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Author
Stevenson, M.L.

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February 1, 1956

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ABSTRACT

The phase space available to the decay products of $\tau \rightarrow 3\pi$, $\Sigma \rightarrow p + \pi$, $\Lambda \rightarrow p + \pi$, and $\Xi \rightarrow \Lambda + \pi$ compared to $\theta \rightarrow 2\pi$ gives the following ratios:

$$\frac{\rho(\tau)}{\rho(\theta)} \sim 10^{-3} \text{ to } 10^{-4}, \quad \frac{\rho(\Sigma)}{\rho(\theta)} = 2.6, \quad \frac{\rho(\Lambda)}{\rho(\theta)} = 1.1, \text{ and } \frac{\rho(\Xi)}{\rho(\theta)} = 0.91\,(2S_{\Lambda} + 1),$$

which are then compared with the following observed decay rates

$$\frac{\tau_{\theta^o}}{\tau_{\tau \rightarrow 3\pi} \approx 10\,\tau_{\tau}} \approx 10^{-3}, \quad \frac{\tau_{\theta^o}}{\tau_{\Sigma}} = 5^{+5}_{-2}, \quad \frac{\tau_{\theta^o}}{\tau_{\Lambda^o}} = 0.4^{+0.3}_{-0.1}, \quad \frac{\tau_{\theta^o}}{\tau_{\Xi}} \sim 1$$

The ratio of $\frac{\rho(\tau)}{\rho(\theta)} \sim 10^{-3}$ leads one to suspect that the Lee-Orear cascade process with the τ as the parent particle is the most likely on the basis of phase space arguments alone. The possibility of explaining the observed relative decay modes, the masses, and the equal lifetimes of the K mesons on the basis of the Lee-Orear scheme, and the parity conjugation scheme of Gell-Mann and of Lee and Yang is considered.

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Heavy-Meson Lifetimes

In an attempt to reconcile the different spin-parity properties of the τ and θ meson[1, 2, 3] with the same apparent lifetimes[4, 5, 6, 7] and approximately equal masses,[8] Lee and Orear[9] (henceforth referred to as LO) have suggested a genetic relationship between the τ and θ. In this scheme, the daughter particle would have a lifetime of the order of 10^{-9} sec or less, very short compared with the lifetime of the parent. The observed apparent lifetimes would then just measure the lifetime of the parent. LO considered two possibilities, one in which the θ was the parent and the τ the daughter; the other in which the roles were reversed.

If the phase space of the decay products is an important factor in determining the relative decay probabilities of the τ and θ, then the ratio of the phase space for 3π decay to that for 2π decay might tell which particle would most likely be the parent.

The result of this calculation is

$$\frac{R_3}{R_2} = \frac{\left(\frac{dN}{dE}\right)_{3\pi}}{\left(\frac{dN}{dE}\right)_{2\pi}} = \frac{1}{2\pi\, 3^{3/2}} \quad \frac{(W)^2}{(cp)(m_k c^2)} \quad \frac{V}{\left(\frac{\hbar}{m_\pi c}\right)^3} \qquad (1)$$

where W is the kinetic energy of the three π's associated with the τ (= 75 Mev) and p is the momentum of either of two π's associated with the θ ($\approx$ 200 Mev/c). The relative decay rate depends upon the volume V in which the reaction takes place. If one chooses a volume $V \approx \frac{4}{3}\pi\left(\frac{\hbar}{m_k c}\right)^3$, then $\frac{R_3}{R_2}$ becomes $\sim 2 \times 10^{-4}$. However, the volume V might conceivably be as large as $\frac{4}{3}\pi\left(\frac{\hbar}{m_\pi c}\right)^3$; then, $\frac{R_3}{R_2} \sim 7 \times 10^{-3}$. Angular-momentum conservation could enhance the 3π decay relative to the 2π decay by a factor of the order of two, in addition to the V factor, if the τ and θ are both assumed to have spin zero. Even if the interaction volume is as large as $\frac{4}{3}\pi\left(\frac{\hbar}{m_\pi c}\right)^3$ the ratio $\frac{R_3}{R_2}$ is still significantly small.

Similar phase-space calculations have been made for the hyperon decays. These are in rough agreement with the observed lifetime ratios and are presented later in this paper.

In addition to the above phase-space considerations, an order-of-magnitude calculation similar to that of Fermi[10] for the π^0 decay was made. The following diagrams show the assumed intermediate states involved in the decay processes. Here e_4 is the constant for the coupling of the K meson to the nucleon. This coupling involves a change in isotopic spin from $I = \frac{1}{2}$, $I_z = \frac{1}{2}$ for the K to $I' = 1$, $I'_z = 1$ for the $p, \bar{n}$, and hence a "strangeness" change of one unit. Further, e_2 is the usual Yukawa coupling constant and p and $\bar{n}$ are the virtual proton-anti-neutron pair.

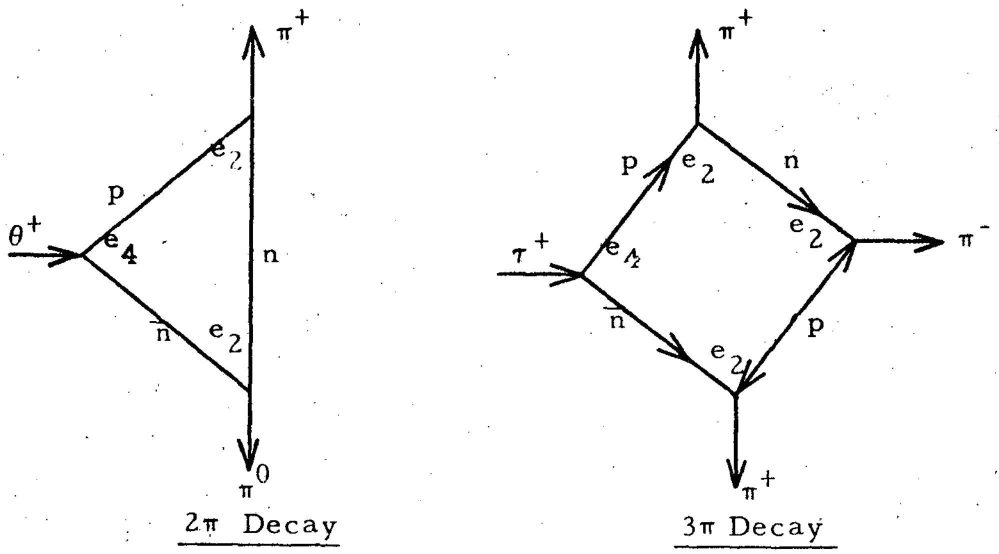

Fig. 1 Decay diagrams.

The result of this calculation is

$$\frac{R_3}{R_2} \approx 2 \times 10^{-4} \left(\frac{e_2^{\,2}}{4\pi\hbar c}\right), \tag{2}$$

where $\frac{e_2^{\,2}}{4\pi\hbar c}$ is of the order of 1/4 to 1. This calculation is of course independent of the interaction volume V and seems to be in better agreement with Eq. (1) with V chosen as $\frac{4}{3}\pi\left(\frac{h}{m_k c}\right)^3$.

The above results, with $\frac{R_3}{R_2} \sim 10^{-4}$ to 10^{-3}, would indicate that the τ and θ would have the properties that LO would require of the parent and daughter respectively. If one assumes that the τ and θ both have the same spin and differ only in their parities then one can make use of the observed ratios of $K_{\mu_2} : K_{\pi_2} : \tau\ (= 3\pi)$ in the emulsion stacks, and of the observed lifetime of the K_{μ_2}, to determine the partial transition rates R_μ, R_3, and R_γ. Here R_μ and R_γ are the transition rates of the τ into a $\mu + \nu$ and θ respectively. In general, the τ and θ might be expected to have different rates of decay into $\mu + \nu$. Bludman,[11] offers the conjecture that these decay rates $R_{\tau\mu}$ and $R_{\theta\mu}$ may be equal. The following figure shows the LO decay scheme according to these assumptions.

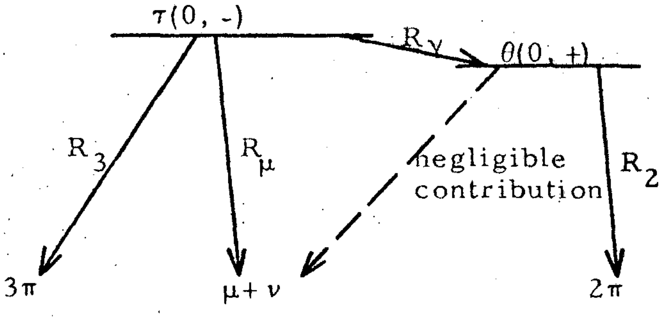

Fig. 2. Decay scheme according to LO.

Whitehead et al.[12] give the amounts for $K_{\mu_2} : K_{\pi_2} : \tau_{\pi_3} : K_{\mu_3} : K_{e_3}$, as 57.3 ± 2.5%, 31.2 ± 3.0%, 6.9 ± 0.7%, 2.5 ± 1.1% and 2.1 ± 1.5% respectively, where an additional 5% error should be compounded with the above errors to take account of possible unknown systematic errors. The lifetime[6, 7] of the K_{μ_2} is taken to be $1.2 \pm 0.15 \times 10^{-8}$ sec. With these data we obtain $R_\mu^{-1} = 2.1 \pm 0.3 \times 10^{-8}$ sec, $R_\gamma^{-1} = 3.9 \pm 0.6 \times 10^{-8}$ sec, and $R_3^{-1} = 17 \pm 3 \times 10^{-8}$ sec.

The above value of R_μ^{-1} is to be compared with the value predicted by Bludman and Ruderman[13] on the basis that the K meson acts like a heavy pion as far as lepton decay is concerned. They obtained $R_\mu^{-1} = 1.8 \times 10^{-8}$ sec.

By using the above-determined value of R_3 and the calculated ratio of $\frac{R_3}{R_2} \sim 10^{-4}$ to 10^{-3} we obtain $R_2^{-1} \sim 0.2$ to 2×10^{-10} sec for the θ lifetime. This is within an order of magnitude of the observed θ^0 lifetime[14] of $1.6 \pm 0.5 \times 10^{-10}$ sec. In the sense of the calculations made in Eqs. (1) and (2) the $\theta^\pm$ and θ^0 would all have the same lifetimes.

Another interesting number can be obtained by comparing the above R_γ with the two-photon emission rate in a (0, +) → (0, -) transition between charged K mesons of mass m_k and mass difference Δm_k. An estimate of R_γ[15] gives

$$R_\gamma \sim \left(\frac{\Delta m_k}{m_k}\right)^3 10^{13} \text{ sec}^{-1} . \qquad (3)$$

From this we obtain $\Delta m_k \sim 13 m_e$, which is probably accurate to within a factor of two. This value ($13 m_e$) can be compared with the observed mass difference of the τ and θ as determined by the measurements on the secondary particles:[12]

$$M_{3\pi} - M_{2\pi} = 6 \pm 2.5 \, m_e ,$$

$$M_{3\pi} - M_{\mu+\nu} = -0.3 \pm 2 \, m_e .$$

The scheme shown in Fig. 2 would indeed give a K_{μ_2} mass the same as the τ mass.

The evidence, though still not conclusive, seems to indicate that the LO scheme with the τ as the parent is the correct one. At present an experiment[6] is in progress that is designed to detect any γ rays that may be present in a $\tau \rightarrow \theta$ transition.

Lee and Yang LY[16] and Gell-Mann GM[17] have recently proposed that there is a new symmetry operation called parity conjugation that turns a τ meson of minus parity into the θ of plus parity. The fact that the masses of these two particles are the same or slightly different just means that parity conjugation is conserved or very nearly conserved. GM[17] points out that parity conjugation is conserved in electromagnetic interactions, and hence the conservation is violated only in the very weak interactions that are responsible for the long lifetimes of the particles. The result of this requirement is that the mass difference is very small compared to the mass differences required by the LO scheme. Therefore, the LO scheme and the GM scheme cannot be compatible.

LY, however, would allow as one of the possibilities that parity conjugation is <u>not</u> conserved in electromagnetic interaction and hence that a mass difference of the order of the $\pi^{\pm} - \pi^{0}$ mass difference could be produced. The parity conjugation scheme would predict equal numbers of τ's and θ's produced in association with hyperons that would themselves differ from each other only in their parity. The lifetimes of the τ and θ would not necessarily be the same in these schemes.

In order to ascertain whether the GM and LY schemes, together with the Bludman hypothesis that $R_{\tau\mu} = R_{\theta\mu} \equiv R_{\mu}$, could be consistent with the experimental data, Bludman suggested the following decay scheme. The author, in collaboration with A. H. Rosenfeld, solved the resulting five simultaneous equations. As the equality of the τ and θ abundance exits only at the target, one must take into account the fact that the detector is about 1.3 x 10^{-8} sec (proper time) from the target.

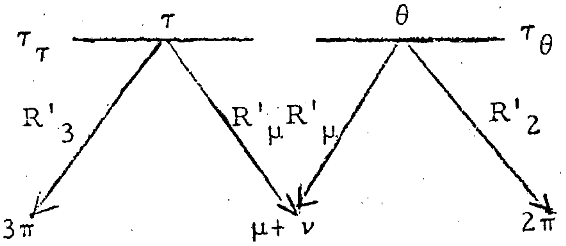

Fig. 3 Decay scheme according to Bludman.

The abundance ratios from Whitehead et al.[12] and the observed K_{π_2} lifetime[6,7] of $\tau_\theta = 1.3 \pm 0.2 \times 10^{-8}$ sec were used to predict the τ lifetime τ_θ and the partial decay rates R'_3, R'_2 and R'_μ on the basis of the above decay scheme. The results gave $\tau_\tau = 0.39 \pm 0.05 \times 10^{-8}$ sec, ${R'_\mu}^{-1} = 2.0 \times 10^{-8}$ sec, ${R'_2}^{-1} = 3.7 \times 10^{-8}$ sec, and ${R'_3}^{-1} = 0.49 \times 10^{-8}$ sec.

The predicted τ lifetime is in disagreement with the observed τ lifetime:

$$\tau_\tau = \begin{array}{ll} 1.27^{+0.12}_{-0.2} \times 10^{-8}\ \text{sec}, & \text{(Fitch and Motley}^{7}\text{)} \\ 1.0^{+0.7}_{-0.3} \times 10^{-8}\ \text{sec}, & \text{(Alvarez and Goldhaber}^{4}\text{)} \\ 0.8^{+0.5}_{-0.2} \times 10^{-8}\ \text{sec}. & \text{(Harris, Orear, and Taylor}^{5}\text{)} \end{array}$$

This could mean that LY and GM are right and Bludman wrong, or vice versa; that both are wrong, or that some other process is occurring.

If one retains the parity conjugation scheme of GM and LY and uses the observed lifetimes of the τ and θ, one can predict (a) that $R_{\tau\mu}/R_{\theta\mu} = 2.5 \pm 0.3$ if the small fractions of $K_{\mu 3}$ and K_{e_3} mesons come equally from the τ and θ; and (b) that $R_{\tau\mu} / R_{\theta\mu} = 2.0 \pm 0.3$ if the $K_{\mu 3}$ and K_{e_3} mesons come only from the τ.

The inconsistency may be resolved for LY and Bludman if one introduces the LO cascade process in addition to the other τ-decay modes and if one takes seriously the calculation that $\dfrac{R_3}{R_2} \simeq 10^{-4}$ to 10^{-3}. The θ's that are produced at the target naturally never reach the emulsions or the counters, since these detectors are approximately 10^{-8} sec (proper time) from the target. These short-lived θ^{+}'s would be seen only near the target. There seems to be conflicting evidence as to the existence of such a short-lived K^+ component in the cosmic ray experiments, where the detectors are near the "target". Arnold et al.[18] and Trilling and Leighton,[19] using cloud chambers, both report a lifetime of charged K-mesons of the order of 5×10^{-10} sec. However, Fretter et al.,[20] with very similar equipment and geometry, do not see evidence for such a short-lived component. They quote a lifetime of $\tau_k = 6.7^{+\infty}_{-5.5} \times 10^{-9}$ sec. There seems to be little evidence in the cosmic ray emulsion stacks for such a short-lived component.[21]

Hyperon Lifetimes

One might suppose that if the phase space is important in determining the relative decay rates of the heavy mesons, then it may be important also in determining the relative decay rates of hyperons not only with respect to each other, but also with respect to K mesons. It may be expected that the same process involving the "strangeness change" coupling constant e_4 occurs in both hyperon and K-meson decay. For instance a crude picture might be that of a nucleon and a K meson bound to form a hyperon.[22] The decay process could be depicted in a fashion similar to the forced μ-decay process of $\mu + P \rightarrow N + \nu$.[10] (See Fig. 4.)

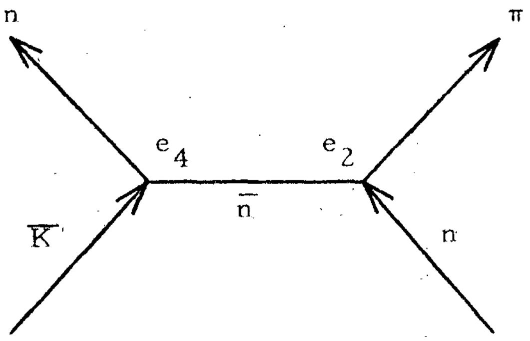

Fig. 4. Hyperon decay diagram.

This is undoubtedly an oversimplified picture of the hyperon decay process,[22] but is used to illustrate the possibility of a common "strangeness change" process in both heavy-meson and hyperon decays. Consequently the ratio of the decay rates of hyperons to K-meson decay rates would be roughly independent of e_4 and would probably be determined primarily by the ratio of the phase spaces. These phase-space ratios are compared below with the observed ratios of the lifetimes.[14]

Phase space ratios	Observed lifetime ratios
$\frac{R_\Sigma}{R_2} = 2.6$	$\frac{\tau_{\theta^0}}{\tau_{\Sigma^\pm}} = 5^{+3}_{-2}$
$\frac{R_\Lambda}{R_2} = 1.1$	$\frac{\tau_{\theta^0}}{\tau_{\Lambda^0}} = 0.4^{+0.3}_{-0.1}$
$\frac{R_\Xi}{R_2} = 0.91\,(2S_\Lambda + 1)$	$\frac{\tau_{\theta^0}}{\tau_\Xi} \sim 1, \; (\tau_\Xi \sim 10^{-10}$ sec)
or	
$\frac{R_\Sigma}{R_\Lambda} = 2.5$	$\frac{\tau_\Lambda}{\tau_\Sigma} = 11 \pm 5$

When one realizes that the strangeness-change process slows the decay rate from $\sim 10^{23}$ sec^{-1} to $\sim 10^{10}$ sec^{-1} it seems quite remarkable that the above phase-space ratios could be as close as they are to the observed lifetime ratios.

Conclusions

The rough agreement of the calculated decay-rate ratios with the observed lifetime ratios of the "strange particles" would suggest that the most important factor in the relative decay rates of the strange particles is the available phase space for the decay products. These comments would not apply to the partial decay rates that involve lepton emission. It appears very likely that the process by which the lifetimes are lengthened is a common process for all the strange particles. This process is associated with the coupling constant $e_4 \sim 10^{-3} e$ to $10^{-4} e$.

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