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Berkeley, California

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Jerome A. Rosenthal

June 6, 1963

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Summary — A pulse modulator that offers several useful features is introduced. Some of its desirable features are that it can respond without delay to a change in input signal and that it is of very simple construction. The modulator can be used as an amplifier, a multiplier, or a divider. It can also be used to control either proportional or on-off systems.

I. INTRODUCTION

This pulse modulator resulted from a desire to build a simple and efficient power amplifier. It has been known for some time that an amplifier with a saturated pulse output can satisfy both of those requirements, since such an amplifier is easy to construct and dissipates very little power in its output stages. Most commonly, pulse-width modulation amplifiers have been built to satisfy these needs. Operationally, a pulse-width modulator periodically samples the amplitude of an input signal and generates a pulse whose width is proportional to this amplitude. In terms of physical components, a pulse-width modulator usually consists of a ramp generator and a comparator.

The pulse modulator discussed in this paper can be of simple construction, requiring only a relay, a resistor, and a capacitor, although more complicated designs will give better performance. The modulator operates by continuously sensing (not sampling) the input signal. The output of the modulator is a pulse train whose frequency and pulse duration both vary with the input signal. By continuously sensing the input signal, the modulator can respond instantaneously and with maximum effort to a change in the input signal. Unlike pulse-width modulation, the output pulses of this modulator have a minimum duration, thereby placing less stringent requirements on the frequency response of the elements in the system. As with other pulse

modulators, this modulator is capable of both proportional and on-off control.

While analyzing the modulator, we discovered that it could easily be converted into an analogue multiplier or divider. All three uses of the modulator (amplification, multiplication, and division) can be performed separately or in combination.

Although of quite dissimilar construction, our modulator and a modulator described by Schaefer¹ produce a similar effect. Schaefer's modulator consists of two current sources and a negative resistance. After Schaefer, we call the type of modulation produced by our modulator pulse-ratio modulation.

II. DESCRIPTION OF THE MODULATOR

The modulator that we describe has several variations. We begin our discussion with the variation that is bistable, bipolar, and has its low pass filter in the feedback loop (Fig. 1).

As can be seen from Fig. 1, the main element of the modulator is a relay (or regenerative switch). There is a feedback path going from the output to the input of the relay; the circuit elements of the feedback path are a low pass filter and an attenuator. The contact voltages of the relay are $+A$ volts and $-A$ volts, the hysteresis of the relay is $2\Delta V$, and the transfer function of the attenuator is B .

In a very general way, certain things can be said of the waveforms at different parts of the circuit. Assume that there is a dc voltage impressed on the input. The output voltage, V_o , oscillates between two values, $+A$ and $-A$. The feedback path passes only the low frequency components of the output and attenuates them by a factor of B . The feedback voltage, V_f , is subtracted from the input voltage to form the error voltage, V_e . Now, the circuit acts to keep the magnitude of the error voltage small. If the magnitude of the error voltage exceeds ΔV , the output changes polarity, thereby decreasing the magnitude of the error voltage. Therefore, if the input is a dc voltage, the amplitude of the error voltage will oscillate between $+\Delta V$ and $-\Delta V$. Since the error voltage is kept small, the feedback voltage, V_f , must

¹ R. A. Schaefer, A New Pulse Modulator for Accurate DC Amplification with Linear or Nonlinear Devices, IRE Transactions on Instrumentation, Vol. I-11, 2, pp. 34-47, September, 1962.

be approximately equal to the input voltage, V_1 . This means that the output of the low pass filter must be approximately equal to $1/B$ times the input voltage. Therefore, the dc value of the output must be approximately equal to $1/B$ times the input voltage. In summary, we see that the modulator has converted the dc input into a signal that oscillates between $+A$ volts and $-A$ volts, and that the dc value of the output is approximately equal to $1/B$ times the input (the gain of the modulator is $1/B$). We also note that the low pass filter in the feedback loop acts as a demodulator of the output voltage.

Figure 2 shows some waveforms that were obtained from an experimental model of the modulator. The results of the experiment are discussed in detail in Sec. V. It might be helpful now to examine the waveforms in order to better understand the operation of the modulator. The upper trace in Fig. 2 is the input waveform, V_1 . The middle trace is the output voltage, V_0 . It can be seen that, for a positive input, the swinger of the relay spends most of its time on the contact that has $+A$ volts on it. For a negative input voltage the swinger spends most of its time on the $-A$ volt contact. The lowest trace is a picture of the feedback voltage, V_f . It looks very much like the input voltage, except that it has a ripple that is approximately equal to $2\Delta V$.

A more precise relationship between the input and the output voltage can be derived with the aid of Fig. 3. As indicated in that figure, the feedback voltage, V_f , will follow an exponential trajectory in time. The trajectory may be either positive going or negative going. Two representative trajectories are shown in Fig. 3.

Consider the steady-state response to a dc input, V_1 . The dwell time on the positive contact equals the time taken to travel from $V_1 - \Delta V$ to $V_1 + \Delta V$ on the positive going trajectory. The dwell time on the negative contact equals the time taken to travel from $V_1 + \Delta V$ to $V_1 - \Delta V$ on the negative going trajectory. These times are functions of V_1 .

The positive and negative dwell times may be derived as follows² (Fig. 3):

² The modulator has a relatively linear gain function for $-AB + \Delta V \leq V_1 \leq AB - \Delta V$. Outside of this range the modulator saturates. Unless otherwise noted, all equations apply only to inputs within the linear range of the modulator.

$$V_1 + \Delta V = -AB(1 - 2\exp -t_1/T) = AB(1 - 2\exp -t_4/T), \quad (1)$$

and

$$V_1 - \Delta V = -AB(1 - 2\exp -t_2/T) = AB(1 - 2\exp -t_3/T), \quad (2)$$

where $T = RC$. From Eqs. (1) and (2), we obtain

$$\frac{V_1 + \Delta V + AB}{V_1 - \Delta V + AB} = \frac{\exp -t_1/T}{\exp -t_2/T}. \quad (3)$$

If the negative dwell time is called t_{off} , then we have

$$t_{\text{off}} = t_2 - t_1 = T \ln \left(\frac{V_1 + \Delta V + AB}{V_1 - \Delta V + AB} \right) = T \ln \left(\frac{1 + \Delta V/(AB + V_1)}{1 - \Delta V/(AB + V_1)} \right). \quad (4)$$

To find the expression for the positive dwell time, t_{on} , we again use Eqs. (1) and (2); we then have

$$\frac{AB - V_1 + \Delta V}{AB - V_1 - \Delta V} = \frac{\exp -t_3/T}{\exp -t_4/T},$$

$$t_{\text{on}} = t_4 - t_3 = T \ln \left(\frac{AB - V_1 + \Delta V}{AB - V_1 - \Delta V} \right).$$

and

$$t_{\text{on}} = T \ln \left(\frac{1 + \Delta V/(AB - V_1)}{1 - \Delta V/(AB - V_1)} \right). \quad (5)$$

Plots of t_{on} , t_{off} and the period, $P = t_{\text{on}} + t_{\text{off}}$ for $\Delta V = 0.01$ V, $AB = 1$ V, and $T = 10^{-3}$ sec, appear in Fig. 4.

An alternative, but only approximate derivation of the positive and negative dwell times (Fig. 5) is based on the assumption that the slope of the exponential is constant during the dwell time. Thus, on the positive going trajectory (Fig. 5), we would have

$$V_1 = AB(1 - 2\exp -t_1/T). \quad (6)$$

The slope of the positive going trajectory at V_1 is

$$m_1 = (2AB/T) \exp -t_1/T. \quad (7)$$

Combining Eqs. (6) and (7), we obtain

$$2AB \exp -t_1/T = AB - V_1 \text{ and } m_1 = (AB - V_1)/T. \quad (8)$$

If we call the approximate value of the positive dwell time that is derived via these equations t_{on}^* , then

$$t_{on}^* = 2\Delta V/m_1. \quad (9)$$

Therefore, we have

$$t_{on}^* = 2\Delta VT/(AB - V_1). \quad (10)$$

To find the approximate expression for t_{off}^* it is necessary to find the slope of the negative going trajectory at V_1 (Fig. 5). We have

$$V_1 = -AB(1 - 2\exp -t_2/T). \quad (11)$$

The slope of the negative going trajectory at V_1 is

$$m_2 = - (2AB/T) \exp -t_2/T. \quad (12)$$

Combining Eqs. (11) and (12), we obtain

$$m_2 = - (V_1 + AB)/T. \quad (13)$$

But we have $t_{off}^* = -2\Delta V/m_2$. So, we see that

$$t_{off}^* = 2\Delta VT/(AB + V_1). \quad (14)$$

Plots of t_{on}^* , t_{off}^* and $P^* = t_{on}^* + t_{off}^*$ for $AB = 1V$, $\Delta V = 0.01V$, and $T = 10^{-3}$ sec, appear in Fig. 6.

Since the approximate expressions for t_{on} and t_{off} [Eqs. (10) and (14)] are more tractable than the exact expressions [Eqs. (4) and (5)], it is convenient to use the approximate expressions. We can justify doing this as follows:

From Eqs. (5) and (10) we have

$$\frac{t_{on}}{T} = \ln \left(\frac{1 + t_{on}^*/2T}{1 - t_{on}^*/2T} \right). \quad (15)$$

and by writing the series expansion for t_{on}/T we obtain

$$t_{on}/T = t_{on}^*/T + 2/3 \left(t_{on}^*/2T \right)^3 + 2/5 \left(t_{on}^*/2T \right)^5 + \dots \quad (16)$$

So if we have $t_{on}^*/2T \ll 1$, then $t_{on} \approx t_{on}^*$.

The relative error is

$$\frac{t_{on} - t_{on}^*}{t_{on}} = \frac{\frac{2}{3} \left(\frac{t_{on}^*}{2T} \right)^3 + \frac{2}{5} \left(\frac{t_{on}^*}{2T} \right)^5 + \frac{2}{7} \left(\frac{t_{on}^*}{2T} \right)^7 + \dots}{\frac{t_{on}^*}{T} + \frac{2}{3} \left(\frac{t_{on}^*}{2T} \right)^3 + \frac{2}{5} \left(\frac{t_{on}^*}{2T} \right)^5 + \dots} \quad (17)$$

When the modulator is in its linear range, V_1 ranges from $-AB + \Delta V$ to $AB - \Delta V$. This range for V_1 corresponds to a range for $t_{on}^*/2T$ [Eq.(10)] that goes from $\Delta V/2AB - \Delta V$ to 1 (where $\Delta V/2AB - \Delta V$ is much less than 1 if $AB \gg \Delta V$). Since we find $t_{on}^*/2T \leq 1$ for all values of V_1 within the linear range of the modulator, and since the infinite series $2/3 + 2/5 + 2/7 + \dots$ diverges, the maximum relative error is 1.

To determine the relationship between t_{off} and t_{off}^* , examine Eqs.(4) and (14):

$$\frac{t_{off}}{T} = \ln \left(\frac{1 + t_{off}^*/2T}{1 - t_{off}^*/2T} \right). \quad (18)$$

and

$$t_{\text{off}}/T = t_{\text{off}}^*/T + 2/3 \left(\frac{t_{\text{off}}^*}{2T} \right)^3 + 2/5 \left(\frac{t_{\text{off}}^*}{2T} \right)^5 + \dots \quad (19)$$

So if we have $t_{\text{off}}^*/2T \ll 1$, then $t_{\text{off}} \approx t_{\text{off}}^*$.

The relative error is

$$\frac{t_{\text{off}} - t_{\text{off}}^*}{t_{\text{off}}} = \frac{\frac{2}{3} \left(\frac{t_{\text{off}}^*}{2T} \right)^3 + \frac{2}{5} \left(\frac{t_{\text{off}}^*}{2T} \right)^5 + \frac{2}{7} \left(\frac{t_{\text{off}}^*}{2T} \right)^7 + \dots}{\frac{t_{\text{off}}^*}{T} + \frac{2}{3} \left(\frac{t_{\text{off}}^*}{2T} \right)^3 + \frac{2}{5} \left(\frac{t_{\text{off}}^*}{2T} \right)^5 + \dots} \quad (20)$$

It can be seen that the expression for the relative error in t_{off} is of the same form as that for t_{on} . Furthermore, $t_{\text{off}}^*/2T$ also ranges between 1 and $\Delta V/(2AB - \Delta V)$. Therefore, the relative error in t_{off} never exceeds 1. A representation of the per cent error in t_{on} as a function of V_1 and as a function of t_{on}^*/T (when $\Delta V = 0.01$ V, $AB = 1$ V, and $T = 10^{-3}$ sec) is shown in Fig. 7. The representation for the per cent error in t_{off} would be of the same shape. As can be seen from Fig. 7, the maximum value of V_1 (with the modulator still being in its linear range) is 0.99V. When V_1 is as high as 0.9V, the error is only 0.6%. We may then conclude that the approximation is a fairly valid one over 90% of the range of the modulator.

In the rest of this analysis we will use the simple expressions for t_{on} and t_{off} and not use the starred notation.

The period of the pulse train is (Fig. 8)

$$P = t_{\text{on}} + t_{\text{off}} = 4\Delta VT \left[\frac{AB}{(AB)^2 - V_1^2} \right] \quad (21)$$

The duty cycle of the pulse train is (Fig. 9)

$$D_0 = \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} = \frac{AB + V_1}{2AB} \quad (22)$$

The dc value of the output voltage is

$$V_0 = AD_0 - A(1 - D_0) = A(2D_0 - 1). \quad (23)$$

Combining Eqs. (22) and (23) we get

$$V_0 = V_1/B. \quad (24)$$

The dc gain of the circuit is

$$K = V_0/V_1 = 1/B. \quad (25)$$

As mentioned at the beginning of this section, our modulator can have several variations. Figure 10 shows a version that is bistable, bipolar, and has the low pass filter in the forward path. The only difference between Fig. 1 and Fig. 10 is the placing of the low pass filter. The operation of these two modulators is almost exactly the same. In fact, all of the equations thus far derived apply to the modulator of Fig. 10. In a later section, we shall see that the main difference between these two modulators is their response to rapidly varying input signals (thus far we have only discussed steady state dc signals).

If the relay in the forward path is a mechanical relay (as opposed to an electronic relay), then we may use the mechanical time constant of the relay as the time constant of the low pass filter. If that is the case, then Fig. 10 shows the simplest version of our modulator. Since the low pass filter is incorporated in the relay, the modulator can reduce to a relay in the forward path and, at most, two resistors in the feedback path.

III. THE TIME RESPONSE OF THE MODULATOR TO OTHER THAN STEADY STATE DC INPUTS

It is difficult to find the time response of the modulator to a generalized input signal. However, it is possible, for certain types of signals, to find useful analytic expressions for the time response.

A. Step Inputs

If the modulator (Fig. 1) receives a step input of amplitude C , it responds by saturating. That is, its initial response will be a step of amplitude A . The output will continue to be A volts until the RC network has integrated to $(C + \Delta V)/B$. At that time, the transient response will have been completed and the modulator will have a steady state output corresponding to an input voltage of C volts.

If we assume that the input step occurs at that instant when $V_f = D$ volts ($|D| \leq \Delta V$), then the transient response of the modulator will be a pulse of width

$$W = T \ln \left(\frac{AB - D}{AB - C - \Delta V} \right). \quad (26)$$

It is interesting that this modulator, unlike other pulse modulators, offers an instantaneous maximum effort transient response. In that regard it is similar to a maximum effort (bang-bang) control system.

B. Response to Slowly Varying Signals

If we assume that the slope of the input signal and the slope of V_f are both constant during the dwell time (a kind of piecewise linear approximation), we can arrive at useful expressions for the response.

Assume that the modulator has just switched positive at the instant (t_1) that the input signal first appears. Label the dwell times $T_1, T_2, \dots, T_n$ (Fig. 11), the positive dwell times having odd subscripts and the negative dwell times having even subscripts. Label the switching instants $t_1, t_2, \dots, t_n$. Now, we have

$$T_n = \Delta V_n / |m_n|, \quad (27)$$

where ΔV_n is the change in V_f during the n th dwell time and m_n is the slope of V_f during the n th dwell time. For odd n (positive dwell times) we have

$$\Delta V_n = 2\Delta V + V_1'(t_n) (t_{n+1} - t_n), \quad (28)$$

and

$$m_n = \frac{2AB}{T} \exp -t_n/T = \frac{AB - V_1(t_n)}{T}. \quad (29)$$

Therefore, we get

$$T_n = t_{n+1} - t_n = \frac{2\Delta VT}{AB - V_1(t_n) - V_1'(t_n)T} \quad (30)$$

For negative dwell times (n even), we have

$$\Delta V_n = 2\Delta V - V_1'(t_n)(t_{n+1} - t_n) \quad (31)$$

and

$$m_n = - \frac{V_1(t_n) + AB}{T} \quad (32)$$

Therefore, we obtain

$$T_n = t_{n+1} - t_n = \frac{2\Delta VT}{AB + V_1(t_n) + V_1'(t_n)T} \quad (33)$$

If $V_1'(t_n) = 0$, Eqs. (30) and (33) reduce to Eqs. (10) and (14), which describe the steady-state dc response of the modulator.

As an example of the application of Eqs. (30) and (33), let V_1 be a ramp with slope a . Then, we get

$$T_n = \frac{2\Delta VT}{AB - V_1(t_n) - aT} \quad (n \text{ odd}),$$

and

$$T_n = \frac{2\Delta VT}{AB + V_1(t_n) + aT} \quad (n \text{ even}),$$

where $t_n = \sum_{k=1}^n T_k$.

The response to a sine input can be found if we let $V_1 = C \sin 2\pi ft$; we then find

$$T_n = \frac{2\Delta VT}{AB - C \sin 2\pi ft_n - TC \cos 2\pi ft_n} \quad (n \text{ odd}),$$

and

$$T_n = \frac{2\Delta VT}{AB + C \sin 2\pi ft_n + TC \cos 2\pi ft_n} \quad (n \text{ even}).$$

C. Response to Rapidly Varying Signals

The two versions of the modulator respond quite differently to rapidly varying signals. For the Fig. 1 modulator, we may consider the feedback path of the modulator open circuited when the signal has a slope much greater than $1/T$. That is, the signal changes faster than the RC network can respond. For such signals, the modulator degenerates to a relay or regenerative switch.

The high-frequency response of the Fig. 10 modulator is much better behaved. Since the low pass filter is in the forward path, the high-frequency response of the modulator is attenuated. Whenever frequencies of the order of $1/T$ or greater are present, the Fig. 10 modulator will be the better modulator of the two.

IV. APPLICATION FOR THE MODULATOR

The modulator has at least four applications. It can be used as a modulator, an amplifier, a multiplier, or a divider.

A. As a Modulator

The operation of the circuit as a modulator has been described in the last two sections, where the response to various kinds of signals has been derived. In comparison to some other pulse modulators, it has two desirable features:

First, it responds instantaneously and with maximum effort to changes in the input. The modulator does not have to wait for sampling instants, and therefore does not exhibit a delay.

Second, t_{on} and t_{off} have minimum values greater than zero. This means that the transmitting medium has to have a bandwidth no greater than

that required to transmit the minimum pulse width. The minimum t_{on} occurs when $V_1 = AB - \Delta V$ and the minimum t_{off} occurs when $V_1 = -AB + \Delta V$. From Eqs. (10) and (14) we have

$$\min t_{on} = \min t_{off} = \frac{2T\Delta V}{2AB - \Delta V} \quad (34)$$

B. As An Amplifier

An undesirable feature of the modulator is that it has a square-wave output (dc value equal to zero) for an input of zero volts. This results in a large amount of dissipation in the load when the input is zero volts. It would be better, from the viewpoint of efficiency, for the output to be identically zero when the input is zero.

There are at least two ways of making the amplifier output be zero volts for an input of zero volts and still maintain a bipolar output. One method is to detect the absolute value of V_1 and gate the output of the amplifier when $|V_1|$ is greater than some threshold voltage. The previous analysis would apply to such an amplifier except that the amplifier would have a dead zone for input signals whose absolute values were below the threshold level.

A second way of achieving the same effect (Fig. 12) is to have a tristable relay in the forward loop, rather than the bistable relay of Figs. 1 and 10. Tristable relays are manufactured by several companies and are usually called bipolar relays. The swinger of a bipolar relay is connected to one of two contacts according to whether the coil voltage is positive or negative. If the coil voltage is insufficient to energize the relay, the swinger comes in contact with neither contact.

A tristable relay can be constructed from two bistable relays by making each relay sensitive to a different polarity input. This can be accomplished by placing a diode in series with each coil. The output of the of the two relays must then be added.

An electronic tristable relay can be constructed with two regenerative switches in parallel, the output of the switches being added.

The sensitivity of the tristable relay can be modified by inserting an amplifier between the summing junction and the relay. This has the effect of

dividing the dead zone and hysteresis of the relay by the gain of the amplifier. The dead zone can be made zero by properly biasing the relay (or regenerative amplifier).

The analytic expressions for the amplifier of Fig. 12 are different than those for the amplifier of Fig. 1. The operation of the tristable amplifier can be explained with the aid of Fig. 13.³ From Fig. 13, we see that

$$V_1 + \Delta V = AB \exp -t_1/T = AB(1 - \exp -t_4/T), \quad (35)$$

$$V_1 = AB \exp -t_2/T = AB(1 - \exp -t_3/T), \quad (36)$$

$$t_2 - t_1 = t_{\text{off}} = T \ln(1 + \Delta V/|V_1|), \quad (37)$$

and

$$t_4 - t_3 = t_{\text{on}} = T \ln \left(\frac{1}{1 - \frac{\Delta V}{|AB| - |V_1|}} \right), \quad (38)$$

where the polarity of the output voltage is the same as the sign of V_1 , and t_{off} is the time during which $V_0 = 0$.

The approximate equations, analogous to Eqs. (10) and (14) of the bistable case, are

$$t_{\text{on}}^* = \frac{\Delta VT}{|AB| - |V_1|} \quad (39)$$

and

$$t_{\text{off}}^* = \frac{\Delta VT}{|V_1|}. \quad (40)$$

The approximate and exact formulas are related by

$$t_{\text{off}}/T = \ln(1 + t_{\text{off}}^*/T) \quad (41)$$

and

$$t_{\text{on}}/T = \ln \left(\frac{1}{1 - \frac{t_{\text{on}}^*}{T}} \right). \quad (42)$$

Therefore, we have

$$t_{\text{off}}/T = \frac{t_{\text{off}}^*}{T} + \frac{1}{2} \left(\frac{t_{\text{off}}^*}{T} \right)^2 + \frac{1}{3} \left(\frac{t_{\text{off}}^*}{T} \right)^3 + \dots$$

³ The amplifier is in its linear range for $\Delta V \leq |V_1| \leq AB - \Delta V$. These equations apply only when the input signal is within the above limits.

which indicates that t_{off}^*/T is close to t_{off}/T for small values of t_{off}^*/T .

We can also observe that

$$\begin{aligned} t_{\text{on}}/T &= \ln \left[1 + t_{\text{on}}^*/T + \left(t_{\text{on}}^*/T \right)^2 + \left(t_{\text{on}}^*/T \right)^3 + \dots \right] \\ &= t_{\text{on}}^*/T + \frac{1}{2} \left(\frac{t_{\text{on}}^*}{T} \right)^2 + \frac{1}{3} \left(\frac{t_{\text{on}}^*}{T} \right)^3 + \dots, \end{aligned}$$

which indicates that t_{on}^*/T is close to t_{on}/T for small values of t_{on}^*/T .

Plots of t_{off} and t_{off}^* are shown in Fig. 14, t_{on} and t_{on}^* in Fig. 15, and $P = t_{\text{on}} + t_{\text{off}}$ and $P^* = t_{\text{on}}^* + t_{\text{off}}^*$ in Fig. 16. The gain of the tristable amplifier is (using the approximate expressions for t_{on} and t_{off})

$$K = \frac{A}{V_1} \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} = \frac{1}{B}. \quad (43)$$

This amplifier is especially well suited for use with transistorized power stages. The advantage results from the fact that the power transistors are always, except during switching, in cutoff or in saturation. While in cutoff the current through the power transistors is small and while in saturation the voltage drop across the transistors is small. Therefore, compared to linear dc amplifiers, relatively little power is dissipated in the transistors for a given power delivered to the load. Since transistors are severely limited in the amount of power that they can dissipate (that is, heat), this type of amplifier results in simpler driving stages.

C. As a Multiplier and as a Divider

If we modify the modulator by placing a saturating amplifier (a regenerative switch or relay would do as well) in the feedback loop, we can use the modulator as a divider, or a multiplier, or both (Fig. 17). The bistable relay in the forward loop switches between +E and -E volts, while the saturating amplifier switches between +A and -A volts. From Eqs.(22) and (23) we have

$$V_0 = EV_1/AB, \quad (44)$$

where Eq. (22) applies directly, while in Eq. (23) the "A" must be changed to "E".

If we allow A to be time varying, the modulator can function as a divider. The modulation will still be of the same type discussed previously. The equations of Secs. II and III still apply [with the exception of Eqs. (24) and (25)]. In equation (23) one must change "A" to "E".

If we cause E to be time varying, the modulator functions as a multiplier. In that case, the type of modulation has changed, since the frequency, width, and amplitude of the pulse train all vary as a function of EV_1 . All the expressions of Secs. II and III apply. In Eq. (23) one must substitute "E" for "A".

It is obvious that the modulator can function simultaneously as a multiplier and as a divider.

V. EXPERIMENTAL RESULTS

Two electronic versions of the modulator were constructed-- a bistable model (Fig. 18) and a tristable model (Figs. 21 and 22). The regenerative switch of the forward loop of the bistable model consists of a dc amplifier with positive feedback (via the 75K resistor) from the output to the base of the input transistor. Important parameters that were determined are $C = 0.5 \mu\text{F}$, $R = 930 \Omega$, $T = 462 \mu\text{sec}$, $AB = 1.1 \text{ V}$, and $2\Delta V = 0.150 \text{ V}$. The gain of the amplifier without regenerative feedback is 300. The potentiometer in the emitters of the differential stage was set for zero dc output with zero input. The positive and negative dwell times for different input voltages were measured with an Eput meter (Fig. 19). The results of these measurements are shown in Fig. 20. Also shown in Fig. 20 are some typical values for the dwell times that were calculated by means of Eqs. (4) and (5). The minimum t_{on} and the minimum t_{off} were both $32 \mu\text{sec}$. Minimum t_{on} occurred when $V_1 = 1 \text{ V}$, and minimum t_{off} occurred when $V_1 = -1 \text{ V}$. Using Eq. (34), one calculates a minimum dwell time of $31.6 \mu\text{sec}$. Some waveforms exhibited by this modulator are shown in Fig. 2.

The results of this experiment were in close agreement with theory. However, the largest discrepancy occurs when V_1 is close in value to AB and when $\Delta V/(V_1 + AB)$ is close in value to 1, because it is for those values that Eqs. (4) and (5) become most sensitive to experimental error in determining ΔV and AB.

The tristable modulator (Figs. 21 and 22) was used to drive a 100-W (output) dc printed-circuit motor. The amplifier delivered as much as 15A at 12 V to the motor. The voltage gain of the modulator section of the amplifier was about 50 and the voltage gain of the power stage was 2. Tachometer feedback was placed around the amplifier and motor and a dc gain of 30, flat to 30 cycles, was achieved (the output velocity being underdamped).

Figure 21 shows each side of the power stage being driven by a signal derived from one regenerative switch. If the modulator should malfunction, there is a danger that all the transistors in the bridge would turn on, destroying the output transistors. One could guard against that mishap by logically operating on the outputs of the two regenerative switches. One side of the power amplifier could be turned on by the logical function $A \cdot \bar{B}$ while the other side would be turned on by the logical function $\bar{A} \cdot B$. This type of operation is possible because the output of the regenerative switch is a train of saturated pulses.

VI. CONCLUSION

It has been shown that a pulse modulator, which can be constructed with simple components, can be used as an amplifier, a divider, or as a multiplier. As an amplifier, the modulator has fairly linear gain over most of its range.

The accuracy of the modulator is mainly determined by how well one can control ΔV . All of the other parameters (T, A, and B) can easily be realized such that they are both precise and drift free. If a relay is used in the forward loop of the modulator, ΔV will not be very stably or accurately determined (although this might not be a limitation for many applications). If a regenerative switch (whose basic elements is a differential amplifier) is used in the forward loop, then ΔV can be much more precisely and stably determined.

Our modulator and Schaefer's⁴ share some advantages over other forms of pulse modulation. They both respond instantaneously with maximum effort to a change in input signal. They both have minimum on and off times (which conserves bandwidth).

⁴ R. A. Schaefer, op cit., reference 1.

The high-frequency gain of one version of the modulator is excessive, although this fault can be mitigated by proper filtering or by using the version shown in Fig. 10. In terms of realizing a variety of configurations our modulator seems to offer many advantages. In his article, Schaefer offers only a unipolar amplifier and it is not obvious how a bipolar model could be simply constructed. Our modulator, besides being used in a variety of bipolar and unipolar configurations, can also be used as a multiplier or divider.

There are still some variations of this circuit that might be worth investigating. One such variation would be to place something other than a low pass filter in the feedback path (e. g., a delay line). It might also be worthwhile to place in the feedback path an integrating network whose time constant is a function of input voltage. This could be accomplished by replacing the R of the RC network with a resistor whose resistance was a function of the voltage across it.

It is perhaps interesting that such a simple circuit lends itself to such varied and useful applications. This is especially striking when one realizes that the modulator is directly analogous to so simple a system as a thermostatically controlled heater. In that analogy, the relay in the forward loop of the modulator is analogous to the bimetallic strip in the thermostat, V_1 is analogous to the setting of the thermostat, the RC element is analogous to the thermal medium coupling the heater to the thermostat, and V_0 is analogous to the pulses of heat that are emitted by the heating element.

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FIGURE LEGENDS

- Fig. 1. The basic modulator (bistable version).
- Fig. 2. Wave forms exhibited in experimental modulator
 - (a) Input voltage Time scale: 2 msec/division
 - (b) Output voltage Voltage scale: (a) and (c) 1 V/division
 - (c) Feedback voltage (b) 10 V/division.
- Fig. 3. Typical operating trajectories for the bistable modulator.
- Fig. 4. t_{on} , t_{off} and $P = t_{on} + t_{off}$ when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec (bistable modulator).
- Fig. 5. Typical trajectories for the bistable modulator (approximate method).
- Fig. 6. t_{on}^* , t_{off}^* and $P^* = t_{on}^* + t_{off}^*$ when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec (bistable modulator).
- Fig. 7. Percent error for t_{on}^* relative to t_{on} when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec.
- Fig. 8. The period of the pulse train when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec (bistable modulator).
- Fig. 9. Duty cycle of pulse train when $AB = 1$ volt (bistable modulator).
- Fig. 10. Modulator with low pass filter in forward loop.
- Fig. 11. Response of the modulator to slowly varying signals.
- Fig. 12. The tristable modulator.
- Fig. 13. Typical trajectories for the tristable modulator.
- Fig. 14. t_{off} and t_{off}^* when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec (tristable modulator).
- Fig. 15. t_{on} and t_{on}^* when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec (tristable modulator).
- Fig. 16. The pulse period and the approximate pulse period when $\Delta V = 0.01$ volts, $AB = 1$ volt, and $T = 10^{-3}$ sec (tristable modulator).
- Fig. 17. The multiplier or divider: $V_0 = (1/B) (EV_1/A)$.
- Fig. 18. Circuit diagram of bistable modulator used in experimental setup.
- Fig. 19. Experimental setup used to test the modulator.
- Fig. 20. The results of the experiment using the bistable modulator.
- Fig. 21. Block diagram of experimental tristable amplifier.
- Fig. 22. The experimental tristable amplifier.

MU-30507

Fig. 1

ZN-3700

Fig. 2

MU-30499

Fig. 3

MU-30502

Fig. 4

MU-30501

Fig. 5

MU-30500

Fig. 6

Fig. 7

MU-30504

Fig. 8

MU-30505

Fig. 9

MU-30498

Fig. 10

MU-30506

Fig. 11

MU-30519

Fig. 12

MU-30509

Fig. 13

MU-30510

Fig. 14

MU-30511

Fig. 15

MU-30512

Fig. 16

MU-30513

Fig. 17

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Fig. 18

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Fig. 19

Fig. 20

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Fig. 21

MU-30516

Fig. 22

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