Children's Cardinality Knowledge Isn't Always Beneficial

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Abstract

A milestone in cognitive development is understanding numerals to represent the *exact* number of discrete items in a set (i.e., the cardinal principle). This development has received much attention, but little is known about its relation to understanding numbers as measures of continuous quantity (e.g., "six blocks long" versus "six blocks"). To investigate this, 90 children were asked to complete two tasks: a give-a-number task, to assess cardinality knowledge, and a novel give-a-line task, to assess measurement knowledge. As expected, accuracy was greater on the give-a-number task than the give-a-line task. More unexpectedly, the quality of performance on the give-a-number task was as often *negatively* associated with quality of performance on the give-a-line task as it was positive correlated. Specifically, when asked to create a line *N*-blocks long, children who gave only approximately correct answers on the give-a-number task often outperformed children who gave exactly correct answers on the same task. These findings indicate that an approximate—and purportedly less mature—understanding of number possesses the hidden strength of being more flexible and suitable for measuring length.

Keywords: children; number; cardinality; continuous

Introduction

The use of numerals to measure discrete and continuous quantities (e.g., six blocks or six blocks long) can be regarded as one of humanity's most fundamental cultural achievements**.** Research on the development of this ability is also a long-standing concern in developmental psychology, since at least Piaget (1960) and Gelman & Gallistel (1978). In this paper, we examined the relations between these two uses of numbers in young children.

Conceptually, the meaning of numbers in counting and measuring is highly similar. In counting, numbers refer to discrete quantities (blocks), and the number "six" should indicate the many-ness of the set (i.e., the cardinality principle). In measures, the numbers refer to continuous quantities (length), and the count of "six" should indicate the many-ness of the units. In both situations, numbers also indicate similar ratio properties. The extensive properties of a set of six identical (non-overlapping) blocks is six-fold the properties of a set of one block, just as the length of six blocks is six-fold the length of one block. These conceptual considerations might lead us to expect that understanding the "sixness" of a set of blocks (i.e., its cardinality) would co-occur with understanding the "sixness" of a length of blocks (i.e., its measure).

One task that has been used in the literature to assess children's understanding of the cardinal meanings of counting words is a give-a-number task (Condry & Spelke, 2008; Fuson, 1988; Le Corre & Carey, 2007; Le Corre et al., 2006; Sarnecka & Gelman, 2004; Sarnecka & Carey, 2008; Schaeffer et al., 1974; Wynn, 1990, 1992). In a typical version of this task, the experimenter repeatedly asks the child to give a specific number of items drawn from a larger set of objects (Wynn, 1990, 1992). For example, the experimenter might ask the child to "Give two fish" from a basket containing 10 or more toy fish to a puppet (LeCorre et al., 2006). Of interest is the range of numbers to which the child gives only the correct number of objects. If the child correctly gives the requested N of objects to "N" and no other, then the child is called N-knower.

Extensive studies using this task have found that it takes several years for children to meet this criterion for exact number knowledge for numbers larger than about 5 (e.g., Carey, 2004, 2009; Sarnecka, 2015; Sarnecka & Carey, 2008; Sarnecka & Lee, 2009; Wynn, 1990, 1992). Children who can count accurately to 20, for example, would fail to meet the criterion of being (say) a "five-knower" because they provide five objects when asked for four and nine objects when asked for eight.

Previous research has shown that meeting the cardinality criterion is associated with other mathematical knowledge. For instance, some researchers believe that the cardinality knowledge is a prerequisite for mastering the "successor function" — that is, any natural number n has a successor defined as n + 1 (Carey, 2004, 2009; Sarnecka & Carey, 2008; but see Cheung, Ruberson, & Barner, 2017). Additionally, it has been shown that understanding of cardinality benefits children's ability to learn the relationship between numbers and even broader mathematics before they start formal school (Geary & vanMarle, 2018; Geary et al., 2018).

Children also have an ability to use number words correctly in measuring continuous magnitudes (Fuson, 1988). In this situation, the entity being measured is a continuous, rather than discrete, quantity (e.g., length), and a unit appropriate for that kind of continuous magnitude (e.g., a block or a centimeter) is given and applied to the continuous quantity until it is depleted. The number word

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indicates the many-ness of the units required to cover the continuous quantity.

The first serious discussion and analyses of young children's use of number words to measure continuous quantities were provided by Piaget et al. (1960). In Piaget's task, twelve and sixteen blocks were arranged in two parallel rows, with the two rows in perfect alignment. One of the rows was then modified by the introduction of angles (e.g., at right angles to one another). Children were always asked whether the two lines were the same length or not. He found that when the two lines were arranged in parallel (where the two lines could be put in one-to-one correspondence), the equality was obvious to children. In contrast, when angles were introduced such that the two lines could not be placed side by side, younger children failed to recognize any conservation but older children could use numbers (i.e., counting the number of squares) to make the indirect comparison (Piaget, Inhelder, & Szeminska, 1960).

Following Piaget et al. (1960)'s work, research focusing on children's understanding of number words in measuring continuous quantities developed independently, without explicit reference to understanding of cardinality (e.g., Carpenter, 1975; Carpenter & Lewis, 1976; Hiebert, 1981, 1984; Levine et al., 2009; Miller, 1989; Solomon, Vasilyeva, Huttenlocher, & Levine, 2015). Generally, there is consensus that development of using number words to describe continuous magnitudes (e.g., length) takes a long time before mastery. However, none of these studies describe children's errors in continuous terms to see if their answers are at least approximately correct (as in the give-a-number task).

Although much is known about understanding of counts and measures, little is known about the relation between these two understandings of number, such as whether exact number knowledge aids or hinders the use of measure numbers. Further, the ratio characteristics of children's use of numbers in these two contexts have not been compared systematically.

The Current Study

The present study aimed to investigate the relation between children's cardinality knowledge and their use of numbers as measures of length. In particular, we classified children by cardinality knowledge based on their performance on the give-a-number task, and we provided them with a novel give-a-line task to assess their use of numbers in measuring continuous quantities. We were specifically interested in whether mastery of the cardinality principle aids or hinders children's use of numbers in continuous quantities.

Methods

Participants

Participants were 90 American children (50 girls), recruited from seven schools in Columbus, OH. They ranged in age from 3 years, 5 months to 6 years, 11 months (mean age 5–

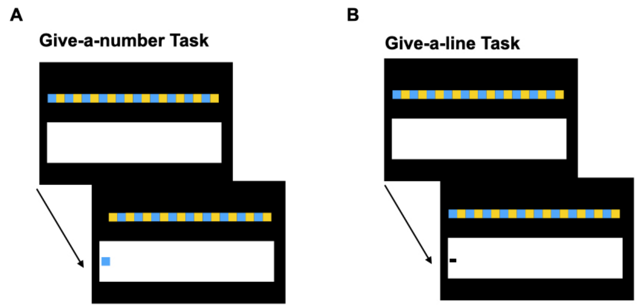

Figure 1. Examples of the give-a-number task (A) and the give-a-line task (B). In task A, children were shown a row of 20 adjacent identical squares at the top of the screen and a blank area at the bottom; they were instructed to put *N* squares in the blank area by depressing a key. Task B was similar to task A, except children were asked to draw a line *N*-squares long, again by depressing a key.

0). Participant ethnicity was similar to that of the community: 74.4% White, 11.1% Black, 10% Asian, and 4.4% Hispanic. An additional five children participated but were excluded from the analysis due to having a primary language other than English (n = 3) or experimenter error (n = 2). All children verbally assented to participate, and parents gave their written permission.

Procedures

Each child met an experimenter individually in a quiet room in the school, with the child sitting before a laptop while the experimenter sat next to the child. Children were given one give-a-number task and one give-a-line task. Children also completed a highest count task that is not included in the present paper (over 90% children could count to 10). The order of the tasks was counterbalanced across children.

Tasks

Both the give-a-number and give-a-line tasks were computerized tasks programed using a custom MATLAB program and presented on a 13-inch MacBook laptop.

Give-a-number task. A non-titrated computerized 'give-a-number' task adapted from Wynn (1992) was used to test whether children understood the cardinality of numbers. Like Wynn (1992)'s study where a heap of objects were presented and children were then asked to "give" a set of *N* objects from the heap, we showed children a row of 20 adjacent identical squares at the top of the screen and a blank area at the bottom, and they were instructed to put *N* squares in the blank area (Figure 1A).

Unlike Wynn's (1992) task where children could physically touch the objects, in our task children were instructed to press buttons to move the objects. Thus, our task prevents children from just grabbing a random group of objects to finish the trial quickly. At the same time, children could still point to the objects (on the screen), which allows them perceptual access to the entire collection of objects to be counted. Specifically, children were instructed to press the L button to put the squares in the blank area and the S button to take squares away. Each L press brought one square from the top to the blank area, and each S press brought one square from the blank area to the top.

As in Wynn (1992), we ensured that children were satisfied with their responses. After the child responded on each trial, the experimenter asked the child "Is that *N* squares? Can you count and make sure?" If the child was not satisfied, he/she could change his or her response.

Given that this could be the first task, a practice trial was shown to children at the beginning. In the practice trial, children were asked to put 'one' square. If the child did not know how to press the buttons, the experimenter showed a demonstration, pressing L and S to show how to put and take away the squares. After the practice trial, the numbers 'one' through 'ten' were requested in a random order within three different blocks, with the size and color different for the squares across the blocks. There were 30 trials in total.

Give-a-line task. The give-a-line task was similar to the give-a-number task except that children were asked to draw a line *N* squares long (Figure 1B). In this task, children were shown a row of 20 adjacent identical squares at the top of the screen and a blank area at the bottom. They were told that "This is one square long, and these squares stick together" as the experimenter moved her finger along the bottom of the first square and then the remaining squares.

Then children were told "I will ask you to draw a line with some squares long. You can press L to make the line longer and press S to make the line shorter. And if you want to draw a long line quickly, you can press and hold the button." Thus, each press led to a very small line (the length of each unit step was around 1/6 of one square long), and by holding the button, they could continuously draw a line with different lengths. As with the give-a-number task, after the child responded, the experimenter asked, "Is that N squares long? Can you count and make sure?" If the child recognized an error, the child could change his or her response.

Given that this task also could be the first computer task, a practice trial was given at the beginning to make sure children understood how L and S buttons worked. In the practice trial, children were asked to draw 'one' square long. If the child didn't know how to press the buttons, the experimenter showed a demonstration, pressing L and S to show how to make the line longer and shorter, and holding L and S to show how to make the line longer and shorter quickly. As with the give-a-number task, after the practice trial, the numbers 'one' through 'ten' were requested in a random order within three different blocks.

Results

Results are organized in two sections. In the first section, we examined performance on the give-a-number task. In the next section, we examined performance on the give-a-line task and its relation to children's performance on the give-a-number task.

1. Children's performance on the give-a-number task.

This task measured children's knowledge of the exact, cardinal meaning of numerals "one" through "ten". To obtain an overall sense of the accuracy of children's performance, we computed each child's percent absolute error (PAE):

$$\left|\frac{Given\ Number - Requested\ Number}{Scale\ of\ Numbers}\right|.$$

For example, if two squares were requested and the child put four squares, then the PAE would be |4-2|/20 =10%. Results showed an interaction between age and set size on PAE ($\beta = -0.01$, $p < .001$), indicating younger children had a better understanding of small numerals than large ones, whereas older children had an equally good understanding of small and large numerals.

We next analyzed children's performance on the give-a-number task by assigning them to different knower level groups (Le Corre, Brannon, Van de Walle, Carey & Sarnecka, 2006; Le Corre & Carey, 2007; Sarnecka & Carey, 2008; Wynn, 1990, 1992). To define children's 'knower-level', the criteria developed by Wynn (1992) was used. According to Wynn, an N-knower must correctly give N objects 2/3 times in response to a request for N, and also it must be the case that 2/3 of cases in which the child gives N are in response to requests for N. For example, a child would be defined as a three-knower if she correctly provided three squares on at least two out of the three trials that three was requested and, of those times that the child provided three, two-thirds of the times she did so it was in response to a request for three.

Consistent with previous literature (Le Corr & Carey, 2007; Sarnecka & Gelman, 2004; Wynn, 1990, 1992), knower-level (0-, 1-, 2-, etc knower) was significantly correlated with age, $r = .57$, $p < .001$. Of the 90 children tested, 53 (58.9%) were 10-knowers. These children ranged in age from 4–0 to 6–11 (mean 5–4). As there were not many individuals in each of the 0-, 1-, 2-, etc. -knower levels in our sample, we grouped them as non-10-knowers. The remaining 37 (41.1%) non-10-knowers ranged in age from 3–5 to 6–0 (mean age 4–6). An independent t-test confirmed that age increased the probability of being a 10-knower compared to being a non-10-knower, $t(2404.9) = 31$, $p < .001$. These results indicated that with age and experience, children's knowledge of cardinality improved.

2. Children's performance on the give-a-line task and its relation to their performance on the give-a-number task.

The give-a-line task measured children's ability of linking number words to continuous magnitudes. As with the give-a-number task, the percent absolute error (PAE) was computed to obtain an overall sense of the accuracy of children's estimates:

$$\left|\frac{Given\ Number - Requested\ Number}{Scale\ of\ Numbers}\right|.$$

For example, if a line of two squares long was requested and the child draw a line four squares long, then the PAE would be |4-2|/20 =10%. Results showed an interaction between age and set size on PAE (β = -0.76, p < .001), indicating that both younger and older children had a better understanding of small numerals than large ones, but the differences of understanding small vs. large numerals were larger for younger children than older children.

A key question was whether there was a relationship between children's cardinality knowledge and their use of number in measuring continuous quantities, and if Yes, what the relation was. To answer these questions, we first compared children's PAE on the give-a-number and give-a-line tasks. Results showed that children gave less accurate responses in the give-a-line task (M = 15.50%, SD = 0.09) than in the give-a-number task (M = 3.95%, SD = 0.08), $t(89)$ = -11.01, p < .001, indicating children's use of number words to represent measures is more erroneous than using them to represent cardinality. Moreover, the individual differences between two tasks were weakly related, r = .18 (p = .07). After controlling for age, the correlation of the PAE between the two tasks was .04 (p = .69).

To further examine the relationship between children's knowledge of discrete quantities and that of continuous quantities, we plotted the distribution of PAE in the give-a-line task by grouping children into 10-knowers vs. non-10-knowers (see Figure 2). As this figure shows, although the average PAE of non-10-knowers was greater than the average PAE of 10-knowers (mean PAE = 18.54% vs. 13.38%, $t(84.32)$ = 3.09, p <. 01), 10-knowers' accuracy was distributed bimodally, whereas non-10-knowers' PAE was distributed normally. The two distinct groups of 10-knowers were either better or worse than non-10 knowers. The below-average 10-knowers made more errors than non-10-knowers (mean PAE = 22.36% vs. 18.54%, $t(49.10)$ = 3.79, p <. 001), indicating 10-knowers do not always have a better understanding of quantities than non-10-knowers. The above-average 10-knowers made less errors than non-10 knowers (mean PAE = 2.52% vs. 18.54%, $t(38.62)$ = -17.03, p < .001). Thus, cardinality knowledge appears to be a double-edged sword in understanding numbers as measurement tools.

To better understand the relation between children's understanding of numbers in discrete and continuous quantities, we regressed each non-10-knower's vs. 10-knower's estimates against their target number in the give-a-line and give-a-number tasks (Figure 3). In this figure, the arbitrary subject number was set to be linearly correlated with children's age, with a higher subject number indicating an older child. This allows us to visually depict the developmental trend (if any).

Figure 3 shows that, first, almost all children showed an approximate understanding of numerical quantities in both continuous and the discrete situations. That is, in 93% of

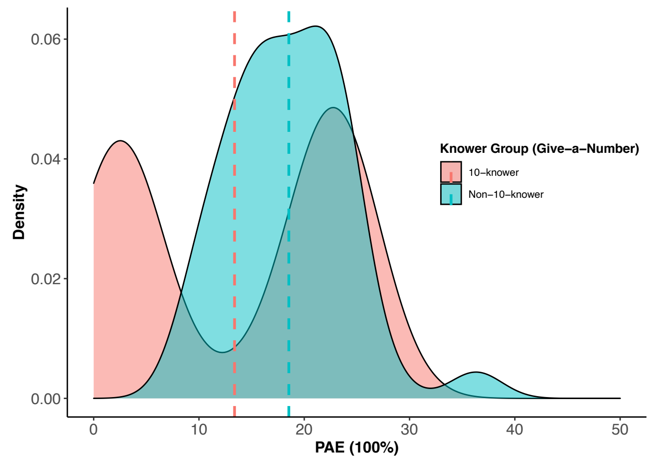

Figure 2. The distribution of PAE for 10-knowers vs. non-10-knowers in the give-a-line task.

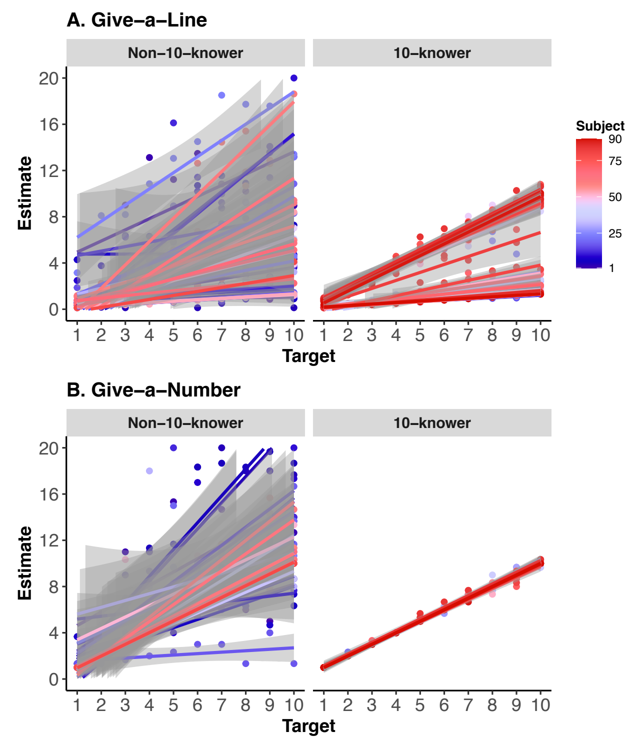

Figure 3. Individual child's estimate by knower groups (non-10-knowers vs. 10-knowers) in the give-a-line and give-a-number task. Subject numbers are linearly related to their ages, with a higher subject number indicating an older child.

children, the quantity given was not independent of the quantity requested at p = .05. Second, the *slope* of 10-knowers' regression line in the give-a-line task showed two distinct patterns. For one group (n = 24), estimates were perfectly fit by a linear function, with both the slope and R^2 equaling to one. In contrast, the remaining 10-knowers' (n = 21) estimates were moderately well-fit by a linear function (mean R^2 = .69), but their slopes were much smaller (mean

= 0.13). An in-depth analysis of their button presses showed that it was caused by their *linking the number of button presses to the number requested* rather than linking *the length of the line to the number requested*. For example, they pressed the L button 3 times when asked to draw a line 3 squares long and pressed the L button 8 times when asked to draw a line 8 squares long. Thus, although these children understood the larger number represented the larger magnitude, they tried to use their knowledge of discrete quantities to solve problems of continuous magnitudes, leading to almost comically large errors. This error appears to be an immature sort of number knowledge. Consistent with this idea, 10-knowers who linked the *number of button presses* to the number requested were indeed younger than 10-knowers who linked line length to the number requested (mean 5–0 vs. mean 5–7, $t(42.11) = -3.08$, $p < .01$).

Discussion

The goal of this study was to assess the relation between young children's cardinality knowledge and their use of numbers in measuring continuous quantities. To assess children's cardinality knowledge, we used a give-a-number task. To assess children's knowledge of using numbers in continuous quantities, we used a give-a-line task. Our core findings are that children's understanding of numbers in these two situations are not equally accurate, and—surprisingly—their knowledge of cardinality is not always beneficial. Specifically, before children successfully understand how to construct a continuous magnitude using numbers, their understanding of cardinality interferes with their ability to approximately link numeric value to continuous extent.

In some respects, our results on children's use of number words in measuring continuous magnitudes are consistent with previous studies (Carpenter, 1975; Carpenter & Lewis, 1976; Hiebert, 1981, 1984; Levine et al., 2009). In previous studies, young children have difficulty understanding the unit of measurement. In our own study, this was evident in their linking numbers to button presses instead of length. Additionally, although children's use of numbers in measuring continuous magnitudes improved with age and development, accurate use of numbers for providing *N* objects occurred earlier than accurate use of numbers for providing *N* length (Fuson, 1988).

The findings of children's understanding of cardinality interfering with their generating continuous magnitudes are somewhat surprising given the conceptual similarities between using numbers to measure discrete and continuous quantities. First, similar to the give-a-number task where children need understand the last counted word refer to the manyness of a whole set of discrete entities, the give-a-line task requires children to understand that last word refer to the manyness of the units filling the continuous quantity. Second, both tasks require children to understand the meaning of "unit" —a discrete quantity in the give-a-number task and a continuous quantity in the give-a-line task. In both tasks, children need to understand the meaning of units and then iterate the single unit to generate a quantity.

Overall, our results suggest a new developmental sequence in children's understanding of numbers. Initially, children start by approximately mapping numbers to spatial extent, regardless of whether it is the spatial extent of objects or the linear length. This state is not ideal because it yields less than perfect accuracy in both situations. Next, children begin to exactly map numbers to discrete quantities, such as objects and button presses. This yields excellent performance on a give-a-number task, but worse performance on the give-a-line task. Finally, children accurately map numbers to both discrete and continuous quantities. This developmental sequence is consistent with the pattern of individual differences in age and performance in our cross-sectional study. However, to test for this developmental sequence more directly, we would need longitudinal or microgenetic data.

Aside from its implications about the development of number understanding, the fact that about half of 10-knowers made comically enormous errors on the give-a-line task shows that cardinality knowledge certainly does not guarantee a very good sense of number. A child who thinks that a line that does not even cover the breadth of *one* square is actually "*six* squares long" seems to indicate a profound misunderstanding of what one and six mean. Depicting such a child as a "10-knower" may be a gross exaggeration of what they know about the meaning of numbers 1 - 10.

Clearly cardinality knowledge is important. Previous studies (Geary & vanMarle, 2018; Geary et al., 2018) have found that children's cardinality knowledge could predict their math achievement at school entry. Our study suggests that many of these children also misunderstand numbers, and assessing their understanding of numbers in a measurement context may improve our ability to project their future achievement.

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References

Carey, S. (2004). Bootstrapping & the origin of concepts. *Daedalus, 133*(1), 59-68.

Carey, S. (2009). *The origin of concepts*. Oxford University Press.

Carpenter, T. P., & Lewis, R. (1976). The development of the concept of a standard unit of measure in young children. *Journal for Research in Mathematics Education, 7*(1), 53–58.

Carpenter, T. P. (1975). Measurement concepts of first-and second-grade students. *Journal for Research in Mathematics Education, 6*(1), 3–13.

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology, 92*, 22–36.

Condry, K. F., & Spelke, E. S. (2008). The development of language and abstract concepts: The case of natural number. *Journal of Experimental Psychology: General. 137*(1), 22.

Fuson, K.C. (1988). *Children's counting and concepts of number*. New York: Springer

Geary, D. C. & vanMarle (2018). Growth of symbolic number knowledge accelerates after children understand cardinality. *Cognition, 177*, 69-78.

Geary, D. C., vanMarle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L. (2018). Early conceptual understanding of cardinality predicts superior school-entry number-system knowledge. *Psychological Science, 29*(2), 191-205.

Gelman, R., & Gallistel, C.R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Hiebert, J. (1981). Cognitive development and learning linear measurement. *Journal for Research in Mathematics Education, 12*(3), 197-211.

Hiebert, J. (1984). Why do some children have trouble learning measurement concepts? *The Arithmetic Teacher, 31*(7), 19-24.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*(2), 395-438.

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of counting principles. *Cognitive Psychology, 52*(2), 130–169.

Levine, S. C., Huttenlocher, J., Kwon, M. K., Deitz, K., & Ratliff, K. (2009). Children's understanding of ruler measurement and units of measure: A training study. In *Proceedings of the Annual Meeting of the Cognitive Science Society*.

Miller, K. F. (1989). Measurement as a tool for thought: The role of measuring procedures in children's understanding of quantitative invariance. *Developmental Psychology, 25*(4), 589-600.

Piaget, J., Inhelder, B., & Szeminska, A. *The child's conception of geometry*. New York: Basic Books, 1960.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108*(3), 662-674.

Sarnecka, B. W., & Gelman, S. A. (2004). Six does not just mean a lot: Preschoolers see number words as specific. *Cognition, 92*(3), 329-352.

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of Experimental Child Psychology, 103*(3), 325-337.

Sarnecka, B. W. (2015). Learning to represent exact numbers. *Synthese*, 1-18.

Schaeffer, B., Eggleston, V. H., & Scott, J. L. (1974). Number development in young children. *Cognitive Psychology, 6*(3), 357-379.

Solomon, T. L., Vasilyeva, M., Huttenlocher, J., & Levine, S. C. (2015). Minding the gap: Children's difficulty conceptualizing spatial intervals as linear measurement units. *Developmental Psychology, 51*(11), 1564.

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36* (2), 155-193.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology, 24*(2), 220-251.