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CP EFFECTS WHEN NEUTRINOS ARE THEIR OWN ANTIPARTICLES *

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Abstract

If neutrinos are their own antiparticles, then in the lepton sector the effects of both CP conservation and CP violation are quite different from what they are in the quark sector. To the extent that CP is conserved, the neutrinos are CP eigenstates possessing intrinsic CP parities. Consequences of these parities are described. If CP is violated, then, for a given number of generations, the leptonic weak interaction can contain more CP-violating phases than can the quark weak interaction. Indeed, the leptonic interaction can already contain a CP-violating phase when there are only two generations. The origin of the additional leptonic CP-violating phases is explained. Examples of CP-violating effects produced by these phases, and the sizes of these effects, are discussed.

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I. INTRODUCTION

If neutrinos have non-zero rest masses, then we must understand why they are so much lighter than the other fundamental fermions—the quarks and charged leptons. One of the most appealing explanations of this fact, the "see-saw mechanism",[1] suggests that neutrinos are Majorana particles; that is, fermions which are their own antiparticles.[2] If neutrinos are their own antiparticles, then there are major consequences relating to CP, whether CP is conserved or not. Suppose it is conserved. Then each Majorana neutrino of definite mass carries a quantum number, its intrinsic CP-parity, which is not carried by a quark or charged lepton. As we shall discuss, the presence of this quantum number has interesting implications in a variety of processes. Now suppose CP is not conserved. Recall that in the standard model, CP violation in the interactions of quarks is attributed to the presence of complex phase factors in the quark mixing matrix. In a similar way, CP violation in the interactions of leptons, although not yet observed, may be attributed to complex phase factors in the lepton mixing matrix. However, if neutrinos are their own antiparticles, then, for a given number of generations, the lepton mixing matrix can contain more CP-violating phases than it could if it were mixing fermions which are not their own antiparticles, such as quarks. Indeed, the quark mixing matrix cannot contain any CP-violating phases at all unless there are at least three generations,[3] but if neutrinos are Majorana particles, then the lepton mixing matrix can already contain a CP-violating phase when there are only two generations. Hence, even if only two of the three known lepton generations mix appreciably, there can still be sizeable CP-violating effects in the leptonic sector.

In this article we shall examine both the CP-conserving and the CP-violating situations. In order to focus on the main points of physics, we shall for the most part discuss only the simplest possible lepton mixing—that involving two generations. By considering several illustrative physical processes, we shall show how the Majorana character of neutrinos affects the CP-related behavior of reactions. In the course of doing this, we shall try to make very clear why the two-generation quark mixing matrix <u>cannot</u> lead to any CP-violating effects, but, if neutrinos are Majorana particles, the equally-small two-generation lepton mixing matrix <u>can</u>. Examples of the resulting CP-violating phenomena will be discussed.

II. CP-PROPERTIES OF MAJORANA NEUTRINOS WHEN CP IS CONSERVED

A Majorana neutrino is its own antiparticle in the sense that it is its own CPT mirror-image.[4] If CP violation may be neglected, as we shall assume in this section, then a Majorana neutrino is also its own CP mirror-image. More precisely, if $|\nu(\vec{p}, h)\rangle$ is a Majorana neutrino with momentum $\vec{p}$ and helicity h, then

$$\mathrm{CP}\,|\nu(\vec{\mathrm{p}},\mathrm{h})\rangle = \tilde{\eta}\,|\nu(-\vec{\mathrm{p}},-\mathrm{h})\rangle\,. \tag{1}$$

Here, the reversal of $\vec{p}$ and h is due to the parity operator in CP, and $\tilde{\eta}$ is a phase factor which represents the intrinsic CP-parity of the neutrino ν. Different neutrinos can have different values of $\tilde{\eta}$, but, interestingly enough, the possible values of this quantum number are not +1 and -1, but +i and -i. An easy way to see this is to consider the decay of the neutral weak boson into a pair of identical Majorana neutrinos: $Z^0 \to \nu\nu$. In the standard model, this decay conserves CP. To find the consequences of this conservation, it suffices to suppose that the outgoing neutrinos are nonrelativistic. Since their state clearly must be antisymmetric, it must be a $^3P_1$ state, this being the only nonrelativistic, antisymmetric state with total angular momentum equal to the spin of the Z^0. Now, from Eq. (1) it follows that if the intrinsic CP-parity of ν is $\tilde{\eta}$, then our $\nu\nu$ final state, with orbital angular momentum $L = 1$, obeys

$$\begin{aligned}\mathrm{CP}\left|\nu\nu;\,{}^3P_1\right\rangle &= \tilde{\eta}^2(-1)^L\left|\nu\nu;\,{}^3P_1\right\rangle \\ &= -\tilde{\eta}^2\left|\nu\nu;\,{}^3P_1\right\rangle.\end{aligned} \tag{2}$$

Hence, since the Z^0 has CP= +1, conservation of CP in $Z^0 \to \nu\nu$ requires that $-\tilde{\eta}^2 = +1$. Thus, the permissible values of the intrinsic CP-parity of a Majorana neutrino are[5]

$$\tilde{\eta} = \pm i. \tag{3}$$

Examples of the role played by this quantity in physical processes will be given in Sections V–VII.

Quite apart from the phase factor $\tilde{\eta}$, the fact that a Majorana neutrino is an eigenstate of CP in the CP-conserving case has physical consequences. For example, suppose there is a very heavy Majorana "neutrino" N which has the decay mode $N \to e^- + X$, where X is some collection of hadrons. If the interaction Hamiltonian H is CP-invariant, and $|N(s)\rangle$ is an N at rest with z-axis spin projection s, then according to Eq. (1) the N decay amplitude obeys

$$\begin{aligned} &\left|\left\langle e^-(\vec{p}_e, h_e)X(\vec{p}_x, h_x)|H|N(s)\right\rangle\right|^2 \\ = &\left|\left\langle e^-(\vec{p}_e, h_e)X(\vec{p}_x, h_x)|(\mathrm{CP})^{-1}\mathrm{H}(\mathrm{CP})|\mathrm{N(s)}\right\rangle\right|^2 \\ = &\left|\left\langle e^+(-\vec{p}_e, -h_e)\bar{X}(-\vec{p}_x, -h_x)|H|N(s)\right\rangle\right|^2 . \end{aligned} \tag{4}$$

Here we have denoted the momenta and helicities of the particles in X collectively by $\vec{p}_x$ and h_x, and $\left|\bar{X}\right\rangle = \mathrm{CP}\,|\mathrm{X}\rangle$. We see that N must also have the decay mode $N \rightarrow e^+ + \bar{X}$. Indeed, summing over final momenta and helicities, we conclude that[6)]

$$\Gamma(N \rightarrow e^+\bar{X}) = \Gamma(N \rightarrow e^-X). \tag{5}$$

As a second example, consider the reaction $e^- + e^+ \rightarrow \nu_1 + \nu_2$, where ν_1 and ν_2 are two different heavy Majorana neutrinos, and we imagine that they can be distinguished by their decay modes. If CP is conserved, then Eq. (1) implies that in the e^-e^+ c.m. frame, where $\vec{p}_{e^-} = -\vec{p}_{e^+} \equiv \vec{p}$, and $\vec{p}_{\nu_1} = -\vec{p}_{\nu_2} \equiv \vec{q}$, the amplitude for the reaction obeys

$$\begin{aligned} &\left|\left\langle \nu_1(\vec{q}, h_1)\nu_2(-\vec{q}, h_2)|T|e^-(\vec{p}, h_-)e^+(-\vec{p}, h_+)\right\rangle\right|^2 \\ = &\left|\left\langle \nu_1(-\vec{q}, -h_1)\nu_2(\vec{q}, -h_2)|T|e^+(-\vec{p}, -h_-)e^-(\vec{p}, -h_+)\right\rangle\right|^2 . \end{aligned} \tag{6}$$

Summing this relation over the final helicities and averaging it over the initial ones, we see that the angular distribution of the outgoing neutrinos can have no front-back asymmetry; as many ν_1 particles must be produced with momentum $-\vec{q}$ as with momentum $+\vec{q}$.[7)]

III. A GENERAL FRAMEWORK

In our discussions of specific physical processes, it will be useful to have a general framework for the treatment of CP effects when neutrinos are Majorana particles. This we develop in this section and the next. We assume that the weak interactions are described by the standard model (in which neutrinos are massless) with a purely-Majorana neutrino mass term added. The neutrinos are then massive, Majorana particles, and there is just one of them per generation. If there are N generations, the mass term is

$$\mathcal{L}_M = -\sum_{f,f'=1}^{N} \overline{(\nu^0_{fL})^c} M_{ff'} \nu^0_{f'L} + \text{ h.c.}. \tag{7}$$

Here $\nu^0_{fL} = \nu^0_{eL}, \nu^0_{\mu L}, \ldots$ is a left-handed "flavor eigenstate" neutrino (we shall distinguish between flavor and mass eigenstates by writing the former with a superscript zero), and c denotes charge conjugation. The matrix M (the mass matrix) is symmetric, and can be diagonalized by a transformation of the form

$$U^T M U = d. \tag{8}$$

In this relation, d is a diagonal matrix whose diagonal elements are the real, positive-definite neutrino masses, and U is a unitary matrix.

The leptons couple to the W boson through the left-handed charged weak current

$$j_\alpha = \sum_{f=1}^{N} i\overline{\ell^0_{fL}}\gamma_\alpha \nu^0_{fL}. \tag{9}$$

Here $\ell^0_{fL} = e^0_L, \mu^0_L, \ldots$ is a gauge or flavor eigenstate charged lepton. Neglecting charged lepton mixing for simplicity, we may identify the flavor eigenstates ℓ^0_f with the familiar mass eigenstates $\ell_f = e, \mu, \ldots$. Then, if we denote the neutrino mass eigenstates by ν_m, $m = 1, \ldots, N$, it can be shown that the weak current may be rewritten in terms of mass eigenstates as

$$j_\alpha = \sum_{f,m=1}^{N} i\overline{\ell_{fL}}\gamma_\alpha U_{fm}\nu_{mL}. \tag{10}$$

In this expression, U, which is called the <u>lepton mixing matrix</u>, is the same $N \times N$ unitary matrix as appears in Eq. (8).

Under charge conjugation, a Majorana field such as the neutrino field ν_m goes into itself apart from a phase factor:[8)]

$$\nu^c_m \equiv C\overline{\nu}^T_m = \lambda^*_m \nu_m. \tag{11}$$

Here C is the charge conjugation matrix, and we shall refer to the phase factor λ_m as the <u>creation phase factor</u>.[5)]

A Majorana neutrino field, like a quark field, may be redefined by multiplying it by a phase factor. Notice, however, that Eq. (11) implies that under such a multiplication, the creation phase factor associated with the Majorana field changes. In particular, if ν_m satisfies Eq. (11) and $\nu'_m \equiv e^{-i\phi_m}\nu_m$, then $(\nu'_m)^c = \lambda'^*_m \nu'_m$ with

$$\lambda'_m = e^{-2i\phi_m}\lambda_m. \tag{12}$$

Now, if the current of Eq. (10) is to remain unchanged when ν_m is multiplied by $e^{-i\phi_m}$, then, for all f, U_{fm} must at the same time be multiplied by $e^{+i\phi_m}$. Thus, when ν_m is multiplied by a phase factor, the quantities

$$\omega_{fm} \equiv \frac{U_{fm}}{U^*_{fm}} \lambda_m, \; f = 1, \ldots, N, \tag{13}$$

do not change. Moreover, since the "rephasing" $\ell_f \to \ell'_f \equiv e^{-i\phi_f} \ell_f$ of the charged lepton field ℓ_f requires the simultaneous rephasing $U_{fm} \to U'_{fm} \equiv e^{-i\phi_f} U_{fm}$ of the U_{fm} for all m, it is clear that the quantities

$$\Omega_{fmm'} \equiv \frac{\omega_{fm}}{\omega_{fm'}}; \quad f, \, m, \, m' = 1, \ldots, N, \tag{14}$$

are invariant under rephasing of either the neutrino or the charged lepton fields. Therefore, physically-meaningful phases can depend on these quantities.

When CP is conserved, the $\Omega_{fmm'}$ have a very simple significance. To see what that is, we note that the weak interaction Hamiltonian in which j_α occurs is

$$\mathcal{H} = \frac{g}{\sqrt{2}} \sum_{f,m=1}^{N} \left[W_\alpha^- i \overline{\ell_{fL}} \gamma_\alpha U_{fm} \nu_{mL} + W_\alpha^+ i \overline{\nu_{mL}} \gamma_\alpha U^*_{fm} \ell_{fL} \right]. \tag{15}$$

Here g is a real coupling constant. Now, under CP,

$$W_\alpha^- i \overline{\ell_{fL}} \gamma_\alpha U_{fm} \nu_{mL} \to \left[\tilde{\eta}(W) \tilde{\eta}(\ell_f) \frac{\tilde{\eta}(\nu_m)}{\lambda_m} \right]^* W_\alpha^+ i \overline{\nu_{mL}} \gamma_\alpha U_{fm} \ell_{fL}, \tag{16}$$

where $\tilde{\eta}(\nu_m)$ is the CP-parity of ν_m defined by Eq. (1), and $\tilde{\eta}(W), \tilde{\eta}(\ell_f)$ are, respectively, CP phases relating W^- to W^+, and ℓ_f^- to ℓ_f^+. Comparing Eqs. (15) and (16), we see that if $\mathcal{H}$ is to be CP-invariant, we must have

$$U^*_{fm} = U_{fm} \left[\tilde{\eta}(W) \tilde{\eta}(\ell_f) \frac{\tilde{\eta}(\nu_m)}{\lambda_m} \right]^*. \tag{17}$$

Now, while $\tilde{\eta}(\nu_m)$ is a physically-significant phase factor, $\tilde{\eta}(W)$, $\tilde{\eta}(\ell_f)$, and λ_m may be chosen to suit our convenience (see Eq. (12), for example). If we choose them so that

$$\frac{\tilde{\eta}(W) \tilde{\eta}(\ell_f)}{\lambda_m} = \tilde{\eta}^*(\nu_m), \tag{18}$$

then, from Eq. (17), the CP-conserving U matrix will be real. This is very similar to the familiar situation in the quark sector, where phases can be (and customarily are) so chosen that the CP-conserving quark mixing matrix is real. Regardless of how we choose the adjustable phase factors in Eq. (17), it follows from that relation that when CP is conserved,

$$\omega_{fm} = \tilde{\eta}(W)\tilde{\eta}(\ell_f)\tilde{\eta}(\nu_m). \tag{19}$$

Thus, when CP is conserved, $\Omega_{fmm'}$ is just the relative CP-parity of the two neutrinos ν_m and ν'_m:

$$\Omega_{fmm'} = \frac{\tilde{\eta}(\nu_m)}{\tilde{\eta}(\nu_{m'})}. \tag{20}$$

IV. THE NUMBER OF CP-VIOLATING PHASES, AND AN ILLUSTRATIVE MATRIX

When the U matrix does not satisfy the constraint (17), it contains CP-violating parameters. How many such parameters can it contain?

Let us first recall the case of quarks. The N-generation quark mixing matrix V which is the analogue of the lepton matrix U appears in the current

$$J_\alpha = \sum_{i,j=1}^{N} i\overline{d_{iL}}\gamma_\alpha V_{ij} u_{jL}. \tag{21}$$

Here $d_i = d, s, b, \ldots$ runs over the negatively-charged quark mass eigenstates, and $u_j = u, c, t, \ldots$ runs over the positively-charged ones. Now, being $N \times N$ and complex, V can be fully specified by $2N^2$ real numbers. However, since V is unitary, these numbers are subject to N^2 unitarity constraints, so only N^2 of them are independent. Furthermore, not all of these N^2 parameters are physically significant, since Eq. (21) involves $2N-1$ arbitrary relative phases between the $2N$ quark fields. If we change these phases, the phases of the V_{ij} must change also so that J_α remains fixed. Thus, V contains $N^2-(2N-1) = (N-1)^2$ physically-meaningful parameters. Now, the quark analogue of Eq. (17) implies that when CP is conserved, V can be taken to be real. Then it is an orthogonal matrix, and can be specified by N^2 real numbers subject to $N+\frac{1}{2}N(N-1)$ orthogonality constraints, so that $\frac{1}{2}N(N-1)$ of them are independent. Thus, the number of CP-violating parameters that V can contain is $(N-1)^2$, the total number of significant parameters in the general case, minus $\frac{1}{2}N(N-1)$, the number of such parameters when CP is conserved. That is,[9]

$$\left\{ \begin{array}{c} \text{Number of CP-violating parameters} \\ \text{in quark mixing matrix} \end{array} \right\} = \frac{1}{2}(N-1)(N-2). \tag{22}$$

As is well known, in the convention where V is real when CP is conserved, the CP-violating quantities in this matrix are complex phase factors. The resulting complexity of V can lead to complex relative phases between physical amplitudes that interfere with each other. It is through these interferences that the CP-violating phases in V make their presence felt and produce observable CP-violating effects.

Like the quark mixing matrix, the lepton mixing matrix U, being unitary, can be specified by N^2 independent parameters. Of these, N are not significant, since U changes if we multiply any of the N charged lepton fields by a phase factor. Thus, $N(N-1)$ significant parameters remain. Of course, the Majorana neutrino fields ν_m can also be multiplied by phase factors, as we have discussed.[10] However, when such multiplications are performed, not only the U_{fm} but also the creation phase factors λ_m change. Now, as we illustrate in the following sections, the λ_m sometimes appear in reaction amplitudes together with elements of U in ν_m-rephasing-invariant quantities such as the ω_{fm} or $\Omega_{fmm'}$ of Eqs. (13) and (14). Moreover, these invariant quantities have observable effects. Thus, while we are indeed free to shift phase factors out of U by rephasing the ν_m, any phase factors that can be removed from U only through such rephasing are actually physically-significant. Their removal from U does not eliminate them from the problem, but merely transfers them to the creation phase factors λ_m. Through their presence in the λ_m, they still have physical consequences, which get transmitted through ν_m-rephasing-invariant quantities such as the ω_{fm}.

With these circumstances in mind, let us now count the number of possible CP-violating phases in the leptonic charged-current weak interaction. As we have seen, the U matrix contains in general $N(N-1)$ significant parameters, allowing for the fact that the charged lepton fields can be rephased. This number of parameters is not further reduced by the possibility of rephasing the neutrino fields because, as we have said, such rephasing only moves a significant phase factor from U to the λ_m. Now, if CP is conserved, we may choose the λ_m so that Eq. (18) is satisfied and U is real. Like the CP-conserving quark mixing matrix, U then contains $\frac{1}{2}N(N-1)$ parameters. Thus, the number of CP-violating phases in the leptonic weak interaction is $N(N-1)$, the number of parameters in the general case, minus $\frac{1}{2}N(N-1)$, the number when CP is conserved. That is,[11]

$$\left\{\begin{array}{l}\text{Number of CP-violating phases in lepton weak} \\ \text{interaction if neutrinos are Majorana particles}\end{array}\right\} = \frac{1}{2}N(N-1). \tag{23}$$

We see that if neutrinos are Majorana particles, then for a given number of generations, the leptonic weak interaction can involve more CP-violating phases than the quark weak interaction. A particularly striking example of this phenomenon occurs when there are only two generations. When $N = 2$, Eq. (22) shows that there can be absolutely no CP-violating phases in the quark sector, but Eq. (23) indicates that there can be a CP-violating phase in the lepton sector. An interesting way to express this state of affairs is to consider the sample 2×2 unitary mixing matrix

$$X = \begin{pmatrix} c & se^{i\delta} \\ -se^{-i\delta} & c \end{pmatrix}, \tag{24}$$

where c and s are the cosine and sine of some mixing angle θ. What we have learned is that if X mixes quarks, then the phase factor $e^{i\delta}$ has no physical consequences. However, if this same matrix mixes leptons, and neutrinos are their own antiparticles, then the factor $e^{i\delta}$ leads in general to CP violation. Now, can we see in some simple, concrete way why this is the case? Indeed we can, and by considering several revealing reactions, we shall.[12]

V. NEUTRINO RADIATIVE DECAY

Assuming that there are just two generations, let us contrast the radiative decay $\nu_2 \to \nu_1 + \gamma$ of a heavy neutrino into a lighter one with the analogous decay $c \to u + \gamma$ of the charmed quark into the up quark. Both decays go through loop diagrams. The loops for the quark decay are shown in Fig. 1. There are two, one with an internal d quark, and the other with an s quark. If the quark mixing matrix V is the matrix X of Eq. (24), then the combination of V matrix elements $V_{dc}V_{du}^*$ to which the d diagram is proportional is $cse^{i\delta}$, while that to which the s diagram is proportional, $V_{sc}V_{su}^*$, is $-cse^{i\delta}$. Thus, the phase factor $e^{i\delta}$ in X is common to the two diagrams, and disappears when their sum is squared. As previously claimed, this factor has no physical consequences. This is, of course, no accidental feature of the particular sample matrix X. As long as V is 2×2, its unitarity requires that $V_{sc}V_{su}^* = -V_{dc}V_{du}^*$. Hence, the two diagrams in Fig. 1 are always real relative to each other; their interference is completely insensitive to any phase factors in V.

What changes when we go to the neutrino decay? The two neutrino diagrams which are the analogues of those in Fig. 1 are depicted together as the diagram S_- in Fig. 2. This diagram is present even if neutrinos are Dirac particles (*i.e.*, not their own antiparticles). However, if they are Majorana particles, then a new diagram, with no analogue in the quark case, is also present. In this diagram, labelled S_+ in Fig. 2, the incoming neutrino, "confused" about whether it is a lepton or an antilepton, turns into an e^+ or μ^+, rather than an e^- or μ^- as in diagram S_-. At each vertex in S_+, the term in the Hamiltonian (15) which acts is the Hermitean conjugate of that which acts at the corresponding vertex in S_-. Thus, where U_{fm} appears in S_-, U^*_{fm} appears in S_+. Indeed, from Fig. 2 we see that for a given charged lepton ℓ_f, the U matrix and creation phase factors impart to S_+ and S_- a relative phase factor

$$\frac{(U_{f1}/U^*_{f1})\lambda_1}{(U_{f2}/U^*_{f2})\lambda_2}. \tag{25}$$

This factor is nothing but the rephasing-invariant quantity Ω_{f12} defined by Eqs. (13) and (14). When $N = 2$, this quantity is independent of f due to the unitarity of U, so let us call it simply Ω. If U is our sample matrix X,

$$\Omega = e^{-2i\delta}\frac{\lambda_1}{\lambda_2}. \tag{26}$$

Obviously, if Ω is complex, the interference between S_+ and S_- will reflect that fact. Furthermore, Eq. (20) implies that when Ω is complex, the weak interactions are CP-violating, and it is not hard to show that the converse is true as well. Thus, when the interactions violate CP, there will be observable consequences arising out of the $S_+ - S_-$ interference. To summarize: Reactions involving Majorana neutrinos can have more diagrams than the corresponding reactions involving quarks or Dirac neutrinos. With this greater number of diagrams comes a greater number of interferences. Through their sensitivity to complex phase factors, the additional interferences can lead to observable violations of CP that would not occur in the Dirac case.[13)]

To what observable effect does a complex Ω lead, and how big is it? First, from Lorentz invariance and conservation of the electromagnetic current J^{EM}_μ, the $\nu_2 \rightarrow \nu_1 + \gamma$ amplitude must always have the form

$$A(\nu_2 \rightarrow \nu_1 + \gamma) = \epsilon^*_\mu \left\langle \nu_1 \left| J^{EM}_\mu \right| \nu_2 \right\rangle = \epsilon^*_\mu i\bar{u}_1 \sigma_{\mu\nu} q_\nu (M + iE\gamma_5) u_2. \tag{27}$$

Here ϵ is the polarization vector of the photon, q is its momentum, u_2 and u_1 are, respectively, Dirac spinors for the initial and final neutrino, and M and E are constants. Now, for a given ℓ_f, the diagram S_- gives to $A(\nu_2 \to \nu_1 + \gamma)$ a contribution of the form (*cf.* Fig. 2)

$$\epsilon^*_\mu \bar{u}_1 U_{f2} U^*_{f1} \Gamma^f_\mu(\gamma_5) u_2, \tag{28}$$

where $\Gamma^f_\mu(\gamma_5)$ is some combination of gamma matrices and coefficients, involving γ_5. From Fig. 2, we see that S_- and S_+ together then yield[5]

$$A(\nu_2 \to \nu_1 + \gamma) = \epsilon^*_\mu \bar{u}_1 \sum_f U_{f2} U^*_{f1} \left[\Gamma^f_\mu(\gamma_5) - \Omega \Gamma^f_\mu(-\gamma_5)\right] u_2. \tag{29}$$

Here the minus sign in front of the S_+ contribution is due to the photon vertex.

Suppose CP is conserved. Then, from Eq. (20), $\Omega = \tilde{\eta}(\nu_1)/\tilde{\eta}(\nu_2)$. Thus, comparing Eqs. (29) and (27), we see that the decay amplitude will be of purely electric dipole ($\sigma_{\mu\nu} q_\nu \gamma_5$) structure if ν_1 and ν_2 have the same CP-parity, and of purely magnetic dipole ($\sigma_{\mu\nu} q_\nu$) structure if they have opposite CP-parity.[14]

Now suppose CP is not conserved, so that Ω is complex. Then Eq. (29) shows that, barring an accident (which does not occur as we shall see shortly), the decay amplitude is neither even nor odd in γ_5. That is, <u>both</u> electric and magnetic dipole terms are present. Furthermore, the simultaneous presence of these two terms in Eq. (27) <u>does</u> have an observable consequence, although not in the photon polarization or angular distribution as one might have thought. To see this, suppose that, in its rest frame, the parent ν_2 is polarized with spin vector $\vec{s}$. Let the angle between the photon momentum $\vec{q}$ and $\vec{s}$ be Θ. The photon can be linearly polarized with its electric field (parallel to $\vec{\epsilon}$) either in the decay plane formed by $\vec{s}$ and $\vec{q}$, or else normal to this plane. The magnetic and electric dipole amplitudes of Eq. (27) to produce these polarization states, when M or E is unity, are given in Table I. We see from this Table that the magnetic and electric dipole terms have almost indistinguishable consequences. Either can produce a

Table I. Amplitudes for production of linearly polarized photons. Common factors are omitted.

Photon electric field	ν_1 helicity	Decay amplitude: Magnetic dipole term	Decay amplitude: Electric dipole term
In plane	+	$-i\cos\frac{\Theta}{2}$	$-i\cos\frac{\Theta}{2}$
	−	$-i\sin\frac{\Theta}{2}$	$i\sin\frac{\Theta}{2}$
Normal to plane	+	$\cos\frac{\Theta}{2}$	$\cos\frac{\Theta}{2}$
	−	$-\sin\frac{\Theta}{2}$	$\sin\frac{\Theta}{2}$

photon with electric field in the decay plane or normal to it. For either of these polarization states, the photon angular distribution, summed over ν_1 helicity, is isotropic whether the decay is induced by the magnetic or the electric dipole term. Now, in principle, a linear combination of magnetic and electric dipole terms could distinguish itself from either term alone by leading to a non-isotropic distribution for linearly-polarized photons. However, it turns out that the particular linear combination which results from the standard model does not do this. It has been shown[15,14] that this model (including now the contributions from its Higgs doublet) gives for the quantity $\Gamma^f_\mu(\gamma_5)$ in Eqs. (28) and (29)

$$\Gamma^f_\mu(\gamma_5) = F(r_f)\sigma_{\mu\nu}q_\nu[M_2(1-\gamma_5) + M_1(1+\gamma_5)]. \tag{30}$$

Here M_2 and M_1 are, respectively, the masses of ν_2 and ν_1, and, in practice, $F(r_f) = M^2_{\ell_f}$ if we disregard irrelevant overall constants. From Eqs. (30) and (29), we find that the rate Γ for decay into photons of polarization $\vec{\epsilon}$ and momentum $\vec{q}$, summed over ν_1 helicity, is given by

$$\Gamma = |G|^2\left(1+\left(\frac{M_1}{M_2}\right)^2 - 2\frac{M_1}{M_2}\,\mathrm{Re}\,\Omega\right)(1+i\vec{\epsilon}\times\vec{\epsilon}^{\,*}\cdot\vec{s}). \tag{31}$$

Here

$$|G|^2 \equiv \left|\sum_{f=1,2} U_{f2}U^*_{f1}F(r_f)\right|^2, \tag{32}$$

and we are again omitting irrelevant overall factors. Note that $|G|^2$ does not depend on the phases in U, due to the unitarity of the latter.

We see from Eq. (31) that neither the photon angular distribution nor its polarization depends on the CP-violating phase Ω. For linearly-polarized photons (real $\vec{\epsilon}$), the angular distribution is isotropic, just as for a pure electric or pure magnetic dipole amplitude. For circularly polarized photons, $i\vec{\epsilon}\times\vec{\epsilon}^{*}\cdot\vec{s} =$ (helicity) $\times\,\hat{q}\cdot\hat{s}$, so the distribution is not isotropic, but it does not depend on Ω. However, Eq. (31) shows that the most easily-measured quantity, the total decay rate, does depend on Ω. Its dependence, through the factor

$$\Gamma_{CP} \equiv 1 + \left(\frac{M_1}{M_2}\right)^2 - 2\frac{M_1}{M_2}\,\mathrm{Re}\,\Omega, \tag{33}$$

can obviously be quite substantial, so long as M_1/M_2 is not too small. For given values of M_1, M_2, and the mixing angle in U (which enters $|G|^2$), $\Gamma_{tot}(\nu_2 \rightarrow \nu_1 + \gamma)$ has its maximum (minimum) value when CP is conserved and $\Omega = \tilde{\eta}(\nu_1)/\tilde{\eta}(\nu_2) = -1(+1)$. When CP is violated, Ω is not real and the decay rate lies somewhere between its extrema.

In the quark sector, a real mixing matrix implies that CP is conserved. With Majorana neutrinos, there is no such implication in the lepton sector. If the mixing matrix U in the leptonic interaction (15) is X, Eq. (24), and we rewrite the interaction in terms of the fields $\nu_2' = e^{i\delta}\nu_2$ and $\mu' = e^{i\delta}\mu$, then X gets replaced by the real matrix

$$X' = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}. \tag{34}$$

However, as Eq. (33) emphasizes, the CP-violating effect in $\nu_2 \rightarrow \nu_1 + \gamma$ depends on the rephasing invariant Ω, given by Eq. (26), and not solely on the phase factor $e^{i\delta}$ in the mixing matrix. It is not necessary for the mixing matrix to be complex in order for CP to be violated. Complexity of Ω is enough. If we make a complex mixing matrix real by rephasing the fields, then any CP-violating phase information will simply be transmitted to Ω through the creation phase factors, rather than through the mixing matrix.[16)]

Can Ω be far from real, so that Γ_{CP} is far from its CP-conserving values? In approaching this question, we recall that in the quark sector, CP-violating effects would be mass-suppressed if all quark masses were small. This is due to the fact that CP violation depends on mixing, and mixing—a mismatch between flavor eigenstates and mass eigenstates—requires that the fermions of a given charge have distinguishable masses. In view of this and the fact that neutrinos are rather light, it is natural to suspect that CP violation in $\nu_2 \rightarrow \nu_1 + \gamma$ must be infinitesimal. Fortunately, this suspicion is unfounded. To be sure, the $\nu_2 \rightarrow \nu_1 + \gamma$ decay rate is neutrino-mass suppressed. However, the ratio between this rate and its CP-conserving values can be far from unity, no matter how light ν_2 and ν_1 are. Equation (33) showed that this ratio is quite sensitive to Ω if M_1/M_2 is not tiny, and we now show that Ω can be far from real for arbitrarily light neutrinos. To do this, we find the values of Ω that may result from the most general 2×2 Majorana neutrino mass matrix,

$$M = \begin{pmatrix} m_{11}e^{i\gamma_1} & m_x e^{i\gamma_x} \\ m_x e^{i\gamma_x} & m_{22}e^{i\gamma_2} \end{pmatrix}, \tag{35}$$

where m_{11}, m_{22}, and m_x are real. A discussion of the mixing matrix which results from diagonalizing this M has been given by Barroso and Maalampi.[17,18] They show that if one follows a diagonalization procedure which yields neutrino mass eigenfields with $\lambda_m = 1$, one obtains a mixing matrix which can be written in the form of the matrix X, Eq. (24). Thus, this "sample" matrix can actually represent the mixing and CP violation engendered by any 2×2 Majorana mass matrix. To see whether CP violation is mass-suppressed, we have amplified the analysis of Barroso and Maalampi to explicitly relate the parameters in X to those in the corresponding M. With β and α defined by

$$\beta = \gamma_x - \frac{\gamma_1 + \gamma_2}{2}, \tag{36}$$

and

$$\tan\alpha = \frac{m_{11} - m_{22}}{m_{11} + m_{22}} \tan\beta, \tag{37}$$

the mixing angle θ in X is given by

$$\tan 2\theta = \frac{2m_x}{m_{22} - m_{11}} \frac{\cos\beta}{\cos\alpha}. \tag{38}$$

Further, with α_1 and α_2 defined by

$$\tan\alpha_1 = \frac{m_{22}\sin^2\theta \sin 2\alpha}{m_{11}\cos^2\theta - m_{22}\sin^2\theta\cos 2\alpha} \tag{39}$$

and

$$\tan\alpha_2 = \frac{m_{11}\sin^2\theta \sin 2\alpha}{m_{22}\cos^2\theta - m_{11}\sin^2\theta\cos 2\alpha}, \tag{40}$$

the phase δ in X is given by

$$\delta = \alpha + \frac{\alpha_1 + \alpha_2}{2}. \tag{41}$$

Now, suppose that m_{11}, m_{22}, and m_x are all proportional to some mass scale w. Then the neutrino masses, the eigenvalues of M, are also proportional to w. However, from Eqs. (37) - (41) we see that if β may vary independently of m_{11}, m_{22} and m_x, then the phase δ will vary with β in a way which is completely independent of w. Numerical examples verify that there is no unexpected cancellation in Eq. (41); for suitable β, δ is very large, no matter how small M_1 and M_2 are. Now, in the present parametrization the $\lambda_m = 1$, so, from Eq. (26), $\Omega = \exp(-2i\delta)$. Hence, Ω can indeed be far from real, and consequently the decay rate for $\nu_2 \to \nu_1 + \gamma$ can be far from its CP-conserving values, even if ν_2 and ν_1 are very light.

Pal and Wolfenstein have pointed out[14)] that if neutrinos have Majorana masses, then the non-standard Higgs multiplets which produce these masses may also lead to significant contributions to the process $\nu_2 \to \nu_1 + \gamma$, beyond those we have considered. Naturally, it will not be possible to use the measured rate for this process to learn about CP violation in the coupling of leptons to the W if non-standard Higgs contributions to the rate are too large. Pal and Wolfenstein considered a model in which these contributions completely dominate those of the W, but expressed the hope that in more attractive theories of neutrino mass the W contributions dominate.

VI. NEUTRINO PAIR PRODUCTION

Let us now consider further the reaction $e^- + e^+ \to \nu_1 + \nu_2$, where ν_1 and ν_2 are two distinct heavy neutrinos, and we imagine they can be distinguished through their decays. If neutrinos are Dirac particles and lepton number is conserved, then this reaction is really either $e^- + e^+ \to \nu_1 + \bar{\nu}_2$ or else $e^- + e^+ \to \bar{\nu}_1 + \nu_2$. For either of these processes, there is in lowest order only one diagram, a W exchange. Any phases at the vertices of this diagram obviously disappear when the amplitude is squared. Thus, there can be no CP violation coming from phases in the U matrix, regardless of the number of generations. By contrast, if neutrinos are Majorana particles, then the reaction is truly $e^- + e^+ \to \nu_1 + \nu_2$, and it arises from two W-exchange diagrams, shown in Fig. 3. The interference between these two diagrams makes CP violation possible. From Fig. 3, we see that relative to the diagram S_{12}, the diagram S_{21} has a phase factor

$$\frac{U_{e2}^{*}U_{e1}\lambda_1}{U_{e1}^{*}U_{e2}\lambda_2} = \Omega_{e12}. \tag{42}$$

This is the same factor as that relating the diagrams S_+ and S_- for $\nu_2 \rightarrow \nu_1 + \gamma$. When $N = 2$, it is just Ω. Its phase will clearly influence the interference between S_{21} and S_{12}. Here, as in neutrino radiative decay, we see that when neutrinos are Majorana particles, there can be more diagrams than when they are Dirac particles, and correspondingly more interferences between diagrams. Through these additional interferences, phase factors such as that in X, which would be of no consequence in the Dirac case, can lead to physical effects.

The relative phase factor (42) between S_{21} and S_{12} is, of course, a ν_m-rephasing-invariant quantity involving the creation phase factors λ_m. One might wonder how the λ_m would come to occur in this relative phase factor if we did not choose to rewrite some of the vertex factors in Fig. 3 in terms of charge-conjugate fields. The answer is that the λ_m appear in the plane wave expansions of the neutrino fields,[5)] and consequently will naturally appear in the Feynman amplitudes for diagrams such as S_{21} and S_{12}.

To what CP-violating effects can the relative phase between S_{21} and S_{12} lead? First, we note that if the energy is large compared to the masses of ν_1 and ν_2, then S_{12} is nonvanishing only if the final state is $\nu_1(-)\nu_2(+)$, and S_{21} only if it is $\nu_1(+)\nu_2(-)$, where the signs in parentheses denote helicity. Hence, unless we measure something fairly exotic, the diagrams do not interfere and there can be no CP violation. Therefore, let us consider energies just above $\nu_1\nu_2$ production threshold.[19)]

Suppose first that CP is conserved. If the electron mass is negligible at $\nu_1\nu_2$ threshold, then S_{12} and S_{21} are nonzero only when the initial state, in the c.m. frame and the notation of Eq. (6), is $|e^-(\vec{p},-)e^+(-\vec{p},+)\rangle$. Now, this state is an eigenstate of CP, and it is easy to see that its CP parity is +1. Note, for example, that in this state the e^- and e^+ have the right momenta and helicities to annihilate into a virtual photon, and the CP of the photon is +1. Turning to the final state, we note that if this state has orbital angular momentum L, then it too is an eigenstate of CP, with CP parity $\tilde{\eta}(\nu_1)\tilde{\eta}(\nu_2)(-1)^L$ (*cf.* Eq. (2)). Hence, if CP is conserved,

$$\tilde{\eta}(\nu_1)\tilde{\eta}(\nu_2)(-1)^L = +1. \tag{43}$$

In applying this constraint, we must remember that the CP parities of Majorana neutrinos are imaginary. Thus, near threshold the ν_1 and ν_2 will be produced in a pure p wave if they have the same CP parity, and in a pure s wave if they have opposite CP parity.

Now suppose that CP is not conserved. From the previous discussion, we expect that ν_1 and ν_2 will then be produced in both s and p waves. To confirm this, we have carried out the somewhat tedious but fairly straightforward calculation of the nonrelativistic limit of the amplitude corresponding to $S_{12}+S_{21}$.[20] We find that apart from irrelevant overall constants, this nonrelativistic limit is given to first order in $\vec{q}$, the c.m. momentum of the outgoing ν_1, by

$$A(e^- + e^+ \rightarrow \nu_1 + \nu_2) = A_s(1-\Omega_{e12}) + A_p(1+\Omega_{e12}), \tag{44}$$

where

$$A_s = \chi_1^\dagger \chi \chi_2^\dagger \chi + \chi_1^\dagger \vec{\sigma}\chi \cdot \chi_2^\dagger \vec{\sigma}\chi, \tag{45}$$

and

$$A_p = -\left[\chi_1^\dagger \frac{\vec{\sigma}\cdot\vec{q}}{2M_1}\chi\chi_2^\dagger\chi + \chi_2^\dagger\frac{\vec{\sigma}\cdot\vec{q}}{2M_2}\chi\chi_1^\dagger\chi \right. \\ \left. + \chi_1^\dagger\frac{\vec{\sigma}\cdot\vec{q}}{2M_1}\vec{\sigma}\chi\cdot\chi_2^\dagger\vec{\sigma}\chi + \chi_2^\dagger\frac{\vec{\sigma}\cdot\vec{q}}{2M_2}\vec{\sigma}\chi\cdot\chi_1^\dagger\vec{\sigma}\chi\right]. \tag{46}$$

Here, χ is the Pauli spinor for both the e^- and e^+ (which have opposite helicities and so identical spins), and χ_1 and χ_2 are, respectively, the spinors for ν_1 and ν_2, whose masses are denoted by M_1 and M_2. Since A_s is independent of $\vec{q}$, it obviously corresponds to s-wave production, while A_p, being linear in $\vec{q}$, corresponds to p-wave production. Thus, Eqs. (44) and (20) show that when CP is conserved, the production is indeed pure p wave (s wave) if ν_1 and ν_2 have like (opposite) CP parity. Furthermore, Eq. (44) confirms that if Ω_{e12} is complex so that CP is violated, there will be both s- and p-wave production.

If there are only two generations, Ω_{e12} is just Ω, Eq. (26), and we have already seen that Ω can be far from ± 1. Thus, Eq. (44) can involve significant amounts of both s and p wave. Consequently, the energy-dependence of the $\nu_1\nu_2$ production cross section near threshold can differ appreciably from the pure s-wave or pure p-wave behavior that corresponds to CP conservation.[21] Here, as in neutrino radiative decay, we see that if neutrinos are Majorana particles, large CP-violating effects can occur, even if there are only two generations.

VII. DOUBLE BETA DECAY AND NEUTRINO OSCILLATION

If neutrinos are Majorana particles, neutrinoless nuclear double beta decay, the process $(A, Z) \rightarrow (A, Z+2) + 2e^-$, can arise from the sum of neutrino exchange diagrams depicted in Fig. 4. (If neutrinos are Dirac particles and lepton number is conserved, these diagrams vanish because the exchanged neutrino must then be a $\bar{\nu}$ at the vertex where it is emitted, but a ν at the one where it is absorbed.) It is well-known that, assuming neutrinos are light, the diagram for exchange of ν_m is proportional to its mass M_m because of a chirality mismatch at the two lepton vertices.[22] Given this fact and the vertex factors shown in Fig. 4, we see that the amplitude for neutrinoless double beta decay ($\beta\beta_{0\nu}$) is proportional to the effective neutrino mass

$$M_{eff} = \left| \sum_m \lambda_m U_{em}^2 M_m \right| = \left| \sum_m \omega_{em} \left| U_{em} \right|^2 M_m \right| . \tag{47}$$

When CP is conserved, ω_{em} is proportional to $\tilde{\eta}(\nu_m)$ (see Eq. (19)), and M_{eff} takes the form[23]

$$M_{eff} = \left| \sum_m \tilde{\eta}(\nu_m) \left| U_{em} \right|^2 M_m \right| . \tag{48}$$

Note that the contributions of neutrinos with opposite CP parity interfere destructively in M_{eff}. Thus, the rate for $\beta\beta_{0\nu}$ can be much smaller than one would naively expect for given values of the masses M_m.[24]

In general, the relative phase factor relating two terms (say, the m and m' terms) in M_{eff} is just $\omega_{em}/\omega_{em'} = \Omega_{emm'}$. When CP is violated, this phase factor can be complex. As a result, the rate for $\beta\beta_{0\nu}$ can differ from what is allowed for given values of the M_m and the $|U_{em}|^2$ when CP is conserved.[25] This effect has been explored quantitatively.[26] Note that it can already occur when M_{eff} contains only two terms; that is, when there are two generations.

Bilenky, Hosek, and Petcov[11] have shown that ordinary neutrino flavor oscillation, $\nu_f \rightarrow \nu_{f'}$ $(f, f' = e, \mu, \tau, \ldots)$, is completely insensitive to the extra CP-violating phases which can occur in U when neutrinos are Majorana particles.[27] However, several authors[28] have pointed out that "antineutrino-neutrino oscillation", $\bar{\nu}_f \rightarrow \nu_{f'}$, is sensitive to these phases. In this type of oscillation, a neutrino mass eigenstate ν_m is born in the reaction $\ell_f^+ + n \rightarrow \nu_m + p$, travels down a neutrino beam line for a time t, and then interacts via the reaction $\nu_m + n \rightarrow \ell_{f'}^- + p$. It is the identities of the initial and final charged leptons that leads one to label the process "$\bar{\nu}_f \rightarrow \nu_{f'}$". Now, if we think of the intermediate mass eigenstate neutrino as a virtual particle, then the diagram for the entire process is just the $\beta\beta_{0\nu}$ diagram of Fig. 4, except that one of the outgoing leptons is replaced by an incoming antilepton (which does not change the vertex factor), and the flavors of the two electrons are generalized to f and f'. Accordingly, the amplitude for $\bar{\nu}_f \rightarrow \nu_{f'}$ is proportional to the quantity in Eq. (47), modified by the flavor generalization $U_{em}^2 \rightarrow U_{fm}U_{f'm}$, and by insertion of the factor $\exp(-iE_m t)$ which describes the propagation of the neutrino. In this factor E_m is the energy of ν_m for a given neutrino momentum p_ν. We have

$$A(\bar{\nu}_f \rightarrow \nu_{f'}) \propto \left|\sum_m \lambda_m U_{fm} U_{f'm} \frac{M_m}{p_\nu} e^{-iE_m t}\right| = \left|\sum_m \omega_{fm} U_{fm}^* U_{f'm} \frac{M_m}{p_\nu} e^{-iE_m t}\right| . \quad (49)$$

We have included a factor of p_ν^{-1} to show the scale of $A(\bar{\nu}_f \rightarrow \nu_{f'})$ relative to the amplitude for ordinary flavor oscillation. The latter is given by

$$A(\nu_f \rightarrow \nu_{f'}) = \sum_m U_{fm}^* U_{f'm} e^{-iE_m t}. \quad (50)$$

We see that $A(\bar{\nu}_f \rightarrow \nu_{f'})$ contains phase factors, the ω_{fm}, not present in $A(\nu_f \rightarrow \nu_{f'})$. When $f' = f$, for example, these factors can plainly lead to physical effects, whereas $A(\nu_f \rightarrow \nu_f)$ is obviously completely insensitive to phases in U. Indeed, when there are only two generations, $A(\nu_f \rightarrow \nu_{f'})$ is insensitive to phases in U for any f, f'. Apart from the factor $\exp(-iE_m t)$, the two terms in $A(\nu_f \rightarrow \nu_{f'})$ when $N = 2$ always have the same or opposite phase, due to the unitarity of U. By contrast, the corresponding two terms in $A(\bar{\nu}_f \rightarrow \nu_{f'})$ have phases whose difference involves the factor Ω. If U is the matrix X and the $\lambda_m = 1$, one finds by squaring Eq. (49) that, for example,[28]

$$P(\bar{\nu}_e \rightarrow \nu_e) \propto \frac{M_1 M_2}{p_\nu^2} \left\{ \frac{M_1}{M_2} c^4 + \frac{M_2}{M_1} s^4 + 2c^2 s^2 \cos[(E_1 - E_2)t + 2\delta] \right\}, \quad (51)$$

where $P(\bar{\nu}_e \rightarrow \nu_e)$ is the probability for $\bar{\nu}_e \rightarrow \nu_e$. Remembering that for relativistic neutrinos $t = x$, the distance of travel, we see that the physical consequence of the CP-violating phase δ is a translation of the oscillation pattern in space. When CP is conserved, so that $\Omega = \exp(-2i\delta) = \tilde{\eta}(\nu_1)/\tilde{\eta}(\nu_2)$, the pattern is either not translated at all, or else translated by half a wavelength. A CP-violating translation of an oscillation pattern would, of course, be very interesting. However, antineutrino-neutrino oscillation may be extremely difficult to observe because it is suppressed by order $(M_m/p_\nu)^2$ relative to ordinary flavor oscillation.[29]

VIII. SUMMARY

Whether CP is violated or not, if neutrinos are of Majorana character, there are significant CP-related implications. If CP is conserved, each neutrino has a well-defined intrinsic CP parity. The relative values of the CP parities of different neutrinos influence, for example, the rates for $\nu_2 \rightarrow \nu_1 + \gamma$ and $\beta\beta_{0\nu}$, the energy-dependence of $e^+ + e^- \rightarrow \nu_1 + \nu_2$, and the character of the $\bar{\nu} \rightarrow \nu$ oscillation pattern. If CP is violated, then, for a given number of generations, there can be more CP-violating phases than are possible in the Dirac case. This results from the fact that when neutrinos are Majorana particles, some processes involve more diagrams, and correspondingly more interferences between diagrams, than they would in the Dirac case. For Majorana neutrinos, there can already be one CP-violating phase when there are only two generations. This phase can lead to rates for $\nu_2 \rightarrow \nu_1 + \gamma$ and $\beta\beta_{0\nu}$, to an energy-dependence for $e^+ + e^- \rightarrow \nu_1 + \nu_2$, and to a $\bar{\nu} \rightarrow \nu$ oscillation pattern that all differ appreciably from what is allowed when CP is conserved.

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FIGURE CAPTIONS

Figure 1 Loop diagrams for the decay $c \to u + \gamma$. The quark mixing matrix element which occurs at each vertex is written next to it. (It is understood that these diagrams are accompanied by similar ones where the photon attaches to the W line.)

Figure 2 Loop diagrams for $\nu_2 \to \nu_1 + \gamma$. The charged lepton ℓ_f can be an e or μ. The term in the Hamiltonian (15) which is active at each vertex is written next to it. In the diagram S_+, this term has been rewritten in terms of the charge-conjugate field ℓ_f^c using the identity $\overline{\nu_{mL}}\gamma_\alpha \ell_{fL} = -\overline{\ell^c_{fR}}\gamma_\alpha \nu^c_{mR}$ and Eq. (11).

Figure 3 Diagrams for $e^- + e^+ \to \nu_1 + \nu_2$. The term in the Hamiltonian (15) which is active at each vertex is written next to it. At vertices where an e^+ is absorbed, this term has been rewritten in terms of the charge-conjugate field e^c.

Figure 4 Neutrino exchange diagrams for neutrinoless double beta decay. The term in the Hamiltonian (15) which is active at both of the lepton vertices is written next to them, but at one of them it has been reexpressed in terms of e^c.

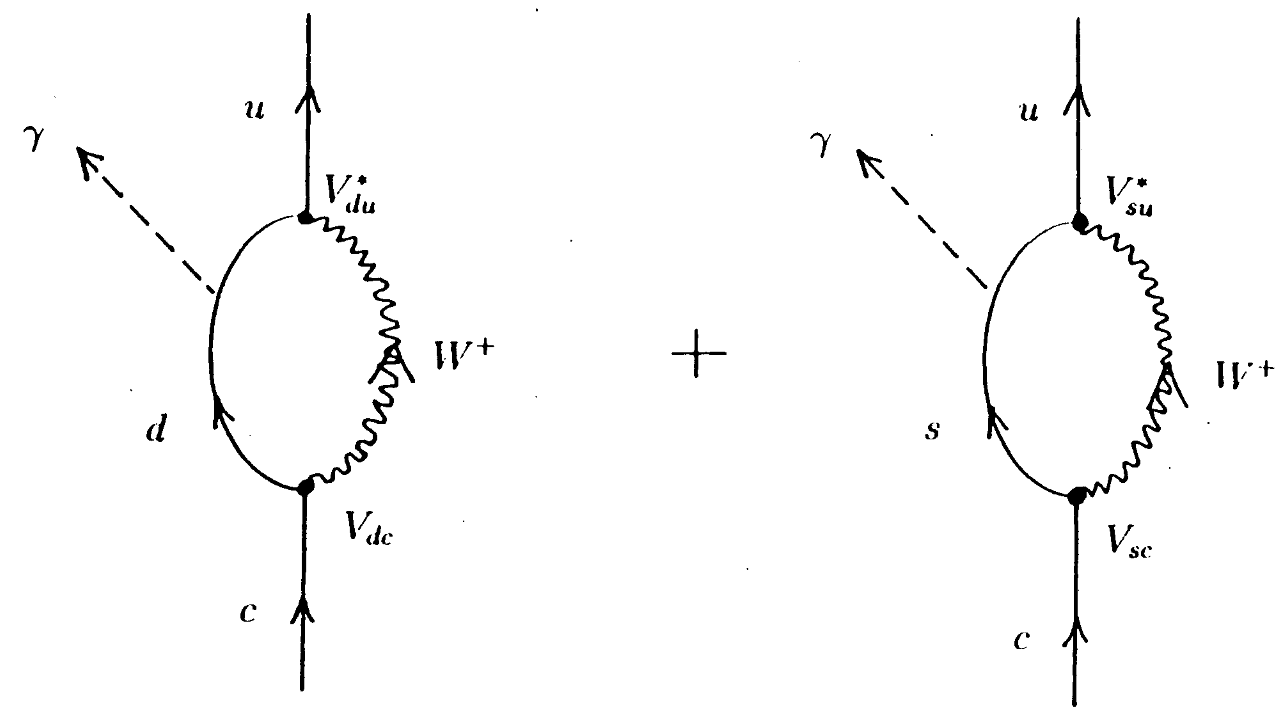
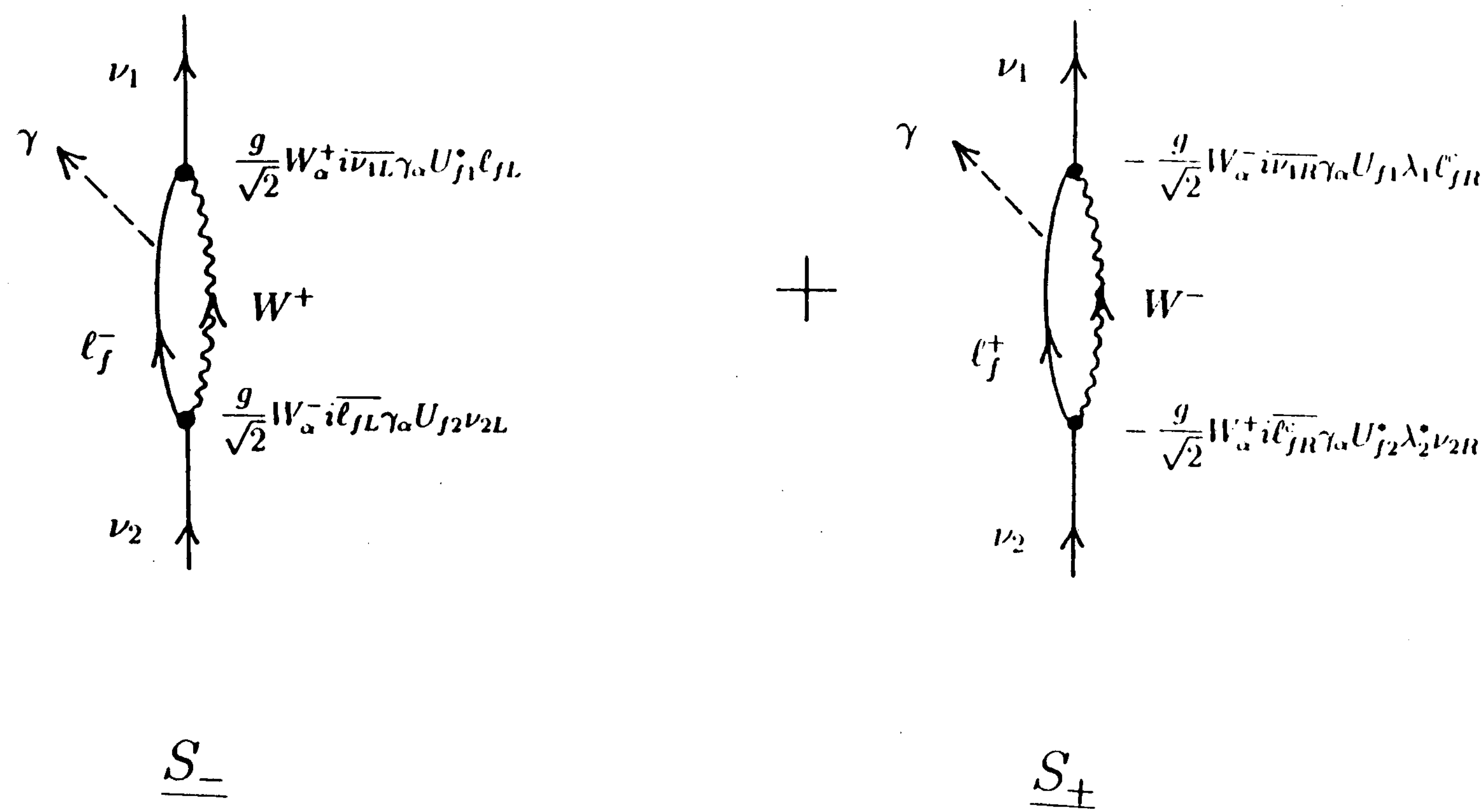
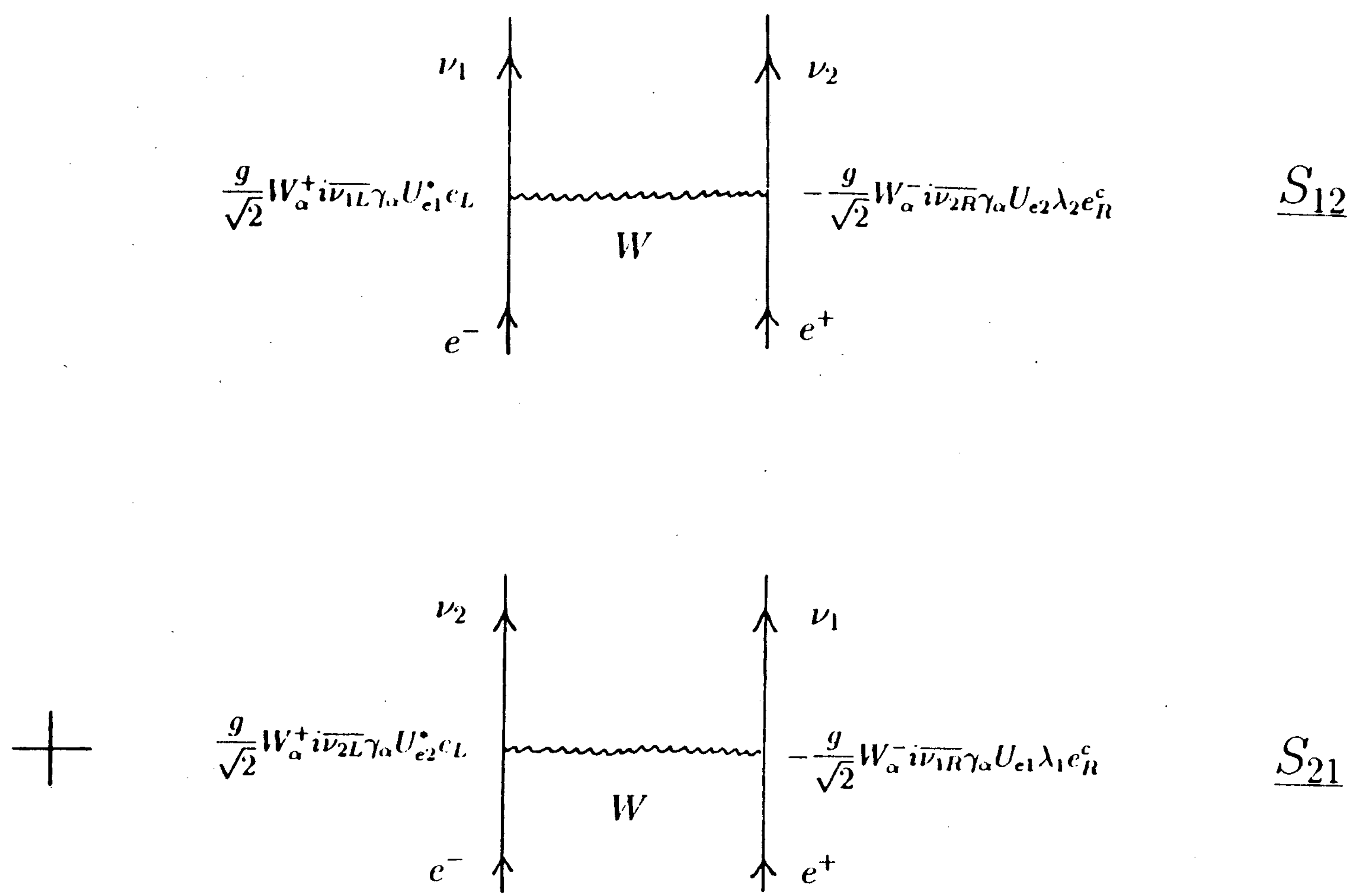


Figure 1



ν1
γ
$\frac{g}{\sqrt{2}} W_\alpha^+ i\overline{\nu_{1L}}\gamma_\alpha U_{f1}^* \ell_{fL}$
W+
ℓ_f^-
$\frac{g}{\sqrt{2}} W_\alpha^- i\overline{\ell_{fL}}\gamma_\alpha U_{f2} \nu_{2L}$
ν2
S−
+
ν1
γ
$-\frac{g}{\sqrt{2}} W_\alpha^- i\overline{\nu_{1R}}\gamma_\alpha U_{f1}\lambda_1 \ell_{fR}^c$
W−
ℓ_f^+
$-\frac{g}{\sqrt{2}} W_\alpha^+ i\overline{\ell_{fR}^c}\gamma_\alpha U_{f2}^*\lambda_2^* \nu_{2R}$
ν2
S±

Figure 2



ν_1
ν_2
$\frac{g}{\sqrt{2}} W_\alpha^+ i\overline{\nu_{1L}}\gamma_\alpha U_{e1}^* e_L$
$-\frac{g}{\sqrt{2}} W_\alpha^- i\overline{\nu_{2R}}\gamma_\alpha U_{e2}\lambda_2 e_R^c$
W
e^-
e^+
S_{12}
$+$
ν_2
ν_1
$\frac{g}{\sqrt{2}} W_\alpha^+ i\overline{\nu_{2L}}\gamma_\alpha U_{e2}^* e_L$
$-\frac{g}{\sqrt{2}} W_\alpha^- i\overline{\nu_{1R}}\gamma_\alpha U_{e1}\lambda_1 e_R^c$
W
e^-
e^+
S_{21}

Figure 3

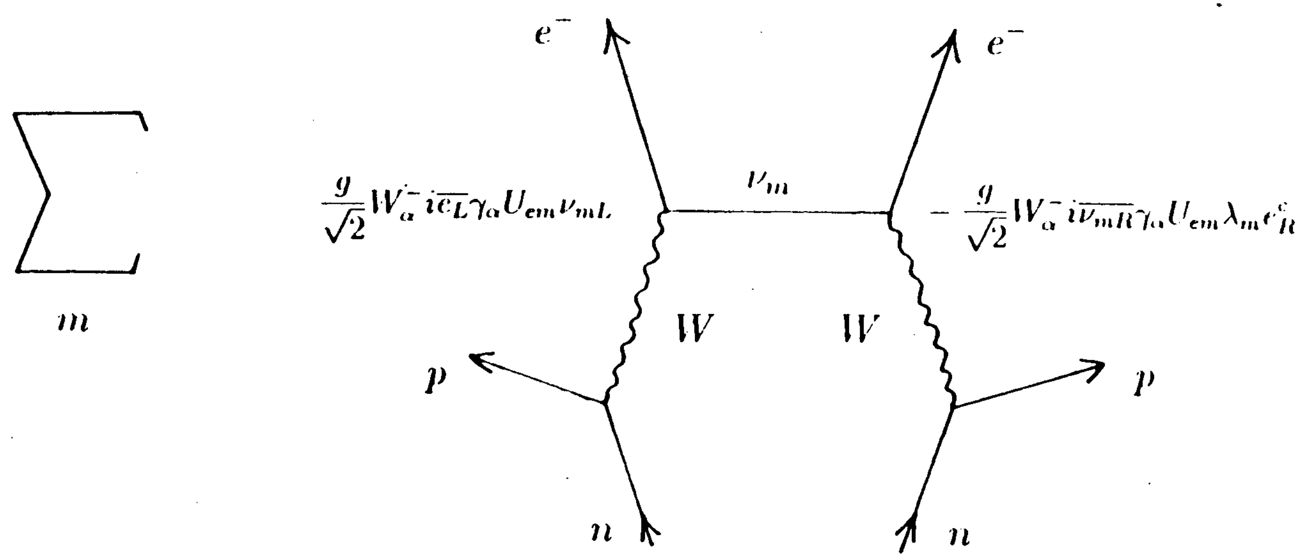


Figure 4

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