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DUALITY CONSTRAINTS ON INCLUSIVE REACTIONS II:
THE ROLE OF THE HARARI-FREUND CONJECTURE

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ABSTRACT

We present a scheme for the generalization of the two component theory of Harari and Freund to multiparticle amplitudes and, in particular, to inclusive reactions. Our scheme is based on duality and borrows the terminology of dual perturbation theory but we do not restrict our generalizations to any specific model. We obtain a criterion for the absence of secondary (non-scaling) Regge contributions in inclusive reactions. We show how inclusive reactions may approach their limiting values from below, in contrast to total cross-sections which always have positive secondary contributions. The scheme suggests a novel interpretation of diffraction dissociation which we discuss in a separate paper.

I. INTRODUCTION

Phenomenological duality¹ has had striking success in accounting for certain features of hadronic interactions, especially for two body processes and, via the optical theorem, for total cross sections. The predictions of duality (supplemented by the absence of exotic resonances) include:

- (1) exchange degeneracy among Regge trajectories and reggeon residues;²
- (2) certain quark model results such as the "magic" ϕ - ω mixing angle;³
- (3) the fall of the total cross-section, $\sigma(a+b \rightarrow \text{anything})$, to its asymptotic value when the channel (ab) is a non-exotic channel, i. e., the positivity of secondary Regge pole contributions;
- (4) The approximate energy independence of the total cross-section, $\sigma(a+b \rightarrow \text{anything})$, at surprisingly low energies when (ab) is an exotic channel, i. e., the absence of secondary Regge pole contributions when (ab) is exotic.

A cornerstone of this approach is the two-component theory of Harari⁴ and Freund⁵ which assumes, at least as a first approximation, that the imaginary part of a two body scattering amplitude may be written as the sum of two terms

$$\text{Im } A(s, t) = P(s, t) + R(s, t) \quad (1)$$

Here, R denotes the contribution of direct channel resonances which build (via finite energy sum rules) the reggeons⁶ in the crossed channel. The first term, P, denotes the contribution of non-resonant background in the direct channel which builds up the crossed channel pomeron contribution. A modified scheme has recently been proposed which includes multiple scattering corrections,⁷ so that:⁸

$$\text{Im } A = P + R + R \otimes P + R \otimes R + R \otimes R \otimes P \quad (2)$$

The success of the above description of two body scattering encourages attempts to generalize this scheme to multiparticle amplitudes. This problem is especially interesting experimentally since the n particle inclusive cross section is related to a discontinuity of the $n + 2$ particle forward scattering amplitude⁹ in much the same way as the total cross section is related to the two-body forward amplitude by the optical theorem.

By assuming Regge theory for the three to three forward elastic amplitude ($abc \rightarrow abc$) this leads to Mueller's expression⁹ for the single particle inclusive cross-section (in the region of a fragmenting into c)

$$\frac{d\sigma}{d p_T^c dx} = f(x, p_T^c) + g(x, p_T^c) \left(\frac{s}{s_0} \right)^{\alpha_{b\bar{b}} - 1} \quad (3)$$

where $x = 2 p_L^c / \sqrt{s}$ and p_T^c, p_L^c are the transverse and longitudinal components of the momentum of c in the center of mass frame. The function f describes the limiting distribution (we have assumed the pomeron to be a factorizable pole with intercept one). The non-scaling term on the right hand side of Eq. 3 is dominated by a Regge pole in $(b\bar{b})$ with intercept $\alpha_{b\bar{b}}$ (usually taken to be ~ 0.5 corresponding to the ρ, ω, f^0, A_2). We refer to such non-scaling contributions as the secondary contributions. It should be noted that the range of validity of Mueller's hypothesis (Eq. 3) is questionable. This is neatly illustrated in dual models as we discuss in Section 6. For all the results of this paper, however, we shall assume that, indeed, the Regge

behaved terms dominate the cross-section. Generalizations of the two-component theory have been suggested in the context of the dual resonance model^{10, 11, 12} supplemented by specific assumptions about the importance of certain terms. In a previous communication¹³ (referred to as I from here on) we have suggested a generalization of the Harari-Freund (HF from here on) conjecture which, while it is in the same spirit as refs. 10, 11, and 12, has very significant differences. In I we were led to several new and powerful phenomenological predictions on the basis of this generalization.

In this paper, we will elaborate on some of the details of the discussion given previously¹⁴ and will obtain some further consequences of our scheme.

Section 2 will be devoted to a detailed discussion of our model for multiparticle meson amplitudes, leading to a set of rules for implementing the Harari-Freund conjecture in inclusive reactions. Our scheme is a sort of pictorial "skeleton" of a complete theory -- we do not have any burdensome formulae, our arguments are based on a sequence of duality diagrams.

The rest of the paper is devoted to the application of this scheme to (mostly single particle) inclusive production spectra. In Section 3, we discuss the conditions for the absence of secondary Regge pole contributions to inclusive cross-sections (i. e., conditions for early scaling). Such conditions have recently been the cause of some controversy and we include a critical discussion of other suggested criteria. A simple criterion for amplitudes involving baryons only exists to the extent that planar duality is reasonable for baryon reactions.

In Section 4, we will show that there is no necessity in our scheme for the secondary (non-scaling) Regge pole contributions in inclusive reactions to be positive. This result should be contrasted with the case of total cross-sections which reach their asymptotic limit from above.

The treatment of pionization, given in Section 5, is a simple extension of the earlier discussion. We also discuss the general n particle inclusive reaction in our scheme.

Finally, in Section 6, we compare our scheme with the Dual Perturbation Theory. We also comment on the validity of Mueller's Ansatz in Dual Models.

The treatment of Diffraction Dissociation has been left to a separate paper¹⁵ (referred to as III). We are led to a novel description of Diffraction Dissociation in which the pomeron-pomeron-reggeon vertex vanishes. In III, we discuss this in detail with substantial experimental comparison.

It is worth observing that the rules for our scheme presented at the end of Section 2 are themselves very simple although their derivation may appear complicated to the reader unfamiliar with duality diagrams. With the aid of these rules, the constraints of duality in multiparticle processes and the essential role of the pomeron may be easily visualized as is demonstrated in the later sections.

II. THE "SKELETON" MODEL OF MULTIPARTICLE AMPLITUDES

Our discussion in I began with a classification of the duality diagrams for a two body scattering amplitude. Examples of such diagrams are given in Figure 1, where we illustrate four equivalent ways of representing the basic duality diagram that contributes to R . The first two emphasize the equivalence between resonances in the s -channel ($ab \rightarrow cd$) with reggeons in the t -channel ($\bar{a}\bar{c} \rightarrow \bar{b}\bar{d}$). The third figure is the corresponding quark duality diagram¹⁶ and the fourth figure is an abbreviated form¹⁷ of the third which is in common usage and we find the simplest and most useful for our purposes. (The diagram in which the quarks circulate in the opposite direction should be added - however we will never need to consider this separately.) In the most naive dual theory this diagram may be associated with a (st) Beta function of the Veneziano model,¹⁸ but our analysis does not depend on this association. (See Section 6 for further discussion of this point.)

Two basic properties of all such duality diagrams are illustrated by Figure 1:

- 1) Quantum numbers: - the $SU(3)$ properties are determined by the quark content.
- 2) Topology: - the shape of the diagram indicates which channels have discontinuities and illustrates precisely the nature of the discontinuity (resonance, Regge pole, pomeron, reggeon-pomeron cut, etc.)

Each diagram is meant to represent some function (of a complete theory) having the analytic structure suggested by the topology of the diagram. For example, it is assumed that the amplitude corresponding to

Figure 1 has resonances in the s -channel and in the t -channel, but not in the u -channel ($a\bar{d} \rightarrow c\bar{b}$). It has Regge asymptotic behavior in s at fixed t , $(-s)^{\alpha(t)}$ and in t at fixed s , $(-t)^{\alpha(s)}$. In addition to the diagrams of Figure 1, one must add the two corresponding to cyclicly inequivalent permutations of the external particles. One has resonances in the u -channel and in the t -channel and behaves, for large u at fixed t as $(-u)^{\alpha(t)}$. When added to Figure 1, this gives the full, signatured Regge pole contribution in the t -channel to the amplitude. The diagram having resonances in s and u is assumed to vanish faster than any power as $s \rightarrow \infty$ for fixed t . It does, however, contribute to the imaginary part of the amplitude in the s -channel and may be important near threshold (see Section 6 for more discussion of such contributions).

Starting from these three basic diagrams, many other types of duality diagrams are required for consistency with unitarity, some of which are shown in Figure 2.¹⁹ We assume that the full scattering amplitude is represented by the sum of all distinct duality diagrams, a feature shared by the dual perturbation theory.²⁰ We shall not assume that the more complicated "higher order" diagrams are small compared with the simpler ones. In this respect, we are not assuming that a perturbative theory is necessarily realistic. We shall later (Section 3) point out that it is likely that a large class of these diagrams must be large for phenomenological reasons. We shall classify all diagrams according to their quantum number and topology content and require that each diagram is consistent with the HF conjecture.

Consider the duality diagram of Figure 2a. This diagram illustrates why the additivity assumption of the HF hypothesis (Eq. 2) is by no means trivial. The particles in the initial state (ab) appear to be able to resonate but the particles in the final state (cd) cannot. The diagram represents a matrix element which mixes a resonance with non-resonant background. Consistency with the HF hypothesis requires that it vanish.²¹ (In fact, a duality diagram was originally termed¹⁶ "illegal" if the quark-antiquark pair from a single external meson annihilated each other, as in Figure 2a.) In general, then, whenever a single external meson is connected to a quark loop, the corresponding duality diagram is assumed to vanish. It is at this point that we diverge most crucially from other treatments.

Figure 2b is the diagram that has classically been identified as the primordial pomeron,^{22, 23, 24} P. It has vacuum quantum numbers in the $a\bar{c}$ channel and in dual perturbation theory^{23, 24} has a j-plane singularity whose position is independent of the external quantum numbers (although in the naive dual perturbation theory it has the wrong intercept and gives a unitarity violating cut). We also identify this diagram as P. The diagram appears to have resonance poles in the t-channel, but any pole is connected to a quark loop to which no other particles are connected. By the discussion of the previous paragraph, such a contribution vanishes so that the residues of the t-channel poles in Figure 2a must vanish. As we emphasized in I, the absence of reggeons in the t-channel is necessary for the maintenance of the HF hypothesis.²⁵

Other diagrams, such as those shown in Figure 3, give reggeon-reggeon cuts. Note that, according to the rule developed above, Figure 3a has no resonances in the direct channel.

Figure 4 illustrates two diagrams obtained from Figure 1 and Figure 2b by adding holes. Holes may contribute to renormalization corrections as in Figure 4a. However, they may also lead to reggeon-reggeon cuts as in Figure 4b. Although the properties of these cuts are not well determined, they do have lower intercepts and may be ignored at high energy. Indeed the whole discussion of constant total cross-sections in exotic reactions assumes cut effects are negligible.²⁶ For the remainder of the paper, we shall therefore ignore them.

In addition, there are further types of diagrams which are constructed by adding handles²⁰ to any of the diagrams so far considered. Two such diagrams are shown in Figure 5, both of which have reggeons and reggeon-pomeron cuts in the t-channel as well as other presumably less important contributions. Figure 5a contributes both to R and $R \otimes P$ terms in equation (2), and so such diagrams will be quite important in general. The addition of a handle may be thought of primarily as an absorption correction. The topology of diagrams with handles is a bit more deceptive than that of the diagrams previously considered. For example, Figure 5b has a discontinuity in the s-channel even though particles a and b are not adjacent on the quark loop boundary. It thus has a reggeon contribution in the t-channel regardless of whether the s-channel is exotic or not, apparently in violation of the HF conjecture.

That this discontinuity is in fact small may be seen as follows: - The diagram has the form $R \otimes P$ where R is purely real in the s -channel and P is taken to be purely imaginary and strongly peaked near $t = 0$ - hence the discontinuity vanishes. This fortuitous vanishing of the discontinuity of Figure 5b guarantees the HF hypothesis for the imaginary part of the amplitude.²⁷ We therefore assume in general that the addition of handles to a diagram does not affect which channels have discontinuities and provides absorptive corrections to resonances or reggeons in those channels.

This discussion suggests how the first assumption of Harari's model,⁷ summarized by Eq. 2 arises in a crossing symmetric, unitary theory. In the imaginary part of the amplitude reggeons plus reggeon-pomeron cuts in the t -channel are dual to s -channel resonances plus their absorption (leaving peripheral resonances prominent, see Figure 6). Our scheme suggests that this picture is only correct to the extent that the pomeron is purely imaginary.²⁸

For the sake of clarity, we summarize rules for constructing our skeleton model:

- 1) Every scattering amplitude can be represented as the sum of all distinct orientable duality diagrams.
- 2) Diagrams having only one external particle attached to a single quark loop are assumed to vanish. Similarly, no resonance (or reggeon) is exchanged between two states unless quarks are exchanged. This is the implementation of the HF conjecture.

- 3) The leading contribution to the exchange mechanism between separate quark loops is the pomeron. The real part of the pomeron is negligible compared to the imaginary part for small momentum transfers.
- 4) Diagrams with handles are assumed to contribute only to the real part of the amplitude in channels formed by non-adjacent particles. Handles contribute primarily to absorptive corrections.
- 5) When the quantum numbers of a pair of particles connected to each other by a quark in a particular duality diagram are exotic then the contribution of that diagram vanishes identically. This rule is implicit in any scheme based on duality diagrams.

These rules have been obtained for meson amplitude. In Section 4 we will discuss the duality diagrams which include baryons.

Our rules are immediately applicable to an arbitrary multiparticle scattering process and provide a generalization of the HF conjecture. In the remainder of this paper, we shall apply this scheme to several multiparticle processes of experimental and theoretical interest.

The simplest diagram of any class of diagrams with a particular quantum number and topology content will be referred to as a primitive diagram. In other words the addition of holes and handles (according to rule 4) to a primitive diagram does not alter which channels have singularities. It is clearly sufficient to study these primitive diagrams only if we wish to discuss the high energy behavior of the amplitude. We have shown that Figures 1 and 2b (plus diagrams formed by noncyclic permutations of

external particles) are the primitive diagrams for the four point function, i. e., Figure 5b need not be considered separately due to rule 4. It remains an assumption that this rule also applies to multiparticle amplitudes with handles.

III. EXOTICITY CONDITIONS FOR EARLY SCALING IN INCLUSIVE REACTIONS

We have derived, in I, a condition for the approximate energy independence of single particle inclusive reactions, $a + b \rightarrow c + x$. We will now elaborate on that discussion.

In the limit corresponding to the fragmentation of a into c, ²⁹ $(a:c|b)$, the leading contributions come from those diagrams for the process $a + b + \bar{c} \rightarrow a + b + \bar{c}$ which have:-

- i) a non-zero foreword discontinuity in $ab\bar{c}$ since this discontinuity gives the inclusive cross section⁹
- ii) either the pomeron or reggeons in the $(b\bar{b})$ channel.

In Figures 7 and 8, we display all such primitive diagrams. Figures 7a, 7b, and 8a all have the pomeron in the $(b\bar{b})$ channel and contribute to the limiting fragmentation. Figure 7b has the pomeron in $(a\bar{a})$ also and is the only diagram contributing to the limiting distribution in the pionization limit (see Section 5). Figure 8 is relevant for diffractive dissociation and will be discussed in more detail in III. All other diagrams displayed have reggeons in the $(b\bar{b})$ channel and are the dominant contributions to the energy dependence in the fragmentation limit. It is easy to check that there is only one criterion that will eliminate all diagrams which have reggeons in $(b\bar{b})$. The criterion is:

(ab) and $(b\bar{c})$ must be exotic

(A)

For nearly all reactions if (ab) and $(b\bar{c})$ are exotic then $(abc\bar{c})$ is also exotic. The exceptions to this are certain reactions in which a , b , and c are all baryons (e. g., $p + \Omega^- \rightarrow \Xi^- + X$). We do not expect our simple dual scheme to work for such processes since the corresponding two body reactions, baryon-antibaryon scattering, cannot be described in a simple dual model without exotic resonances.

In any case, consideration of reactions involving baryons only gives criterion A if planar duality is applicable to baryons. The assumption of planar duality is common to most approaches (e. g., refs. 30, 31).

From our presentation, which involved examining all the relevant diagrams for the six point function, the existence of any simple criterion may seem fortuitous. However, we shall see in Section 5 how a general criterion can be obtained for an n point function without having to draw any diagrams explicitly, just by using special ordering properties that duality imposes on quark lines.

We stress that our criterion A eliminates all secondaries in the whole fragmentation region. In certain limited regions of phase space, we may expect other, weaker, criteria to be good to some approximation. Thus, for instance, in the triple Regge region (x near 1, where x is the Feynman scaling parameter) Figure 7c may be expected to be the leading non-scaling diagram (if $(a\bar{c})$ is not exotic) since this has the triple Regge behavior. To the extent that Figure 7c (the tree diagram) does dominate in this region, the condition $(abc\bar{c})$ exotic is sufficient to ensure early scaling. This is the condition proposed by Chan, et al.^{30, 31} who, however, suggest a larger region of validity. It is true that the range of validity of the triple Regge formula is

not well established but it cannot be appropriate when ($s_{a\bar{c}}$) is large. The region in which $s_{a\bar{c}}$ is large is however included in the fragmentation region of a into c which is defined as a limit at large s_{ab} in which s_{bc}/s_{ab} and $s_{a\bar{c}}$ are held fixed. We have seen that in general there is only one criterion appropriate to the whole fragmentation region. Chan and Hoyer³¹ suggest that "higher order loop corrections" are small compared with tree diagrams in order to justify their criterion for a large region of phase space. However, the pomeron is a loop diagram and it is known to have couplings of the same order of magnitude as normal Regge trajectories.³² We emphasize this point, which seems to cause a lot of confusion, by listing some total cross-section data³³ in Table I. This table shows that the pomeron contribution is larger than the secondary contributions at remarkably small energies - typically of the order of 700 MeV center of mass kinetic energy. It is therefore exceedingly dangerous to ignore "loop contributions" at the energies under discussion in inclusive reactions.

The criterion suggested by Ellis, Finkelstein, Frampton, and Jacob³⁴ which says that reactions scale when $(a\bar{b}\bar{c})$ and (ab) are exotic amounts to neglecting certain of our diagrams since they arbitrarily neglect the $b\bar{c}$ channel. They assert that since $s_{b\bar{c}}$ is negative in the physical region, the singularities in $s_{b\bar{c}}$ are negligible. However, in a dual model the poles at positive values of $s_{b\bar{c}}$ imply a smooth energy dependent secondary contribution at negative values of $s_{b\bar{c}}$ which will be just as large as the average energy dependence arising from s_{ab} (which is positive). It is easy to see from our diagrams that the channel $(b\bar{c})$ is just as important as (ab) in

determining secondary contributions. The present data is not adequate to separate secondary contributions with sufficient accuracy to determine which criterion is correct. We feel that a discussion of the HF conjecture must be a prerequisite for any exoticity criterion since it is the crucial element in the discussion of total cross sections - it is the lack of such a discussion in refs. 30, 31, and 34 that makes their conclusions unsatisfactory. We should point out that in order to determine which reactions scale early it is necessary to have a precise study of a reaction over a range of relatively low energies (3-10 GeV/c, say) and values of x not too close to 1. This is not so useful, of course, for observing limiting distributions and establishing properties of the pomeron which has been subject to most attention up till now. In I, we listed some reactions which distinguish our criterion, the most interesting case being $K^+ p \rightarrow \pi^- + x$. This should scale at low energies in both fragmentation regions according to previous rules, but only in the proton fragmentation region according to ours.

To conclude this section, we would like to discuss a very recent proposal by Tye and Veneziano.¹² In their scheme the inclusive cross sections are built from the square of production amplitudes. A crucial assumption is that as far as possible one should only include tree diagrams in the production amplitudes. This leads to the same primitive diagrams for total cross-sections as we consider. However, at the level of the six point function one loop pomeron diagrams must be added in the production amplitudes to allow for diffraction dissociation (the 8th component in ref. 12). In addition, many interference terms are neglected for simplicity in Ref. 12 while our rule 2 only kills a subset of them. A subtler difference is that our primitive diagrams

are supposed to include handles (corresponding to absorption) and therefore we do not predict that even the diagonal terms are positive, while their most important prediction is based on positivity of each of the seven components in their scheme. On the other hand as they do not assume rule 2 they do not predict the vanishing of secondaries when the s-channel is exotic (they call this the strong HF hypothesis as opposed to the weak form that they obtain) and therefore they do not have exchange degeneracy. This allows a diagonal diagram in which there is a pomeron and a secondary reggeon in the same channel. This secondary reggeon may give a negative contribution since the pomeron is positive and dominates (since it has a higher intercept) ensuring positivity for the diagram as a whole. Such negative secondaries are known to be necessary¹² and are discussed in detail in the next section.

IV. THE SIGN OF SECONDARY CONTRIBUTIONS

Before turning to diagrams having reggeons in ($b\bar{b}$) we would like to remind the reader of the third consequence of the HF hypothesis for two body reactions which was mentioned in the opening paragraph of the paper viz. the fall of total cross-sections to their asymptotic values. This result depends on the fact that the discontinuity of the resonance component of the forward elastic amplitude must be positive as shown in Fig. 9a, and that interference diagrams, such as Fig. 2a, vanish. It is not obvious, a priori, that absorption corrections of the form $R \otimes P$ cannot make the overall secondary component negative. For instance, in the square of Fig. 9b, the $R \otimes P$ term may be negative although the net contribution of the square (which involves pomeron terms like P and $P \otimes P$), is, of course, positive. It is a phenomenological

fact that these absorption corrections are not strong enough to cause the total cross-sections to rise to their asymptotic limits.

Let us now examine single particle inclusive reactions. In order to gain some insight into the kinds of processes that build up each contribution, we will look at the missing mass discontinuity explicitly. The contributions that build the pomeron in $(\bar{b}\bar{b})$ when squared and summed over X, Y, and Z are illustrated in Figure 10a, b, and c and so each of these individually contributes a positive component to the limiting distribution.

Since it was a useful approximation in the two body case, we might ask whether, in the absence of absorption corrections, all single particle inclusive cross-sections necessarily fall to their limiting values in our scheme. If this were so, it would be catastrophic since it can be shown rigorously, by use of energy conservation sum rules,³⁵ that some inclusive reactions must rise to their limiting values. To examine this, we return to a consideration of those primitive diagrams for the fragmentation $(a:c|b)$ having reggeons in the $(\bar{b}\bar{b})$ channel. Fig. 7c and all of Fig. 8 arise from the square of the production mechanisms shown in Fig. 11a. The crucial point is that although the square of the first two diagrams in Figure 11a and their interference with the third diagram have reggeons in $(\bar{b}\bar{b})$, the square of the third gives Fig. 8a, having only a pomeron in $(\bar{b}\bar{b})$. Since this pomeron term is positive in $(\bar{b}\bar{b})$ there is no reason why the interference terms, Figs. 8b, could not be negative. If so, they will dominate over the other contributions with reggeons in $(\bar{b}\bar{b})$ for a sufficiently large ratio of s_{ab} to s_{abc} (see the discussion of diffraction dissociation in III), leading to an approach to limiting fragmentation from below. Similarly, there are further interference terms (Figs. 7d

and 7e) which arise from squaring Fig. 11b and occur in processes in which a and c have different quantum numbers. Once again, the square of the third diagram gives the pomeron in $(\bar{b}\bar{b})$ and is positive so that it dominates over the other terms in the square of Fig. 11b, which have reggeons in $(\bar{b}\bar{b})$. This means that the interference terms (Fig. 7e) which need not be positive may dominate over the other terms with reggeons in $(\bar{b}\bar{b})$ and the possibility again arises of the fragmentation limit being reached from below.

In summary, the primitive diagrams suggest that the inclusive reaction might rise to its asymptotic value in two circumstances:

- 1) If a or b can diffractively dissociate into c , or
- 2) If the quantum numbers of a , b , and c do not force the interference terms of Figure 7e to vanish.

These results only apply to reactions involving baryons if we only need to include planar diagrams. It is clear that in the case of nonplanar diagrams there are so many possible interference terms that a much weaker set of conditions would allow negative secondary terms.

The above considerations have completely neglected the possible effects of absorption. As we have noted, absorption does not change the sign of secondaries in total cross-sections but there is no a priori reason why it could not. Thus, for the six point function, where the possibilities for absorption are much more complicated, the negative non-scaling secondaries may arise from this mechanism.

V. PIONIZATION AND THE GENERAL N-POINT FUNCTION

In the pionization limit, $(a|c|b)$, the dominant contributions have either a pomeron or a reggeon in both the $(a\bar{a})$ and $(b\bar{b})$ channels. All other diagrams are assumed to vanish exponentially - this is equivalent to assuming the Mueller hypothesis.⁹ The surviving diagrams are shown in Figure 12. In Table II, we indicate each type of diagram and state which criterion eliminates it.³⁶ (The diagrams are labelled as in Figure 12.)

From Table II, we see, for instance, that $(a\bar{c})$ exotic kills the contribution which has a pomeron in $(b\bar{b})$ and a reggeon in $(a\bar{a})$. This is a leading secondary and behaves like $s_{ab}^{\alpha_{a\bar{a}}/2 - 1/2}$ in the pionization limit.³⁷ So the condition to kill both leading secondaries is $(a\bar{c})$ and $(b\bar{c})$ exotic. The condition that kills all secondaries (including terms with reggeons in both $(a\bar{a})$ and $(b\bar{b})$) is (ab) , $(a\bar{c})$, and $(b\bar{c})$ exotic.

These conditions for the absence of secondaries in fragmentation and pionization may be seen in a general way without examining particular diagrams as mentioned earlier. We make use of the following properties of all duality diagrams for an n particle inclusive reaction, $a+b \rightarrow c_1+c_2+\dots+c_{n-2}+X$. (The relevant diagrams are for the $a+b+\bar{c}_1+\bar{c}_2+\dots+\bar{c}_{n-2} \rightarrow \bar{a}+\bar{b}+c_1+c_2+\dots+c_{n-2}$ forward amplitude.)

- a) Each diagram consists of the external particles (made of quark-antiquark pairs) attached to quark loops.
- b) There is a strict ordering of particles around each quark loop such that all incoming particles can be separated from all outgoing particles as in Figure 13a.

- c) Any group of particles which have low relative momenta and are separated from other particles by a pomeron or reggeon must be adjacent on a quark loop or on separate quark loops. For example, in Figure 13b, the particles $b\bar{b}c\bar{c}$ are arranged so that this diagram contributes to the region of b fragmenting into c .

From these simple rules, the general prescription for the absence of secondary contributions in an arbitrary process is manifest:

No secondary Regge poles contribute to an inclusive reaction if, and only if, each channel whose subenergy becomes large is exotic.

VI. RELATION TO DUAL PERTURBATION THEORY AND MUELLER'S HYPOTHESIS

We first wish to clarify the connection between our model and Dual Perturbation Theory (DPT) based on Feynman-like diagrams.²⁰

Although the classification of diagrams is exactly the same, the philosophy of our approach is rather different. In DPT, at least in principle, one has a definite prescription³⁸ for calculating the diagrams based on unitarity and the assumption that the Veneziano amplitude is the Born approximation. However, since the Veneziano formula does not correspond too well to the physical world, it is hard to believe in this prescription. Our approach is to abstract from the DPT certain properties that we expect will be solved by a more realistic model and we feel free to modify all other aspects. To this we add certain phenomenological information, essentially the HF conjecture and some features of absorption. Finally, we take the optimistic attitude that the whole scheme is consistent with known rigorous constraints (such as sum

rules³⁵). This is certainly non-trivial since, for example, our manner of implementing the HF conjecture has been very simple minded and is surely not the only possible one (see, for example, ref. 12).

To illustrate our approach consider Figure 2b which we have identified with the pomeron and have assumed behaves up to logarithms like $s^{\alpha(t)}$ as $s \rightarrow \infty$ for fixed t and that $\alpha(0) = 1$. In fact Freund²² argued in favor of this identification before the prescription for calculating loops was established, solely on the basis of duality. DPT shows a suggestive J-plane^{23, 24} singularity for this diagram but with a different intercept. Furthermore, for a physical J the singularity becomes a cut in s that violates unitarity. However, it is more or less accepted that while the existence of such a singularity is model independent (i. e., the topology arguments of ref. 39) its intercept and the fact that it is a cut are not.²⁴ Therefore we only use DPT to tell us which classes of diagrams can have a pomeron-like singularity and in this way we are led to rule 3.

However, DPT also shows that whenever a diagram has the pomeron in a certain channel that the same diagram will have the f or f' pole in that channel. We have argued that this feature would break EXD and the HF hypothesis and therefore have established rule 2. This is not necessarily incompatible with DPT if we consider that the diagrams of the skeleton model correspond to an infinite set of diagrams in DPT.

An interesting and rather controversial diagram is the one of Figure 8a which has been used by some authors⁴⁰ as evidence in favor of the vanishing of the triple pomeron vertex. Contrary to common myth, a thorough calculation

of this diagram following the methods of ref. 23 reveals that it does have a triple pomeron singularity and the pomeron in the $b\bar{b}$ channel is dual to resonances in the abc channel.

Of course, as is well-known, in a unitary theory the triple pomeron vertex must vanish when $s_{a\bar{c}} = s_{\bar{a}c} = 0$ if the pomeron is a pole with unit intercept. This would only be true in DPT for the completely summed series.

The observation that resonances build the pomeron in diffraction dissociation in DPT is very important and independent of the manner of implementation of the HF hypothesis. It is discussed in detail in III.¹⁵

A more difficult point to understand in the context of DPT is the 4th rule which states that diagrams with handles have discontinuities in the same channels as ones without them. We are not sure if this rule is too strong or even consistent with unitarity. For the four point function, as discussed earlier, the rule is true if the pomeron is purely imaginary. We would hope that in any case it would be true to at least the leading power of s although this is very much an open question.

We now turn to a discussion of the validity of Mueller's hypothesis that the pomeron and secondary reggeons dominate inclusive cross-sections. Our whole analysis has assumed this to be true at laboratory energies. The Dual Resonance Model indicates a possible source of failure of this assumption. Recall that the Veneziano model is the sum of three beta functions

$$V = B_{st} + B_{ut} + B_{su}$$

As $s \rightarrow \infty$, for fixed t , the first two terms lead to Regge behaviour (at least outside of a wedge) and the third term, B_{su} , has indefinite oscillating

phase. It is universally assumed that in the fully unitarized theory the resonances will develop widths in such a manner as to lead to Regge behaviour along the real s axis and lead to exponential damping of the third term.

An analogous situation holds in inclusive processes⁴¹ for which diagrams in which reggeons carry large mass (analogous to B_{su} for the four point function) are damped by an exponential which depends on the imaginary part of the trajectory function. If these terms contributed significantly at present laboratory energies, they would undermine the whole notion of reggeon-resonance duality as well as the applicability of Mueller's Ansatz of Regge behaviour of the six point function. Whereas the usual correlation length is determined from the difference between the intercepts of the pomeron and the leading reggeon these extra terms might lead to an entirely different kind of correlation. Even if resonances have widths that grow with $(\text{mass})^2$, these terms may lead to significant correlation at present energies since the actual rate of decrease of the exponential is unknown. This would be an unhappy situation for most reggeists.

ACKNOWLEDGEMENTS

Two of us (M. B. G. and M. A. V.) are grateful to Professor G. F. Chew for extending the hospitality of the Lawrence Berkeley Laboratory to them in the summer of 1971 when this work was started. M. B. G. also thanks Carl Kaysen for the hospitality of The Institute for Advanced Study.

REFERENCES AND FOOTNOTES

1. For reviews of duality see, e. g., H. Harari, Lectures at the Brookhaven National Laboratory Summer School, August 1969; H. Harari, Lectures at the Ettore Majorana Summer School, 1970.
M. Jacob, Lectures at the Herceg-Novi Summer School, 1970.
J. D. Jackson, Invited talk at the International Conference on Duality at Tel-Aviv, Israel (1970).
2. See e. g., C. Michael in "Proceedings of the Ruhstein Meeting on Low Energy Hadron Interactions and Coupling Constants," Springer Tracts.
3. C. B. Chiu and J. Finkelstein, Phys. Letters 27B, 510 (1969).
4. H. Harari, Phys. Rev. Letters 20, 1395 (1968).
5. P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968).
6. In this paper, the terms reggeons and Regge pole should be understood to exclude the pomeron.
7. H. Harari, Invited talk at the International Conference on Duality at Tel-Aviv, Israel (1971).
H. Harari and A. Schwimmer, SLAC-PUB-952 (August, 1971) and references therein.
See also the review by R. J. N. Phillips, Rapporteur's talk at the Amsterdam Conference on Elementary Particles (July, 1971).
8. Here the symbol P is meant to represent the full pomeron contribution which, in many models (including the one under discussion in this paper), is made of a primordial pomeron plus some kind of absorption. Symbolically " $P \equiv P + P \otimes P + \dots$ "

9. A. H. Mueller, Phys. Rev. D2, 2963 (1970). See also, H. P. Stapp, UCRL 20623; C. I. Tan, Phys. Rev. D4, 2412 (1971); J. C. Polkinghorne, Nuovo Cimento 7A, 555 (1972).
10. A. DiGiacomo, S. Fubini, L. Sertorio, and G. Veneziano, Phys. Letters 33B, 171 (1970).
11. G. Veneziano, talk presented at the International Conference on Duality and Symmetry in Hadron Physics, Tel-Aviv (1971).
12. S-H. H. Tye and G. Veneziano, Phys. Letters 38B, 30 (1972).
13. M. B. Einhorn, M. B. Green, and M. A. Virasoro, Phys. Letters 37B, 292 (1971).
14. Although this paper repeats some of I, we shall at times refer back to I for the sake of brevity.
15. M. B. Einhorn, M. B. Green, and M. A. Virasoro, Duality Constraints in Inclusive Reactions III. Duality and Diffraction Dissociation, IAS preprint (March 1971).
16. H. Harari, Phys. Rev. Letters 22, 562 (1969); J. Rosner, Phys. Rev. Letters 22, 689 (1969); T. Matsuoka, K. Ninomiya, S. Sawada, Prog. Theor. Phys. 42, 56 (1969).
17. D. Neville, Phys. Rev. Letters 22, 494 (1969).
18. G. Veneziano, Nuovo Cimento Letters 1, 681 (1971).
19. We need never consider "non-orientable" diagrams. These diagrams contain states with forbidden quark orientations. See J. Shapiro, Phys. Rev. D4, (Comments and Addenda) 1249 (1971) for a rigorous proof that such diagrams never contribute.

20. K. Kikkawa, B. Sakita, and M. A. Virasoro, Phys. Rev. 184, 1701 (1969);
K. Kikkawa, S. A. Klein, B. Sakita, and M. A. Virasoro, Phys. Rev. D1,
3258 (1970).
21. By vanish, we mean of course, negligible compared with other diagrams
which do contribute.
22. P. G. O. Freund, Nuovo Cimento Letters 4, 147 (1970); P. G. O. Freund
and R. J. Rivers, Phys. Letters 29B, 510 (1969).
23. D. J. Gross, A. Neveu, J. Scherk, and J. H. Schwarz, Phys. Rev. D2,
697 (1970); C. S. Hsue, B. Sakita, and M. A. Virasoro, Phys. Rev. D2,
2857 (1970); G. Frye and L. Susskind, Phys. Letters 31B, 589 (1970).
24. C. Lovelace, Phys. Letters 34B, 500 (1971).
25. In I, we first noticed this fact and then deduced that Figure 2a has to
vanish. For variety, we chose to emphasize the converse here.

The correspondence between the couplings of the pomeron and those of
the f and f' Regge poles (see R. Carlitz, M. B. Green, and A. Zee, Phys.
Rev. D4, 3439 (1971)) is maintained by Figure 2b even if we assume that
the f and f' poles in Figure 2b have very small residues.
26. For a discussion of Regge-Regge cuts in phenomenology, see H. Harari,
Phys. Rev. Letters 26, 1079 (1971).
27. This property is shared by reggeon-multiple pomeron cuts ($R \times P \times P$,
etc.) obtained by adding several handles.
28. Harari⁷ also assumes the pomeron to be purely imaginary but for very
different reasons.

29. This notation was introduced in R. Cahn and M. B. Einhorn LBL-30, Phys. Rev. D4, 3337 (1971) ($a:c_1|c_2|c_3:b$) means the scattering process $a + b \rightarrow c_1 + c_2 + c_3 + X$ in the region of a fragmenting into c_1 , b fragmenting into c_3 and with c_2 a pionization product.
30. H. M. Chan, C. S. Hsue, C. Quigg, J-M. Wang, Phys. Rev. Letters 26, 672 (1971).
31. H. M. Chan, P. Hoyer, Phys. Letters 36B, 79 (1971).
32. See for example R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. Letters 26, 1515 (1971).
33. For a summary of total cross-section data, see the Particle Data Tables. The most recent data from Serpukhov are in Denisov, et al., Phys. Letters 36B, 528 (1971).
34. J. Ellis, J. Finkelstein, P. H. Frampton, M. Jacob, Phys. Letters 35B, 227 (1971).
35. G. Veneziano, Seminar at the International School of Physics "Enrico Fermi," Varenna, August 1971, M. I. T. preprint. L. Caneschi, CERN preprint, Ref. Th. 1416 (October 1971).
36. M-S. Chen and F. E. Paige, Brookhaven National Laboratory preprint BNL 15906.
37. H. D. I. Abarbanel, Phys. Letters 34B, 69 (1971).
38. C. Lovelace, Phys. Letters 32B, 703 (1970); V. Alessandrini, Nuovo Cimento 2A, 321 (1971); M. Kaku and L. P. Yu, Phys. Letters 33B, 166 (1970).
39. V. Alessandrini and D. Amati, Nuovo Cimento 4A, 793 (1971).

40. R. C. Brower and R. E. Waltz, CERN Preprint Ref. TH-1335.
P. H. Frampton, Phys. Letters 36B, 591 (1971).
41. M. A. Virasoro, Phys. Rev. D3, 2834 (1971); C. E. DeTar, K. Kang,
C. I. Tan, and J. H. Weis, Phys. Rev. D4, 4251 (1971).

TABLE CAPTIONS

Table I A demonstration of the great importance of the pomeron (nearly constant) contributions to total cross-sections at rather low kinetic energies.

Table II Terms contributing to the pionization cross-section. The labeling in the first column corresponds to that of Figure 12. The last column indicates the conditions that force specific terms to be absent.

TABLE I

<u>Total Cross-Section</u>	<u>Type</u>	<u>$\sigma(\infty)$ (mb)</u>	<u>Dependence on Center of mass kinetic energy, T(MeV)</u>
$K^+ p$	Exotic	18	Constant for $T \geq 600$
$K^- p$	Non-Exotic	18	Falls below 36 for $T \geq 500$
$\pi^+ p$	Non-Exotic	23-24	Falls below 46 for $T \geq 800$
$\pi^- p$	Non-Exotic	23-24	Falls below 46 for $T \geq 700$
pp	Exotic	39-40	Constant for $T \geq 800$
$\bar{p}p$	Non-Exotic	39-40	Falls below 78 for $T \geq 800$

TABLE II

<u>Type of Contribution</u> <u>(labelled by Figure 12)</u>	<u>Exchange Regge Singularity</u>		<u>Condition for Absence</u> <u>of Contribution</u>
	<u>in (aa)</u>	<u>in (b\bar{b})</u>	
(a)	P	P	
(b)	R	P	(a \bar{c}) exotic
(c)	P	R	(b \bar{c}) exotic
(d)	R	R	(ab) exotic
(e)	R	R	(a \bar{c}) or (b \bar{c}) or (ab \bar{c}) exotic

FIGURE CAPTIONS

- Figure 1 Equivalent ways of representing a duality diagram.
- Figure 2 a) This diagram violates exchange degeneracy and the Harari-Freund conjecture.
b) The pomeron diagram. Secondary reggeons must be absent if HF is satisfied.
- Figure 3 Diagrams giving rise to Regge-Regge cuts.
- Figure 4 a) A renormalization correction
b) Example of a diagram which has renormalization loop that builds Regge cuts.
- Figure 5 a) This diagram has a discontinuity in all channels.
b) We argue that the s_{ab} discontinuity of this diagram is very small.
- Figure 6 An illustration of absorbed reggeons dual to peripheral resonances.
- Figure 7 Contributions to the six point function in the region of a fragmenting to c excluding diagrams contributing to the diffraction dissociation of a into c.
- Figure 8 Contributions to the diffraction dissociation of a into c.
- Figure 9 a) Illustration of the positivity of resonance contributions.
b) The square of this contribution contains $P \otimes R$ term that may be negative.

Figure 10 Terms contributing to the pomeron in the inclusive reaction

(a:c|b). There are no interference terms between 10a, b, and c by rule 2.

Figure 11 a) When squared, this sum gives rise to Figure 7c and Figure 8.

b) Diagrams contributing to the interference terms 7e and 7f.

Figure 12 Diagrams that contribute to the pionization region (a|c|b).

Figure 13 a) Illustration of the ordering of particles implied by duality.

The dashed line can always be drawn to separate the incoming from outgoing particles.

b) Diagram contributing to the region of b fragmenting into c

in $a+b \rightarrow c+d_1+d_2+d_n+X$, where d_i are also detected.

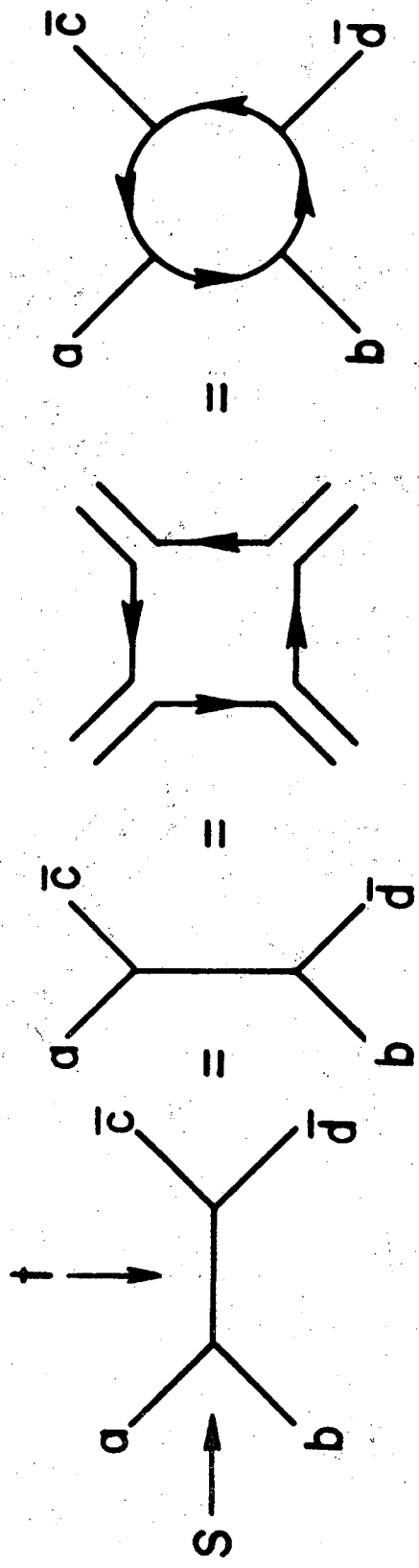
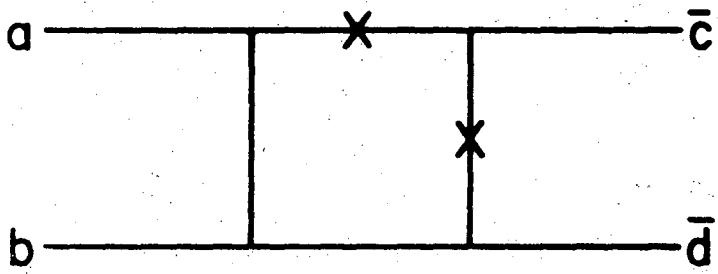
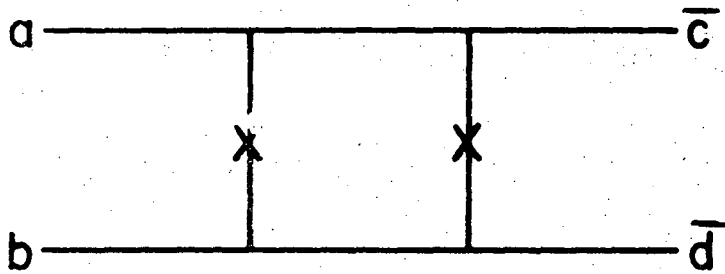
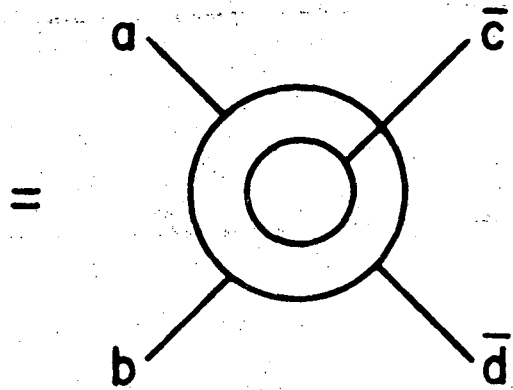


FIG. 1



(a)



(b)

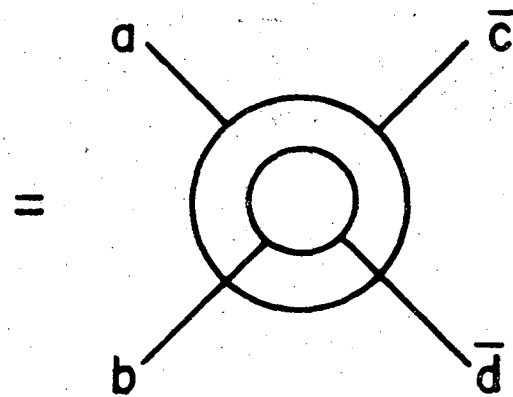
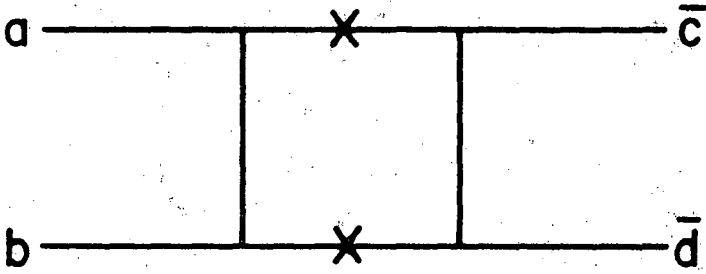
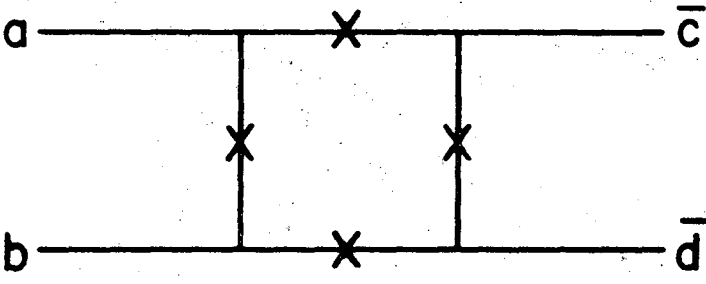
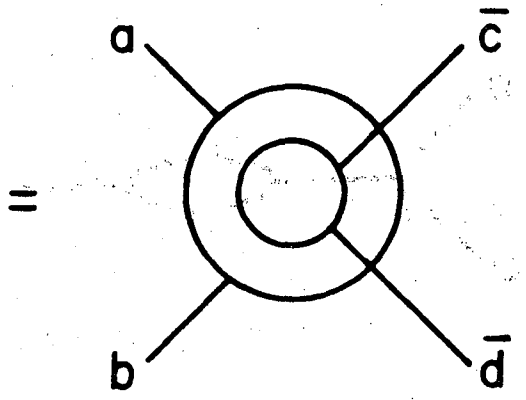


FIG. 2



(a)



(b)

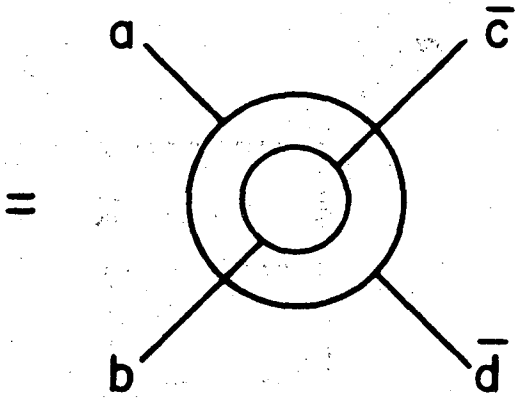
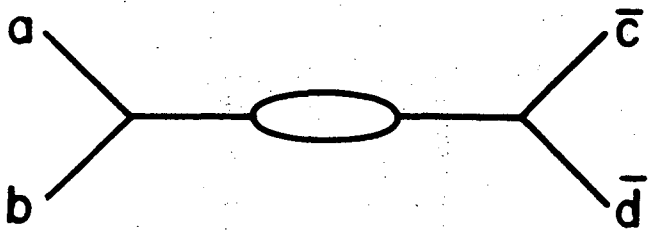
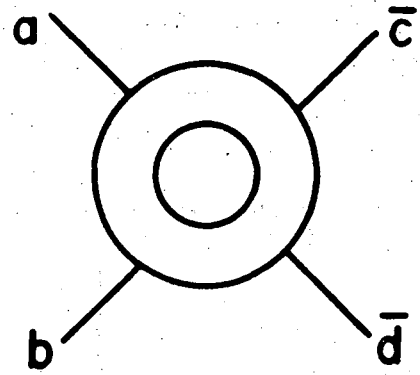


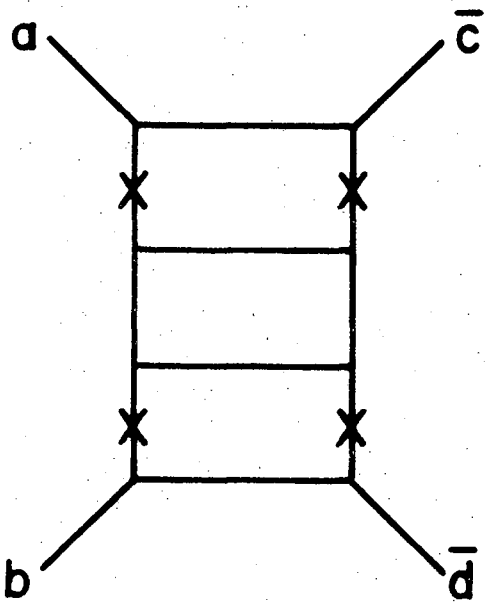
FIG. 3



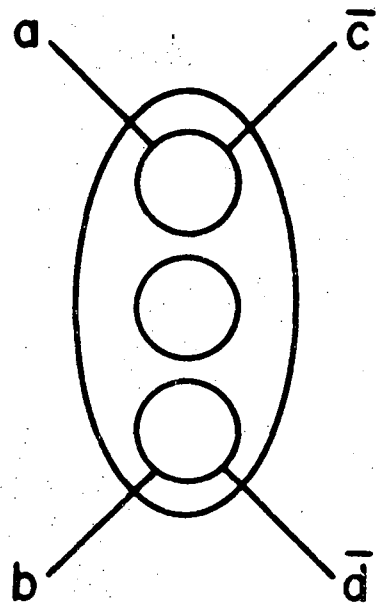
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(a)



=



(b)

FIG.4

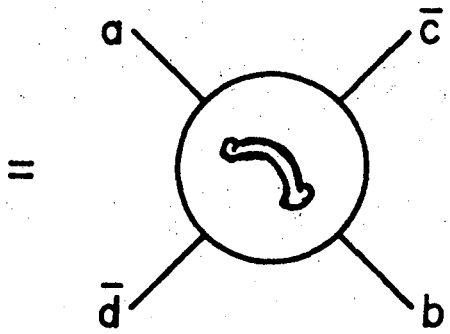
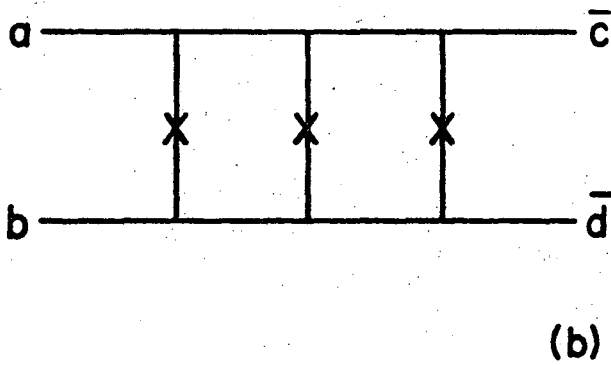
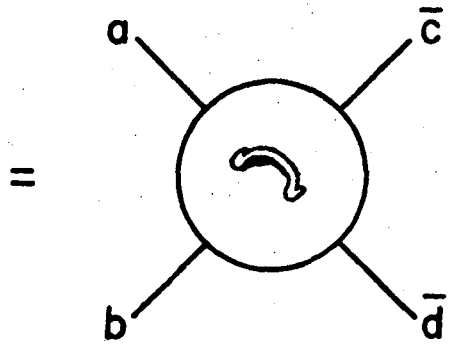
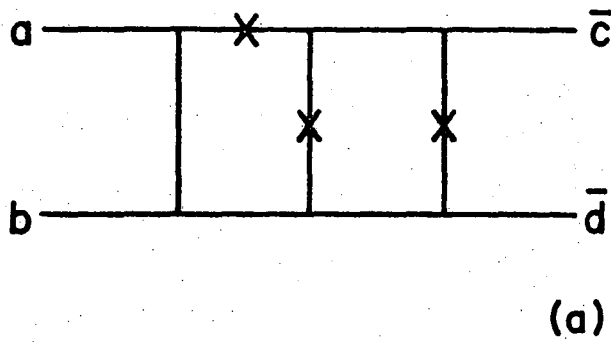


FIG.5

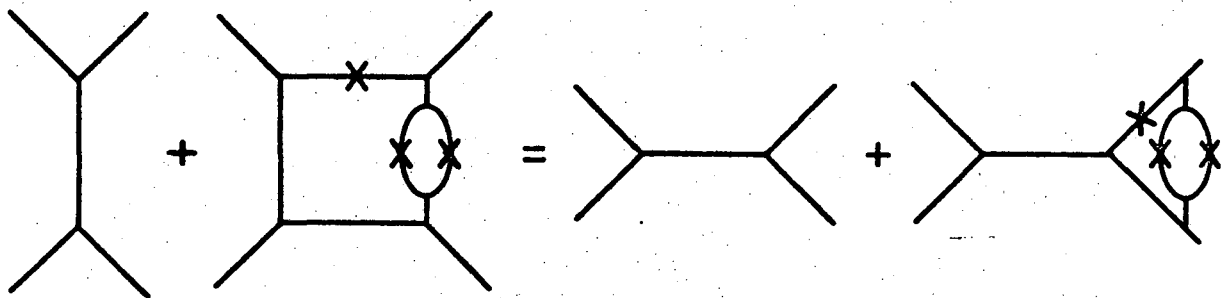


FIG. 6

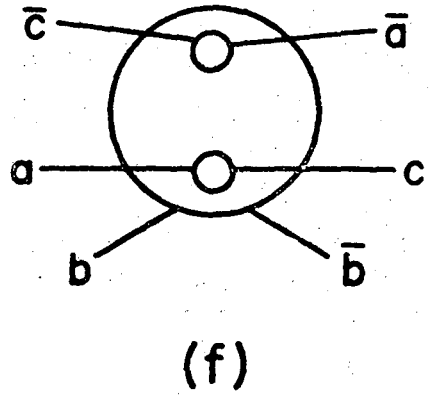
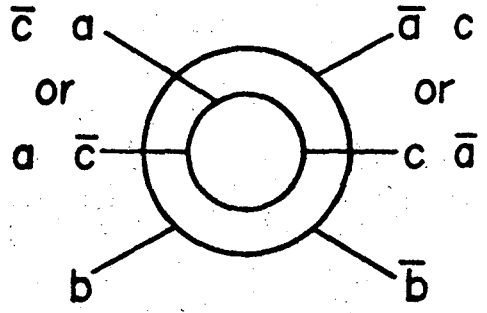
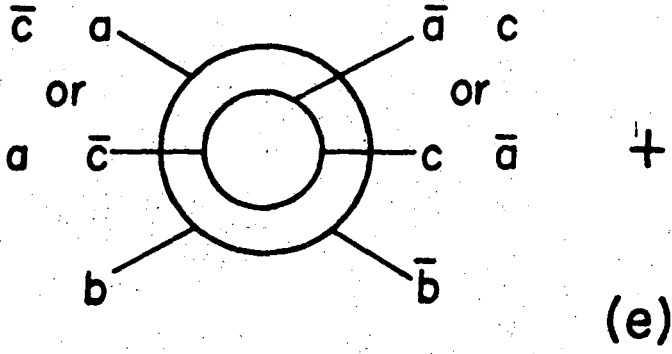
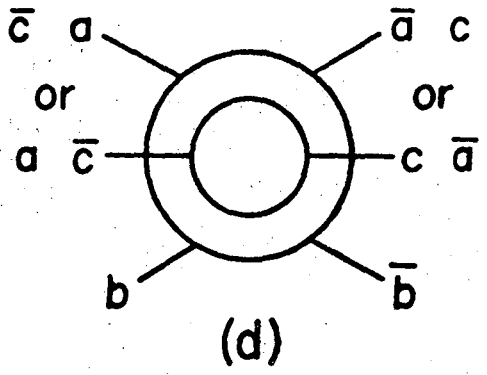
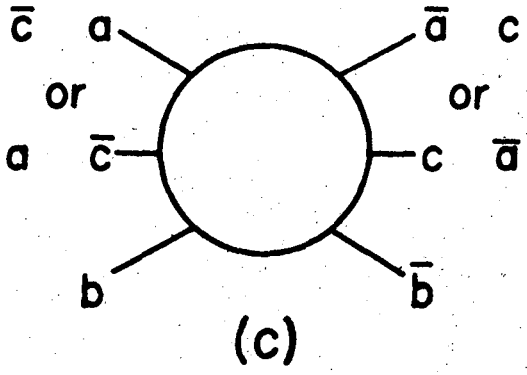
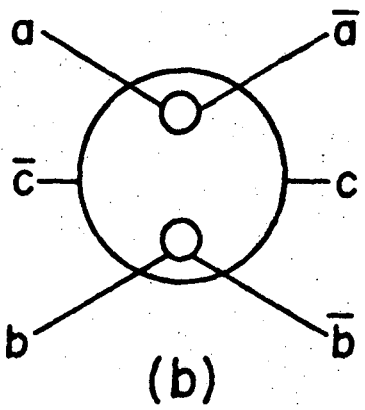
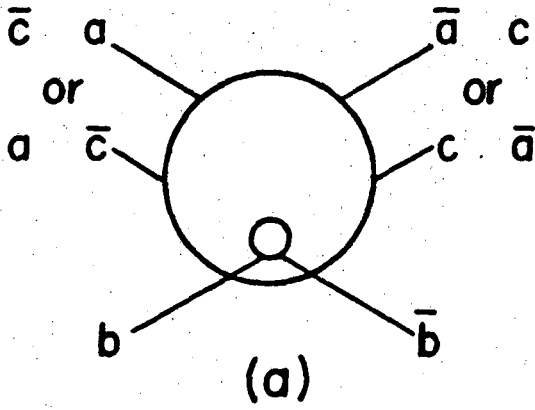
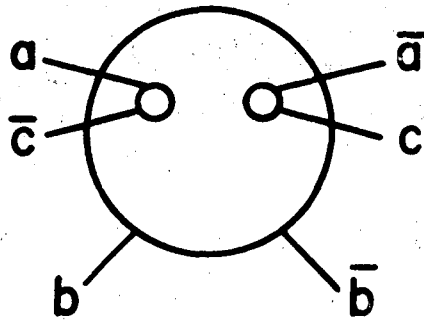
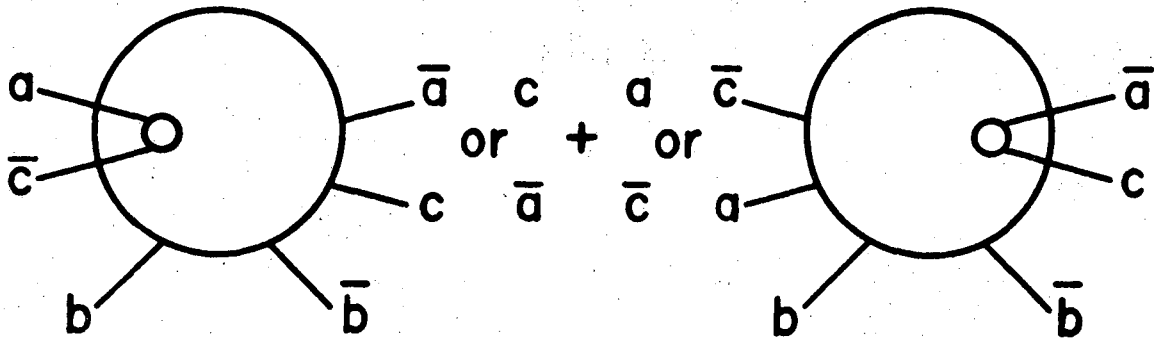


FIG. 7

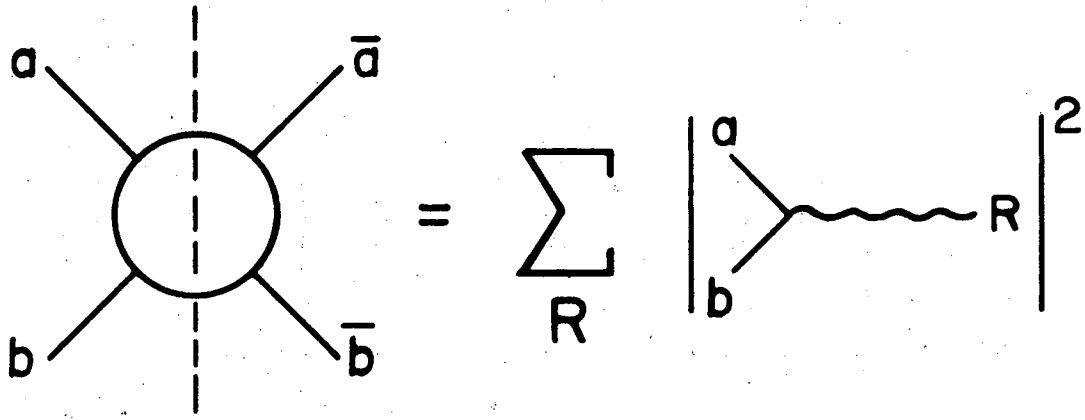


a

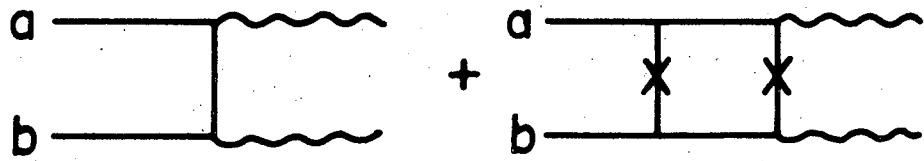


b

FIG. 8

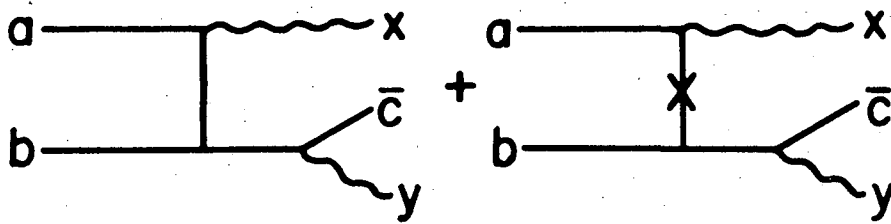


(a)

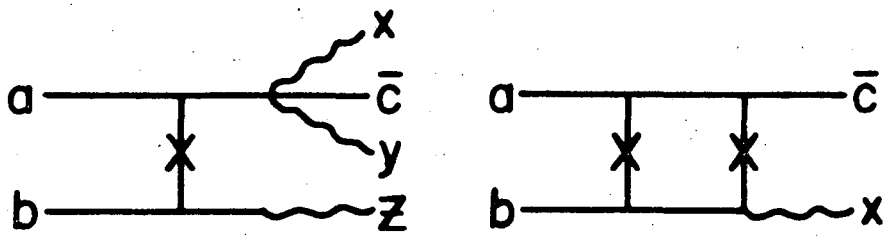


(b)

FIG. 9



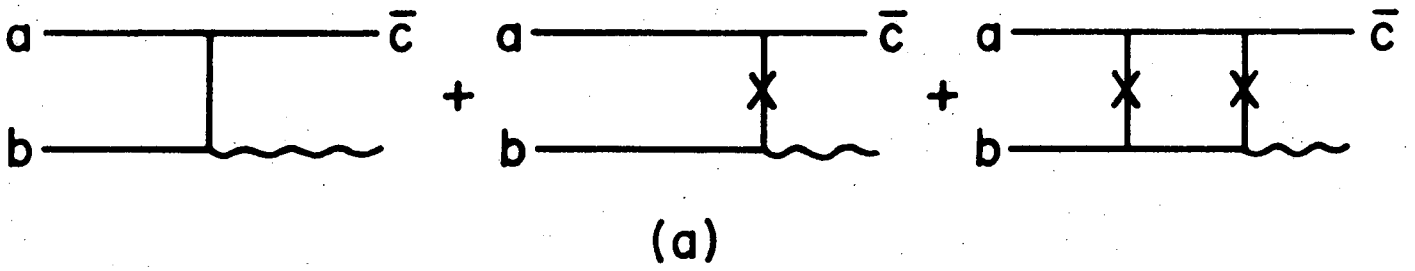
(a)



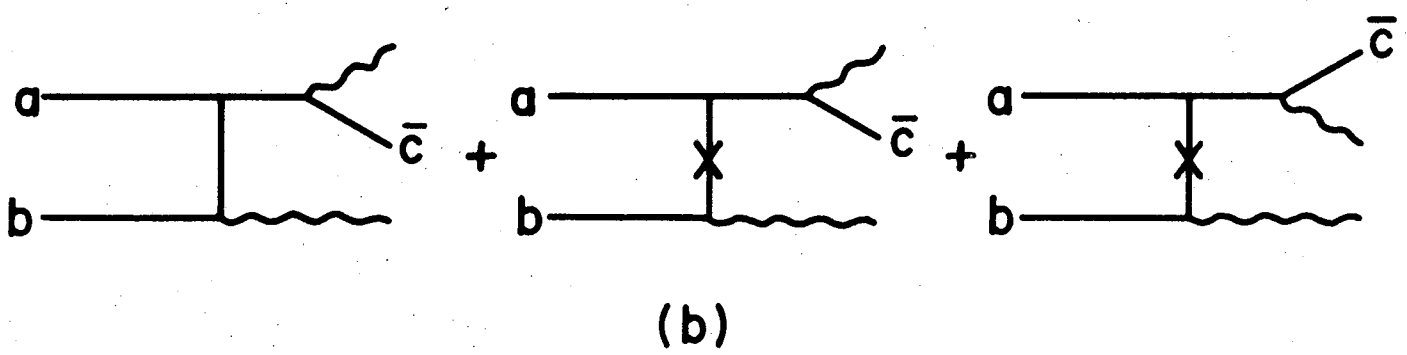
(b)

(c)

FIG. 10



(a)



(b)

FIG. 11

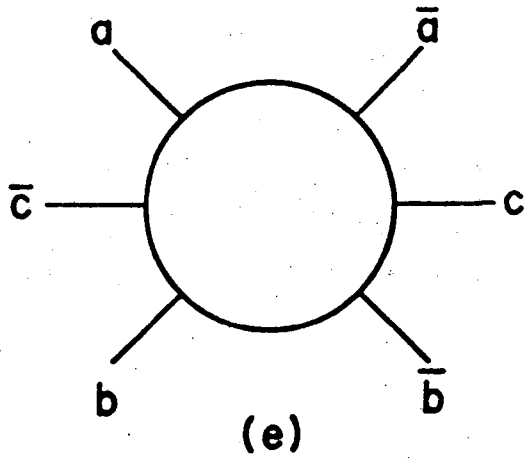
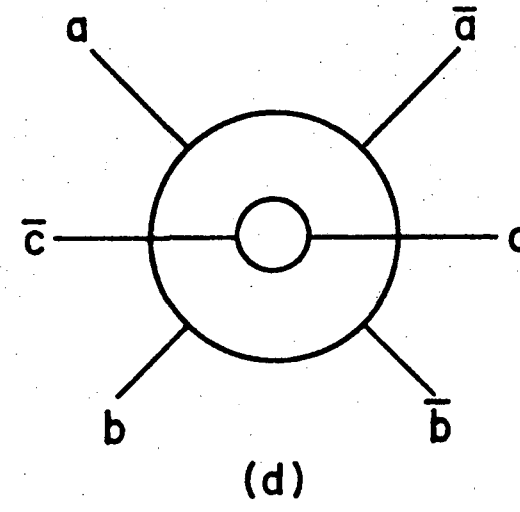
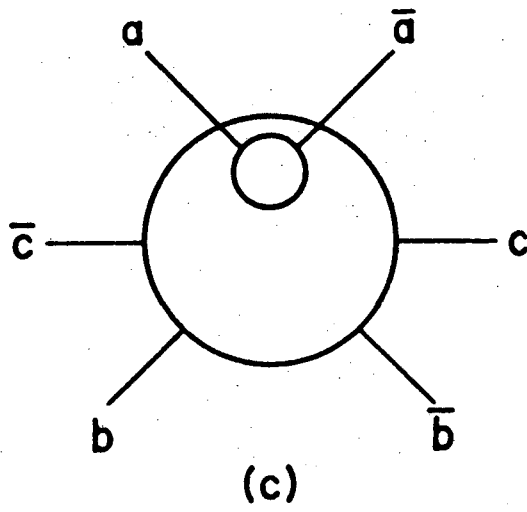
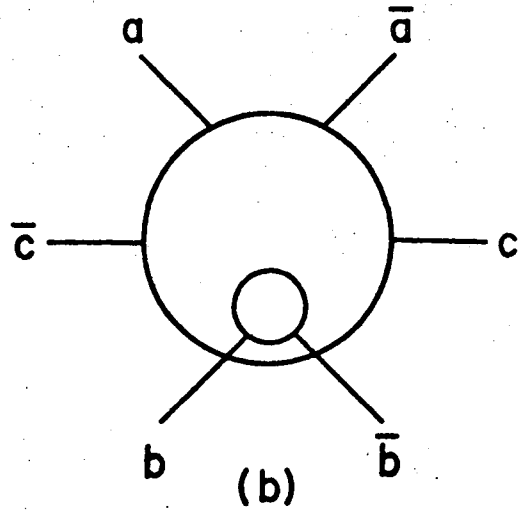
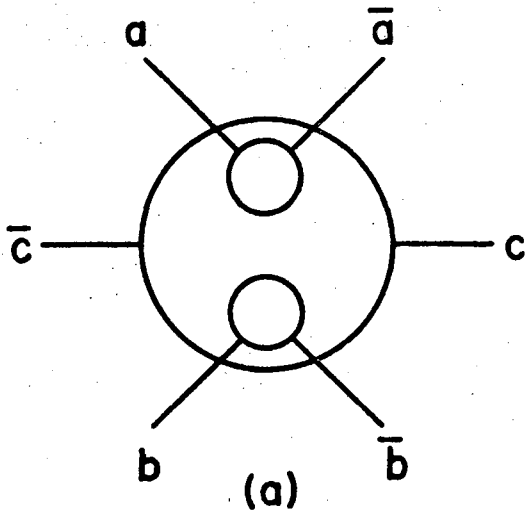


FIG. 12

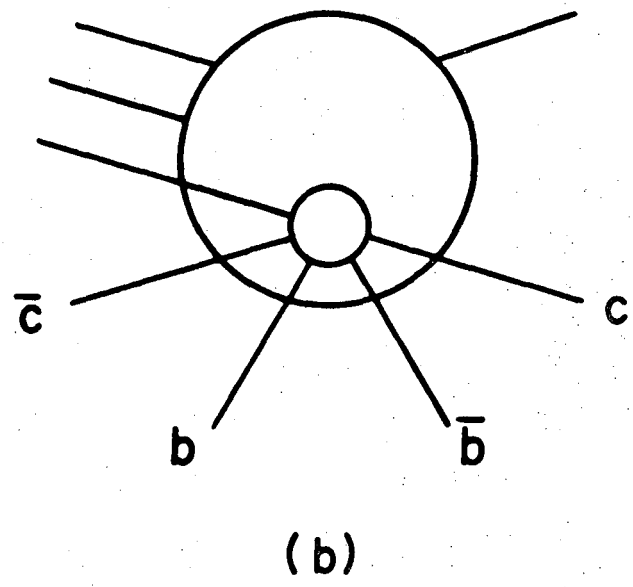
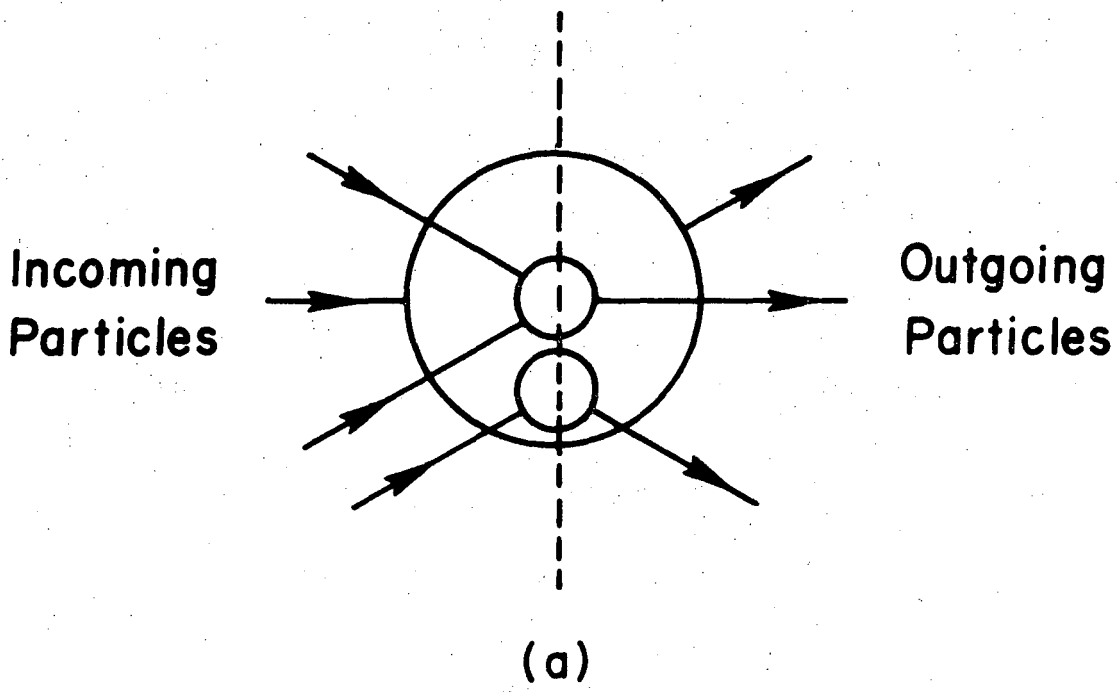


FIG. 13

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