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LETTER TO THE EDITOR

A theoretical analysis of resonant scattering of free carriers by electron-hole droplets in germanium

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Abstract. The longitudinal magnetoconductivity of electrons in the presence of electron-hole droplets is calculated using the Born approximation. The magnetoconductivity has an oscillatory structure with period:

$$\Delta B^{1/2} = \frac{\pi}{2a} \left(\frac{m_e \hbar}{2m_z e} \right)^{1/2}$$

where m_e and m_z are the masses for cyclotron motion and for motion parallel to the magnetic field respectively. This analysis applied to recent experimental data on Ge at 2 K gives a drop radius of 1.1 μm .

In a recent publication (Eaves *et al* 1976), we described resonances observed in the longitudinal magnetoconductivity of optically excited pure germanium at low temperatures. The resonances consist of an oscillatory variation of the conductivity as a function of magnetic field B , the oscillations having a constant periodicity when plotted against $B^{1/2}$. The observed structure arises from elastic scattering of electrons by a random distribution of electron-hole droplets which can be represented by hard-edged spherical potential wells of depth V_0 and radius a . In this Letter, we present in greater detail the calculations which are summarised in our earlier paper.

Using the method formulated by Roth and Argyres (1966) the longitudinal magnetoconductivity σ_{zz} can be written, neglecting spin, as

$$\sigma_{zz} = - \frac{e^2}{\Omega} \sum_{nk} \frac{df}{d\epsilon_{nk}} \tau(\epsilon_{nk}) (v_{nk}^z)^2 \quad (1)$$

where f is the electron distribution function, Ω is the volume of the crystal, τ is the relaxation time and v^z is the carrier velocity along the direction of the magnetic field. The scattering rate is assumed to consist of two parts

$$\tau^{-1}(\epsilon_{nk}) = \tau_r^{-1}(\epsilon_{nk}) + \tau_{ep}^{-1}(\epsilon_{nk}) \quad (2)$$

where τ_{ep}^{-1} is due to deformation potential scattering by acoustic phonons and τ_r^{-1} is the resonant scattering by electron-hole droplets.

For the Landau gauge $A = (0, Bx, 0)$, the electronic eigenstates are

$$|n, k\rangle = (L_y L_z)^{-1/2} \phi_n(x - X_k) \exp(i(k_y y + k_z z)) \quad (3)$$

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where L_y and L_z are the crystal dimensions and where $\phi_n(x - X_k)$ are eigenfunctions of a simple harmonic oscillator centred at $X_k = -\hbar k_y / m\omega_c$.

The resonant scattering rate is then given by

$$\tau_r^{-1}(\epsilon_{nk}) = \frac{2\pi n_i}{\hbar\Omega} \sum_{n'k'q} |V(q)|^2 \left(1 - \frac{k'_z}{k_z}\right) G(\mu)_{nn'} \Delta(\epsilon, k_y, k_z) \quad (4)$$

where

$$\Delta(\epsilon, k_y, k_z) = \delta(k_y - k'_y - q_y) \delta(k_z - k'_z - q_z) \delta(\epsilon_{nk} - \epsilon_{n'k'}) \quad (4a),$$

$$V(q) = 4\pi V_0 \left(\frac{\sin qa - qa \cos qa}{q^3} \right) \quad (4b),$$

$$G(\mu)_{nn'} = \left| \int_{-\infty}^{\infty} dx \exp(iq_x X) \phi_n(x + l^2 k_y + q_y) \phi_{n'}(x + l^2 k_y) \right|^2 \quad (4c),$$

$$\mu^2 = l^2(q_x^2 + q_y^2) = l^2 q_p^2 \quad (4d),$$

$$l^2 = \hbar/eB \quad (4e)$$

and n_i/Ω is the droplet concentration. The $G(\mu)_{nn'}$ integrals are given in the paper of Shin *et al* (1973).

The δ -functions make the integrations over k'_y and q_z trivial. The integration over k'_z and q_y is summarised in the Appendix and is given by equation (A.2). The following values were used to calculate τ_r^{-1} : $V_0 = 4$ meV; $n_i/\Omega = 10^{10} \text{ cm}^{-3}$; $a = 1-2 \mu\text{m}$. The depth of the well is assumed to equal the binding energy of an electron inside the drop.

The non-resonant part of the scattering rate τ_{ep}^{-1} is given by

$$\tau_{ep}^{-1}(\epsilon_{nk}) = \frac{2\pi E_1^2 k T dN}{\hbar \mu v_s^2 d\epsilon} \quad (5)$$

where $dN/d\epsilon$ is the density of states in the Landau levels, E_1 is the deformation potential, μ is the mass density and v_s is the velocity of sound. We have used the estimate of Fukai *et al* (1964), equation (27a), to evaluate this contribution to $\tau^{-1}(\epsilon_{nk})$.

The longitudinal magnetoconductivity was calculated by evaluating equations (4) and (5) numerically and substituting the resulting value of τ_{nk} into equation (2). Thus

$$\sigma_{zz} = \frac{e^2}{(2\pi l)^2 m^2 k T} \frac{\hbar^2 \exp(\epsilon_F/kT)}{l^3} \sum_n \int_{-\infty}^{\infty} \tau(\epsilon_{nk}) x^2 \exp(-\epsilon_{nk}/kT) dx$$

where $x = lk_z$ so that

$$\epsilon_{nk} = \hbar\omega_c \left(n + \frac{x^2 + 1}{2} \right). \quad (6)$$

This integral has been evaluated at 2 K and its derivative $\partial\sigma_{zz}/\partial B$ is plotted in figure 1 for two different drop radii. According to this simple spherical band model the oscillations have a constant periodicity given by

$$\Delta B^{1/2} = \frac{\pi}{2a} \left(\frac{\hbar}{2e} \right)^{1/2}$$

The physical origin of the oscillations is explained in figure 2 which shows the possible elastic transitions for electrons in state $|n, k\rangle$. Because of the infinite density of states at

Figure 1. Calculated oscillatory magnetoconductivity (plotted as $\partial\sigma/\partial B$) for $a = 1.0$ and $2.0 \mu\text{m}$. The calculation assumes a mixture of elastic acoustic phonon and resonant droplet scattering and a spherical band so that $m_z = m_c$ (see text).

Figure 2(a): The Landau level spectrum of electrons in a magnetic field. At points such as A and B the elastic scattering rate increases rapidly due to the singularity in the density of states of the next Landau level. In the text $A'A = \bar{q}_z = 2\bar{k}_z$. **(b):** Sawtooth variation of the scattering time in the n th Landau level as a function of k_z . The scattering potential $V(q)$ is an oscillatory function of q , the sharp cut-off in τ at $lk_z = \sqrt{2}$ gives rise to oscillations in $\sigma_{zz}(B)$.

the bottom of each Landau level, the transition rate becomes infinite whenever elastic scattering into the $(n + 1)$ th band is possible. Thus the scattering time $\tau(\epsilon_{nk})$ of level n has a saw-tooth variation when plotted against k_z . The longitudinal magnetoconductivity is dominated by carriers with long relaxation times corresponding to regions just below the discontinuities in the saw-tooth. The momentum transfer q for the intra-Landau level transition at the first discontinuity has a z component given by

$$\bar{q}_z = 2\bar{k}_z = \frac{2\sqrt{2}}{l} \quad (7)$$

since

$$\frac{\hbar^2 \bar{k}_z^2}{2m} = \hbar\omega_c = \frac{\hbar eB}{m}.$$

Numerical calculation shows that most of the oscillatory structure arises from carriers close to this energy.

For a simple scattering process such as that arising from acoustic phonons which is a smooth function of q , the singular structure in $\tau(\epsilon_{nk})$ will not be evident in the total conductivity since this involves an integration over k_z . However, for the oscillatory scattering potential given by equation (4b), extrema in the conductivity occur whenever

$$\bar{q}_z a = (N + \phi)\pi \quad (8)$$

where $N = 0, 1, 2, 3, \dots$ and ϕ is a phase factor.

Equations (7) and (8) give the resonant condition

$$a2\sqrt{2}/l = (N + \phi)\pi$$

or

$$\Delta B^{1/2} = \frac{\pi}{2a} \left(\frac{\hbar}{2e} \right)^{1/2} \quad (9)$$

As discussed by Eaves *et al* (1976), this resonance condition combined with the observed periodicity of the oscillations gives a droplet radius of 2.3 μm . It should be noted that equation (8) is nearly the same as the simple-minded dimensional resonance condition $a = nl$, ($n = 1, 2, 3 \dots$); that is, an integral number of cyclotron radii correspond to the size of the drop. The fit to our data is shown in figure 3.

Figure 3. A comparison of the experimental data (lower trace) and calculation (upper trace). Both curves plot the differential magnetoresistance $\partial\rho/\partial B$. The theoretical curve is for $a = 2.3 \mu\text{m}$ and assumes a spherical band. With the correction for the ellipsoidal conduction band structure of germanium this gives a drop radius of 1.1 μm .

The complex electron energy band structure of germanium slightly modifies the resonance condition. In a magnetic field, the eigenvalues are given by Miller and Omar (1961):

$$\epsilon = (n + \frac{1}{2})\hbar\omega_c + \hbar^2 k_z^2 / 2m_z \quad (10)$$

where $\omega_c = eB/m_c$,

$$\frac{1}{m_c^2} = \frac{\cos^2 \theta}{m_t^2} + \frac{\sin^2 \theta}{m_l m_t},$$

$$m_z = m_l \sin^2 \theta + m_t \cos^2 \theta$$

and θ is the angle between the magnetic field and the principal axis of the ellipsoid. In germanium $m_t = 0.082 m_0$ and $m_l = 1.6 m_0$. When m_z and m_c are different the critical

value of q_z becomes

$$q_z = \frac{2\sqrt{2}}{l} \left(\frac{m_z}{m_c} \right)^{1/2}$$

giving

$$\Delta B^{1/2} = \frac{\pi}{2a} \left(\frac{\hbar m_c}{2m_z e} \right)^{1/2} \quad (11)$$

For germanium $m_z = 0.59 m_0$ and $m_c = 0.135 m_0$ so that equation (11) gives a drop radius of $1.1 \mu\text{m}$ instead of the value of $2.3 \mu\text{m}$ quoted by Eaves *et al* (1976).

Our calculation neglects possible scattering of holes by electron-hole droplets since the hole contribution to the conductivity is likely to be less than that of the electrons. In any case, the complex form of the hole Landau level spectrum would greatly complicate the analysis.

It should be noted that the structure in figure 1 grows steadily in amplitude as a function of magnetic field whereas the experimental recordings showed a strong damping of the resonances at fields above about 0.6 kG . It is possible that this damping of the oscillations arises from a distribution of droplet sizes or shape which is not taken into account in this simple calculation.

Appendix

After integrating over k_y and q_z

$$\tau_r^{-1}(\epsilon_{nk}) = \left(\frac{n_i}{2\pi\hbar} \right) \sum_{n'} \int dk'_z dq_\rho |V(q)|^2 G(\mu)_{nn'} \left(\frac{k_z - k'_z}{k_z} \right) \delta(\epsilon_{nk} - \epsilon_{n'k'}). \quad (\text{A.1})$$

Now

$$\epsilon_{nk} - \epsilon_{n'k'} = \frac{\hbar^2}{2m} (\bar{k}_z^2 - k_z'^2)$$

where

$$\bar{k}_z^2 = k_z^2 - q_0^2$$

and

$$\frac{\hbar^2 q_0^2}{2m} = \hbar\omega_c(n' - n).$$

Thus

$$\begin{aligned} & \int dk'_z \delta(\epsilon_{nk} - \epsilon_{n'k'}) F(q, k_z, k'_z) \\ &= \frac{2m}{\hbar^2} \int dk'_z \left\{ \frac{1}{2\bar{k}_z} [\delta(\bar{k}_z - k'_z) + \delta(\bar{k}_z + k'_z)] \right\} F(q, k_z, k'_z) \end{aligned}$$

giving, for $k_z^2 > q_0^2$,

$$\tau_r^{-1}(\epsilon_{nk}) = C \sum_{n'} \left[\frac{k_z - \bar{k}_z}{k_z \bar{k}_z} I(k_z - \bar{k}_z) + \frac{k_z + \bar{k}_z}{k_z \bar{k}_z} I(k_z + \bar{k}_z) \right] \quad (\text{A.2})$$

and zero for $k_z^2 < q_0^2$.

In this equation

$$C = \frac{mn_i}{2\pi\hbar^3}$$

and

$$I(q_z) = \int_0^\infty dq_\rho q_\rho |V(q)|^2 G(\mu)_{nn}.$$

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