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ABSTRACT

The reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+$ has been studied at 1170 Mev/c in the 72 inch liquid hydrogen bubble chamber. An analysis of 251 events gave a total cross section of 0.205 ± 0.014 mb and values of $\alpha^0 \overline{P}_{\Sigma^+} = 0.62 \pm 0.19$, $\alpha^+ \overline{P}_{\Sigma^+} = -0.18 \pm 0.13$. Based on the differential cross section and Σ^+ polarization, the s-, p-, and d-wave amplitudes were determined:

$$s_{1/2} \equiv a = 0.061 \pm 0.016\ (\mathrm{mb/sr})^{1/2},$$

$$\frac{p_{1/2} + 2p_{3/2}}{s_{1/2}} \equiv \frac{b \exp i\chi_b}{a} = (2.14 \pm 1.13) \exp (34.7 \text{ deg} \pm 29.2 \text{ deg})i,$$

$$\frac{p_{3/2} - p_{1/2}}{s_{1/2}} \equiv \frac{c \exp i\chi_c}{a} = -(1.46 \pm 1.78) \exp (20.5 \text{ deg} \pm 26.1 \text{ deg})i,$$

$$\frac{3d_{5/2} + 2d_{3/2}}{s_{1/2}} \equiv \frac{d \exp i\chi_d}{a} = -(1.02 \pm 0.97) \exp (-7.4 \text{ deg} \pm 60.2 \text{ deg})i,$$

and

$$\frac{d_{5/2} - d_{3/2}}{s_{1/2}} \equiv \frac{e \exp i\chi_e}{a} = -(0.72 \pm 0.63) \exp (91.9 \text{ deg} \pm 25.8 \text{ deg})i.$$

An alternative set of solutions exists for the transformations $\chi_b \rightarrow -\chi_b$, $\chi_c \rightarrow 180 \text{ deg} - \chi_c$, $\chi_d \rightarrow -\chi_d$, $\chi_e \rightarrow 180 \text{ deg} - \chi_e$. Based on extrapolation of s and p amplitudes obtained near threshold in a previous experiment in which angular distribution, polarization, and energy dependence were used, an attempt has been made to resolve the Minami ambiguity in the amplitudes of

this experiment. Currently available data of Σ^- and Σ^0 production by (π^-, p) interaction at 1220 Mev/c were used to test the charge-independence hypothesis. Within the limits of statistical accuracy, no evidence of a violation of charge independence was observed.

PRODUCTION OF Σ^+K^+ BY (π^+, p) INTERACTION AT 1170 Mev/c*

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I. INTRODUCTION

As a continuation of a previous analysis of the reaction $\pi^+ + p \to \Sigma^+ + K^+$ near threshold (1020 Mev/c)[1] the same reaction has been studied for an average incident laboratory-system momentum of 1170 Mev/c. From a scan of 35 000 pictures obtained with the 72-inch liquid hydrogen bubble chamber, approximately 400 events have been identified. After removal of events that would have led to experimental biases, a sample of 251 events was used for a determination of the s-, p-, and d-wave production amplitudes based on the observed angular distribution and sigma polarization. By comparing the s- and p-wave amplitudes obtained near threshold with those at 1170 Mev/c, an attempt has been made to remove the Minami ambiguity in the identification of these amplitudes. The present data, along with those available for the production of Σ^- and Σ^0 at about the same incident pion momentum, have been compared in order to test the charge-independence hypothesis.

II. EXPERIMENTAL DETAILS

The conditions under which the pictures have been obtained were very similar to those of the previous experiment.[1] The bubble chamber was operated in a magnetic field of 18 kgauss. The incident momentum was such that by energy loss by ionization in the hydrogen the events were produced in a momentum interval of 1153 to 1183 Mev/c. The beam resolution (half width at half-maximum) was about 3 Mev/c. A precise knowledge of the beam resolution was not required, as it is in threshold analysis, since the energy dependence of the amplitudes has not been taken into account.

Using the same technique as described previously,[1] the beam contamination was found to be 3.0± 1.6% for the combined positron and muon background, and 2.1± 1.2% for the proton background. The kinematical upper limits of the energy of the δ ray as produced by 1170-Mev/c pions, muons, and positrons are 68, 113, and 1170 Mev/c respectively. The theoretical cross sections for producing δ rays of energy 68 through 113 Mev/c by 1170-Mev/c muons and positrons are 0.33 mb and 1.38 mb respectively.[2] The corresponding cross section for δ rays with energy greater than 113 Mev/c produced by positrons is 1.53 mb. To determine the proton contamination, the following total cross sections have been used: 15.3± 1.5 mb for the elastic (π^+, p) scattering,[3] and 25.2±0.8 mb for the elastic (p, p) scattering.[4] In order to remove the forward-scattering ambiguity, the two-prong events were accepted for a c.m. angle of the proton greater than 36 deg. The corresponding cross sections were evaluated, using the above elastic cross sections and elastic differential angular distributions available for neighboring incident energies.[3,4]

III. RESULTS

For an average pion energy of 1170 Mev/c, the laboratory-system momentum of the Σ^+ varies from about 550 Mev/c for those sigmas produced in the backward direction in the c.m. system, to about 950 Mev/c for those produced in the forward direction. In order to avoid errors arising from a bias against short sigmas, only events in which the sigma track length exceeds a certain limit were accepted for the final analysis, and an appropriate correction factor was introduced for the interpretation of the experimental results. From a preliminary analysis of the data as a function of the various parameters upon which the scanning and measurement efficiencies can depend, the following selection criteria were adopted: (a) all acceptable beam tracks were required to enter through the bubble chamber thin window ($-4.0 \leq x_w \leq 10.0$ cm, $13.0 \leq z_w \leq 19.0$ cm), where x_w and z_w are the two coordinates normal to the incident beam direction, measured at the window; (b) it was required that the beam meet criteria in azimuth ($108.5 \text{ deg} \leq \phi_w \leq 111.5$ deg) and dip ($-1.0 \leq \lambda_w \leq 2.5$ deg) at the window, so that acceptable beam tracks were well collimated in the chamber; (c) the fiducial volume was furhter defined by $-70.0 \leq y \leq 55.0$ cm, where y is the dimension parallel to the beam with y=0 at the center of the chamber; (d) the angle of the sigma decay plane with respect to the optic axes of the bubble chamber cameras was restricted to be greater than 20 deg for the $\Sigma^+ \rightarrow p + \pi^0$ decay mode, and greater than 10 deg for the $\Sigma^+ \rightarrow n + \pi^+$ decay mode, since events with decay angles smaller than these were observed to be lost by scanners because they were edge-on with the line of sight of the cameras and interpretable as two-prong events; (e) the forward-backward distribution of the decay pion in the Σ^+ rest frame for the $\Sigma^+ \rightarrow n + \pi^+$ decay mode showed no evidence of scanning loss for events satisfying the above criteria. For the $\Sigma^+ \rightarrow p + \pi^0$ decay mode, a cutoff of $+0.90 \leq \cos\psi$ (proton) ≤ -0.97 was required, where ψ is measured in the Σ^+ rest frame; this included all events in which the Σ^+ decayed at an angle of less than about 5 deg in the laboratory system; (f) the distribution of the lengths of the sigma tracks for the events satisfying the above criteria indicated significant loss for a length less than 3.5 mm in the $\Sigma^+ \rightarrow n + \pi^+$ decay mode and 7.5 mm for the $\Sigma^+ \rightarrow p + \pi^0$ decay mode. Accordingly, each accepted event was weighted with a factor equal to

$$+ \exp\left[\frac{\ell_{min} M_{\Sigma^+}}{3p_{\Sigma^+} \tau_{\Sigma^+}}\right] ,$$

representing the inverse of the probability for the observation of a sigma track with a length greater than ℓ_{min}. Here $\overline{\tau}_{\Sigma^+}$, the mean lifetime of the sigma (in 10^{-10} sec), has been taken equal to 0.75,[1] p_{Σ^+} is the laboratory-system momentum of the Σ^+, and M_{Σ^+} its mass (1189 Mev).

Figure 1 shows the production c.m. angular distribution, corrected for short sigma tracks. Figure 2 shows the angular distribution of the proton in the rest frame of the Σ^+ with respect to the normal to the production plane for the $\Sigma^+ \rightarrow p + \pi^0$ decay mode. A similar distribution for $\Sigma^+ \rightarrow n + \pi^+$ is given in Fig. 3. Figure 4 gives the decay asymmetry in $\Sigma^+ \rightarrow p + \pi^0$, measured with respect to the normal to the production plane as a function of the sigma c.m. production angle. The production angular distribution has been fitted satisfactorily within the statistical limits of the data by a polynomial of order five in $\cos\theta$. No significant improvement in the fit could be achieved with a polynomial of higher order. The data of Figs. 2 and 3 have been fitted to the function $(1 + a \cos\phi)$, using the maximum-likelihood method. Results obtained for the polarization are $\alpha^0 \overline{P} = + 0.62 \pm 0.19$, and $\alpha^+ \overline{P} = -0.18 \pm 0.13$. The total cross section for $\pi^+ + p \rightarrow \Sigma^+ + K^+$ at 1170 Mev/c was measured to be 0.205 ± 0.014 mb, using the information contained in Table I. The numbers of corrected events corresponding to the two decay modes were determined separately. By adding all events, each corrected by the weighting factor

$$+ \exp\left(\frac{\ell_{min} M_{\Sigma^+}}{3p_{\Sigma^+} \tau_{\Sigma^+}}\right) ,$$

the number of events corrected for short sigma tracks was obtained. For the $\Sigma^+ \rightarrow p + \pi^0$ decay mode, the number of events has been further corrected by assuming that the forward-backward angular distribution in the rest frame of the Σ^+ is isotropic. No such correction was required for the $\Sigma^+ \rightarrow n + \pi^+$ decay mode. Correction for the events whose decay planes made too small an angle with the optic axes was made by assuming a continuous distribution of the events as a function of that angle. Finally, the correction for scanning efficiency, determined separately for the two decay modes, was made. The resultant final corrected numbers of events correspond to a decay branching ratio of

$$\frac{(\Sigma^+ \rightarrow p + \pi^0)}{(\Sigma^+ \rightarrow p + \pi^0) + (\Sigma^+ \rightarrow n + \pi^+)} = 0.49 \pm 0.05,$$

in good agreement with the current best value of 0.50 ± 0.02.[5]

IV. PARTIAL-WAVE ANALYSIS

Baltay et al. have studied $\pi^+ + p \rightarrow \Sigma^+ + K^+$ at neighboring incident momenta of 1110 and 1220 Mev/c.[6] The angular distribution obtained at 1220 Mev/c by these workers was fitted satisfactorily within the accuracy of the statistics by Legendre polynomials P_0, P_1, P_2, and P_3 consistent with the presence of at least d waves. We fitted the data of Baltay et al. at 1220 Mev/c to a third-order polynomial in $\cos\theta$, with the following results: $d\sigma/d\Omega \propto 1 + (1.63 \pm 0.10) \cos\theta - (0.17 \pm 0.18) \cos^2\theta - (1.71 \pm 0.17) \cos^3\theta$ with $\chi^2 = 2.7$ for seven degrees of freedom. The data of Fig. 1 at 1170 Mev/c have been fitted with polynomials of orders three, four, and five with the following results: third order, $d\sigma/d\Omega \propto 1 + (1.58 \pm 0.25) \cos\theta + (0.21 \pm 0.21) \cos^2\theta - (1.50 \pm 0.40) \cos^3\theta$, with $\chi^2 = 14.1$ for seven degrees of freedom; fourth order, $d\sigma/d\Omega \propto 1 + (1.71 \pm 0.32)\cos\theta + (0.74 \pm 0.72)\cos^2\theta - (1.62 \pm 0.43)$

$\cos^3\theta$ - $(0.64\pm0.81)\cos^4\theta$, with χ^2 = 13.7 for six degrees of freedom; fifth order, $d\sigma/d\Omega \propto 1 + (2.44\pm0.41)\cos\theta + (0.23\pm0.65)\cos^2\theta - (5.45\pm1.55)\cos^3\theta - (0.14\pm0.76)\cos^4\theta + (3.59\pm1.40)\cos^5\theta$ with χ^2 = 5.6 for five degrees of freedom. No additional powers of $\cos\theta$ were required for a better fit. Considering the results of the χ^2 tests stated above on the data in Fig. 1, the presence of at least f waves cannot be excluded conclusively. Since only 95 events in the mode $\Sigma^+ \rightarrow p + \pi^0$ were observed, the polarization distribution vs $\cos\theta$ of Fig. 4 does not permit any further comment on the possible presence of f waves. Nevertheless the following analysis has been done under the assumption that only s, p, and d waves are present in the production mechanism. A determination of the s, p, and d partial-wave production amplitudes was carried out, using the production c.m. angular distribution and the distribution of the decay pion with respect to the normal to the production plane. The differential cross section $d\sigma/d\Omega$ and the sigma polarization P have been expressed in terms of the nonspin flip amplitude, $g(\theta)$, and the spin flip amplitude $h(\theta)$[7] as

$$d\sigma/d\Omega = g(\theta)^2 + h(\theta)^2 , \tag{1a}$$

and

$$P\, d\sigma/d\Omega = 2\mathrm{Im}h^*(\theta)\, g(\theta) \tag{1b}$$

where

$$g(\theta) = \sum_\ell \left[(\ell+1)f_\ell^+ + \ell f_\ell^-\right] P_\ell(\cos\theta) , \tag{1c}$$

$$h(\theta) = \sin\theta \sum_\ell \left[f_\ell^- - f_\ell^+\right] \frac{dP_\ell(\cos\theta)}{d\cos\theta} . \tag{1d}$$

Here $f_\ell^\pm$ are the partial-wave amplitudes for total angular momentum $J = \ell \pm 1/2$. In our analysis, the contribution of angular momentum ℓ larger than two has been neglected; that is, the nonspin flip and spin flip amplitudes take the form

$$g(\theta) = S_{1/2} + (2P_{3/2} + P_{1/2})\cos\theta + 1/2\,(3D_{5/2} + 2D_{3/2})(3\cos^2\theta - 1), \qquad (2a)$$

$$h(\theta) = (P_{3/2} - P_{1/2})\sin\theta + 3(D_{5/2} - D_{3/2})\sin\theta\,\cos\theta. \qquad (2b)$$

The s, p, and d wave amplitudes were determined by the maximum-likelihood method. Considering for each event the c.m. production angle, θ, and the angle ϕ between the decay pion and the normal to the production plane, $\hat{n}$, — defined by

$$\hat{n} = \frac{\underset{\sim}{P}_\pi \times \underset{\sim}{P}_\Sigma}{|P_\pi|\,|P_\Sigma|\,\sin\theta_{\pi\Sigma}}\,, \qquad (3)$$

the probability for the observation of an individual event is

$$f = A_0 + A_1\cos\theta + A_2\cos^2\theta + A_3\cos^3\theta + A_4\cos^4\theta$$
$$+ \alpha\cos\phi\sin\theta\,(A_5 + A_6\cos\theta + A_7\cos^2\theta + A_8\cos^3\theta)\,, \qquad (4)$$

where α is the decay asymmetry parameter of the Σ^+ (α^0 for $p\pi^0$ and α^+ for $n\pi^+$). The coefficients A_n are related to the partial-wave amplitudes by the expressions:

$$\begin{aligned}
A_0 &= a^2 + d^2/4 - ad\cos\chi_d + c^2\,, \\
A_1 &= 2ab\cos\chi_b - bd\cos(\chi_d - \chi_b) + 6ce\cos(\chi_c - \chi_e)\,, \\
A_2 &= b^2 - c^2 + 9e^2 - 3d^2/2 + 3ad\cos\chi_d\,, \\
A_3 &= 3bd\cos(\chi_d - \chi_b) - 6ce\cos(\chi_c - \chi_e), \\
A_4 &= 9d^2/4 - 9e^2\,, \\
A_5 &= -2ac\sin\chi_c + cd\sin(\chi_c - \chi_d)\,, \\
A_6 &= -2cb\sin(\chi_c - \chi_b) - 6ea\sin\chi_e + 3ed\sin(\chi_e - \chi_d), \\
A_7 &= -3cd\sin(\chi_c - \chi_d) - 6eb\sin(\chi_e - \chi_b), \\
A_8 &= -9ed\sin(\chi_e - \chi_d)\,,
\end{aligned} \qquad (5)$$

where

$$
\begin{aligned}
S_{1/2} &\equiv a \\
2\,P_{3/2} + P_{1/2} &\equiv b \exp[i\,\chi_b] \quad , \\
P_{3/2} - P_{1/2} &\equiv c \exp[i\,\chi_c) \ , \\
3\,D_{5/2} + 2D_{3/2} &\equiv d \exp[i\,\chi_d] \\
D_{5/2} - D_{3/2} &\equiv e \exp[i\,\chi_e] \ .
\end{aligned}
\qquad (6)
$$

Using the probability function f, given in Eq. (4), the production amplitudes have been determined by maximizing the likelihood function,

$$
\mathcal{L} = \prod_{i=1}^{N=251} \frac{f^{(i)} \exp\left[\dfrac{\ell_{min} \times M_{\Sigma^+}}{3\, p_{\Sigma^+}{}^{(i)}\, \overline{\tau}_{\Sigma^+}}\right]}{\displaystyle\int_{-1}^{+1}\int_{-1}^{+1} f \exp\left[\dfrac{\ell_{min} \times M_{\Sigma^+}}{3\, p_{\Sigma^+} \times \overline{\tau}_{\Sigma^+}}\right] d\cos\theta \, d\cos\phi} \ , \qquad (7)
$$

The decay asymmetry parameters α^0 and α^+ have been taken equal to +1 and zero respectively. Table II gives the set of amplitudes that fits our data. These results do not change appreciably if α^0 is assumed to have a value somewhat different from + 1.00. For comparison, Table II includes also the results obtained for $\alpha^0 = +0.8$. The differences from the previous results are small compared with the errors. It is interesting to mention that the value of $\alpha^0 = +1.0$ fits the experimental data better than $\alpha_0 = +0.8$, as also was the case in the previous experiment. In Table III are found the A_n, as computed from Eq. (5). An alternative solution exists also for χ_b replaced by $-\chi_b$, χ_c replaced by 180 deg $- \chi_c$, χ_d by $-\chi_d$, and χ_e by 180 deg $-\chi_e$. The absence of an a priori energy-dependent representation of the amplitudes outside the threshold region has led to a determination of values of the amplitudes averaged over the energy interval investigated. The angular distribution based on the amplitudes is in complete agreement with the solid curve of Fig. 1, obtained by an independent maximum-likelihood fit. In the same manner, the average polarization, $\alpha^0 \overline{P}$, has been computed from the amplitudes of Table II. The following value, $\alpha^0 \overline{P} = 0.55$, has been obtained, also in agreement with the average polarization, $\alpha^0 \overline{P} = 0.62 \pm 0.19$, determined from the separate maximum-likelihood fit to the form $(1+a\cos\phi)$, with the data of Fig. 2. The solid curve of Fig. 4 represents the s, p, and d amplitude prediction for $P(\theta)$, in agreement with the data. The form of the curve is given by the following ratio.

$P(\theta) =$

$$\sin\theta[-(0.0064\pm0.0055) - (0.0157\pm0.0167)\cos\theta$$
$$-(0.0211\pm0.0191)\cos^2\theta + (0.0243\pm0.0312)\cos^3\theta\,,$$

divided by

$$(0.0164\pm0.0086) + (0.0266\pm0.0151)\cos\theta + (0.0094\pm0.0064)\cos^2\theta$$
$$-(0.0256\pm0.0156)\cos^3\theta - (0.0087\pm0.0155)\cos^4\theta,$$

with $\alpha^0 = +1.0$.

It must be noted that the interpretation of the amplitudes of Table II in terms of individual partial-wave amplitudes is ambiguous. It is well known that the set of amplitudes obtained by a Minami transformation followed by the complex conjugation operation corresponds to the same angular distribution and polarization (including the sign);[7, 8] in this experiment this means that the amplitudes $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, $d_{5/2}$, as obtained by the angular distribution and polarization, could equally represent amplitudes $p_{1/2}^*$, $s_{1/2}^*$, $d_{3/2}^*$ $p_{3/2}^*$, and $f_{5/2}^*$ respectively. Only with additional dynamical information can one attempt to find the correct set of solutions. That is, assuming that the amplitudes are reasonably well behaved and continuous functions of energy, one could eliminate one set of solutions if that set were in considerable disagreement with those extrapolated from a neighboring energy. In an attempt to do this, the s- and p-wave amplitudes determined near threshold in the previous experiment[(1)] have been extrapolated to 1170 Mev/c, using the assumed energy-dependence of the amplitudes in that analysis, including the factor p_{th}^{*}/p_{1170}^{*}, where p^{*} represents the production c.m. momentum of the colliding pion. Since energy dependence was taken into account in the threshold experiment, no Minami ambiguity exists in the amplitudes that have been extrapolated to 1170 Mev/c. The results of this comparison are indicated in Table IV; here the extrapolated s and p amplitudes, along with the regular and Minami transform solutions at 1170 Mev/c, have been given. In each of the three cases, two sets of solutions are given corresponding to the transformations $\chi_b \rightarrow -\chi_b$ and $\chi_c \rightarrow 180\text{ deg} - \chi_c$. According to the previously adopted convention, the $S_{1/2}$ phases of the Minami solutions have been set to zero; phases of the amplitudes have been rotated so thatthe amplitudes are all positive. Since the errors on the amplitudes and phases are rather large, it is not possible to select that set of amplitudes determined at 1170 Mev/c, either regular or Minami transform, which unambiguously agrees with the extrapolated amplitudes.

Figure 5 represents the final corrected production differential cross section of this experiment (at 1170 Mev/c), along with the corrsponding minimum values as allowed by the charge-independence hypothesis[9] from the data obtained for Σ^- and Σ^0 at 1220 Mev/c by Crawford et al.[10] Based on this comparison, there is no evidence for a violation of the charge-independence hypothesis. A similar conclusion has been arrived at by Baltay et al. at 1220 Mev/c[6]

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Table I. Scanning information and total cross section for $\pi^+ + p \rightarrow \Sigma^+ + K^+$ at 1770 Mev/c.

1.	Total number of pictures scanned.	34 738
2.	Total track length scanned, corrected for contamination, window criteria, and attenuation due to strong interactions. The latter correction is based on a total cross section of 29.7±0.44 mb, as given by Kopp et al. (reference 3), and made at the center of the chamber.	$(611.91 \pm 14.56)\times 10^3$ m
3.	Scanning efficiency for events meeting selection criteria on fiducial volume, decay plane orientation, window coordinates of beam track, and proton decay angle of 5 deg.	$p\pi^0$ = 94± 3% $n\pi^+$ = 95± 3%
4.	Number of observed events	$p\pi^0$ = 96 $n\pi^+$ = 155
5.	Number of corrected events	$p\pi^0$ = 217±22 $n\pi^+$ = 223±17
6.	Total cross section, based on above (2) and (5), and a target density of 0.0586 g$=$/cm^3.	0.205±0.014 mb

Table II. Maximum-likelihood solution to s-, p-, and d-wave amplitudes as defined by Eq. (6) for $a^0 = +1$, $a^0 = +0.8$, $a^+ = 0$, $\ell_{min}(\Sigma^+ \to p + \pi^0) = 0.75$ cm, and $\ell_{min}(\Sigma^+ \to n + \pi^+) = 0.35$ cm, with alternate solutions obtained by the transformations $\chi_b \to -\chi_b$, $\chi_c \to 180\text{ deg} - \chi_c$, $\chi_d \to -\chi_d$ and $\chi_e \to 180\text{ deg} - \chi_e$, and including the complete error matrix (angles are expressed in radians unless otherwise indicated).

	$a^0 = +1.0$	$a^0 = +0.8$
b/a	$= +2.14 \pm 1.13$	+2.06
χ_b	$= +34.7 \pm 29.2$ deg	+35.8 deg
c/a	$= -1.46 \pm 1.78$	-1.31
χ_c	$= +20.5 \pm 26.1$ deg	+26.9 deg
d/a	$= -1.02 \pm 0.97$	-1.11
χ_d	$= -7.4 \pm 60.2$ deg	-11.9 deg
e/a	$= -0.72 \pm 0.63$	-0.74
χ_e	$= +91.9 \pm 25.8$ deg	+98.9 deg
$\ell n\, \mathcal{L}$	$= -59.58$	-59.81
a	$= 0.061 \pm 0.016\ (\text{mb/sr})^{1/2}$	

Error matrix

d/a	0.948	0.962	0.333	0.255	-0.165	0.187	-0.173	0.319
χ_d		1.105	0.389	0.288	-0.270	0.256	-0.002	0.361
e/a			0.396	0.242	-0.530	0.138	0.746	0.094
χ_e				0.203	-0.300	0.096	0.398	0.108
b/a					1.272	-0.282	-1.879	0.007
χ_b						0.264	0.342	0.146
c/a							3.130	-0.157
χ_c								0.206

Table III. The coefficients of Eq. (4) computed from Eq. (5) and the s, p, and d amplitudes of Table II: (all values in units of mb/sr).

A_0	$= 0.0164 \pm 0.0086$
A_1	$= 0.0266 \pm 0.0151$
A_2	$= 0.0094 \pm 0.0064$
A_3	$= -0.0256 \pm 0.0156$
A_4	$= -0.0087 \pm 0.0155$
A_5	$= -0.0064 \pm 0.0055$
A_6	$= -0.0157 \pm 0.0167$
A_7	$= -0.0211 \pm 0.0191$
A_8	$= 0.0243 \pm 0.0312$

Table IV. The s and p amplitudes of this experiment compared with those obtained by extrapolation from threshold. The regular solutions plus the Minami-transform solutions of this experiment are given; solution Regular I corresponds to the amplitudes and phases given in Table II, while Regular II corresponds to these amplitudes with the transformations $\chi_b \to -\chi_b$, $\chi_c \to$ 180 deg - χ_c already made. The corresponding Minami-tranform solutions are given as Minami I and Minami II respectively. Solutions Regular A and Regular B correspond to the solutions obtained at threshold and these with $\chi_b \to -\chi_b$ and $\chi_c \to$ 180 deg - χ_c respectively, extrapolated to 1170 Mev/c, using the assumed energy dependence of that experiment. The $S_{1/2}$ phases of the Minami solutions have been set to zero according to previously adopted convention. Amplitude phases have been rotated so that the amplitudes are all positive. There exists no Minami ambiguity in the threshold extrapolation solutions. Errors shown on the Minami I solutions are typical of errors on all Minami solutions.

	This experiment				Extrapolation	
	Regular I	Regular II	Minami I	Minami II	Regular A	Regular B
$a\left(\frac{mb}{sr}\right)^{1/2}$	0.061±0.016		0.102±0.094	0.036	0.083±0.014	
b/a	2.14± 1.13		0.65 ±0.65	1.72	1.88 ±0.65	
$\chi_b{}^0$	34.7±29.2	325.3±29.2	328.8±40.0	301.3	52.1±8.0	307.9±8.0
c/a	1.46± 1.78		0.77	2.14	1.55 ±0.58	
$\chi_c{}^0$	200.5±26.1	339.5±26.1	227.5	192.4	260.8±27.2	279.2±27.2

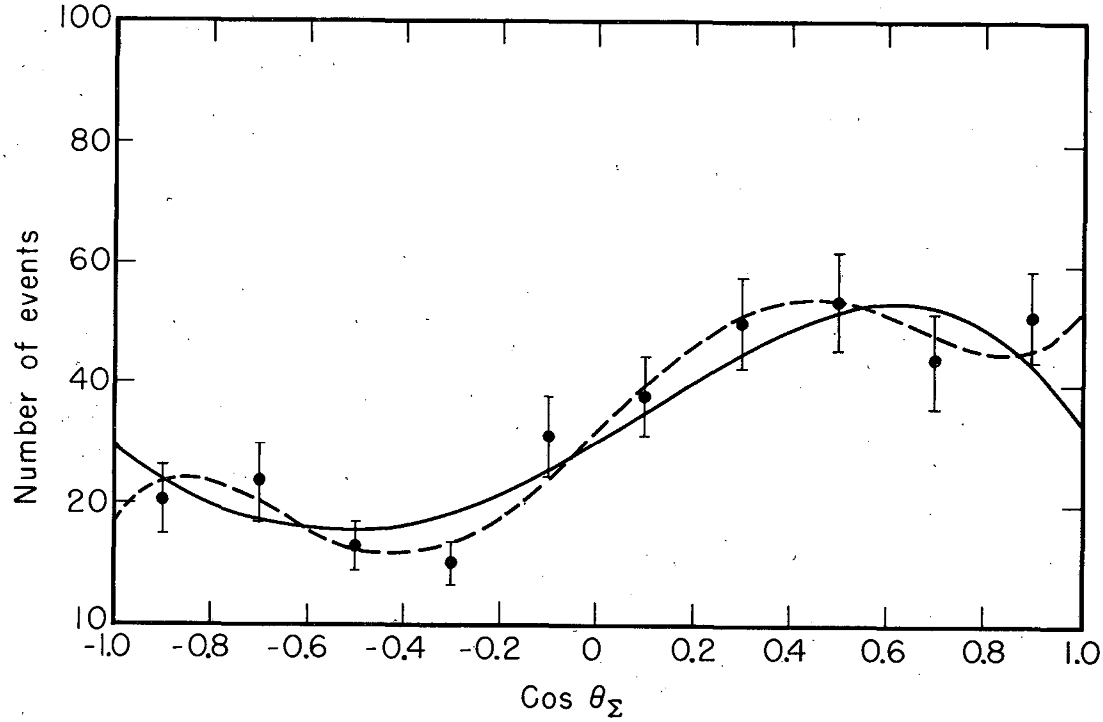

Fig. 1. Production c.m. angular distribution for the observed events corrected for sigma minimum-length cutoff. The solid curve represents the prediction of the s-, p-, and d-wave amplitudes of Table II, with the form $1+(1.71\pm 0.32)\cos\theta+(0.74\pm 0.72)\cos^2\theta - (1.62\pm 0.43)\cos^3\theta - (0.64\pm 0.81)\cos^4\theta$, with $\chi^2=13.7$ for six degrees of freedom. The dashed curve represents an independent maximum-likelihood fit to a fifth-order polynomial in $\cos\theta$ normalized to 336 events with the form $1+(2.44\pm 0.41)\cos\theta+(0.23\pm 0.65)\cos^2\theta-(5.45\pm 1.55)\cos^3\theta - (0.14\pm 0.76)\cos^4\theta+(3.59\pm 1.40)\cos^5\theta$ with $\chi^2=5.6$ for five degrees of freedom.

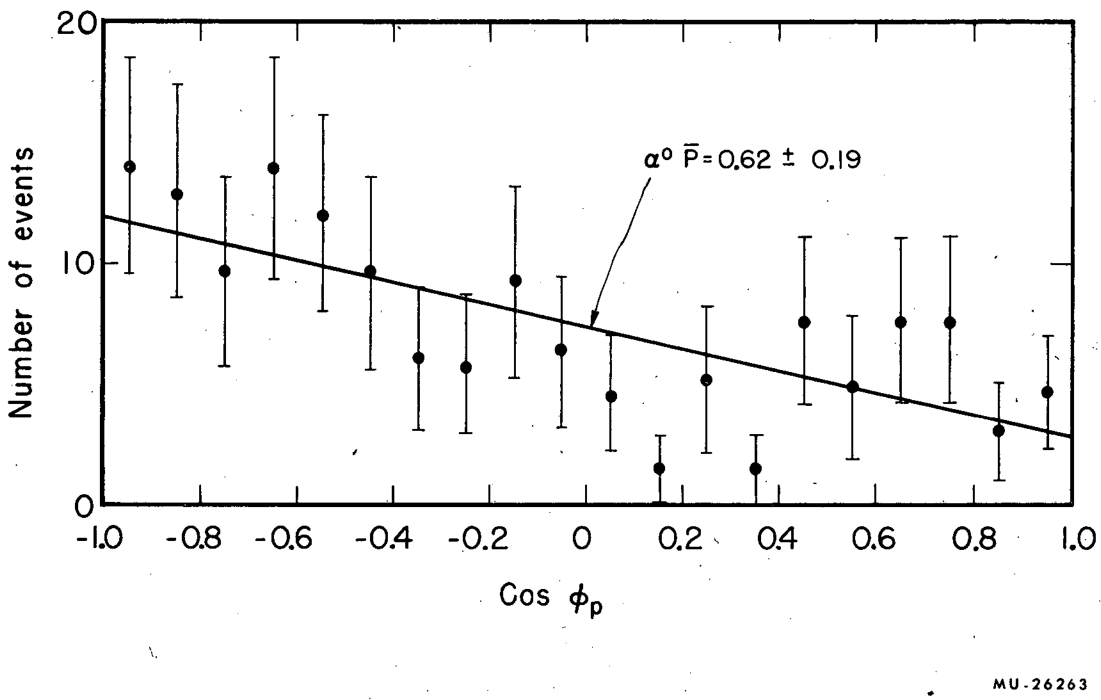

Fig. 2. Decay proton angular distribution for $\Sigma^+ \rightarrow p + \pi^0$, measured in the Σ^+ rest frame with respect to $\hat{n}$. The curve represents a maximum-likelihood fit to the form $1-\alpha^0\overline{P}\cos\phi_p$ normalized to 148 events, including the correction for short sigma tracks.

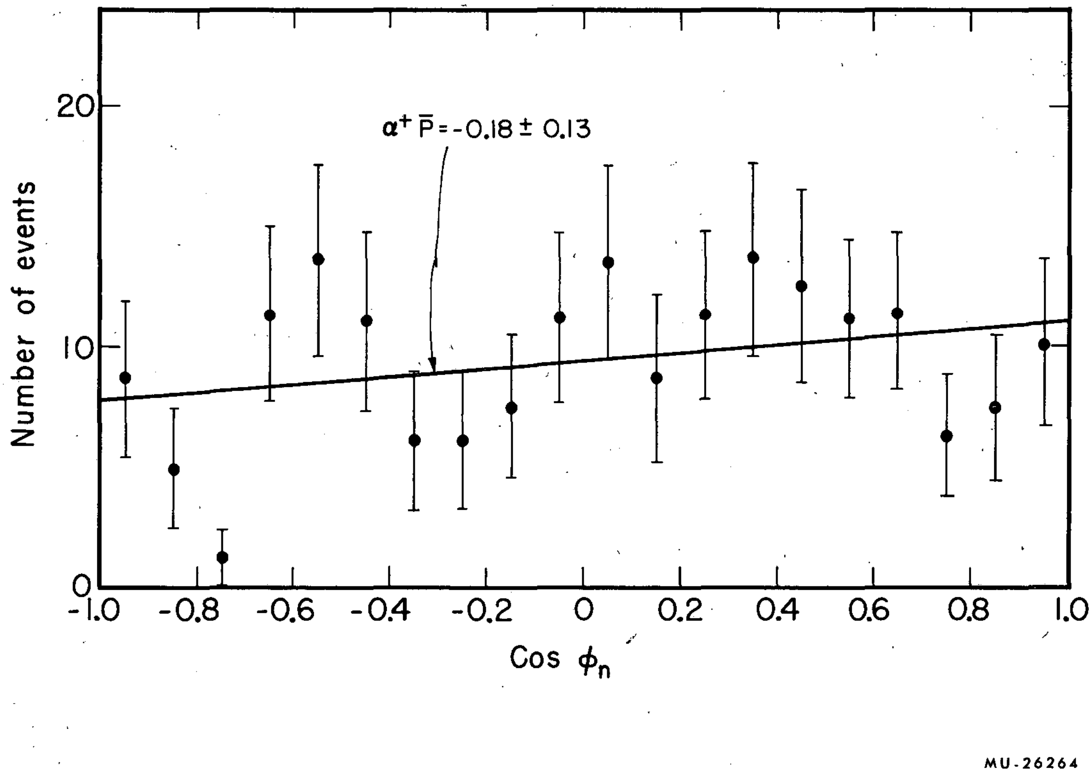

Fig. 3. Decay neutron angular distribution for $\Sigma^+ \rightarrow n + \pi^+$, similar to that of Fig. 2. The fit to $1-\alpha^+\overline{P}\cos\phi_n$ has been normalized to 188 corrected events.

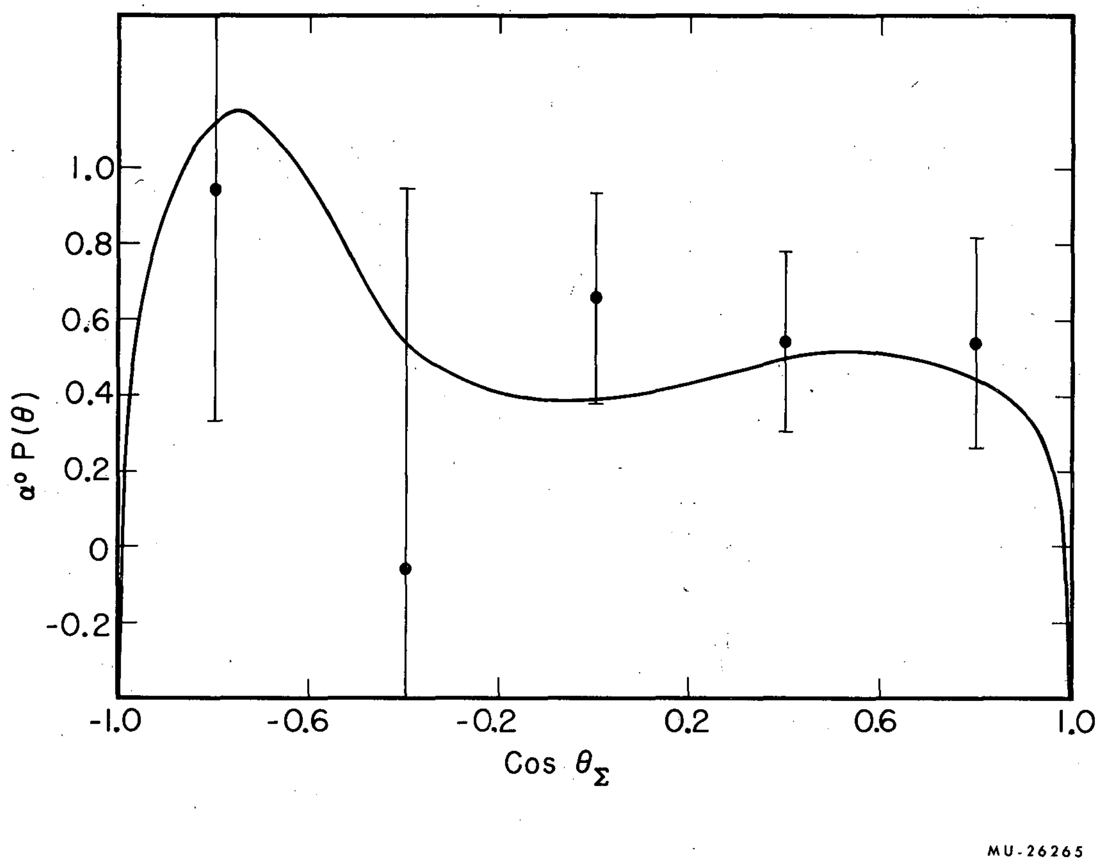

Fig. 4. Decay asymmetry with respect to $\hat{n}$ for $\Sigma^+ \rightarrow p + \pi^0$ based on 95 observed events. The solid curve represents the prediction of the s, p, and d amplitudes of Table II, with $\chi^2 = 1.6$ for one degree of freedom.

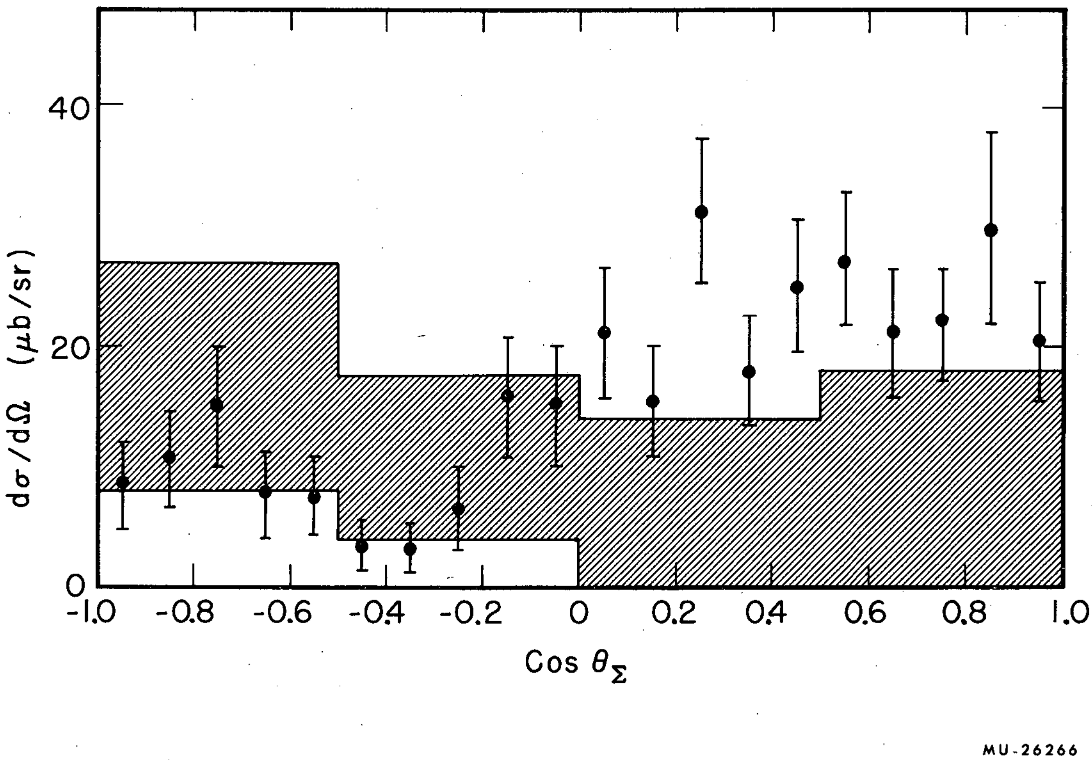

Fig. 5. The corrected absolute differential cross section of this experiment. The cross-hatched areas represent minimum Σ^+K^+ cross sections and their errors, as allowed by charge independence and the data of Crawford et al. at 1220 Mev/c.[10]