UCLA
Recent Work

Title
Efficiency of Asset Markets with Asymmetric Information

Permalink
https://escholarship.org/uc/item/1xz210d9

Authors
Bernardo, Antonio
Judd, Kenneth

Publication Date
1997-04-01

#16-97

Efficiency of Asset Markets with Asymmetric
Information

August 1997

Antonio Bernardo
Anderson Graduate School of Management
University of California, Los Angeles
Los Angeles, CA 90095-1481

and

Kenneth Judd
Hoover Institution
Stanford University
and NBER

Efficiency of Asset Markets with Asymmetric Information[*]

Antonio E. Bernardo[†]   Kenneth L. Judd[‡]

August, 1997

Abstract

We examine the welfare effects of costly information acquisition in
a version of the Grossman-Stiglitz (1980) exchange economy in which
all traders are fully rational. We find, as emphasized by Hirshleifer,
that information gathering leads to suboptimal risk sharing. Fur-
thermore, information gathering worsens the equilibrium risk-return
tradeoff faced by investors. We show that *all* investors would be bet-
ter off if the information were not available and that the welfare costs
of information gathering are greater when (i) the cost of acquiring
information is lower; (ii) the prior return variance of the risky asset is
higher; and (iii) the quality of the information is higher. We also show
that the relative welfare costs to investors depends on the combination
of their endowment and preferences.

Conventional economic analysis focuses on two properties of equilibrium: existence and efficiency. When markets are perfect, equilibria exist and are Pareto efficient. Matters become much more complex in markets with informational asymmetries. Hirshleifer (1971), for example, examined an exchange economy in which the number of informed agents is exogenous and the uninformed traders do not use the information contained in equilibrium prices and found that improved public information can make everyone worse off because it removes the possibility of value increasing risk-sharing opportunities. In this paper, we build on the work of Allen (1984) and examine the efficiency of equilibrium in a version of the Grossman and Stiglitz (1980) exchange economy in which some investors acquire costly private information and the number of informed traders is determined endogenously. In contrast to Allen (1984), however, *all* traders in our economy are fully rational, have well-defined preferences, and use the information conveyed by equilibrium prices. In this setting, we compute the equilibrium allocation of information, the informativeness of equilibrium prices, and show that all would be better off if the information were not available.

Grossman and Stiglitz (1980) began a long literature on the analysis of security markets with asymmetric information and endogenous information acquisition. They introduced "noise traders" to get a model in which some traders would rationally expend resources to acquire information in a rational expectations equilibrium. Unfortunately, "noise" or "liquidity" trader demands are insensitive to prices and their information content and to the degree of adverse selection they face in the market. Together, these restrictions substantially limit the generality of the results and the range of questions which can be addressed. In particular, they cannot evaluate efficiency questions because the noise traders' preferences are unmodelled.[1] Verrecchia (1982) also developed a model with rational expectations and costly information acquisition. Rather than introduce noise traders, he assumed that agents' endowments of the risky asset are random and i.i.d. One problem

---

[1]Many previous attempts to examine welfare issues in models with liquidity traders have adopted *ad hoc* criteria for measuring their welfare. For example, it has been assumed that liquidity traders are better off if the expected value of their endowment is greater (e.g., Allen, 1984); the expected costs of trading with the informed is lower (e.g. Admati and Pfleiderer, 1988, and Rochet and Vila, 1994); and the variance of trading costs is lower (e.g., Leland, 1992).

2

with this approach is that if the variance of each endowment is bounded then as the number of agents grows the per capita supply becomes constant by the law of large numbers. Thus, to get a model in which some traders rationally expend resources to acquire information in a large, competitive market, the variance of each endowment must be unbounded.

In this paper, we use the general computational approach developed in Bernardo and Judd (1997) to investigate the welfare consequences of endogenous information acquisition in a noisy rational expectations equilibrium without noise traders or restrictive endowment shocks. We examine the Grossman-Stiglitz model of endogenous acquisition of costly information, but replace noise traders with a group of rational investors whose tastes are not common knowledge. In our formulation, equilibrium prices are noisy because the risk preferences of some traders is random, thus uninformed investors are unsure if a high price is due to good information or to a high aggregate risk tolerance in the economy. This approach to generating noisy rational expectations equilibria is very similar to Ausubel (1990) who assumed that the preferences of one group of traders is random. The main advantage of this approach is that it allows us to examine the welfare effects of costly information acquisition because all traders are fully rational and have well-defined preferences.[2]

We compute comparative statics results relating the equilibrium proportion of informed traders and the informativeness of equilibrium prices to key parameters of the model such as the risk aversion of the potentially informed traders, the quality of the information, and the payoff variance of the risky asset. Our results are consistent with those in Grossman and Stiglitz (1980). We then find that the equilibrium acquisition of costly information results in a Pareto *reduction* in welfare. This result contrasts with Allen (1984), who found that the welfare effects of costly information acquisition on liquidity traders was ambiguous.[3] When all traders have well-defined preferences, and choose their demands accordingly, we find that all traders are worse off when

---

[2]In Section 2, we briefly discuss several other approaches that have been proposed in the literature to generate noisy rational expectations without noise traders.

[3]To measure the welfare of liquidity traders, Allen (1984) endows them with a utility function which is increasing in $p \times l$ where p is the equilibrium price and l is the the random endowment. However, the demands of liquidity traders are not related to this utility function nor conditioned on equilibrium prices; instead, they are given exogenously. In particular, demands are unaffected by the equilibrium proportion of informed traders.

information can be acquired.

There are several complementary explanations for this strong welfare result. First, information acquisition is socially costly because the private incentives to gather information include rent-seeking which is of no social value. Second, informed traders have speculative motives for trade which leads to sub-optimal risk sharing in the trading round. Third, information gathering resolves some uncertainty before investors have a chance to share the associated risk optimally; thus, value-increasing risk-sharing opportunities are destroyed.[4] These explanations are consistent with our finding that the welfare costs to all traders is greatest when (i) the cost of information is low; (ii) the variance of the risky asset payoff is high; or (iii) the information is precise. Thus, when information is more useful, in the sense of reducing the conditional variance of the risky asset payoff, it is more harmful to the welfare of all traders. In other words, when markets are more informationally efficient they *reduce* economic welfare in pure exchange.

We also demonstrate how the early resolution of uncertainty affects the welfare of investors in different ways depending on their endowments: if an investor is endowed with more than the optimal risk-sharing holdings, they bear too much wealth risk; while if an investor is endowed with less than the optimal risk-sharing holdings, they face a poorer investment opportunity set in the trading round. The early release of information affects the equilibrium risk-return tradeoff in the trading round because it reduces the conditional variance of the risky asset's payoff (which is similar to a reduction in the supply of the risky asset) and lowers the equilibrium risk premium. Thus, investors who want to buy stock in the trading round will face poorer terms of trade. This distinction between wealth risk and changes in the investment opportunity set helps to explain our finding that, in addition to making everyone worse off, costly information acquisition makes risk averse investors worse off than more risk-tolerant investors when they have large stock endowments and risk tolerant investors worse off than more risk-averse investors when they have small stock endowments. The former result follows from the fact that a large endowment subjects investors to wealth risk which hurts risk averse traders disproportionately. The latter result follows from

---

[4]In a rational expectations equilibrium, information gathering by the informed also resolves some uncertainty for the uninformed because they observe the equilibrium price. The assumption of rational expectations is critical to the analysis.

the fact that a small endowment subjects investors to changes in the equilibrium risk-return tradeoff which hurts risk tolerant investors (who wich to buy more stock) disproportionately.

There have been several other papers that have studied the welfare consequences of information in models with traders that are fully-rational. Hakansson et al. (1982) established various conditions under which public information may or may not be socially valuable in a pure exchange model. Their paper differs from ours in several respects: in particular, they studied a model with public information rather than with costly information acquisition, all traders in their model are symmetrically informed, and agents in their model do not have rational expectations. Ausubel (1990) evaluated the welfare effects of informed trading in a production economy with fully-rational traders. In his model, however, informed agents are endowed with information whereas we consider a model in which the number of informed agents is endogenously determined. Dow and Gorton (1995) evaluated the welfare effects of informed trading with fully-rational traders but also does not consider endogenous information acquisition. They also use an alternative equilibrium concept to rational expectations which explicitly allows for purely information-based trading. Diamond (1985) used a version of the Verrecchia (1982) model to study welfare issues related to the optimal release of public information by firms and came to some of the same conclusions we do here about the welfare effects of private information acquisition.

The paper is organized as follows: first we describe our generalization of Grossman and Stiglitz (1980) without noise traders but with a group of investors whose tastes are unknown *ex ante*. Section 2 describes the numerical strategy used to compute approximate equilibria, and tests the accuracy of our approximations. Section 3 demonstrates some comparative statics results related to information acquisition and equilibrium informational efficiency of stock prices and compares them to those in Grossman and Stiglitz (1980). Section 4 demonstrates that costly information acquisition leads to Pareto *inferior* allocations, and provides economic intuition for the numerical results. Section 5 considers the robustness of the welfare results to alternative modelling assumptions: in particular, we discuss the extension of these results to an economy with endogenously determined bond prices and with production. Section 6 concludes.

1 A Model with Rational "Noise" Traders

We consider a two-period economy in which risk-averse investors allocate their wealth between a risky asset (stock) and a riskless asset (bond) in the first period, and consume in the second period. The bond pays a certain amount R dollars in the second period for every dollar invested in the first period; the stock pays a random amount $\tilde{Z}$ dollars per share in the second period. We also assume that all investors have an initial endowment of cash and shares. The bond is in perfectly elastic supply and, for simplicity, we normalize its first-period price to unity. We also normalize the supply of shares to equal unity.

We assume that there are two groups of traders in this economy. The first group of traders are the "potentially informed" group who can acquire information about the payoff on the stock at a fixed cost of c. Those who choose to acquire information are called type I while those who are uninformed are called type U. The proportion of each type is determined endogenously. The other group of traders are called "rational noise traders." These are uninformed traders, who may have either high risk aversion (type H) or low risk aversion (type L). A key feature of model is that the total size of groups L plus H is known to all investors, but the relative frequency of type L and H investors is not known. The balance of L and H traders affects the risky asset's price, but does not provide any information about $\tilde{Z}$, the asset's payoff. Therefore, it is important to know if a high price is due to a high frequency of type L traders or to good information observed by informed investors. Let λ be the the ratio of type L investors to the sum of types L and H. We assume that the value of λ is chosen by Nature from a uniform $[0, 1]$ density. This process is known by all investors, but no one knows the realized value of λ. Even though for type L and H traders one's own type is informative about aggregate risk preferences, they do not know the realization of λ so they will also be uncertain about how to interpret a high or low price.[5]

The assumption that there exists a class of traders with unobserved preferences is just one way to generate noisy rational expectations equilibria without noise traders. There have been many other approaches used to break down the full-revelation properties of the simple model without re-

---

[5]In particular, if a trader finds that he is of type L, his posterior belief about λ is a beta distribution with $\alpha = 2$ and $\beta = 1$.

sorting to exogenous liquidity trade. The key to all of these approaches is to introduce a second source of uncertainty so that a single equilibrium price will not reveal all information. For example, Diamond and Verrecchia (1981), Glosten (1989), Bhattacharya and Spiegel (1991), and Paul (1994) assumed that some or all traders have random endowments; Spiegel and Subrahmanyam (1985) assumed the existence of a group of traders who trade to hedge their endowment risk; and Wang (1994) assumed stochastic returns to private, non-tradeable investment opportunities that generate hedging demands for the traded asset. Our approach is most similar to Ausubel (1990) who assumed that one group of traders have random preferences. For the questions we are interested in, the main advantage of our approach over models that rely on noise traders is that all traders have well-defined preferences which allows us to analyze the welfare effects to all traders of information acquisition.

It is important to note that our model is partial equilibrium since we take the price of bonds as given, and allow an arbitrary net aggregate position in bonds. If we make the bond price endogenous, the model is complicated by the fact that the bond price also conveys information about investor preferences. While this is an important assumption of our model, we follow the tradition of this literature and make the bond price exogenous. We discuss the likely effects of endogenizing the bond price in Section 5.

If an investor is endowed with W_i dollars in cash and $\bar{\theta}_i$ shares of stock, and ends the first period with θ_i shares of stock which trade at a price of p dollars per share, his random second-period consumption is

$$\tilde{c}_i = [W_i - (\theta_i - \bar{\theta}_i)p] R + \theta_i \tilde{Z}.$$

Investors are assumed to be expected utility maximizers, solving the maximization problem

$$\max_{\theta_i} E[u_i(\tilde{c}_i) \mid \mathcal{F}_i],$$

where $\mathcal{F}_i$ is the investor's information set. The first-order condition for the choice of θ_i will be

$$E[u_i'(\tilde{c}_i)(\tilde{Z} - pR) \mid \mathcal{F}_i] = 0. \tag{1}$$

We make several parametric assumptions to facilitate comparisons with the standard models. We assume that (i) all traders have exponential utility

7

functions of the form $u_i(c) = -e^{-a_i c}$, where a_i is the coefficient of absolute risk aversion; (ii) the stock payoff is normally distributed, $\tilde{Z} \sim N(\mu, \sigma_z^2)$; and (iii) informed investors observe information $\tilde{Y} = \tilde{Z} + \tilde{\epsilon}$, where $\tilde{\epsilon} \sim N(0, \sigma_\epsilon^2)$ is independent of $\tilde{Z}$.

For the type I investors, $\mathcal{F}_I = \{p, y\}$, but p contains no payoff-relevant information that is not contained in y. For all other investors, the information set is the equilibrium price alone. Let $p(y, \lambda)$ be the asset price function, $\theta_I(y)$ be the demand rule for the informed investors, and $\theta_j(p)$ be the demand rule for the uninformed, type $j = U, L, H$ investors. A rational expectations equilibrium is a collection of price and demand rules such that the market for the stock clears in every state of the world, and θ_j satisfies the type j first-order condition, (1). Furthermore, since the potentially informed traders are identical *ex ante,* equilibrium requires that the expected utility achieved by those who choose to gather information, net of information gathering cost c, must be equal to the expected utility achieved by those who do not. Because agents have exponential utility it follows that q, the equilibrium proportion of the potentially informed who choose to become informed, solves:

$$\ln \left[\frac{U_U(q)}{U_I(q)} \right] = c \tag{2}$$

where $U_U(q)$ is the expected utility of the uninformed and $U_I(q)$ is the expected utility of the informed *gross* of information costs, and the expectation is computed by integrating over all possible realizations of y and λ. Because expected utilities take negative values the term in the square brackets is greater than unity. Finally, since the utility functions are concave, the second-order conditions are automatically satisfied.

2 Numerical Solutions

The rational expectations equilibrium described above cannot be solved in closed-form. To find approximate solutions to equilibrium, we employ the projection method described in Judd (1992) and Bernardo and Judd (1997). Below we describe the most important details of this approach. For more details the interested reader should refer to Bernardo and Judd (1997).

8

2.1 Computing Conditional Expectations

The first-order condition in (1) implies that our equilibrium concept involves a conditional expectation. Numerical implementation of the conditional expectation conditions is the most challenging aspect of this problem. We use Gaussian quadrature methods combined with basic projection ideas to implicitly compute conditional expectations[6].

To solve this problem, we use the following definition of conditional expectation:

$$Z(X) = E[Y \mid X]$$

if and only if

$$E[(Z(X) - Y)f(X)] = 0$$

for all continuous bounded functions, $f(X)$, of X. Intuitively, this says that the prediction error of the conditional expectation, $E[Y \mid X]$, is uncorrelated with any continuous function of the conditioning information, X.

With this formulation, a conditional expectation is equivalent to an infinite number of unconditional expectations. In practice, we approximate $Z(X)$ by finitely parameterizing $Z(X)$ and imposing a finite number of the unconditional expectation conditions to identify the free parameters.

2.2 Implementation

To implement the first-order condition in (1) we use the definition of conditional expectation given earlier: $E[Y \mid X] = Z(X)$ *if and only if* $E[(Z(X) - Y)f(X)] = 0$ *for all bounded, continuous* f. The projection method, described in Bernardo and Judd (1997), solves for an approximate equilibrium by finitely parameterizing $p(y, \lambda)$ and $\theta(p)$ and imposing a finite number of the conditions implicit in our definition of equilibrium.

More precisely, we approximate the price law with the polynomial

$$p(y, \lambda) = \sum_{k=1}^{N_p} \sum_{l=1}^{N_p} a_{kl} H_k(y) T_l(\lambda) \tag{3}$$

---

[6]One could use basic regression methods combined with Monte Carlo simulation, as in Marcet's (1989) parameterized expectations approach. That approach would be inefficient and impractical (unless one is satisfied with very low accuracy results) in these problems even using a supercomputer.

where H_k denotes the degree k Hermite polynomial, T_l is the degree l Chebyshev polynomial, and N_p represents the total degree of the polynomial approximation. These polynomials are natural in this setting because they are mutually orthogonal with respect to the normal density with mean zero in the y dimension and variance of one half and the uniform density in the λ dimension.[7]

An informed investor's demand rule can be solved in closed-form because of the parametric assumptions. In particular,

$$\theta_I(y) = \frac{E[Z|Y = y] - pR}{a_I Var[Z|Y = y]}$$

where $E[Z|Y = y] = \beta y + (1-\beta)\mu$, $Var[Z|Y = y] = (1-\beta)\sigma_z^2$, and $\beta = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\epsilon^2}$.

The demand policies for the uninformed investors, however, cannot be solved in closed form. We approximate the demand rule by the polynomial

$$\theta_j(p) = \sum_{m=1}^{N_\theta} b_m^j H_m(p), \quad j = U, L, H. \tag{4}$$

Our goal then is to determine the unknown a_{kl} and b_m^j coefficients.

If we let $N_\theta = N_p = 3$, for example, the number of unknown coefficients is twenty-eight: sixteen for the price function and four for each of the three policy functions. To determine the unknown coefficients we impose projection conditions on the investors' first-order conditions and market clearing. The total number of conditions will equal the number of unknown coefficients, hoping that they are sufficient to fix the unknown coefficients. More generally, we could impose more conditions than unknowns and find coefficient values which minimize some squared error criterion. In this paper, our approach is, to use an econometric term, to exactly identify the unknown coefficients by imposing an equal number of projections.

The first-order-condition for an uninformed investor is given by

$$E[u_j'(\tilde{c}_j)(\tilde{Z} - pR) \mid p\} = 0, \quad j = U, L, H. \tag{5}$$

Using the definition of conditional expectation given above we numerically approximate the first-order-condition with the conditions

$$E[u_j'(\tilde{c}_j)(\tilde{Z} - p(y, \lambda)R)H_m(p(y, \lambda))\} = 0, \quad m = 1, ..., N_\theta. \tag{6}$$

---

[7]See Judd (1992) for a discussion of the advantages of orthogonal bases in projection methods.

10

The projection conditions given in equations (6) are only part of the conditional expectation condition given in equation (5). According to the definition we would need to project on all bounded functions of the conditioning information to get equivalence.

Equilibrium also requires market clearing. We cannot impose market clearing in each and every state and simultaneously have all agents follow rules measurable in their individual information. Therefore, we assume that deviations from market clearing are orthogonal to several of our basis elements, as in

$$E\left\{\left(\sum_j \theta_j - 1\right) H_k(y) T_l(\lambda)\right\} = 0 \quad j = I, U, L, H \text{ and } k, l = 1, ..., N_p. \quad (7)$$

Combining (6) and (7) results in a finite system of nonlinear equations which are used to "identify" the unknown price and policy coefficients (a, b). The equilibrium value of q, the proportion of the potentially informed who become informed, is trivially solved using (2). We have thus reduced an infinite dimensional functional problem to a finite-dimensional algebraic problem.[8] Below we will make diagnostic checks of our approximations. Overall, our experience is that this method is fast and reliable.

2.3 Accuracy Measures

The disadvantage of numerical methods is that any answer is necessarily approximate. To avoid important errors, we must test any proposed numerical solution to ensure that the apparent errors are economically insignificant. In this section we test the accuracy of our numerical algorithm by testing it on some simple problems.

2.3.1 No-Trade Results

Suppose that all investors have the same utility function and endowment. Then, if the information were common knowledge, each type would hold the same portfolio independent of the information. Since this fact is common

---

[8]The interested reader is referred to Bernardo and Judd (1997) for details on how to implement this algorithm.

knowledge, the uninformed investors will, in a rational expectations equilibrium, trade to this point in all states of the world, no matter what the distribution of private information (see Milgrom-Stokey (1982) for an elaboration of this result), and the price will be the full-information equilibrium price. Numerical calculation of the full-information prices is trivial, involving only numerical integration which in this case (because of the smooth functions involved) will be very accurate. While we may know these facts, the algorithm does not, and instead approaches the problem in the general way. Therefore, we can check our algorithm on these cases.

We used the algorithm above to compute equilibrium for cases covered by the Milgrom-Stokey theorem with a wide variety of utility functions and returns. By comparing it with the full-information calculations, we found that this method generated the correct prices and holding strategies to within at least five significant digits, as long as we used degree-three polynomials for the pricing and demand functions. We also found the correct solution even when our initial guesses were poor, indicating the stability of the method. Furthermore, the solutions took at most a few seconds to find.

Table 1 displays a typical example of the no-trade result. In this example all investors have the same constant absolute risk aversion of $a = 1$ and the same endowment, $\bar{\theta} = 0.5$. The column labelled Y represents the realization, in standard deviations, of the signal observed by the informed. Thus, if $Y = -1$ the informed investors observed a signal one standard deviation below the mean. The column labelled λ represents the realization of the proportion of low risk-aversion types. Because both the L and H types have the same risk aversion in this example, prices should be independent of λ, as observed. The Milgrom-Stokey theorem predicts that equilibrium shareholdings will equal the endowment of 0.5 and that the equilibrium price will equal the full-information price. The table verifies that the algorithm is correct to six significant digits.

Table 1: No-Trade Results

				Equilibrium Shareholdings	
Y	λ	Computed Price	Full-information Price	Informed	Uninformed
-1	0.1	0.901533	0.901533	0.500000	0.500000
-1	0.5	0.901533	0.901533	0.500000	0.500000
-1	0.9	0.901533	0.901533	0.500000	0.500000
0	0.1	1.038835	1.038835	0.500000	0.500000
0	0.5	1.038835	1.038835	0.500000	0.500000
0	0.9	1.038835	1.038835	0.500000	0.500000
1	0.1	1.313440	1.313440	0.500000	0.500000
1	0.5	1.313440	1.313440	0.500000	0.500000
1	0.9	1.313440	1.313440	0.500000	0.500000

The high accuracy of our method in this problem gives us confidence that our algorithm is reliable more generally. However, we do not rest with these examples, but also include tests which can be used in arbitrary problems.

2.3.2 Euler Equation Errors

We would also like to know the accuracy of our approximations and how many polynomial terms we need to get a good approximation. While one way to check an algorithm is to apply it to cases where the solution is already known (as in the cases above), we would like to evaluate the approximation in all cases, not just in those special cases where we know the answer.

There are two ways to do this. First, one could find another way to solve the problem and compare the solutions resulting from the two procedures. In our case, this could be done by using different basis functions and different integration formulas. A second approach is to compute a measure of inaccuracy. In this problem, we could ask how much better an agent could do if he used more information than is implicit in the equilibrium conditions. It is important to recall that the equilibrium conditions force him to choose decision rules which yield Euler equation residuals which are orthogonal to a restricted set of basis functions.[9] We can compute the wealth equivalent of the Euler equation residual when projected in other directions. This is the

---

[9]This is, essentially, allowing him to do only a limited regression analysis of the data.

consumption error, that is, the difference, in consumption units from following the equilibrium rule versus following a rule which uses more information in making inferences from the price. We operationalize this by taking the equilibrium law and subjecting it to a more refined regression analysis and asking how much an agent will gain if he is allowed to use the better inference rule. The results in Table 2 suggest that the Euler equation residual errors are very small when projected in directions not used to approximate the equilibrium. For example, when a fourth-degree polynomial approximation is used to compute the equilibrium, an investor with a parameter of absolute risk aversion of 1.5 gains approximately 1/100,000 of her wealth by using a fifth-degree inference rule.[10]

<div align="center">Table 2: Projection Errors</div>

approx. degree	# quad. nodes	CARA	$\log_{10}$ projection errors: 5th order	6th order	7th order
1	3	1.5	-4.35	-4.28	-4.21
1	3	2.5	-4.01	-3.94	-3.86
1	3	3.5	-3.76	-3.68	-3.60
2	5	1.5	-4.45	-4.40	-4.35
2	5	2.5	-4.25	-4.20	-4.15
2	5	3.5	-4.15	-4.11	-4.06
3	7	1.5	-4.81	-4.76	-4.70
3	7	2.5	-4.60	-4.56	-4.50
3	7	3.5	-4.51	-4.46	-4.41
4	10	1.5	-5.06	-5.02	-4.96
4	10	2.5	-4.97	-4.92	-4.88
4	10	3.5	-4.92	-4.88	-4.83

We find that the quality of our approximation increases with the degree of the polynomial approximation and with the number of quadrature nodes used to compute our integrals. We also found that the optimal number of quadrature nodes is approximately twice the polynomial degree. Furthermore, the quality of our approximation decreases when investors are more risk averse, that is, our approximate policy rules are less precise when there is greater curvature in the utility function.

---

[10]We have normalized consumption to be equal to unity, thus the parameter of absolute risk aversion is equal to the parameter of relative risk aversion.

2.3.3 Bounded Rationality Interpretations

This procedure suggests a bounded rationality interpretation of these calculations. If these computational methods produce a policy function with small optimization errors, then that approximate policy function is as compelling a description of behavior as the equilibrium policy function since it is unclear why individuals would bother making the nontrivial effort to find the "true" policy function if the gain is small. From this perspective, the challenge of numerical economic modelling is not in finding the perfectly accurate description of the mathematical equilibrium, but in finding the collection of behaviors which are approximately rational.[11] In our examples, the deviations from full rationality generally turn out to be a dollar per \$10,000 to \$1,000,000 of wealth, a small error by any reasonable standard.

3 Endogenous Information Acquisition

We now turn to examples which use the novel features of our model. We first choose some parameter values which are plausible descriptions of the U.S. stock market. We do not intend to estimate such parameters here nor do we assert that any one set of values is best. We choose a range of values for our calculations so that they describe sensible examples for us to examine.

3.1 Calibration

We calibrate the model so that the period corresponds to a year. The riskless interest rate is set to one percent, which is approximately the average annual real return on U.S. Treasury bills. We choose $\sigma_z^2 = 0.1$ and $\sigma_\epsilon^2 = 0.1$. Together, these parameter choices yield standard deviations of equilibrium stock returns of roughly 20% per period (year) which is approximately the standard deviation of a broad index of U.S. stocks. Our choices for a_i, the constant absolute risk aversion coefficient, lie between 0.5 and 4.5. Initial wealth parameters are chosen so that average consumption is near unity, so

---

[11]There is a strong similarity between this procedure and the approach of Anderson and Sonnenschein (1982). They assume that agents run regressions and use them when making decisions. They prove existence of equilibrium when agents are restricted in the regressors they use.

these measures of absolute risk aversion coincide with parameters of *relative* risk aversion between 0.5 and 4.5. We also assume that the "potentially informed" are endowed with 100% of the stock. This is an important assumption and we will relax it later. We demonstrate comparative statics results for all of the key parameters in this model below.

3.2 Equilibrium Information Acquisition

In Figures 1 through 4 we compute the equilibrium fraction of the "potentially informed" who choose to become informed as a function of the cost of acquiring information, c. In all of the figures there is a negative relation between the cost of acquiring information and the equilibrium proportion of investors who choose to become informed, as we would expect. We consider how this equilibrium relation varies as we vary other key parameters in the model. Figure 1 demonstrates that, holding c fixed, more traders will become informed when the "potentially" informed are more risk averse. The intuition is that risk-tolerant investors will act more aggressively on information, which leads to more information being revealed in prices. Consequently, the marginal value of observing the signal (rather than the price) is smaller and reduces the value of purchasing the information. In Figure 2 we show that, holding c fixed, more traders become informed as the return variance increases. This follows from the fact that information of fixed precision is more valuable as the return variance increases in the sense of Bayesian updating. In Figure 3, however, we find that, holding c fixed, more traders become informed as the precision of the signal *decreases*. This may seem counterintuitive, but the logic of this result is as follows: the value of acquiring the signal depends on the marginal value of observing the precise signal rather than the noisy equilibrium price. Thus, holding the informativeness of the price fixed, we would expect that more traders will become informed when the precision of the signal increases. However, as the quality of the information decreases, the equilibrium price system becomes less informative and the marginal information value of the signal may increase, *in equilibrium,* even though the quality of the information decreases. In Figure 4, we find that, holding c fixed, more traders become informed as the risk aversion of the high-type rational noise traders increases. Intuitively, when the gap between the risk aversion of the high-type and low-type rational noise traders increases, equilibrium prices become more noisy and, consequently,

16

information becomes more valuable.

3.3   The Informativeness of Equilibrium Prices

We now examine how the informativeness of equilibrium prices depends on various parameters in the economy. To do so, we first compute the full-information price that would obtain in an economy in which *all* investors received the information signal. We then compute the equilibrium price when the number of informed are endogenously determined. In Figures 5 through 8, we show how the mean absolute deviation, in percent, of the equilibrium price from the full-information price depends on various key parameters in the economy. We interpret a greater deviation to mean that equilibrium prices are less informative. We report the results for all values of c, the cost of information, which yield an interior solution for the endogenous proportion of potentially informed who become informed in equilibrium.

In all of the figures, we find that as the cost of information increases, the equilibrium price becomes less revealing because fewer of the potentially informed will choose to gather more costly information. In Figure 5 we analyze how the risk-aversion of the potentially informed affects the informativeness of equilibrium prices. We find that, holding the cost of information fixed, the equilibrium price is less revealing when the potentially informed are more risk averse. The intuition for this result is not obvious because, on one hand, we showed in Figure 1 that, for fixed c, a greater proportion of the potentially informed will choose to gather information when they are more risk averse, suggesting that prices will be more informative. On the other hand, the potentially informed trade less aggressively on their information when they are more risk averse, suggesting that prices will be less informative. As in Grossman and Stiglitz (1980), the latter effect dominates.

Figures 6 and 7 demonstrate that equilibrium prices are more informative when the prior return variance and the signal variance, respectively, are smaller. This is true even though more investors become informed in equilibrium when the return variance and signal variance increase. In figure 8 we show that the informativeness of equilibrium prices is largely independent of the difference between the risk aversion parameter of the high and low types. For a fixed number of informed traders, the greater the gap in the risk aversion of the high and low type, the more noisy equilibrium prices will be. However, the value of information is greater when the gap is greater, induc-

17

ing more traders to gather information. These two effects largely offset one another. Interestingly, Grossman and Stiglitz (1980) found the same result with non-rational liquidity traders. In their model, the equilibrium informativeness of the price system is independent of the variance of the liquidity trades because the two effects described above *exactly* offset one another. We find the same result, even though all traders are rational in our model.

4   The Social Value of Information Acquisition

We next examine whether endogenous information acquisition is of value to society. Hirshleifer (1971) and others have argued that information acquisition can destroy valuable risk-sharing opportunities because it resolves some uncertainty before investors can trade to optimally share the associated risk. Furthermore, there will be too much acquisition of such information, since the private incentives include rent-seeking which is of no social value in an exchange economy. Allen (1984) examined a model similar to ours except all traders are not rational thereby limiting the ability to study the welfare effects of information acquisition. Diamond (1985) studied the effects of the release of public information on the costly acquisition of private information and finds that public information releases can enhance social welfare. He uses a model with stochastic endowments to generate noisy rational expectations equilibria.

Since all of our traders have utility functions, we can examine the *ex ante* (prior to the observation of y) value to prohibiting informed trading for all types of agents. Because all traders have exponential utility it is straightforward to express the welfare costs in dollar units. In particular, an agent would be willing to pay $s = \ln \left[\frac{EU_{INF}}{EU_{NO}} \right]$ dollars to have informed trading prohibited where EU_{INF} and EU_{NO} represent expected utility with and without informed trading, respectively. Note that expected utility takes on negative values so the term in square brackets is greater than unity (and the willingness to pay is positive) if expected utility is greater without informed trading. In Figures 9-13 we display the welfare consequences of information acquisition to the different types of agents, expressed as a fraction of wealth, as a function of key parameters in the economy. All of our comparative statics

18

results use our base-case parameterization as a starting point. The general finding is that *all* traders gain by prohibiting informed trading as long as the cost of information leads to an equilibrium in which some rational traders buy information. This finding holds for *all* of the parameterizations we have studied. This is significantly stronger than saying that some agents may lose from the introduction of costly information acquisition. In Figures 10-13 we fix the cost of information at $c = 0.01$ and in Figures 9-12 we endowed the potentially informed with all of the stock. In Figure 13, we vary the endowment held by one group of traders to determine its effect on welfare. While changes in the initial endowment distribution do not change the result that all traders are made worse off, they do change the *relative* costs to different traders. The following statement summarizes our findings:

Numerical Results: *Suppose that the cost of information is sufficiently low that some of the potentially informed agents buy information. Then all traders would be better off ex ante if there were no such information available. Furthermore, the welfare costs to all traders are greater when*

(i) *the cost of acquiring information is lower;*

(ii) *the potentially informed are more risk averse;*

(iii) *the prior variance of the stock payoff is higher; and*

(iv) *the information is of higher quality.*

The relative welfare costs to different types of traders depends on their specific combination of endowments and preferences. In particular,

(v) *more risk-averse investors are relatively worse off when their share endowment exceeds the optimal risk sharing holdings; and*

(vi) *more risk-tolerant investors are relatively worse off when their share endowment is less than the optimal risk sharing holdings.*

This kind of welfare analysis illustrates an important strength of the methods used in this paper: welfare analysis is impossible in models with noise traders but with these methods we can do welfare analysis because all of our traders are fully rational. We will now provide some economic intuition for our numerical results.

19

4.1 Suboptimal Risk Sharing

Hirshleifer (1971) argued that releasing information may not be socially valuable because it destroys valuable risk-sharing opportunities. Hirshleifer derived this result in the context of a model in which the number of informed agents is exogenously given and the uninformed do not use the information conveyed by equilibrium prices. We now examine the effect of information on optimal risk sharing in the context of our model with endogenous information acquisition and rational expectations. The assumption of rational expectations is important because it is through the informativeness of the equilibrium price that uninformed traders resolve some uncertainty.

Information acquisition affects risk sharing in two ways: first, because some traders have an information advantage in the trading stage, they will take speculative positions which cause shareholdings to deviate from the optimal risk-sharing holdings. Second, information gathering resolves some uncertainty before trading begins: if the endowments prior to trading are not the optimal risk-sharing holdings, then valuable insurance opportunities are lost. We will consider each of these two effects in turn.

4.1.1 Speculative Trading

In Figures 14-17 we investigate the first effect by comparing the equilibrium shareholdings in the trading round with endogenous information acquisition to the optimal risk-sharing holdings and report the mean absolute deviation of the difference. In each state, the optimal risk-sharing holdings are given by $\theta_j^* = \frac{r_j}{\sum_j r_j}$, where $r_j = \frac{1}{a_j}$, investor j's parameter of risk tolerance. However, because more informed investors will take speculative positions, the actual shareholdings will differ from the optimal risk-sharing holdings. Higher values of this difference suggest that information acquisition is more costly in terms of lost risk-sharing opportunities. We report the differences for both the high and low-type rational noise traders.[12]

In Figure 14 we see that as the cost of acquiring information increases the equilibrium shareholdings are closer to the optimal risk-sharing holdings. As c increases, fewer of the potentially informed become informed in equilibrium

---

[12]Since the potentially informed have the same risk aversion as the low types in these exercises, the uninformed among the potentially informed will have differences which mimic the low types and thus are excluded from the diagrams.

and shareholdings are closer to the optimal risk-sharing holdings which would prevail when there is no information acquisition. In Figure 15, we see that as the risk aversion of the potentially informed increases, the difference between the equilibrium shareholdings of the rational noise traders and the optimal risk-sharing holdings increases. This effect is consistent with the welfare results in Figures 9 and 10 which showed the social costs of informed trading are greater when the cost of information is lower and when the risk aversion of the potentially informed is greater.

Figures 16 and 17 demonstrate that increasing the prior return variance and the signal precision has little effect on the difference between the equilibrium shareholdings and the optimal risk-sharing holdings. In Figure 16 an increase in the return variance increases the difference only slightly. Similarly, in Figure 17 an increase in the signal precision increases the difference only slightly. Both figures are consistent with Figures 11 and 12, but it is unlikely that suboptimal risk sharing in the trading round provides a complete explanation for these welfare results. We now explore the second effect.

4.1.2 Early Resolution of Uncertainty

Figures 14 through 17 demonstrate that information acquisition leads to suboptimal risk sharing in the trading round. It is also true, however, that information gathering leads to the early resolution of uncertainty which destroys valuable risk-sharing opportunities. To develop intuition, consider the following simple model: Consider the economy described above except that (i) all investors costlessly observe the public signal y; and (ii) there are N investors with potentially different risk aversion parameters, a_j. It is easy to show that, in equilibrium, $\theta_j^* = \frac{r_j}{\sum_j r_j}$, or that trader j's holdings equal the optimal risk-sharing holdings. The *ex ante* (prior to observing y) equilibrium expected utility of a trader j is given by:

$$V_j^{FI} = -\exp\left\{-a_j(\mu\bar\theta_j + W_j) + a_j(\bar\theta_j - \theta_j)\frac{\sigma_z^2(1-\beta)}{h} + \frac{a_j^2\theta_j^2\sigma_z^2(1-\beta)}{2} + \frac{a_j^2\bar\theta_j^2\beta\sigma_z^2}{2}\right\},$$

where $\bar\theta_j$ is trader j's stock endowment, W_j is trader j's cash endowment, $h \equiv \left(\sum_{j=1}^{N} r_j\right)$ and $\beta = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\epsilon^2}$. Furthermore, it can be shown that if no signal is observed then the equilibrium expected utility of trader j is:

$$V_j^{NI} = -\exp\left\{-a_j(\mu\bar{\theta}_j + W_j) + a_j(\bar{\theta}_j - \theta_j^*)\frac{\sigma_z^2}{h} + \frac{a_j^2\theta_j^{*2}\sigma_z^2}{2}\right\}.$$

Consequently, the ratio of expected utilities under the two regimes is:

$$\frac{V_j^{FI}}{V_j^{NI}} = \exp\left\{\frac{(\bar{\theta}_j - \theta_j^*)^2\beta\sigma_z^2 a_j^2}{2}\right\}.$$

Since $V_j^{FI} \leq 0$ and $V_j^{NI} \leq 0$ it follows that $V_j^{NI} \geq V_j^{FI}$. Thus, all traders are better off when there is no public signal. It is important to note that this welfare result is not due to suboptimal risk-sharing in the trading round because optimal risk sharing is achieved in the trading round whether or not the signal y is observed by all traders. The decrease in welfare is due rather to the early resolution of uncertainty which does not permit investors to share all risks optimally. Notice that if investors happen to be endowed with the optimal risk-sharing holdings $\bar{\theta}_j = \theta_j^*$ then the welfare cost of the early resolution of uncertainty is zero because the associated wealth risk is shared perfectly. If the investors are not endowed with the optimal risk-sharing holdings the risk introduced by this early resolution of uncertainty cannot be hedged because there are no trading opportunities prior to the release of the public information. In reality, investors have many opportunities to trade so it is reasonable to expect that shareholdings will be close to the optimal risk-sharing holdings at any time. Nonetheless, the greater the divergence of an investor's endowment from the optimal holdings, the greater is the welfare cost of information.

From this simple model we also see that the welfare cost associated with the early resolution of uncertainty is greater if the public signal is more precise (i.e. β is greater) and/or if the prior variance of the stock payoff is greater, because this introduces more non-tradeable risk, and more valuable risk sharing opportunities are destroyed. This intuition helps to explain the numerical results in Figures 9, 11 and 12 which we now try to explain more fully.

4.2 Investment Opportunity Set Effect

From the simple model above we see that the early resolution of uncertainty caused by the release of public information makes all investors worse off.

In particular, investors with too much or too little endowment in the risky asset are made worse off by the early resolution of uncertainty. This may seem puzzling at first glance. One would expect that the early resolution of uncertainty would only make those endowed with too much of the stock worse off because it exposes them to an unhedgeable wealth risk. But why should an investor endowed with none of the stock care about the early resolution of uncertainty?

The answer is that the early resolution of uncertainty also affects the equilibrium investment opportunity set faced by net buyers of the stock in the trading round. We shed more light on this by examining the equilibrium risk-return tradeoff which, in the simple model above, is summarized by the equilibrium Sharpe ratio since the risky asset payoff is normally distributed. In Figure 18, we show that the investment opportunity set consists of a security market line in mean-standard deviation space with intercept given by the fixed, exogenous risk-free rate, and slope given by the equilibrium Sharpe ratio. Because the risk-free rate is fixed exogenously, a higher slope (i.e. a higher equilibrium Sharpe ratio) yields a strictly preferred investment opportunity set. It can easily be shown that the equilibrium Sharpe ratio in the simple model is an increasing function of the conditional variance:

$$SR = \left(\sum_{j=1}^{N} \frac{1}{a_j} \right)^{-1} \sigma_Z \sqrt{1 - \beta}.$$

Thus, investors face a more favorable investment opportunity set when the conditional variance of the stock payoff increases and when the aggregate risk tolerance increases. In particular, this implies that the investment opportunity set worsens for investors who want to buy the stock in the trading round if public information is released. The intuition for this result is that a reduction in σ_z^2 or a reduction in σ_ϵ^2 is similar to a reduction in the supply of the risky asset, and leads to a higher price for the asset, a lower risk premium, and a lower Sharpe ratio. Similarly, an increase in the aggregate risk tolerance shifts out the demand curve for the risky asset, and leads to an increase in the price of the risky asset, a lower risk premium, and a lower Sharpe ratio. To sharpen intuition, consider the case when $\sigma_\epsilon^2 = 0$ in which case all investors know the future stock payoff with certainty upon the release of public information. In this case, the risky security becomes a riskless security with an exogenously fixed return in the trading round. Thus, in

23

Figure 18, the investment opportunity set shrinks to a point. This makes all net buyers of the stock strictly worse off because, in the absence of the public information, they would choose to buy a positive amount of the risky asset but, in the presence of the public information, they can only invest in a riskless asset with exogenously fixed return. In this extreme case, it is easy to see that the release of information worsens the investment opportunity set of individuals who wish to buy stock in the trading round.

We now examine this effect in our model. In the following experiments we fix the cost of acquiring information at $c = 0.01$ and compute the equilibrium Sharpe ratios in economies with and without information acquisition for various values of σ_z^2 and σ_ϵ^2. The results are shown in Table 3.

Table 3: Equilibrium Sharpe Ratios

	With Informed Trading	No Informed Trading
$\sigma_z^2 = 0.05$	0.1166	0.1407
$\sigma_z^2 = 0.10$	0.1510	0.2056
$\sigma_z^2 = 0.20$	0.2189	0.3112
$\sigma_\epsilon^2 = 0.05$	0.1245	0.2056
$\sigma_\epsilon^2 = 0.10$	0.1510	0.2056
$\sigma_\epsilon^2 = 0.20$	0.1722	0.2056

The results are consistent with the intuition above. In particular, we see that (i) the equilibrium Sharpe ratio increases when the unconditional (and conditional) variance of the stock payoff increases; (ii) the equilibrium Sharpe ratio is greater for fixed values of σ_z^2 and σ_ϵ^2 when there is no informed trading; and (iii) the difference in the equilibrium Sharpe ratios with and without informed trading is exacerbated when the prior return variance is higher and when the signal precision is higher.

These results help to explain the welfare results in Figures 9, 11 and 12 where we found that information gathering is more costly to all investors when the information is less costly, the stock is more risky, and the signal is more precise, respectively. In our model, information gathering resolves uncertainty early for all investors, not just informed investors, because the equilibrium price is informative in rational expectations equilibria. Thus, observing the price reduces the aggregate risk in the economy. When information is less costly to acquire, more investors acquire information which, among other

things, has the effect of resolving more uncertainty before traders can optimally share the associated wealth risks. Investors endowed with too much of the stock face too much wealth risk while investors endowed with too little of the stock get a smaller risk premium for bearing risk after the trading round. To explain the results in Figure 11, we know that information of fixed quality reduces the conditional variance, in percentage terms, by a greater amount when the prior return variance is greater. Consequently, information acquisition resolves more uncertainty earlier which exposes investors endowed with too much of the stock to more wealth risk and hurts investors endowed with little stock because information gathering has a more dramatic effect on the equilibrium investment opportunity set when the prior variance is high. The intuition for the welfare result in Figure 12 is similar to Figure 11: for a fixed return variance a more precise signal reduces the conditional variance, in percentage terms, by a greater amount than a less precise signal.

4.3   The Endowment Effect

Our discussion above suggests that the welfare effects of the early resolution of uncertainty depend on the investor's endowment. If an investor is endowed with too much of the stock, the early resolution of uncertainty exposes them to considerable wealth risk. On the other hand, if an investor is endowed with too little of the stock, the early resolution of uncertainty worsens the equilibrium risk-return tradeoff faced in the trading round. Thus, our analysis suggests the relative welfare costs to the different types of traders will depend on the level of their stock endowment. Figure 13 verifies this intuition. In this experiment we consider our rational noise trader model using the base case parametrization and the cost of acquiring information fixed at $c = 0.01$. We then compare the value of prohibiting informed trading to the high risk aversion rational noise traders as a function of their endowment of stock for various levels of risk aversion. There are three important things to notice. First, for a fixed a_H, the value of prohibiting informed trading decreases then increases in the level of the endowment just as we described above. The minimum is achieved at the optimal risk-sharing holdings. Second, for large values of the share endowment, the value of prohibiting informed trading is greater for more risk-averse traders. The intuition is that more risk-averse traders are made relatively worse off by the wealth risk introduced by the early resolution of uncertainty than more risk-tolerant investors. Third, for

25

small values of the share endowment, the value of prohibiting informed trading is greater for the more risk-tolerant investors. The intuition is that more risk-tolerant investors would like to buy relatively more stock in the trading round, thus the poorer investment opportunity set caused by informed trading hurts them more than more risk-averse traders.

We conclude from this experiment that while costly information acquisition makes everyone worse off, it should make more risk-averse investors relatively worse off when they are endowed with a lot of the stock and it should make more risk-tolerant investors relatively worse off when they are endowed with very little of the stock. Figures 9 through 12 reinforce this intuition. In these experiments, the rational noise traders were endowed with no stock, and we would expect that information gathering by the potentially informed would hurt the more risk tolerant rational noise traders. This is exactly what we find. We ran similar experiments with the rational noise traders endowed with all of the stock, but omitted the results due to space limitation. In these cases, the relative costs to the low and high risk-averse traders were reversed, exactly as we would expect.

5 Extensions

Costly information acquisition hurts all investors because it leads to speculaton in the trading round, which is costly to the uninformed, and it resolves uncertainty before it can be optimally shared, which is costly to everyone. Thus, a Pareto superior allocation could be achieved if no one gathered information. However, if no one gathered information, investors would have a private incentive to gather information to speculate in the trading round which destroys the possibility of a no-information-gathering equilbrium in a decentralized economy. The potentially informed compares the cost of acquiring information to the private benefit which is purely rent-seeking. As we have shown, this creates a negative externality by resolving some uncertainty early and by destroying risk-sharing opportunities in the trading round; consequently, there is too much information acquisition in equilibrium. In this section we discuss the likely robustness of our welfare results to important generalizations of our model.

26

5.1 Endogenous Bond Returns

The comparative statics results related to the investment opportunity set are unambiguous in our model because the bond is in perfectly elastic supply and its return is fixed exogenously. As with virtually all the literature in this area, we assumed an exogenously fixed bond price because an endogenously determined bond price would also reveal information about investor preferences and complicate our analysis considerably. In reality, however, we would expect that changes in many of the exogenous variables in our model would also affect the equilibrium price (and returns) of bonds, in which case both the slope *and the intercept* of the security market line shift. For example, if information gathering has the effect of increasing the price of the stock and decreasing the price of the bond (lowering the return) then the investment opportunity set cannot be ranked, and it is possible that the welfare of net lenders of bonds could increase.

5.2 Production Economies

In this paper we have studied a version of the Grossman-Stiglitz (1980) exchange economy in which investors expend resources to gather information about the value of a risky asset. We showed that information acquisition is costly to *all* investors in the economy. Hayek (1945), however, stressed the importance of informationally efficient prices for productive decisions. Ausubel (1990) found that informed trading can improve the welfare of all traders by improving the allocation of productive resources. Recently, Dow and Gorton (1996) and Titman and Subrahmanyam (1997) have stressed the importance of informationally efficient stock prices for efficient capital budgeting decisions. In these cases, as in Ausubel (1990), the benefits of improved productive decisions may exceed the social costs of suboptimal risk sharing.

6 Conclusions

In this paper we examined a version of the Grossman-Stiglitz (1980) model in which some traders rationally choose to expend resources to gather information. Since all agents are rational in our model, we could evaluate the welfare effects of various information regimes. In our model, with unknown

27

risk preferences, we found that all agents would be better off *ex ante* if there were no acquisition of costly information. Moreover, the social costs of information acquisition are greatest when information is more useful in the sense of reducing the conditional variance of the asset payoff. We also derived many comparative statics results including how the relative welfare costs to the various traders depends on their combination of preferences and endowments. All of these results were derived in a pure exchange economy with an exogenously fixed bond return. We believe that the no-information regime may not Pareto dominate the information acquisition regime if information revelation is allowed to affect productive decisions and/or bond yields.

We used the general computational approach in Bernardo and Judd (1997) to find approximate solutions to rational expectations equilibrium. This approach gives us the flexibility to solve a model which does not rely on the existence of irrational noise traders to generate noisy equilibrium prices. Furthermore, we can solve more complex models than can be solved with analytic solutions. For example, we hope to use our numerical approach to study an extension of our model with endogenous bond pricing and production. This would be particularly useful for normative analyses such as the welfare consequences of insider trading when insiders make productive decisions, or the efficacy of a transactions tax or a tax on information gathering.

References

[1] Admati, A., and P. Pfleiderer, 1988, A Theory of Intraday Trading Patterns: Volume and Price Variability, *Review of Financial Studies* 1, 3-40.

[2] Allen, F., 1984, The Social Value of Asymmetric Information, *University of Pennsylvania Working Paper*.

[3] Anderson, R. M. and H. Sonnenschein, 1982, On the Existence of Rational Expectations Equilibrium, *Journal of Economic Theory* 26, 261-278.

[4] Ausubel, L. M., 1990, Insider Trading in a Rational Expectations Economy, *American Economic Review* 80, 1022-1041.

[5] Ausubel, L. M., 1991, Partially-Revealing Rational Expectations Equilibrium in a Competitive Economy, *Journal of Economic Theory* 50, 93-126.

[6] Bernardo, A. E., and K. L. Judd, 1997, Asset Market Equilibrium with General Tastes, Returns, and Informational Asymmetries, *U.C.L.A. Working Paper*.

[7] Bhattacharya, U., and M. Spiegel, 1991, Insiders, Outsiders, and Market Breakdowns, *Review of Financial Studies* 4, 255-282.

[8] Diamond, D. W, 1985, Optimal Release of Information by Firms, *Journal of Finance* 40, 1071-1094.

[9] Diamond, D. W. and R. E. Verrecchia, 1981, Information Aggregation in a Noisy Rational Expectations Economy, *Journal of Financial Economics* 9, 221-235.

[10] Dow, J., and G. Gorton, 1995, Profitable Informed Trading in a Simple General Equilibrium Model of Asset Pricing, *Journal of Economic Theory* 67, 327-369.

[11] Dow, J., and G. Gorton, 1997, Stock Market Efficiency and Economic Efficiency: Is there a Connection?, *Journal of Finance* 52, 1087-1129.

29

[12] Glosten, L. R., 1989, Insider Trading, Liquidity, and the Role of the Monopolist Specialist, *Journal of Business* 62, 211-235.

[13] Grossman, S. J., 1976, On the Efficiency of Competitive Stock Markets Where Agents Have Diverse Information, *Journal of Finance* 18, 81–101.

[14] Grossman, S. J., and J. E. Stiglitz, 1980, On the Impossibility of Informationally Efficient Markets, *American Economic Review* 70, 393–408.

[15] Hakansson, N., J. G. Kunkel, and J. A. Ohlson, 1982, Sufficient and Necessary Conditions for Information to have Social Value in Pure Exchange, *Journal of Finance* 37, 1169-1181.

[16] Hirshleifer, J., 1971, The Private and Social Value of Information and the Reward to Inventive Acitivity, *American Economic Review* 61, 561–574.

[17] Hayek, F. A., 1945, The Use of Knowledge in Society, *American Economic Review* 35, 519–530.

[18] Jordan, J., 1983, On the Efficient Markets Hypothesis, *Econometrica* 51, 1325–1344.

[19] Judd, K. L., 1992, Projection Methods for Solving Aggregate Growth Models, *Journal of Economic Theory* 58, 410-452.

[20] Laffont, J. -J., 1985, On the Welfare Analysis of Rational Expectations Equilibria with Asymmetric Information, *Econometrica* 53, 1-29.

[21] Leland, H. E., 1992, Insider Trading: Should it be Prohibited?, *Journal of Political Economy 100,* 859-887.

[22] Paul, J., 1994, Information Aggregation without "Exogenous" Liquidity Trading, *University of Michigan Working Paper.*

[23] Rochet, J.-C., and J.-L. Vila, 1994, Insider Trading without Normality, *Review of Economic Studies* 61, 131-152.

[24] Spiegel, M., and A. Subrahmanyam, 1995, Informed Speculation and Hedging in a Noncompetitive Securities Market, *Review of Financial Studies* 5, 307-329.

30

[25] Titman, S. and A. Subrahmanyam, 1997, IPOs, Spinoffs, and the Development of Financial Markets, *U.C.L.A. Working Paper.*

[26] Verrecchia, R., 1982, Information Acquisition in a Noisy Rational Expectations Economy, *Econometrica* 50, 1415-30.