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Peer Instruction as a Way of Promoting Spontaneous Use of Diagrams When Solving Math Word Problems

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Abstract

Although using diagrams is generally considered one of the most effective strategies for solving math word problems, research and educational practice reports indicate that students lack spontaneity in their use of diagrams. In an attempt to address this problem, the present study investigated the hypothesis that creating situations whereby students have to teach other students how to solve math word problems with the use of diagrams would promote those students' subsequent spontaneous use of diagrams. Experimental classes were conducted with 57 8th-grade students for five days. Participants in the experimental condition were all given opportunities to explain to other students in their group their way of solving the math word problems given. In contrast, in the control condition, only some of the students were given opportunities to make presentations in front of the class about their way of solving the problems. In both conditions, the teacher encouraged diagram use during the instructions provided. The main finding was that, in the post-instruction assessment, those in the experimental condition evidenced greater spontaneous use of diagrams in attempting to solve the math word problems given. This finding suggests that, as a consequence of the peer teaching experience – which provided opportunities for the use of diagrams as communication tools – participants internalized diagrams as tools for problem solving. The protocol of peer interaction was also analyzed to better understand the mechanisms involved in this effect.

Keywords: Diagram Use, Math Word Problem Solving, Peer Instruction

Introduction

Diagrams are generally considered as one of the most effective tools in problem solving. For example, Larkin and Simon (1987) showed that diagrammatic representations are superior to sentential representations because they help reduce mental computational loads associated with searching and processing. Other studies have also empirically demonstrated that diagrams effectively promote the performance of many types of tasks (e.g., Ainsworth & Th Loizou, 2003; Cheng, 2002; Koedinger & Terao, 2002; Larkin & Simon, 1987; Mayer, 2003; Polya, 1945; Schoenfeld, 1985; Stern, Aprea, & Ebner, 2003).

As a sub-category of diagrams, those that are self-constructed are especially effective. Although some studies have suggested that self-constructed diagrams do not always promote success in problem solving (e.g., Van Essen & Hamaker, 1990), the findings of recent studies mostly indicate that self-constructed diagrams provide powerful reasoning strategies when some prerequisite conditions are met (e.g., Stern, et al., 2003; see also the review by Cox, 1999).

It appears, however, that school students do not appreciate the efficiency that diagram use brings to problem solving as much as teachers and researcher do. A number of problems about students' use of diagrams have been reported, which include lack of spontaneity in their use (e.g., Ichikawa, 1993, 2000; Uesaka, Manalo, & Ichikawa, in press), inappropriate construction (e.g., Cox, 1996; Uesaka & Manalo, 2006), and failure to draw correct inferences even when appropriate diagrams have been used (e.g., Cox, 1996).

These problems concerning students' use of diagrams have not been sufficiently examined in previous studies. The present study focused on the lack of spontaneity with which students use diagrams in math word problem solving: a teaching method for promoting spontaneity was developed and evaluated.

1.1 Students' Lack of Spontaneity in Diagram Use

Research reports relating to educational practices suggest that students' failure to spontaneously use diagrams occur despite plenty of exposure to appropriate use of diagrams. Ichikawa (1993), for example, reported in a tutoring case study an 8th-grade girl who did not spontaneously use diagrams in a test situation and failed to solve the problem given – despite previously being taught how to solve similar problems with the use of diagrams. Ichikawa (2000) described the extent to which this problem is found among students. Dufour-Janiver, Bednarz, & Belanger (1987) also noted the lack of spontaneity in diagram use among students and observed that this problem occurs even though math teachers use a lot of diagrams in class. Uesaka, Manalo, & Ichikawa's (in press) empirical study confirmed this tendency particularly among Japanese students.

If students do not (or cannot) use diagrams in situations where they would be deemed appropriate, they (the students) would clearly be disadvantaged. Although in some educational contexts suitable diagrams are provided or suggested, in the majority of cases – including 'real world' situations – students are required to construct and use diagrams by themselves according to necessity. In light of the previously identified problems, it would seem that interventions additional to traditional classroom instructions

are necessary to effectively promote student use of diagrams in problem solving.

However, most of the research studies that have been done about diagrams have focused on their effects and functions (e.g., Ainsworth & Th Loizou, 2003; Cheng, 2002, 2004; Koedinger & Terao, 2002; Larkin & Simon, 1987; Mayer, 2003) and few have empirically examined possible teaching methods that would promote spontaneity of their use when problem solving. Uesaka (2003) is one exception in that her report proposed an actual teaching method for enhancing spontaneous use of diagrams. She found evidence that diagram use can be promoted via a combination of two interventions – verbal encouragement from the teacher (to enhance students' perceptions about the efficiency that diagram use brings), and training in drawing (to improve their diagram construction skills).

Although Uesaka (2003) succeeded in identifying instructional components that promote the spontaneous use of diagrams, the method she proposed requires a fair amount of environmental control that is difficult to achieve outside of experimental situations. Thus a necessary next step is to develop a teaching method that can more naturally be employed in real classroom teaching situations. This constituted the main aim of the present study.

1.2 Diagrams as Communication Tools

Diagrams are effective tools not only for problem solving but also for communication (e.g., Lyon, 1995; Tversky, Lozano, Heiser, Lee, & Daniel, 2005; see also review by Sacchett, 2002). For example, Lyon (1995) reported that diagrams facilitated communication with adults who, because of aphasia, found communication through verbal means difficult. Moreover, as pointed out by Dufour-Janiver et al. (1978), math teachers use a lot of diagrams for explaining how to solve problems in class, and this practice contributes to the promotion of students' understanding of the processes involved. Diagrams thus work equally well as tools of explanation as they do as tools for problem solving.

Although previous studies have examined these two aspects of diagram use separately, data gathered by Uesaka et al. (in press) suggest the possibility that viewing diagrams as tools of communication promotes spontaneous use of diagrams when problem solving. Uesaka et al. found greater use of diagrams among students in New Zealand compared to Japan, and this difference appeared related to differences in the students' experiences about the use of diagrams as communication tools. They pointed out that the New Zealand school curriculum included the objective of providing "opportunities for students to develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalizations" (New Zealand Ministry of Education, 1992, p. 10). Such an objective is absent from the Japanese school curriculum which focuses primarily on the math content that students should acquire.

If the learning environment is such that students have to use visual representations such as diagrams when providing explanations (even in cases, for example, where they can solve the problems given without the use of diagrams), other benefits associated with the use of diagrams are likely to follow. Firstly, as Ainsworth and Th Loizou (2003) pointed out, explaining with the use of diagrams improves understanding of the nature of the problem. It contributes to enhancing students' appreciation of the efficacy that diagram use brings. Secondly, more experiences of constructing diagrams from the math word problems given would be provided and, as a result, diagram drawing skills would be promoted. There is clear congruence therefore between Uesaka's (2003) finding that improving perceptions and skills associated with diagram use promotes actual diagram use, and the notion that if diagrams are seen and employed as communication tools their use would likewise increase.

A similar line of argument exists in the area of reading strategy research: this suggests that using strategies during collaborative learning situations promotes the spontaneous use of those strategies when students subsequently work on their own. For example, Palincsar and Brown's (1984) study on reciprocal teaching showed increases in students' spontaneous use of reading strategies when their teacher encouraged them to use those same strategies when listening and attempting to understand explanations provided by other students.

However, the applicability to natural settings of the two instructional components that Uesaka (2003) identified has not previously been examined; nor has the notion that "using diagrams as communication tools leads to greater appreciation of their benefits and thus increases in their actual use" been properly investigated. The present study therefore sought answers to the questions arising from these through the use of experimental classes in which the "jigsaw method" for learning (Aronson, 1978) was employed and all students had to provide explanations to other students with the use of diagrams. The main hypothesis of the present study was that students who receive opportunities to explain the use of diagrams to their peers would evidence increases in their own spontaneous use of diagrams when solving math word problems.

Method

2.1 Participants

Fifty-seven 8th-grade students (female = 25, male = 32), who were recruited from public junior high schools in two wards of Tokyo and a junior high school affiliated with the University of Tokyo, participated voluntarily in this study.

Participants were assigned to one of the two conditions (experimental and control) by using a randomized block design, in which they were grouped according to their achievements at school. Information about the participants' achievements was collected from parents through the use of a questionnaire that was mailed out to them.

2.2 Materials

Math Word Problems Used in Instruction Sessions. The study was carried out over the course of six days, three of which – Days 2 to 4 – were devoted to instruction. In each of the instruction days, two math word problems, with similar story contexts and requiring similar types of diagrams for their correct solution, were used. 'Arrangement problems', for which constructing both a table and drawings to represent the situations described in the problems was deemed helpful, were used on Day 2. 'Mobile phone problems', for which the construction of graphs was deemed effective towards arriving at their correct solutions, were used on Day 3. 'Area problems', for which the use of tables was effective, were used on Day 4. As illustrative examples, the two problems that constituted the 'mobile phone problems' are shown in Appendix 1. Examples of diagrams constructed by participants are shown in Fig. 1.

(A) Arrangement Problem   (B) Mobile Phone Problem
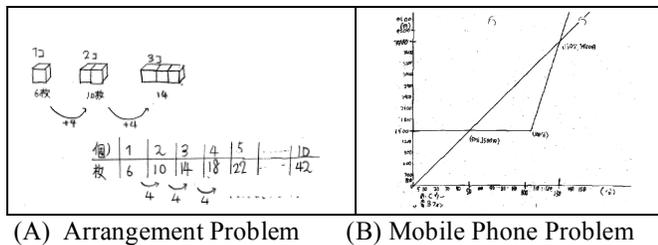
Fig. 1. Typical Diagrams Participants Produced during the Instruction Sessions. (A) is for one of the arrangement problems, and (B) for one of the mobile phone problems.

Math Word Problem Solving Assessment. A math word problem solving assessment was administered at the end of the experiment to examine the quantity and quality of the diagrams spontaneously produced by the participants. In this assessment, four more math word problems were used. These problems were comparatively more difficult than the ones used during the instruction sessions, and their level of difficulty was intended to convey to the students the need to use visual representations (e.g., diagrams) when attempting to solve them. Solving two of the problems ('tile problem' and 'pentagon problem') would have been facilitated by the use of tables, and the other two problems ('water problem' and 'travel problem') would have been facilitated by the use of graphs. Examples of the problems used are shown in Appendix 2. The participants were given a total of 45 minutes to work on the problems. Four university colleagues, including a qualified math teacher, independently considered the most effective kinds of diagrams to use in attempting to solve these problems, and all concurred on the kinds of diagrams noted above.

Basic Skills Assessment. The basic skills assessment was administered on the first day of the study to check the equivalence of the participants in the experimental and control conditions as far as their skills in constructing tables and graphs were concerned. The assessment consisted of two questions which required the participants to construct a table and then a graph according to information provided in the form of sentences. Two versions of booklet were made, each version being given to half of the participants in each condition. The participants were allowed 5 minutes to work on each of the two questions in the booklet.

2.3 Procedure

Data collection and instruction sessions were conducted over 6 days at the University of Tokyo. Pre-instruction assessment was carried out on the first day, and post-instruction assessment on the final two days. As noted earlier, the instruction sessions were provided on the second to fourth days. These instruction sessions lasted approximately 50 minutes each day, and participants assigned to the same condition took the classes together.

During the instruction sessions, the teacher presented two math word problems each day and employed a consistent teaching procedure for each group. Firstly, the teacher encouraged the students to carefully read and think about the problems given so that they would understand the nature of these problems. During this time, the teacher asked the students in the small group (of usually 4) to split into two smaller groups and assigned one of the two problems to each. Secondly, the teacher asked the students to solve the problem they had been assigned by themselves. However, prior to letting the students attempt solving the problem, the teacher explicitly encouraged them to use diagrams – pointing out their usefulness for solving problems. Participants in the experimental condition were also told that they would be asked to explain to another student later and were encouraged to use diagrams when explaining. During this period, the teacher also provided as much help as the students needed. For example, the teacher encouraged students who wanted to receive hints to gather in front of the board where the teacher then provided hints, as well as demonstrated steps leading to the correct solution of the problem and the use of appropriate diagrams.

Thirdly, in the control condition, some of the participants were asked to present their ideas about how to solve the problem they were assigned in front of the class. This kind of presentation of one's ideas about how a problem might be solved is quite common in typical Japanese classrooms. The teacher contributed as necessary to each presentation to ensure that it led to an appropriate approach to solving the problem. In contrast, in the experimental condition, the students were asked to explain to another student in the group how to solve the problem they had been assigned. After this, the teacher asked a student from the class to present his or her idea for solving the problem to the class. Like in the control condition, the teacher contributed to the presentation as was necessary in each case.

On the final two days, the students' spontaneous use of diagrams in problem solving was evaluated by asking them to individually solve some math word problems. All representations, except words and equations, generated by the participants were judged as 'spontaneous diagram use'.

Results

Only participants who attended all sessions were included in the analyses. The final number of participants included in the analyses was 42 (for the experimental condition, n =21; for the control condition, n =21).

3.1 Math Word Problem Solving Assessment Findings

The students' spontaneous use of diagrams in the math word problem solving assessment was analyzed. Before conducting the analyses, the participants' responses to the problems were scored. A diagram was defined as any representation of the problem other than words (on their own), sentences, or numerical formulas. Tables were also counted as diagrams for the purposes of this study and defined as a depiction of at least a pair of values arrayed to represent two related variables. If a participant constructed at least one diagram when solving a problem, the participant's response to that problem was coded as "used diagrams (1)". Otherwise, the response was coded as "no diagram (0)".

A t-test was used to compare the average numbers of problems in which participants "used diagrams" in the two conditions. As depicted in Fig. 2, the result indicates significantly more spontaneous use of diagrams in the experimental condition compared to the control condition ($t_{(40)} = 2.86$, $p < .01$). This finding supports the main hypothesis of this study.

An analysis was also undertaken to compare the quality of diagrams produced by participants in the two conditions. A diagram was categorized as "appropriate" if it matched the type of diagram (i.e., table, or graph) the four colleagues (noted earlier) considered most appropriate for the problem given. Otherwise the response was categorized as being "inappropriate".

Again a t-test was used to compare the average number of problems in which the participants constructed "appropriate diagrams" in the two conditions. The result, also depicted in Fig. 2, indicates that the participants in the experimental condition produced significantly more appropriate diagrams compared to those in the control condition ($t_{(40)} = 4.36$, $p < .01$). Together, these findings suggest that explaining how to solve math word problems to other students (with the use of diagrams), promotes not only the spontaneous use of diagrams in subsequent attempts at solving problems, but also the construction of better – more appropriate kinds of – diagrams.

3.2 Basic Skills Assessment Findings

To check the equivalence of the experimental and control conditions at the beginning of the experiment, participants' responses to the basic skills assessment that was administered on the first day (prior to any instructions being provided) were analyzed and compared. For each question, a perfectly correct answer was given full credit (2 points), an answer which was mostly correct but contained a small
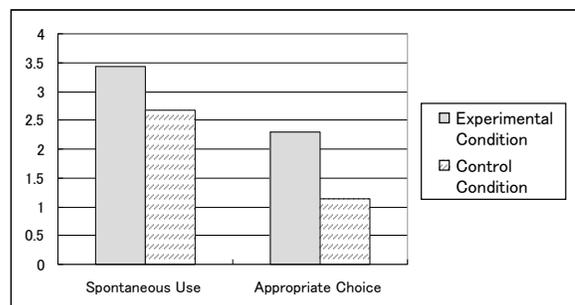
Fig 2. The Mean Number of Problems in which Diagrams Were Spontaneously Used (left), and in which Appropriate Diagrams Were Constructed (right) in the Math Word Problem Solving Assessment.

Fig 3. Examples of Diagrams Produced During the Post-Instruction Sessions.

mistake was given partial credit (1 point), and the rest were given no credit (0 point). The participants' total scores for the two questions were compared using a t-test. No significant differences were found between the two conditions ($t_{(40)} = .62$, n.s). This finding suggests that participants in the two conditions were equivalent in their diagram construction skills at the beginning of the study.

3.3 Example of Protocol During Classes

The findings of this study suggest that using diagrams as tools for explanation subsequently promoted the spontaneous use of diagrams as tools for problem solving. However, the kind of mechanism involved in this process was not clear. Therefore, the present study also analyzed the typical protocol of the experimental condition and considered the instructional features that the participants in these groups received via the manipulations made.

An example of protocols during the explanation time in the instruction sessions provided is shown in Fig. 4. In this protocol, S1 and S3 tried to explain how to solve "mobile phone problem 2" with diagrams to S2 who was earlier assigned to do the other problem ("mobile phone problem 1"; see the Appendix 1). Firstly, S1 explained her idea about

how to solve the problem using diagrams she sketched, and S2 questioned her. S3 afterwards participated in this communications in order to add information to address the question that S2 asked. They communicated with each other with the use of diagrams. Actually S3 originally did not use diagrams for solving the problem (when he was working on his own), but he constructed diagrams to explain how to solve the problem to S2.

This case – and many others like it observed during the study – suggests the possibility that encouraging students to explain how to solve math word problems to other students promotes the likelihood that participants would actually construct diagrams. It is unlikely that, in the example protocol described, S3 would have constructed diagrams if they had not been asked to explain to another student how to solve the problem (which he could solve correctly without the use of diagrams). He constructed the diagram because he experienced difficulty in explaining to S2 how to solve the problem with the use of only words. The experimental manipulation used here of asking students to explain to other students increases the likelihood of students 'needing' to use diagrams – and therefore promotes diagram construction and the enhancement of skills associated with it (e.g., drawing, thinking, communicating).

It is also possible that the opportunities to explain to others helped the participants to understand and appreciate better the efficiencies that diagram use brings. Responses written on the review sheets (which, among other things, asked participants what they learnt in the classes they attended) suggest this. S2, for example, noted that "when a graph is drawn, the problem becomes dramatically easier". S1 also commented that "graphs and tables are convenient tools for solving problems".

S1: Up to the 100th minute, it takes 1500 yen. After that, the cost would gradually increase as shown in this diagram (she pointed to a diagram she had drawn).
S2: I know we can explain only with diagrams.
S1: Yes.

S1: Until this point, plan B is cheaper. But after this point, the cost gradually increases and plan C becomes a little bit cheaper after this crossing point … Moreover, after this next crossing point, plan B becomes cheaper again.

S2: It looks like plan C is cheaper only for such a short period …?
S1: Yes, only this period.
S2: Very short …
S1: Don't worry about that.
S3: Well, actually, this diagram shows this period is very short, but actually the period is much longer (he pointed to his diagram which was more accurately drawn compared to S1's, on a sheet of grid paper. His graph was shown as an example in Fig. 1).

Fig 4. Example of Protocols During the Instruction Session in the Experimental Condition.

Discussion

The main finding of the present study was that students in the experimental condition, who were given opportunities to teach their peers how to solve the math word problems they were assigned, evidenced greater spontaneous use of diagrams in subsequent assessment compared to those in the control condition. There were also higher proportions of the diagrams produced by those in the experimental condition that matched the kinds of diagrams deemed most appropriate for use with the problems given. These findings suggest that two functions of diagrams – as tools for problem solving and as tools for communication – which have been viewed separately in the past, may in fact be related.

The possibility of the differences found between the experimental and control groups coming from some other variable other than the instruction sessions provided is clearly minimal in light of the students' performance on the basic skills assessment. This assessment, which was administered at pre-instruction, clearly showed that there were no significant differences between the experimental and control groups in their abilities to construct diagrams – more specifically, graphs and tables – at the beginning of the study, prior to any instructions being provided.

The examination of the protocols during the student-to-student teaching/explanation sessions suggested that teaching other students provided more practice in constructing and using diagrams, as well as possibly more opportunities to consider and appreciate the efficiencies and other benefits that diagram use brings. As noted earlier, the value of practice and appreciation of the benefits of diagram use were similarly identified in Uesaka (2003). As the examination of the protocols involved in the peer explanation/teaching sessions undertaken in this study was only at an exploratory level, a more thorough and systematic investigation into these processes is needed in future studies. Such an investigation would need to address the actual nature of the relationship between using a tool like diagrams for communication purposes and incorporating the tool into one's personal repertoire of problem solving skills.

One important contribution of the present study to math education is that it suggests what appears to be a viable and effective method for promoting students' use of diagrams in math word problem solving – a method that can easily be adopted in natural, real-life classroom settings. The findings suggest that the commonly-used method (in some countries like Japan) of simply asking students to make short presentations about their ideas in front of the class is not as effective in promoting the desired behaviors – in this case using diagrams. However, giving students opportunities to explain to other students appears to promote the desired behavior. This finding is congruent with earlier findings about the benefits of peer tutoring and collaborative learning, and it highlights the various components of learning that are facilitated (e.g., reflection, comparison, insight, understanding, practice) when such procedures are used – not only for the tutee, but most importantly also for the tutor.

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Acknowledgement

Appendix 1: Examples of Math Problems Used in the Instruction Sessions

Mobile Phone Problem (1)

You are a clerk in a mobile phone shop which sells two types of mobile phone plans. When a customer asks your advice, which plan would you recommend as being cheaper depending on calling time?

Plan A: The basic fee is 4500 yen, and no extra cost is charged.

Plan B: There is no basic fee, and no free calling time. The cost of calls is 30 yen per minute.

Mobile Phone Problem (2)

In another shops, two different types of mobile phone plans are sold. When a customer asks your advice, which plan would you recommend as being cheaper depending on calling time?

Plan B: There is no basic fee, and no free calling time. The cost of calls is 30 yen per minute.

Plan C: The basic fee is 1500 yen including 100 minutes of free calling time, with 80 yen per minute charged thereafter.

Appendix 2: Examples of Math Problems Administered at Post-Instruction Assessment

Water Problem

The head of a company asked Taro to find out which of three countries – A, B, or C – would be best for establishing a factory which uses water. The different charging methods of each country are described below. Please imagine you are Taro, and come up with an explanation for the head of the company.

Country A: 1000 yen is charged as a basic fee, but you can use water without additional charge up to 100 litres. After 100 litres, 40 yen/litre is charged.

Country B: There is no basic fee. Water cost is 20 yen/litre.

Country C: In addition to 2400 yen as a basic fee, there is a charge of 4 yen/litre of water used.

Pentagon Problem

There are many sheets of paper in the shape of a regular pentagon, with each side being 1 cm. These sheets are arranged one by one with the rule that a new sheet shares only one side with already arranged sheets. Find the circumference when arranging 1, 5, 10 and 20 sheets.