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Rivalrous Benefit Taxation:
The Independent Viability
of Separate Agencies or Firms

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Abstract

We ask when firms with increasing returns can cover their costs independently by charging two-part tariffs (TPTs), a condition we call *independent viability*. To answer, we develop notions of substitutability and complementarity that account for the total value of goods and use them to find the maximum extractable surplus. We then show that independent viability is a sufficient condition for existence of a general equilibrium in which regulated natural monopolies use TPTs. Independent viability also guarantees efficiency when the increasing returns arise solely from fixed costs. For arbitrary technologies, it ensures that a Second Welfare Theorem holds.

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1. Introduction

Increasing returns abound. Examples include roads and bridges, electricity and telephone services, and much of manufacturing. However, the provision of such goods is left unaddressed by the competitive paradigm of an Arrow-Debreu economy.

Accordingly, noneconomists may puzzle at the vitality of the Arrow-Debreu model. This vitality no doubt lies in its tractability: The convexity of production, budget, and upper-contour sets makes demand and supply correspondences convex-valued and upper-hemi continuous, so that marginal-cost prices exist that support a general equilibrium; the efficiency of such an equilibrium is guaranteed; and the economist can justify separating efficiency from equity considerations, because a Second Welfare Theorem holds.

Increasing returns imply that production sets are nonconvex; even when the firm's problem is well defined, since supply correspondences are not generally convex valued, competitive equilibria may not exist. This is unfortunate since we know that competitive equilibria are efficient, and there is no such guarantee for other obvious non-competitive equilibrium concepts. For instance, pricing at marginal cost as in marginal-cost-pricing equilibria—although necessary for optimality—no longer ensures optimality with increasing returns. For these reasons, the question of how increasing-returns firms should price and who should run them is of long standing.

Hotelling [23] and Lerner [25] argued that these natural monopolies should be regulated to price at marginal cost. However, such pricing creates two problems that Coase [12] connected: it causes losses when average costs are decreasing, and it allows inefficiencies. Covering these losses—or put differently, giving a reasonable return to capital—is a central problem in regulation, and in our paper. As we will explain, in some circumstances, covering these losses in the marketplace as Coase [12] advocated is sufficient to guarantee efficiency.

Hotelling argued that the regulator should tax to cover these losses, but he met two compelling objections in the "marginal cost pricing controversy." First, Coase [12], Ruggles [32], and Wald [35] argued that lump-sum taxes are fanciful: real taxes cause inefficiencies that may be at least as large as those from pricing above marginal cost. Second, Coase went on to argue that even if lump-sum taxes were feasible, marginal-cost pricing would not ensure efficiency. For, while such pricing gives households appropriate

trade-offs at the margin between the monopoly good and other goods, households never need to consider the full cost of producing the good; therefore, nothing ensures that the product's value exceeds its cost. This . Coasian source of inefficiency is an early glimpse of the individual-rationality or willingness-to-pay problem that later appeared in Brown and Heal [8].

Unfortunately, these objections to Hotelling's solution are not addressed in the recent mathematical literature on the existence of marginal-cost-pricing equilibria by Mantel [27], Beato and Mas-Colell [2], Brown, Heal, Kahn, and Vohra [6], Bonnisseau and Cornet [3], [5], and on nonoptimality by Guesnerie [21] and Brown and Heal [8]. Much of the literature attributes losses to full-liability shareholdings, often with a fixed structure of revenues. Even when the revenue structure is variable, as in Bonnisseau and Cornet [5], individual rationality is not considered. Since these papers present equilibrium concepts that do not account for Coase's individual-rationality objection, inefficiencies can arise.[1]

These considerations motivate our paper, which attends to individual rationality by developing conditions such that the costs of two monopoly firms can be covered through nonlinear pricing.[2] We extend the model of Brown, Heller, and Starr [7], in which a single regulated monopolist produces a good under increasing returns and covers its costs by charging households individualized two-part tariffs. In order to consume a monopoly firm's good at all, households must pay one tariff, a hookup; added to this, the household must pay a second tariff proportional to consumption. (For an often cited partial equilibrium study of such two-part tariffs, see Oi [29].) Notable general equilibrium papers that use two-part tariffs or other nonlinear pricing to address individual-rationality concerns are Vohra [33,34], Kamiya [24], Clay [11], and Moriguchi [28].

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[1]We will see that at least in fixed-cost models, satisfying Coase's objection is sufficient to guarantee efficiency. Unfortunately, with other technologies, satisfying Coase's objection is neither necessary nor sufficient for efficiency.

[2]We do so for situations in which reselling entails prohibitively large transaction costs.

When firms cover costs with two-part tariffs in general equilibrium, two potential problems come into tension: demand must be continuous, and hookups must cover costs. Hookups of zero leave demand continuous, but will not cover costs. Positive hookups may cover costs but give households nonconvex budget sets, which can cause discontinuities. Brown, Heller, and Starr [7] found the maximum hookups that leave demand continuous; they show that when these hookups cover costs, equilibria exist. Their solution is simply that the maximum hookups equal willingness to pay.

Since our model has two regulated firms charging nonlinear tariffs, the situation is more complicated. The continuity of total demand requires more than just the independent continuity of demand for each regulated firm with respect to that firm's nonlinear price. The budget set remaining after paying for one firm's goods is nonconvex, so changes in that firm's prices may cause discontinuous purchases from the other firm. Such discontinuities are eliminated if the hookups of both firms are always paid.[3]

Unlike Brown, Heller, and Starr's model [7], in our context with more than one firm, households may not pay hookups, even when they sum to less than total willingness to pay. Our paper identifies the maximum hookups that two separate firms can charge. When these hookups lead to sufficient revenue to cover the costs of their production plans, we say that these plans are independently viable. Since these maximum hookups are continuous, we can use our independent viability definition to state conditions under which general equilibria exist.

These maximum hookups are linked to our new conceptions of substitutability and complementarity, notions that measure the total valuation of goods in the spirit of Coase [12].[4] When goods are value substitutes, not all the willingness to pay can be gathered with independent hookups.

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[3]Other papers like Moriguchi [28] rightly identify the problem as finding continuous individually-rational hookup functions which cover losses; but they do not analyze when this will be possible.

[4]Since the sort of nonconvexities and consequent discontinuities discussed above are inherent whenever two principals compete for an agent's surplus, these definitions are useful to demonstrate existence and evaluate equilibria in that setting as well (see Epelbaum [20]).

Therefore when products or services are close value substitutes, their marketing should probably be joint if it is efficient to produce both. Although our discussion focuses on the case of two firms charging two-part tariffs, the methodology might be expanded to n firms, as demonstrated in our later paper, Edlin and Epelbaum [16].

To ensure that hookups are voluntarily paid, we follow Brown, Heller, and Starr [7] by varying them in accord with tastes. We recognize that individualized hookups usually require a prohibitive amount of information, but we interpret them as a stylized version of discriminatory pricing schemes used in such diverse circumstances as health clubs, toll roads, rail passes, and higher education (on the last, see Edlin [19]).

In addition to having more than a single regulated increasing returns firm, our work extends Brown, Heller, and Starr [7] by allowing the joint production of multiple goods.[5] Our final extension is to prove existence in a model in which the regulated firms also charge nonlinear tariffs to nonregulated firms that buy regulated goods for inputs. This additional revenue source allows welfare enhancement since a wider range of increasing returns goods may be supported than could be supported by household demand alone.

We divide the remainder of the paper into three sections. Section 2 develops notions of normality, complementarity and substitutability; we link these to standard notions, and use them to identify when two firms are independently viable. Finally, we explain how these ideas apply to a broad class of partial equilibrium settings. Section 3 shows that assuming independent viability over the set of production equilibria is sufficient to prove the existence of a general equilibrium. Section 4 shows that independent viability implies that a Second Welfare Theorem holds, and explains that when increasing returns arise solely from fixed costs, independent viability implies that the First Welfare Theorem holds as well. Finally, we

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[5]At the 1990 Stanford Institute on Theoretical Economics, Frank Hahn impressed upon us the importance of having a model with more than one good produced per firm. He argued that for the general-equilibrium increasing-returns models of regulation to be useful, products must at least be distinguished by their time of delivery.

explain how the regulatory authority might reduce regulation, privatize the ownership of the increasing-returns technology, and provide a profit motive.

2. Independent Viability

2.1 Surpluses, Substitutes, and Complements

The model has c goods, F firms and H households. Of the c goods, we call the first m "IR" goods because they may be produced with increasing returns. We call the remaining $c - m$ goods "DR" goods because they are produced with decreasing returns. The IR goods are produced by two regulated monopoly firms, indexed 1 and 2, and it is technologically impossible for any of the remaining $F - 2$ firms to produce IR goods.

We require a number of definitions to proceed. For now, we assume prices are strictly positive, i.e. $\mathbf{p} \in \Re^c_{++}$; to prove existence, we will abandon this assumption and use the standard technique of temporarily restricting productions and consumptions within some huge compact box.

In order to write down sufficient conditions for existence parallel to Brown, Heller, and Starr [7], we need to find out how much households and DR firms will pay for the IR goods. We assume that each DR firm $f > 2$ has a production technology $\mathbf{Y}^f \subset \Re^c$ formed by the free disposal of some compact set $\mathbf{K}^f \subset \Re^c$. Let $\mathbf{M} = \{1, \ldots, m\}$, be the indices of the IR goods produced under increasing returns.

When firm $f > 2$ is only permitted to utilize IR goods in some subset $\mathbf{M}_1 \subseteq \mathbf{M}$, its restricted profit possibilities are:

$$\Pi^f_{\mathbf{M}_1}(\mathbf{p}) = \max_{\mathbf{z}} \mathbf{p} \cdot \mathbf{z}$$

$$\text{s.t.} \quad \mathbf{z} \in \mathbf{Y}^f$$
$$z_i = 0 \quad \forall i \in \mathbf{M} \setminus \mathbf{M}_1$$

(1)

Since $\mathbf{p} \in \Re^c_{++}$, the maximization is effectively over the compact set $\mathbf{K}^f$. Therefore the $\Pi^f_{\mathbf{M}_1}$'s exist, and their continuity follows from Berge's Maximum Theorem.

The value to firm f of adding set $\mathbf{M}_2$ to its permissible IR good set $\mathbf{M}_1$ is given by

$$S^f_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}) = \Pi^f_{\mathbf{M}_1 \cup \mathbf{M}_2} - \Pi^f_{\mathbf{M}_1}.$$

(2)

We call $S^f_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}$ firm f's surplus or willingness to pay for the right to purchase from $\mathbf{M}_2$.

To make an analogous definition for households, let the function $U^h: \mathfrak{R}^c_+ \to \mathfrak{R}$ be the household's continuous, strictly monotonic, and strictly quasi-concave utility function. Let $\overline{V}^h_{\mathbf{M}_1}(\mathbf{p}, I)$ denote the household's restricted indirect utility, the maximum utility level achievable with income I when only IR goods in set $\mathbf{M}_1$ are available. By Berge's Maximum Theorem, $\overline{V}^h_{\mathbf{M}_1}(\mathbf{p}, I)$ is continuous when defined as follows:

$$\overline{V}^h_{\mathbf{M}_1}(\mathbf{p}, I) = \max_{\mathbf{x} \in \mathfrak{R}^c_+} U^h(\mathbf{x}), \quad s.t. \quad \mathbf{p} \cdot \mathbf{x} \le I, \tag{3}$$
$$x_i = 0 \ \forall i \in \mathbf{M} \setminus \mathbf{M}_1.$$

Define the household's expenditure function conditional upon only being allowed to consume IR goods in set $\mathbf{M}_1$ as follows (again, this function is continuous by the Berge's Maximum Theorem):

$$E^h_{\mathbf{M}_1}(\mathbf{p}, V) = \min_{\mathbf{x} \in \mathfrak{R}^c_+} \mathbf{p} \cdot \mathbf{x} \tag{4}$$
$$s.t. \quad U^h(\mathbf{x}) \ge V, \quad x_i = 0 \ \forall i \in \mathbf{M} \setminus \mathbf{M}_1.$$

When $\mathbf{M}_1$ is the permissible set, the household's surplus from adding set $\mathbf{M}_2$ to its choice set is given continuously by

$$S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}, I) = I - E^h_{\mathbf{M}_1 \cup \mathbf{M}_2}(\mathbf{p}, \overline{V}^h_{\mathbf{M}_1}(\mathbf{p}, I)). \tag{5}$$

Where confusion is unlikely, we will generally suppress prices $\mathbf{p}$ from the surplus functions.

We can now define *weak viability* for a single firm. Suppose firm 1 has a production set $\mathbf{Y}^1 \subset \mathfrak{R}^c$ and produces all m IR goods.

Definition 1: *When firm 1 produces all the monopoly goods and firm 2 does not produce, we say* $\mathbf{y} \in \mathbf{Y}^1$ *is a **weakly viable** production for incomes* (I^h) *and prices* $\mathbf{p} \in \mathfrak{R}^c$ *if* $\sum_h S^h_{\varnothing \to \mathbf{M}}(I^h) + \sum_{f > 2} S^f_{\mathbf{M}} > -\mathbf{p} \cdot \mathbf{y}$.

Consider firm 1 pricing with two-part tariffs. If the production is weakly viable, then hookups can be charged which cover (or exceed) the losses remaining after the input-output list $\mathbf{y}$ is bought and sold at the linear prices $\mathbf{p}$. We call the production *weakly* viable as a remainder that the definition includes no guarantee that the firm can sell its output at these prices, as would be desirable in many contexts; nonetheless we will often simply write "viable."

With two firms producing, identifying when losses can be covered is less straightforward. Their productions are not made viable, merely because total surplus exceeds total costs: Two hookups that would

voluntarily be paid together for a bundle of opportunities will not necessarily both be paid when the opportunities and hookups are unbundled. With two IR firms, the surplus from adding one firm's goods to the economy depends upon the availability of the other's goods. As a consequence, two projects may fail to be self-supporting when priced separately, even though they would be viable if priced jointly.

Consider the monopolists each producing sets of IR goods $M_i \subset M$, $i=1,2$. There are four potentially relevant surpluses from transitions related by the following identity and depicted in Figure 1:

$$
\begin{aligned}
S^h_{\varnothing \to M_2}(I) + S^h_{M_2 \to M_2 \cup M_1}\left(I - S^h_{\varnothing \to M_2}(I)\right) &= \\
S^h_{\varnothing \to M_2 \cup M_1}(I) &= S^h_{\varnothing \to M_1}(I) + S^h_{M_1 \to M_2 \cup M_1}\left(I - S^h_{\varnothing \to M_1}(I)\right).
\end{aligned}
\tag{6}
$$

[Insert Figure 1]

The middle quantity is the most household h with income I is willing to give up to be allowed to buy from the IR goods in $M_1 \cup M_2$. After paying this amount as a hookup, it can achieve the same utility $\overline{V}^h_\varnothing(p, I)$ that it could with initial income I, but without consuming any of the IR goods. Consider the left-hand expression. The first term on the left-hand side is what the household would pay to add M_2 to its initial consumption set of DR goods, since this payment would leave h with its reservation utility $\overline{V}^h_\varnothing(p, I)$. The second term is what the household would pay to add M_1 to its permissible set M_2 given that it has income $I - S^h_{\varnothing \to M_2}(I)$ —the minimum income sufficient to achieve $\overline{V}^h_\varnothing(p, I)$ subject to permissible set M_2. Thus, if the total quantity of the left-hand expression is taken from household h it buy any goods in $M_1 \cup M_2$, its utility will be $\overline{V}^h_\varnothing(p, I)$. Therefore, the left-hand quantity must equal the middle quantity. The argument for the right-hand side is perfectly analogous.

This identity will assist us in deducing some useful facts about *value substitutes* and *complements*

Definition 2: *For household h, the set M_i is a value substitute for the set M_j at income I if*
$$S^h_{\varnothing \to M_j}(I) > S^h_{M_i \to M_i \cup M_j}\left(I - S^h_{\varnothing \to M_i}(I)\right).$$

Definition 3: *For household h, the set M_i is a value complement for the set M_j at income I if*
$$S^h_{\varnothing \to M_j}(I) \leq S^h_{M_i \to M_i \cup M_j}\left(I - S^h_{\varnothing \to M_i}(I)\right).$$

For firm f, we analogously define $\mathbf{M}_i$ as a *value substitute (resp. value complement)* for the set $\mathbf{M}_j$, if $S^f_{\varnothing \to \mathbf{M}_j} > S^f_{\mathbf{M}_i \to \mathbf{M}_i \cup \mathbf{M}_j}$ (resp. $S^f_{\varnothing \to \mathbf{M}_j} \leq S^f_{\mathbf{M}_i \to \mathbf{M}_i \cup \mathbf{M}_j}$).[6]

These definitions take into account not just local preferences for goods, but global characteristics of preferences. They are consequently well suited to determine what hookups can be charged. Gas and electricity are good examples of value substitutes, since when cold, you probably will pay more to be hooked up for access to gas heat than when already warmed by electric heat. Water, in contrast may be a value complement for electricity, since one important use for electricity is to heat water. (And as we show below, if this is true then electricity is necessarily a value complement to water.)

Consider the definitions for households. Observe first that they are exclusive and exhaustive, since they only differ by the direction of their inequalities. Also, an inspection of identity (6) reveals that they are symmetric, just as are the definitions of Hicksian substitutability or complementarity. For instance, if $\mathbf{M}_i$ is a value substitute for $\mathbf{M}_j$ then $\mathbf{M}_j$ is a value substitute for $\mathbf{M}_i$. To see why, compare the first term in the left expression of (6) with the second in the far right; we see that if $\mathbf{M}_2$ yields more surplus when $\mathbf{M}_1$ is denied than when $\mathbf{M}_1$ is allowed, then to preserve the identity, the same must be true if the numerals "1" and "2" are exchanged. (Note that this symmetry—analogous to that of Hicksian notions of substitutability and complementarity—results because our definitions also hold utility constant.) Also, observe that for value complements, the two first legs in Figure 1 sum to (weakly) less than the total surplus, while for value substitutes they sum to more. In fact, goods are value complements if and only if $S^h_{\varnothing \to \mathbf{M}_1}(I) + S^h_{\varnothing \to \mathbf{M}_2}(I) \leq S^h_{\varnothing \to \mathbf{M}_1 \cup \mathbf{M}_2}(I)$. Finally, note that for many preferences, as prices and income change, goods can switch from value substitutes to complements and vice-versa. Perfectly analogous results hold for firms.

One consequence of these definitions taking into account the total value of goods is that if $\mathbf{M}_1$ and $\mathbf{M}_2$ each included only one good, they might be value complements at some prices and income, even though

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[6]We thank Jonathan Paul for pointing out that these latter definitions are exactly parallel to those of Admati and Pfleiderer [1], who study the values of different kinds of information.

they were Hicksian substitutes. This can happen because of the global verses local nature of these concepts. Although it is by no means anomalous, we can show, however, that such a relationship cannot hold everywhere:

THEOREM 1: *If two goods are Hicksian complements everywhere then they are value complements everywhere; if two goods are Hicksian substitutes everywhere then they are value substitutes everywhere.*

Proof: See Appendix A. ‖

Justified in part by the relationship shown in Theorem 1, we will find it convenient to drop the term "value" when referring to these conceptions of substitutability and complementarity.

Finally, to account for income effects and to define independent viability simply, we require one more definition.

Definition 4: *When the IR goods are divided into sets* $\mathbf{M}_i$ *and* $\mathbf{M}_j$, $\mathbf{M}_i \cup \mathbf{M}_j = \mathbf{M}$, *the set of goods* $\mathbf{M}_j$ *is* **surplus normal**, *if for all prices, higher incomes imply a higher surplus from introducing* $\mathbf{M}_j$ *to the choice set. That is, the set of goods* $\mathbf{M}_j$ *is surplus normal if,*

$$S^h_{\mathbf{M}_i \to \mathbf{M}_i \cup \mathbf{M}_j}(I) \ge S^h_{\mathbf{M}_i \to \mathbf{M}_i \cup \mathbf{M}_j}(I-q), \quad \forall I > q > 0.$$

(There is no counterpart to surplus normality for firms, because their surpluses do not depend on income.)

If a set of goods is surplus normal, then willingness to pay for access to it increases with income. Theorem 2 relates surplus normality to ordinary normality.

THEOREM 2: *Let the IR goods be divided into sets* $\mathbf{M}_i$ *and* $\mathbf{M}_j$, $\mathbf{M}_i \cup \mathbf{M}_j = \mathbf{M}$. $\mathbf{M}_j$ *is surplus normal if each of its constituent goods is normal in the usual sense that its demand at linear prices increases with income.*

Proof: See Appendix A. ‖

We expect that for most goods, surplus increases with income. However, it will increase at a rate less than one, as can be verified by considering equation (5). Consequently, gathering surplus from DR firms allows us to run IR firms in more circumstances. Consider charging some nonregulated firm an extra dollar in a hookup to buy the IR goods. If that dollar had instead been distributed to shareholders as income, only a fraction of it would have been translated into extra household willingness to pay, the rest being spent on other goods.

2.2 Individually Rational Hookups

Different surpluses are relevant to hookups for the cases of substitutes and complements. For complements, hookups summing to the total surplus $S^h_{\varnothing \to M_i \cup M_j}(I)$ may be charged as follows:

THEOREM 3: *MAXIMUM HOOKUP THEOREM FOR COMPLEMENTS. Paying both hookups is individually rational if a household h is confronted with hookups*

$$q^h_{M_1} \le S^h_{\varnothing \to M_1}(I) + \alpha\left(S^h_{\varnothing \to M_1 \cup M_2}(I) - S^h_{\varnothing \to M_1}(I) - S^h_{\varnothing \to M_2}(I)\right)$$

$$q^h_{M_2} \le S^h_{\varnothing \to M_2}(I) + (1-\alpha)\left(S^h_{\varnothing \to M_1 \cup M_2}(I) - S^h_{\varnothing \to M_1}(I) - S^h_{\varnothing \to M_2}(I)\right)$$

where M_1 *and* M_2 *are value complements at income I and surplus normal.*[7]

P R O O F : Let $\tilde{S}^h_1 = S^h_{\varnothing \to M_1}(I) + \alpha\left(S^h_{\varnothing \to M_1 \cup M_2}(I) - S^h_{\varnothing \to M_1}(I) - S^h_{\varnothing \to M_2}(I)\right)$ and $\tilde{S}^h_2 = S^h_{\varnothing \to M_2}(I) + (1-\alpha)\left(S^h_{\varnothing \to M_1 \cup M_2}(I) - S^h_{\varnothing \to M_1}(I) - S^h_{\varnothing \to M_2}(I)\right)$. Observe that $\tilde{S}^h_1 + \tilde{S}^h_2 = S^h_{\varnothing \to M_1 \cup M_2}(I)$. If one or both inequalities is strict, then paying no hookup is clearly not utility maximizing, since $q^h_{M_1} + q^h_{M_2} < S^h_{\varnothing \to M_1 \cup M_2}(I)$. We need to show that if either hookup is paid, then paying the other is individually rational. This fact follows from surplus normality combined with the observation that because the goods are value complements both hookups will be paid when both are maximal. The relevant chain of inequalities to see that hookup 2 will be paid in the case where hookup 1 has been paid is

$$S^h_{M_1 \to M_1 \cup M_2}(I - q^h_{M_1}) \ge S^h_{M_1 \to M_1 \cup M_2}\left(I - \tilde{S}^h_1\right) \ge \tilde{S}^h_2 \ge q^h_{M_2} .$$

The first inequality again follows from surplus normality, and the third follows from the theorem's predicate. The second inequality follows because value complementarity implies that $\tilde{S}^h_1 \ge S^h_{\varnothing \to M_1}(I)$, which in turn implies that $V^h_{M_1}(I - \tilde{S}^h_1) \le V^h_\varnothing(I)$. Hence, given the opportunity to buy M_1 and the income $I - \tilde{S}^h_1$, the household would willingly pay a hookup of $\tilde{S}^h_2$ to firm 2 since that would give it utility $V^h_{M_1 \cup M_2}(I - \tilde{S}_1 - \tilde{S}_2) = V^h_\varnothing(I) \ge V^h_{M_1}(I - \tilde{S}_1)$. If the inequalities bind as equalities, then the household is indifferent between paying no hookup and paying both hookups. These options are again weakly preferred to only paying one of the hookups, so paying both does not violate utility maximization. ⫾

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[7]In previous versions, we considered the case of $\alpha=0$. Clay [11] proposed hookups equivalent to $\alpha=.5$, which inspired this generalization.

Below we design hookups for the case of substitutes:

THEOREM 4: HOOKUP THEOREM FOR SUBSTITUTES. Paying both hookups is individually rational if a household h is confronted with hookups $q_{M_1}^h \leq S_{M_2 \to M_1 \cup M_2}^h(I - S_{\varnothing \to M_2}^h(I))$ *a n d* $q_{M_2}^h \leq S_{M_1 \to M_1 \cup M_2}^h(I - S_{\varnothing \to M_1}^h(I))$ *where* M_1 *and* M_2 *are value complements at income I and surplus normal.*

PROOF: By construction and normality, if the household consumes either set, then paying the other hookup to consume the other set is maximizing behavior. And, paying both hookups is superior to paying none since the theorem's hypothesis of substitutability implies that $q_{M_1}^h + q_{M_2}^h < S_{\varnothing \to M_1 \cup M_2}^h(I)$. ‖

Analogous theorems are easily verified for hookups charged to DR firms.

In the case of DR firms, no more surplus can be collected with both hookups being paid than hookups equal to the second legs (as are charged to households in Theorem 4). However, due to income effects more surplus can be extracted from households. Theorem 4' below gives the maximum total surplus that can be captured from households. The example at the end of the section shows that the difference in the total surplus extracted between Theorems 4 and 4' can be significant. The following Lemma allows us to specify the maximum extractable surplus in Theorem 4'.

L e m m a : *Let the first hookup for the household be defined implicitly b y* $\tilde{q}_1 = S_{M_2 \to M_1 \cup M_2}^h(I - S_{M_1 \to M_1 \cup M_2}^h(I - \tilde{q}_1))$ *and let its second hookup be* $\tilde{q}_2 = S_{M_1 \to M_1 \cup M_2}^h(I - \tilde{q}_1)$, *where* M_1 *and* M_2 *are value substitutes at income I and surplus normal. Then:*

1) $\tilde{q}_1$ and $\tilde{q}_2$ are well defined and continuous functions of income and prices.

2) $\tilde{q}_1$ and $\tilde{q}_2$ are always voluntarily paid.

3) $\tilde{q}_1$ and $\tilde{q}_2$ are uniquely maximal hookups in the sense that for all q_1 and q_2 which are voluntarily paid, $q_1 + q_2 < \tilde{q}_1 + \tilde{q}_2$.

PROOF: See Appendix A. ‖

THEOREM 4': MAXIMUM HOOKUP THEOREM FOR SUBSTITUTES. Paying both hookups is individually rational if a household h is confronted with hookups $q_{M_1}^h \leq \tilde{q}_1^h$ *and* $q_{M_2}^h \leq \tilde{q}_2^h$ *where* M_1 *and* M_2 *are value substitutes at income I and surplus normal, and where $\tilde{q}_1^h$ and $\tilde{q}_2^h$ are determined by the Lemma.*

PROOF: Immediate from the Lemma together with surplus normality. ‖

Theorem 4' extracts more surplus than theorem 4. Nevertheless, for substitutes the hookups must sum to less than $S^h_{\varnothing \to M_j \cup M_j}(I)$, whereas for complements, all the surplus may be collected.

2.3 Independent Viability

Taking care to maintain continuity as goods switch from complements to substitutes, we are finally prepared to define appropriate surpluses which gather the maximum total surplus for both cases; if these are sufficiently large, the two firms are independent viability. Let

$$
S^f_1 = \begin{cases} S^f_{\varnothing \to M_1} & \text{if the goods are complements} \\ S^f_{M_2 \to M_1 \cup M_2} & \text{if the goods are substitutes} \end{cases} \tag{8}
$$

$$
S^f_2 = S^f_{M_1 \to M_1 \cup M_2}, \, f > 2.
$$

$$
S^h_1 = \begin{cases} S^h_{\varnothing \to M_1}(I^h) & \text{if the goods are complements} \\ \tilde{q}_1 \text{ defined in Lemma} & \text{if the goods are substitutes} \end{cases} \tag{9}
$$

$$
S^h_2 = \begin{cases} S^h_{M_1 \to M_1 \cup M_2}(I^h - S^h_{\varnothing \to M_1}(I^h)) & \text{if the goods are complements} \\ \tilde{q}_2 \text{ defined in Lemma} & \text{if the goods are substitutes} \end{cases}
$$

(Note that in the case of complements, we have not utilized the full power of Theorem 3, since we have fixed $\alpha = 0$).

Observe that the S^f_i's, $i = 1,2$, are continuous since when goods switch from being substitutes to complements, or vice-versa, the two quantities defining them are equal. The same is true of the S^h_i's, $i = 1,2$, because when $S^h_{\varnothing \to M_1}(I^h) = S^h_{M_2 \to M_1 \cup M_2}(I^h - S^h_{\varnothing \to M_2}(I^h))$ then $\tilde{q}_1 = S^h_{\varnothing \to M_1}(I)$ and $\tilde{q}_2 = S^h_{M_1 \to M_1 \cup M_2}(I - S^h_{\varnothing \to M_1}(I))$.

Definition 5: *A pair of productions* $\mathbf{y}^1 \in Y^1$ *and* $\mathbf{y}^2 \in Y^2$ *is* **weakly independently viable** *at prices* $\mathbf{p}$ *and incomes* (I^h) *if the sum of each firm's extractable surplus is larger than losses, i.e.,*

$$
\sum_{f>2} S^f_i + \sum_h S^h_i > -\mathbf{p} \cdot \mathbf{y}^i, \quad i = 1,2.
$$

When two productions or projects $\mathbf{y}^1$ and $\mathbf{y}^2$ are weakly independently viable, marketing can be separate; if they are not, then a different α might be tried, or the projects might be run with a single hookup provided that together they are at least weakly viable. As with the definition of weak viability, we include "weak" as a reminder that market clearing is not required. In the future we often drop this reminder.

There are many applications outside of the general equilibrium literature for our notions of weak independent viability, together with our notions of substitutability and complementarity. Consider for example, the partial equilibrium literatures on organizations or on the public finance of projects. In the transfer pricing literature, it is important to know when two divisions can be run separately, each marketing to a third division, or when the two must be combined to sell together. In the public finance literature, one may ask when two projects can be supported separately by user fees, or when they must be supported together. Alternatively, in club theories of local public finance, one might ask when two clubs can collect membership fees to cover needed services or when the clubs must merge, coordinating their membership policies.

When goods are complements, as we have explained, two firms may independently charge hookups that sum to the whole of surplus. Unfortunately, however, when goods are substitutes, they can't charge all surplus; consequently many productions cannot be undertaken with independent marketing, even though they might well be viable if undertaken together. Such a situation does not necessarily mean that one of the projects should be shut down. Quinzii [31] has shown that from an efficiency standpoint, when there are increasing returns, some projects with insufficient willingness to pay are nonetheless efficient. This could easily hold true for one of the two firms.

2.3 Example

Below we present an example which calculates the possible hookups specified in Theorems 3, 4 and 4'. It shows that the surpluses made available by the maximum hookup scheme in Theorem 4' can far exceed those from Theorem 4.

Consider a household with preferences $U(x_1, x_2, z) = z^{1/2} + (1 + x_1)^{1/2}(1 + x_2)^{1/2}$, where the x_i, $i=1,2$, are the IR goods and z is a composite DR good. For any given prices and income what are the largest hookups that the household is willing to pay? To answer we need to know the restricted expenditure functions. (We will restrict attention to prices p that lead to interior consumptions of all permissible goods, since for other prices, there is no willingness to pay for one or both goods and so building hookups is of no interest).

$$E_{\{i\}}[\mathbf{p}, V] = \frac{p_i}{1+p_i}\left[V^2 - 1 - p_i\right], \; i = 1,2 \text{ and } E_{\{1,2\}}[\mathbf{p}, V] = 2V(p_1 p_2)^{1/2} - (p_1 p_2 + p_1 + p_2).$$

where we have normalized the price of z to one, so that $\mathbf{p} = (p_1, p_2, 1)$.

Observe that $\overline{V}_{\{\varnothing\}}(\mathbf{p}, I) = I^{1/2} + 1$, and let $V_o = \overline{V}_{\{\varnothing\}}(\mathbf{p}, I)$. The first leg in Figure 1 is

$$S^h_{\{\varnothing\}\to\{i\}}(I) \equiv I - E_{\{i\}}[\mathbf{p}, V_o] = I - \frac{p_i}{1+p_i}\left[I + 2I^{1/2} - p_i\right]; \text{ while total surplus is}$$

$$S^h_{\{\varnothing\}\to\{1,2\}}(I) \equiv I - E_{\{1,2\}}[\mathbf{p}, V_o] = I - 2(I^{1/2} + 1)(p_1 p_2)^{1/2} - (p_1 p_2 + p_1 + p_2).$$

Using identity (6), the second leg is $S^h_{\{i\}\to\{1,2\}}(I - S^h_{\{\varnothing\}\to\{i\}}(I)) = S^h_{\{\varnothing\}\to\{1,2\}}(I) - S^h_{\{\varnothing\}\to\{i\}}(I)$.

From these expressions, we can calculate the extractable surpluses in the hookup theorems. Let I = 25 and suppose that $p_1=2$ and $p_2=2$. Then $S^h_{\{\varnothing\}\to\{i\}}(I) = 3$, $i=1,2$; $S^h_{\{\varnothing\}\to\{1,2\}}(I) = 9$; and $S^h_{\{i\}\to\{1,2\}}(I - S^h_{\{\varnothing\}\to\{i\}}(I)) = 6$, $i=1,2$. Notice that $S^h_{\{\varnothing\}\to\{i\}}(I) < S^h_{\{-i\}\to\{12\}}(I - S^h_{\{\varnothing\}\to\{-i\}}(I))$, $i=1,2$, which implies that for this income and these prices, the goods are value complements.

Following Theorem 3, we can define hookups that sum to total surplus. Choosing $\alpha=0$, the first hookup equals the first leg, 3, and the second hookup equals the second leg, 6. Increasing α reduces the second hookup and increases the first without changing their sum. Hence surplus can be transferred across firms without cost. Nonetheless, the degree to which this can be done is limited, since not all hookups summing to 9 would be paid (in other words, Theorem 3 is not trivial). For instance, if $q_1 < 3$, then after paying this hookup and consuming good 1, the household would have utility greater than V_o. Therefore if $q_2 = 9 - q_1$, hookup 2 would not be paid. An implication is that if one of the firms has larger losses than the other, a hookup scheme that extracts the full willingness-to-pay potentially cannot provide for the proper division of willingness-to-pay, but one that extracted less total willingness to pay might.

Now consider changing prices to $p_1=p_2=1/2$. Each of the first legs becomes $13\frac{1}{2}$ and the sum of the first and second legs becomes $20\frac{1}{4}$, implying that each of the second legs is $6\frac{3}{4}$. Observe that for these prices, the goods have switched to value substitutes. Theorem 4 therefore sets hookups equal to the second legs, in this case $6\frac{3}{4}$, yielding a total extractable surplus for both firms of $13\frac{1}{2}$, much less than the full willingness-to-pay of $20\frac{1}{4}$.

Significantly more surplus can be captured with the definition of Theorem 4'. The hookups in Theorem 4' can be approximately $9\frac{2}{3}$, which yields a total extractable surplus of about 19, instead of $13\frac{1}{2}$. Theorem 4' shows that in this example almost the entire surplus can be extracted despite the fact that the goods are value substitutes. This difference shows that accounting for income effects can be important in this context.

Notice finally, that unlike in the case of complements, transferring surplus from one firm to another results in "wasted" surplus. This follows from the uniqueness of the maximum hookups found in the Lemma.

3. General Equilibrium: Existence

In this section we connect independent viability with the existence of a general equilibrium. We make the following assumptions about households and firms.

3.1 Households

Each household h possesses a strictly quasiconcave, strictly monotonic, and continuous utility function $U^h: \mathfrak{R}_+^c \to \mathfrak{R}$. Each household is endowed with some $\omega^h \in \mathfrak{R}_+^c$ and limited liability shares $(\theta^{fh}) = (\theta^{1h},...,\theta^{Fh})$. Households are not endowed with any of the IR goods, an asumption that could be removed if restricted utilities accounted for other endowments.

3.2 Firms

Following Beato and Mas-Colell [2] each firm f has a production set formed by the free disposal of a compact set $\mathbf{K}^f$ containing the origin: $\mathbf{Y}^f = \mathbf{K}^f - \mathfrak{R}_+^c$, $f = 1,...,F$. $\mathbf{K}^f$ is convex for $f > 2$, but the two IR firms are allowed arbitrary technologies, potentially with increasing returns. As in the previous section, firm 1 produces goods in $\mathbf{M}_1$ and firm 2 produces goods in $\mathbf{M}_2$, $\mathbf{M}_1 \cup \mathbf{M}_2 = \mathbf{M}$. The DR firms cannot produce IR goods. The $\mathbf{K}^f$'s may be viewed as proxies for the attainable production sets as discussed by Bonnisseau and Cornet [3]; compactness is useful because we show existence by mapping the boundary of the production set into itself. Observe that since the $\mathbf{K}^f$ are compact, $\exists r$ such that $\forall f$, $\mathbf{K}^f \subset \left\{ -re^c + \mathfrak{R}_+^c \right\}$, where e^c $=(1,...,1)$. There exist homeomorphisms η^f from the simplex into the boundary of the production set:

$\eta^I : \Delta^c \rightarrow \partial \mathbf{Y}^I \cap \left\{ -re^c + \mathfrak{R}_+^c \right\}$ which preserve the natural orientation of faces (see Beato and Mas-Collel [2]):

Figure 2 illustrates the geometry.

[Insert Figure 2 here]

Since we allow for nonsmooth production technologies with increasing returns, we need a generalization of marginal-cost pricing (and of the first-order necessary conditions for profit maximization). The Clarke normal cone is the best choice since it is always closed and convex and its intersection with the simplex is an upper-hemi-continuous map ζ^f from the boundary of the production set $\partial \mathbf{Y}^I \cap \left\{ -re^c + \mathfrak{R}_+^c \right\}$ into the simplex Δ^c ; if the production set is differentiable, the Clarke normal cone becomes the ordinary normal vector, pointing outward from the surface of the production set (if the technology is convex, it is the normal cone). For discussions of the Clarke normal cone, see Clarke [10], Brown [9], and Bonnisseau and Cornet [5].

3.3 Existence of a General Equilibrium

At prices $\mathbf{p}$ and hookups q_1^I, q_2^I the DR firm's continuous distributable profit function is

$\Pi^I = \Pi_{M_1 \cup M_2}^I - q_1^I - q_2^I$, $f = 3, \ldots, F$. Observe that $\Pi^f \geq 0$ whenever q_i^f's are chosen weakly less than S_i^f's. Since the IR firms will charge hookups big enough to cover losses, distributable profits for the IR firms at output y are $\Pi^f(\mathbf{y}, \mathbf{p}) \equiv \max[0, \mathbf{p} \cdot \mathbf{y}^f]$, $f = 1, 2$.

Hence each household's share of profits, at production $\mathbf{y} \equiv (\mathbf{y}^1, \ldots \mathbf{y}^F)$ and prices $\mathbf{p}$, is $\Pi^h(\mathbf{y}, \mathbf{p}) \equiv \sum_f \theta^{fh} \Pi^f$, making total household income,

$$I^h \equiv \mathbf{p} \cdot \omega^h + \Pi^h(\mathbf{y}, \mathbf{p}) \qquad (10)$$

Definition 6: *A Two-Part Marginal-Cost Price Equilibrium (TPMCPE) for the economy* $\mathcal{E} \equiv ((U^h), (\theta^{hf}), (\omega^h), (\mathbf{Y}^f))$ *described above is a list of outputs* $\mathbf{y} \equiv (\mathbf{y}^1, \ldots \mathbf{y}^F)$, *(* $\mathbf{y}^f \in \mathfrak{R}^c$ *); prices* $\mathbf{p} \in \Delta^c$ *; consumptions* $(\mathbf{x}^h \in \mathfrak{R}^c)$; *and nonnegative individualized hookups* (q_1^h), (q_2^h), (q_1^I), (q_2^I), *h=1,...,H f=3,..., F, such that:*

(1)   *Both monopolies are marginal-cost pricing and all convex firms are maximizing profits:* $\mathbf{y}^I \in \mathbf{Y}^I$ *and* $\mathbf{p} \in \zeta^f(\mathbf{y}^f) \; \forall f$.

(2)   *It is rational for DR firms to pay both hookups:* $\Pi^f_{M_1 \cup M_2} - q^f_1 - q^f_2 \geq \Pi^f_{M_1} - q^f_1$

$$\Pi^f_{M_1 \cup M_2} - q^f_1 - q^f_2 \geq \Pi^f_{M_2} - q^f_2$$

$$\Pi^f_{M_1 \cup M_2} - q^f_1 - q^f_2 \geq \Pi^f_{\varnothing}, \quad f > 2.$$

(3)   *It is rational for each household h to pay both hookups and consume* x^h:

$$x^h \in \arg\max\{U^h(x) \mid p \, x \leq I^h - q_1 - q_2\} \text{ and}$$

$$U^h(x^h) \geq V^h_{M_1}(I^h - q_1)$$

$$U^h(x^h) \geq V^h_{M_2}(I^h - q_2)$$

$$U^h(x^h) \geq V^h_{\varnothing}(I^h)$$

where I^h is given above in equation 10.

(4)   Both IR firms are producing.

(5)   Markets clear, i.e., $\sum (x^h - \omega^h) \leq \sum y^f$, and if $\sum (x^h_i - \omega^h_i) < \sum y^f_i$ then $p_i=0$.

(6)   Hookups cover losses, $\sum q^h_j + \sum_{f>2} q^f_j = \max\left[0, -p \cdot y^j\right] \quad j = 1,2.$

Let $\mathbf{B} \subset \Re^c_+$ be a huge convex and compact box containing $\Re^c_+ \cap \sum K^f + \sum \omega^h$. So that household demand and surplus functions will be defined when prices are zero, in the definitions below, and in the proof of existence, we restrict the household's maximizing choices of goods in all problems to $\mathbf{B}$. This is possible since the K^f's are compact.

Under the following two assumptions equilibria exist:

(IVA) We assume that at all production equilibria — that is, (p,y), for which $y^f \in Y^f$ and $p \in \zeta^f(y^f) \, \forall f$ — surpluses are nonzero and such that productions are weakly independently viable, where surpluses are defined at incomes

$$I^h = p \cdot \omega^h + \sum_{f>2} \theta^{hf}\left[p \cdot y^f - S^f_1 - S^f_2\right] + \theta^{h1} \max\left[0, p \cdot y^1\right] + \theta^{h2} \max\left[0, p \cdot y^2\right].$$

(SA) We assume that the incomes defined above are positive at all production equilibria.

The survival assumption (SA), in one form or another, is standard in the literature. Assumption SA would be automatically satisfied if $\omega^h \gg 0$, which we might assume if we suitably modify the restricted utility functions. The independent viability assumption (IVA) ensures that the surplus is enough to pay hookups and that a fixed point of the map we construct below corresponds to an equilibrium. It asserts that at any candidate equilibrium (production equilibrium) the monopoly firms are weakly independently viable. IVA makes no assertion that the firm can sell its whole output, but that if it could, it could cover its costs.

Assumption **IVA** is in the same spirit but is more concrete than the abstract minimum income assumptions that others use to prove existence.[8]

We measure surplus at the income we have chosen because this income, which considers the complete confiscation of surplus from each DR firm, makes viability most apt to hold. For, as pointed out earlier, it is a "waste" of surplus to leave the DR firms with surplus to distribute to shareholders—only a fraction would then appear as extra household surplus. It should be noted that **IVA** could be weakened, as Vohra [34] does in the one-firm case to be an assumption over all marginal valuation equilibria instead of production equilibria. Since our present purpose is to explain the difference between independent viability and viability for a single IR firm, we have not focused our attention on the set over which viability must be assumed.[9]

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[8]Papers such as Bonnisseau and Cornet [3], [5] have abstract household income rules which do not separate out the ownership of profits. If incomes were formed by share ownership in each firm as Bonnisseau and Cornet suggest at one point, then assuming each household has positive income everywhere requires that after paying for its share of the IR firm's losses, it still has positive income. This assumption does not link a household's payment for the monopoly losses to its income from endowment and dividends; instead it assigns losses in constant shares, without regard to who consumes the monopoly goods. Our assumption links payments to income, and as prices change, the distribution of income shifts. It changes the load from those with lower income to those with higher income. In some sense this makes it more likely to hold. Still, one could come up with circumstances in which either one might hold while the other doesn't. Other papers such as Kamiya [24] make an assumption on preferences that would guarantee that our assumption is met whenever the value of the economy is positive. He assumes $\forall x^h, \tilde{x}^h$ s.t. $x^h \in \partial \Re_+^c$ and $\tilde{x}^h \in \Re_{++}^c$, $U^h(x^h) < U^h(\tilde{x}^h)$. With these preferences, households are willing to pay their entire incomes for access to the monopoly goods. Benefit taxation reduces to income taxation, and existence follows easily because **IVA** must then hold if **SA** holds.

[9]For more discussion of this sort of viability assumption, see Brown, Heller, and Starr [6, p.63] or our subsequant paper Edlin and Epelbaum[16].

If **IVA** did not hold but the analogous assumption of weak viability (distinct from weak *independent* viability) did hold, then while no equilibrium might exist with the firms pricing independently, one will exist if the firms price together.

THEOREM 5: *If assumptions IVA and SA hold and sets* **M**$_1$ *and* **M**$_2$ *are surplus normal, then there exists a TPMCPE .*

PROOF: To prove existence, we construct a convex-valued upper-hemi-continuous correspondence $\Phi:$ **D** $\rightarrow$ **D**, where **D** $= (\Delta^c)^{2F+1}$ X **B**^{H} X $[0,1]^2$. Φ thus has $2F+H+3$ components. A typical member of the domain **D** is $d = ((\mathbf{z}^f),(\mathbf{s}^f),\mathbf{p},(\mathbf{x}^h),\lambda_1,\lambda_2)$ where $(\mathbf{z}^f) \in (\Delta^c)^F,(\mathbf{s}^f) \in (\Delta^c)^F, \mathbf{p} \in \Delta^c,(\mathbf{x}^h) \in \mathbf{B}^H,(\lambda_1,\lambda_2) \in [0,1]^2$. Since **D** is the Cartesian product of convex compact sets, it itself is convex and compact; hence Φ must have a fixed point. This fixed point will define an equilibrium. Our map is an amalgam of our own ideas with those of Brown, Heller, and Starr [7], Beato and Mas-Colell [2], and an anonymous referee.

The prices **p** allow definitions of continuous surpluses S_1^f and S_2^f for DR firms as in Section 2. Interpreting the λ's as the benefit taxation rates to cover the costs of the monopoly firms , we are motivated to define continuous hookup functions $q_1^f = \lambda_1 S_1^f$ and $q_2^f = \lambda_2 S_2^f$ and from these continuous distributable profits $\Pi^f = \Pi_{\mathbf{M}_1 \cup \mathbf{M}_2}^f - q_1^f - q_2^f$, $f = 3,...F$. The profit functions for the monopoly firms are given by $\Pi^f(\mathbf{y},\mathbf{p}) = \max[0,\mathbf{p}\cdot\mathbf{y}^f]$, $f = 1,2$. These provide continuous income functions for each household, $I^h = \mathbf{p}\cdot\omega^h + \Pi^h$, as defined in equation (10). These incomes define the household problems described in Section 2 so that continuous surpluses S_1^h and S_2^h can be calculated (subject to being in **B**). From these surpluses we define continuous hookup functions: $q_i^h = \lambda_i S_i^h$ $i = 1,2$.

We define the map Φ in five parts, i.e. $\Phi = (\Phi_1,\Phi_2,\Phi_3,\Phi_4,\Phi_5)$, where $\Phi_1 \in (\Delta^c)^F, \Phi_2 \in (\Delta^c)^F, \Phi_3 \in \Delta^c, \Phi_4 \in \mathbf{B}^H, \Phi_5 \in [0,1]^2$. Let $\mathbf{y}^f = \eta^f(\mathbf{z}^f)$. Define Φ as follows:

Marshallian output adjustment,

$$\Phi_{1,f}^i((\mathbf{z}^f),(\mathbf{g}^f),\mathbf{p},(\mathbf{x}^h),\lambda_1,\lambda_2) = \frac{z_i^f + \max\left[0,p_i - g_i^f\right]}{1 + \sum_{j=1}^{c}\max\left[0,p_j - g_j^f\right]}, \quad \text{firm } f = 1,...,F, \text{good } i = 1,...,c;$$

marginal-cost pricing,

$$\Phi_{2,f}((z^f),(g^f),p,(x^h),\lambda_1,\lambda_2) = \zeta^f(y^f) \ ;$$

Walrasian price adjustment,

$$\Phi_3^i((z^f),(g^f),p,(x^h),\lambda_1,\lambda_2) = \frac{p_i + \max\left[0, \sum_h \left(x_i^h - \omega_i^h\right) - \sum_f y_i^f\right]}{1 + \sum_{j=1}^c \max\left[0, \sum_h \left(x_j^h - \omega_j^h\right) - \sum_f y_j^f\right]}, \ \text{good } i = 1,\dots c \ ;$$

utility-maximizing demands,

$$\Phi_{4,h}((z^f),(g^f),p,(x^h),\lambda_1,\lambda_2) = \begin{cases} \operatorname{argmax}\{U^h(x) \mid x \in B, p \cdot x \le I^h - q_1^h - q_2^h\} \\ \qquad\qquad \text{when } I^h - q_1^h - q_2^h > 0 \\ B \qquad \text{when } I^h - q_1^h - q_2^h \le 0 \end{cases} \ ;$$

and break-even benefit taxation (or covering losses),

$$\Phi_5^i((z^f),(g^f),p,(x^h),\lambda_1,\lambda_2) = \begin{cases} \min\left[1, \dfrac{-\min[0,p\cdot y^f]}{\sum S_i^h + \sum_{f>2} S_i^f}\right] & \text{when } \sum S_i^h + \sum_{f>2} S_i^f \ne 0 \\ [0,1] & \text{when } \sum S_i^h + \sum_{f>2} S_i^f = 0 \end{cases} \quad i=1,2.$$

To apply a fixed-point theorem observe that Φ_1 is a single-valued continuous function. Φ_2, the Clarke normal cone, is convex-valued and upper-hemi-continuous. Φ_3 is a single-valued continuous function. Φ_4 is convex-valued and upper-hemi-continuous because when $I^h - q_1^h - q_2^h \le 0$, $\Phi_4 = B$, and otherwise Φ_4 equals demand, which is convex-valued and upper-hemi-continuous. Finally Φ_5 is convex-valued and upper-hemi-continuous, since it is either a single-valued continuous function or the whole interval $[0,1]$.

By Kakutani's fixed point theorem, this convex-valued upper-hemi-continuous map from one compact space to another has a fixed point. Appendix B proves that any such fixed point corresponds to an equilibrium. ‖

4. Welfare, Decentralization, and Deregulation

4.1 Optimality Under Fixed Costs

Two-part marginal-cost pricing satisfies two conditions related to optimality: prices at the margin equal marginal cost, and Coase's objection to marginal-cost pricing does not apply. This avoids the individual

rationality problems of Brown and Heal [8], but some TPMCPE are still inefficient (see, e.g., Vohra [34]). Efficiency depends upon whether the aggregate production possibility set is cut by the social indifference surface, the surface which bounds the better-than set (those points that can be allocated to Pareto-dominate equilibrium allocations). TPMCPE are efficient for a broad class of technologies, namely those where the increasing returns arise solely from moving to positive production from zero production (i.e., fixed-cost technologies).

Consider the case of a single monopolist with a production set formed by the free disposal of the set $\{\mathbf{0}\} \cup \left(\{\phi^1\} + \mathbf{C}^1\right)$, where ϕ^1 is the fixed cost of setting up a factory and $\mathbf{C}^1$ is some ordinary *convex* production set representing the factory's production possibilities. In a TPMCPE, prices p support the better-than set. They also support the convex portions of production, the set of productions such that the monopoly firm does not shut down. Hence these sets are separated by p. The existence of voluntary hookups that cover the firm's losses is a statement of the production's viability, according to Definition 1. It implies that the nonconvex portion of the production set—those productions where the monopoly is shut down—does not cut the social better-than set (see Edlin [18], Kamiya [24], and Moriguchi [28]).[10] This result expands upon the intuition behind Vohra's [34, Theorem 5.1] fixed-cost efficiency result for two goods and a single firm.

When there are two fixed cost technologies, the issue of efficiency is more delicate, but independent viability provides us with a tool to identify efficient equilibria. Consider the case of two fixed-cost

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[10]Edlin [18] and Kamiya [24] proved this efficiency result for the case of a single monopolist; Moriguchi [28] provided an elegant proof of this result for multiple monopolists. The next section shows that any TPMCPE can be converted into an equilibrium in which a monopolist is allowed to maximize revenue subject to a prescribed production level. If the technology is of the simple fixed-cost type, then the new equilibrium would be one in which the monopolist is a perfect discriminator maximizing profits. Edlin [18], Heller [22], and Edlin, Epelbaum, and Heller [17] show that all these equilibria are efficient and this paper relates them to TPMCPE.

production technologies which allow productions in $\mathbf{M}_1$ and $\mathbf{M}_2$: $\mathbf{Y}^1 = \left\{\mathbf{0} \cup \left(\{\phi^1\} + \mathbf{C}^1\right)\right\}$ and $\mathbf{Y}^2 = \left\{\mathbf{0} \cup \left(\{\phi^2\} + \mathbf{C}^2\right)\right\}$, where ϕ^1 and ϕ^2 are the fixed costs and $\mathbf{C}^1$ and $\mathbf{C}^2$ are convex. When a *single*-hookup is charged for access to both sets $\mathbf{M}_1$ and $\mathbf{M}_2$, the equilibrium might not be efficient. The nonindependent viability of Definition 1 is not sufficient to imply optimality, and the one-firm efficiency results of Kamiya [24] and Edlin [18] do not apply here, because the set of positive productions is nonconvex. If $\mathbf{M}_1$ and $\mathbf{M}_2$ are *independently viable*, however, the equilibrium is efficient. For, with independent viability, we know that the hookups could be charged separately while still covering losses. And Moriguchi [28] has shown in the fixed-cost case that such a TPMCPE is efficient even with multiple monopolists.

To gain some intuition about why the one-firm efficiency results extend to the case of multiple monopolists, consider a thought experiment with two fixed-cost monopolists as in our model. Consider a TPMCPE. We know that if the aggregate production set cut the better-than set when both firms shut down, it would also cut it if there were a single monopolist with a "combined-fixed-cost technology" $\mathbf{Y}^1 = \left\{\mathbf{0} \cup \left(\{\phi^1 + \phi^2\} + \mathbf{C}^1 + \mathbf{C}^2\right)\right\}$, for this technology is the same when both monopolists shut down. But, according to the one-firm efficiency result, such a TPMCPE is efficient, so the aggregate production set cannot cut the better-than set where both firms are shut down.

The aggregate production set also cannot cut the better-than set with only one firm shut down, because as we shall see the one-firm efficiency result prevents that as well. To understand why, imagine a modified economy in which the second firm had technology $\mathbf{C}^2$ without the fixed cost ϕ^2. Modify the questionably efficient equilibrium by not charging households hookups q_2^h and not charging DR firms hookups q_2^f; instead subtract $\left(q_2^h / (\mathbf{p} \cdot \phi^2)\right)\phi^2$ from the endowment of each household h, and $\left(q_2^f / (\mathbf{p} \cdot \phi^2)\right)\phi^2$ from the production set of each DR firm. Now firm 2 no longer needs to be regulated and the old consumption plans and production plans yield us a TPMCPE in this new one-monopoly-with-fixed-cost economy. And, as we have emphasized, such equilibria are efficient. Moreover, any proposed Pareto superior allocation in the original economy which involved firm 2 producing could be replicated in this economy. Since no Pareto-superior allocations exist in the modified economy, no Pareto-superior allocations exist in the original economy with firm 2 producing. Repeating this argument for firm 1 shows that

TPMCPE are efficient with two monopolists in the fixed-cost case. This efficiency argument extends to any number of monopolists each of whom has fixed costs followed by decreasing returns to scale, i.e., it extends to explain the intuition behind the *n*-firm efficiency result first proved by Moriguchi [28].

We have shown that when a single monopolist's technology consists of two fixed-cost technologies, then independent viability implies that the equilibrium is Pareto optimal. It would be a mistake, however, to infer that when there is no independent viability, one of the technologies should be shut down. The Guesnerie-Quinzii counterexample (see Quinzii [31]) can be extended to show this fact. The lesson is only that one test for whether a production is efficient is to isolate all the fixed costs, view each fixed cost technology as separate, and then ask if the production is independently viable.

4.2 Decentralization: A Second Welfare Theorem

If a Pareto optimum is viable, in the sense of Definition 1, the optimum can be decentralized with appropriate transfers if the IR firms are run together charging a single hookup. Brown, Heller, and Starr [7] prove such a theorem in their model. In our model, if a Pareto optimum is *independently* viable, then it can be decentralized with the IR firms run separately.

THEOREM 6: SECOND WELFARE THEOREM. Let $\{(x^h),(y^f)\}$ *be a Pareto optimum and let* $p \neq 0$ *be prices that support* (x^h) *and are in the Clarke normal cones of productions* (y^f) *(Bonnisseau and Cornet [4, Theorem 2.1 and Proposition 2.1] implies such a p exists). Suppose* y^1 *and* y^2 *are weakly independently viable at prices p and incomes* $(I^h = p \cdot x^h)$. *Then the Pareto optimum can be decentralized as a TPMCPE with appropriate transfers.*

PROOF: Recall the surpluses $S_i^h(I^h)$ and S_i^f defined in Section 2, equations. (8) and (9). Define hookups for competitive firms by multiplying these surpluses by the ratio of losses to total surplus:

$$q_i^f = \left(\frac{-\min[0, p \cdot y^i]}{\sum S_i^h(p \cdot x^h) + \sum_{f>2} S_i^f} \right) S_i^f \quad \text{and} \quad q_i^h = \left(\frac{-\min[0, p \cdot y^i]}{\sum S_i^h(p \cdot x^h) + \sum_{f>2} S_i^f} \right) S_i^h(p \cdot x^h) \qquad , i=1,2.$$

Weak viability implies that $q_i^h < S_i^h(p \cdot x^h)$ and $q_i^f < S_i^f$. From Section 2, we know that it is rational for DR firms to pay these hookups. Also, if households are given the income $p \cdot x^h + q_1^h + q_2^h$, then surplus normality together with Theorems 3 and 4' ensure that the households voluntarily pay both hookups; the fact that p supports consumptions x^h and productions $y^{f>2}$, implies that after paying hookups, DR

firms produce y^f as prescribed and households consume x^h (expenditure minimization implies utility maximization since strict monotonicity guarantees $p \gg 0$). The only remaining question is whether some tax and transfer scheme will give households income $p \cdot x^h + q_1^h + q_2^h$. Define transfers $\tau^h = q_1^h + q_2^h + p \cdot x^h - p \cdot \omega^h - \sum_{f>2} \theta^{hf} \left(p \cdot y^f - q_1^f + q_2^f \right) - \sum_{f=1,2} \theta^{hf} \max[0, p \cdot y^f]$. Such transfers sum to zero and leave households with appropriate incomes. (Since we assumed strictly monotonic preferences and $\{(x^h), (y^f)\}$ is a Pareto optimum, $\sum x^h - \sum \omega^h = \sum y^f$.) ‖

From the proof, it may be surmised that due to income effects, the viability assumption in the theorem can be weakened. All that is necessary is that there be sufficient surplus under some distribution of income so that households and firms are willing to pay hookups that sum to the losses of the monopolies and that after paying their hookups they be left with enough income to buy their prescribed bundle.

4.3 Regulation With a Profit Motive

Due to incentive problems, economists and policymakers alike doubt that regulated firms can produce efficiently, minimizing costs. The economies of Russia and Eastern Europe are rife with support for this hypothesis. It is therefore worth pointing out that not only can viable Pareto optima be decentralized as Theorem 6 states, but for many technologies, if in addition to lump-sum taxation, we allow the regulator to "privatize the firms," distributing ownership shares as she pleases, then monopolists can be partially deregulated: The discriminating monopolists can be given output targets, but allowed to charge hookups that maximize revenue subject to meeting those targets. When the iso-production sets are convex, the prices p in Theorem 6 can be chosen so that the monopolies are also minimizing costs at the Pareto optimum. Therefore, such monopolies only need be told their output targets, and can be allowed to maximize their profits subject to these targets. This result makes incentive compatibility in the regulated general equilibrium literature somewhat less troubling than it would otherwise be. Moreover, if technology were of the simple form discussed above, with increasing returns arising only from fixed costs, then the equilibrium could actually be fully decentralized with the monopoly firms maximizing profits in an unconstrained way. (With such technologies, when prices are in the normal cone, maximizing revenue implies profit maximization.)

What really needs to be demonstrated in the above argument is that with a suitable assignment of shares in the monopolies, the allocations of a TPMCPE can be replicated in an equilibrium in which the monopolists charge hookups that maximize revenues . We now show that this can be done for a TPMCPE with a *single* monopolist, provided that the derivative of household surplus S^h with respect to income is uniformly bounded below one.[11] We leave the argument for two monopolists to the reader, as it involves additional notation and is not difficult when armed with Theorems 3 and 4'.

Consider a simplified version of a TPMCPE as defined in this paper, but with only one IR firm: $\left((\mathbf{y}^f),(\mathbf{x}^h),(q^f)_{f>1},(q^h),\mathbf{p}\right)$. We assume that in the TPMCPE, the regulated monopolist, firm 1, charges hookups such that it breaks even when producing $\mathbf{y}^1$ at prices p. (The reader may extend the result to cases where the regulated monopolist initially made profits.) If he monopoly firm raises hookups to capture all surplus and make profits, we must decide how to assign shareholdings so as not to disturb the equilibrium allocations.

Consider first what happens when the monopolist raises its hookups to DR firms from q^f to S^f. This increases monopoly profits by $\sum_{f>1} S^f - q^f$ which equals the sum of the decreases in each of the DR firm's profits. Distribute these monopoly profits to households according to what the households would have received if the DR firms had retained the profits they had before their hookups were increased to S^f. Thus each household gets $\sum_{f>1}(S^f - q^f)\theta^{hf}$ from the monopoly firm as a dividend to exactly replace its lost dividend income from the DR firms. Since the households have the same incomes, they will buy the same bundle $\mathbf{x}^h$ as in the TPMCPE under consideration. We have therefore found a new sort of equilibrium which yields the same allocations as our original TPMCPE. However, we have not yet deregulated the monopoly firm's marketing, because the monopolist would profit from raising the households' hookups . Let's see how household surplus can be captured without upsetting the equilibrium allocations.

In the original TPMCPE, households received incomes $I^h = q^h + \mathbf{p} \cdot \mathbf{x}^h$. In order to capture all household surplus without changing equilibrium allocations, the new household hookups must satisfy

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[11]Recall that the derivative is strictly less than one.

$q^{h\cdot} = S^h(\mathbf{p}, I^{h\cdot})$, where $I^{h\cdot} = q^{h\cdot} + \mathbf{p} \cdot \mathbf{x}^h$. Such hookups would increase the monopoly firm's profits by $\sum(q^{h\cdot} - q^h)$. If each household h receives an increased distribution from the monopolist in the amount of $q^{h\cdot} - q^h$, it will have income $I^{h\cdot}$. After voluntarily paying its hookup it will have $\mathbf{p} \cdot \mathbf{x}^h$ left over, and can do no better than buy $\mathbf{x}^h$. As illustrated in Figure 3, we can surely find satisfactory hookups ($q^{h\cdot}$) because of our assumption that the derivative of surplus is uniformly bounded away from 1.

[Insert Figure 3]

This demonstrates that if each household h is given shares in the monopoly

$$\theta^{h1} = \frac{\sum_{f>1}(S^f - q^f)\theta^{hf} + q^{h\cdot} - q^h}{\sum_{f>1}S^f - q^f + \sum q^{h\cdot} - q^h},$$

then the TPMCPE $\left((\mathbf{y}^f), (\mathbf{x}^h), (q^f)_{f>1}, (q^h), \mathbf{p}\right)$ can be suitably modified: The monopoly can charge hookups that capture all surplus $q^{f\cdot} = S^f(\mathbf{p})$ and $q^{h\cdot} = S^h(\mathbf{p}, I^{h\cdot})$, where $I^{h\cdot} = q^{h\cdot} + \mathbf{p} \cdot \mathbf{x}^h$; each DR firm f maximizes profit by producing $\mathbf{y}^f$ and paying hookup $q^{f\cdot}$; each household h has income $I^{h\cdot} = q^{h\cdot} + \mathbf{p} \cdot \mathbf{x}^h$ and maximizes utility by paying hookup $q^{h\cdot}$ and buying $\mathbf{x}^h$. Since the monopolist is charging hookups equal to surplus, the marketing side of its problem has been deregulated as we sought.

As outlined at the beginning of this section, this result implies that when the monopolist has convex iso-production sets, then a viable Pareto optimum can not only be decentralized into a TPMCPE, but also partially deregulated, where the firm only receives an output target. When in addition, the increasing returns arise solely from a fixed cost, the Pareto optimum can be fully deregulated, allowing firms to maximize profits. Similar revenue-maximizing or profit-maximizating equilibria may be found for two firms as well; however this requires a more sophisticated fixed-point argument involving theorems 3 and 4'.

Such partial regulation is often preferable to full regulation. Informational difficulties will of course usually prevent a monopolist from setting individualized hookups. Nonetheless as we discussed in the Introduction, price discrimination is often quite effective. And, although inefficiency may result when the number of different hookups is exceeded by the diversity of willingness to pay, with reasonable amounts of discrimination these inefficiencies may be much smaller than those from trying to cover losses outside the market, or than from hoping a regulated firm will minimize costs.

V. Conclusions

We derived conditions under which two-part-tariff marginal-cost pricing equilibria (TPMCPE) exist with two monopolists. The conditions involved assuming that productions were *independently viable* over the set of production equilibria. In order to determine when two firms would be independently viable, we needed to distinguish between goods sets that were *value complements* and *value substitutes*. The distinction was important because the maximum surplus that could be extracted with independent hookups was lower for substitutes than for complements.

Brown, Heller, and Starr [7] emphasized that unlike in the partial equilibrium literature, such two-part tariffs do not guarantee efficiency in general equilibrium. Efficiency is guaranteed, however, when the increasing returns arise only from fixed costs, and when isolating each fixed cost, productions are independently viable.

APPENDIX A

Throughout this section we assume differentiability.

Proof of Theorem 1: Denote the Hicksian demands as $\mathbf{x}^c(\mathbf{p},V) = \arg\min \mathbf{p}\cdot\mathbf{x}$ s.t. $U(\mathbf{x}) \geq V$. We seek to prove that $\frac{\partial x_1^c}{\partial p_2} > 0 \Rightarrow S_{\varnothing\to\{1\}}(I) > S_{\{2\}\to\{1,2\}}(I - S_{\varnothing\to\{2\}}(I))$ and $\frac{\partial x_1^c}{\partial p_2} \leq 0 \Rightarrow S_{\varnothing\to\{1\}}(I) \leq S_{\{2\}\to\{1,2\}}(I - S_{\varnothing\to\{2\}}(I))$. As we have observed, not being allowed to consume a good is equivalent to that good having a price of infinity. Therefore, defining $V_0 \equiv V_\varnothing(\mathbf{p}, I)$. we can write

$$S_{\varnothing\to\{1\}}(I) = E_{\{1,2\}}(\infty,\infty,V_0) - E_{\{1,2\}}(p_1,\infty,V_0) = \int_{p_1}^\infty \frac{\partial E_{\{1,2\}}(r_1,\infty,V_0)}{\partial r_1}\,dr_1 = \int_{p_1}^\infty x_1^c(r_1,\infty,V_0)\,dr_1 \qquad (A1)$$

and

$$S_{\{2\}\to\{1,2\}}(I - S_2(I)) = E_{\{1,2\}}(\infty,p_2,V_0) - E_{\{1,2\}}(p_1,p_2,V_0) = \int_{p_1}^\infty x_1^c(r_1,p_2,V_0)\,dr_1, \qquad (A2)$$

where we have suppressed the price vector $\mathbf{p}$ from the surplus functions and prices $(p_3,\dots,p_c)$ from the expenditure functions. Subtract (A1) from (A2) to get

$$S_{\varnothing\to\{1\}}(I) - S_{\{2\}\to\{1,2\}}(I - S_{\varnothing\to\{1\}}(I)) = \int_{p_1}^\infty \{x_1^c(r_1,\infty,\overline{V}) - x_1^c(r_1,p_2,\overline{V})\}\,dr_1 = \int_{p_1}^\infty\int_{p_2}^\infty \frac{\partial x_1^c(r_1,r_2,\overline{V})}{\partial p_2}\,dr_2\,dr_1.$$
$$(A3)$$

The right-hand side of (A3) is positive when goods 1 and 2 are Hicksian substitutes, since then $\frac{\partial x_1^c}{\partial p_2} > 0$. Similarly, it is non-positive when goods 1 and 2 are Hicksian complements. ‖

Proof of Theorem 2: We prove the theorem for the case where $\mathbf{M}_j = \mathbf{M}$, leaving suitable modification to other $\mathbf{M}_j$ to the reader. We therefore seek to show that if the Marshallian demands for each good $i \in \mathbf{M}$ is increasing with income I, then $S_{\varnothing\to\mathbf{M}}(I)$ is also increasing. To begin, observe that similar to identity (6), we can decompose the surplus as follows: $S_{\varnothing\to\mathbf{M}}^h \equiv \sum_{k=1}^m S_{\mathbf{M}_{k-1}\to\mathbf{M}_k}(E_{\mathbf{M}_{k-1}}(V_o))$, where $\mathbf{M}_0 \equiv \{\varnothing\}$, $\mathbf{M}_k \equiv \{1,\dots,k\}$. and $V_o \equiv \overline{V}_\varnothing(\mathbf{p}, I)$. (Note that again we have suppressed prices from the surplus and expenditure functions.) As in the proof of theorem 1, $S_{\varnothing\to\{n\}}(I) = \int_{p_n}^\infty x_{i.}^c(t_n,V_o)\,dr_n$, where the modified price vector t_n is given component-wise by $t_n^l = \begin{cases} \infty \; \text{if } l < m, l \neq n \\ p_l \; \text{if } l > m \\ r_n \; \text{if } l = n \end{cases}$, $l=1,\dots,c$. Putting together these two facts we can decompose surplus as follows:

$$S_{\varnothing\to\mathbf{M}}^h = \sum_{k=1}^m \int_{p_k}^\infty x_{k.}^c(t_k V_o)\,dr_k, \qquad (A4)$$

where the modified price vector t_k is given by
$$t_k^l = \begin{cases} r_k & \text{if } l = k \\ p_l & \text{if } l < k \\ \infty & \text{if } k < l < m \\ p_l & \text{if } l > m \end{cases}, \quad l = 1, \dots, c.$$

When goods are normal the Hicksian compensated demand x_k^c is an increasing function of V_O, and since V_O increases in I, $S^h{}_{\varnothing \to \mathbf{M}}(I)$ increases in I. ‖

Note finally that when $\mathbf{M}_j$ is a singleton, the theorem can be stated as an if-and-only-if theorem.

PROOF OF LEMMA: 1) Let $F(I,q) \equiv S^h_{\mathbf{M}_2 \to \mathbf{M}_1 \cup \mathbf{M}_2}(I - S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(I - q)) - q$. Then $\tilde{q}_1$ is defined implicitly by $F(I, \tilde{q}_1) = 0$. Notice that the continuity of F follows from the continuity of the surplus functions. Note also that $F(I,0) \geq 0$ and $F(I,I) \leq 0$ so by the Intermediate Value Theorem $\tilde{q}_1$ is well-defined. Moreover $\dfrac{\partial F(I,q)}{\partial q} = \dfrac{\partial S^h_{\mathbf{M}_2 \to \mathbf{M}_1 \cup \mathbf{M}_2}}{\partial I} \dfrac{\partial S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}}{\partial I} - 1$. Since each derivative of surplus must be strictly less than one, $\dfrac{\partial F(I,q)}{\partial q} < 0$. This implies that $\tilde{q}_1$ is unique given I and $\mathbf{p}$; and also that $\tilde{q}_1$ is continuous by the Implicit Function Theorem. Hence $\tilde{q}_1$ and $\tilde{q}_2$ are both well-defined continuous functions.

2) We prove something stronger than claim 2. We prove that the set of pairs of hookups that are individually rational is given by:

$$\mathbf{Q} \equiv \left\{ (q_1, q_2) \,\middle|\, q_1 \leq S^h_{\mathbf{M}_2 \to \mathbf{M}_1 \cup \mathbf{M}_2}(I - q_2) \text{ and } q_2 \leq S^h_{\mathbf{M}_2 \to \mathbf{M}_1 \cup \mathbf{M}_2}(I - q_1) \right\}.$$

It is trivial that a pair of hookups outside of $\mathbf{Q}$ is not individually rational, but are all hookups in $\mathbf{Q}$ voluntarily paid? It is immediate from construction that if either hookup is paid, it is individually rational to pay the other hookup. But is it possible the household pays no hookups? No. Suppose it did. Then $q_1 > S^h_{\varnothing \to \mathbf{M}_1}(I)$ or q_1 would be paid. However, this inequality implies that $q_2 \leq S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(I - q_1) \leq S^h_{\mathbf{M}_1 \to \mathbf{M}_1 \cup \mathbf{M}_2}(I - S^h_{\varnothing \to \mathbf{M}_1}(I)) \leq S^h_{\varnothing \to \mathbf{M}_2}(I)$ or that it is individually rational to pay q_2. The second inequality comes from surplus normality and the third is a direct application of the definition of value substitutes. Paying both hookups is therefore rational.

3) Let $(q_1', q_2') \in \arg\max_{(q_1, q_2) \in Q} q_1 + q_2$. We will show that $(q_1', q_2') = (\tilde{q}_1, \tilde{q}_2)$. First we need to show that $q_1' = S^h_{M_2 \to M_1 \cup M_2}(I - q_2')$ and $q_2' = S^h_{M_1 \to M_1 \cup M_2}(I - q_1')$. q_1' and q_2' cannot both be less than the corresponding surpluses, for then each could be raised slightly with out leaving Q. But is it possible that only one hookup be less than the surpluses (the other being equal)? To see why this cannot happen, suppose $q_1' = S^h_{M_2 \to M_1 \cup M_2}(I - q_2')$ but $q_2' < S^h_{M_1 \to M_1 \cup M_2}(I - q_1')$. If q_2 is raised by a small amount, then q_1' may start to exceed $S^h_{M_2 \to M_1 \cup M_2}(I - q_2)$, driving us out of Q. Nonetheless, since $\dfrac{\partial S^h_{M_2 \to M_1 \cup M_2}}{\partial I} < 1$, some combination of raising q2 and lowering q1 would leave us in Q while raising q1+q2. Therefore we know that $q_1' = S^h_{M_2 \to M_1 \cup M_2}(I - q_2')$ and $q_2' = S^h_{M_1 \to M_1 \cup M_2}(I - q_1')$, which implies that $q_1' = S^h_{M_2 \to M_1 \cup M_2}(I - S^h_{M_1 \to M_1 \cup M_2}(I - q_1'))$. However since $\tilde{q}_1$ is the only solution to this equation $q_1' = \tilde{q}_1$ and $q_2' = \tilde{q}_2$.

Therefore we see that $\forall (q_1, q_2) \in Q$, and $(q_1, q_2) \neq (\tilde{q}_1, \tilde{q}_2)$, $q_1 + q_2 < \tilde{q}_1 + \tilde{q}_2$. $\|$

APPENDIX B

We now show that if $((\mathbf{z}^f),(\mathbf{s}^f),\mathbf{p},(\mathbf{x}^h),\lambda_1,\lambda_2)$ is a fixed point of Φ, the map defined in Section 3, then an equilibrium is given by outputs $y^f = \eta^f(\mathbf{z}^f)$, prices $\mathbf{p}$, hookups $q_1^f = \lambda_1 S_1^f, q_2^f = \lambda_2 S_2^f$ for DR firms and $q_1^h = \lambda_1 S_1^h, q_2^h = \lambda_2 S_2^h$ for households (where surpluses are measured at income $I^h = \mathbf{p} \cdot \omega^h + \Pi^h$).

Step 1: Show $\mathbf{p} \in \zeta(\mathbf{y}^f) \, \forall f$.

First observe that it suffices to show that $\mathbf{g}^f = \mathbf{p} \, \forall f$, since at a fixed point of Φ_2, $\mathbf{g}^f \in \zeta(\mathbf{y}^f) \, \forall f$. Analogous to the use of Walras' law in an ordinary Walrasian output adjustment argument, we use the boundary condition of Bonnisseau and Cornet [3]. They show that $\left[y_j^f \leq -r \right] \wedge \left[\mathbf{g}^f \in \zeta(\mathbf{y}^f) \right] \Rightarrow \left[g_j^f = 0 \right]$. That is, if you are at the free-disposal part of the frontier, the price of the good being disposed is 0. Since η^f preserves the natural orientation of faces, $\left[z_j^f = 0 \right] \Rightarrow \left[y_j^f = -r \right]$. At a fixed point, the boundary condition reduces to: $\left[z_j^f = 0 \right] \Rightarrow \left[g_j^f = 0 \right]$.

Consider firm f and some good j'. Viewing Φ_1 at a fixed point, we see: $z_j^f \sum \max\left[0, p_j - g_j^f \right] = \max\left[0, p_{j'} - g_{j'}^f \right]$. Suppose that $p_{j'} > g_j^f$ it follows that $z_j^f > 0$. Also, now for all j $\left[z_j^f > 0 \right] \Rightarrow \left[p_j > g_j^f \right]$ and $\left[z_j^f = 0 \right] \Rightarrow \left[p_j \leq g_j^f \right]$. Combining this latter result with the boundary condition, $\left[z_j^f = 0 \right] \Rightarrow \left[p_j = g_j^f = 0 \right]$.

To summarize, if $p_{j'} > g_{j'}^f$ for some j' then $p_j \geq g_j^f \, \forall j$. The fact that $\mathbf{p}$ and $\mathbf{g}$ both lie on c-simplex's ensures that $p_j = g_j^f \, \forall j$. Repeating the argument $f=1,...,F$ shows $\mathbf{g}^f = \mathbf{p} \, \forall f$, and $\mathbf{p} \in \zeta(\mathbf{y}^f) \, \forall f$. Criterion 1 for TPMCPE is satisfied.

Step 2: Hookups are Paid Voluntarily and Markets Clear

Since $\lambda_1 \leq 1$ and $\lambda_2 \leq 1$, households get at least as much income as in **(IVA)**. By normality then the surplus conditions of **(IVA)** are met. Because we showed above that $\mathbf{p} \in \zeta(\mathbf{y}^f) \, \forall f$, all conditions of **(IVA)** hold, and so at a fixed point, $\Phi_5^i = -\min[0, \mathbf{p} \cdot \mathbf{y}^1] / \left[\sum S_i^h + \sum_{f>2} S_i^f \right] i = 1,2$ and hookups cover losses $\sum q_i^h + \sum_{f>2} q_i^f = -\min[0, \mathbf{p} \cdot \mathbf{y}^i], i = 1,2$.

By **(SA)** incomes exceed 0, and since **(IVA)** is defined so that surplus strictly exceeds losses, $\lambda_1 < 1$ and $\lambda_2 < 1$. Therefore $I^h - q_1^h - q_2^h > 0$ and, Φ_4 ensures that

$x^h = \arg\max\{U^h(x) \mid x \in B, p \cdot x \le I^h - q_1^h - q_2^h\}$. The conditions of the theorems in Section 2 for rational payment of both hookups are met (hookups are less than the relevant surpluses), so households rationally pay both hookups. These two observations imply the x^h's are utility-maximizing demands for the households given the box, their incomes, the prices, and the hookups.

Clearly, if DR firms choose to pay their hookups, they are profit maximizing at y^f since $p \in \zeta(y^f) \; \forall \; f$. For them also there is no reason to deviate from paying both hookups since they are less than relevant surpluses. Therefore, criterion 2 for TPMCPE is satisfied.

Critically, the hookups constructed meet losses, $\sum q_i^h + \sum_{f>2} q_i^f = -\min[0, p \cdot y^i], i = 1, 2$, so Walras' law holds, $p \cdot \sum (x^h - \omega^h) = p \cdot \sum y^i$, and so a standard argument, e.g., Debreu [14], shows that markets must clear. Hence criterion 5 for TPMCPE is satisfied. Moreover since the surplus for monopoly goods is positive, their demand is positive, and since markets clear, the IR firms produce and criterion 4 for TPMCPE is satisfied.

Finally, by standard arguments (see Debreu [14], section 5.7, equation 6), the box B can be removed and $x^h = \arg\max\{U^h(x) \mid p \cdot x \le I^h - q_1^h - q_2^h\}$. Hence criterion 3 for TPMCPE is satisfied and the fixed point corresponds to a TPMCPE.

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FIGURE 1

Only IR firm one's goods

$\mathbf{M_1}$

$S_{\{\varnothing\} \mapsto \mathbf{M_1}}$

$S_{\mathbf{M_1} \to \mathbf{M_2} \cup \mathbf{M_1}}$

$\{\varnothing\}$

First legs   Second legs

$\mathbf{M_1} \cup \mathbf{M_2}$

No IR goods

Both IR fims' goods

$S_{\{\varnothing\} \mapsto \mathbf{M_2}}$

$S_{\mathbf{M_2} \to \mathbf{M_2} \cup \mathbf{M_1}}$

$\mathbf{M_2}$

Only IR firm two's goods
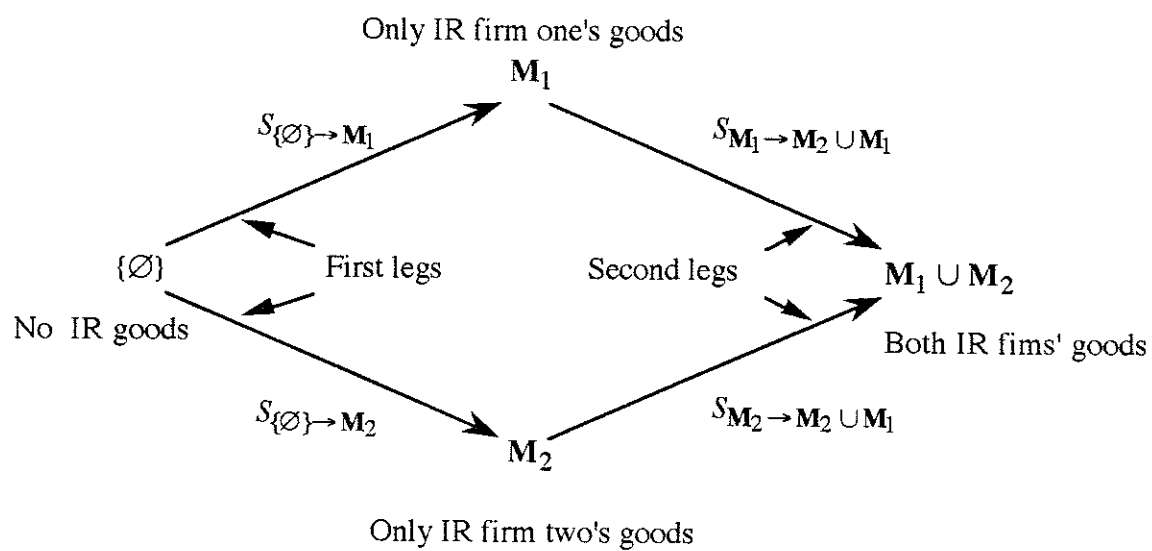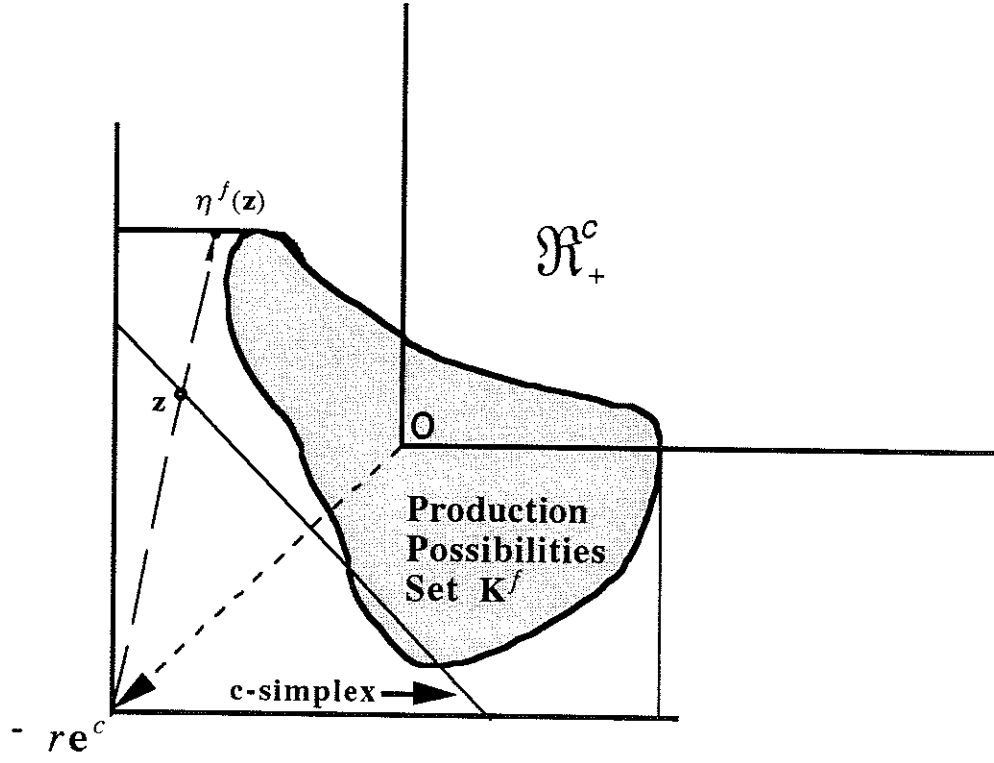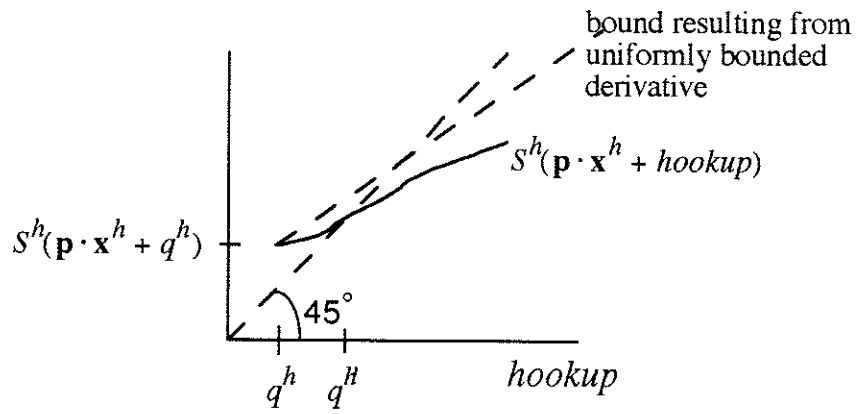
FIGURE 2

FIGURE 3

Caption for Figure 1: Each arrow in the figure represents the transition of adding an additional set of goods to the household's choice set. The arrow from $\{\varnothing\}$ to $\mathbf{M}_1$ represents adding set $\mathbf{M}_1$ when no monopoly goods are allowed and the arrow from $\mathbf{M}_2$ to $\mathbf{M}_1 \cup \mathbf{M}_2$ represents adding set $\mathbf{M}_1$ when only set $\mathbf{M}_2$ is allowed.

Caption for Figure 2: η^f maps the simplex into the boundary of the production set.

Caption for Figure 3: The fact that the derivative of surplus is uniformly bounded away from one implies that a $q^{h\prime}$ exists, such that $q^{h\prime} = S^h(\mathbf{p}\mathbf{x}^h + q^{h\prime})$.

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