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THE ANALYSIS OF Pi MESON PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

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November 7, 1950

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ABSTRACT

A phenomenological analysis of meson production in nucleon-nucleon collisions is proposed. On the basis of an hypothesis that the production takes place for collisions whose impact parameters tend to be less than the range of nuclear forces a partial wave analysis of the scattering matrix is given. The theory seems capable of describing the experimental results in a simple manner. It is shown, on the assumption that the π-meson is pseudoscalar, that angular momentum and parity considerations play an important role in interpreting the experimental results. If processes involving mesons are related to nuclear forces, then the hypothesis of charge symmetry in nuclear phenomena should receive a crucial test in experiments concerning meson production.

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I. Introduction

The data which has at present been obtained on the production of π-mesons in the collisions of two nucleons is very incomplete, yet it is sufficient to establish a number of interesting features of these processes. Indeed, there seems to be enough quantitative information to warrant the development of a systematic and unified interpretation of the phenomena of meson production in nucleon collisions, and it is the purpose of the present paper to sketch the outline of such a means of interpretation on the basis of a phenomenological theory. Although this type of analysis is less satisfying than one based on a fundamental theory of elementary particles, the lack of any satisfactory form of a basic theory makes it necessary to fall back on a phenomenological approach in the hope of obtaining a unified picture of the processes under consideration. The theory developed here should also be of assistance in the study of meson production in complex nuclei (which is not considered in the present paper) and in the comparison with the inverse processes of meson absorption.

Some of the qualitative experimental information on meson production which has been obtained at this laboratory is given in Table I. From these results, with the corresponding cross sections, it is possible to deduce approximately the nucleon-nucleon cross sections for meson production given in Table II.

We shall present in Section II a formalism for giving an analysis of the experiments on meson production in terms of a partial wave analysis of the scattering matrix. Such a study establishes naturally relationships between the energy spectrum of the mesons, their angular distribution, and the excitation function for the cross section. It is particularly useful, as shown by Brueckner, Serber and Watson[1], in the study of the inverse processes of meson absorption.

It has been pointed out by Brueckner, Chew and Hart[2] that the production of mesons in nucleon-nucleon collisions is strongly dependent on the interaction of the nucleons in the final state. In the course of applying the theory of the present paper, the calculations of these authors have been extended. These considerations are discussed in Section III.

In Section IV cross sections deduced from the preceeding development are given, and in Section V these are compared with and fitted to the experimental results on the production of π^+-mesons in proton-proton collisions. In Section VI evidence is given that the observed lack of π^0-mesons produced in proton-proton collisions implies a selection rule prohibiting this process.

It is shown in Section VII that angular momentum and parity conservation are of critical importance in the analysis of the cross sections. In particular, the assumption that the π-meson is pseudoscalar (in accordance with present evidence) is used to make a somewhat more concrete analysis of the scattering matrix.

A summary of the results obtained in the present paper is given in the final two sections.

II. Formal Development

Above the threshold for meson production, the study of nuclear collisions is complicated by the interdependence of the elastic and the inelastic (meson production) scattering. This fact throws doubt on the usefulness of a Schrodinger equation, in which phenomenological interaction are assumed, for the study of either the elastic or inelastic scattering at energies above the threshold for meson production. That is, an Hermitian potential energy for the two nucleon interaction leads to a scattering event in which the energy of the nucleons is conserved with a probability of unity, whereas it is known experimentally that this probability is less than unity above the threshold for meson production. The addition of an interaction term to produce mesons leads in turn to an additional potential energy, etc. To avoid these difficulties we shall not attempt a dynamical description of meson production by means of a Schrodinger equation, but rather shall employ directly the notion of the scattering matrix in the analysis of the experiments. Use will be made of a nucleon potential to describe elastic scattering only at energies well below the threshold for meson production.

We introduce a matrix operator, R, to describe the creation (or absorption) of a meson in the collision of two nucleons. In terms of R the transition probability, P, for carrying a system from a state I to a state F is (we use as units $\hbar = c = 1$)

$$P = 2\pi \left| (F \mid R \mid I) \right|^2 . \quad (1)$$

We make the assumption that R describes transitions which are reversible in time, so that we may have a means of comparing direct and inverse processes.

By a canonical transformation, we can represent R as a matrix in coordinate space of the form:

$$(\underline{z}, \underline{x}_1', \underline{x}_2' \mid R \mid \underline{x}_1 \ \underline{x}_2) \qquad (2)$$

where $\underline{x}_1$ and $\underline{x}_2$ (and $\underline{x}_1'$, $\underline{x}_2'$) are the coordinates of the two nucleons and $\underline{z}$ is the meson coordinate. R can also be expected to depend upon the nucleon spin and isotopic spin coordinates (as well as the meson spin, if the charged meson actually has a spin). Alternatively, R can be represented in momentum space by the variables $\underline{p}$, $\underline{p}'$ describing the relative momenta of the two nucleons before and after the collision, respectively, $\underline{P}$ and $\underline{P}'$ representing the respective total momentum of the two nucleons before and after the collision, and $\underline{q}$, the momentum of the created meson.

The collision is most simply described in the center of mass system, so we restrict ourselves to this coordinate system and set $\underline{P} = 0$. Then R has the form

$$R \equiv \delta(\underline{q} + \underline{P}') \ (\underline{q}, \underline{p}' \mid R_o \mid \underline{p}) \qquad (3)$$

R_o in Eq. (3) can be expected, in general, to be a complicated function of its arguments, so some condition must be found to impose simplifying restrictions on it. For energies sufficiently near the threshold for meson production (i.e., for energies presently available) such a condition is at hand, for (in the center of mass system) the greater part of the nucleon bombarding energy is found in the meson rest-mass in the final state. This implies relatively low kinetic energy for the particles in the final state ($\simeq$ 20 Mev for the Berkeley cyclotron), enabling us to assume that

$$q, p' << p \quad .$$

This condition is better fulfilled than might at first appear, because it happens that most of the mesons have nearly all the available kinetic energy, leaving little for the two remaining nucleons.

These arguments can be made more explicit by returning to the coordinate representation for R given by expression (2). In particular, in the center of mass coordinate system the sub-matrix R_o (Eq. (3)) becomes

$$R_o \equiv (\underline{u}, \underline{r}' \mid R_o \mid \underline{r}), \tag{3'}$$

where $\underline{r} \equiv \underline{x}_1 - \underline{x}_2$, $\underline{r}' \equiv \underline{x}_1' - \underline{x}_2'$, and $\underline{u} \equiv \underline{z} - \dfrac{\underline{x}_1' + \underline{x}_2'}{2}$.

The large momentum transfer ($\simeq p$) of the nucleons during the collision and meson production process suggests that the meson production takes place for very close encounters at distances of $O(\frac{1}{p})$. This implies that the effective distances for which R_o will contribute to the production process correspond to $u, r' < \frac{1}{p}$ --or that R_o will have a range of $O(\frac{1}{p})$ for the particles in the final state. (R_o is, of course, not a function of p, but it represents a velocity dependent interaction with a range dependent on the bombarding energy.) Since the deBroglie wave lengths of the outgoing particles in the final state are appreciably greater than $1/p$ (for the energies of interest to us), these arguments suggest that a partial wave analysis in terms of the angular momentum substates for the outgoing particles will prove fruitful.

To make such an analysis it is convenient to perform a partial wave decomposition of the operator R_o of Eq. (3'). This can be written in the form:

$$R_o = (|\underline{u}|, |\underline{r}'| \, \Big| R_1 \Big| \, \underline{r}) + (\nabla_u)_i \, (|\underline{u}|, |\underline{r}'| \, \Big| R_2 \Big| \, \underline{r})_i$$

$$+ (\nabla_{r'})_i \, (|\underline{u}|, |\underline{r}'| \, \Big| R_3 \Big| \, \underline{r})_i$$

$$+ \Big[(\nabla_u)_i (\nabla_u)_j - \tfrac{1}{3} \nabla_u^2 \, \delta_{ij} \Big] (|\underline{u}|, |\underline{r}'| \, \Big| R_4 \Big| \, \underline{r})_{ij}$$

$$+ \ldots\ldots\ldots, \qquad (4)$$

where a summation over repeated indices is implied. Here the various R_n $(n = 1, 2, \ldots.)$ are functions of the magnitudes only of $\underline{u}$ and $\underline{r}'$ and each is considered to have the short range of interaction discussed above.

Eq. (4) involves no approximations, but implies the assumption that in calculating cross sections from it the contribution from successive terms will decrease rapidly enough that it will provide a practical means of analysis of the experiments. However Eq. (4) is still more complicated than we wish to use at present, so we shall because of the zero range approximation, the matrix element of R_o for the transition will have the form (cf. Eq. (5')):

$$(F \,|R_o|\, I) = \sum_i \Gamma_i(p)\,(F\,|O_i|\,I)\,\left((2\pi)^{\frac{3}{2}}\,\psi_F^*(0)\right) \tag{6}$$

where ψ_F^* is evaluated at $\underline{r}' = 0$ and the $(F\,|O_i|\,I)$ are just the matrix elements of the operators $\underline{\sigma}$ and $\underline{\tau}$ as they occur multiplied by the vectors $\underline{q}$ and $\underline{p}$ in Eq. (5').

The cross section for meson production is then

$$d\sigma = \frac{(2\pi)^4\, dJ\, \sum |(F\,|R_o|\,I)|^2}{v_r} \tag{7}$$

where v_r is the relative velocity of the incoming nucleons, dJ is the volume in momentum space accessible to the particles in the final state, and $\sum$ means a summation over final spin states and an average over initial spin states. If the final nucleons are not bound to each other,

$$dJ = \sqrt{2}\,(2\pi)\,(M\mu)^{3/2}\,\left(1+\frac{T}{2\mu}\right)^{\frac{1}{2}}\left[T(T_{max} - (1.072)T)\right]^{\frac{1}{2}} dT\, d\Omega_q \tag{8}$$

Here M is the nucleon mass, μ is the meson mass, T is the meson kinetic energy, $d\Omega_q$ is an element of solid angle about the direction of $\underline{q}$, and T_{max} is the initial kinetic energy of the nucleons minus the meson rest-mass energy (plus or minus the neutron-proton mass difference, if there is a change of nucleon isotopic spin state). The factor of (1.072) results from taking $\mu/M = .144$. In deriving

Eq. (8) the nucleons in the final state are treated non-relativistically.

If the nucleons in the final state are bound (i.e., as a deuteron),

$$dJ = (.932)\, q \left[q^2 + \mu^2\right]^{1/2} d\Omega_q \qquad (8')$$

In this case the mesons have fixed energy.

Consistent with the form (5') for R_o, we can write** (cf. Eq. (7))

$$\left|(F \,|\, R_o \,|\, I)\right|^2 = (2\pi)^3 \left|\psi_F(0)\right|^2 \Big\{ \Gamma_1^2\, g_1(F,I)\, q^{2n} + \Gamma_2^2\, g_2(F,I)\, q^{2n'} \cos^2\theta + \Gamma_3^2\, g_3(F,I)\, q^{2n''} \cos^4\theta \;\ldots\ldots \qquad (9)$$

where the n's are positive integers and θ is the angle between $\underline{q}$ and $\underline{p}$. The $g_i(F,I)$ are numerical constants depending on the initial and final spin and isotopic spin states. The Γ's are numerical functions of p only and can be deduced from the Γ's of Eq. (5') once the operators O_i are specified.

** When the incoming nucleons are not identical, there will in general be odd powers of cos θ in Eq. (9). As we are primarily interested in p-p collisions in the present paper, we disregard such terms.

In the following sections we shall investigate individually the contribution of the following terms in Eq. (9).

$$\begin{aligned}
&\text{Type IA} &\longrightarrow\ & \Gamma^2 \ g_{IA}(F,I) \\
&\text{Type IIA} &\longrightarrow\ & \Gamma^2 \ g_{IIA}(F,I)\ q^2 \\
&\text{Type IB} &\longrightarrow\ & \Gamma^2 \ g_{IB}(F,I)\ \cos^2\theta \\
&\text{Type IIB} &\longrightarrow\ & \Gamma^2 \ g_{IIB}(F,I)\ q^2 \cos^2\theta
\end{aligned} \tag{10}$$

If we retain the terms in Eq. (5) which are linear in p', we obtain two more expressions which will be of interest in discussing the cross sections. These are

$$\begin{aligned}
&\text{Type III} &\longrightarrow\ & \Gamma^2 \ g_{III}(F,I)\ p'^2 \\
&\text{Type IV} &\longrightarrow\ & \Gamma^2 \ g_{IV}(F,I)\ p'^2\ q^2 .
\end{aligned} \tag{10'}$$

In obtaining these terms, it is assumed that the final state wave function for the two nucleons is a plane wave. The justification for this is that the terms in Eq. (5) involving p' linearly couple only to nucleons in p-states (relative orbital angular momentum). For p-states, the effect of the nuclear potential on the cross section will be small. It should be noted that our cross sections can contain no interference terms involving $\underline{p}'$ linearly, because we integrate over the angles of $\underline{p}'$ (corresponding to the experimental conditions under which the cross sections are measured).

The linear combinations of these terms which are compatible with the experimental cross sections will be discussed. Higher powers of q than the second do not seem necessary at present (indeed they predict a cross section apparently incompatible with the experimental results unless their contribution is small). Strictly speaking, the term of type IB is not consistent with our deduced form of R_0, but it is included because it is of use in estimating the sensitivity of the angular distributions in the laboratory coordinate system with respect to those in the center of mass system.

Finally, since the Γ's are numerical functions of p only, they are constant for any given beam energy. They are also, presumably, much more slowly varying functions of beam energy than are the other factors in the cross section. We thus assign them constant values compatible with the 340 Mev beam energy at Berkeley. Deviations of total cross sections at other energies from those here calculated will then give the dependence of these quantities on p.

There is one qualification to be noted at this point, however. The zero range approximation used in deducing the form of R_o given in Eq (5) is valid only if the wave function $\psi_F(r)$ is nearly constant for $o < r < 1/p$. This is indeed true for potentials which do not show a strong short range singularity. However, a repulsive core with a radius of $O(\frac{1}{p})$, such as that suggested by R. Jastrow[6], would make it necessary to use explicitly a finite range of interaction in Eq. (4). According to Jastrow's analysis, permissible cores for the triplet state two-nucleon potential are of too short a range $(\ll \frac{1}{p})$ to lead to an appreciable modification in our analysis, so on the basis of his model our zero range approximation (with the neglect of the core) is justifiable for final triplet states of the two nucleons. However, Jastrow's model involved a core of radius of $O(\frac{1}{p})$ for the singlet potential. Such a core would necessitate a more careful consideration of the finite range of the interaction. Indeed, explicit calculations with a perfectly "hard" core (i.e., a potential that is infinitely repulsive for distances less than the core radius) have been made, but indicate little modification in the calculated cross sections. On the other hand, it might be expected that a core which is not "hard" would modify somewhat the energy spectrum of the mesons. however, these considerations are probably not crucial in the present analysis, as we are primarily interested in final states containing a neutron and proton, for which a strong admixture of triplet state seems to be needed to explain the experimental results. Experiments on the production of mesons with two identical nucleons in the final state (singlet S-state) may throw more light on this question. Indeed, meson production may provide a useful means of studying nuclear forces at small distances.

IV. Calculated Cross Sections

It will be convenient to introduce the following type of notation to designate the various meson production processes; for instance,

$$(t, s; p\ p, \pi^+)$$

is taken as indicating the collision of two protons in an initial triplet spin state to produce a π^+-meson, leaving the resulting two nucleons in a singlet spin state. Similarly $(s, s; n\ p, \pi^0)$ indicates a singlet to singlet scattering of a neutron and proton to produce a π^0 meson, etc.

In the present section we will give the cross sections corresponding to the four types of terms occurring in expressions (10), and in later sections will consider what linear combinations give the best agreement with experiments. Then for the present we need only enumerate the final nucleon states in calculating meson cross sections, since an examination of expressions (10) shows that the various initial nucleon states (i.e., charge and spin states) enter only through the multiplicative constants g (F, I). Values of $\Gamma^2 g$ were arbitrarily chosen to normalize the total cross sections (disregarding deuteron formation) to $2.60(10)^{-28}$ cm^2. The values used for Γ^2 g are given in Table III for the four cases of expressions (10).

There are then three types of final states to be considered. The first is an n-p triplet state with the neutron and proton left as free particles. The corresponding cross section will be designated as

$$d\,\sigma^{t}_{IA} = \sigma^{t}_{o\,IA}(\theta, T)\, d\Omega_q\, dT \qquad (12)$$

where IA refers to type IA of the expressions (10), etc. The second is an n-p triplet state with the neutron and proton bound to form a deuteron, with a cross section:

$$d\sigma^{d}_{IA} = \sigma_{o\,IA}^{d} (\theta,T)\, d\Omega_q\, dT \qquad (13)$$

etc., where

$$\sigma_{o\,IA}^{d} (\theta,T) = \delta(T - (.937)(T_{max} + E_d))\ \sigma_{o_{o IA}} (\theta,T), \qquad (14)$$

since the mesons created by deuteron formation have constant energy. Here E_d is the deuteron binding energy. The third case is an n-p or n-n or p-p singlet state (as we are assuming the same singlet potential for all nucleons and are neglecting the small Coulomb correction for the p-p final state). This cross section is designated for type IA, etc., as

$$d\sigma^{s}_{IA} = \sigma^{s}_{o_{IA}} (\theta,T)\, d\Omega_q\, dT \qquad (15)$$

These cross sections are defined with the values of $\Gamma^2 g$ given in Table III. The units of σ_o in eachcase are cm^2 (Mev-steradian)$^{-1}$.

To facilitate comparison with the experiment, the differential cross sections (12) and (15) (i.e., σ_o) have been transformed to the laboratory system and are given in Figs. 1 and 2 for a beam energy of 343 Mev. The results are plotted as the meson energy spectrum at various angles. The corresponding values of $\sigma^{d}_{o\ o}$ (Eq. (14)) are given in Table IV with their respective meson energies. For comparison with experiment the δ-function in Eq. (14) should be replaced by a function of finite extent corresponding to the energy resolution of the detection apparatus and the spread in beam energy, and the resulting values of $d\sigma^d$ (Eq. (13)) added to $d\sigma^t$ (Eq. (12)). Experiments at Berkeley seem to indicate that about one-half the mesons for the (pp, π^+) process are accompanied by deuteron formation with a beam energy of 340 Mev.

The tendency of the curves in Figs. 1 and 2 to have a peak near the maximum possible meson energy is due to the rapid increase in $|\psi_F(o)|^2$ with increasing meson energy--a dependency with predominates over the variation of the phase space factor, dJ, (Eq. (7)) for large meson energies.

The variation of the total cross section with energy is given in Figs. 3 and 4 for final singlet and triplet (deuteron formation included) states for types I and II (constant and q-dependence, respectively, in the transition operator R_o). The cross sections fall off much more slowly with energy than would be expected on the basis of phase space arguments alone. The production with deuteron formation for a final triplet state causes the triplet cross section to be appreciably larger than the singlet at low energies. The cross sections were arbitrarily normalized to $8(10)^{-28}$ cm^2 at 340 Mev and constant values of $\Gamma^2 g$ were assumed. Deviations in observed cross sections at higher energies can be used to deduce the dependence of Γ^2 on p, the relative momentum of the initial nucleons in the center of mass system.

The cross sections corresponding to expressions III and IV (expressions (10')) are given in Fig. 5 for mesons in the forward direction. Here we make no distinction between final singlet and triplet states as we have neglected the nuclear interaction. There is, of course, no deuteron formation for these expressions. The striking difference in the energy spectrum of the produced mesons between these cross sections and those for which the nucleons come out in s-states is noted by a comparison of Fig. 5 and Figs. 1 and 2. The corresponding cross sections for expression (10') are designated as $d\sigma_{III}$ and $d\sigma_{IV}$.

V. Comparison with the Experimental Data on (pp, π^+) Production

The most detailed experimental data available is for (pp, π^+) production with a beam energy of 341 ± 2 Mev. The experiments of Cartwright and Whitehead[7] and Cartwright, Richman, Whitehead and Wilcox[8] give the meson energy spectrum at zero degrees (± 5°) with respect to the beam direction. Further results concerning the energy spectrum at 30° have been obtained by Peterson[9] and at 18° by Peterson, Iloff and Sherman[10].

Because of the meager experimental information available and because of its limited accuracy, it was thought better to analyze first the meson energy spectrum and then the angular distribution, to indicate the limitations on the conclusions drawn. When this study is complete, the results will be pieced together in an attempt to obtain as complete a picture as possible of the experimental cross section.

The points in Fig. 6 represent the experimental meson energy spectrum at zero degrees[11]. Referring to the cross sections resulting from expressions (10) (see Section IV), we note that in each case a final singlet spin state of the nucleons gives too few mesons in the high energy peak, whereas a final triplet state (with deuteron formation) gives too many. It is thus clear that for each of the expressions (10) we can determine a unique admixture of singlet and triplet contributions to the cross section by specifying only a single condition relating to the size of the peak to be met in satisfying the experimental data. The fact that the experimental peak in the cross section occurs for mesons with an energy greater than about 65 Mev suggests that the experimental ratio, r , of the number of mesons produced with an energy greater than 65 Mev to the number with an energy less than 65 Mev will provide such a condition. Choosing the ratio of triplet to singlet contributions in the cross section to give this same ratio, r , we obtain the cross sections given below.

The resulting, properly normalized cross section for type IA (See expressions (10). For IB the right side of Eq. (16) should be multiplied by 1/3. The energy scale for the theoretical cross sections is readjusted to fit experimental energies.) is:

$$d\sigma_{IA} = (0.950)\left[d\sigma^{t}_{IA} + d\sigma^{d}_{IA}\right] \tag{16}$$

where $d\sigma^{t}_{IA}$ and $d\sigma^{d}_{IA}$ are the expressions of Eqs. (12) and (13), respectively. Here no singlet contribution to the cross section is necessary (i.e., no admixture of $d\sigma^{s}_{IA}$, Eq. (15)). However, the estimates of the experimental error are consistent with as much as 15% contribution to the cross section from $d\sigma^{s}_{IA}$ in Eq. (16).

For type IIA (for type IIB a factor of 1/3 should be introduced on the right side of Eq. (17)) the cross section is:

$$d\sigma_{IIA} = (0.532)\left[d\sigma^{t}_{IIA} + d\sigma^{d}_{IIA} + (2.52)d\sigma^{s}_{IIA}\right] \tag{17}$$

(See Eqs. (12), (13) and (15)). Eq. (17) corresponds roughly to a 25% contribution to the cross section from the final singlet spin state (i.e., $d\sigma^{s}_{IIA}$). Because of the greater tendency of the cross sections of type II than of type I to peak at the high energy limit of the meson spectrum less triplet state is needed here. The experimental error is consistent with a relative amount of singlet admixture between 15% and 35% in Eq. (17).

The cross sections of Eqs. (16) and (17) are plotted for $\theta = 0^{\circ}$ (forward direction) in Figs. 6 and 7 respectively. The cross sections for mesons with energies greater than 65 Mev are averaged uniformly over the 10 Mev interval from 65 to 75 Mev, since the measured spectrum in this interval presumably depends only on the characteristics of the detecting equipment and the spread in beam energy.

Figs. 6 and 7 indicate that the experimental uncertainties in the cross section make it impossible on the basis of the presently known meson energy spectrum to decide between the cross sections of types I and II (momentum independent and momentum dependent matrix elements, respectively). The effect which does seem clearly established, however, is the importance of the nuclear interaction in the final states and the accompanying deuteron formation. The complete incompatibility of the meson energy spectra in Fig. 5 for nucleons in final p-states (expressions (10'); the spectrum would be even worse for higher angular momentum states) with the experimental spectrum (Fig. 10) is an indication that the predominant process leads to nucleons in final s-states--and thus provides evidence for our assertion that the range of interaction for which mesons are produced is very short.

As mentioned previously, calculations assuming a q^4 dependence in the cross section indicate that this type of term is incompatible with the experiments unless its contribution to the cross section is small (i.e., is a small correction in the series of Eq. (9)).

It is possible to have some contribution to the cross section from the terms given by expressions (10'), which correspond to a final p-state for the two nucleons. It is clear that the fraction of the cross section that can come from this type of term is quite limited, since there is no high energy peak of mesons associated with this interaction type. Thus, for a cross section of type I (Eq. (16)) only a very few percent of type III or type IV (expressions (10')) can be added, because of the difficulty of obtaining enough mesons in the high energy peak. However, for the type II cross section of Eq. (17), considerably more mixture of type III or IV cross sections can be added by correspondingly decreasing the amount of $d\sigma_{IIA}^{s}$ admixture. In particular, if no $d\sigma_{IIA}^{s}$ admixture is permitted, as much as 20% contribution from $d\sigma_{III}$ or $d\sigma_{IV}$ gives a very satisfactory fit to the experimental meson energy spectrum.

Because of the limited experimental data, we cannot at present settle the question of the amount of admixture of nucleon p-states. It is shown, however, in Section VII that angular momentum and parity restrictions help in removing some of the ambiguity once a definite spin and parity is assigned to the π-meson.

We now turn to a consideration of the angular distribution of the produced mesons. Designating the production cross section per steradian at an angle θ' with respect to the beam direction by

$\sigma(\theta')$ (i.e., the area under the meson spectrum curve), we have the experimental results given in the first column of Table V. The corresponding results deduced from the cross sections of Eqs. (16) and (17) are also given in Table V. The most immediate conclusion drawn from a comparison of these values is that the cross section is not spherically symmetrical in the center of mass system. On the other-hand, the type B cross sections ($\cos^2 \theta$ distribution in the center of mass system) are quite consistent with the experimental results. We thus conclude that most of the mesons produced at 341 Mev beam energies are emitted into p-states with a $\cos^2 \theta$ angular distribution (in the center of mass system). This implies that the leading term in the cross section is of type IIB (c.f. Eq. (17)).

The experimental error given in Table V is consistent with a maximum of about 25% spherically symmetric contribution to the total cross section at 341 Mev--although, of course, this contribution may be much less. Experiments are now in progress to measure the cross section at 60°. Since the contribution from the $\cos^2 \theta$ distribution is quite small at this angle, these experiments should give a fairly sensitive indication of the amount of spherical symmetry in the cross section.

VI. Other Types of Processes

Experiments by Bjorkland, Crandall, Moyer and York[12] indicate that the cross section for π^{o}-meson production in p-p collisions, if nonvanishing, is less than 1/20 the cross section for p-p production of π^{+}-mesons at 340 Mev. (They observed no production in p-p collisions, the factor 1/20 representing their estimated experimental uncertainty.) The present calculations indicate p-p cross sections not much less than about 1/3 the (p-p, π^{+}) cross section can be expected on the basis of the interaction of the particles in the final state if the transition operators are the same. It thus appears that there is some selection rule prohibiting π^{o} production in p-p collisions. Very little is known about the cross section for meson production in n-p collisions.

VII. Angular Momentum and Parity Relationships

The analysis of the experiments made in Section V was given without regard to angular momentum and parity restrictions. The assumption of a given spin and parity for the π-meson considerably limits the freedom of choice of final states, however. In the present section we shall investigate these limitations on the assumption that the π-meson is pseudoscalar. At the time of writing, this seems to be the most reasonable choice, since it is known that the π^0 cannot have spin one[13], and since the experiments on the absorption of π^--mesons in deuterium[14] indicate that the charged π-meson is not scalar. If future experimental results should lead to contrary evidence, an analysis such as that given here can be made for any given spin and parity of the meson.

For the production of a π-meson, the initial state containing two nucleons may have even or odd parity and be in a singlet or triplet spin state. Under actual experimental conditions, of course, the initial state will be a combination of these states, restricted only by the Pauli principle. The final state will again contain two nucleons in a mixture of the allowed spin states, but with zero relative angular momentum according to the approximation leading to Eq. (5'). There will also be a meson present, which we assume to be pseudoscalar, and which may be in an even parity state (odd angular momentum) or an odd parity state (even angular momentum). In table VI are summarized all the permissible transitions to produce a pseudoscalar meson which are consistent with parity and total angular momentum conservation and zero relative angular momentum for the two nucleons in the final state. It is clear that if the hypothesis that the π-meson is pseudoscalar is to be maintained, the experimental results must be consistent with table VI. For example, if the (pp, π^+) process leads to mesons in p-states (evidence for this was given in Section V), the final neutron and proton must be in a triplet spin state only.

Again, if the production of mesons into p-states represents a universal type of coupling for all production processes in nucleon-nucleon collisions, reference to Table VI indicates the existence of a general selection rule prohibiting the (pp, π^0) process--which is consistent with our deductions in Section VI that such a selection rule must exist. Had we assumed the meson to be scalar, we would not have obtained such a selection rule from parity and angular momentum considerations. This seems to give further evidence that we can obtain a self-consistent picture of meson processes with the assumption that the π-meson is pseudoscalar.

Our analysis can be made more explicit by considering the limitations imposed by symmetry conditions on the operators $O(\underline{\sigma}, \underline{\tau}, \underline{q}, \underline{p})$ of Eq. (5'). These must be invariant with respect to rotations in coordinate space but must change sign under a coordinate reflection, since we have assumed the meson to be pseudoscalar. The operators, O , must also be symmetric with respect to an interchange of the coordinates of the two nucleons.

The following products of $\underline{\sigma}$, $\underline{p}$ and $\underline{q}$ are possible (we designate one nucleon by the superscript "1" and the other by"2"):

$$A_1(1) = (p \cdot q)^{n-1} \; \underline{\sigma}^{(1)} \cdot \underline{p}$$

$$A_2(1) = (p \cdot q)^{n} \; \underline{\sigma}^{(1)} \cdot \underline{q}$$

$$A_3(1) = (p \cdot q)^{n-2} \; \underline{\sigma}^{(1)} \cdot p \;\; \underline{\sigma}^{(2)} \cdot (\underline{p} \times \underline{q})$$

$$A_4(1) = (p \cdot q)^{n-1} \; \underline{\sigma}^{(1)} \cdot \underline{q} \;\; \underline{\sigma}^{(2)} \cdot (\underline{p} \times \underline{q})$$

$$A_5(1) = \frac{1}{2} (p \cdot q)^{n-1} \; \underline{\sigma}^{(1)} \times \underline{\sigma}^{(2)} \cdot \underline{p}$$

$$A_6(1) = \frac{1}{2} (p \cdot q)^{n} \; \underline{\sigma}^{(1)} \times \underline{\sigma}^{(2)} \cdot \underline{q} \qquad (18)$$

Similar quantities $A(2)$ can be obtained by interchanging superscripts "1" and "2" and replacing $\underline{p}$ by $-\underline{p}$. Further factors of the form $(\underline{p} \times \underline{q})^2$ may also be introduced, but lead to no new results.

A_1 (with $n = 1$) is characteristic of pseudoscalar meson theory with pseudoscalar coupling while a particular linear combination of A_1 (with $n = 2$) and A_3 (with $n = 2$) is obtained with pseudoscalar theory with pseudovector coupling. For further details, the paper of Brueckner[2] should be consulted.

For the isotopic spin dependence we choose the following combination of τ-operators:

$$\tau_i^{(1)} \, T \quad \text{and} \quad \tau_i^{(2)} \, T \quad (i = 1, 2, 3; \ \text{or} \ i = 1, 2, 4) \qquad (19)$$

where

$$T = \tau_1^{(1)} \tau_1^{(2)} + \tau_2^{(1)} \tau_2^{(2)} + \beta \left[\tau_3^{(1)} \tau_3^{(2)} + d \, \tau_4^{(1)} \tau_4^{(2)} \right] \qquad (20)$$

The τ_i ($i = 1, 2, 3$) are the usual isotopic spin operators and τ_4 is the unit two-dimensional matrix. The index "i" on the τ_i in expressions (19) represents the isotopic spin type of the meson emitted (or absorbed). β and d are arbitrary parameters. The choice of τ_3 or τ_4 in expressions (19) for the emission of a neutral meson corresponds to the well known ambiguity in the coupling of neutral mesons to nucleons.

The form (20) for T implies a symmetry with respect to the interchange of isotopic spin states that is characteristic of meson theory. If this is not borne out by experiment, additional factors of the type $(1 \pm \tau_3)$ will have to be introduced. Complete charge symmetry is obtained by setting $\beta = 1$ in Eq. (20) and using the τ_3-coupling in expression (19).

We now define the operators 0 of Eq. (5') to within a phase factor as:

$$
\begin{aligned}
0 = & \left[\tau_i^{(2)} \, T \, A(1) + \tau_i^{(1)} \, T \, A(2)\right] \\
& + \rho\left[\tau_i^{(1)} \, T \, A(1) + \tau_i^{(2)} \, T \, A(2)\right] \\
& + e\left\{\left[T \, \tau_i^{(2)} \, A(1) + T \, \tau_i^{(1)} \, A(2)\right]\right. \\
& \left. + \rho\left[T \, \tau_i^{(1)} \, A(1) + T \, \tau_i^{(2)} \, A(2)\right]\right\}
\end{aligned}
\qquad (21)
$$

where the A's are any of the quantities of expressions (18). ρ is an arbitrary parameter and e is either ± 1, as we wish to restrict ourselves to an Hermitian combination of the γ's.

In accordance with Eq. (5'), R_o can be expected in general to be a sum of O's of the type given by Eq. (21) with various different A's of the form given in Eqs. (18). This form of R_o is not analyzed in terms of partial wave states for the meson, but seems to be simpler to use as we are neglecting the effects of a meson-nucleon interaction in the final state. (See the Appendix for a more complete discussion of this point.)

To investigate the allowed transitions permitted by the various A's of Eqs. (18) it will be convenient, for the moment, to assume that R_o contains only one of these A's. Then writing

$$Q \equiv \sum \left| (F \; | R_o | \; I) \right|^2$$

(see Eqs. (5') and (7)), we have

$$Q_1 \equiv \Gamma_1^2 \; q^{2(n-1)} \; [\cos\theta]^{2(n-1)} \; g_1(F,I)$$

$$Q_2 \equiv \Gamma_2^2 \; q^{2(n+1)} \; [\cos\theta]^{2n} \; g_2(F,I)$$

$$Q_3 \equiv \Gamma_3^2 \; q^{2(n-1)} \; [\cos\theta]^{2(n-2)} \sin^2\theta \; g_3(F,I)$$

$$Q_4 \equiv \Gamma_4^2 \; q^{2(n+1)} \; [\cos\theta]^{2(n-1)} \sin^2\theta \; g_4(F,I)$$

$$Q_5 \equiv \Gamma_5^2 \; q^{2(n-1)} \; [\cos\theta]^{2(n-1)} \; g_5(F,I)$$

$$Q_6 \equiv \Gamma_6^2 \; q^{2(n+1)} \; [\cos\theta]^{2n} \; g_6(F,I) \qquad (22)$$

where θ is the angle between $\underline{p}$ and $\underline{q}$. In these equations the subscripts on the Q's refer to the subscripts on the A's of Eqs. (18). The Γ's are numerical functions of p only (see Eq. (5')) and the g's are numerical constants depending on the initial and final spin and isotopic spin states and are the g's of Eq. (9). The cross sections for meson production are obtained by substituting any one of the Q's of Eq. (22) into Eq. (7) for $\sum |(F \mid R_o \mid I)|^2$. If R_o contains a sum of several of the O's, there will in general be interference terms in the cross section. The total cross section for an initially unpolarized beam is of course 3/4 that for an initial triplet state plus 1/4 that for an initial singlet state.

The g's of Eqs. (22) fall into two classes depending upon whether n is even or odd. (This corresponds to even or odd parity, respectively, for the initial two-nucleon state from which the transition takes place. See Table VI.) This evenness or oddness can be designated by an additional subscript on the g's; i.e., "e" or "o", respectively. Then we write $g_{1,e}$, $g_{1,o}$, etc. The values of the g's are given in Tables VII and VIII, where the following abbreviations are used.

$$a \equiv 8/3\ \beta^2 (1+d)^2 (1+e)^2 (1+\rho)^2$$

$$b^{\pm} \equiv 8/3\ \left[2 \pm e\beta(1+d) \mp \beta(1-d)\right]^2 (1 \pm \rho)^2$$

$$c \equiv 8/3\ \left[\beta(1-d)(1+e) + 2(1-e)\right]^2 (1-\rho)^2$$

$$d^{\pm} \equiv 8/3\ (1+e)^2 \left[\beta(1-d) \pm 2\right]^2 (1+\rho)^2 \qquad (23)$$

Those g's having the same value are grouped in one column in the tables. The values of g_5 and g_6 are not given, but are the same as those in the tables with the exception that they vanish for triplet to triplet transitions. The values in Table VII correspond to τ_3 coupling of the neutral mesons (expressions (19)). The use of the τ_4 coupling for neutral mesons changes only the cross sections for neutral meson production. The corresponding values of the g's with τ_4 coupling are given in Table VIII for neutral meson production. Tables VII and VIII together include all the allowed transitions given by Table VI.

If charge symmetry is assumed and thus β is set equal to unity, there remain only two adjustable constants, ρ and d, for each O. In this case we have the identical relations $b^- \equiv c$ and $b^+ \equiv a$ for the constants in Table VII (charge symmetry implies that we use the τ_3 - coupling). The observed $\cos^2 \theta$ angular distribution implies that we must choose the "e" column in Table VII as the dominant term in the cross section. We are then left with only one parameter, $b^- \equiv c$, to fix the magnitude of all possible meson production processes if charge symmetry is to be maintained. By a combination of interaction types consistent with the $\cos^2 \theta$ angular distribution we have in general one parameter for all final triplet spin states and one for all final singlet spin states on the charge symmetry hypothesis.

VIII. Summary of Results

We are now in a position to piece together the results which we have deduced from the experimental data. This will be useful in determining the limitations which present experimental material places on our model as well as in suggesting further experiments which should give crucial information.

In Section V we concluded that the observed $\cos^2 \theta$ center of mass angular distribution for the (pp, π^+) mesons indicates that at least 75% of the mesons are emitted into p-states. This implies that the leading term in the cross section is of type IIB (expressions(10)).

A difficulty is now met in that the type IIB cross section of Eq. (17) gives a 25% singlet contribution. This is inconsistent with Table V, which indicates the existence of a selection rule prohibiting any final nucleon singlet state for pseudoscalar mesons in p-states. The difficulty is that the final triplet state does not give enough low energy mesons (see the experimental spectrum of Fig. (10)). Although the low energy part of the spectrum is least well known experimentally, it seems not unlikely that the type IIB (triplet) cross section does deviate significantly from the experimental energy spectrum.

There is, however, neither experimental nor theoretical evidence that the $\cos^2 \theta$ angular distribution represents the entire cross section. In fact, the absorption of π^--mesons in deuterium rather conclusively suggests that there must be some mesons produced into s-states (since the absorption in deuterium presumably is from an s-state). We can also suppose that there are relatively small admixtures of higher angular momentum states of both the nucleons and mesons. Indeed, as little as 15% admixture of some state containing no deuteron formation is probably consistent with the experimental results (see discussion following Eq. (17)), so we need not be outside the experimental error in the angular distribution in fitting the energy spectrum.

In fact, we are faced with more possibilities for doing this than can be resolved by present experimental results. We may, for instance, add 15% to 20% of type IA (singlet) cross section (with a correspondingly small amount of IA (triplet) to be consistent with the absorption in deuterium--this actually happens in pseudoscalar meson theory[15]). Again, we may add the same amount of type III or type IV cross section (expressions (10'), Fig. 5. Type III corresponds to nucleons in p-states, mesons in s-states; type IV to nucleons and mesons both in p-states). The energy spectra resulting from a 20% admixture of type III or type IV cross sections are shown in Fig. 8, where the solid curve represents type III and the dotted curve type IV (the deuteron peak is, of course, the same for both and has been plotted as a Gaussian distribution in Fig. 8). Present experimental results do not permit us to make even a qualitative determination of the relative amounts of these various terms. The experimental results at 60°, when available, may be of considerable use in resolving this ambiguity, since at this angle the $\cos^2 \theta$ term contributes relatively little to the cross section.

None of the additional terms just discussed leads to the selection rule mentioned in Section VII prohibiting the (pp, π^o) process. We are thus faced with the choice of supposing that there is an additional selection rule for these additional terms, or (more probably) that the limits on the experimental error[12] were somewhat overly optomistic--by a factor of about 2 to 4. In any case, the general conclusion of a considerably smaller (pp, π^o) cross section than (pp, π^+) cross section is accounted for by the lack of deuteron formation for the former process plus the natural selection rule for mesons in p-states. Measurements of the (pp, π^o) cross section are now being repeated at this laboratory in order to obtain a better ratio of these two cross sections.

Turning to the interaction types given by Eq. (18), we see that only A_1 and A_5 with $n = 2$ will give the leading term in the cross section with a $\cos^2 \theta$ angular distribution and mesons in p-states. Further assignment of interaction types, and in particular a test of the charge symmetry hypothesis, must await additional experimental information. Of particular importance are experiments involving meson production in n-p collisions.

The relation of our conclusions to meson theory is of some interest. This is so, in particular, since pseudoscalar meson theory has in many cases given reasonable qualitative agreement with experiment. It appears, however, that the $\cos^2 \theta$ angular distribution presents a real difficulty, since Brueckner[2] found that scalar, pseudoscalar, and vector meson theories predicts a spherically symmetric center of mass angular distribution for the mesons. (This was true of both the results from perturbation theory and the phenomenological theory of Marshak and Foldy[16]). This would seem to be rather a fundamental discrepancy. It is thus our opinion that conclusions drawn from meson theory concerning both production and absorption of mesons (inverse process) are of doubtful validity--and thus that the present phenomenological approach is a safer--if less spectacular--means of studying processes involving meson production and absorption.

IX. Conclusions

We have concluded from our analysis and a previous consideration[1] that if the charged and neutral π-mesons are of the same type, then they are very probably pseudoscalar. These conclusions have been drawn without recourse to meson theory (on the whole, conclusions drawn from pseudoscalar meson theory have shown better agreement with experiment than have those drawn from the other versions of meson theory). The evidence that charged and neutral mesons are of the same type is certainly not conclusive, but is very definitely suggested by their nearly equivalent mass and production cross sections.

Then on the assumption that the π-meson is pseudoscalar, we have concluded from the experimental results that for the (pp, π^+) process most of the mesons are produced into p-states from an initial even parity (singlet) state of the two protons, while in the final state the resulting neutron and proton are left in a ^{3}S-state (and bound to form a deuteron for more than half the production events at 340 Mev). The assumption that this represents a universal type of coupling for all mesons produced in nucleon-nucleon collisions leads naturally to a selection rule prohibiting (pp, π^0) production (as observed) for pseudoscalar--but not for scalar--mesons.

The simple process given in the last paragraph will certainly not prove sufficient to describe the finer details of meson production; but an analysis of these must await more detailed experimental results.

The hypothesis of charge symmetry for both nuclear forces and meson production (presumably these are not independent) will receive a crucial test when the production in n-p collisions can be compared with that for p-p collisions. In particular, a comparison of the energy spectrum of π^+ and π^- mesons produced in n-p collisions will give a direct comparison of n-n and p-p singlet potentials through the dependence of the cross sections on $|\psi_F(o)|^2$ (Eq. (9)). Further information concerning the properties of nuclear forces at close distances may very well be obtained from such experiments.

We wish to express our appreciation to Professor R. Serber, with whom much of the present work has been discussed and to whom we are indebted for many valuable suggestions. We are also indebted to Dr. C. Richman, Dr. H. A. Wilcox, Mr. W. Cartwright, Miss M. Whitehead, Dr. V. Z. Peterson and their collaborators for aid in interpreting their experiments and for permission to quote their results in advance of publication. Further appreciation is expressed to Mr. D. Clark and Mr. W. Noh for their considerable assistance with the numerical calculations.

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A. Free nucleons	B. Complex nuclei
1) $P + P \rightarrow \pi^{+}$ (allowed)[a]	1) $P + (P,N) \rightarrow \pi^{+}$ (allowed)[c]
2) $P + N \rightarrow \pi^{+}$ (unobserved)	2) $P + (P,N) \rightarrow \pi^{-}$ (allowed)[c]
3) $N + N \rightarrow \pi^{-}$ (unobserved)	3) $P + (P,N) \rightarrow \pi^{0}$ (allowed)[b]
4) $N + P \rightarrow \pi^{-}$ (unobserved)	4) $N + (P,N) \rightarrow \pi^{+}$ (allowed)[d]
5) $N + N \rightarrow \pi^{0}$ (unobserved)	5) $N + (P,N) \rightarrow \pi^{-}$ (allowed)[d]
6) $N + P \rightarrow \pi^{0}$ (unobserved)	6) $N + (P,N) \rightarrow \pi^{0}$ (unobserved)
7) $P + P \rightarrow \pi^{0}$ (forbidden)[b]	

Table I. Qualitative experimental results for processes involving production of charged and neutral mesons by nucleons bombarding free nucleons or complex nuclei.

(a) Cartwright, Richman, Whitehead and Wilcox, Phys. Rev. <u>78</u>, 823 (1950)

(b) Bjorkland, Crandall, Moyer and York, Phys. Rev. <u>77</u>, 213 (1950)

(c) Richman and Wilcox, Phys. Rev. <u>78</u>, 496 (1950)

(d) Bradner, O'Connell and Rankin, private communication

I. Process		II. Experiment	III. Total cross sections ($\times 10^{28}$ cm^{-2})	
			Complex nuclei	Free
1) $P + P \rightarrow \pi^+$	allowed	A(1), B(1)	3.3 ± 1.0	4.0 ± .8*
2) $P + P \rightarrow \pi^0$	forbidden	A(7)	?	0.1 ± 0.1
3) $N + P \rightarrow \pi^+$	allowed	B(4), B(2)	0.8 ± 0.4	?
4) $N + P \rightarrow \pi^-$	allowed	B(2)	0.8 ± 0.4	?
5) $N + P \rightarrow \pi^0$	allowed	A(7) + B(3)	1.7 ± 0.9	?
6) $N + N \rightarrow \pi^-$	possible	B(5)	?	?
7) $N + N \rightarrow \pi^0$	unobserved		?	?

Table II. Nucleon-nucleon cross sections for production of charged and neutral mesons. The experimental results given in Table I from which the cross sections are deduced are indicated in column II.

---

* Deduced from the cross section in the forward direction with the angular distribution found in Section V.

	IA (Triplet)	IA (Singlet)	IIA (Triplet)	IIA (Singlet)
$\Gamma^2 g$	$1.94(10)^{-44} \frac{cm^2}{(Mev)^5}$	$2.01(10)^{-44} \frac{cm^2}{(Mev)^5}$	$4.90(10)^{-48} \frac{cm^2}{(Mev)^7}$	$4.36(10)^{-48} \frac{cm^2}{(Mev)^7}$
	IB (Triplet)	IB (Singlet)	IIB (Triplet)	IIB (Singlet)
$\Gamma^2 g$	$5.82(10)^{-44} \frac{cm^2}{(Mev)^5}$	$6.03(10)^{-44} \frac{cm^2}{(Mev)^5}$	$1.47(10)^{-47} \frac{cm^2}{(Mev)^7}$	$1.31(10)^{-47} \frac{cm^2}{(Mev)^7}$

Table III. Values of the arbitrary constants $\Gamma^2 g$ adjusted to give a total cross section (neglecting deuteron formation) at 343 Mev of $2.60(10)^{-28}$ cm^2.

θ	IA	IB	IIA	IIB	Meson energy (Mev)
0°	1.75	5.25	2.87	8.61	73
30°	1.42	1.45	2.32	2.38	56
60°	0.82	0.19	1.34	0.32	28
90°	0.28	0.50	0.46	0.83	11

Table IV

$d\sigma/d\Omega$ deuterons

Differential cross section in units of 10^{-28} cm^2 per unit solid angle for production of a positive π-meson and a deuteron in a 343 Mev p-p collision. θ is the angle between the directions of the initial nucleon beam and meson momenta in the laboratory system. The columns headed IA, IB, etc. are for transition operators of the type IA, IB, etc. respectively (cf. expressions (10)).

Table V

The angular distribution for (pp, π^+) mesons is analyzed on the basis of experimental data at 0°, 18° and 30° with respect to the beam direction (laboratory frame of reference). In column (a) are given the experimental ratios of the cross sections at 0° to 18° and 0° to 30°. The corresponding ratios calculated from the cross sections of Eqs. (16) and (17) are given in columns (b)........(e) . Type "A" refers to spherical symmetry, type "B" to a $\cos^2 \theta$ dependence in the center of mass system.

	(a)	(b)	(c)	(d)	(e)
Ratio *	Experimental	IA	IIA	IB	IIB
$\frac{\sigma(0^\circ)}{\sigma(30^\circ)}$	4.40 ± .7	1.34	1.30	5.10	4.75
$\frac{\sigma(0^\circ)}{\sigma(18^\circ)}$	1.62 ± .25	1.16	1.15	1.70	1.64

* $\sigma(0^\circ) = 2.64 \pm .3(10)^{-28}$ cm²/steradian

$\sigma(18^\circ) = 1.63 \pm .2(10)^{-28}$ cm²/steradian

$\sigma(30^\circ) = .58 \pm .07(10)^{-28}$ cm²/steradian

(References are given at beginning of Section V.)

Table VI

The allowed spin and parity relations for the production of pseudoscalar π-mesons. Column (a) gives the parity and spin of the nucleons in the initial state; column (b) shows whether the meson angular momentum state is even or odd; column (c) gives the spin of the nucleons in the final state assuming that they have zero relative orbital angular momentum. The abbreviation "e" is used for "even"; "o" for "odd"; "s" for "singlet"; and "t" for "triplet".

Type of process	a	b	c
$pp \rightarrow \pi^+$ or	e, s	o	t
$nn \rightarrow \pi^-$	o, t	e	t or s
$pp \rightarrow \pi^0$ or	e, s	forbidden	
$nn \rightarrow \pi^0$	o, t	e	s
$np \rightarrow \pi^+$ or	e, t	o	s
$np \rightarrow \pi^-$	e, s	forbidden	
	o, t	e	s
	o, s	forbidden	
$np \rightarrow \pi^0$	e, t	o	t or s
	e, s	o	t
	o, t	e	t or s
	o, s	e	t

Table VII. Values of the constants, g, for the various processes of meson production. The definitions of a, $b^{\pm}$, and c are given by Eq. (23). The first column gives the type of process; the second column gives the initial and final nucleon spin states as singlet (s) or triplet (t). Values of g_5 and g_6 are the same except that they vanish for t ⟶ t transition.

		g_1, o; g_2, o g_3, o; g_4, o	g_1, e; g_2, e; g_3, e; g_4, e.
$pp \to \pi^+$ or	t ⟶ t	$2b^-$	0
$nn \to \pi^-$	t ⟶ s	b^+	0
	s ⟶ t	0	$3b^-$
	s ⟶ s	0	0
$pp \to \pi^0$ or	t ⟶ t	0	0
$nn \to \pi^0$	t ⟶ s	a	0
	s ⟶ t	0	0
	s ⟶ s	0	0
$np \to \pi^+$ or	t ⟶ t	0	0
$np \to \pi^-$	t ⟶ s	$\frac{1}{2}b^+$	$\frac{1}{2}b^-$
	s ⟶ t	0	0
	s ⟶ s	0	0
$np \to \pi^0$	t ⟶ t	c	0
	t ⟶ s	0	$\frac{1}{2}c$
	s ⟶ t	0	$\frac{3}{2}c$
	s ⟶ s	0	0

Table VIII. Values of the constants, g, for neutral meson production when $\mathcal{T}_4$ is used as the coupling. Value of d is given in Eq. (23). Values of g_5 and g_6 are the same as those given, except that they vanish for $t \to t$ transitions. (See Table VII for terminology.)

		g_1, o; g_2, o; g_3, o; g_4, o	g_1, e; g_2, e; g_3, e; g_4, e.
$pp \to \pi^o$	$t \to t$	0	0
$nn \to \pi^o$	$t \to s$	a	0
	$s \to t$	0	0
	$s \to s$	0	0
$np \to \pi^o$	$t \to t$	0	d^+
	$t \to s$	$\frac{1}{2}d^-$	0
	$s \to t$	$\frac{3}{2}d^+$	0
	$s \to s$	0	0

Appendix

The quantities $(|\underline{u}|, |\underline{r}'| \; | R_n | \; \underline{r})$ of Eq. (4) can be formally expanded as

$$(|\underline{u}|, |\underline{r}'| \; |R_n| \; \underline{r}) = \sum_{\lambda, \mu} (R_n^{\lambda\mu} | \underline{r}) \nabla_u^{2\lambda} \nabla_{r'}^{2\mu} \delta(\underline{u}) \; \delta(\underline{r}') \; .$$

These expansions, when substituted into Eq. (4), correspond to a Taylor expansion of the momentum representation of R_o in $\underline{q}$ and $\underline{p}'$. Such an expansion can be useful, however, only if the final states are plane wave states. This is clearly not the case for the nucleon wave function. As long as plane wave states are used for the meson wave function it is only a matter of convenience whether one uses a partial wave analysis or a power series in $\underline{q}$ in studying the properties of the produced mesons.

Fig. 1. Differential cross section for meson production in the laboratory system in units of 10^{-31} cm^2 per Mev per unit solid angle (designated as σ_o in Eqs. (12) and (15)) at 343 Mev for the incident nucleon and the meson rest-mass taken as 140 Mev. The final nucleons are assumed to be in a triplet spin state. The energy scale in Mev refers to the meson kinetic energy; the angles indicated are the angle between the meson and incident nucleon momenta. The labeling IA, IIA, etc., refers to the type of transition operator defined in Eq. (10).

Fig. 2. Differential cross section for meson production with a beam energy of 343 Mev. The final nucleons are assumed to be in a singlet spin state, but other wise the notation is the same as for Fig. 1.

Fig. 3. Variation of the total cross section for meson production with the energy of the incident nucleon. The transition operator R_o (see Eq. (5')) is assumed to be independent of the meson momentum. The cross sections are arbitrarily normalized to $8(10)^{-28}$ cm^2 at 340 Mev. The solid curve is for a final nucleon singlet state, the dashed curve is for a final nucleon triplet state and includes the possibility of deuteron formation.

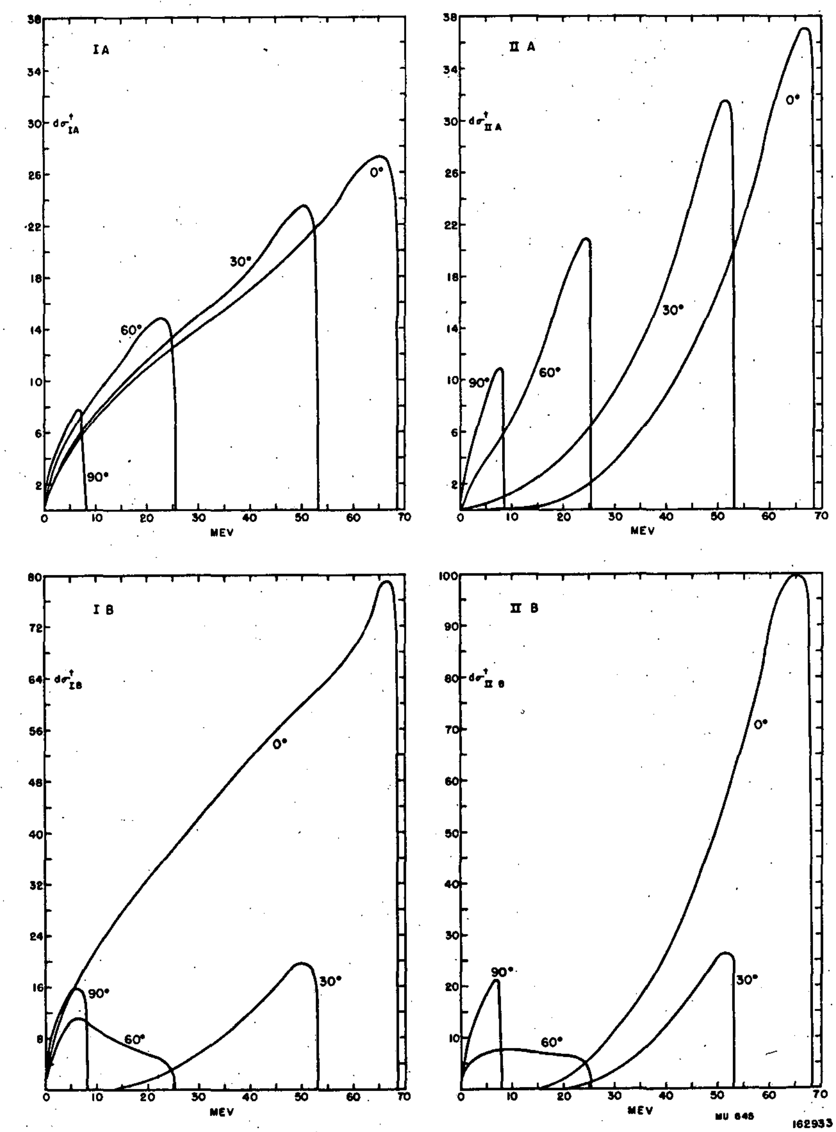
Fig. 4. Variation of the total cross section for meson production. The definitions and symbols are the same as for Fig. 3 except that the transition operator R_o is assumed to depend linearly on the meson momentum.

Fig. 5. Differential cross section (in the laboratory system) in the direction of the beam for producing mesons in 343 Mev nucleon collisions when the nucleons in the final state are in p-states. The energy scale refers to the meson kinetic energy; the ordinate is given in arbitrary units. Cross sections of type III and type IV refer to the respective expressions (10'). The lack of high energy peak is readily apparent from a comparison with Figs. 1 and 2.

Fig. 6. Differential cross section for meson production in the direction of the beam at 340 Mev. The transition operator R_0 (Eq. (5')) is of the form leading to Eq. (16); i.e., independent of meson momentum and leading to a final nucleon triplet spin state. The cross section for mesons with energies greater than 65 Mev (including the delta function contribution for deuteron formation at 70 Mev) is averaged uniformly over the energy interval of 65 to 75 Mev. The points indicated are from the experimental results of Cartwright, Richman, Whitehead and Wilcox[7] and Cartwright and Whitehead[6].

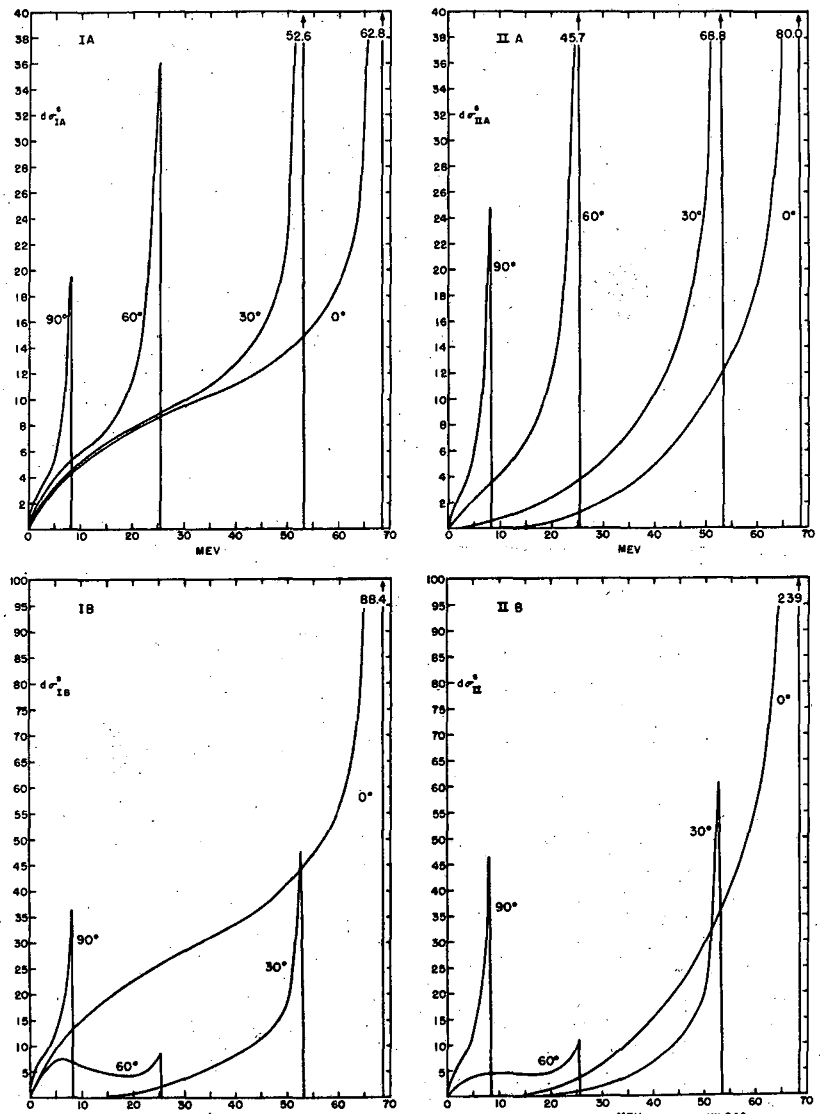
Fig. 7. Differential cross section for meson production at 340 Mev. The definitions and symbols are the same as for Fig. 6, except that the transition operator is assumed to be of the form given in Eq. (17); i.e., linearly dependent on meson momentum and leading to a mixture of singlet and triplet spin states.

Fig. 8. Differential cross section for meson production at 340 Mev. Definitions are similar to those for Fig. 6, except that the transition operator is of the form IIB (triplet) (expressions (10)) with 20% of type III cross section (solid curve) and 20% of type IV cross section (dotted curve) (expressions (10')). The deuteron peak is spread over a Gaussian error curve.



I A
$d\sigma^{\dagger}_{IA}$
II A
$d\sigma^{\dagger}_{II A}$
I B
$d\sigma^{\dagger}_{IB}$
II B
$d\sigma^{\dagger}_{II B}$
0°
30°
60°
90°
MEV
MU 646
162933

Fig. 1



IA
$d\sigma^{s}_{IA}$
52.6
62.8
90°
60°
30°
0°
MEV
II A
$d\sigma^{s}_{IIA}$
45.7
66.8
80.0
90°
60°
30°
0°
MEV
IB
$d\sigma^{s}_{IB}$
88.4
90°
60°
30°
0°
MEV
II B
$d\sigma^{s}_{II}$
239
90°
60°
30°
0°
MEV
MU 646
162943

Fig. 2

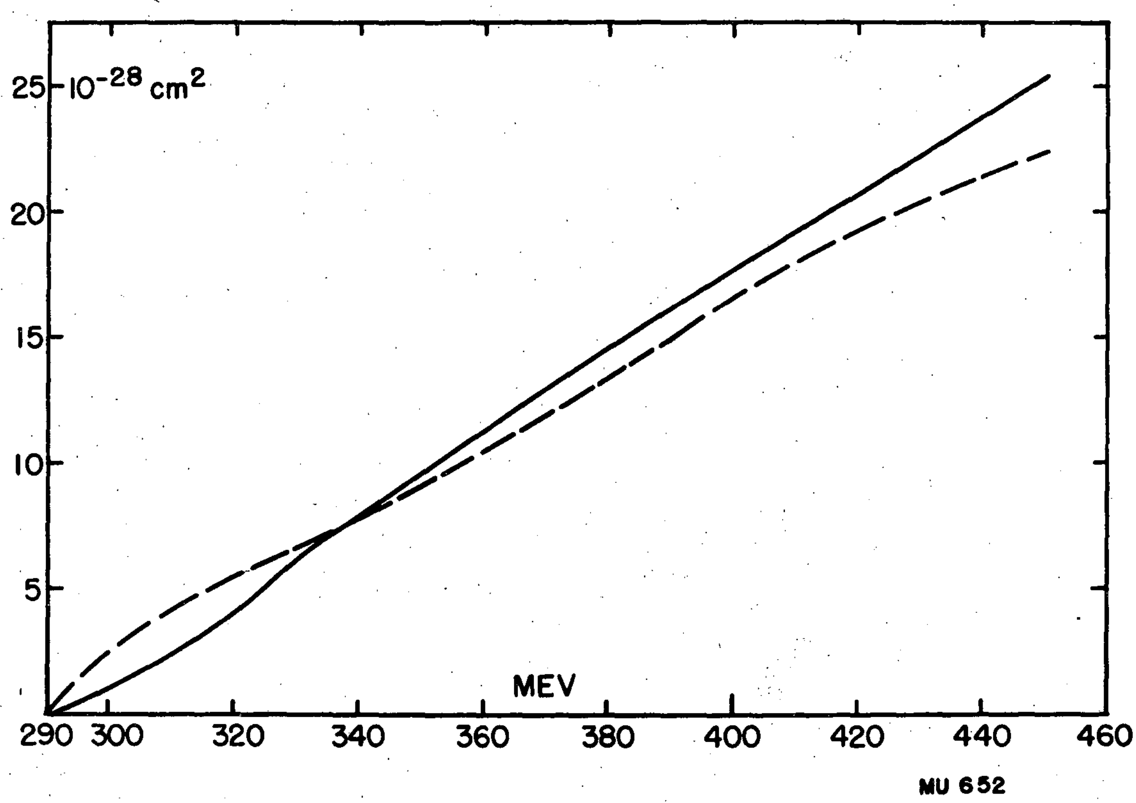

25
20
15
10
5
10^{-28} cm^2
MEV
290
300
320
340
360
380
400
420
440
460
MU 652

Fig. 3

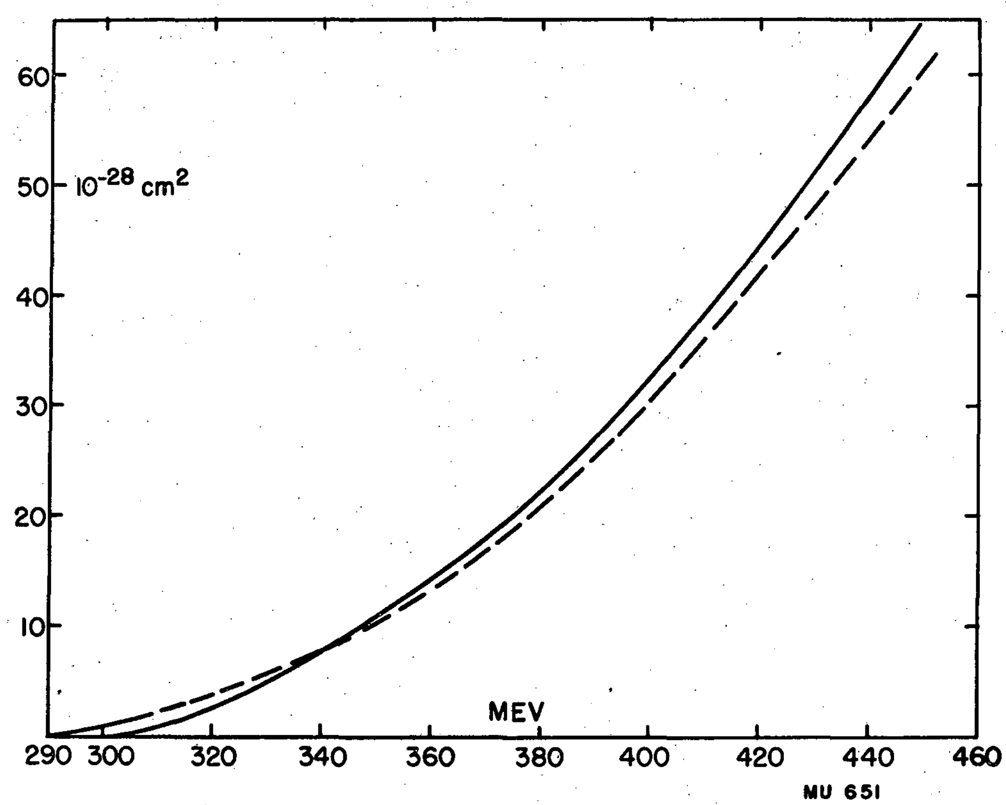
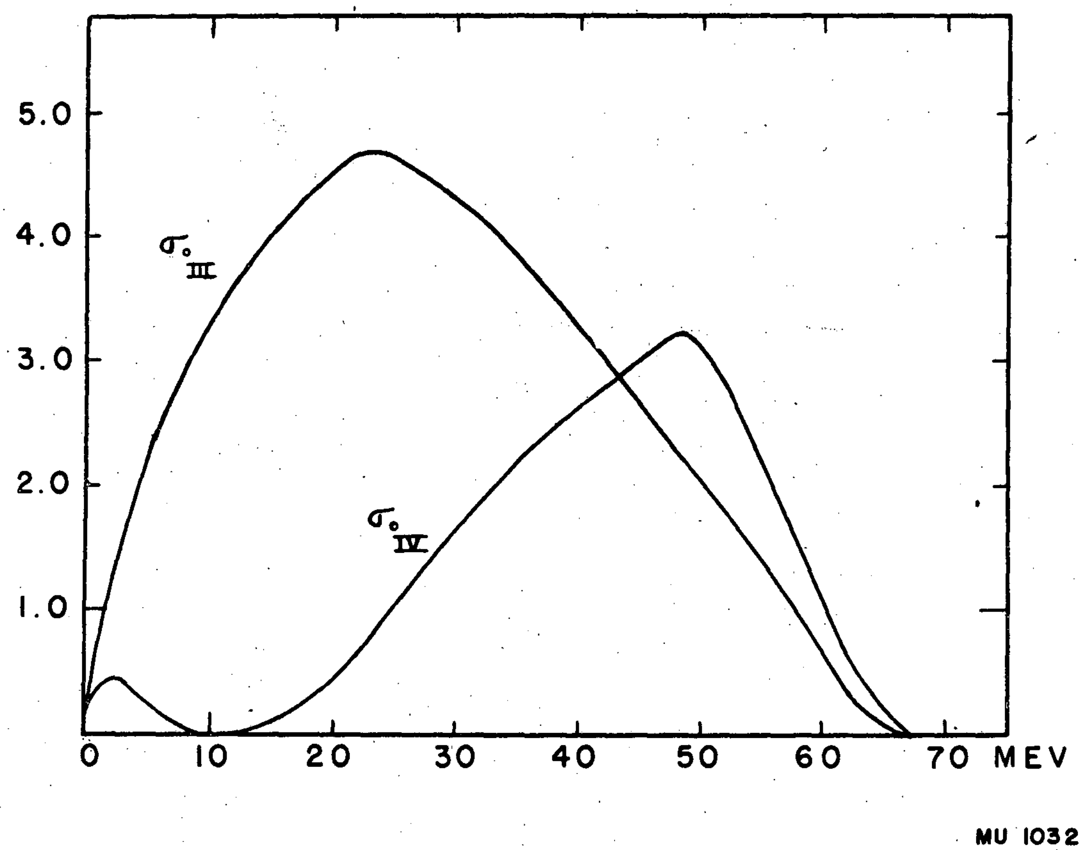
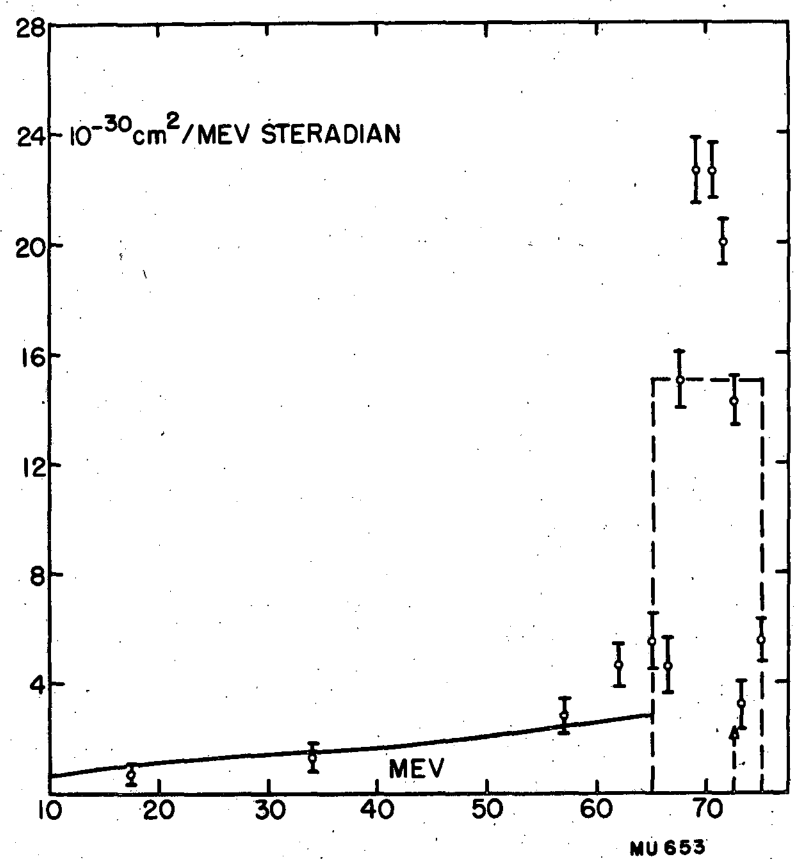

60
50
40
30
20
10
10^{-28} cm^2
MEV
290
300
320
340
360
380
400
420
440
460
MU 651

Fig. 4



5.0
4.0
3.0
2.0
1.0
$\sigma_{o_{III}}$
$\sigma_{o_{IV}}$
0
10
20
30
40
50
60
70 MEV
MU 1032

Fig. 5



28
24
$10^{-30}cm^2$/MEV STERADIAN
20
16
12
8
4
MEV
10
20
30
40
50
60
70
MU 653

Fig. 6

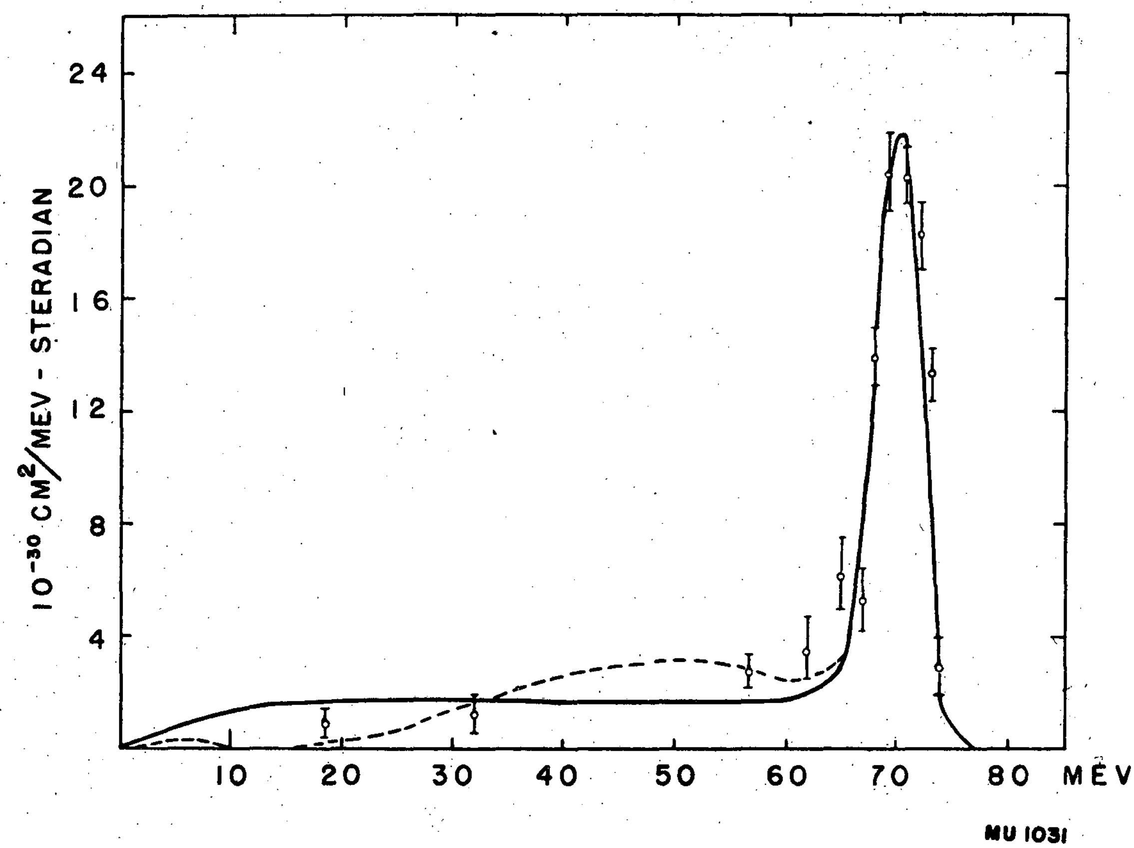

24
20
16
12
8
4
10^{-30} CM^2/MEV - STERADIAN
10
20
30
40
50
60
70
80 MEV
MU 1031

Fig. 8