optimistic about their students' educational attainment (i.e., college-seeking) but less optimistic about their Black students' prospects than their White students. These same scholars found that teachers' expectations did predict their students' college attendance. Starck et al. (2020) found that teachers are just as prone to explicit and implicit pro-White racial biases as adults in other fields. Thus, although teachers have a wealth of information at their disposal to make accurate math ability judgments about their students, they still can and do fall prey to racial and gender biases that may impact these judgments, making teacher ratings prone to some inaccuracies and part of an imperfect diagnostic system. Nevertheless, teachers' ratings of students' academic skills are a primary tool for schools' early and/or continued identification of students with MD. Teachers are typically the first to flag a child for potentially needing services, acting as advocates for their students (Ikeda & Gustafson, 2002). As teachers do play a major role in the detection of students' mathematics difficulties which directly impacts services and supports that students are (or are not) provided, we include teacher ability judgments as a potentially important predictor of detection of MD. We consider whether teachers' ratings of students' academic skills adequately align with their displayed levels of academic achievement.

Theoretical Accounts of Math Growth: Matthew Effect Versus Compensatory Growth Model

As previously mentioned, students display considerable variability in their math skills and achievement early on in their formal schooling (e.g., G. J. Duncan et al., 2007). Indeed, earlier work, from a large nationally representative sample suggests that children who began formal schooling with the lowest mathematical knowledge among their peers demonstrated the smallest gains between kindergarten and third grade (Bodovski & Farkas, 2007). This finding is especially troubling because children who began kindergarten with the lowest mathematical knowledge also received the greatest instructional supports in math but displayed the lowest active engagement with said supports. Additional accounts of early learning difficulties are also evidenced by the aforementioned variability in math skills documented between students from mid-to-high versus low-income families and those from minoritized groups.

Based on the literature, two theoretical accounts exist for future academic outcomes for students who display early MD: the Matthew effect and the compensatory growth model. According to the Matthew effect in education (Walberg & Tsai, 1983), students who begin school with stronger academic performance continue to refine their skills because they can build upon their strong foundation. Those with lower initial demonstrated skills continue to fall behind their peers resulting in the "the rich get richer, and the poor get poorer" phenomenon. This has primarily been examined among students with reading disabilities (e.g., Sideridis, 2011), with relatively less known about whether and how it applies to students with MD.

Alternatively, the compensatory growth model (e.g., Baumert et al., 2012) suggests that students with initial lower levels of performance will demonstrate resilience and experience faster rates of skill growth with additional educational instruction, thus allowing them to catch up to their peers. Therefore, students who demonstrate early MD may no longer display MD or may display it to a lesser extent later on in their schooling. However, results in the literature to support one theoretical account versus the other for students with MD are inconclusive (e.g., Morgan et al., 2011; Scammacca et al., 2020). Additionally, it is possible that one theoretical account cannot capture the complex development of skills for students with MD and that a combination of both theoretical accounts holds true. For example, this would be displayed if a student with MD is able to initially keep afloat in their math courses using their compensatory skills but starts to fall further behind their peers in subsequent years as math instruction becomes more challenging.

Math Growth Trajectories

Previous research demonstrates students with MD display variability in their math achievement growth, with some students making more progress than others. For example, Morgan et al. (2009) examined math achievement growth among four predefined subgroups of students with MD and found that their rate of growth varied based on their timing of MD in kindergarten. Students with MD in the fall and spring of kindergarten displayed the lowest levels of growth over time, followed by those with MD in the spring of kindergarten only, and then those with MD in the fall of kindergarten only; children who did not display MD in either the fall or spring of kindergarten demonstrated the highest levels of growth. It is possible that those who displayed MD only at the start of the school year had less exposure to mathematics prior to kindergarten and may have benefitted from formal math instruction, whereas other students possessed good foundational early math skills but began to struggle as instruction shifted from general school readiness (i.e., shapes, counting) to more formal math skills. Judge and Watson (2011) revealed similar variability in math achievement growth based on the timing of a students' initial diagnosis of an LD (i.e., early-emerging LD; emerging LD; late-emerging LD; non-LD). Although the gap in math performance between the LD groups and their non-LD peers consistently widened over time, differences in levels of performance between LD groups changed, with students in the late-emerging LD demonstrating just as low final math achievement scores in the end of fifth grade as those in the early-emerging LD group. Thus, students can display considerable variability in their patterns of math growth despite displaying similar starting or ending levels of performance. Although these aforementioned studies provide insight on the nature and types of patterns of math achievement among struggling students and those with MD, they predefined groups or subtypes of MD prior to their growth modeling, thus constraining the types of math achievement trajectories observed among this subpopulation of students. In the current study, we are interested in examining whether similar patterns of growth emerge among students with early MD when the formation of growth trajectory groups is directly informed by the longitudinal data.

Studies that use a more person-centered method for modeling growth, such as latent class growth analysis (LCGA), show struggling students and those at-risk for MD display heterogeneous patterns of growth for the development of various math skills, including fractions and early number sense (e.g., Gesuelli & Jordan, 2023; Hansen et al., 2015; Jordan et al., 2006; Tikhomirova et al., 2019). Across these studies, researchers identified distinct growth trajectories (i.e., latent growth classes), typically consisting of a consistently low-performing class, a consistently high-performing class, and one or more classes of students that display a change in level of their growth (e.g., start low but end with high performance). We note this because we suspect similar trajectories exist for students with early MD when examining growth based on latent factors, where some initially struggling students catch up to their peers in later grades but others experience sustained difficulties. These patterns of growth in the aforementioned LCGA studies also align with two theoretical accounts of math growth for struggling students (i.e., compensatory growth model and the Matthew effect). Use of LCGA methods may therefore help provide empirical evidence to support whether the compensatory growth model and/or the Matthew effect hold true for students with early MD. If students with MD demonstrate the Matthew effect in the development of math skills during elementary school, we would expect one or more growth classes to display lower initial math performance (i.e., intercept) and a less steep positive slope compared to the other identified classes and/or their typical peers (i.e., those without MD). Conversely, if students with MD display compensatory growth, we would expect the identified growth classes to possess similar intercepts, but for one or more classes to display a steeper positive slope relative to each other and/or their typical peers.

LCGA has also helped identify and uncover subgroups of students with MD and/or disabilities. For example, Geary et al. (2009) revealed four classes of math achievement growth (i.e., high achieving, typically achieving, low achieving, and math learning disabilities) from kindergarten through third grade using LCGA. LCGA in this study not only uncovered a group of students with MD but also demonstrated the presence of two empirically distinct patterns of growth among students with low math performance (i.e., low achieving and math learning disabilities). Thus, use of LCGA might help uncover if certain students with early MD are at risk of later developing a math LD. In addition, Wong et al. (2014) identified the presence of distinct growth of a low-performing class and separate class of students with math learning disabilities among a sample of young Chinese children. Multinomial logistic regression (MLR) analyses demonstrated the two groups of low-performing students differed in number-specific and cognitive processes, further highlighting important differences among students who perform on the lower end of the math continuum and how student characteristics can influence math growth (Wong et al., 2014). These findings indicate that struggling students may require different types of supports and focus on bolstering different underlying skills to help promote their future math achievement. Similar patterns of growth have also been identified in students' math achievement using LCGA. For instance, Salaschek et al. (2014) identified the presence of overall high and low math achievement classes in addition to a "catch up" class characterized by students who start out low but make growth over time ending with high levels of achievement. However, the authors assessed students' math growth using a researcher-designed assessment of math achievement and only examined students' growth over the course of first grade. Thus, it is unclear if similar individual variability and the presence of this "catch up" group remains when using standardized assessments and examining math achievement growth for students with early MD over the course of several years.

The Current Study

A considerable number of students experience MD in the United States, with instances often first identified in early elementary school. However, both the persistence of MD (i.e., continued low levels of math performance) and factors that impact the mathematics growth among students who display early math difficulties is unclear. In addition, schools play a large role in facilitating the academic success of struggling students and those with MD through the early identification of learning difficulties and providing the necessary academic supports. Yet, the adequate detection and provision of services by schools is uncertain, as is the role it and other background characteristics play in the initial and future success for students with MD.

The present study thus seeks to address the following research questions:

> *Research Question 1 (RQ1):* What types of growth trajectories do students with early MD demonstrate in their math achievement from kindergarten through fourth grade?
>
> *Research Question 2 (RQ2):* What role do student characteristics (i.e., academic, cognitive, and demographic factors) and schools' initial detection of math difficulties and provision of academic supports play in differentially predicting these growth trajectories?
>
> *Research Question 3 (RQ3):* Are there differences in students' outcomes in relation to their academic skills and school supports in fifth grade based on these empirically derived growth trajectories, and to what extent do schools continue to adequately detect and provide academic supports for students with MD at the end of elementary school (i.e., fifth grade), especially among those in greatest need?

To address RQ1, we examine individual variability in math achievement among students who display early MD from kindergarten through fourth grade using data from the Early Childhood Longitudinal Study - Kindergarten Class of 2010–2011 (ECLS-K: 2011). As such, we use a person-centered approach of LCGA that allows us to examine patterns of growth within and between students over the course of five time points between kindergarten and fourth grade. Given that other studies have identified heterogeneous patterns of growth in specific math skills for young struggling students and those with MD (e.g., Jordan et al., 2006), we anticipate several distinct patterns of math achievement growth will also exist for students who display early MD.

To address RQ2, we assess the utility of background characteristics and schools' detection of learning difficulties and whether provision of academic supports in kindergarten differentially predict patterns of math achievement growth for students with early MD using MLR. To do so, we examine several demographic variables (i.e., race/ethnicity, income status), reading achievement, cognitive variables (i.e., working memory and cognitive flexibility), teacher's ratings of students' math skills, and academic supports (i.e., IEP, math instructional support) in predicting a student's likelihood of being a member of a certain growth class.

Examination of these factors is critical given that cognitive skills are shown to differentially predict MD (see Kroesbergen et al., 2023 for a review), while demographic variables such as race, and income status are shown to predict differences in students' current and future math performance (Sonnenschein & Galindo, 2015). However, due to the conflicting findings in the over- versus underrepresentation of URM students and low-income students in special education and related provision of academic supports, we do not have specific hypotheses about URM or low-income status as predictors of math achievement growth class membership. Conversely, given that students with comorbid MD and RD display lower rates of math growth than those with math difficulties only (Jordon et al., 2002), we anticipate that among students with early MD, those with lower reading achievement in kindergarten would display lower future growth. Additionally, difficulties in executive functioning serve as a characteristic trait and risk-factor for MD. Thus, we hypothesize students with lower working memory and cognitive flexibility in kindergarten would also be more likely to display lower math achievement growth. Lastly, based on disparate findings of the accuracy of teachers' rating of students' academic skills in relation to their actual academic achievement, we do not have specific hypotheses about teacher's rating of students' math skills in kindergarten as a predictor of growth class membership. However, we anticipate that students with IEPs and those who receive math instructional support in kindergarten will display higher levels of growth since early intervention is shown to reduce students' experience of persistent academic difficulties (e.g., Perez-Johnson & Maynard, 2007).

To address RQ3, we examine whether patterns of math achievement growth differentially predict several student outcomes in the spring of fifth grade, including teacher's ratings of students' math skills, and academic supports (i.e., IEP, math instructional support). Again, we did not have specific hypotheses about the relation between math achievement growth and teacher's ratings of students' math skills given discrepancies in the literature. However, based on previous findings on rates of math achievement growth for students with versus without additional RD, we anticipated that students who demonstrate higher levels of math achievement growth will demonstrate higher levels of reading achievement in fifth grade. Conversely, we anticipate that lower levels of growth between kindergarten and fourth grade will be associated with a greater probability of having an IEP and math instructional support in fifth grade, as the continued need for academic supports along with low growth, may indicate students with more overarching academic difficulties.

Method

Participants

Participants were recruited for the ECLS-K: 2011, a nationally representative sample of 18,174 kindergarteners drawn from 968 schools. Participants were recruited from 3,141 counties in the United States based on the 2007 Census Bureau population estimates. A full description of the sampling procedure can be found elsewhere (i.e., Tourangeau et al., 2015). We restricted the analytic sample to students who met the criteria for MD in the spring of kindergarten, which was the first time point of our study. Students with MD are defined as those who scored at or below the 25th percentile based on the distribution of item response theory (IRT) scale scores on the ECLS-K Mathematics Test. We selected the 25th percentile as this is a common cut-point used in the literature to identify students with more considerable MD (Lewis & Fisher, 2016). We opt to use the term MD as opposed to math learning disabilities, as our selection criteria only reflects low math achievement and cannot replace exhaustive psychoeducational or clinical evaluations to diagnose an LD. Among the 18,174 participants, 4,287 met the criteria for MD (i.e., at or below the 25th percentile) with a math IRT scale score ≤ 40.32 in the spring of kindergarten. Summary demographics for the overall sample and MD subsample are displayed in Tables 1 and 2, respectively.

Procedure and Design

Data were collected for the ECLS-K study through direct child assessment and parent interviews in the fall/spring of kindergarten and spring of first through fifth grade. In the present study, we used the data collected on students' math achievement in the spring

Table 1
Descriptive Demographics for Overall ECLS-K Kindergarten Sample (N = 18,174)

Characteristic	% or *M* (*SD*)
Sex	
Male	51.22
Female	48.78
Race	
White	47.30
Black	12.68
Hispanic	25.49
Asian	8.31
Native Hawaiian/Pacific Islander	0.57
American Indian/Alaska Native	0.83
Two or more races	4.83
URM	44.67
Low income (participation in free and reduced lunch spring of kindergarten)	39.67
IEP (spring of kindergarten)	8.78
M_{age} (months)	73.44 (4.47)

Note. All reported values are for data collected at the start of the study in the fall of kindergarten unless listed otherwise. ECLS-K = Early Childhood Longitudinal Study Kindergarten data set; URM = underrepresented minorities; IEP = Individualized Education Program.

of kindergarten (T1), first grade (T2), second grade (T3), third grade (T4), and fourth grade (T5) to allow for consistent time intervals between assessment points in the main LCGA model. Data collected from the fall and/or spring of kindergarten served as predictors of latent growth class membership, while those collected in the spring of fifth grade were used for estimating distal outcomes. Participants completed assessments for reading, math, and executive function in a standardized order in a quiet school setting. In the fall of the first and second grade, only a subset of students who were representative of the full sample completed assessments (Grade K: $n =$ 18,174, Grade 1: $n = 15{,}620$; Grade 2: $n = 14{,}412$; Grade 3: $n =$ 13,519, Grade 4: $n = 12{,}080$, Grade 5: $n = 11{,}426$). Selection procedures are described elsewhere (Tourangeau et al., 2018).

Table 2
Descriptive Demographics for ECLS-K MD Subsample (N = 4,287)

Characteristic	% or *M* (*SD*)
Sex	
Male	52.63
Female	47.37
Race	
White	32.62
Black	20.61
Hispanic	36.97
Asian	4.35
Native Hawaiian/Pacific Islander	0.75
American Indian/Alaska Native	1.29
Two or more races	3.41
URM	63.03
Low income (participation in free and reduced lunch spring of kindergarten)	57.52
IEP (spring of kindergarten)	17.01
M_{age} (months)	72.42 (4.71)

Note. All reported values are for data collected at the start of the study in the fall of kindergarten unless listed otherwise. ECLS-K = Early Childhood Longitudinal Study Kindergarten data set; MD = math learning difficulties; URM = underrepresented minority; IEP = Individualized Education Program.

Dependent Measure

Math Achievement

Students' math achievement was estimated using the ECLS-K Mathematics Test, a standardized assessment that contained items designed to measure children's conceptual understanding, procedural knowledge, and problem-solving skills. Items assessed children's skills in six general domains: number sense; measurement; geometry and spatial sense; data analysis, statistics, and probability; and patterns, algebra, and functions. The assessment was administered on an easel and questions were read to reduce the likelihood that math assessment was dependent on reading skills. Participants completed a set of routing items at each assessment, which routed them to a second block of items of low, medium, or high difficulty. Reliability and validity have been reported elsewhere (Tourangeau et al., 2015). We used IRT scale scores, which are ideal for assessing growth over time (Tourangeau et al., 2013), with scores ranging from 0 to 148 for each wave of data collection. These scores represent the estimated number of questions the child would have correctly answered if they had been administered all possible questions on the assessment at a given time point (Tourangeau et al., 2015). Reliability of IRT scale scores was considered high at each time point and ranged from .92 to .94.

Predictors

We used three performance measures (i.e., reading achievement, working memory, and cognitive flexibility), two measures of academic support (i.e., IEP status and math instructional support), one measure of academic skill ratings (i.e., ratings of students' math skills) and two demographic measures (i.e., URM and low-income status) collected in kindergarten as predictors of growth class membership.

Low-Income Status

The measure for income status was created by the ECLS-K team from parent interview data using five items: Parent 1's highest level of education, Parent 2's or highest level of education, Parent 1's occupational prestige, Parent 2's occupational prestige, and household income. The ECLS-K team coded Parent 1 as being the mother in a two parent mother–father household, the father in a single-parent household, or the female partner that lived with the father. Parent 2 was coded as the father in a two parent mother–father household, or the father of the household that also resided with a female partner. In our analyses, low-income status was determined based on whether these reports indicated if the child qualified for free or reduced lunch during the spring of kindergarten.

URM Status

Based on reports from parent interviews in the fall of kindergarten, we derived a binary categorical variable to indicate whether children's parents identified them as either Black, Hispanic/Latine, Native Hawaiian/Pacific Islander, American Indian/Alaska Native, or two or more races (1), which are underrepresented groups within mathematics, as compared to White or Asian (0), who are not underrepresented within mathematics.

Reading Achievement

Students' reading achievement was estimated using IRT scale scores from the ECLS-K Reading Test in the spring of kindergarten.

The test was a standardized reading assessment that measured children's basic skills (e.g., print familiarity, letter recognition) and sight vocabulary, decoding, vocabulary, and passage comprehension using a variety of literary genres. Test items were designed to tap a broad range of skills that are typically taught, important skills for that grade, and consistent with national and state standards. A Spanish version of the assessment was administered to children who did not pass the screener test in English. IRT modeling was used to create scale scores that could be compared across time, with scores ranging from 0 to 83 (Tourangeau et al., 2013).

Executive Function

Executive function was assessed using two assessments: The dimensional change card sort (DCCS; Zelazo, 2006) and Numbers Reversed (Mather & Schrank, 2001) that measured cognitive flexibility and working memory, respectively. Though the field acknowledges a third component of executive function, inhibitory control (Miyake et al., 2000), the ECLS-K data set does not currently have an assessment for inhibitory control. Furthermore, it is difficult to disentangle some cognitive flexibility tasks from inhibitory control, and thus, it is likely that some inhibitory control is required. We detail each of these assessments below.

Cognitive Flexibility (DCCS). Children's cognitive flexibility was assessed using the DCCS (Zelazo, 2006) task in the spring of kindergarten. The task instructs participants to sort cards into different piles based on a changing set of rules. Participants were first instructed to sort cards based on color (i.e., red or blue), then by shape (i.e., rabbit or boat), and finally to sort cards with a black border by color and without a black border by shape. Participants moved on to the last rule if they were correct for four of six items during the shape rule. Scores were based on the total number correct out of 18 trials. Previous research shows that the DCCS has good test–retest reliability (intraclass correlation = .92).

Working Memory (Numbers Reverse). Working memory was assessed in the spring kindergarten using the Numbers Reversed task from the Woodcock–Johnson III (WJ-III) Test of Achievement (Mather & Schrank, 2001; $\alpha = .90$). Participants were instructed to verbally repeat an increasingly long string of orally presented numbers in reverse order, such that if the child was told the numbers "1 … 5" a correct response would be "5 … 1." Administration stopped when the child got three consecutive sequences incorrect or completed all sequences. We used W-ability scores (W scores) computed based on standardized scores from the WJ-III for our analyses. W scores reflect a child's ability as well as related item difficulty and provide an equal-interval scale that is considered ideal for longitudinal and regression analyses (Tourangeau et al., 2015).

IEP Status

A student's IEP status was determined based on their school records. This information was collected each year by the ECLS-K team at the time of preassessment by calling the school's coordinator and asking if the student had an IEP or similar program plan listed on file. We recoded this information for IEP status into a binary categorical variable to indicate the presence (1 = *yes*) or absence (0 = *no*) of an IEP or similar program on file for each student. It should be noted that publicly available data on this variable does not indicate the specific reason (i.e., the specific type of disability or special education classification for which child is receiving an IEP). Instead, we use this variable as a general proxy of a child's provision of special education services by their school.

Teacher Ratings of Students' Math Skills

Teacher academic ratings of students' math skills were used as a proxy of a school's ability to detect children with MD in the spring of kindergarten. Ratings were collected as part of the ECLS-K child-level teacher questionnaire completed by the student's general classroom teacher. The questionnaire was self-administered and designed to collect individual information pertaining to the child's performance and experiences in the classroom, which included their academic abilities. Teachers rated each student's math skills as being "far below average," "below average," "average," "above average," or "far above average" with respect to grade level. These rating categories differed from those used in later waves of data collection, where students' skills. To allow for easier comparisons across time, we collapsed the five rating categories into three binary categorical variables to indicate the presence (1 = *yes*) or absence (0 = *no*) of a given math skills rating. Ratings of "far below average" and "below average" were recoded "below grade level"; ratings for "average" were recoded as "at grade level" (1 = *yes*, 0 = *no*), and ratings for "above average" and "far above average" were recoded as "above grade level."

Math Instructional Support

In the current analyses, we derived the variable for receiving math instructional support using two categorical variables for math instruction from the existing ECLS-K data set: student received individual tutoring in math and/or pull-out small group in math in kindergarten. These two types of instruction were not mutually exclusive, and many students received both forms of additional instruction. Data for both variables were collected as part of the child-level teacher questionnaire completed by the student's general classroom teacher in the spring of kindergarten. This information was used to create a single binary variable to indicate the presence (1 = *yes*) or absence (0 = *no*) of individual math tutoring and/or pull-out small group math.

Distal Outcome Measures

We used two measures of academic support (i.e., IEP status and extra math support), and one measure of academic skill ratings (i.e., ratings of student's math skills), collected in the spring of fifth-grade variables to explore potentially differential outcomes based on growth class membership. Measures for IEP status and reading achievement were the same as those previously described for the predictor measures. Variation in the outcome variables for extra math support and ratings of student's math skills is further described below.

Ratings of Students' Math Skills

Math skill ratings were collected as part of the self-administered ECLS-K child-level teacher questionnaire completed by the student's math teacher in the spring of fifth grade. Teachers rated each student's math skills as being "below grade level," "about on grade level," and "above grade level." This information was represented as three separate binary categorical variables indicating the presence (1 = *yes*) or absence (0 = *no*) for each rating category.

We recoded the categorical variable for "about on grade level" as being "at grade level" to align with the categories used for the predictor measure of teachers ratings of students' math skills in the spring of kindergarten.

Math Instructional Support

Data for math instructional support were collected as part of the self-administered child-level teacher questionnaire completed by the student's math teacher. We derived this measure similarly to that described for corresponding predictor measure, with the exception that the outcome measure only consisted of a single binary variable to indicate the presence ($1 = yes$) or absence ($0 = no$) of individual math tutoring. That is, data for pull-out small group math instruction were not provided in the spring of fifth grade.

Data Analytic Plan

LCGA

The current study seeks to determine the subgroups or patterns of growth students with MD display in their math achievement during kindergarten through the fourth grade. To address RQ1, we use a person-centered approach of LCGA based on students' performance on the ECLS-K Mathematics Test over time. LCGA, like other types of growth mixture models (GMMs), permits the examination of longitudinal patterns of change in a dependent measure using estimated growth parameters (e.g., B. O. Muthén, 2002), which are subsequently used to describe latent class variables or subpopulations of growth (e.g., Kaplan, 2002). However, unlike traditional GMMs, LCGA models fix the factor loads from the growth parameters to zero, as this model assumes no individual variation around the mean for the growth estimate of each class (e.g., B. O. Muthén, 2004; Jung & Wickrama, 2008). Thus, LCGA serves to identify underlying heterogeneity but assumes uniform growth among participants in a given latent class.

In addition, LCGA models can be specified with varying functions to fit different shapes or patterns of growth that reflect the specific function requested. Researchers typically start by fitting a linear growth function and progress to using more complex forms of polynomial growth (e.g., quadratic, cubic, quartic) based on the observed shape of the data and significance of the growth terms as higher order growth parameters are added (Ram & Grimm, 2007). However, it is possible that the pattern of growth does not follow a specific uniform function across time. The use of a partially constrained slopes model is beneficial in this latter situation, as it allows the growth trajectories to assume the shape that best fits the data (T. Duncan et al., 2006). We assessed both possibilities of growth in the current study by fitting a series of fully constrained linear and partially constrained slope models and then comparing the two best fitting models (i.e., best fitting linear vs. partially constrained slopes).

Model Specification. We used students' IRT scale scores on the ECLS-K Mathematics Test across five time points (i.e., spring of kindergarten [T1], spring of first grade [T2], spring of third grade [T3], and spring of fourth grade [T5]) as indicators of growth class membership in the LCGA model. Time was coded in years with the intercept set at the spring of kindergarten (i.e., T1). We used the standard approach to modeling LCGA, which is to define and estimate a two-class solution and then successively increase the number of classes in the model by one until a well-fitting and theoretically meaningful set of growth classes emerges.

In addition, we considered a number of fit indices simultaneously to determine the optimal number of latent growth classes in the LCGA model, including the Akaike information criterion (AIC), Bayesian information criterion (BIC), and Vuong–Lo–Mendell–Rubin likelihood ratio test (VLMR-LRT), adjusted Lo–Mendell–Rubin likelihood ratio test (LMR-LRT; Lo et al., 2001), and bootstrapped likelihood ratio test (BLRT). Lower AIC and BIC values signify better model fit and are particularly useful when comparing nonnested models (Geiser, 2013). The VLMR-LRT and LMR-LRT indices compare nested models (i.e., $k-1$ vs. k classes), where a significant p value indicates inclusion of an additional class (i.e., k classes) provides improved model fit (Nylund et al., 2007). The BLRT similarly evaluates nested models, in which a significant p-value signifies better model fit for the more complex model as compared to the simpler model. The entropy index, which represents the certainty in the classification of all individuals across the latent growth classes and degree of separation between identified growth classes, was also considered. Entropy values range from 0 to 1 with those greater than 0.80 representing good class separation and classification accuracy (Geiser, 2013). Additionally, the number of participants displaying a most likely latent growth class, for each given class within the model, should be large enough in size to be considered substantially meaningful. Although there is no set standard, a common rule of thumb is to ensure that a solution is found in which no growth class is less than 1%–5% of the total sample (e.g., Hill et al., 2000; Jung & Wickrama, 2008). We selected the more conservative 5% cutoff to reduce potential issues with model convergence (Nylund et al., 2007). Moreover, within each profile, participants should have a high probability of displaying one profile and low probability of displaying all others. Finally, the conceptual interpretability of the solution, based on the research questions and theoretical context, was jointly considered with the above indices to determine the ideal number of latent growth classes (Jung & Wickrama, 2008).

MLR

Next, we examine the predictive utility of student background factors (i.e., demographic-, academic-, and cognitive variables), school-based academic supports, and math academic ratings in the spring of kindergarten in predicting the likelihood of displaying a particular growth trajectory (i.e., latent growth class) among children who meet early criteria for MD. More specifically, we assess whether qualifying for free or reduced lunch (a proxy of low-SES), being an underrepresented minorities (URM) student (i.e., all those who were not White or Asian), reading achievement, working memory, and cognitive flexibility predict the likelihood of displaying one type of growth class over another. We also examine whether a student's provision of additional academic supports by their school in the form of an IEP, and math instructional support, as well as ratings of student's math in the spring of kindergarten are predictive of their growth class membership. Ratings of students' math skills were represented as two binary variables, "above grade level" and "at grade level," with ratings of "below grade level" serving as the referent group. To address this second aim, we add predictors into our LCGA model by regressing each latent growth class onto the set of predictors using MLR. We specifically apply the simultaneous three-step approach (R3STEP) using the R3STEP command in Mplus to prevent predictors from steering the development of the growth classes (e.g., Asparouhov & Muthén, 2014).

DCAT

To address the third research aim of determining whether student outcomes in fifth grade differ based on growth class membership, and the extent to which schools continue to adequately detect and provide academic supports to students with MD at the end of elementary school, we use the DCAT method. The DCAT method (Lanza et al., 2013) is recommended for the investigation of binary distal outcomes, including categorical variables (Asparouhov & Muthén, 2014), and uses the Wald test to examine significant mean differences between growth classes while simultaneously retaining consistent class membership (Lanza et al., 2013).

Using DCAT, we first examine whether students who demonstrate varying growth class memberships significantly differ on four categorical variables of interest, including ratings of their math skill ratings (i.e., above grade level or at grade level), whether certain classes are more likely to have an IEP, and/or whether certain classes are more likely to receive math instructional support in the spring of fifth grade.

To address the second part of our third aim, we use descriptive statistics by growth class and chi-square analyses. More specifically, we assess schools' ability to adequately detect and provide supports to students with MD by comparing teacher ratings of students' math skills and the proportion of students receiving academic supports (i.e., IEP and math instructional support) across classes. We consider to what extent these ratings and supports align with students' actual level of math achievement in the spring of fifth grade.

Missing Data

LCGA in Mplus uses full-information maximum likelihood (FIML) estimation. The FIML estimation method uses all information available in the data set to estimate parameters without requiring imputation or deleting participants with missing values. Based on these advantages, FIML is the suggested method for handling missing data in latent variable modeling when data are missing completely at random (MCAR) or missing at random (MAR) (Acock, 2012; Cham et al., 2017). Among the data set, 34.2% of participants were missing data on the ECLS-K Mathematics Test at one or more time points. Data were not MCAR based on Little's (1988) Test, $\chi^2(40) = 82.30$, $p < .001$; however, additional patterns of missingness indicate that they were MAR. That is, the probability of missing data for the latent variable indicators was related to other observed variables (i.e., math skill ratings, IEP status, math instructional support, URM status, and reading achievement) rather than unobserved variables in the data set, thus we used FIML as our estimation method when defining our LCGA model.

In contrast, MPlus uses listwise deletion when predictors are added into a mixture model. Listwise deletion or complete-case analysis (CCA) is only appropriate when data are MCAR. CCA is problematic outside of MCAR in that it substantially reduces sample size, thus reducing power and it can also introduce bias in estimates as the reduced sample is no longer as representative of the original target population. Thus, to retain as much information as possible, multiple imputation was used for examining predictors in the LCGA model for Aim 2 (Rubin, 1987; Schafer, 1997). We chose not to impute demographic characteristics (e.g., race, SES) due to ethical considerations. That is, due to systematic marginalization of URM students and those from low-income homes in the United States that leads to social stratification and widening opportunity gaps, imputing demographic characteristics such as race and SES may risk the replication and amplification of any performance differences in our sample due to opportunity gaps. We therefore used the percentage of missing data and analysis of patterns of missingness on the remaining predictors to guide the number of necessary imputed data sets (Woods et al., 2021). Overall, 39.05% of the sample were missing data on one or more predictors (i.e., IEP status, math instructional support, math skill ratings (above-, at-, or below grade level), and reading achievement). Given that we are using a secondary data set, the reasons for missingness are unknown. Separate MCAR tests were completed for continuous and categorical variables. Categorical data were MCAR based on Little's (1988) test, $\chi^2(19) = 28.00$, $p = .084$. Continuous predictors were not MCAR—Little's MCAR test: $\chi^2(11) = 21.70$, $p = .027$; however, additional patterns of missingness indicate that the data were MAR. That is, the probability of missing data for the predictors was related to other observed study variables (i.e., IEP status, URM status, and reading achievement). For MCAR or MAR data, Graham et al. (2007) suggest using 100 imputed data sets for a missingness rate of 30%–50% to obtain statistical power of at least .78. Thus, 100 data sets were imputed using Markov chain Monte Carlo simulation and results from the pooled data sets are presented in the LCGA model with predictors.

Mplus similarly uses complete case analysis for the handling of missing data when examining distal outcomes. Overall, 72.20% of the sample was missing data on one or more distal outcomes. Analysis of the missingness revealed that outcome data were MCAR—Little's MCAR test: $\chi^2(9) = 4.77$, $p = .854$. We suspect that the missingness for this data set, and the other missing data analyses, partially reflects the fact that only a representative subset of children from the initial student sample were selected to complete cognitive and academic assessments in each subsequent wave of data collection (Tourangeau et al., 2018). Nevertheless, due to the large percentage of missing data, we opted not to impute outcomes and instead used complete case analysis (CCA) for the estimation of distal outcomes. We consider the limitations of this analysis decision in the discussion.

Transparency and Openness

Based on JARS guidelines (Kazak, 2018), we thoroughly describe our methods for recruiting participants, how we derived our analytic sample, and how we handled any missing data or exclusion of data (if any). Although children from all 50 states were recruited in both studies, neither sample should be considered nationally representative per se because of intentional oversampling of low incidence groups to obtain reliable estimates (e.g., American Indians; private school students). However, both the study sample and the MD subsample fall well above the minimum recommended sample size of 300 participants necessary for the proper function and power (i.e., $\geq .80$; Cohen, 1988) of GMMs (e.g., Kim, 2012). In addition, we provide detailed information regarding participant demographics, manipulation of data, descriptions of all predictors, distal outcomes, dependent measures, as well as their associated reliability. Descriptive and chi-square analyses were performed using SPSS Statistics (Version 29.0; IBM Corp, 2023), missing data analyses (i.e., MCAR tests) were performed using the naniar R package (Tierney & Cook, 2023), all other analyses were performed using the Mplus 8.3 software program (L. K. Muthén & Muthén, 1998–2017). Analysis scripts will be uploaded to Open Science Framework upon publication. This study was not preregistered.

Results

Descriptive statistics and rates of missingness for all predictors, latent indicators, and outcome variables are provided in Table S1 in the online supplemental materials. Correlations among all study variables are presented in Table S2 in the online supplemental materials. For regression analyses, all continuous predictors were transformed to standardized z-scores to allow for comparison across variables and ease of interpretation. All descriptive statistics and correlations are based on the MD subsample unless noted otherwise.

Latent Growth Class Analysis: Identification of MD Growth Trajectories

To investigate the types of growth trajectories students with MD exhibit in kindergarten through fourth grade, we iteratively defined a series of two- to six-class LCGA models. We fit a series of both linear constrained and partially constrained slope models and determined the best fit was the three-class model with partially constrained slopes. Cubic models were also examined (see the online supplemental materials for fit indices); however, based on the shape of the data, poor entropy values (i.e., <.70), and a nonsignificant quadratic growth term for several classes in the various models, we did not explore the cubic or other higher order growth models any further.

For the partially constrained LCGA models, we defined a series of two to six class solutions by fixing the first (T1) and second (T2) time points, to allow for model identification, and allowing the slope loading for the remaining time points (T3–T5) to be freely estimated (T. Duncan et al., 2006; Scammacca et al., 2020). A summary of the fit indices for the various partially constrained slope models is presented in Table 3. Several of the partially constrained models demonstrated better fit and interpretability than all of the fully constrained linear models. More specifically, for the fully constrained linear slopes models, the three- to six-class solutions possessed low entropy (i.e., <.80) and all six models possessed higher AIC/BIC values than the best fitting than the three-class model with partially constrained slopes, thus indicating overall poorer data fit.

Among the various partially constrained models, the two-class solution demonstrated good class separation (i.e., entropy = .87) and significant LRT values (i.e., BLRT, LMR-LRT, VLMR-LRT), indicating improved model fit with the addition of a second class relative to the single class baseline model. The classes for the two-class model were also substantial in size (i.e., Class 1: 12.29%, Class 2: 87.71%). The three-class solution similarly demonstrated high entropy (i.e., .84), lower AIC/BIC values, and significant BLRT, VLMR-LRT, and LMR-LRT values signifying improved fit with the addition of a third class relative to the two-class solution. It also demonstrated good classification membership probabilities (Table 4) and classes that were substantial in size (i.e., Class 1: 7.16%; Class 2: 24.03%; Class 3: 68.81%). These classes were characterized by students who displayed mild, moderate, and extensive

Table 3
ECLS-K Math Achievement LCGA With Partially Constrained Slopes: Model Fit Statistics

Model and class	Count (n)	Prop.	Entropy	AIC/BIC	BLRT (p)	LMR-LRT (p)	VLMR-LRT (p)
Two classes							
1	527	.12	.87	117,678.32/ 117,780.13	<.001	<.001	<.001
2	3,760	.88					
Three classes							
1	307	.07	.84	117,234.74/ 117,355.64	<.001	<.001	<.001
2	2,950	.24					
3	1,030	.69					
Four classes							
1	1,146	.27	.81	117,112.44/ 117,252.43	<.001	.002	.002
2	2,442	.57					
3	459	.11					
4	240	.06					
Five classes							
1	221	.05	.81	117,009.24/ 117,168.32	<.001	.002	.002
2	2,055	.48					
3	675	.16					
4	194	.05					
5	1,142	.27					
Six classes							
1	1,939	.45	.84	116,903.20/ 117,081.37	<.001	<.001	<.001
2	691	.16					
3	259	.06					
4	1,172	.27					
5	202	.05					
6	24	.01					

Note. ECLS-K = Early Childhood Longitudinal Study Kindergarten data set; LCGA = latent class growth analysis; Prop. = proportion; AIC = Akaike information criterion; BIC = Bayesian information criterion; BLRT = bootstrapped likelihood ratio test; LMR-LRT = Lo–Mendell–Rubin likelihood ratio test; VLMR-LRT = Vuong–Lo–Mendell–Rubin likelihood ratio test.

Table 4
Classification Membership Probabilities by Latent Growth Class

Growth class/MD categorization	1	2	3
1. C1: extensive MD	.92	.08	<.001
2. C2: moderate MD	.02	.83	.15
3. C3: mild MD	<.001	.04	.96

Note. C1 = Class 1; C2 = Class 2; C3 = Class 3; MD = math learning difficulties.

levels of MD based on their patterns of growth (see below for further description).

The four-, five-, and six-class models all displayed high entropy (i.e., >.80) and significant LRT-values. These models also displayed continuously decreasing AIC/BIC values but resulted in additional classes that were similar in growth patterns to classes in the three-profile solution (see online supplemental materials for further description). For example, the four-class model seems to split the class of students with moderate levels of MD in the three-class solution into two subclasses (i.e., one slightly higher and one slightly lower) of moderate levels of MD (see Figure S1 in the online supplemental materials). In addition, the six-class solution contained a class that was smaller in size than the 5% minimum guidelines suggested by Hill et al. (2000). Thus, based on this combination of fit indices and theoretical interpretability, we retained a three-profile solution.

Characterization of Patterns of Growth in Math Achievement

Normative and Average Math Development

To better understand and characterize the three empirically distinct growth trajectories in the LCGA model, we first examined normative development in students' math achievement by plotting the mean performance over time (i.e., T1–T5) of students without MD (i.e., those above the 25th percentile) from the original ECLS-K sample. As displayed in the left panel of Figure 1, non-MD students displayed an average IRT scale score of 55.32 (67th percentile) on the ECLS-K Mathematics Test in the spring of kindergarten (T1). These students progressed over time, making steady positive growth (slope [*SE*] = 22.08 [0.09]; $p < .001$), ending with an average IRT scale score of 117.71 in the spring of fourth grade (T5) which corresponds to the 54th percentile on the ECLS-K Mathematics Test. Thus, the normative samples' overall IRT scores went up but their relative performance (i.e., compared to all other students in the sample) in terms of percentile somewhat decreased. We similarly examined average development of students' math achievement by plotting mean performance over time of all students in the ECLS-K sample. The overall student sample displayed an average math IRT scale score of 49.78 (i.e., 50th percentile) in the spring of kindergarten. Students progressed over time, showing similar growth magnitude (slope [*SE*] = 22.15 [0.07], $p < .001$) as those in the normative sample (see right panel in Figure 1). However, the average student sample ended with slightly lower math performance in the spring of fourth grade, with an average math IRT scale score of 111.48, which corresponds to the 42nd percentile. Thus, both students without MD and the average student sample display significant positive growth in their math skills during elementary school, ending with levels of performance that fall slightly above and below the 50th percentile, respectively, in the spring of fourth grade. Descriptive statistics for the performance of non-MD and the overall sample on the ECLS-K Mathematics Test from T1-T5 are displayed in Table 5.

Patterns of development in math achievement among the empirically distinct growth classes in the LCGA model with partially constrained slopes are displayed in the right panel of Figure 1 (see Table 6 for corresponding model growth estimates). Descriptive statistics for mean levels of performance as well as differences in performance relative to the normative and average student samples are displayed in Table 5.

Class 1: Extensive MD

We named Class 1 extensive MD ($n \sim 307$; 7.16%), which was characterized by scores falling well below the MD cutoff criteria (i.e., IRT scale score ≤ 40.312) in the spring of kindergarten ($M = 19.29$; < first percentile). These students made significant growth (slope = 19.13 [0.42], $p < .001$) ending with an average IRT scale score of 69.78 (first percentile). Despite their improvement, students in Class 1 displayed very low levels of math performance across time that fell far below the 25th percentile cutoff criteria and were also substantially lower than those demonstrated by the normative and average student samples, thus indicating substantial math difficulties. This was the smallest of the three classes.

Class 2: Moderate MD

We named Class 2 moderate MD ($n \sim 1{,}030$; 24.03%) as it was characterized by students displaying scores that fell below the MD range across all five time points, although not as low as those displayed by students in Class 1. Despite low levels of performance, students in Class 2 also made steady gains (slope = 22.15 [0.25], $p < .001$) between the spring of kindergarten (Math IRT Scale Score: $M = 29.03$; fifth percentile) and fourth grade (Math IRT Scale Score: $M = 88.12$; 11th percentile).

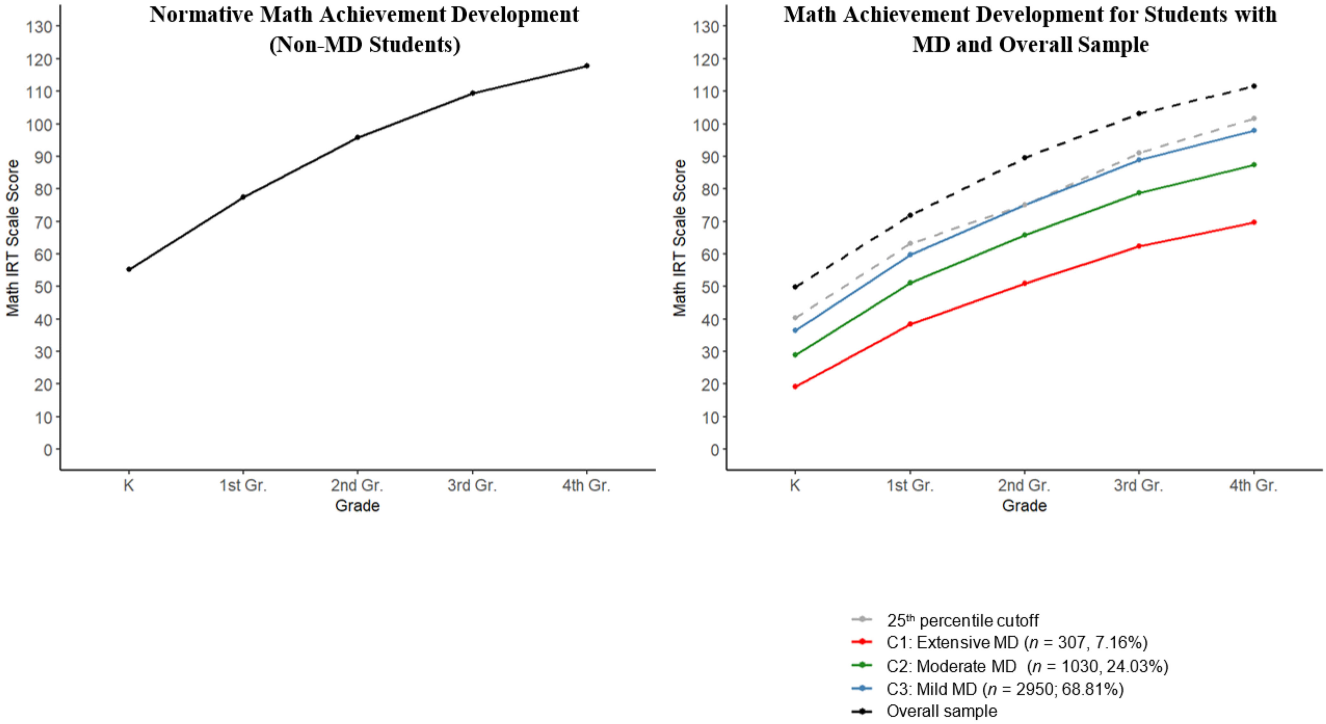
Class 3: Mild MD

We named Class 3 mild MD ($n \sim 2{,}950$; 68.81%), as it was characterized by students displaying scores that fell below the MD range in the spring of kindergarten (Math IRT Scale Score: $M = 36.52$; 16th percentile). These students also made significant growth over time but continued to fall slightly below the MD cutoff criteria, ending with an average math IRT scale score of 97.99 (i.e., 20th percentile). These levels of performance from the spring of kindergarten to fourth grade were consistently higher than those displayed by Classes 1 and 2 but still considerably lower than the normative and average student samples, indicating the continued, yet mild, presence of MD. This was the largest of the three classes.

Predictors of Math Achievement Growth Membership

Next, we aimed to determine the predictive utility of student background characteristics and schools' detection and provision of academic supports during kindergarten in differentially predicting growth class membership for children with MD. We used an MLR to examine the likelihood of students displaying different growth classes based on their values on the range of spring of kindergarten predictors previously described. We simultaneously entered predictors

Figure 1
Latent Class Growth Analysis of Student Math Achievement From Kindergarten Through Fourth Grade



Note. MD = math learning difficulties; IRT = item response theory; Gr = grade; K = kindergarten; C1 = Class 1; C2 = Class 2; C3 = Class 3. See the online article for the color version of this figure.

for reading achievement, cognitive flexibility, working memory, URM status, low-income status, IEP status, ratings of students' math skills above grade level (below grade-level referent), ratings of students' math skills at grade level (below grade-level referent), and math instructional support into the model using R3STEP. Results of the MLR analyses are displayed in Table 8. Additional descriptive statistics on predictors by growth class are presented in Table 7.

Predicting Membership in Class 1: Extensive MD

We examined predictors of displaying extensive MD (Class 1) compared to moderate (Class 2) or mild (Class 3) MD. Students with higher initial cognitive flexibility (Class 2 *OR*: 1.43; Class 3 *OR*: 1.83), working memory (Class 2 *OR*: 2.99; Class 3 *OR*: 6.23), and reading achievement (Class 2 *OR*: 6.66; Class 3 *OR*:

Table 5
ECLS-K Mathematics Test Scores in Kindergarten Through Fourth Grade for Students With Versus Without MD

Growth class/MD categorization	Spring of kindergarten (T1), *M*	Spring of first grade (T2), *M*	Spring of second grade (T3), *M*	Spring of third grade (T4), *M*	Spring of fourth grade (T5), *M*
Class 1: extensive MD	19.29	38.42	51.01	62.32	69.78
Class 2: moderate MD	29.03	50.92	65.52	79.07	88.12
Class 3: mild MD	36.52	59.81	75.13	88.91	97.99
All students (i.e., average performance)	49.78	71.92	89.53	103.14	111.48
Non-MD students	55.32	77.40	95.83	109.47	117.71

	Spring of kindergarten (T1)		Spring of first grade (T2)		Spring of second grade (T3)		Spring of third grade (T4)		Spring of fourth grade (T5)	
Growth class/MD categorization	Non-MD	All students	Non-MD	All students	Non-MD	All students	Non-MD	All students	Non-MD	All students
Change in mean scores										
Class 1: extensive MD	36.03	30.49	38.98	33.50	44.32	38.52	47.15	40.82	48.93	42.70
Class 2: moderate MD	26.29	20.75	26.48	21	30.31	24.01	30.40	24.07	29.59	23.36
Class 3: mild MD	18.80	13.26	17.59	12.11	20.70	14.40	20.56	14.23	19.72	13.49

Note. Test scores are presented as IRT scale scores. ECLS-K = Early Childhood Longitudinal Study Kindergarten data set; MD = math learning difficulties; IRT = item response theory; T1–T5 = Time 1–Time 5.

Table 6
Growth Estimates: Three Class Partially Constrained Slope LCGA Model

Growth class/sample	Intercept, Est. (*SE*)	Slope, Est. (*SE*)
Class 1: extensive MD	19.29 (0.27)**	19.13 (0.42)**
Class 2: moderate MD	29.02 (0.20)**	22.15 (0.25)**
Class 3: mild MD	36.52 (0.08)**	23.29 (0.16)**
Normative student sample	55.32 (0.09)**	22.08 (0.09)**
Average student sample	49.78 (0.10)**	22.15 (0.07)**

Note. LCGA = latent class growth analysis; Est. = estimate; MD = math learning difficulties.
** $p < .001$.

22.95) were more likely to be in Class 2 (moderate MD) or Class 3 (mild MD) as compared to Class 1 (extensive MD). Furthermore, teachers were more likely to rate students in Class 2 (*OR*: 3.40) and Class 3 (*OR*: 7.79) at grade level than Class 1. Teachers were more likely to rate Class 3 students with math skills above grade level (*OR*: 6.22) compared to Class 1. Students with an IEP in the spring of kindergarten were less likely to display membership in Class 2 and Class 3 (Class 2 *OR*: 0.41; Class 3 *OR*: 0.28) than Class 1.

Predicting Membership in Class 3: Mild MD

We also examined predictors of displaying mild MD (Class 3) compared to moderate MD (Class 2). Students with higher cognitive flexibility (*OR*: 1.28), working memory (*OR*: 2.08), and reading achievement (*OR*: 3.45) in kindergarten were more likely to display mild MD (Class 3) than moderate MD (Class 2). Also, those displaying mild MD were more likely to be rated as above grade level than below grade level (*OR*: 7.96) compared to those who displayed moderate MD in kindergarten. However, it is important to note that students in the mild MD class were not at grade level according to their average percentile score on the ECLS math assessment ($M = 16$th percentile). Conversely, students from low-income backgrounds (*OR*: 1.66) were more likely to display membership in Class 2 than Class 3. See Table 8 for full estimates.

Distal Outcomes

Last, we used the DCAT method to address the third aim which focused on comparing latent growth classes on several fifth-grade outcome measures and proper detection of MD and allocation of academic supports. That is, to what extent are schools aware of the difficulties their students have with mathematics and, relatedly, do schools provide supports needed for those with varying levels of MD. Thus, we examined four binary distal outcomes which included (a) ratings of students' math skills as either above or (b) at grade level (as compared to below grade level) in fifth grade, (c) IEP status in fifth grade, and (d) receiving math instructional support in fifth grade. As previously mentioned, we used listwise deletion to handle missing data on distal outcome measures. This resulted in 1,195 students (27.87%) with available data on ratings of students' math skills (i.e., above or at grade level), 1,192 students (27.84%) with data for math instructional support, and 2,373 (55.35%) with IEP data. Descriptive statistics on distal outcome measures by growth class are presented in Table 9. As demonstrated in Table 10, latent growth classes varied significantly on all four distal outcomes. We describe each of the four outcomes in turn.

Teacher Ratings of Students' Fifth-Grade Math Skills

Ratings of math skills at or above grade level significantly varied across all three classes. First, all three classes had a generally low probability of being rated above grade level in fifth grade (probabilities = <.001–.05). That is, they were all highly likely to be rated as below grade level (probabilities = .97–1.00), which is in alignment with their status as students with MD. However, within these low values, there was enough variability to detect significant differences; Class 3 was most likely (probability = .05; mild MD) Class 2 was second most likely (probability = .03; moderate MD) and Class 1 was least likely (probability < .001; extensive MD) of being rated above grade level in math in fifth grade. The pattern was the same for at grade level with Class 3 being most likely (probability = .48), Class 2 being second most likely (probability = .27) and Class 1 being least likely (probability = .16).

Table 7
Descriptive Statistics for Spring of Kindergarten Predictors by Growth Class

Predictor measure	Class 1 extensive MD, *M* (*SD*) or %	Class 2 moderate MD, *M* (*SD*) or %	Class 3 mild MD, *M* (*SD*) or %
Cognitive flexibility	−0.78 (1.24)	−0.18 (1.09)	0.144 (0.88)
Working memory	−0.75 (0.25)	−0.43 (0.72)	0.23 (1.04)
Reading achievement[a]	−1.51 (.71)	−0.53 (0.78)	0.34 (0.85)
IEP	43.68	23.50	12.17
Math instructional support	44.49	31.94	18.82
Rating of math skills above grade level	1.53	2.68	9.30
Rating of math skills at grade level	11.45	31.76	59.28
Demographics	(%)	(%)	(%)
Low income	66.50	67.04	53.17
URM	67.97	69.94	60.10

Note. Descriptives are based on nonimputed data. MD = math learning difficulties; IEP = Individualized Education Program; URM = underrepresented minority.
[a] Reading achievement scores are represented as standardized z-scores.

Table 8
Multinomial Logistic Regression: Effects of Predictors on Growth Class Membership

Referent: predicting profile membership	C1: extensive MD				C3: mild MD	
	C2: moderate MD		C3: mild MD		C2: moderate MD	
	Logit (*SE*)	*OR* [95% CI]	Logit (*SE*)	*OR* [95% CI]	Logit (*SE*)	*OR* [95% CI]
Predictors						
Cognitive flexibility	0.35 (0.10)**	1.43** [1.18–1.73]	0.60 (0.11)**	1.83** [1.47–2.27]	−0.25 (0.07)**	0.78** [0.68–0.90]
Working memory	1.10 (0.35)**	2.99** [1.51–5.91]	1.83 (0.35)**	6.23** [3.13–12.42]	−0.78 (0.10)**	0.48** [0.38–0.60]
URM	0.45 (0.27)	1.57 [0.93–2.64]	0.15 (0.29)	1.17 [0.67–2.05]	0.30 (0.17)*	1.34* [0.97–1.87]
Low income	0.02 (0.26)	1.02 [0.62–1.69]	−0.49 (0.28)	0.62 [0.36–1.06]	0.51 (0.16)*	1.66** [1.21–2.28]
IEP	−0.90 (0.27)**	0.41** [0.24–0.69]	−1.27 (0.30)**	0.28** [0.16–0.51]	0.37 (0.19)	1.44 [0.99–2.10]
Math instructional support	0.04 (0.26)	1.04 [0.63–1.73]	0.02 (0.29)	1.02 [0.58–1.81]	0.03 (0.18)	1.02 [0.71–1.45]
Math skills rated above grade level	−0.25 (0.75)	0.78 [0.18–3.37]	1.83 (0.65)*	7.79* [3.12–19.46]	−2.07 (0.51)**	0.13** [0.05–0.34]
Math skills rated at grade level	1.22 (0.45)*	3.40* [1.41–8.20]	2.05 (0.47)**	7.79** [3.12–19.46]	−0.83 (0.16)**	0.44** [0.32–0.60]
Reading achievement	1.90 (0.26)**	6.66** [4.04–10.98]	3.13 (0.28)**	22.95** [13.28–39.65]	−1.24 (0.12)**	0.29** [0.23–0.37]

Note. C1 = Class 1; MD = math learning difficulties; C3 = Class 3; C2 = Class 2; CI = confidence interval; URM = underrepresented minority; IEP = Individualized Education Program.
$^{*}p < .05$. $^{**}p \leq .002$.

Instructional Supports in Fifth Grade

All three classes also significantly varied in their probability of having an IEP in fifth grade. Class 1 had the highest probability of having an IEP (probability = .85), followed by Class 2 (probability = .53) and then Class 3 (probability = .22). Patterns were slightly different for the probability of receiving math instructional support with Class 3 (probability = .40) being significantly less likely than Classes 1 (probability = .58) and 2 (probability = .50) to receive supplemental math instructional support. Classes 1 and 2 did not differ from each other. That is, Class 3 showed the lowest probabilities of continued instructional supports and IEPs in fifth grade despite low levels of math achievement (i.e., below the 25th percentile) 1 year prior. This suggests that these students may be more likely to be performing just "under the radar" (i.e., being incorrectly labeled as not needing additional instructional supports). Class 1 (extensive MD) showed the highest probabilities of being detected as below grade level and being offered additional instructional supports (i.e., an IEP and supplemental math instruction).

Supports for Minoritized Students

To further address the third aim of whether schools adequately provide supports to students in need at the end of elementary school, we conducted descriptive statistical analyses to determine whether student IEP status and math instructional support among the three growth classes varied by student demographic factors (Table 11). We were particularly interested in examining differences based on low income and URM status, given that these students are disproportionately represented in special education. Chi-square analyses indicated a significantly smaller proportion of URM students received IEP supports compared to non-URM in all three MD classes: Class 1—extensive MD, $\chi^2(1) = 4.46, p = .041$; Class 2—moderate MD, $\chi^2(1) = 17.21$, $p < .001$; Class 3—mild MD, $\chi^2(1) = 19.45$, $p < .001$. The likelihood of receiving IEP supports did not vary by income status across the three classes (*p*s = .078–1.00). Furthermore, we did not find differences in the proportion of students who received math instructional support based on URM status or income status across the three classes (*p*s = .523–1.00).

Discussion

We examined patterns of longitudinal mathematics achievement among a large nationally representative sample of students from kindergarten through fourth grade to better understand growth trajectories of children with early MD. LCGA revealed considerable variability in

Table 9
Descriptive Statistics for Fifth-Grade Distal Outcomes by Growth Class

Distal outcome measures	Class 1 extensive MD (%)	Class 2 moderate MD (%)	Class 3 mild MD (%)
Academic supports and ratings			
IEP	82.89	48.81	24.13
Math instructional support	60.00	49.10	40.70
Rating of math skills above grade level	<0.001	3.57	5.18
Rating of math skills at grade level	17.65	27.86	46.87
Demographics			
Low income	66.50	67.04	53.17
URM	67.97	69.94	60.10

Note. Descriptives are based on nonimputed data. MD = math learning difficulties; IEP = Individualized Education Program; URM = underrepresented minority.

Table 10
Mean Differences in Distal Outcomes in the Spring of Fifth Grade in Relation to Growth Class

Growth class	IEP ($N = 2,373$) Prob. (*SE*)	*OR* [95% CI]	Math instructional support ($N = 1,192$) Prob. (*SE*)	*OR* [95% CI]	Rating of math skills above grade level ($N = 1,195$) Prob. (*SE*)	*OR* [95% CI]	Rating of math skills at grade level ($N = 1,195$) Prob. (*SE*)	*OR* [95% CI]
C1	.85 (0.03)	5.27 [3.25, 8.56]	.58 (0.06)	1.31 [0.76, 2.24]	<.001 (<0.001)	<0.001 [<0.001, <0.001]	.16 (0.04)	0.52 [0.26, 1.05]
C2	.53 (0.03)	1.00 [1.00, 1.00]	.51 (0.03)	1.00 [1.00, 1.00]	.03 (0.01)	1.00 [1.00, 1.00]	.27 (0.03)	1.00 [1.00, 1.00]
C3	.22 (0.01)	0.25 [0.19, 0.33]	.37 (0.01)	0.56 [0.41, 0.78]	.05 (0.01)	1.68 [0.70, 4.00]	.48 (0.02)	2.53 [1.65, 3.86]
χ^2								
C1 versus C2	72.61**		0.96		7.76*		3.89**	
C1 versus C3	441.01**		12.91**		91.89**		49.76**	
C2 versus C3	93.80**		11.94 **		1.80		22.05**	

Note. IEP = Individualized Education Program; Prob. = probability; CI = confidence interval; C1 = Class 1; C2 = Class 2; C3 = Class 3.
* $p < .05$. ** $p \leq .001$.

patterns of math achievement growth revealing three distinct growth classes. Most students who met MD criteria in kindergarten made significant growth and consistently scored slightly below the MD cutoff range (i.e., 25th percentile), ending with levels of math achievement in the 20th percentile (~69%). However, notable proportions of students displayed moderately low (~24%) and very low (~7%) levels of math achievement over time. Although both groups of students made significant gains in their IRT math scores across subsequent grades, they displayed end levels of performance that fell well below the MD cutoff, scoring in the 11th and first percentile, respectively, in the spring of fourth grade. Collectively, results from our LCGA suggest that students with early MD in kindergarten continue to experience math difficulties 4 years later, highlighting the importance and increased need for supporting students with MD during their early formative school years. Practical and theoretical implications of our findings are further detailed below.

Math Growth Trajectories Among Students With Early MD

Our RQ1 focused on detecting and describing the types of growth trajectories that students with early MD demonstrate in their math achievement from kindergarten through fourth grade. Our use of person-centered LCGA methods revealed three growth classes with significantly different intercepts and slopes. Class 1 (extensive MD), which was the lowest performing class in kindergarten, demonstrated substantially slower growth (slope = 19.13 [0.42]) than Class 2 (moderate MD; slope = 22.15 [0.25]) and Class 3 (mild MD; slope = 23.29 [0.16]), as well as those levels displayed by the normative (slope = 22.08 [0.09]) and average student samples (slope = 22.15 [0.07]). In contrast, students in Classes 2 and 3 performed considerably lower than the normative and average student samples both initially and over time but displayed similar levels of relative growth, as evidenced by their slopes (slopes ranging from 22.08 to 23.29). Descriptive statistics and visual inspection of the growth trajectories additionally show that the differences in levels of math performance between Class 2 and Class 3 relative to the normative and average student samples remains relatively stable overtime but increases for Class 1. Taken together, we view these patterns of growth (i.e., intercepts, slopes, differences in math performance levels) as support of the Matthew effect for students in Class 1. That is, students who started with the lowest math performance in the spring of kindergarten continued to perform lower

Table 11
Fifth-Grade Academic Supports Based on Student Demographics and Growth Class

Demographic characteristic	Class 1: extensive MD IEP (%)	Math instructional support (%)	Class 2: moderate MD IEP (%)	Math instructional support (%)	Class 3: mild MD IEP (%)	Math instructional support (%)
URM	78.51	57.69	43.15	51.95	20.33	40.69
Non-URM	90.77	65.63	62.82	48.00	29.83	40.72
Low income	82.22	65.00	47.33	47.90	23.22	41.76
Mid-high income	86.00	63.64	50.00	46.15	28.13	41.37

Academic supports	Class 1 URM versus non-URM	Low income versus mid-high income	Class 2 URM versus non-URM	Low income versus mid-high income	Class 3 URM versus non-URM	Low income versus mid-high income
Difference test						
IEP	$\chi^2 = 4.46$, $p = .04$	*ns*	$\chi^2 = 17.21$, $p = <.001$	*ns*	$\chi^2 = 19.45$, $p = <.001$	ns
Math instructional support	*ns*	*ns*	*ns*	*ns*	*ns*	*ns*

Note. Class membership probability and descriptive statistics are based on nonimputed data. MD = math learning difficulties; IEP = Individualized Education Program; URM = underrepresented minority; *ns* = not statistically significant.

than the other MD classes and their non-MD peers and that gap widened over time. However, given that (a) the slopes for Classes 2 and 3 were similar to those of the normative students and (b) differences in their levels of performance compared to non-MD students did not substantially decrease or widen over time, we view patterns of growth for these classes as neither supporting the compensatory growth model nor the Matthew effect. These findings largely align with those by Morgan et al. (2009) who identified four parallel growth trajectories among kindergarten students (i.e., three MD groups with parallel growth and one typically achieving group). That is, similar to Morgan et al. (2009), the class of MD students with the lowest initial performance in our study continued to demonstrate lower levels of math achievement relative to the other classes over time, followed by those who started with the second lowest levels of math achievement, and those with the highest initial math achievement continuing to display the highest performance several years later in fourth grade. Nevertheless, the current study adds to the literature by demonstrating the presence of three distinct MD trajectories based on individual differences in growth among students who meet criteria for MD in kindergarten. Below we discuss how students' individual cognitive/academic skills and the supports they receive further shape these observed math achievement trajectories.

Predictors of MD Growth

Our RQ2 examined the role that student characteristics (i.e., academic, cognitive, and demographic factors) and schools' initial detection of math difficulties and provision of academic supports play in differentially predicting these growth trajectories.

In addition to teacher math skill rating in predicting growth for students with early MD, we found that future growth for students with MD in kindergarten was also predicted by individual differences in reading and executive functions. If executive function skills underlie these challenges, it may be that children who struggle in one area will struggle in both. This idea is consistent with prior work demonstrating an underlying executive function challenge for both RD and MD (Morgan et al., 2017). Using the same ECLS-K data set used herein, Morgan et al. (2017) demonstrated consistent challenges in executive function when controlling for demographic variables for students with RD and MD, particularly working memory. These findings have also been demonstrated in other samples, particularly for students with attention challenges (Barnes et al., 2020), which were not included in the current analyses. Second, reading is likely a prerequisite for completing math tasks. That is, even for nonword problem items, children are still tasked with reading other math language and data, such as reading instructions to solve math problems. It is logical that RD would thus also impede performance in math.

It is possible that these early cognitive skills (i.e., working memory and cognitive flexibility), if developed and supported at school entry, can protect against MD. Indeed, some work suggests that executive function may facilitate learning early math skills (Ribner, 2020), indicating that children with stronger executive function skills learn math skills with greater ease (e.g., Ribner et al., 2017). Children who demonstrate challenges on executive function assessments also demonstrate fact retrieval difficulties in mathematics, many of whom have MD (Geary et al., 2012). Thus, it may be the case that executive function challenges prevent children from recalling important math information (e.g., strategies, facts, and concepts) to solve problems. However, relations at kindergarten between executive function and math performance are concurrent; thus, the direction of this finding is unclear. That is, do improvements in early math lead to greater improvements in early executive function or do improvements in early executive function lead to greater improvements in early math? (e.g., Zhang et al., 2023). Another possibility is that these improvements happen in a bidirectional manner. Directional work, such as early interventions that manipulate early math skills or early executive function skills to determine the transfer of improvements on each outcome would help us better understand these results.

One curious finding was that initial math instructional support (i.e., small group/individual instruction) in the spring of kindergarten was not predictive of growth membership. That is, there was no difference in rates of additional instructional supports offered by MD Class. There are a few possibilities for this finding. This could indicate that schools and teachers may struggle to differentiate early on which children are in need of additional academic supports despite being able to adequately gauge their overall skill level. However, this finding is not entirely surprising due to the skills/academic focus in kindergarten (i.e., general school readiness skills) relative to later grades when academic focus and rigor ramps up. In addition, schools do not typically begin RTI supports for students until the spring of kindergarten, which was our first time point. Thus, it is possible that more students, especially those in Class 1 who continued to display very low levels of math performance, are receiving supports later on or are being identified as needing supports later in their elementary education. Nevertheless, given that teachers' ratings of students' math skills aligned with early MD students' performance and predicted their future growth but that math instructional support did not, suggests teachers are able to adequately detect those students with or at risk for MD based on their current math skills, but struggle to differentially allocate those support early on to those in need.

One encouraging finding was that income and URM status did not result in increased likelihood of displaying Class 1 (extensive MD). Though some work suggests that URMs are less likely to have specific learning disabilities like MD (Fletcher & Navarrete, 2003) others show no difference in the likelihood of Black and Hispanic/Latine children in displaying MD (and RD) than White children (Morgan et al., 2017). More recently, a 2018 study by Morgan and colleagues found no evidence for URMs being overrepresented in special education; it is possible that this finding can be extended to work on MD as well. One possibility for these conflicting findings is that Fletcher and Navarrete's (2003) synthesis of the special education literature does not control for ELL status, which is also associated with the overrepresentation of students with specific learning disabilities. Thus, it is likely that these findings relate more to the issues of the overidentification of ELL students as having an LD rather than URM status. Furthermore, many of these studies cited by Fletcher and Navarrete (2003) date back to the 1980s and 1990s prior to the more current definitions and criteria used for diagnosis of specific learning disabilities, many of which were changed to reduce overrepresentation of certain student populations (IDEA, 2004).

Differential Outcomes Related to MD Students' Growth

Our RQ3 examines first whether there are differences in students' academic outcomes as well as the supports they receive in fifth grade by growth trajectories. We also consider to what extent schools continue to adequately detect and provide academic supports for students with MD at the end of elementary school (i.e., fifth grade), especially among those in greatest need. We assessed teacher ratings of students' math skills, receipt of math instructional supports, and IEP status. First, it is important to note that all three classes appropriately had a low probability of being rated either at or above average in their math skills. However, within those low probabilities, Class 3 (mild MD) was more likely to be rated at or above grade level than those in Class 1 (extensive MD) or Class 2 (moderate MD), suggesting there is still some room for improvement in teacher rating accuracy. Students with early MD (i.e., all three classes) continued to display overall levels of math achievement that fell very low to slightly below the 25th percentile even in cases where they demonstrated growth. These findings indicate that teachers are generally accurate in terms of their ranking of ratings (i.e., aware of the relative ordering of performance) but may in fact be overestimating the math performance of students with mild MD. As teachers in the elementary grades spend the majority of their day working with one classroom of students and administering both formative and summative assessments, it is not surprising that they would be accurate in the relative ratings of math skills.

Interestingly, even though most children were rated as being below grade level in math in fifth grade, there was still variability in terms of which of these classes were most likely to receive additional supports. Classes also differed in the supports they were likely to receive in fifth grade. Class 1, which had the highest likelihood (probability = .85) of having an IEP and was characterized by persistent very low levels of math achievement, was the smallest class within the sample (~7%). Class 2 was the second largest class (~24%) and was characterized as having moderate MD. Their probability of having an IEP in fifth grade (probability = .53) was higher than Class 3 but lower than Class 1. However, they were equally likely to receive additional math instructional supports as Class 1 (and more likely than Class 3). Class 3 (~69%) was characterized as having mild MD in kindergarten and levels of math achievement that were slightly below the 25th percentile in fifth grade, which were closer to but still well below the sample average. Their probability of having an IEP in fifth grade was surprisingly low (probability = .22) considering their observed performance. They also differed in their probability of whether or not they were receiving math supports, with Class 1 (extensive MD) having a higher probability of receiving additional math instructional support (probability = .58) than those in Class 3 (mild MD; probability = .37). It is important to note that these students had a generally low probability of receiving supports with Classes 1 and 2 both having a probability of ~.50 and even less so for Class 3. This indicates that a large portion of students in these classes are underserved in their needs for additional supports in fifth grade.

Finally, we observed differences in supports for URM versus non-URM students for all three classes. More specifically, URM students were significantly less likely than non-URM students to receive IEP supports even within Class 1, the class demonstrating the lowest mathematics performance. This suggests that many URM students may be going undetected in terms of their need for supports and overall disproportionately in special education placement and provision of related supports. However, it is important to note that additional mathematical supports did not vary by URM status. Thus, it is possible that the formal designation of needing an IEP may vary by URM status but the math supports provided may not. One explanation of this finding could be that recent focus of over-identification and disproportionate designations of URM students as having MD may also lead to practitioner hesitations surrounding the designation in particular. That is, there may be either resistance to or lack of supports for URM students in getting the formal diagnoses needed to support an IEP, especially considering that these are often documented by schools or districts in relation to URM status. Yet the formal designation of having an IEP is undoubtedly an important first step in getting the proper supports to meet each students' educational needs. Thus, it is possible that fewer URM students would experience extensive levels of MD during elementary school if there was greater equity in provision of academic supports. Further research with more detail on specific supports provided may assist in clarifying these findings.

Methodological Strengths and Unique Contributions of Study

Collectively, our study possesses several strengths. First, our study uses a person-centered approach of LCGA in examining the math achievement growth among students with early MD, which allows us to examine underlying heterogeneity in patterns of performance among this subpopulation of students. This method is particularly fitting given the large variability in skill performance demonstrated by struggling students and those with learning difficulties. In addition, previous studies examining MD growth have primarily used traditional growth curve modeling methods such as hierarchical linear modeling; however, these types of analyses require the researcher to predefine groups of initial interest that are then tracked longitudinally, which can constrain the number of unique patterns of growth observed. Our use of LCGA in the current study addresses this issue by letting the data inform the makeup of the patterns (i.e., class) of observed growth using mixture modeling. Second, our study is the first to date to look at distal outcomes of MD growth in terms of schools' ability to continue to detect and provide support to students in need. Furthermore, by examining the role of school supports as both predictors and distal outcomes, we approach our understanding of MD growth in the context of larger structural factors rather than focusing solely on individual student-level factors. By using this alternative lens to the traditional deficit-based approach, we provide critical insight into potential ways inequalities in math achievement growth among students with early MD are maintained.

Limitations and Future Directions

Despite its strengths, we also note several limitations of our study. First, we acknowledge the large amount of missing data in later waves of data collection, which resulted in the examination of distal outcomes for only a portion of the original sample. ECLS-K documentation specifically notes that a representative subsample of the study sample completed assessments at each subsequent time point (Tourangeau et al., 2018). However, this does not account for all of the missing data observed in the spring of fifth grade. Examination of missingness revealed that data were MCAR; thus, our use of listwise deletion is unlikely to have resulted in biased estimates for our distal outcomes regarding our targeted population. Nevertheless, we feel that any resulting biases are considerably less than those that would result from the imputation of such large rates of missingness.

A second limitation is that in examining mathematics skill growth, we were only able to assess MD students' general math achievement based on the math assessment used in this secondary data set. Thus, we were unable to make distinctions between children who have learning difficulties in specific areas of math. It is possible that additional types of MD exist during early-to-mid elementary school that reflect weaknesses in specific areas of math that may be prone to be more or less growth over time. Indeed, other studies have identified various subtypes of math learning disabilities that reflect specific skill weaknesses in different areas, such as number line magnitude, approximate number system, spatial working memory, counting and numerosities, or multiple areas (e.g., Bartelet et al., 2014). In addition, under IDEA (2004), students can be classified with different types of math learning disabilities, including those for problem-solving and/or calculation skills. Future replication and expansion of this study using both general and more specific math measures would be informative and ensure all types of MD are being captured.

A third limitation of this study is that we did not control for student IQ, a proxy for general scholastic ability. Furthermore, exclusionary criteria for IQ are typically used for the differential diagnosis of a specific LD and learning difficulty like MD versus an intellectual disability (ID) (IDEA, 2004), the latter of which reflects students with more global cognitive and academic difficulties. Because we did not have access to a student's specific disability or reason for an IEP, it is possible that individuals identified in the MD profile may have ID, which could influence the patterns of growth we observed, as students with ID often experience more negative long term academic outcomes (Caffrey & Fuchs, 2007; Harrison & Holmes, 2014).

The findings herein provide further support for using a variety of information to identify children for MD to provide early supports. Schools and teachers may provide small or individual math instruction, perhaps through RTI, to potentially buffer against persistent MD or persistent low math achievement. However, one potential barrier to providing this support could be teacher comfort with early math. Indeed, it is well documented that elementary school teachers often do not have a specialization in math (Bastian et al., 2023), and they themselves may have math anxiety and it may transfer to their students (Beilock et al., 2010). Further initiatives may circumvent this in numerous ways: (a) examine differences in school structure and math supports provided (b) determine teacher comfort and willingness in providing additional math support in early childhood, and (c) document how this additional support influences math scores during this critical period.

Finally, in the present study, we provide insight on the types of growth displayed by students with early MD during elementary school from kindergarten through fourth grade. However, we did not explore patterns of growth in later grades, when an additional wave of changes in students' display of MD, skill level, and/or performance disparities may occur. For example, many students who did not experience past difficulties in math begin to struggle when they transition to learning algebra (Herscovics & Linchevski, 1994). Additionally, there is a trend of increased rates of student dismissal from special education (e.g., Walker et al., 1988), potentially due to the development of new or additional compensatory skills. However, the use of school tracking systems is also more common during secondary school with the placement of lower achieving students in remedial-level classes, which can hinder their academic expectations and achievement (e.g., Ansalone, 2010; Oakes, 2013). That is, rather than participating in general math classroom instruction and receiving additional math instruction, students are placed in separate classrooms, where the pace and math content is often reduced in difficulty to focus on remedial skills. Future work should thus examine the continued growth of math achievement among those with early MD students in secondary school. Together, findings from this study along with future research at the secondary school level can provide a holistic understanding of the patterns of mathematics growth and role of related academic supports for MD throughout students' core academic years.

Conclusions

Strong mathematical skills support a range of long-term personal and academic benefits. Unfortunately, many students exhibit early MD. Using LCGA, we demonstrated that students with early MD exhibit continued math difficulties throughout elementary school. Most students with MD consistently display skills that fall slightly below 25th percentile cutoff for MD, indicating that they experience more mild math difficulties. However, a sizable proportion of students display very low levels of performance in the spring of kindergarten and fall increasingly behind their normative peers. In addition, findings suggest that URM students with early MD are more likely to go undetected in late elementary school, resulting in them receiving the wrong kind of academic supports, the correct dosage, or both. This issue is particularly concerning for students with consistently moderately low and very low levels of MD and may exacerbate their persistent math difficulties. Nevertheless, an important finding from the current study is that teachers appear to have seemingly accurate assessments of students' current and future levels of math achievement in terms of relative rank to their peers, indicating the utility of teachers' ratings as a useful method to detect students with or at risk for MD. Thus, schools appear to adequately detect students with MD but struggle to continue to provide supports for those students who are in greatest need in late elementary school. Finally, these findings make the case for supporting further intervention work, as early as kindergarten (e.g., Clarke et al., 2019) but also in later years (e.g., Barbieri et al., 2020; L. S. Fuchs et al., 2021) to maintain mathematic skills in ways that leverage cognitive learning strategies to possibly help facilitate greater math achievement growth among children with early MD.

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