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# A Defense of Shareholder Favoritism

Stephen J. Choi\*      Eric L. Talley†

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**Preliminary Draft**

## Abstract

This paper considers the efficiency implications of managerial “favoritism” towards block shareholders of public corporations. While favoritism can take any number of forms (including the payment of greenmail, diversion of opportunities, selective information disclosure, and the like), each may have the effect (if not the intent) of securing a block shareholder’s loyalty in order to entrench management. Accordingly, the practice of making side payments is commonly perceived to be contrary to other shareholders’ interests and, more generally, inefficient. In contrast to this received wisdom, we argue that when viewed *ex ante*, permissible acts of patronage toward block shareholders may play an important efficiency role that benefits all shareholders alike. We demonstrate that the prospect of having to share rents with a third party may itself have a deterrent effect on managerial self-dealing — an off-equilibrium benefit that would not be readily apparent if one looked only at instances where favoritism actually occurs in practice.

## 1 Introduction

A central debate in corporate law concerns whether regulation or competition is best suited to address managerial agency costs in public corporations. While doctrine has traditionally sided with immutable mandates as a remedy for misaligned incentives, courts and commentators are increasingly placing greater faith in various market mechanisms to accomplish the task. Output markets, for example, impose continuous pressure on managers to choose the least cost methods of production. Labor markets constrain firms’ abilities to increase profits through wage decreases and layoffs. Likewise, reputation markets can play

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an important role in deterring managers from engaging in repeat acts of self dealing.

Of these various mechanisms, however, perhaps none has received more attention than the oft-celebrated “market for corporate control” as a source of managerial discipline. By providing a persistent acquisition risk, the takeover market creates a powerful incentive for managers to constrain their own rapacity in the interests of self-preservation. Consequently, the argument goes, courts have little reason to interfere when the market for corporate control remains active, robust and competitive.

It is hardly surprising, therefore, that even those who champion the market incentives over regulation grow suspicious of practices that stifle competition in the acquisitions market. One practice that has garnered significant attention in this regard is managerial “favoritism” towards large block shareholders. Favoritism can take any number of forms: A manager may, for instance, sell discounted shares to select shareholders, thereby giving preferential rights to the corporation’s profits. A manager might alternatively seek to divert corporate opportunities and other favorable business prospects toward particular shareholders. Or, she may seek to procure block shareholders’ quiescence more directly, through express payments of cash or property in exchange for their shares or their acquiescence in a managerial voting trust.

Regardless of its manifestation, the true intent of these apparent acts of largess is to retard the competitiveness of the acquisitions market, reducing a block shareholder’s incentives to mount a takeover, or even to monitor management very closely. Perhaps accordingly, a number of scholars from both economics and the legal academy have criticized such practices as inefficient or morally objectionable. In the context of vote buying, for example, Easterbrook and Fischel (1984) argue that separating votes from the parties holding the residual claim on the corporation’s profits will lead to distorted incentives on the part of managers not to maximize share value. For example, block shareholders able to sell their votes to managers may do so at the expense of minority shareholders. The law, as well, prohibits various forms of side payments. The Delaware Supreme Court has consistently held that non pro rata dividend distributions violate state law fiduciary duties. Recently, the Securities and Exchange Commission promulgated new rules aimed at curtailing the ability of companies to disclose information selectively to favored block shareholders.<sup>1</sup>

In contrast to the conventional wisdom, this paper questions whether managerial favoritism towards select shareholders is necessarily undesirable. We argue that it is not, and that under plausible conditions, the ability of managers to pay off block shareholders can make *all* shareholders better off and managers worse off compared to where side payments are legally prohibited.

Our analysis emanates from a simple observation: While a bona fide threat of an acquisition can certainly deter managerial opportunism, executing a takeover nonetheless requires a substantial investigation and investment on the part of

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<sup>1</sup>See Securities and Exchange Commission, Selective Disclosure and Insider Trading, 65 Fed. Reg. 51,716 (Aug. 24, 2000) (to be codified as 17 C.F.R. § 243.100-.103).

an outside party. Consequently, except for cases of extreme mismanagement, potential acquirers might remain on the sidelines, fearing that the payoff from a takeover does not justify its significant cost. If, however, investors were allowed to extract value through favoritism, it would give them an enhanced incentive to assemble ownership blocks in the first place. Indeed, a large block would now confer two valuable benefits on its owner: (1) a more credible threat to acquire the firm (since the cost of assembling the initial block will subsequently be viewed as sunk)<sup>2</sup>; and (2) the ability to use this threat to hold up the manager, credibly promising a takeover if she is unwilling to render patronage. This latter option, moreover, requires only that the investor build modest (i.e., less than controlling) stake in the firm, saving her from a more substantial investment when (as is often the case) the marginal cost of assembling a block of shares increases with the size of a block.<sup>3</sup>

However, the enhanced attraction for investors to form toeholds is but half the story. Anticipating this incentive, a corporate manager must select one of two (relatively unappetizing) strategies. On the one hand, she could simply accommodate whatever block shareholders emerge, securing their quiescence through side payments or other acts of patronage. While acquiescence *might* increase the amount of private benefits the manager can safely appropriate from the firm,<sup>4</sup> it also requires her to share whatever surplus she gleans with others. On the other hand, the manager could choose the path of deterrence, so constraining her own ability to appropriate value as to deter the formation of any block. In this Article, we argue that under many plausible circumstances, managers would favor deterrence, preferring to consume all of a small pie than a meager portion of a large one. In so doing, managers would voluntarily commit themselves to appropriating even less firm value than they would in a world where favoritism was effectively prohibited (and outside investors had to make a binary choice between inaction and outright acquisition). Furthermore, in conventional market settings where multiple outside blocks could potentially form (each demanding patronage from managers) this incentive to deter entry by block shareholders grows even stronger. As such, playing favorites with block shareholders may, ironically, be in all shareholders' interests.

Figure 1 helps to tease out the intuition behind our argument. The figure considers the possible strategies of an incumbent manager faced with an outside investor (i.e., a potential block shareholder) who can assemble a control block of shares. The horizontal axis illustrated in the figure represents the fraction of firm value that a manager ultimately appropriates, and thus ranges between zero and one. The notation above the horizontal axis represents equilibrium behavior when side payments are prohibited, while the notation below the axis

<sup>2</sup>This first benefit, of course, would be present even if side payments were not possible.

<sup>3</sup>Indeed, investors tend to pay a significantly larger per share premium as the size of the block they are purchasing increases. See Part III, *infra*.

<sup>4</sup>As our model below demonstrates, even this proposition is questionable. Indeed, if the manager significantly increases the amount she appropriates from the firm, she might inadvertently make it into such a strong takeover target that it is impossible to deter the outside investor with patronage. See *infra* \_\_\_\_.

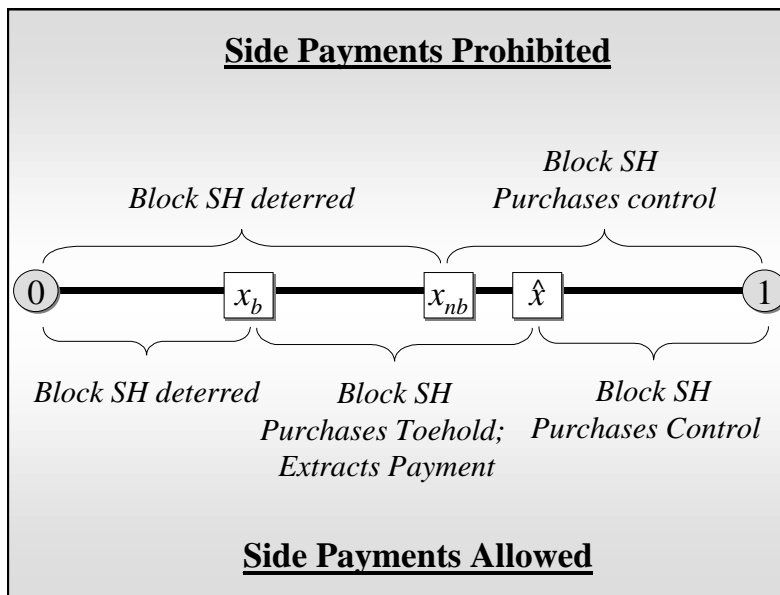


Figure 1: Basic Intuitions

corresponds to equilibrium behavior when they are allowed.

When side payments are prohibited (represented by the top portion of the figure), the value  $x_{nb}$  represents the equilibrium fraction of value appropriation that deters a takeover.<sup>5</sup> If the manager appropriated any greater value, the outside investor would find it profitable assemble a control block and stage a takeover, depriving the managers of any benefits whatsoever. Conversely, should manager appropriate up to  $x_{nb}$ , no shareholder would find assembling a control block to be profitable. As such, a rational, utility-maximizing manager will choose the maximal level of  $x$  that still deters the outside investor, and thus would choose  $x = x_{nb}$ .

When, conversely, side payments are allowed, the manager must choose between acquiescence and deterrence. Here, the outside investor may choose to form a less-than controlling block of shares to extract a side payment. Indeed, forming such a block is far less costly than an outright acquisition, and it gives the investor faces a reduced incremental cost to complete a takeover, which in turn makes the threat of a takeover more credible. If the manager cuts a deal with the block shareholder (through some sort of side payment), she both extinguishes the current threat and enhances the chances that no future takeover can ever occur.<sup>6</sup> With the added security of this collusive agreement,

<sup>5</sup>The subscript *nb* stands for “no bargaining.” In the formal model, we assume that  $x$  is not contractible, but rather that the manager can nonetheless “tie her own hands” by committing to some upper bound on value dilution.

<sup>6</sup>Indeed, once a block of shares is locked up, any subsequent outside investor seeking to

the manager *may* increase their level of private benefits to a (possibly) higher level,  $\hat{x} \geq x_{nb}$ .<sup>7</sup> At the same time, however, the manager will have to share a portion of these rents with the block shareholder (according to their relative bargaining strengths).

Anticipating this downstream division of benefits, the incumbent manager might consider an alternative strategy – selecting a more modest level of appropriation,  $x_b$ , just small enough to dissuade the outside investor from ever entering the picture. Note that because the investor’s cost of building a credible is smaller than engineering a takeover, the manager has to work harder to deter entry, and therefore  $x_b < x_{nb}$ . Our analysis demonstrates that for many plausible ranges of bargaining power between managers and the block shareholder, managers will prefer to deter the formation of blocks altogether. The equilibrium level of value diversion is  $x_b$ , and thus the manager consumes fewer private benefits than she would have if side payments were not allowed.

Others have discussed the relationship between block shareholders and private benefits of control. Zwiebel (1995), for example, analyzes the incentives of investors with varying endowments of wealth to distribute their share ownership across firms to construct blocks in an effort to capture private benefits of control. Zwiebel, however, takes the level of private benefits as exogenous and does not examine the relationship between the presence of a block shareholder and the incentive of managers to extract private benefits from the corporation. Stulz (1988) examines the effect of a manager-owned block or blocks in coalition with management on the actions of potential tender offer bidders. Stulz’s model predicts that a larger management-controlled block will result in both a decreased likelihood of a tender offer bid and an increase in the bid premium. Stulz, however, does not address the formation of the blocks and the effect of such blocks on the overall level of private benefits.

Macey and McChesney (1985) discuss the beneficial effect of greenmail paid in the context of a change in control to a potential acquirer.<sup>8</sup> They argue that

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engage in a takeover faces an even larger (or even prohibitive) cost to assemble a control block than the block shareholder would have in the no-side-payments case.

In the extreme, where managers form alliances with multiple block holders, leaving insufficient shares in the hands of disperse shareholders to assemble into a control block, managers effectively cut off the possibility of any takeover.

In the model below, we shall assume that an alliance with the block shareholder renders infeasible any other takeover attempts. We comment on relaxing this assumption later in the paper.

<sup>7</sup>Interestingly, within our framework, the value of  $\hat{x}$  never exceeds  $x_{nb}$ , except in the perverse case where the manager is an efficient value appropriator (in the sense that she values private benefits more than the average shareholder values her abstinence).

<sup>8</sup>Shleifer and Vishny (1986) provide a similar argument in support of greenmail. They argue that greenmail allows managers to eliminate a low-value bidder and thereby encourage more higher value bidders to expend resources investigating a potential takeover target. Greenmail also provides a target company a credible means to signal to the market that no “White Knight” exists ready to purchase the firm, encouraging other potential bidders to expend resources in preparing a bid.

In contrast, Gordon and Kornhauser (1986) argue against allowing greenmail. They argue that many forms of greenmail are shareholder-value decreasing. Moreover, Gordon and Kornhauser argue that distinguishing between different cases of greenmail is difficult for a court

greenmail benefits shareholders because it allows managers to stop a lower-value bid for the corporation and encourage higher value bidders to enter into an auction for control. Greenmail also compensates initial tender offer bidders for providing the market with information on profitable takeover targets.<sup>9</sup> Significantly, unlike Macey and McChesney, our current argument extends to situations where the side payment does not help instigate an auction for control. Managers may enter into a coalition with a block shareholder that prevents an auction market from ever developing; nevertheless, the prospect of making side payments as part of the coalition will lead managers to appropriate a lower level of private benefits. As well, irrespective of any information signaling effect from making a side payment as discussed in Macey and McChesney, the practice of side payments may enhance shareholder welfare.

Our analysis proceeds as follows. Section 2 discusses three critical assumptions that drive our results. Section 3 then presents a more formal framework for studying patronage to block shareholders, endogenizing both the decision to appropriate private benefits on the part of a manager and an outside investor's decision to assemble a block of shares. Section 4 characterizes and discusses the equilibria that emerge from this framework when side payments are prohibited. Section 5 then characterizes equilibria when side payments are allowed. Section 6 compares the results and discusses possible extensions to the model. Section 7 concludes.

## 2 Critical Assumptions

Before proceeding with the formal analysis, it is appropriate to state up front the three critical assumptions that drive our argument. They are as follows.

- *Managers can commit ex ante to bind themselves to some maximal value of appropriation.*

Our analysis presupposes that it is possible for managers to commit ex ante to bind themselves to some upper bound on the fraction of firm value they may appropriate. Although a number of different real-world mechanisms exist to make this commitment credible, a few examples are especially salient. First, managers may utilize various corporate governance devices to control the possibility of managerial opportunism. For example, managers may install a board of directors consisting of outside independent directors with a reputational interest in monitoring managers. The credibility of such a board over time, moreover, may be enhanced through the use of a staggered board structure that limits the ability of managers and shareholders to remove certain directors without significant delay. Alternatively, the corporation could employ a confidential voting policy aimed at increasing the willingness of shareholders to vote against

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and therefore all greenmail should be prohibited.

<sup>9</sup>The very act of paying greenmail also sends a credible signal to the market on the manager's own perception of the value of the company and thereby may facilitate an auction market for takeovers. Macey and McChesney (1985).



management during a proxy contest. Although managers may later attempt to reverse such corporate governance choices, reversal may be difficult. Reversal, for example, may send a negative signal to the market reducing share value and attracting the attention of potential corporate acquirers. Managers may also commit to certain devices through a corporate charter amendment to reduce the risk of reversal.

A second means of commitment is to utilize state corporate law more directly. Managers may incorporate the firm in a state with more stringent fiduciary duty standards or weaker derivative suit demand requirements on shareholders. To the extent reincorporation requires a shareholder vote, managers lose the ability to exit unilaterally from the state law fiduciary duty protections. Managers may similarly choose to initiate the process to opt-out of state law antitakeover devices. State law fiduciary duties also often place great importance on the status quo. Given the business judgment rule, managers enjoy great leeway in the amount of private benefits they may appropriate from the firm. Courts lack the expertise to assess directly the value of particular managerial decisions. Once a particular management team chooses to appropriate a particular level of private benefits, however, courts then have a benchmark for the level of shareholder value possible in the firm. To the extent the managers choose to increase drastically the amount they appropriate from the firm resulting in a large drop in corporate profits, courts and shareholders may use this as a signal that managerial opportunism has increased. Other factors, of course, may cause a drop in corporate profits; nevertheless, the signal may be particularly strong when a new control block shareholder immune to subsequent takeover assumes control over the firm.

Third, managers may choose to have the firm take on a higher level of debt financing. A greater amount of debt forces the firm to pay out its free cash flow to the debtholders. To the extent managers seek to avoid financial distress, managers will then have an incentive to engage in projects that generate cash flow instead of projects more geared to their own personal preferences. Managers, as well, will be forced to pay out this cash flow to the debtholders rather than re-invest the cash into a value-reducing project that increases the managers' own welfare.

Fourth, managers may install long-term executive compensation packages that rely on options and other means of aligning the incentives of managers and their shareholders. Managers with option-based compensation, for example, possess a reduced incentive to appropriate private benefits of control to the extent their options suffer a reduction in value as a result.

Finally, managers may enter into long-term contracts with particular customers and suppliers that penalize the firm for failing to meet certain targets. For example, a contract with a customer may require the delivery of a set amount of products at a fixed quality level. Failure to meet the terms of the contract may result in a large penalty payment that reduces the amount of value available for managers to appropriate or places the firm at risk of financial distress. Such a contract may therefore force managers to operate the firm at the minimum level of efficiency necessary to ensure that the customer's contract

terms are met. Managers seeking to appropriate value from the firm through a reduction in work effort, for example, may find a decreased ability to do so given a long-term supply contract with large penalty terms. Managers that seek to appropriate value through the diversion of production to their own benefit, as well, may face a reduced ability to make such diversion.

Given the ability to commit to a particular maximum level of private benefits, one might wonder why firms leave managers any flexibility to set the maximum level. Put another way, why wouldn't the incorporators and promoters of the firm simply pre-commit to the maximum level of private benefits at the time of the initial corporate charter? To the extent private benefits of control result in value transferred from shareholders to managers, the argument goes, shareholders that initially invest in the firm will reduce their willingness to pay for the firm's shares. The incorporators of the firm may increase the amount they receive from the initial sale of securities to the public through the adoption of limits on such private benefits.

Thus, if managerial value diversion were inefficient, this reasoning asserts, the initial charter would prohibit it. While this argument is well taken, we find it to be of somewhat limited practical usefulness. Indeed, the needs of most corporations vary over time, and most entrepreneurs lack the ability to predict with precision the firm's prospective needs. As such, it is virtually impossible to design complete governance structures that come close to maximizing firm value far into the future without also allowing for some managerial flexibility over governance.<sup>10</sup> Of course, this flexibility may also allow managers to exploit governance "gaps" by self-dealing. But we assert it is precisely in such circumstances where the market for corporate control becomes an important deterrent. Our analysis offers insights about how such a market can be made to operate more effectively, providing durable and continuous benefits well after a firm's initial public offering.

- *The cost of assembling a block of shares increases with the size of the block.*

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<sup>10</sup>For example, one method of monitoring managers for private benefits of control is to install a completely independent board of directors. Such a board, however, may not prove optimal for firms where the firm seeks to induce investments on the part of managers in firm-specific human capital. Managers that need to make firm-specific human capital investments may fear that ex post, the board acting on behalf of the shareholders may attempt to hold up the managers. Shleifer and Summers (1988). Installing more managers on the board, therefore, may work to induce value-increasing investments on the part of managers in firm-specific human capital. Firms, moreover, may not know the importance of firm-specific investments in human capital for their specific situation until well after the initial incorporation and sale of securities to the public. Likewise, when a firm changes its capital mix to include more debt, it provides a strong incentive on managers to generate free cash flow and pay this cash flow out to the debtholders, reducing the amount available for private benefits of control. For some firms with unstable cash flows, however, the fear of financial distress may counsel against adopting high levels of debt. More debt for such firms may reduce shareholder value through the costs associated with possible financial distress. Customers, for example, may choose not to deal with a firm near financial distress for fear of reduced product quality or poor after-purchase support. Moreover, the cash flow situation of a particular firm may vary after the time of the initial incorporation.

A central characteristic that drives our arguments below is that for parties considering assembling a block of shares, shares become marginally more costly to acquire as the size of the block increases. Put another way, parties face an upward sloping supply curve for shares. Without an upward sloping supply curve for shares, managers would lack the ability to appropriate value without inducing an outside investor to assemble a control block of shares to displace the managers. To the extent managers appropriate even a marginal amount of value from the corporation, outside investors benefit through the purchase of all the corporation's shares and the displacement of the managers. Faced with an upward sloping supply curve, on the other hand, outside investors will not immediately displace managers that appropriate private benefits of control. Instead the outside investors will weigh the cost of assembling a full control block against the benefit from displacing management. Managers that understand this dynamic, in turn, will appropriate just up to the amount where outside investors are indifferent with respect to assembling a control block.

Significantly, in the case where managers are allowed to give side payments to block shareholders, the upward sloping supply curve gives outside investors an incentive to assemble small, non-control blocks of shares. Assembling a non-control block of shares reduces an outside investor's remaining cost to assemble a full control block. An outside investor with a non-control block of shares then poses an increased takeover threat to managers. In response, managers may make a side payment to the outside investor, splitting their private benefits of control or, in the alternative, managers may attempt to commit to a lower level of private benefits to deter outside parties from forming a non-control block of shares in the first place.

Although canvassing the various reasons for an upward sloping supply curve is beyond our ken for current purposes, such characteristics could emerge under a variety of circumstances. In our particular model, shareholders face a range of different tax liabilities for the sale of their shares. Some shareholders may be non-profit organizations and face no tax liability. Other shareholders may hold shares primarily purchased over one year in the past and therefore enjoy long-term capital gains preference on any appreciation in their shares. Still other shareholders may hold shares purchased within one year and face taxation at ordinary income rates for the appreciation in their shares. Shareholders who face differential tax liability upon the sale of their shares will require differential prices to induce them to sell, a trait that leads directly to an upward sloping supply curve for shares.

We conjecture (but do not prove) that our results are less sensitive to the exact reason for an upward sloping supply curve than they are to its existence. Other possible reasons, for example, may explain an upward sloping supply curve. Shareholders might, for instance, have differential liquidity needs, and thus may hold out for different prices. Alternatively, shareholders may subscribe to heterogeneous beliefs about the fundamental value of the shares, again causing the lower valuers to sell first. Regardless of the reason for the increase in per share cost for outside parties assembling a control block as the block size grows, the upward sloping supply curve both gives managers the ability to ex-

tract private benefits of control and makes a non-control block a credible threat of an increased takeover risk.

- *The bargaining power of the manager is not “too large” in relation to that of the block shareholder.*

Key to our theoretical arguments are the terms of the bargain managers may strike with non-controlling block shareholders. As discussed above, outside investors may choose to assemble a toehold block precisely to extract a side payment from managers. Importantly, when the managers suffer from modest or weak relative bargaining power, they will have to forego a larger portion of their rents under a collusive transaction, and would therefore favor committing ex ante to a reduced level of private benefits so as to deter the formation of the block of shares in the first instance.

In contrast, where managers enjoy strong bargaining power relative to the toehold block shareholder, the managers will have to share only a small proportion of their private benefits of control. In such instances, the manager will tend to prefer collusion to deterrence, and would thereby fail to induce the decreased equilibrium levels of value appropriation by managers.<sup>11</sup>

The relative bargaining strength of managers and block shareholders, therefore, is pivotal to the paper’s prescriptive results. Nevertheless, for a wide range of bargaining power allocations between the non-control block shareholder and managers, the paper demonstrates plausible conditions under which managers maximize their welfare by opting for deterrence over collusion. As such, we contend, scholars, courts, and regulators should take seriously the possibility that allowing side payments to block shareholders is efficiency enhancing on the aggregate.

That said, we may now turn to specifying a modeling framework for analyzing the effects of shareholder favoritism, in the form of side payments.

### 3 Framework and Preliminaries

Consider a business entity (or “firm”) that is controlled by a single, risk-neutral manager (“she”), denoted hereinafter as  $M$ .<sup>12</sup> Following standard convention, we assume that ownership of the firm diverges (at least initially) from control. In particular, the firm is owned not by  $M$ , but rather (at least initially) by a continuum of risk-neutral public shareholders, indexed by  $\tau \in [0, 1]$ , each owning an infinitesimal claim  $d\tau$  on the corporation.<sup>13</sup> Although we shall discuss the shareholders’ characteristics at greater length below, we suppose throughout

<sup>11</sup>At the same time, however, within our theoretical framework, so long as value diversion is inefficient, every equilibrium where bribes are allowed result in weakly less value diversion by managers.

<sup>12</sup>Player  $M$  might also represent a “management team” that acts as though it is a unified, coordinated team. Our analysis in such a case would be virtually identical.

<sup>13</sup>It is possible to generalize this framework to assume that each shareholder owns a fraction  $g(\tau)d\tau$  of the corporation, where  $g(\tau) > 0 \forall \tau$ , and  $\int_0^1 g(\tau)d\tau = 1$ . We assume that  $g(\tau) = 1$  for expositional purposes, however.

our analysis that no *existing* shareholder possesses sufficient individual wealth to engineer a takeover of the firm, and, moreover, that coordination costs among existing shareholders are sufficiently high to rule out group lobbying or effective proxy contests. Nevertheless, there exists a third party investor and potential block shareholder (“he”), denoted hereinafter as  $B$ , who begins with no ownership stake in the firm but is sufficiently liquid to purchase a non-trivial block of shares if it is profitable for him to do so.<sup>14</sup> Should  $B$  purchase a sufficient fraction of shares to cross the threshold of control for the firm, denoted by  $c \in (0, 1)$ , he can displace  $M$  and substitute himself as manager.

Total firm value,  $\tilde{V}$ , is realized only at the very end of the model, at which point the firm is liquidated and the proceeds distributed to the shareholders according to their individual ownership stakes. We assume this value to be non-contractible.<sup>15</sup> Nevertheless, if managed selflessly by  $M$ , the the expected value of the firm would be equal to  $\bar{V} > 0$  dollars. However, the manager is in a position to expropriate private benefits of control in the form of a fraction  $x_M \in [0, 1]$  of the value of the firm. Consequently, the expected value of the firm in the presence of  $M$ 's value diversion is equal to  $(1 - x_M) \cdot \bar{V} \equiv V$ . While diverting value certainly helps the manager, it is nonetheless inefficient. Explicitly, for each dollar's worth of firm value she appropriates, the manager is assumed to benefit by  $\mu$  dollars,<sup>16</sup> where  $0 < \mu \leq \frac{1}{2}$ . Consequently, the first best solution would set  $x_M = 0$ .

Nevertheless, we assume (for reasons outside the model) that the governance structure of the firm is sufficiently flexible to permit managerial moral hazard. Thus, the manager has the tools at her disposal to appropriate up to the entire value of the firm if she so desires. However, the manager is able to take actions (such as issuing highly-monitored debt, reputational bonding, and the like) that bound her ex ante ability to expropriate value from above at a level<sup>17</sup> denoted by  $\bar{x} \in [0, 1]$ , so that  $x_M \in [0, \bar{x}]$ . Quite obviously, absent the threat of outside intervention,  $M$  would always want to expropriate as much as she can, and would therefore be inclined to leave herself unconstrained, at  $\bar{x} = 1$ . But selecting some  $\bar{x} < 1$  carries at least two potential advantages for  $M$  (explored more formally below). First, it induces existing shareholders to place greater value on their existing ownership stakes, thereby increasing the costs of a takeover by  $B$ ; and second, the value of  $\bar{x}$  chosen by  $M$  also serves to constrain the ability of an acquirer (i.e.,  $B$ ) to appropriate value, thereby decreasing the potential benefits of a takeover.<sup>18</sup> As to this second factor, we assume that after taking over the

<sup>14</sup>Nothing turns on  $B$ 's lack of an initial stake in the firm. Our analysis persists when  $B$  also begins with ownership of an infinitesimal stake in the firm.

<sup>15</sup>In order to concentrate on the role of the takeover market—rather than incentive pay—as a device for addressing agency costs, we assume that  $\tilde{V}$  is not contractible, and that the manager simply receives a flat wage for her services, normalized without loss of generality to be zero.

<sup>16</sup>We bound  $\mu$  above by  $\frac{1}{2}$  rather than one for reasons that are explained below. Our assumptions about existing shareholders in fact require that  $\mu \leq \frac{1}{2}$  for managerial value appropriation to be inefficient from an organizational standpoint.

<sup>17</sup>We assume that this  $\bar{x}$  is chosen endogenously by  $M$  in a manner described below.

<sup>18</sup>This depends, of course, at least in part on the manner in which  $M$  commits herself.

firm,  $B$  may find that he would like to extract private benefits from control. In particular, subsequent to a takeover,  $B$  observes his own marginal benefit of control, denoted by  $k \in [0, 1]$  drawn from a uniform distribution. For each dollar  $B$  appropriates,  $B$  benefits by  $k$  dollars.  $B$  may then choose his own level of value diversion, denoted by  $x_B \in [0, \bar{x}]$ .<sup>19</sup>

In order to acquire a control share,  $B$  must buy out existing shareholders who are willing to sell. Quite obviously, then,  $B$  would have to offer a price that is sufficiently attractive to induce the marginal shareholder to cash out now rather than later. Although the next subsection derives the precise cost function for assembling various blocks of shares, for current purposes we highlight one factor (in addition to the expected value of the firm) that plays a pivotal role in shareholders' willingness to tender. Each shareholder faces a distinct marginal tax rate, which we assume (for notational ease) to coincide with the her index designation,  $\tau \in [0, 1]$ .<sup>20</sup> Each shareholder is assumed to begin with a tax basis of 0, and thus her tax liability on a given payoff  $z > 0$  is equal to  $\tau z$  (once the payoff is realized for tax purposes).<sup>21</sup> Consequently, a shareholder will sell her shares only if the price offered by  $B$  is sufficiently high for her to justify foregoing the benefits of tax deferral, and instead reinvest her after-tax sales price at the prevailing rate  $r$  for one period.<sup>22</sup>

Reiterating, then, our framework can be summarized in the following table:

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If she uses various "hard" mechanisms (such as corporate debentures or durable governance structures), then acquirors are much more likely to be stuck with the same constraints than if  $M$  used more personal reputational measures to bond.

<sup>19</sup>For  $B$ , of course, this decision of how to set  $x_B$  is somewhat more complicated than it was for  $M$ , since  $B$ 's newly-purchased ownership share gives him a countervailing stock-price incentive to forebear from expropriating the firm. We model this tradeoff more formally below.

<sup>20</sup>Consequently, this assumption is tantamount to a situation where the marginal tax rate is 100%, while the bottom is 0%. At the cost of additional notation, one could generalize our model to assume that  $\tau$  is distributed on  $[\underline{\tau}, \bar{\tau}] \subseteq [0, 1]$ . However, the core intuitions we develop below would be substantially the same. We should note, of course, that because shareholders often differ regarding the basis they may claim in the securities they own, the continuous distribution of "effective" marginal tax rates is not a bad approximation.

<sup>21</sup>In addition, we shall assume that the potential block shareholder  $B$  faces a tax rate of zero.

<sup>22</sup>As noted above, we will generally assume throughout what follows that  $M$ 's marginal private benefits of control are less than one half (i.e.,  $\mu < \frac{1}{2}$ ). Indeed,  $\mu > \frac{1}{2}$  would correspond to the (seemingly perverse) case where value diversion by the manager is actually efficiency enhancing after accounting for the tax effects in the text.

To see this, suppose that no block shareholder exists and that  $M$  expects to appropriate an  $x$ -share of the firm's value. The present discounted payoff for the manager is thus  $\bar{V} \cdot (\mu x) / (1 + r)$ . The payoff of each shareholder holding fraction  $d\tau$  of the firm's  $\frac{\bar{V}}{(1+r)} (1-x)(1-\tau) \cdot d\tau$ . Summing across shareholders, their aggregate after-tax payoff to is given by  $\int_0^1 \bar{V} (1-x)(1-\tau) / (1+r) \cdot d\tau = \frac{\bar{V}}{(1+r)} \cdot (\frac{1-x}{2})$ . Comparing this to the manager's payoff, it is clear that value diversion is efficient on the margin whenever  $\mu > \frac{1}{2}$ . We therefore exclude this possibility in our analysis. Note that this assumes managers are not taxed on their private benefits. Where managers are taxed (e.g., on higher wage compensation) then value diversion may be inefficient even when  $\mu > \frac{1}{2}$ .

Obviously, the above definition of "efficiency" does not take into account any external benefits of the tax revenue raised from the shareholders. However, it is an appropriate definition from the standpoint of organizational design.

$\bar{V}$	=	Maximal value of corporation (absent any self-dealing)
$V$	=	Realized value of corporation (more below...)
$x_i$	=	Fraction of firm value actually converted; $i \in \{M, B\}$
$\bar{x}$	=	Maximal expropriation level (chosen by $M$ ); $x_i \in [0, \bar{x}]$
$\mu$	=	$M$ 's marginal benefit for each unit of $V$ converted ( $\mu \leq \frac{1}{2}$ )
$k$	=	$B$ 's marginal benefit for each unit of $V$ converted ( $k \in U[0, 1]$ )
$\delta$	=	$B$ 's initial block of shares
$\gamma$	=	$B$ 's final block of shares
$c$	=	Minimum fraction of outstanding shares required for control
$r$	=	Discount rate for future payoffs
$\tau$	=	Shareholder's "type" (denoting marginal tax rate)
$d\tau$	=	Initial infinitesimal ownership share of each shareholder

**Table I: Summary of Notation**

### 3.1 Shareholder Preferences

Of central importance to both the block shareholder and the manager is the market structure of the shareholders, since this determines  $B$ 's costs of assembling a block of shares. In order to analyze meaningfully the manager's choices, then, we must first characterize the structural characteristics of the "supply curve" for shares.<sup>23</sup> The first step in doing so is to characterize the conditions under which a shareholder would be willing to tender her shares rather than holding onto them until the end of the game. Thus, consider a shareholder of type  $\tau$  who does not tender. Viewed at the time of her decision, the discounted expected payoff for such a shareholder is:

$$\frac{E(V) \cdot (1 - \tau)}{1 + r} \cdot d\tau, \quad (1)$$

where  $E(V)$  denotes the shareholder's equilibrium expectation about the eventual value of the firm.<sup>24</sup> In contrast, consider a shareholder who tenders her ownership claim at per-share price  $p$ . The payoff of such a shareholder no longer turns on her expectations about the value of the firm. Instead we assume that she would reinvest the after-tax purchase price at rate  $r$  (also subject to later capital gains tax). The present-discounted expected payoff for the tendering shareholder is therefore:<sup>25</sup>

<sup>23</sup>As noted above, a key characteristic of this supply curve is that it be upward sloping. We confirm this within our framework below.

<sup>24</sup>As is conventional, we shall later impose the condition that the shareholder's expectations be part of a rational expectations equilibrium.

<sup>25</sup>Note that we assume that the shareholder faces the same tax rate for capital gains on the sale of her shares as for the one period investment return on the sale proceeds. Although somewhat unrealistic, this assumption simplifies the analysis without changing the qualitative point that shareholders must receive compensation for the loss in tax deferral they experience when they sell their shares earlier rather than later.

$$\frac{p \cdot (1 - \tau) \cdot (1 + r \cdot (1 - \tau))}{1 + r} \cdot d\tau \quad (2)$$

Combining (1) and (2), a shareholder of type  $\tau$  will sell her shares at per-share price  $p$  if and only if the following condition holds:

$$p \geq \frac{E(V)}{1 + r \cdot (1 - \tau)} \quad (3)$$

The interpretation of (3) is quite intuitive. It says that the shareholder will sell her ownership stake if and only if the price offered is sufficiently attractive to offset the benefits of tax deferral. Intuitively, then, the minimal price at which the shareholder would tender (expressed in the left hand side of the above inequality) is increasing in  $E(V)$  and  $\tau$ , but decreasing in  $r$ .

The reservation price of a *particular* shareholder, however, does not yet reveal what the *market supply curve* for the market will look like. To characterize the latter, note first that (3) is equivalent to the following condition on the shareholder's tax rate  $\tau$ :

$$\tau \leq \frac{1}{r} \left( (1 + r) - \frac{E(V)}{p} \right)$$

(Note that the right hand side of the above expression need not be positive, such as for relatively small values of  $p$  or relatively large values of  $E(V)$ ). Consequently, to derive the market supply curve, one must merely sum up all individual shareholders who would be willing to tender at price  $p$ . Assuming all shareholders have the same beliefs in equilibrium about the expected value of the firm, the market supply curve is given by:

$$\begin{aligned} Q_S(p) &= \int_0^{Max\{0, \frac{1}{r}(1+r) - \frac{E(V)}{p}\}} d\tau \\ &= Max\left\{0, \frac{1}{r} \left( (1 + r) - \frac{E(V)}{p} \right)\right\} \end{aligned} \quad (4)$$

A plot of this supply curve appears below, for  $r = 0.20$ ,  $E(V) = 0.3$ .



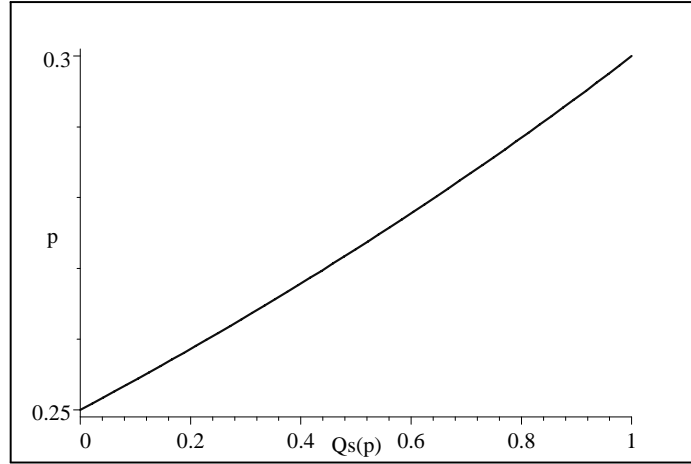


Figure 2

Note that the supply curve is increasing in price offered, and that a price of  $p = E(V)$  will induce all shareholders to part with their shares. (Of course, absent some private benefits from control or change in management, it would never be optimal for someone to offer such a high price, since the present value of the purchased firm would be  $\frac{E(V)}{1+r} < E(V)$ ).

In some of the computations below, it will be helpful to vary this supply function to account for the possibility that the bidder has already obtained a  $\delta$ -block of shares from outside shareholders. Assuming that this existing  $\delta$ -block was originally obtained from the lowest tax-rate shareholders,<sup>26</sup> the supply curve will shift back to the following:

$$Q_S(p; \delta) = \text{Max} \left\{ 0, \frac{1}{r} \left[ (1+r) - \frac{E(V)}{p} \right] - \delta \right\} \quad (5)$$

(Note that equation (4) is simply a special case of the expression in (5) for the case of  $\delta = 0$ ).

So suppose that someone owning a  $\delta \geq 0$  fraction of the firm wanted to increase his ownership to  $\gamma > \delta \geq 0$ . The purchaser would have to offer enough to induce the *marginal* shareholder of type  $\tau = \gamma$  to forego the benefits of tax deferral. He would therefore have to offer a per-share price of<sup>27</sup>

$$p(\gamma) = \frac{E(V)}{1+r \cdot (1-\gamma)}.$$

All told, then, the cost borne by an existing block shareholder to increase her fractional holding from  $\delta$  to  $\gamma$  is given by the following.

<sup>26</sup>Formally, of course, this conjecture must be verified in the equilibrium portion of the paper below.

<sup>27</sup>Note how this amount is independent of  $\delta$ .

$$\begin{aligned}
T(\gamma, \delta) &= p(\gamma) \cdot (\gamma - \delta) \\
&= \left( \frac{E(V)}{1 + r(1 - \gamma)} \right) \cdot (\gamma - \delta)
\end{aligned} \tag{6}$$

A few details about  $T(\delta, \gamma)$  are worth mentioning at this point. Holding  $E(V)$  constant, note that  $\frac{\partial T}{\partial \gamma} = E(V) \frac{1+r-r\delta}{(1+r-r\gamma)^2} > 0$ , making block acquisitions increasingly more expensive at the margin. In fact, note also that  $\frac{\partial^2 T}{\partial \gamma^2} = 2 \frac{(r+1-r\delta)r}{(r+1-r\gamma)^3} E(V) > 0$ , suggesting that the marginal cost of assembling a block of size  $\gamma$  increases at an increasing rate due to the upward sloping supply curve. Second, note that  $T(\cdot)$  is decreasing in  $\delta$ , signifying the fact that  $B$  treats his existing toehold as a sunk cost in assessing the incremental cost of increasing that ownership share to  $\gamma > \delta$ . As such, it will cost him less to take a control share than if he had no toehold.

### 3.2 B's Demand-side Preferences

Although we have fully specified the supply side characteristics of the model (up to  $E(V)$ ), we have yet to say anything about demand-side characteristics. For this we need to specify what motivates the block shareholder. To fill this in, we suppose that two things motivate  $B$  in her purchases of shares. First, and most obviously, if  $B$  purchases an ownership stake in the firm, she stands to earn later rents when the firm is liquidated. Thus, purchasing a fractional share  $\gamma$  in the firm will yield a payoff of  $\gamma \cdot E(V)$  for the block shareholder. This payoff accrues regardless of whether the block shareholder ever purchases control of the firm.

Second, and as noted above, the block shareholder *may* also want to extract private benefits of control from the firm in the event that she successfully mounts a control transaction. Recall that the block shareholder observes his own marginal private benefit from control of  $k \in [0, 1]$  dollars for each unit of firm value that she appropriates, whose realization is learned only after a successful takeover. The realized value of  $k$  is what determines how, after a control transaction (i.e.,  $\gamma \geq c$ ), the block shareholder will select his own level of value diversion,  $x_B$ . In particular, if  $\gamma \geq k$ , player  $B$  would control the firm, but would never choose to expropriate value, since he loses more as an owner than he gains as an expropriator. In this case,  $B$  would set  $x_B = 0$ , and his payoff is simply be  $\gamma \cdot E(V) = \gamma \cdot \bar{V}$  — the value of her fractional ownership of the enterprise under efficient management. On the other hand, if  $k > \gamma$ , the insider would willingly expropriate the maximum possible fraction of firm value, setting  $x_B = \bar{x}$ . In this later case, her payoff would be equal to  $[\gamma(1 - \bar{x}) + k\bar{x}]\bar{V}$ .<sup>28</sup>

<sup>28</sup>Explicitly, it is equal to

$$\begin{aligned}
\gamma \cdot E(V) + k \cdot (\bar{V} - E(V)) &= \gamma(1 - \bar{x})\bar{V} + k\bar{x}\bar{V} \\
&= [\gamma(1 - \bar{x}) + k\bar{x}]\bar{V}.
\end{aligned}$$

### 3.3 Equilibrium Expectations:

Finally, before pressing on, it is worth taking note that the cost of assembling a control share (or any other fractional share for that matter) turns critically on shareholders' equilibrium expectations about the expected future value of the firm, or  $E(V)$ . Consequently, it is important to be clear about our assumptions pertaining to these expectations. Perhaps most natural is to assume that people share "rational expectations" about the future value of the firm — i.e., their expectations must square with subsequent equilibrium play, at least given current information. Thus, suppose the posited equilibrium involves the manager retaining control, after which she is expected to appropriate a  $x_M$  fraction of the firm in the form of private benefits. Here, all players must share the following expectations about the value of the firm:

$$E(V) = (1 - x_M) \cdot \bar{V}.$$

If, however, the posited equilibrium involves the block shareholder successfully amassing a  $\gamma \geq c$  share, then expectations will shift to reflect the equilibrium value of the firm under her stewardship. Here, then, the expected value of the firm becomes:

$$E(V) = (1 - x_B) \cdot \bar{V},$$

Of course,  $x_M$  and  $x_B$  may differ from one another, and their values depend on the equilibrium play of the game. This analysis appears in the ensuing sections. We now proceed to characterize the equilibrium in the absence and presence of side payments, *ad seriatim*.

## 4 Side Payments Prohibited

We begin by considering the model under the assumption that  $M$  and  $B$  are prohibited from negotiating standstill agreements with one another. Assume the following structure, in which the first two stages take place at an initial period,  $t_0$ , and the third and fourth take place at  $t_1$ , one period later:

1. (Commitment Stage) Player  $M$  commits to a maximal value of expropriation  $\bar{x} \leq 1$ . We assume that  $\bar{x}$  represents the maximal value of expropriation whether  $M$  or  $B$  controls the firm.
2. (Takeover Stage) Player  $B$  decides how much of an ownership stake in the firm to purchase denoted by  $\gamma \geq 0$ . If  $\gamma \geq c$ , player  $B$  assumes control, but otherwise player  $M$  remains as manager.<sup>29</sup>
3. (Expropriation Stage) Should  $M$  retain control, she selects a value of  $x_M \in [0, \bar{x}]$ . Should  $B$  obtain control, however, he observes the realized value of  $k$ , and then selects his own value of  $x_B \in [0, \bar{x}]$ .

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<sup>29</sup>The next section allows  $B$  to build an initial "toehold" ownership share of  $\delta > 0$ , from which he may bargain with  $M$  for a side payment. Since this section excludes the possibility of side payments, however, we collapse the toehold purchase and control purchase into one decision (implicitly assuming that  $\delta = 0$ ). This is done without loss of generality.

4. (Realization Stage) The firm realizes a value equal in expectation to  $(1 - x_i) \cdot \bar{V}$ , which is liquidated and distributed to existing shareholders on a pro rata basis.

To characterize the equilibria of this model, we employ standard backwards induction techniques. We begin with stage 3 (since stage 4 involve no strategic choices by the players), and then move back to stages 2 and 1, *ad seriatim*.

#### 4.1 Expropriation stage:

Beginning with stage 3, suppose first that  $M$  has managed to retain control of the firm (that is,  $B$  has purchased less than a control share, or  $\gamma < c$ ). Now,  $M$  must choose some  $x_M \in [0, \bar{x}]$ . Her private payoff is equal to  $\mu \cdot x_M \cdot \bar{V}$ , which is obviously maximized at  $x_M = \bar{x}$ . And since  $M$  has no ownership stake of the firm, her taking of perquisites comes without cost to her. Thus, when no takeover has occurred,  $M$  will maximally expropriate the firm, giving her a payoff measured ex-ante and in terms of present discounted value (PDV) of  $\bar{V} \cdot \left(\frac{1}{1+r}\right) \cdot \mu \bar{x}$ , and shareholders an aggregate ex ante expected discounted payoff of  $\bar{V} \cdot \left(\frac{1}{1+r}\right) \cdot (1 - \bar{x})$ . In this case, the block shareholder earns only his pro rata share (if any) of the shareholders' aggregate payoff less his now-sunk acquisition cost of his ownership stake.

Now, suppose instead that  $B$  has captured control of the firm in stage 3 (that is,  $\gamma \geq c$ ), has observed the realization of  $k$ , and now  $B$  must choose some  $x_B \in [0, \bar{x}]$ . Her private payoff is equal to  $[\gamma(1 - x) + kx] \bar{V}$ , which constitutes the sum of her pro rata ownership share and private benefits of control. Clearly,  $B$  will set  $x_B = 0$  whenever he observes  $k \leq \gamma$ , since  $B$  would gain more on a pro rata basis by acting selflessly than by expropriating the firm. In this case,  $B$ 's payoff consists simply of his pro rata payoff under efficient management, or  $\gamma \cdot \bar{V}$ . On the other hand, if he observes  $k > \gamma$ ,  $B$  will choose to set  $x_B$  maximally, at  $x_B = \bar{x}$ , since his marginal private benefits now outweigh his marginal pro rata losses. In this case,  $B$ 's payoff consists of the sum of (1) any remaining pro rata benefits (or,  $\gamma(1 - \bar{x}) \bar{V}$ ), plus (2) his own his private benefits of control (or,  $k \bar{x} \bar{V}$ ). Just before the realization of  $k$  obtains, then, a controlling block shareholder can expect the following payoff:

$$\begin{aligned}
E[\pi_B(\gamma \geq c)] &= \Pr(k \leq \gamma) \cdot \underbrace{[\gamma \cdot \bar{V}]}_{\text{Pro Rata Benefits}} \\
&\quad + \Pr(k > \gamma) \cdot \left( \underbrace{\gamma \cdot (1 - \bar{x}) \bar{V}}_{\text{Pro Rata Benefits}} + \underbrace{E(k|k > \gamma) \cdot \bar{x} \bar{V}}_{\text{Private Benefits}} \right) \\
&= (\gamma) \cdot [\gamma \cdot \bar{V}] + (1 - \gamma) \cdot \left( \gamma(1 - \bar{x}) \bar{V} + \left(\frac{1 + \gamma}{2}\right) \cdot \bar{x} \bar{V} \right) \\
&= \bar{V} \cdot \left( \gamma + \frac{1}{2} \bar{x} (1 - \gamma)^2 \right)
\end{aligned}$$

Discounted to ex-ante present value, player  $B$ 's *expected* payoff subsequent to a takeover is equal to  $\frac{\bar{V}}{1+r} \cdot \left( \gamma + \frac{1}{2}\bar{x}(1-\gamma)^2 \right)$ , while player  $M$ 's payoff is simply zero.

## 4.2 Takeover Stage

Given these expected payoffs at the expropriation stage we now back up one period to consider  $B$ 's purchase options at Stage 2, keeping in mind that  $B$  does not know with certainty at this point about the eventual realization of  $k$ . There are three qualitative possibilities to consider, corresponding, respectively to situations in which  $B$  purchase a sub-controlling block of shares, an exact controlling block of shares, and a super-controlling block of shares.

### 4.2.1 Case 1: $\gamma < c$ .

Consider first whether  $B$  will purchase a sub-controlling share, or  $\gamma < c$ . In this case, it is common knowledge that no takeover will occur, and accordingly,  $B$ 's discounted benefit for purchasing  $\gamma$ -stake will be  $\gamma \cdot \frac{E(V)}{1+r} = \gamma \cdot \frac{(1-\bar{x})\bar{V}}{1+r}$ . The cost of the purchase would be  $T(\gamma, 0) = \left( \frac{(1-\bar{x})\bar{V}}{1+r(1-\gamma)} \right) \cdot \gamma$ . Subtracting the cost from the benefit yields  $B$ 's net gain from purchasing a sub-control share:

$$\begin{aligned} \Phi(\gamma|\gamma < c) &= \gamma \cdot \frac{(1-\bar{x})\bar{V}}{(1+r)} - \left( \frac{(1-\bar{x})\bar{V}}{1+r(1-\gamma)} \right) \cdot \gamma \\ &= - \left( \frac{\bar{V} \cdot (1-\bar{x}) \cdot r}{(1+r)(1+r-r\gamma)} \right) \cdot \gamma^2 < 0 \end{aligned}$$

Notice that this expression is strictly negative for any  $\gamma > 0$ . And thus, the entire term is maximized at the boundary by setting  $\gamma = 0$ . Consequently, we know that  $B$  would prefer inaction (which gives a zero payoff) over purchasing shares at any level that falls short of taking control. Intuitively, where  $B$  does not obtain control,  $B$  receives the same pro rata value as all other shareholders based on the managers' selection of  $\bar{x}$ . Because  $B$  must pay a premium to assemble even a non-control block of shares due to the upward sloping supply curve for shares,  $B$  suffers a negative return.

### 4.2.2 Case 2: $\gamma = c$ .

Second,  $B$  can attempt an exact takeover of the firm by making a tender offer for exactly  $\gamma = c$  shares. For an exact takeover,  $B$  potentially receives private benefits, and thus the present value of constructing an exact control block of size  $c$  (as derived above) is:

$$\left( \frac{1}{1+r} \right) \cdot E[\pi_B(c)] = \bar{V} \cdot \left( \frac{1}{1+r} \right) \cdot \left( c + \frac{1}{2}\bar{x}(1-c)^2 \right)$$

The price  $B$  offers must be sufficiently large so that the pivotal shareholder of type  $\tau = c$  is willing to tender, and is thus given by:

$$p(c) = \lim_{\gamma \rightarrow c^-} \frac{E(V)}{1+r(1-\tau)} = \frac{\bar{V}(1-\bar{x})}{1+r(1-c)}$$

In deriving the value for  $E(V)$ , recall that for an exact takeover, the *marginal* shareholder (of type  $\gamma = c$ ) is also the *pivotal* shareholder, and thus her failure to tender will cause the takeover to fail, garnering only  $c - d\tau$  shares for  $B$ . Because this shareholder is pivotal, then, she will realize that if she refuses to tender, the firm will be governed by current management. Thus, the pivotal shareholder will tender if the price exceeds what she would expect the firm to be worth in the absence of an acquisition.<sup>30</sup> As such,  $B$ 's cost of purchasing  $c$  is  $T(c, 0) = \left(\frac{E(V)}{1+r(1-c)}\right) \cdot (c) = \left(\frac{\bar{V}(1-\bar{x})}{1+r(1-c)}\right) \cdot (c)$ . Combining the expected benefits and costs,  $B$ 's net gain is given by:

$$\Phi(c) = \bar{V} \cdot \left( \frac{2c + \bar{x}(1-c)^2}{2(1+r)} - \frac{c(1-\bar{x})}{1+r(1-c)} \right), \quad (7)$$

and thus Player  $B$  would prefer an exact takeover over inaction if and only if  $\Phi(c) > 0$ .

Note that  $\Phi(c)$  might be either positive or negative, depending on the value of  $\bar{x}$  chosen by  $M$ . Indeed, when  $\bar{x} = 1$ , (7) reduces to:

$$\Phi(c)|_{\bar{x}=1} = \bar{V} \cdot \left( \frac{1+c^2}{2(1+r)} \right) > 0,$$

implying that  $B$  would prefer to mount a takeover. Conversely, if  $\bar{x} = 0$ ,  $B$ 's expected net payoff from an acquisition becomes:

$$\Phi(c)|_{\bar{x}=0} = \bar{V} \cdot \left( \frac{-rc^2}{(1+r-rc)(1+r)} \right) < 0,$$

implying that  $B$  would prefer to do nothing. By virtue of the linearity of  $\Phi(c)$  in  $\bar{x}$ , there exists a unique value of  $\bar{x}$  that just deters a takeover on the margin. In particular,  $M$  could make  $B$  indifferent between inaction and a takeover by committing to an  $\bar{x} = x_{nb}$  such that  $\Phi(c)|_{\bar{x}=x_{nb}} = 0$ . Solving for  $x_{nb}$  yields:

$$x_{nb} = \frac{2rc^2}{(1-c)^2(1+r-rc) + 2c(1+r)}, \quad (8)$$

and thus  $x_{nb} \in (0, 1)$ . For technical convenience (and without loss of generality), we assume hereinafter that whenever  $B$  is indifferent, he always prefers inaction to a takeover; and thus, any  $\bar{x} \leq x_{nb}$  will effectively deter a takeover.

<sup>30</sup>Note that our analysis implicitly assumes that the block shareholder is able to make a take-it-or-leave-it offer to the pivotal shareholder. Where the pivotal shareholder chooses not to take the offer, for example, the block shareholder may infinitesimally increase his offer to induce the shareholder with the next higher tax rate to sell her shares.

### 4.2.3 Case 3: $\gamma > c$ .

Finally, consider the case where  $B$  attempts to acquire  $\gamma > c$  shares. We analyze this case separately from an exact takeover (where  $\gamma = c$ ) because the purchasing of a super-controlling block of shares has subtler pricing effects. Indeed, unlike the exact takeover, here *the marginal shareholder (type  $\gamma = \tau$ ) is no longer pivotal to the takeover*. This observation is important, because the marginal (but not pivotal) shareholder expects the takeover to occur regardless of whether she tenders, and thus her expected value of the firm implicitly presumes a successful takeover will occur. (Put another way, the marginal shareholder can simply free ride on any increase in value due to the takeover if she refuses to tender her shares). Any tender offer on  $B$ 's part, therefore, must include an increased premium to compensate her for the post-takeover expected value of the firm. When  $\gamma > c$ , then, the marginal shareholder's expectation of ex post firm value is as follows:

$$\begin{aligned} E(V|\gamma > c) &= \gamma \bar{V} + (1 - \gamma)(1 - \bar{x}) \bar{V} \\ &= \bar{V} \cdot (1 - (1 - \gamma)\bar{x}) \end{aligned}$$

Note that this clearly exceeds  $(1 - \bar{x})\bar{V}$ , the pivotal shareholder's expectation of firm value under an exact takeover. Now, the price to acquire from such a marginal shareholder is equal to:

$$p(\gamma) = \frac{E(V)}{1 + r(1 - \tau)} \Big|_{\tau=\gamma > c} = \frac{\bar{V}(1 - (1 - \gamma)\bar{x})}{(1 + r - r\gamma)}$$

Note that  $\lim_{\gamma \rightarrow c^+} p(\gamma) > p(c)$ , indicating an upward discontinuity of required price for a super-controlling takeover. This observation should not be terribly surprising, given that the marginal shareholder must receive compensation for the benefits she would receive from free-riding on  $B$ 's acquisition.

All told, then, the net benefit of purchasing a greater-than control share ( $\gamma > c$ ) is:

$$\begin{aligned} \Phi(\gamma|\gamma > c) &= \bar{V} \left( \left( \frac{2\gamma + \bar{x}(1 - \gamma)^2}{2(1 + r)} \right) - (\gamma) \left( \frac{1 - (1 - \gamma)\bar{x}}{1 + r - r\gamma} \right) \right) \quad (9) \\ &= \bar{V} \left( \frac{-2r\gamma + \bar{x}(1 - \gamma)(1 + r + \gamma + r\gamma^2)}{2(1 + r)(1 + r - r\gamma)} \right) \end{aligned}$$

Differentiating with respect to  $\gamma$  confirms that this expression is strictly decreasing in  $\gamma$ . Moreover, the limit of  $\Phi(\gamma)$  as  $\gamma$  approaches  $c$  from above is:

$$\begin{aligned} \lim_{\gamma \rightarrow c^+} \Phi(\gamma|\gamma > c) &= \bar{V} \left( \left( \frac{2c + \bar{x}(1 - c)^2}{2(1 + r)} \right) - (c) \left( \frac{1 - (1 - c)\bar{x}}{1 + r - rc} \right) \right) \quad (10) \\ &< \bar{V} \cdot \left( \frac{2c + \bar{x}(1 - c)^2}{2(1 + r)} - (c) \frac{(1 - \bar{x})}{1 + r(1 - c)} \right) = \Phi(c) \end{aligned}$$

And thus, not only does  $\Phi(\gamma)$  have a downward discontinuity at  $\gamma = c$ , but it is also a strictly decreasing function in  $\gamma$  thereafter.

Clearly, then,  $B$  would never rationally attempt to purchase shares greater than the minimum necessary for control. Indeed, doing so would require  $B$  to increase discontinuously the premium he pays to all shareholders once a takeover is certain. Furthermore,  $B$ 's incentive to engage in a takeover in part rests with  $B$ 's ability to appropriate potentially his own private benefits.  $B$ 's ability to appropriate private benefits does not increase with block size once  $B$  has obtained control over the firm. In contrast, as  $B$ 's share ownership increases past  $c$ ,  $B$  will bear more of the cost of appropriating private benefits, reducing the likelihood that  $B$  will in fact profit from the possibility of such benefits.

#### 4.2.4 Synthesis

As the above analysis makes clear,  $B$ 's expected net payoff,  $\Phi(\gamma)$  is strictly decreasing for all  $\gamma < c$ , discontinuous at  $\gamma = c$ , and strictly decreasing once again for all  $\gamma > c$ . Consequently, the only actions that  $B$  can plausibly take are (1) to purchase nothing at all ( $\gamma = 0$ ), or (2) to engineer an exact takeover ( $\gamma = c$ ). Drawing on the analysis from subsection 4.2.2 above, then, we arrive at following Lemma:

**Lemma 1** *Suppose  $M$  has previously committed to a maximal level of value expropriation,  $\bar{x}$ . The unique equilibrium of the Takeover-Stage subgame is as follows:*

- If  $\bar{x} \leq x_{nb}$ ,  $B$  will purchase no shares whatsoever, yielding respective payoffs for  $M$  and  $B$  of:

$$(\pi_M, \pi_B) = \left( \frac{\mu \bar{x} \bar{V}}{1+r}, 0 \right).$$

- If  $\bar{x} > x_{nb}$ ,  $B$  will make a successful tender offer for exactly  $c$  shares, at price  $p = \frac{(1-\bar{x})\bar{V}}{1+r-rc}$ , yielding payoffs of:

$$(\pi_M, \pi_B) = (0, \Phi(c)).$$

### 4.3 Commitment Stage

Finally, consider the initial stage of the game where  $M$  commits to a maximal level of appropriation,  $\bar{x}$ . Recall from the previous subsection that the manager's payoff is:

$$\pi_M(\bar{x}) \begin{cases} \frac{\mu \bar{x} \bar{V}}{1+r} & \text{if } \Phi(c) \leq 0 \\ 0 & \text{else} \end{cases}$$

Clearly, then,  $M$  would like to select the largest value of  $\bar{x}$  that still makes a takeover unprofitable to  $B$ . Equivalently,  $M$  will choose the largest  $\bar{x}$  that ensures  $\Phi(c) \leq \Phi(0) = 0$ . As demonstrated above, this value is given by  $\bar{x} =$



$x_{nb}$ , where  $x_{nb}$  is given by (8). The plot of  $x_{nb}$  appears below, as a function of  $r$  and  $c$ :

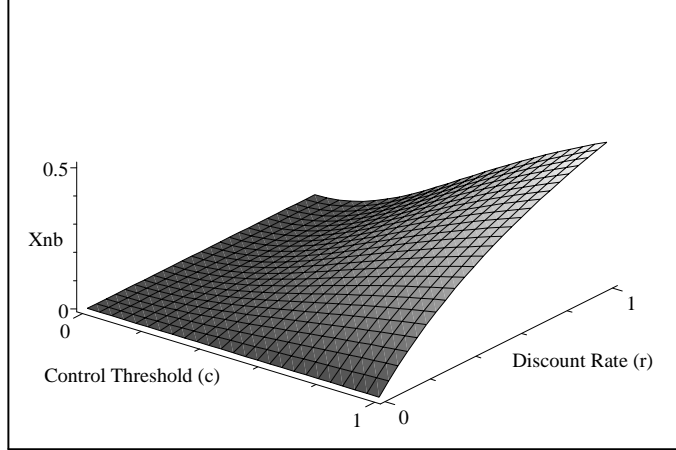


Figure 3

Simple differentiation immediately yields the result that  $x_{nb}$  is strictly increasing with  $c$  and  $r$ , so that  $x_{nb}$  reaches a maximal value of  $\frac{1}{2}$  at  $r = c = 1$ . Higher levels of  $c$  both increase the cost to  $B$  from purchasing a control block of shares and reduce the likelihood that  $B$  will profit from his own private benefit appropriation once in control. Higher levels of  $r$ , in turn, reduce the present discounted value to  $B$  of obtaining control over the firm (note that firm value is realized only at the end of the model). Managers faced with increased levels of  $c$  or  $r$  may therefore raise the maximal level of expropriation  $x_{nb}$  to which they commit without incurring a takeover, all other things being equal.

#### 4.4 Equilibrium

Having constructed the equilibria for each proper subgame, we are now in a position to state more precisely the equilibrium of the entire game when side payments are prohibited. This statement is embodied in Proposition 1.

**Proposition 1:** *The unique subgame-perfect equilibrium strategies and payoffs for the game in which side payments between  $M$  and  $B$  are prohibited are as follows:*

- *Commitment Stage: Player  $M$  commits to an upper bound on expropriation at  $\bar{x} = x_{nb} = \frac{2rc^2}{(1-c)^2(1+r-rc)+2c(1+r)}$ , which is strictly increasing both in the required control threshold ( $c$ ) and in the rate of discounting ( $r$ ).*
- *Takeover Stage: Player  $B$  purchases no shares (i.e.,  $\gamma = 0$ ), and thus  $M$  retains control.*

- *Expropriation Stage:* Player  $M$  expropriates the maximal amount private benefits, setting  $x_M = x_{nb}$ ,
- *Realization Stage:* The total realized value of the firm is equal to  $\bar{V} \cdot (1 - x_{nb})$ .
- *The expected payoffs of the parties are:*

Initial SHs	$\int_0^1 \frac{\bar{V} \cdot (1 - x_{nb}) \cdot (1 - \tau)}{1 + r} d\tau = \frac{\bar{V}}{(1 + r)} \cdot \left(\frac{1}{2} - \frac{x_{nb}}{2}\right)$
Player M	$\frac{\mu \bar{V}}{(1 + r)} \cdot x_{nb}$
Player B	0
Aggregate Surplus	$\frac{\bar{V}}{(1 + r)} \left(\frac{1}{2} - x_{nb} \left(\frac{1}{2} - \mu\right)\right)$

Proposition 1 makes clear the deterrent benefits of the market for corporate control. Representing a persistent threat of a takeover for self-serving managers, the block shareholder provides an incentive for the manager to regulate her own ability to appropriate value. Indeed, in the game without side payments, the manager places a voluntary cap on her own ability to appropriate value at  $x_{nb}$  in order to deter entry by  $B$ . In turn, this voluntary constraint works in the interests of existing shareholders, and of economic efficiency.<sup>31</sup>

## 5 Side Payments Permitted

As noted in the introduction, the practice of favoritism towards block shareholders is conventionally viewed with great suspicion, as an overt attempt to undermine the disciplining forces of the takeover market studied above. To evaluate this claim, we switch gears in this section to consider how the predictions of the model might change if the players were allowed to bargain with one another, and the manager could make a side payment to the block shareholder in order to secure her acquiescence and complicity. We find, contrary to received wisdom, that permitting such side payments may (in many plausible circumstances) tend to have a salubrious effect on aggregate corporate welfare.

In order to analyze the effects of side payments, we introduce two additional stages into the existing framework, expanding the game so that it now has six stages, the first four of which take place at  $t_0$ , with the final two occurring at  $t_1$ , after one period elapses.

1. (Commitment Stage) Player  $M$  decides whether to commit to a maximal  $\bar{x}$ , and if she so chooses, she selects some value from  $[0, 1]$ . We again assume that  $\bar{x}$  represents the maximal value of expropriation whether  $M$  or  $B$  controls the firm.
2. (Toehold Stage) Player  $B$  decides whether to amass a toehold in the firm, denoted by  $\delta$ . If  $\delta \geq c$ , player  $B$  assumes control, and the game skips to step (5) below.

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<sup>31</sup>Note once again that value appropriation by the manager will be inefficient only if  $\mu \leq \frac{1}{2}$ , as per our earlier assumption.

3. (Bargaining Stage) Players  $M$  and  $B$  bargain over a possible side payment to  $B$ , which would allow  $M$  to retain control. We suppose that the players bargain over the available surplus according to the Nash (1950) program with respective bargaining powers  $\theta$  for  $M$  and  $(1 - \theta)$  for  $B$ , where  $\theta \in [0, 1]$ .
4. (Takeover Stage) Should no bargain be struck in (3), player  $B$  decides whether to increase his holdings in the firm to  $\gamma \geq \delta$ . If  $\gamma \geq c$ , player  $B$  assumes control, and otherwise player  $M$  remains as manager.
5. (Expropriation Stage) Should  $M$  retain control, she selects a value of  $x_M \in [0, \bar{x}]$ , and pays  $B$  the amount (if any) contracted for earlier. Should  $B$  obtain control, however, he observes the realized value of  $k$ , and then selects his own value of  $x_B \in [0, \bar{x}]$ .
6. (Realization Stage) One period passes, and the firm realizes an expected value of  $(1 - x) \cdot \bar{V}$ , which is subsequently liquidated and distributed to existing shareholders on a pro rata basis.

Note that in addition to the bargaining stage (stage 3), we have also introduced an initial investment stage (stage 2), where  $B$  can build a “toehold” from which to bargain. It is the possibility of this toehold stage that allows  $B$  to extract additional rents, because it allows her to reduce the incremental cost of completing a takeover, thereby becoming a credible bargainer with  $M$ .<sup>32</sup> We once again employ standard backwards induction techniques to characterize the subgame perfect equilibrium. We begin once again with the penultimate expropriation stage, and then work backwards.

## 5.1 Expropriation stage

This stage is virtually identical to its analog in the no bargaining game above. As before, when no takeover has occurred (i.e.,  $\gamma < c$ ),  $M$  maximally expropriates the firm, setting  $x_M = \bar{x}$ , which yields an ex ante payoff of  $\bar{V} \cdot \left(\frac{1}{1+r}\right) \cdot \mu\bar{x}$ . Conversely, if  $B$  captures control of the firm (that is,  $\gamma \geq c$ ), he will receive a payoff of  $[\gamma(1 - x_B) + kx_B]\bar{V}$ , and will set  $x_B = \bar{x}$  if and only if  $k < \gamma$ , and will otherwise set  $x_B = 0$ . Viewed one period earlier (and before the realization of  $k$  is known), the expected payoff for an acquiring player  $B$  is therefore equal to  $\bar{V} \cdot \left(\frac{1}{1+r}\right) \cdot \left(\gamma + \frac{1}{2}\bar{x}(1 - \gamma)^2\right)$ .

## 5.2 Takeover Stage

In light of these expropriation payoffs, we now move back to consider the takeover decision. Thus, suppose that  $M$  has set  $\bar{x}$ , that  $B$  has amassed some

<sup>32</sup>It is theoretically possible to include a toehold stage in the version of the game where side payments are prohibited. Doing so, however, adds nothing to the analysis, since no strategic interactions would separate the toehold from the takeover stage.

$\delta \in [0, c)$  share of the firm, that bargaining has failed, and that  $B$  is now considering increasing her ownership stake to  $\gamma \geq \delta$ .

Although we relegate most of the majority of the technical analysis to the appendix, worth highlighting the particular case where  $B$  executes an exact takeover, increasing his holdings from  $\delta$  to exactly  $\gamma = c$ . The incremental gain to  $B$  (over the status quo) from such a purchase is:

$$\text{Incremental Gain} = \bar{V} \cdot \left( \frac{2c + \bar{x}(1-c)^2}{2(1+r)} - \frac{\delta(1-\bar{x})}{(1+r)} \right) \quad (11)$$

Note that this incremental gain is smaller than the analogous gain of  $\bar{V} \cdot \left( \frac{2c + \bar{x}(1-c)^2}{2(1+r)} \right)$  in the no-bargaining case, where the outside investor did not already possess the value of the toehold shares. However, the incremental cost of engineering a takeover also contains a sunk component. Given that the marginal tenderer is pivotal, equilibrium expectations of firm value are  $E(V) = (1-\bar{x})\bar{V}$  if the pivotal shareholder does not tender (since the tender offer will fail). Hence, the cost of buying control starting from a toehold of  $\delta$  is:

$$T(c, \delta) = \bar{V} \cdot \left( \frac{(1-\bar{x})}{1+r-rc} \right) (c-\delta), \quad (12)$$

which is also decreasing in  $\delta$ . Summing expressions (11) and (12), the *net incremental benefit* to  $B$  of increasing his sub-control share ( $\delta$ ) to a control share ( $c$ ) is equal to:

$$\begin{aligned} \Omega(c, \delta) &= \bar{V} \cdot \left( \frac{2c + \bar{x}(1-c)^2}{2(1+r)} - \frac{\delta(1-\bar{x})}{(1+r)} \right) - \bar{V} \cdot \left( \frac{(1-\bar{x})}{(1+r-rc)} \right) (c-\delta) \\ &= \Phi(c) + \bar{V} \cdot \delta \left( \frac{(1-\bar{x})rc}{(1+r-rc)(1+r)} \right) \end{aligned}$$

Note that  $\Omega(c, \delta)$  is strictly increasing in  $\delta$ , indicating that the sunk cost component of starting from a toehold swamps the sunk benefit component. This observation is significant, because it means the incremental cost of a takeover goes down with a toehold by even more than does the incremental gain. Hence, if a takeover is profitable without any existing toehold, it must always be profitable with one. As such,  $B$  may be able to make his later bargaining position stronger by first obtaining a toehold, whereupon his cost of building the toehold is sunk. (We return to this consideration shortly).

Comparing this payoff to those of other possible actions by  $B$  yield the following Lemma, whose proof can be found in the Appendix.

**Lemma 2.** *Suppose that  $B$  has an installed toehold of  $\delta$  shares, and  $M$  has committed to a maximal level of value expropriation,  $\bar{x}$ . The unique equilibrium of the Takeover-Stage subgame is as follows:*

- If  $\Omega(c, \delta) > 0$ ,  $B$  engineers a takeover by purchasing exactly  $\gamma = c$  shares at price  $p(c) = \bar{V} \cdot \left( \frac{(1-\bar{x})}{1+r-rc} \right)$ . In this case, the parties' continuation payoffs are:

$$(\pi_M, \pi_B) = (0, \Omega(c, \delta)).$$

- If  $\Omega(c, \delta) \leq 0$ ,  $B$  does not engineer a takeover, and instead simply remains at  $\gamma = \delta$ . In this case, the parties' continuation payoffs are:

$$(\pi_M, \pi_B) = \left( \frac{\mu \bar{x} \bar{V}}{1+r}, 0 \right).$$

### 5.3 Bargaining Stage

Inducting backwards one stage further, suppose now that  $B$  has built a toehold of  $\delta$ , but has yet to engineer a takeover attempt, and is allowed to bargain with  $M$  about a standstill agreement. The parties' ability to reach an agreement turns crucially on whether there are any gains from trade available. This will not always be the case, since in some situations  $B$ 's continuation payoff from forging ahead with a takeover is non-positive, and thus he is not a credible bargainer. In other situations,  $B$ 's takeover payoff is so large that it exceeds  $M$ 's own private benefits, and thus  $M$  is not a credible bargainer. Nevertheless when  $B$ 's takeover payoff is positive but still less than  $M$ 's private benefits, bargaining will occur. We consider each possible case, ad seriatim.

Suppose first that  $\Omega(c, \delta) \leq 0$ . Here, the parties continuation payoffs are  $(\pi_M, \pi_B) = \left( \frac{\mu \bar{x} \bar{V}}{1+r}, 0 \right)$ , and  $B$  does not pose a credible threat to appropriate. Consequently, since  $M$  has no reason to fear a subsequent takeover, she will refuse to bargain.<sup>33</sup> In this case, then, the status quo prevails and the parties simply receive the continuation payoffs noted above.

Suppose instead that  $\Omega(c, \delta) > \frac{\mu \bar{x} \bar{V}}{(1+r)}$ , so that  $B$  earns more from a takeover than  $M$  can earn from the status quo. Here as well, there are no gains from trade available since there exists no payment that prevents a takeover and also leaves  $M$  with a nonnegative payoff. Consequently, bargaining fails and the parties receive continuation payoffs associated with a takeover of  $(\pi_M, \pi_B) = (0, \Omega(c, \delta))$ .

Finally, consider the case where  $0 < \Omega(c, \delta) \leq \frac{\mu \bar{x} \bar{V}}{(1+r)}$ , so that in the absence of bargaining a takeover will occur, but  $M$  stands to lose more than  $B$  gains in the process. Here there are gains from trade available with a negotiated outcome in which  $M$  pays  $B$  to stand down from any future takeover attempt. Under the Nash bargaining protocol with bargaining shares  $\theta$  for  $M$  and  $(1 - \theta)$  for  $B$ ,  $M$ 's post-bargaining payoff is equal to:

$$(\theta) \cdot \left( \frac{\mu \bar{x} \bar{V}}{(1+r)} - \Omega(c, \delta) \right)$$

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<sup>33</sup>We assume implicitly, of course, that  $M$  cannot sell his job to  $B$ . Thus, the only mechanism by which  $B$  can gain control is through a tender offer (in which case  $M$  is cut out of the picture entirely).

Analogously,  $B$ 's bargaining payoff (when her share of the surplus is added to her reservation value) is given by:

$$\theta \cdot \Omega(c, \delta) + (1 - \theta) \cdot \frac{\mu \bar{x} \bar{V}}{(1 + r)}$$

Summarizing the bargaining stage, then, the following equilibrium payoffs emerge (as functions of  $\Omega(c, \delta)$ ):

$$(\pi_M, \pi_B) = \begin{cases} \left( \frac{\mu \bar{x} \bar{V}}{(1+r)}, 0 \right) & \Leftrightarrow \Omega(c, \delta) \leq 0 \\ \left( \theta \cdot \left[ \frac{\mu \bar{x} \bar{V}}{(1+r)} - \Omega(c, \delta) \right], \theta \cdot \Omega(c, \delta) + (1 - \theta) \cdot \frac{\mu \bar{x} \bar{V}}{(1+r)} \right) & \Leftrightarrow \Omega(c, \delta) \in \left( 0, \frac{\mu \bar{x} \bar{V}}{(1+r)} \right] \\ (0, \Omega(c, \delta)) & \Leftrightarrow \Omega(c, \delta) > \frac{\mu \bar{x} \bar{V}}{(1+r)} \end{cases}$$

Notice that only when  $\Omega(c, \delta) > \frac{\mu \bar{x} \bar{V}}{(1+r)}$  does a takeover actually occur in equilibrium at this stage. Notice also that as  $\Omega(c, \delta)$  becomes crosses zero,  $B$ 's payoff increases and  $M$ 's decreases discontinuously. Whether  $M$  will allow  $B$  to reach this threshold is therefore an important question.

## 5.4 Toehold Stage

Suppose now that the manager has set  $\bar{x}$ , and  $B$  is deciding about how large of a toehold to purchase. As before, it is useful to consider the same three regions as those explored above:

### 5.4.1 Case 1: $\Omega(c, \delta) \leq 0$ .

Consider first the case where  $\Omega(c, \delta) \leq 0$ , and thus there is no takeover threat. Because  $\Omega$  is increasing in  $\delta$ , for any  $\delta > 0$  it must also be true that  $\Omega(c, 0) = \Phi(c) < 0$ , which will constitute our starting point in determining whether there is any incentive to increase  $\delta$  beyond  $\delta = 0$ .

First, we ask whether it pays for  $B$  to purchase some  $\delta$  size block of shares such that  $\Omega(c, \delta)$  remains non-positive, and thus a takeover is not credible. If  $B$  makes such a purchase, it is clear from the above that bargaining will not occur,  $M$  will retain control, and  $B$ 's continuation payoff will be zero. Thus  $B$ 's sole benefit is the pro rata market value of his purchase (assuming  $M$  retains control), or,  $\left( \frac{(1-\bar{x})\bar{V}}{(1+r)} \right) \cdot \delta$ . This benefit comes at a cost of  $p(\delta) \cdot \delta = \left( \frac{(1-\bar{x})\bar{V}}{1+r(1-\delta)} \right) \cdot \delta$ , so that the net benefit of this non-threatening toehold purchase is equal to:

$$\left( \frac{(1-\bar{x})\bar{V}}{(1+r)} \right) \cdot \delta - \left( \frac{(1-\bar{x})\bar{V}}{1+r(1-\delta)} \right) \cdot \delta = -\delta^2 \left( \frac{r \cdot (1-\bar{x})}{(1+r-r\delta)(1+r)} \right) \cdot \bar{V}$$

Since this expression is negative for all  $\delta > 0$ ,  $B$  strictly prefers inaction to purchasing a non-threatening toehold that falls short of making him a credible bargainer with  $M$ .

**5.4.2 Case 2:**  $\Omega(c, \delta) > \frac{\mu\bar{x}\bar{V}}{(1+r)}$ .

Now suppose that for some  $\delta \geq 0$ , we have  $\Omega(c, \delta) > \frac{\mu\bar{x}\bar{V}}{(1+r)}$ . In this case, there will be no bargaining, since  $B$ 's takeover payoff of  $\Omega$  strictly exceeds  $M$ 's non-takeover payoff. Given that a takeover is inevitable in this region, any toehold that  $B$  wished to build would cost him<sup>34</sup>  $\frac{(1-\bar{x})\bar{V}}{1+r-rc} \cdot \delta$ . As such, the payoff associated with a toehold purchase of  $\delta$  in this region is:

$$\begin{aligned} & \Omega(c, \delta) - T(\delta) + \frac{(1-\bar{x})\bar{V}}{(1+r)} \cdot \delta & (13) \\ = & \bar{V} \cdot \left( \frac{2c^2 + (1-c^2)\bar{x}}{2(1+r)} - \frac{\delta(1-\bar{x})}{1+r} \right) - \left( \frac{(1-\bar{x})\bar{V}}{1+r-rc} \right) \cdot (c-\delta) \\ & - \frac{(1-\bar{x})\bar{V}}{1+r-rc} \cdot \delta + \frac{(1-\bar{x})\bar{V}}{(1+r)} \cdot \delta \\ = & \bar{V} \cdot \left( \frac{2c^2 + (1-c^2)\bar{x}}{2(1+r)} - \frac{c(1-\bar{x})}{1+r(1-c)} \right) = \Phi(c). \end{aligned}$$

Note that this expression does not depend on  $\delta$ . This observation comports well with one's intuition: Once  $\Omega(c, \delta) > \frac{\mu\bar{x}\bar{V}}{(1+r)}$ , a takeover is inevitable. Consequently,  $B$  is indifferent between proceeding with a takeover at the initial stage, or waiting for the second purchase opportunity. We shall assume arbitrarily (but without loss of generality) that when  $B$  is indifferent, he puts it off a takeover as long as possible.<sup>35</sup> Thus, once  $\Omega(c, \delta) > \frac{\mu\bar{x}\bar{V}}{(1+r)}$ , there is no incentive for  $B$  to increase  $\delta$  any further.

**5.4.3 Case 3:**  $\Omega(c, \delta) \in \left(0, \frac{\mu\bar{x}\bar{V}}{(1+r)}\right]$ .

Finally, suppose that for some  $\delta \geq 0$ , we have  $0 < \Omega(c, \delta) \leq \frac{\mu\bar{x}\bar{V}}{(1+r)}$ . In this case, as shown above, bargaining would occur between  $M$  and  $B$ . Our first question is whether  $B$  has an incentive to purchase a sufficient toehold to make himself a credible bargainer—that is, so that  $\Omega(c, \delta)$  is just positive. We then turn to ask whether  $B$  would have any incentive to increase his toehold even further within this region.

Consider first whether there is sufficient incentive for  $B$  to purchase a toehold  $\delta$  sufficient to push him into this region—i.e., just past the point where  $\Omega(c, \delta) = 0$ . So doing would exploit a discontinuity in  $B$ 's payoff function, because  $B$

<sup>34</sup>This computation assumes equilibrium expectations of a later control purchase at  $\gamma = c$ , and pricing for the pivotal shareholder.

<sup>35</sup>On a similar note, Bris (2000) presents evidence from a sample of tender offers in the United States from 1985 to 1998 that only 3.2 percent of the bidders had a prior toehold stake. Bris hypothesizes that assembling a toehold prior to a bid signals to the market the possibility of a takeover and thereby may lead to a run-up in the secondary market price, making the purchase of subsequent shares in the target more expensive for the bidder and decreasing the probability of a successful takeover.

would now become a credible bargainer with  $M$  and can extract a portion of  $M$ 's rents. Defining  $\delta^c$  as the value of  $\delta$  which solves the equation  $\Omega(c, \delta) = 0$ , we obtain the following<sup>36</sup>:

$$\delta^c = \left( \frac{2c^2r - x \left( (1+c^2)(1+r) - (1-c)^2(rc) \right)}{2rc(1-x)} \right) \quad (14)$$

(Note that this value is interior only if  $\bar{x} \leq x_{nb}$ . Indeed, if  $\bar{x} > x_{nb}$ , then  $\Omega(c, \delta) > 0$  for all values of  $\delta$ . Thus, when  $x > x_{nb}$ , player  $B$  need not amass any toehold to have credibility as a bargainer, and  $\delta^c = 0$ ).

Thus, if  $B$  sets  $\delta$  infinitesimally above<sup>37</sup> that specified in (14), then he will ensure himself of an approximate net continuation benefit through bargaining of:

$$\Omega(c, \delta^c) + (1-\theta) \cdot \left( \frac{\mu \bar{x} \bar{V}}{(1+r)} - \Omega(c, \delta^c) \right) = (1-\theta) \mu \cdot \left( \frac{\bar{x} \bar{V}}{(1+r)} \right) > 0$$

At the same time, however,  $B$ 's toehold purchase comes at a cost. Assuming  $B$ 's equilibrium strategy is to purchase just enough shares to become a credible bargainer (i.e., infinitesimally more than  $\delta^c$ ), then it will be common knowledge that no takeover will ever occur, since  $B$  will be simply paid off by  $M$  once he obtains a credible toehold. Thus, as  $B$  is purchasing his toehold, the market's expectation of firm value will remain at  $(1-\bar{x})\bar{V}$ , and thus the price of purchasing  $\delta^c$  is equal to  $p(\delta^c) = \bar{V} \cdot \left( \frac{(1-\bar{x})}{1+r-r\delta^c} \right)$ . Netting out the pro-rata benefit of buying a  $\delta^c$  share of the firm, then, the net cost of purchasing the toehold is:

$$\begin{aligned} T(\delta) - (\text{Pro Rata Ben.}) &= \left( \frac{(1-\bar{x})\bar{V}}{1+r-r\delta^c} \right) \cdot \delta^c - \left( \frac{(1-\bar{x})\bar{V}}{(1+r)} \right) \cdot \delta^c \quad (15) \\ &= (1-\bar{x})\bar{V} \cdot \left( \frac{r \cdot (\delta^c)^2}{(1+r-r\delta^c)(1+r)} \right) > 0 \end{aligned}$$

Let  $x_b$  denote the value of  $\bar{x}$  that equates the benefits and costs of such a purchase, thereby making  $B$  indifferent. Solving for  $x_b$  yields:

$$x_b = 2c^2r \cdot \left( \frac{\lambda_{rc} - \mu(1-\theta)(1+r-rc) \cdot \left( \sqrt{1 + \frac{2(1+c^2)(1+r)}{\mu(1-\theta)(1+r-rc)}} - 1 \right)}{(\lambda_{rc})^2 - 2\mu(1-\theta)rc(1-c)^2(1+r-rc)} \right) \quad (16)$$

<sup>36</sup> Here, even though it is hypothetical (that is, off the equilibrium path), we use the pivotal shareholder ( $\tau = c$ ) to characterize the pricing rule for moving from  $\delta$  to  $c$ .

<sup>37</sup> We say "infinitesimally above" because of the underlying behavioral assumption that when  $B$  is indifferent, he will not engineer a takeover. Thus, to be a credible acquirer,  $B$  must amass slightly above  $\delta^c$ , though we can approximate the equilibrium arbitrarily close by using the exact value of  $\delta^c$ .



where  $\lambda_{rc} \equiv (1 - c)^2 (1 + r - rc) + 2c(1 + r)$ .

Perhaps the most salient question for current purposes concerns the relationship between the  $x_b$  computed above and  $x_{nb}$ , the value that deters a takeover in a world without bargaining. Lemma 3 (whose proof appears in the Appendix) describes this relationship:

**Lemma 3** *The commitment level that deters a takeover when bargaining is prohibited,  $x_{nb}$ , is strictly greater than the unique commitment level that deters the entry of a block shareholder when bargaining is allowed,  $x_b$ .*

The intuition behind this lemma is relatively simple. When bargaining is allowed, an outside investor contemplating an acquisition will consider not only the benefit of engaging in a takeover, but also the possibility of extracting rents from the manager under the threat of a takeover. Because even a modest toehold can render this latter “holdup” threat a credible one,  $B$  can capture rents with only a de minimis investment. Consequently, the manager must work even harder to deter this lower-cost form of entry, choosing  $x_b < x_{nb}$ . It is important to note, however, that Lemma 3 does not predict that  $M$  will *always* choose to deter entry; rather, it states that should she decide to do so, she will commit to a lower ceiling on private benefits than she would if bargaining were prohibited. (We defer until the next subsection the question of whether  $M$  will favor deterrence over acquiescence).

A plot of  $x_b$  as a function of  $c$  appears below for the case of  $r = 1/5$ ,  $\mu = 2/5$ , and  $\theta = 1/2$ . (On the same graph,  $x_{nb}$  is shown for the same parameter values in a gray perforated line).

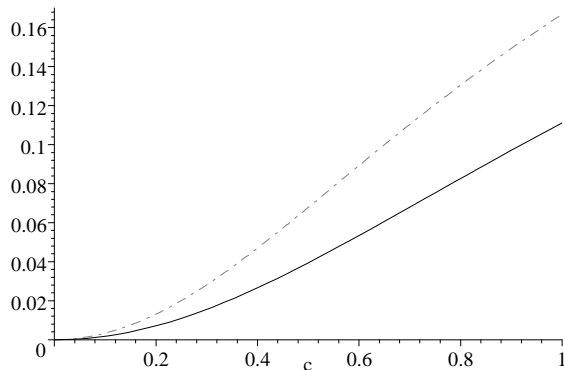


Figure 4

Generalizing to three dimensions (for the case of  $\mu = 2/5$  and  $\theta = 1/2$ ), the plot appears as follows. Also appearing on the graph is  $x_{nb}$  (represented by the higher manifold).

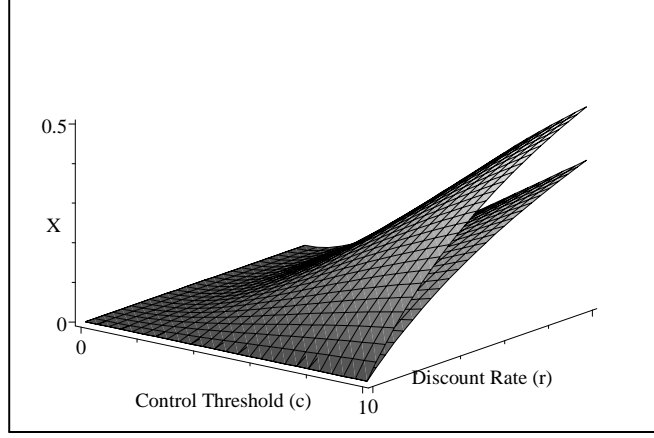


Figure 5

Consider now whether  $B$  has any marginal incentive to increase his sub-control toehold *even further* within this region. On first blush, this might seem like a good strategy for  $B$ , since he can perhaps capture a larger fraction of the surplus by “moving” the lower end of the bargaining range upwards. As it turns out, however, this would never be in  $B$ ’s interests. To see why, consider a value of  $\delta$  that is arbitrarily close to (but infinitesimally larger than)  $\delta^c$ , so that  $\Omega(c, \delta) > 0$ . Recall that the post-bargaining benefits that  $B$  will obtain in this subregion is  $\left(\theta \cdot \Omega(c, \delta) + (1 - \theta) \cdot \frac{\mu \bar{x} \bar{V}}{(1+r)}\right)$ , while the total cost<sup>38</sup> of amassing  $\delta$  shares to begin with is  $\left(\frac{(1-\bar{x})\bar{V}}{1+r-r\delta}\right) \cdot \delta$ . Subtracting the latter from the former yields  $B$ ’s net benefit:

$$\theta \cdot \left[ \Phi(c) + \bar{V} \cdot \delta \left( \frac{(1-\bar{x})rc}{(1+r-rc)(1+r)} \right) \right] + (1-\theta) \cdot \frac{\mu \bar{x} \bar{V}}{(1+r)} - \left( \frac{(1-\bar{x})\bar{V}}{1+r-r\delta} \right) \cdot \delta$$

Differentiating this expression with respect to  $\delta$  yields  $B$ ’s net marginal benefit from building a toehold in this region:

$$\begin{aligned} & \theta \cdot (1-\bar{x})\bar{V} \left( \frac{rc}{(1+r-rc)(1+r)} \right) - (1-\bar{x})\bar{V} \left( \frac{(1+r)}{(1+r-r\delta)^2} \right) \\ &= \frac{(1-\bar{x})\bar{V}}{(1+r)} \cdot \left[ \left( \frac{\theta rc}{(1+r-rc)} \right) - \left( \frac{1+r}{1+r-r\delta} \right)^2 \right] \\ &< 0 \end{aligned}$$

$B$ ’s post-bargaining profit is therefore strictly decreasing in  $\delta$ , and hence it is never optimal for  $B$  any more shares at the toehold stage than is necessary to

<sup>38</sup>This cost is derived assuming that no takeover will occur in equilibrium. Indeed within this region,  $M$  and  $B$  merely bargain to a standstill agreement, and thus no takeover occurs.

be a credible bargainer (that is, infinitesimally more than  $\delta^c$ ). Intuitively, the principal gain to  $B$  from expanding a toehold derives from the increased profit  $B$  would obtain from a takeover launched after  $B$  assembles the toehold. As part of Nash bargaining,  $M$  must compensate  $B$  for the increased takeover profit available once  $B$  owns a toehold. The compensation to  $B$ , however, necessarily reduces the surplus remaining for  $M$  and  $B$  to divide. To the extent  $B$  has some bargaining power,  $B$  will therefore expect a reduced amount of surplus due to his larger toehold. Therefore, for each \$1 that  $B$  expends in increasing his toehold during the toehold stage,  $B$  will expect \$1 more in compensation for  $B$ 's foregone takeover profits as well as  $\$(1 - \theta)$  less in surplus in the negotiation stage, leading  $B$  to expect a net loss.

#### 5.4.4 Summary of Toehold Stage

From the above arguments, it is clear that equilibrium play of the toehold stage hinges crucially on the value of  $\Omega(c, 0) = \Phi(c)$  — i.e., the expected takeover payoff starting from a zero toehold. If, on the one hand,  $\Phi(c) \leq 0$  (a condition that is equivalent to  $\bar{x} \leq x_{nb}$ ), then whether  $B$  builds a toehold turns on a comparison of  $\bar{x}$  to  $x_b$ :

- If  $\bar{x} \leq x_b$ , then  $B$  builds no toehold, never engineers a takeover, and the payoffs of the parties will be:

$$(\pi_M, \pi_B) = \left( \frac{\mu \bar{x} \bar{V}}{(1+r)}, 0 \right)$$

- If  $\bar{x} > x_b$ ,  $B$  will build a toehold of (infinitesimally more than):

$$\delta^c = \left( \frac{2c^2r - x \left( (1+c^2)(1+r) - (1-c)^2(rc) \right)}{2rc(1-x)} \right).$$

$B$  and  $M$  will negotiate a standstill agreement, and the parties' resulting payoffs will be

$$(\pi_M, \pi_B) = \left( \theta \cdot \frac{\mu \bar{x} \bar{V}}{(1+r)}, (1-\theta) \cdot \frac{\mu \bar{x} \bar{V}}{(1+r)} - (\delta^c)^2 \left( \frac{r \cdot (1-\bar{x})}{(1+r-r\delta^c)(1+r)} \right) \cdot \bar{V} \right)$$

If, on the other hand,  $\Phi(c) > \frac{\mu \bar{x} \bar{V}}{(1+r)}$ ,  $B$  will take over the firm with no bargaining, and the parties' payoffs will be

$$(\pi_M, \pi_B) = (0, \Phi(c))$$

Finally, if  $\Omega(c, 0) = \Phi(c) \in \left( 0, \frac{\mu \bar{x} \bar{V}}{(1+r)} \right]$ ,  $B$  will build no additional toehold,  $M$  will retain control, and the parties will bargain to a standstill agreement, such that their respective payoffs are equal to:

$$(\pi_M, \pi_B) = \left( \theta \cdot \left[ \frac{\mu \bar{x} \bar{V}}{(1+r)} - \Phi(c) \right], \theta \cdot \Phi(c) + (1-\theta) \cdot \frac{\mu \bar{x} \bar{V}}{(1+r)} \right).$$

## 5.5 Commitment Stage

With the toehold, bargaining, takeover, expropriation, and realization stages all accounted for, we now consider the commitment stage of the game. Building on the results from the previous subsection, we once again consider three cases. Note that the regions studied above and below turn on the value of  $\Omega(c, 0) \equiv \Phi(c)$ , which recall is defined as:

$$\Phi(c) = \bar{V} \cdot \left( \frac{2c + \bar{x}(1-c)^2}{2(1+r)} - \frac{c(1-\bar{x})}{1+r(1-c)} \right)$$

Significantly, this is itself a function of the maximal level of expropriation,  $\bar{x}$ . Thus, it is necessary to remain mindful throughout about what values of  $\bar{x}$  are required to support the regions analyzed.

### 5.5.1 Case 1: $\Phi(c) \leq 0 \Leftrightarrow \bar{x} \leq x_{nb}$

Consider first the case where  $\Omega(c, 0) = \Phi(c) \leq 0$ , corresponding to a situation where  $\bar{x} \leq x_{nb}$ . Recall that when bargaining is not allowed,  $\Phi(c) \leq 0$  is sufficient to ensure that no takeover would occur. But when side payments are allowed,  $M$  has to worry about the possibility that  $B$  will purchase just large enough of a toehold to make  $\Omega(c, \delta^c)$  positive, thereby affording him with the credibility to hold up  $M$  for a side payment.

Consequently, within this region, the manager has essentially two options. First, she might attempt to set  $\bar{x}$  low enough to circumvent any such toehold entry by  $B$ . As was noted above, this option would entail setting  $\bar{x} = x_b < x_{nb}$ . Should  $M$  choose this route, the parties' respective payoffs will be equal to:

$$(\pi_M, \pi_B) = \left( x_b \cdot \frac{\mu \bar{V}}{(1+r)}, 0 \right)$$

Alternatively, of course,  $M$  can simply accommodate entry and split the profits with  $B$ . If she opts for this route, then  $B$  will purchase infinitesimally more than  $\delta^c$ , so that his takeover threat remains credible. When this happens,  $B$  will be able to capture a  $(1-\theta)$  share of  $M$ 's rents, leaving  $M$  with a corresponding payoff of  $\theta \cdot \bar{x} \cdot \frac{\mu \bar{V}}{(1+r)}$ . Clearly, then, if the manager chooses accommodation, she will want to set  $\bar{x}$  as high as possible within this region, at  $\bar{x} = x_{nb}$ , so as to maximize her post-bargaining rents at:

$$\theta \cdot x_{nb} \cdot \frac{\mu \bar{V}}{(1+r)}.$$

Comparing the two payoffs above, the optimal choice of  $\bar{x}$  within this region is as follows:

$$\bar{x} = \begin{cases} x_b & \text{if } x_b \geq \theta \cdot x_{nb} \\ x_{nb} & \text{else} \end{cases}$$

**5.5.2 Case 2:**  $\Phi(c) > \frac{\mu\bar{x}\bar{V}}{(1+r)} \Leftrightarrow \bar{x} > \hat{x} > x_{nb}$

Now consider the case where  $\Phi(c) > \frac{\mu\bar{x}\bar{V}}{(1+r)}$ , a contingency that is equivalent to:

$$\bar{x} > \hat{x} = \frac{2rc^2}{(1-c)^2(1+r-rc) + 2c(1+r) - 2\mu(1+r-rc)} > x_{nb} \quad (17)$$

Note first that  $M$  would never rationally choose to set  $\bar{x} > \hat{x}$ .<sup>39</sup> Indeed, so doing would simply induce  $B$  to take over the firm without bargaining, thereby leaving  $M$  with a zero payoff. In such a situation, the players' respective payoffs would be:

$$(\pi_M, \pi_B) = (0, \Phi(c))$$

Because she receives a zero payoff everywhere in this subregion,  $M$  is indifferent about where she sets  $\bar{x} \in (\hat{x}, 1]$ . We will assume in such a situation, however, that if  $M$  is forced to select an  $\bar{x}$  in this region, her personal indifference leads her to do the best thing for shareholders, and  $M$  will thus set  $\bar{x}$  arbitrarily close to the lower boundary of this region,  $\hat{x}$ .

Clearly, however, this subregion is never an equilibrium, since  $M$  always can always select a smaller  $\bar{x}$  and earn a strictly positive payoff. In particular, if possible,  $M$  would like to avoid this region by setting  $\bar{x}$  such that either  $\Phi(c) \leq \frac{\mu\bar{x}\bar{V}}{(1+r)}$  or, even better,  $\Phi(c) \leq 0$ . We return to this question at the summary of this stage, below.

**5.5.3 Case 3:**  $\Phi(c) \in \left(0, \frac{\mu\bar{x}\bar{V}}{(1+r)}\right] \Leftrightarrow \bar{x} \in (x_{nb}, \hat{x}]$

Finally, consider the case where  $0 < \Phi(c) \leq \frac{\mu\bar{x}\bar{V}}{(1+r)}$ , and thus  $M$  has set  $\bar{x} \in (x_{nb}, \hat{x}]$ . Here, bargaining occurs, after  $B$  builds only a minimal toehold, and  $M$  pays  $B$  not to take over the firm. As noted above, the payoffs for the players will be:

$$(\pi_M, \pi_B) = \left( \theta \cdot \left[ \frac{\mu\bar{x}\bar{V}}{(1+r)} - \Phi(c) \right], \theta \cdot \Phi(c) + (1-\theta) \cdot \frac{\mu\bar{x}\bar{V}}{(1+r)} \right)$$

hence, in this region,  $M$  would like to set  $\bar{x}$  in such a way that maximizes  $\theta \cdot \left[ \frac{\mu\bar{x}\bar{V}}{(1+r)} - \Phi(c) \right]$ . Recalling once again that:

$$\Phi(c) = \bar{V} \cdot \left( \frac{2c + \bar{x}(1-c)^2}{2(1+r)} - \frac{c(1-\bar{x})}{1+r(1-c)} \right),$$

$M$  will choose  $\bar{x}$  so as to maximize the non-negative quantity:

$$\theta \cdot \left[ \frac{rc^2}{(1+r-rc)(1+r)} + \bar{x} \left( \frac{2\mu(1+r-rc) - (1-c)^2(1+r-rc) - 2c(1+r)}{2(1+r)(1+r-rc)} \right) \right],$$

<sup>39</sup> Although  $\mu \leq \frac{1}{2}$  ensures that  $\hat{x} > 0$ , nothing ensures that  $\hat{x}$  is less than one. When it is greater than one, this region simply fails to exist and need not be analyzed.

which is decreasing strictly in  $\bar{x}$ , since  $\mu \leq \frac{1}{2}$ . Thus,  $M$  would choose the smallest possible value of  $\bar{x}$  in this region, at  $\bar{x} = x_{nb}$ .<sup>40</sup>

Intuitively, when  $M$  commits to a higher level of private benefits  $\bar{x} > x_{nb}$ , she experiences two countervailing effects. First,  $M$  can expect a larger surplus from forming a coalition with  $B$  (of which  $M$  will obtain a  $\theta$  share). Because of the inefficiency of expropriating private benefits, however,  $M$  will only enjoy a  $\mu$  fraction of her  $\theta$  share of the increased surplus. Second, the increased level of private benefit expropriation will raise the potential takeover profit to  $B$  in the negotiation stage. To the extent  $B$  expects to benefit pro rata with other shareholders from the takeover and not through  $B$ 's own private benefit expropriation,  $B$  faces no inefficiency from appropriating private benefits. Even where  $B$  expects to gain through the expropriation of his own private benefits, because  $B$  takes into account the cost to the shares in  $B$ 's control block from the expropriation,  $B$  will only take private benefits only where  $B$ 's efficiency is relatively high (e.g.,  $k > c$ ). During the negotiation stage, managers seeking to form a coalition must then compensate  $B$  for her foregone takeover profits due to the higher level of private benefits  $\bar{x} > x_{nb}$ . The higher the level of inefficiency in private benefit expropriation on the part of  $M$  (indicated through a lower value of  $\mu$ ), the more likely the need to increase compensation to  $B$  outweighs the gain to managers from the elevated private benefits.<sup>41</sup>

## 5.6 Equilibrium

Collecting all of the above analysis, we are now in a position to describe with precision the equilibrium of the game when side payments are allowed. This description is contained in Proposition 2, below.

**Proposition 2:** *The subgame perfect equilibrium strategies and payoffs for the game in which side payments between  $M$  and  $B$  are allowed are characterized as follows:*

- *Commitment Stage:* If  $\theta \leq \frac{x_b}{x_{nb}}$ , then Player  $M$  sets  $\bar{x} = x_b$ . If  $\theta > \frac{x_b}{x_{nb}}$ , then  $M$  sets  $\bar{x} = x_{nb}$ .
- *Toehold Stage:* If  $\theta \leq \frac{x_b}{x_{nb}}$ , Player  $B$  assembles neither a toehold nor a controlling share (i.e.,  $\gamma = \delta = 0$ ). If  $\theta > \frac{x_b}{x_{nb}}$ , Player  $B$  assembles a minimal toehold of  $\delta^c$ .
- *Bargaining Stage:* If  $\theta \leq \frac{x_b}{x_{nb}}$ , no bargaining occurs. If  $\theta > \frac{x_b}{x_{nb}}$ ,  $M$  and  $B$  then reach a standstill agreement under which  $B$  receives a

<sup>40</sup>Although this interval has an open lower support, the continuity of  $M$ 's payoff at  $x_{nb}$  implies that the value maximizing choice is  $\bar{x}$  is at the lower limit of this region.

<sup>41</sup>A side note/caveat: If we were to relax the assumption that  $\mu \leq \frac{1}{2}$ , and supposed instead that  $\mu > \frac{1}{2}$ , then the above derivative could go the other way, in which case it would be optimal for  $M$  to maximally expropriate the firm at  $\bar{x} = 1$ . Indeed, when the coefficient on  $\bar{x}$  above is positive, it is also the case that  $\hat{x} = 1$ . Notice, however, that it makes sense from an efficiency standpoint to exclude this possibility, since it would then be efficient for  $M$  to maximally expropriate the firm (once you account for after tax payoffs). Only when  $\mu \leq \frac{1}{2}$  is value diversion inefficient.

payment of  $(1 - \theta) \cdot \left(\frac{\mu\bar{V}}{(1+r)}\right) x_{nb}$  to refrain from additional acquisitions.

- *Takeover Stage:* Player B purchases no additional shares, and thus M retains control.
- *Expropriation Stage:* Player M expropriates the maximal amount private benefits, setting  $x_M = x_b$  if  $\theta \leq \frac{x_b}{x_{nb}}$  and  $x_M = x_{nb}$  if  $\theta > \frac{x_b}{x_{nb}}$ .
- *Realization Stage:* The total realized value of the firm is equal to  $\bar{V} \cdot (1 - x_b)$  if  $\theta \leq \frac{x_b}{x_{nb}}$  and  $\bar{V} \cdot (1 - x_{nb})$  if  $\theta > \frac{x_b}{x_{nb}}$ .
- *The expected payoffs of the parties are:*

	$\theta \leq \frac{x_b}{x_{nb}}$	$\theta > \frac{x_b}{x_{nb}}$
Initial SHs	$\frac{\bar{V}}{(1+r)} \cdot \left(\frac{1}{2} - \frac{x_b}{2}\right)$	$\frac{\bar{V}}{(1+r)} \cdot \left(\frac{1}{2} - \frac{x_{nb}}{2}\right)$
Player M	$\frac{\mu\bar{V}}{(1+r)} \cdot x_b$	$\theta \cdot \left(\frac{\mu\bar{V}}{(1+r)}\right) x_{nb}$
Player B	0	$(1 - \theta) \cdot \left(\frac{\mu\bar{V}}{(1+r)}\right) x_{nb}$
Aggregate Surplus	$\frac{\bar{V}}{(1+r)} \left(\frac{1}{2} - x_b \left(\frac{1}{2} - \mu\right)\right)$	$\frac{\bar{V}}{(1+r)} \left(\frac{1}{2} - x_{nb} \left(\frac{1}{2} - \mu\right)\right)$

## 5.7 Observations and Discussion

Comparing Propositions 1 and 2, it appears far from clear that the existence of side payments hurts non-block shareholders or is inefficient. Indeed, as Proposition 2 illustrates, so long as the block shareholder is a sufficiently powerful bargainer against the manager (that is,  $\theta$  is sufficiently small), the manager will choose to commit to a value of  $\bar{x} = x_b$  that makes shareholders strictly better off than they would be when side payments are disallowed. Moreover, even in situation where  $\theta$  is large, collusion between the manager and the block shareholder may do no worse than replicate the outcome that obtains when bargaining is disallowed ( $\bar{x} = x_{nb}$ ). This intuition directly generates the following:

**Corollary 2.1** *When  $\theta \leq \frac{x_b}{x_{nb}}$ , both non-block shareholders and aggregate firm value are largest if side payments are permitted. Conversely, when  $\theta > \frac{x_b}{x_{nb}}$ , non-block shareholders and aggregate firm value are the same regardless of whether side payments are permitted.*

Of course, the determination of whether side payments confer laudable efficiency benefits in practice may well come down to an empirical assessment of bargaining power. Although such bargaining parameters are not easy to measure in practice, we conjecture that there may be a few indirect means of measuring the parties' relative bargaining power. For example, in industries where managers suffer significant reputational losses after a hostile takeover, the block shareholder is likely to possess a large amount of bargaining power. Similarly, the existence of well-capitalized block shareholders (such as institutional investors) is another hallmark of significant bargaining power. Or alternatively, the empirical frequency with side payments occur in the absence of regulation

might be a good measure of the bargaining power block shareholders possess on average.

Complicating the analysis is the fact that the “beneficial” effects of side payments in Proposition 2 *only occur off of the equilibrium path*. Indeed, note from the proposition that the spectre of side payments induces the manager to select a modest level of private benefits, which deters the emergence of a block shareholder completely. Opportunistic outside investors seeking to hold up managers act as an ever present threat hanging over the heads of managers in addition to the possibility of a takeover, much like the sword of Damocles. Conversely, the actual execution of side payments occurs *on the equilibrium path*, and each time a side payment is successfully made, it represents a loss to shareholders. Consequently, it would be a mistake for scholars to attempt to diagnose the overall effects of side payments by examining only those cases where side payments are made. Indeed, our model predicts that in such instances, the side payment does not make shareholders better off. However, our principal argument is that off-equilibrium benefits of such transactions may well swamp the equilibrium costs of them. Thus, in order to conduct a balanced and more coherent analysis, scholars should keep in mind that observed bad effects of such collusive transactions present them with a biased diagnostic sample.

## 6 Caveats and Extensions

Several caveats and extensions are possible to the analysis presented in this paper. Without presenting a formal analysis, the section provides a brief discussion of them.

- *Commitment to a maximal level of private benefits not binding on B.*

Our model assumes that the manager may pre-commit to a maximal level of private benefits that binds not only the manager but also the outside investor that takes control of the corporation. Some forms of commitments, however, may dissipate after a change in control. For example, to the extent the manager relies on her own personal reputation as a commitment not to expropriate private benefits, a change in control that replaces the manager will also remove the commitment. Likewise, some firms may use the composition of the board of directors to commit to a maximal level of private benefits, employing outside directors with reputations for monitoring managers. With a change in control, the outside investor often will have the ability to put in place its own slate of directors.<sup>42</sup>

Where the manager adopts a commitment device that does not bind successors, our analysis is changed along at least two dimensions. First, the outside investor faces a potentially greater return from engaging in a takeover. The paper’s analysis indicates that part of the return an outside investor expects

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<sup>42</sup>Note, nevertheless, that a staggered board structure reduces the ability of an outside investor to change the composition of the board of directors immediately. Instead, the outside investor may have to wait years before obtaining majority control over the board.



to receive from assembling a control block depends on the possibility that the outside investor may extract his own high levels of private benefits. Where the manager's commitment to  $\bar{x}$  also binds the outside investor, this commitment reduces the expected return from extracting private benefits to the outside investor that obtains control. In contrast, when  $\bar{x}$  is not binding, the outside investor may obtain the full expected value from private benefit expropriation when he obtains control. Facing a higher risk of a takeover, all other things being equal, the manager will then face greater pressure to commit to a reduced level of private benefits to the benefit of aggregate corporate welfare. So doing can increase the price that existing shareholders would demand before they sell out.

Second, however, a countervailing effect may occur (at least in the extreme). As noted above, a non-binding value of  $\bar{x}$  dramatically increases the block shareholder's return to an acquisition. It may simply be impossible for the manager to deter a takeover, even by severely constraining herself. In such a case, the manager can do nothing to prevent an acquisition of the firm. Thus, allowing a commitment level that is not binding on successors has ambiguous effects on our results.

- *Timing of the commitment to a maximal level of private benefits*

In the paper's formal model, the manager makes the initial decision to commit to a fixed maximal level of private benefits  $\bar{x}$ . Significantly, this decision is made without any ability on the part of the manager to negotiate with the outside investor. Because managers must commit to  $\bar{x}$  prior to negotiations with a block shareholder, managers are put in a position of negotiating weakness. Once a toehold block appears, for example, the toehold's threat to the manager consists of undertaking a takeover. To the extent credible, the threat leaves managers removed from control with zero rents. Zero rents, therefore, represents the bottom end of the negotiating range for managers seeking to form a coalition with a toehold block, exposing much of the expropriated private benefits for division with the toehold block.

A completely different negotiation game occurs where the manager is able to negotiate with an outside investor prior to setting the maximal level of private benefits  $\bar{x}$ . The manager that negotiates prior to setting the maximal level of private benefits may threaten to set  $\bar{x}$  such that the outside investor will not find it profitable to form a toehold block at all ( $x_b$  from the paper's analysis above). Even without an agreement, the lowest level of rents managers will receive then is  $x_b$  and not zero, establishing a higher lower bound for the manager's bargaining range with the block shareholder. The manager that forms a coalition with the block shareholder then gains a greater share of the private benefits expropriated and correspondingly is more likely to favor such coalitions as opposed to deterring the formation of toehold blocks altogether.

Despite the possibility of the manager engaging in negotiations with the outside investor prior to making a pre-commitment to  $\bar{x}$ , for many companies such negotiations are infeasible. Many of the commitment devices are possible to implement at only certain specified times and moreover may not be instantaneous.

Directors, for example, are elected at only certain times of the year. Taking on more debt, likewise, can take time to put into effect. To negotiate with an outside investor prior to making a commitment, therefore, managers must know the identity of a potential acquirer before the acquirer assembles a toehold.<sup>43</sup> Where shareholders are dispersed, however, the manager will face large costs in identifying outside investors that credibly have the ability and desire to assemble a block. Talk is cheap. Any dispersed shareholder may voice an intention to assemble a block to receive a share of the manager's private benefits of control. The manager, therefore, may necessarily have to wait for a block actually to form before commencing negotiations and therefore must make a commitment to a particular level of private benefits prior to negotiations.

Where pre-existing blocks are present in the market, managers in fact may have the ability to negotiate prior to committing to a maximal level of private benefits. Policy makers, therefore, may wish to reduce prohibitions on opportunistic side payments to shareholders disproportionately for firms that lack the presence of significant pre-existing blocks. As discussed below, nevertheless, even where a pre-existing block is present, managers face the threat of other, unknown outside investors forming new toehold blocks. So long as the pre-existing block is not too large, the paper's analysis on the beneficial ex ante value of allowing opportunistic side payments to shareholders remains valid.

- *Multiple potential block shareholders.*

The paper's formal model assumes the existence of only one outside investor with the capability of assembling a block of shares. For any given company, nevertheless, multiple outside investors with the capability of assembling a significant block of shares may exist. A firm's manager that seeks to form a coalition with one toehold block, therefore, continues to face the risk that other outside investors may form another toehold.

Where the supply curve for shares is upward sloping, a coalition between the manager and the first toehold block will raise the cost of to a second outside investor of forming another toehold. Because the second toehold will also face a higher cost of completing a full control block due to upward sloping supply curve, the credibility of the second toehold's threat to engage in a takeover is also reduced, all other things being equal. Nevertheless, in many situations, the threat of a takeover on the part of a second toehold, may still be credible and thereby force managers to enter into a coalition with the second toehold.

Where bargaining power is equal among all market participants, the presence of a second toehold in the manager's coalition forces managers to share the surplus from private benefit expropriation three ways (among the manager, the first toehold, and the second toehold). Managers facing the need to share a

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<sup>43</sup>The manager, of course, may simply choose not to commit to any maximal level of private benefits prior to the formation of a toehold block. However, because implementing a commitment device takes time, managers will lack the ability to implement a new commitment upon the entry of a block. The outside investor will therefore view a manager that fails to commit as equivalent to a manager that commits to the highest possible maximal level of private benefit expropriation (or  $\bar{x} = 1$  in the model).

larger fraction of their private benefit surplus, therefore, are likely to favor even more actions that deter the entry of any block of shares through the commitment to the  $x_b$  maximal level of private benefits.

## 7 Conclusion

Many view with skepticism the managerial practice of paying patronage to shareholders in an effort to obviate an acquisition. Received wisdom holds that such payments work to decrease the risk managers face from either a takeover or a proxy contest, exacerbating agency costs by allowing them to expropriate higher levels of private benefits, reducing the aggregate welfare of all corporate participants. While other areas of law (such as fiduciary duties) also constrain the ability of managers to extract benefits, litigation is costly to invoke and likely snares only the most easily identifiable transactions where managers extract a large amount of value. As such, shareholder patronage is thought to render mute one of the most effective market forces of managerial discipline. Courts and regulators have therefore responded with a series of prohibitions on managerial favoritism. Managers, for example, may not declare a non pro rata dividend to shareholders of the same class.<sup>44</sup> Managers are also prohibited from engaging in selective disclosures of non-public material information to favored shareholders.<sup>45</sup> Other examples abound.<sup>46</sup>

Received wisdom, however, is not always right. This paper has demonstrated that allowing managers the ability to make even opportunistic payments to curry the favor of large block shareholders may improve on aggregate corporate welfare under plausible circumstances. Managers that form a coalition with a large block shareholder may be able to expropriate significant value; however, in doing so the manager must often share this benefit with the block shareholder. Anticipating the size of the transfer payment needed to maintain the coalition, a manager may prefer instead to commit to a reduced maximal level of private benefits to deter the formation of blocks of shares altogether.

A significant virtue of the benefit that we have highlighted is that it is ongoing and persistent as the corporation evolves over time. Consequently, the incentives providing by permitting favoritism do not require that corporate promoters and entrepreneurs anticipate these incentive problems ex ante and draft inflexible governance schemes or compensation plans to avoid them. Moreover, the incentive structure we have highlighted remains even after a firm goes public and shareholders are widely dispersed.

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<sup>44</sup>See Del. Code Ann. tit. 8, § 170 (1998); Revised Model Bus. Corp. Act § 6.40. See 11 Fletcher Cyclopedia of Private Corp. § 5352 (perm. ed. rev. vol. 1995); Edward P. Welch & Andrew J. Turezyn, *Folk on Delaware Corporation Law: Fundamentals* § 170.2, at 340 - 41 (Little Brown ed. 1993).

<sup>45</sup>In 2000, the SEC promulgated new rules under Regulation FD that greatly restricted the ability of public corporations to make disclosures of non-public material information selectively to favored analysts and shareholders. See Securities and Exchange Commission, *Selective Disclosure and Insider Trading*, Rel. No. 34-43154, Aug. 15, 2000, 2000 WL 1239722 (S.E.C.).

<sup>46</sup>See Choi & Talley (2001) for a more complete description.

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## 9 Appendix

This appendix contains proofs for some of the results stated in the text.

**Lemma 2.** *Suppose that B has an installed toehold of  $\delta$  shares, and M has committed to a maximal level of value expropriation,  $\bar{x}$ . The unique equilibrium of the Takeover-Stage subgame is as follows:*

- *If  $\Omega(c, \delta) > 0$ , B engineers a takeover by purchasing exactly  $\gamma = c$  shares at price  $p(c) = \bar{V} \cdot \left(\frac{1-\bar{x}}{1+r-rc}\right)$ . In this case, the parties' continuation payoffs are:*

$$(\pi_M, \pi_B) = (0, \Omega(c, \delta)).$$

- If  $\Omega(c, \delta) \leq 0$ ,  $B$  does not engineer a takeover, and instead simply remains at  $\gamma = \delta$ . In this case, the parties' continuation payoffs are:

$$(\pi_M, \pi_B) = \left( \frac{\mu \bar{x} \bar{V}}{1+r}, 0 \right).$$

**Proof:** As in the no-bargaining analysis, consider three cases, corresponding to sub-control, exact-control, and super-control block purchases by  $B$ , respectively. Consider them ad seriatim.

**Case 1:**  $\gamma \in [\delta, c)$ :

Consider first whether  $B$  would have an incentive to purchase short of a control stake (i.e.,  $\delta \leq \gamma < c$ ). Since a takeover would never occur here,  $B$ 's benefit from such a purchase consists solely of his increased pro rata claim on the firm, or  $(\gamma - \delta) \cdot \frac{(1-\bar{x})\bar{V}}{1+r}$ . On the other hand, the cost of this purchase would be  $T(\gamma, \delta) = (\gamma - \delta) \cdot \left( \frac{(1-\bar{x})\bar{V}}{1+r(1-\gamma)} \right)$ . The difference between these expressions represents the net benefit of a sub-control purchase:

$$\begin{aligned} \Omega(\gamma, \delta | \delta \leq \gamma < c) &= (\gamma - \delta) \cdot \frac{(1-\bar{x})\bar{V}}{(1+r)} - (\gamma - \delta) \cdot \left( \frac{(1-\bar{x})\bar{V}}{1+r-r\gamma} \right) \\ &= -(\gamma - \delta) \cdot \left( \frac{\gamma r (1-\bar{x})}{(1+r)(1+r-r\gamma)} \right) \cdot \bar{V} \end{aligned}$$

Note that this expression is negative for any  $\gamma > \delta$ , and thus  $B$  would strictly prefer purchasing no additional shares ( $\gamma = \delta$ ) over purchasing additional shares to a level that falls short of taking control.

**Case 2:**  $\gamma = c$

This case is discussed at length in the text.  $B$ 's incremental gain from an exact takeover is

$$\Omega(c, \delta) = \Phi(c) + \bar{V} \cdot \delta \left( \frac{(1-\bar{x})rc}{(1+r-rc)(1+r)} \right)$$

If this expression is strictly positive,  $B$  would prefer an exact takeover to inaction.

**Case 3:**  $\gamma > c$ .

Finally, consider a super-control share, where  $\gamma > c$ . Like in the no-bargaining case, there is an upward discontinuity in price when purchasing  $\gamma > c$  shares, since the marginal shareholder is no longer the pivotal one. As such, the price to acquire a super-controlling interest is:

$$p(\gamma) = \bar{V} \cdot \left( \frac{\gamma + (1-\gamma) \cdot (1-\bar{x})}{1+r-r\gamma} \right) > \bar{V} \cdot \left( \frac{(1-\bar{x})}{1+r-rc} \right) = p(c)$$

Using this pricing correspondence,  $B$ 's net payoff becomes:

$$\Omega(\gamma, \delta) = \bar{V} \cdot \left( \frac{2\gamma + (1-\gamma)^2 \bar{x}}{2(1+r)} - \frac{\delta(1-\bar{x})}{1+r} \right) - \bar{V} \cdot \left( \frac{\gamma + (1-\gamma) \cdot (1-\bar{x})}{1+r-r\gamma} \right) (\gamma - \delta) \quad (18)$$

Differentiating  $\Omega(\gamma, \delta)$  with respect to  $\gamma$  yields:

$$\frac{d}{d\gamma}\Omega(\gamma, \delta) = -\bar{V} \cdot \left( \frac{\gamma r^2 (1 - \gamma) (1 - x + \gamma x) + \gamma^2 x r + r (\gamma - \delta x) + (r + r^2 + x) (\gamma - \delta)}{(1 + r - \gamma r)^2 (1 + r)} \right) < 0,$$

and consequently,  $B$  would never rationally raise his purchase above  $\gamma = c$ .  $B$  would therefore constrain his actions to choosing between  $\gamma = \delta$  and  $\gamma = c$ , choosing the latter if and only if  $\Omega(\gamma, \delta) > 0$ . ■

**Lemma 3** *The commitment level that deters a takeover when bargaining is prohibited,  $x_{nb}$ , is strictly greater than the unique commitment level that deters the entry of a block shareholder when bargaining is allowed,  $x_b$ .*

**Proof:** From the expressions in the text, the net benefit associated with purchasing a block of (slightly above)  $\delta^c$  is given by:

$$\begin{aligned} & (1 - \theta) \mu \cdot \left( \frac{\bar{x} \bar{V}}{(1 + r)} \right) - (1 - \bar{x}) \bar{V} \cdot \left( \frac{r \cdot (\delta^c)^2}{(1 + r - r\delta^c)(1 + r)} \right) \quad (19) \\ & = (1 - \theta) \mu \cdot \bar{x} - r (\delta^c)^2 \cdot \left( \frac{(1 - \bar{x})}{(1 + r - r\delta^c)} \right) \end{aligned}$$

where  $\delta^c = \text{Max} \left\{ 0, \left( \frac{2c^2 r - x((1+c^2)(1+r) - (1-c)^2(rc))}{2rc(1-x)} \right) \right\}$ . If  $\bar{x} > x_{nb}$ , we know that  $\delta^c = 0$ , and (19) simplifies to  $(1 - \theta) \mu \cdot \bar{x} > 0$ , so there cannot be any roots of (19) that are larger than  $x_{nb}$ . Conversely, if  $\bar{x} = 0$ , the expression in (19) becomes  $-r (\delta^c)^2 \cdot \left( \frac{1}{(1+r-r\delta^c)} \right) < 0$ , and thus  $B$  will never purchase a toehold. Since (19) is continuous for all  $\bar{x} \in [0, 1]$ , it must have at least one root on  $(0, x_{nb})$ . Solving (19) for  $\bar{x}$  yields two roots,  $x^{high}$  and  $x^{low}$ , which correspond (respectively) to the (+) and (-) manifestations of the following:

$$2c^2 r \left( \frac{\lambda_{rc} + (1 - \theta) \mu (1 + r - rc) \left( 1 \pm \sqrt{1 + \frac{2(1+c^2)(1+r)}{(1-\theta)\mu(1+r-rc)}} \right)}{(\lambda_{rc})^2 - (1 - \theta) \mu [2rc(1 - c)^2 (1 + r - rc)]} \right)$$

It is easily confirmed that  $x^{high} > x_{nb}$ , and thus from the above reasoning, this root can be excluded. The lower root ( $x^{low}$ ) is strictly less than  $x_{nb}$ , and is the solution reported in the text. Because it is the only root of (19) lying in  $[0, x_b]$ , it is therefore the unique value of  $\bar{x}$  that deters a toehold from being formed. It is easily confirmed that  $x_b \equiv x^{low} < x_{nb}$ . ■