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Modeling and Optimization of Aerospace Tensegrity Structures

THESIS

submitted in partial satisfaction of the requirements
for the degree of

MASTER OF SCIENCE

in Mechanical and Aerospace Engineering

by

Shyam Patel

Thesis Committee:

Distinguished Professor John Michael McCarthy, Chair

Professor Mark E. Walter

Assistant Professor Penghui Cao

2021

DEDICATION

To my parents, Ashok Patel and Usha Patel, and my brother, Ritesh Patel, for their continued unconditional love and support of my interests and endeavors.
And to my grandmother, Shantaben Patel, for always watching over me from above.

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ABSTRACT OF THE THESIS

Modeling and Optimization of Aerospace Tensegrity Structures

By

Shyam Patel

Master of Science in Mechanical and Aerospace Engineering

University of California, Irvine, 2021

Distinguished Professor John Michael McCarthy, Chair

This thesis presents an exploration and analysis of the feasibility of two tensegrity systems through a series of optimization studies. First, double-helix tensegrity systems are defined for their application in a package delivery processes as payload carriers that provide impact attenuation for their payload. The architecture of the established double-helix tensegrity payload carrier is defined and modified to enhance flexibility in optimization studies. A dynamic model to simulate ground impact for the payload carriers is derived such that the physical and material characteristics of the carriers are simulated. The dynamic simulation calculates stress and strain in all members of the payload carriers and outputs all member failures. The simulation also performs checks on the carrier to ensure that it has not deformed to flip inwards on itself. Finally, the simulation ensures that the payload within the carrier does not experience forces greater than $10 g_E$ to ensure that no damage occurs to the payload from carrier impact. The dynamic simulation is then used to perform a full optimization analysis to optimize the mass and impact attenuation of the payload carrier by varying topological parameters and applying penalties to configurations that experience physical or material failures during the simulation. The MATLAB genetic algorithm is used to conduct the optimization using a fitness score to penalize any configurations that contain material or physical failures as well as any configurations that have a maximum payload deceleration greater than $10 g_E$. An optimization study is conducted using three shapes,

cylinder, ellipsoid, and parabola, each carrying a 5 kg payload dropped at a height of 2 m, to find the optimal geometry and topology for a payload delivery application. The parabolic double-helix tensegrity payload carrier obtained the lowest mass of the three carrier types with a structure mass of 2.33 kg while its payload experienced a maximum deceleration during impact of $8.99 g_E$. The ellipsoidal carrier provided the second lightest structure mass at 3.88 kg with a maximum payload deceleration of $6.15 g_E$. Finally, the cylindrical carrier converged to a mass of 5.44 kg with a maximum payload deceleration of $8.71 g_E$. A similar optimization technique is applied to a tensegrity morphing wing that actuates using a twisting tensegrity column design. The tensegrity morphing wing has a greater lift-to-drag ratio than a conventional wing that uses flaps to actuate due to the reduction of induced drag from the elimination of control surfaces. The tensegrity morphing wing replaces control flaps with a tensegrity torsional mechanism to twist the entire wing and create a change in angle of attack. A finite element model of the wing is developed in ABAQUS to simulate a Carl Goldberg Falcon 56 Mk II R/C unmanned aerial vehicle with a wingspan of 142.24 cm and a chord length of 25.31 cm. The finite element model evaluates the maximum twist achievable at an air speed of 20 m/s given that no components of the wing fail from material or structural failure, as determined by ABAQUS. A multi-objective optimization study is conducted to maximize twist angle and minimize wing mass. The multi-objective optimization provided a range of solutions in the form of a Pareto front. Three designs are chosen from the Pareto front to determine the effectiveness of the optimization. Compared to previous design of experiment studies, the optimization was able to provide wing configurations with higher twist angles and lower masses. The highest twist wing design obtained a maximum wing twist of 18.68° while obtaining a low mass of 1.88 kg.

Chapter 1

Introduction

¹ This chapter provides background to the topics presented in this thesis. A literature review and motivation for the double-helix tensegrity payload carriers is presented in Section 1.1 and Section 1.2. These sections also detail the differentiation of tensegrity systems from other design principles and how they can be beneficial in aerospace applications. A literature review and motivation for work pertaining to morphing wing research is presented in Section 1.3.

1.1 Introduction to Tensegrity Systems

Tensegrity structures are truss-like systems of compressive bars that are stabilized by a network of tensile string members [49, 51]. Tensegrity structures are also pre-stressable, which is a property that enables them to have deployable and morphing capabilities [11, 21, 61]. Tensegrity structures have been proposed as suitable candidates for load-bearing struc-

¹Portions of this section are reprinted or adapted from [Shyam A. Patel and Edwin A. Peraza Hernandez, 2021, “Design of Double-Helix Tensegrity Payload Carriers for Impact Load Attenuation,” AIAA Scitech 2021 Forum. <https://doi.org/10.2514/6.2021-0563>]; reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.

tures [2, 18]. The deformation characteristics of tensegrity structures make them suitable candidates for reducing force experienced by a payload during an impact. Low mass and high load capabilities make tensegrity structures ideal for payload protection for aerial and space applications.

All tensegrity structures are bound by design principles in which rigid members loaded in compression are supported by string members that are loaded in tension as defined by [51]. The combination of compressive and tensile load distributions make the tensegrity design both low-mass and high strength since there are no bending moments on the members. Tensegrity design is similar in principle to truss structures which distribute loads in both tension and compression [66]. The replacement of tensile load carrying members with cables enables mass-efficiency for known loads. The applications of tensegrity design have been especially relevant to low mass bridge structures as investigated by [5, 50]. The simple design principles in tensegrity means that a wide range of designs under the umbrella term of tensegrity exist. Their common design principle ensures that most geometries will retain similar loading characteristics. A challenge lies in determining which of these designs are relevant to the problem at hand. Algorithmic optimization is an essential technique for developing effective tensegrity devices. Masic *et al.* provide a method for optimization of tensegrity structures for specified mass-to-stiffness ratios in [31]. The benefit of using evolutionary algorithms is apparent in [46] where irregular structures are optimized for real world manufacturing using such a technique. Furthermore, optimization techniques have provided a method to determine optimal pre-stress states for tensegrity devices [67] which is important as pre-stress states affects tensegrity impact dynamics.

Key benefits to tensegrity design over other design principles are compliance and mass-efficiency. The exploration of tensegrity systems has grown in recent years and the concept encompasses a wide variety of research fields. The idea of using tensegrity as a compliant protective exoskeleton has also been explored for human use as the compliance found in

structures made from tensile and compressive members often mimic the dynamics of the human musculoskeletal system [25]. Tensegrity design has gone as far as to act as a meaningful model for human biomechanics [26, 27].

Tensegrity structures have also been explored in small scale structures as lattice metamaterials for applications that undergo high loads that would benefit from tuneable material stiffness [1]. A tensegrity lattice design also enables lattice composites that have thermal expansion properties, expanding their use cases [2, 6].

It is important to note that generating tensegrity structures is non-trivial. While the tensegrity design principle is simple, finding stable tensegrity designs is a challenge. Research into finding analytical methods to create tensegrity morphologies and mathematically describe node positions that connect compressive and tensile members has been ongoing [8, 24, 30].

1.2 Tensegrity Payload Carrier Motivation

For the case of payload carriers, minimizing mass ensures higher benefits for these applications which rely on weight reductions to travel longer distances. Minimizing the acceleration experienced by the payload upon impact is essential to safety of the payload when it is deployed.

Many current aerial package delivery methods involve a landing operation that the UAV must make to deliver the payload. This was discussed in detail by [9, 16]. Tseng *et al.* [58] found that landing and takeoff sequences are points of peak current draw during drone flight. These landing sequences are currently needed for the safety of packages and accuracy of delivery. An alternative to landing and releasing a package from a delivery drone is to drop it from the sky. These drones drop packages from large heights and often employ parachutes to slow package descent. Parachutes need favorable weather conditions to ensure that a package does

not veer off-target during its descent, restricting their operating abilities to calm climates or specific seasons in a year, as discussed in [65]. Dropping parachute-equipped packages from shorter heights does not solve the drop accuracy problem since parachutes have minimum height requirements for deployment and deceleration.

Current package protection methods such as the cardboard box offer little impact load attenuation. Thus, dropping packages from height without a parachute could result in damage of the delivery payload. This paper focuses on an alternative solution during the final delivery portion of the drone delivery sequence that employs a tensegrity payload carrier to protect a payload as it is dropped over the target delivery area. An example of such a delivery method can be seen in Fig. 1.1. A challenge therein lies in finding an optimal tensegrity carrier topology which is able to successfully absorb and dissipate the impact energy.

One popular tensegrity design is the D-bar construction. These have been studied in detail and been shown by [17] and [19] to be useful as mass-efficient energy absorption devices. While the D-bar proves the concept of mass-efficient energy absorption devices, the D-bar shape is limited in geometry and is thought to be most useful in static loads. These structures have proven useful in lattices, as shown by [2]. Static load-bearing structures for buildings and bridges are some other applications of the static energy dissipation characteristics of D-bar members. [10] shows that there is great promise for tensegrity structures in earthquake-proof building design. Alternatively, helical tensegrity columns, as seen in [28] and similar to those used in this paper, have been studied as dampening systems in large structures. The deformation that enables energy dissipation in these structures can be controlled and tuned to various energy dissipation needs [41, 69].

The SUPERball v2 in [60] is an example that highlights the tune-able deformation characteristics of tensegrity structures. The SUPERball v2 uses its real-time tension adjustment as method of movement by deforming specific portions of the tensegrity structure to enable a continuous rolling motion. Similar concepts have even been applied at smaller scales in the

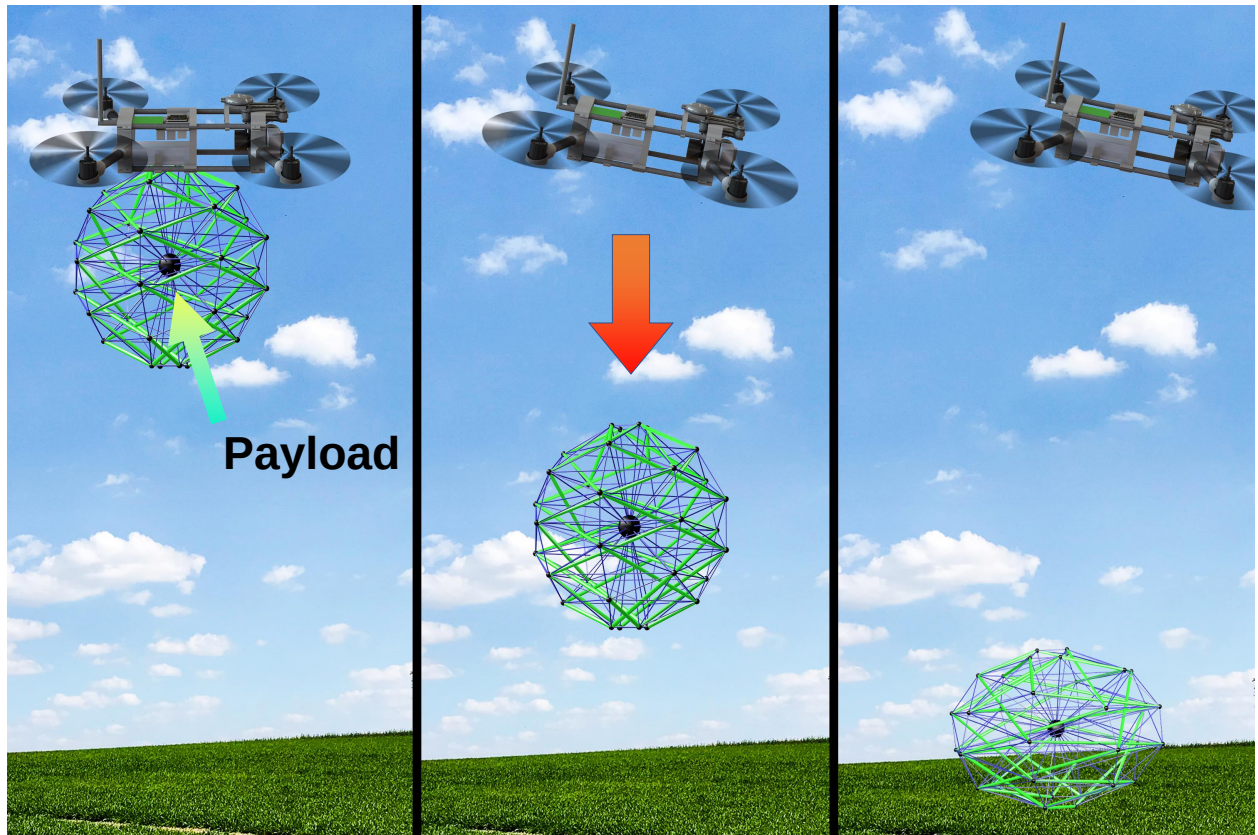


Figure 1.1: Diagram of a package delivery drone dropping an ellipsoidal tensegrity payload carrier form height with a payload attached in the center. Absorption of impact energy into the tensegrity carrier can be seen through deformation in the carrier body in the final frame.

form of bio-mimicking soft-robots [45, 63]. [29] takes the adjustable deformation of tensegrity structures further by implementing a programmatic control of deployment of folded tensegrities. A combination of mass-efficiency and deployability also enable tensegrity designs suitable in space applications [12, 56]. An additional characteristic to consider comes from an explorational study in [47] that shows how tensegrity systems may be able to maintain favorable energy dissipation characteristics even after critical buckling failure. This enables tensegrity designs to have lower factors of safety when performing one-time operations.

Tensegrity planetary landers have been explored thoroughly for characteristics mentioned above [4, 15]. A modification of D-bar tensegrities demonstrate a proof-of-concept for tensegrity-based lander devices in [42]. Due to the simple definition of tensegrity structures, a challenge forms in finding optimal designs to meet specific design goals. While many tensegrity structures exist, it is important to find robust designs that can be tuned and optimized to suit any challenge.

The great promise shown for tensegrity landers on other worlds poses a question of whether they can be effective here on Earth. The same properties which make the tensegrity design promising in other applications can be applied in the creation of a payload protecting structure for aerial drone deliveries. To improve performance, elastic and viscoelastic material choices can enable energy absorption and dissipation characteristics.

1.3 Tensegrity-based Morphing Wing Motivation

Reducing aerodynamic drag to increase efficiency is an enduring quest for every aircraft designer. Conventional aircraft designs employ discrete control surfaces such as flaps, ailerons, rudders, and elevators. These control components introduce gaps on the surface of the wing, where turbulent vortices can form and significantly increase drag [55, 57]. Ways to reduce

induced drag by modifying these control surfaces is being researched. Physical limitations to performance occur when these modifications are made [64]. One method of induced drag reduction is developing a morphing control surface that is able to adapt to flight requirements [48]. An additional benefit to morphing wings is the flexibility in their methods of actuation, leading to reduction of complexity and weight [3, 53]. Challenges in semi-compliant control surfaces arise due to the stiffness requirements to combat aerodynamic forces during flight. Removing and replacing separated control surfaces without sacrificing maneuverability and stability has gained interest within both academic and industrial circles. Aircraft with highly flexible and tailorable structures offer a potential solution to improve efficiency. Approaches have been taken to mechanically modify wing properties such as camber, twist, and thickness while in flight [14]. The variable camber continuous trailing edge (VCCTE) flap system presented in [39] is an example of such structures. In this study, computer simulation demonstrated over 50% of drag reduction. Similarly, Phillips, Alley, and Goodrich designed “twisterons,” a system that acts as both flaps and ailerons while minimizing induced drag [44]. Vos and Barrett also introduced a pressure-adaptive honeycomb seamless flap system that delivered up to a 0.3 increase in the lift coefficient [62]. At a mechanism level, compliant lattices have been proposed [23] to enable real time camber changes during flight. At a larger scale, entire-wing morphing capabilities are also being developed and studied. For example, Gabor, Koreanschi, and Botez demonstrated an impressive 26.73% reduction drag through the use of embedded upper skin morphing technology [13]. Finally, research into the creation of materials to reduce wrinkling in skin during wing twist is ongoing to further increase drag reduction [40].

Scholars and industry experts are also investigating and developing optimization schemes to improve existing concepts. Molinari, Arrieta, and Ermanni simultaneously consider the aerodynamic characteristics, the morphing configurations, and the underlying compliant structure in an optimizing strategy [33]. The result was a morphing wing with embedded actuators that delivered a maximum of 18% increase in efficiency. In 2010, the NASA

Innovation Fund Award project titled “Elastically Shaped Future Air Vehicle Concept” using VORVIEW—a vortex-lattice computational code that is used extensively in the design, analysis, and optimization for the development of elastically shaped aircraft concepts and Cart3D—a computational fluid dynamic (CFD) tool—to generate three different concepts that optimize the wing shape during cruise to minimize induced drag [38]. As the previous examples have demonstrated, due to the coupled interactions between the aircraft aerodynamic characteristics, flight stability, and structural soundness, achieving a morphing wing demands a multidisciplinary study [59].

In the past few years, researchers have looked into utilizing tensegrity structures to actuate wing morphing. By varying the rest length of the tensile members, the shape, stiffness, and vibration response of the overall structure can be tailored. Due to the mentioned attributes, tensegrity structures are deemed as suitable underlying mechanisms for morphing wings which need to both carry aerodynamic loads, vibration, and vary their configuration [36]. During literature review, tensegrity-based morphing wing concepts have came up in various research. A submarine vehicle with tensegrity-based morphing wings introduced by Moored and Bart-Smith demonstrated the feasibility and capability of utilizing tensegrity-based morphing structures for vehicles that travel through a fluid medium [34]. This research was preceded by Moored with a study on the effectiveness of tensegrity structures in biomimetic applications [35]. In the aerospace spectrum, Ref. [22] provided a control algorithm with an output regulator that actuate tensile members of the underlying tensegrity structure to vary the external profile from one NACA airfoil to another. Similarly, Djouadi *et al.* provides a method of active actuation for tensegrity structures [7]. Alternatively, entire-wing morphing is illustrated in the deployable, variable-span wing presented by Skelton and de Oliveira [52]. In Ref. [32], a lightweight tensegrity wing demonstrated similar load-bearing capability to that of a conventional wing which solidifies its potential.

This research presents the modeling and optimization of a novel tensegrity-based twisting

wing—one without discrete control surfaces enabled by an internal torsional tensegrity mechanism. The underlying torsional tensegrity mechanism is inspired by a twisting tensegrity tower that can rotate by 40° from top to bottom without changing the structural height, presented in Ref. [52]. The motion was enabled by activating the network of shape memory alloy (SMA) wires—which contracted due to the increase in temperature—via electrical current.

In this work, the twisting tensegrity tower was adapted into a wing to create a torsional tensegrity mechanism that enables twisting motion to control airflow and lift distribution in the spanwise direction. Previously published work demonstrated the feasibility of the concept through means of finite element modeling and simulation. Furthermore, by conducting computational fluid dynamics (CFD) analysis, the design was justified when it delivered lower drag than the baseline, flapped wing [20, 43]. Figure 1.2(a) depicts a preliminary tensegrity wing prototype that was built by undergraduate students from the Mechanical and Aerospace Engineering Department of the University of California, Irvine, [54]. Figure 1.2(b) is a depiction of the twisting motion, caused by shape memory alloy (SMA) wires [20].

1.4 Summary of Contributions and Outline of the Thesis

The main contributions to research in this work are as follows:

- i Implementing modifications to double-helix tensegrity design structure to add enhanced variability to double-helix tensegrity carrier designs for payload carrier exploration and parameterizing double-helix tensegrity payload carrier topological parameters to enable optimization study (axial complexity, radial complexity, axial node concentration, payload height, radius-to-length ratio)

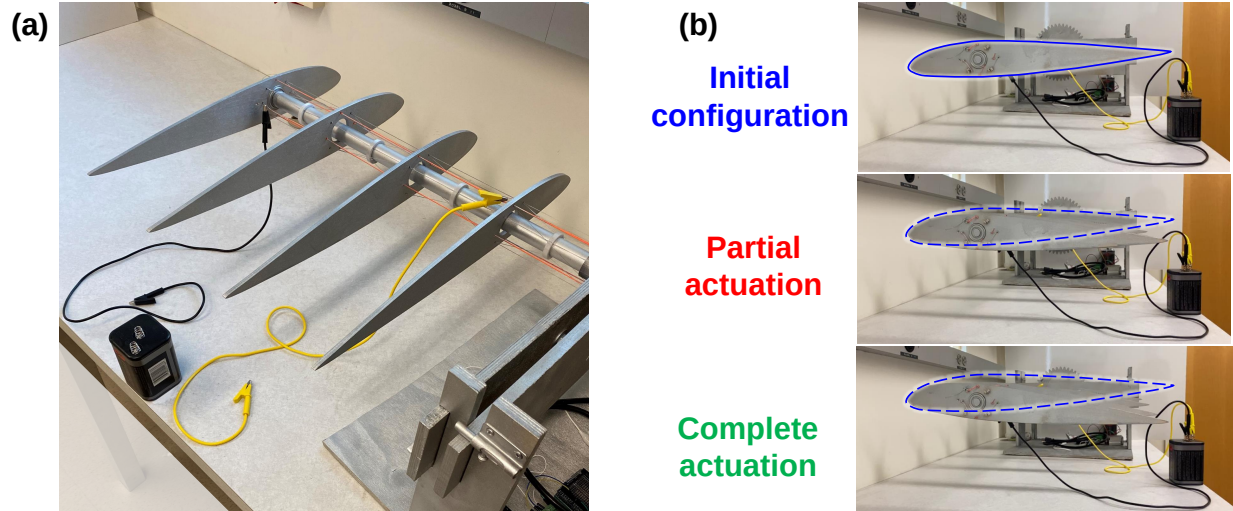


Figure 1.2: Photo of an early physical prototype of the tensegrity-based twisting wing with actuation configurations labeled. This physical prototype is actuated using shape memory alloy actuators, providing a lightweight and strong actuator. Credit: Undergraduate researchers from the Tensegrity Wing Design Team in the Department of Mechanical and Aerospace Engineering at the University of California, Irvine [54].

- ii Development on dynamic tensegrity simulation to provide accurate stress and failure measurements of simulated double-helix tensegrity payload carrier
- iii Parametrization of failure modes in dynamic tensegrity simulator to enable penalties for failed designs
- iv Parametrization of tensegrity morphing wing design principles
- v Conducted performance optimization of morphing tensegrity wing design using multi-objective optimization methods to generate a Pareto front

Chapter 2

Double-helix Tensegrity Payload Carrier

¹ This chapter provides a review of the topology of the double-helix tensegrity (DHT) payload carrier and presents modifications made to increase the variability in its the design structure. The composition of the DHT payload carriers for each carrier shape is depicted in Fig. 2.1 and is made up of five components: Central payload, bar members, free-moving joints, internal string members, and surface string members. Section 2.1 presents the origins of the double-helix tensegrity architecture. In Section 2.2, the axial and circumferential complexity of the DHT design structure is presented. Section 2.3 derives the parametrization of axial payload position. Section 2.4 presents the derivation of the second order axial node distribution modification.

¹Portions of this section are reprinted or adapted from [Shyam A. Patel and Edwin A. Peraza Hernandez, 2021, “Design of Double-Helix Tensegrity Payload Carriers for Impact Load Attenuation,” AIAA Scitech 2021 Forum. <https://doi.org/10.2514/6.2021-0563>]; reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.

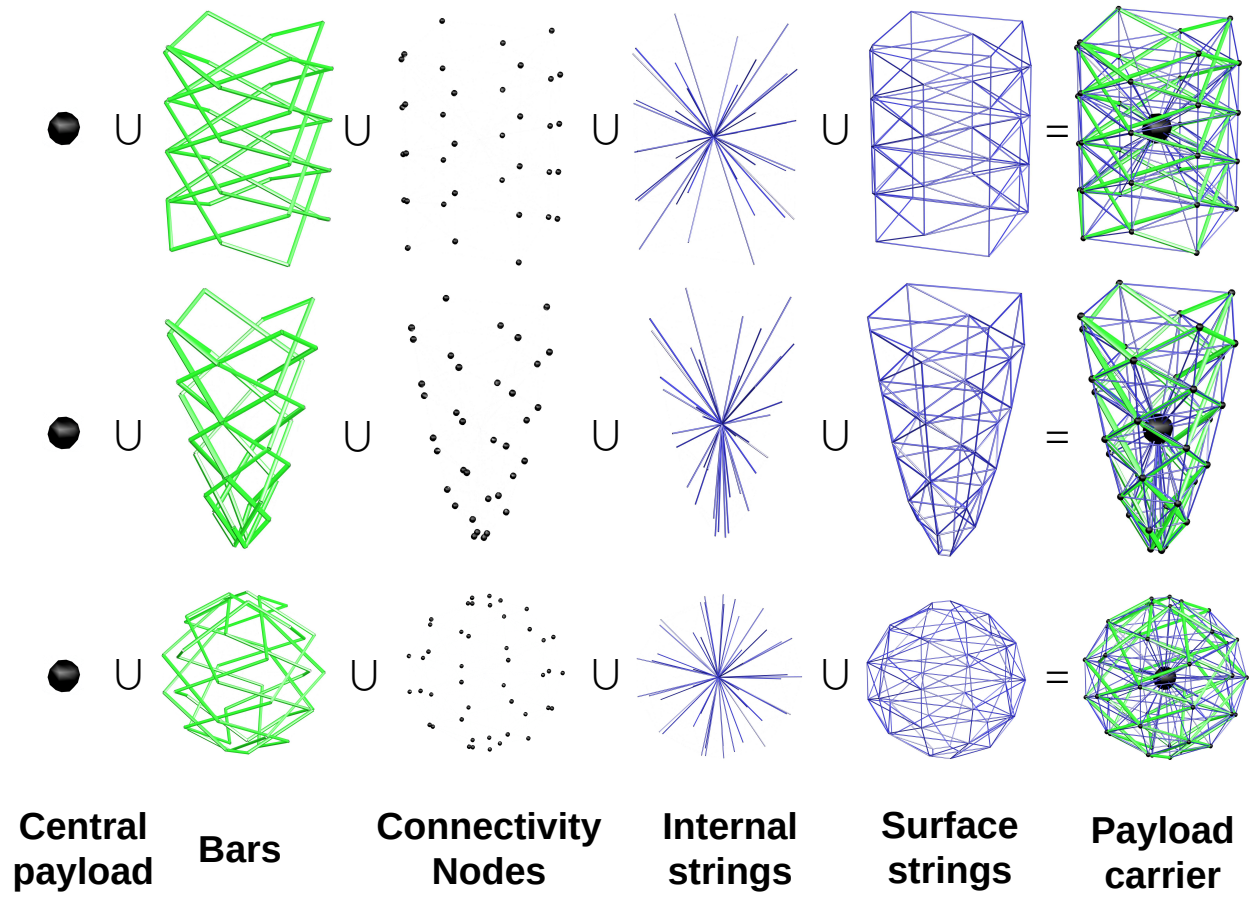


Figure 2.1: Internal components of DHT carriers. Left to right: Central payload, bars, free-moving joints, internal strings, surface strings, complete DHT carrier. Top to bottom: Cylindrical, parabolic, and ellipsoidal DHT carriers.

2.1 Yoshimura Fold Pattern

The DHT payload carriers are modifications of the double-helix tensegrities derived previously by Nagase and Skelton in [37]. The double-helix tensegrity design is derived from the Yoshimura fold pattern as seen in Fig. 2.2 and adapts the sequences of the Yoshimura folds to a tensegrity bar and string arrangement. The complexity and number of folds in the Yoshimura origami pattern is defined by terms p and q which determine the number of axial and radial units as seen in Fig. 2.3. In the double-helix tensegrity, these values determine the position of the nodes and connectivity matrix of the the strings and bars.

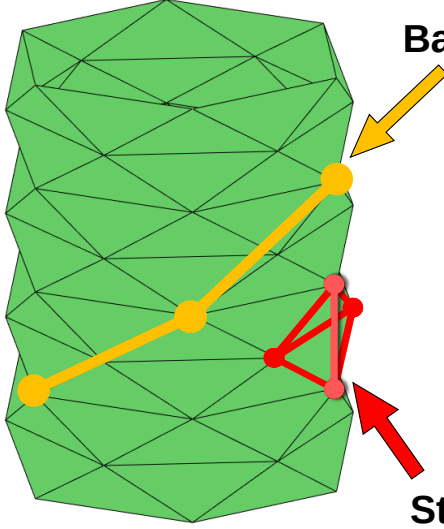
2.2 Axial and Circumferential Complexity

The DHT overall structure is dictated by the number of units that form along the axial and circumferential directions of each structure. The axial and radial DHT units are defined by the parameters p and q , respectively. Seen in Fig. 2.4, higher values of p and q increase the number of free-moving joints and bars. Increased complexity also increases the length of the surface strings and decreases the length of bar members. Increased complexity in either direction also increases structure mass since each free-moving joint is given a mass of 10 g .

2.3 Axial Payload Position

In previous work [68], the position of the payload with respect to the structure was axially centered. Increasing or decreasing the payload height impacts the optimizer designs. If the payload height is higher, the optimizer is able to evaluate designs that have larger deformations during impact without risking a collision between the payload and the ground. The payload height also affects the length of the internal strings which influences the acceleration

Yoshimura Pattern



Double Helix Tensegrity

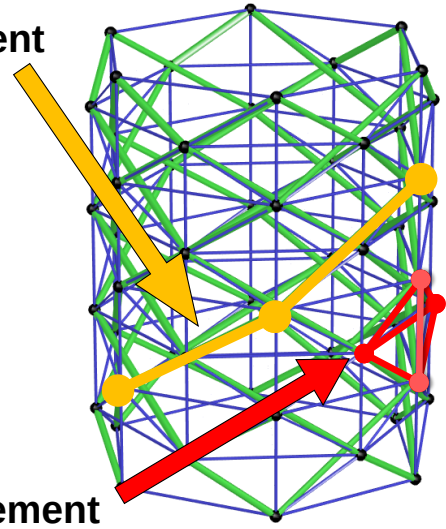


Figure 2.2: Translation from the Yoshimura origami fold pattern to the double-helix tensegrity(DHT) structure.

experienced by the payload acceleration during impact. A vector $\mathbf{n}_P$ defines position coordinate of the payload and has a vector component, $n_P \in \{-0.45, 0.45\}$ in the y -axis which adjusts the height of the payload with respect to the length, as follows:

$$\mathbf{n}_P = \begin{bmatrix} 0 \\ n_P L \\ 0 \end{bmatrix}, \quad (2.1)$$

If n_P is negative, the payload is situated below the axial center. If n_P is positive, it is situated above the axial center. The payload is centered within the payload carrier when $n_P = 0$. Fig 2.5 shows some possible payload height configurations and their respective values.

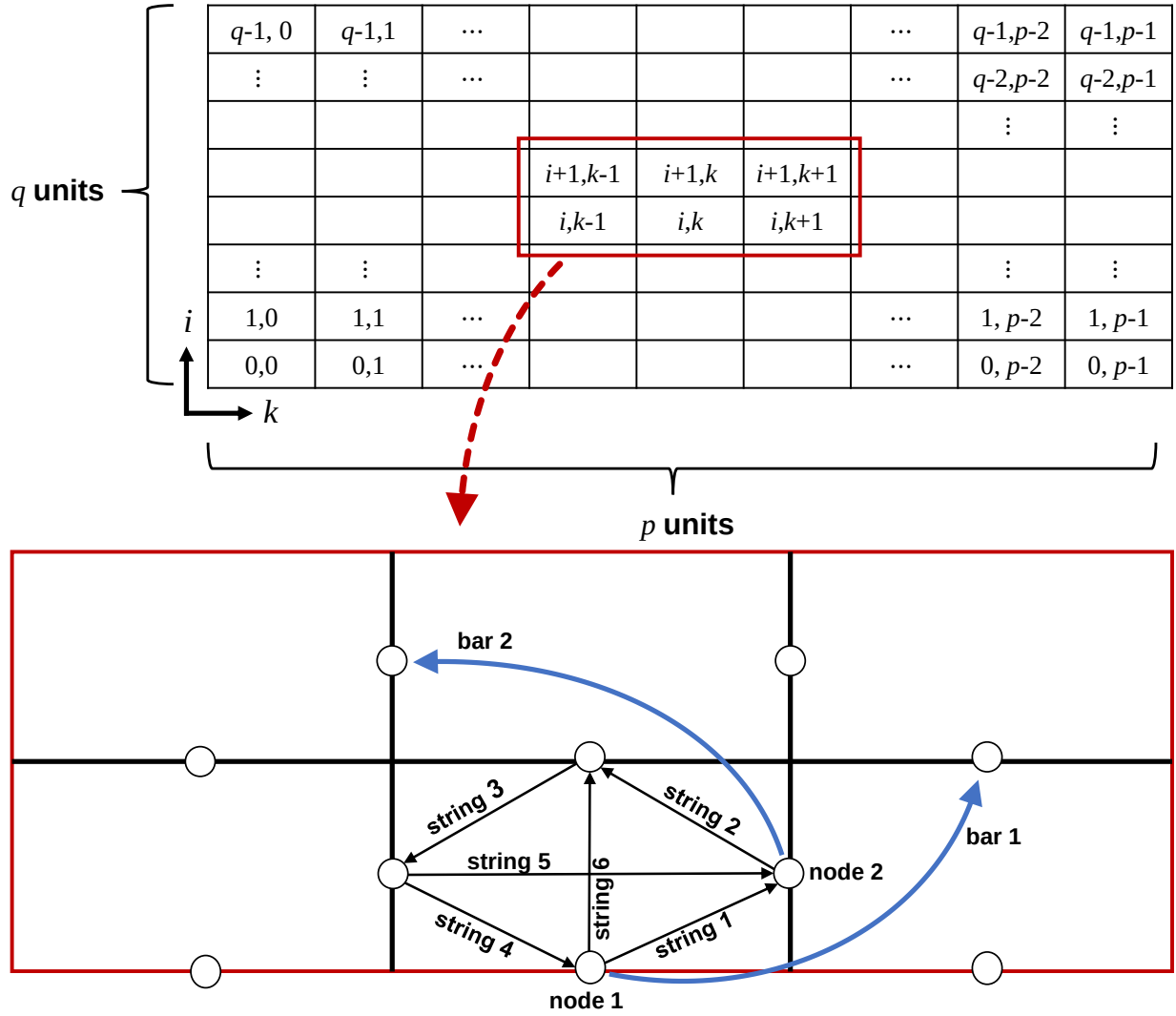


Figure 2.3: String and bar propagation pattern for double-helix tensegrity structures in a $p \times q$ grid.

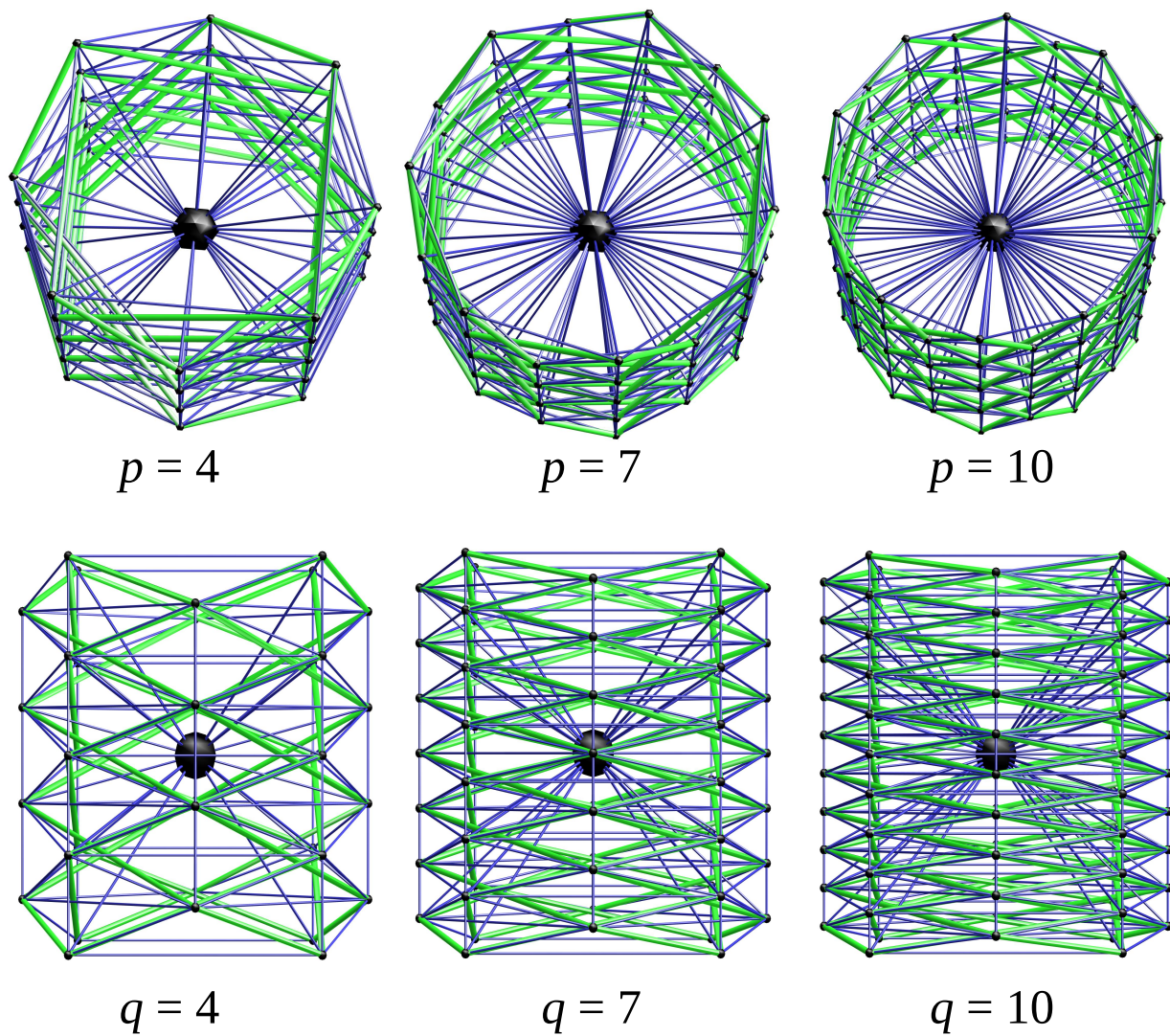


Figure 2.4: (Top) Left to right: Increasing circumferential complexity, p , while holding axial complexity, q , constant at $q = 4$. (Bottom) Left to right: Increasing axial complexity, q , while holding circumferential complexity, p , constant at $p = 4$.

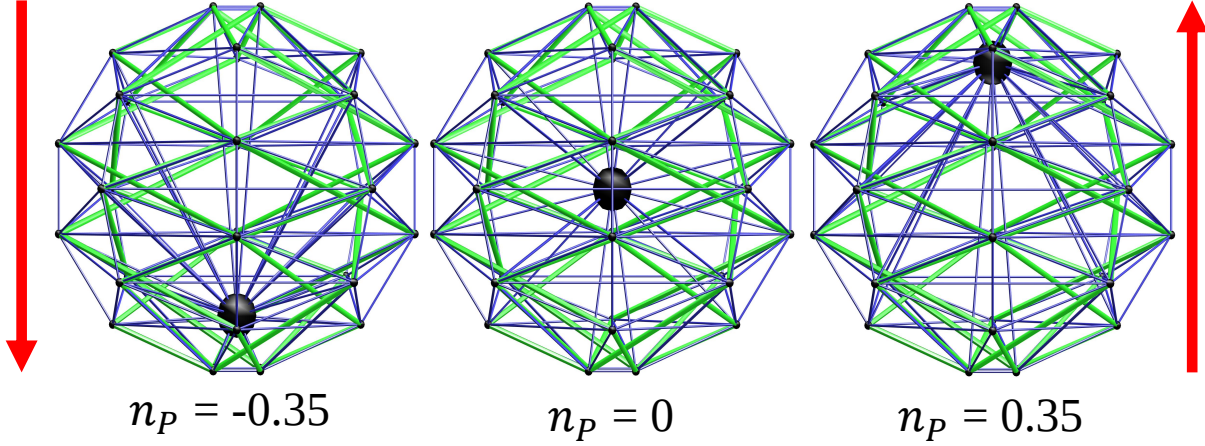


Figure 2.5: Variations in axial payload position, n_P , across three different DHT payload carriers.

2.4 Axial Node Distribution

The original DHT structures, defined by Nagase and Skelton, are defined with even distributions of nodes along both axial and circumferential directions. The effects of uneven node distributions in the axial direction are explored by deriving a second-order equation that changes the axial positions of the structure nodes. The equation for formulating axial position of the nodes in [37] is as follows:

$$n_l(i, k) = y \frac{L}{2q}, \quad (2.2)$$

where y is the first-order distribution of axial nodes as follow:

$$y = (2i + (l - 1)) \frac{L}{2q}, \quad (2.3)$$

where i is the axial unit from bottom to top and l indicates whether a node is the start or end of an axial unit. The second order distribution is bounded by the start and end

conditions of the original distribution where the start condition is when $(i = 0, l = 1)$ and the end condition is when $(i = q, l = 1)$. Based off of these conditions, a general second order distribution equation is formed as follows:

$$y = c_0 + c_1(2i + l) + c_2(2i + l)^2, \quad (2.4)$$

where c_0 , c_1 , and c_2 are general constants. The start $(i = 0, l = 1)$ and end $(i = q, l = 1)$ conditions are then substituted into the general second order equation in Eq. (2.3) as follows:

$$0 = c_0 + c_1 + c_2, \quad (2.5)$$

$$2q = c_0 + c_1(2q + 1) + c_2(2q + 1)^2. \quad (2.6)$$

Both equations are then combined to form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2q + 1 & (2q + 1)^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2q \end{bmatrix}. \quad (2.7)$$

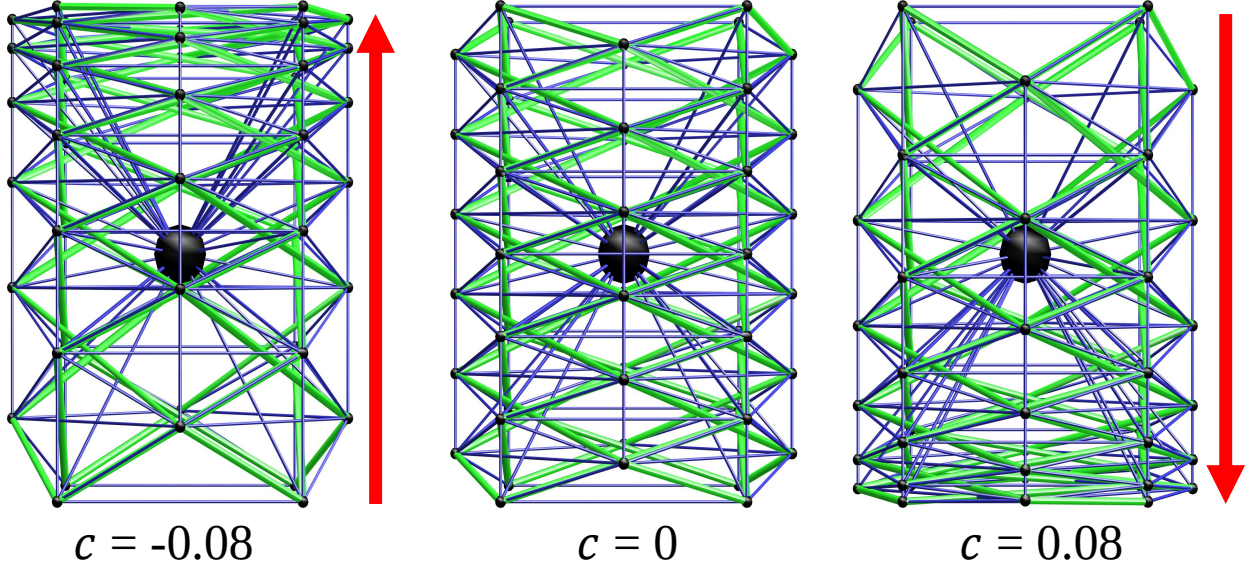


Figure 2.6: Various concentrations of axial nodes and their respective second-order concentration coefficient, or c , values.

After solving for y , the second-order axial distribution equation becomes:

$$y = c + 2cq - 1 + (1 - 2cq - 2c)(2i + l) + c(2i + l)^2, \quad (2.8)$$

where c_2 is referred to as c for simplicity as it is the only parameter used to modify the second-order distribution. As seen in Fig. 2.6, values where $c < 0$ concentrate the axial nodes towards the top of the DHT carrier. Alternatively, a value of c where $c > 0$ concentrates the axial nodes towards the bottom of the carrier and a value of c where $c = 0$ distributes the axial nodes evenly across the carrier.

2.5 Radius to Length Ratio

The size of the carriers are defined by their length, L , and radius, R . The radius, R , is dependent on a radius-to-length parameter, R/L while the length of the carrier can be modified independently with L . Variations of the R/L ratio are seen in Fig. 2.7. Varying the length and radius of the carriers effectively changes its mass and energy absorption characteristics. Larger carriers also have longer bar members which reduces the Euler critical load, leading to a design that is more likely to fail from buckling.

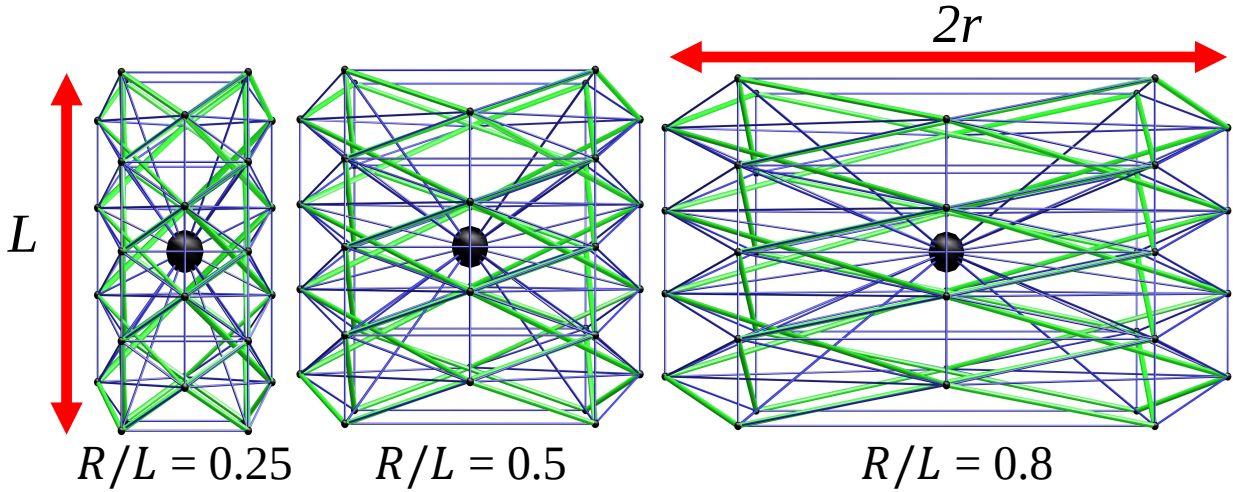


Figure 2.7: Various aspect ratios of DHT carriers and their corresponding R/L ratios.

Chapter 3

Double-helix Tensegrity Performance Model

¹ A model to calculate the dynamics of tensegrity structures is detailed in this section. The initial implementation of the model was presented in [68]. This model determines the stresses in each bar and string member as well as the forces experienced by each node in the structure, including the payload. The model also adheres to the material and structural dynamics of the tensegrity models, whether they are elastic or viscoelastic and whether they are strings or bars.

3.1 Ground Plane

The ground plane in the model is a flat surface with a frictional coefficient, ν , stiffness coefficient, p_c , and a damping coefficient, c_c . The y -axis is modeled to be normal to the

¹Portions of this section are reprinted or adapted from [Shyam A. Patel and Edwin A. Peraza Hernandez, 2021, “Design of Double-Helix Tensegrity Payload Carriers for Impact Load Attenuation,” AIAA Scitech 2021 Forum. <https://doi.org/10.2514/6.2021-0563>]; reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.

ground plane. The carriers are evaluated from a distance of 2 m from the bottom-most node on the carrier to the ground plane so the DHT carriers are given an initial height, h , at the start of the simulation. A visualization of these parameters can be found in Fig. 3.1. This height is added to the DHT carrier node coordinates to define the initial position vector as follows:

$$\mathbf{n}_i = \begin{bmatrix} 0 \\ h \\ 0 \end{bmatrix} + \begin{bmatrix} n_{ix_0} \\ n_{iy_0} \\ n_{iz_0} \end{bmatrix}, \quad (3.1)$$

where n_{ix_0} , n_{iy_0} , and n_{iz_0} are the positional coordinates of the i^{th} node with respect to the center of the DHT carrier.

3.2 DHT Position, Velocity, and Acceleration

The position vector of all nodes in the DHT carrier, $\mathbf{n} \in \mathbb{R}^{3n_n}$, is defined by the following:

$$\mathbf{n} = \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_{n_n} \end{bmatrix}, \quad (3.2)$$

where the vectors to n_i are positional coordinates of each node like in Eq. (3.1) and n_n is the total number of nodes in the DHT carrier. The initial velocity, v , of the nodes is zero

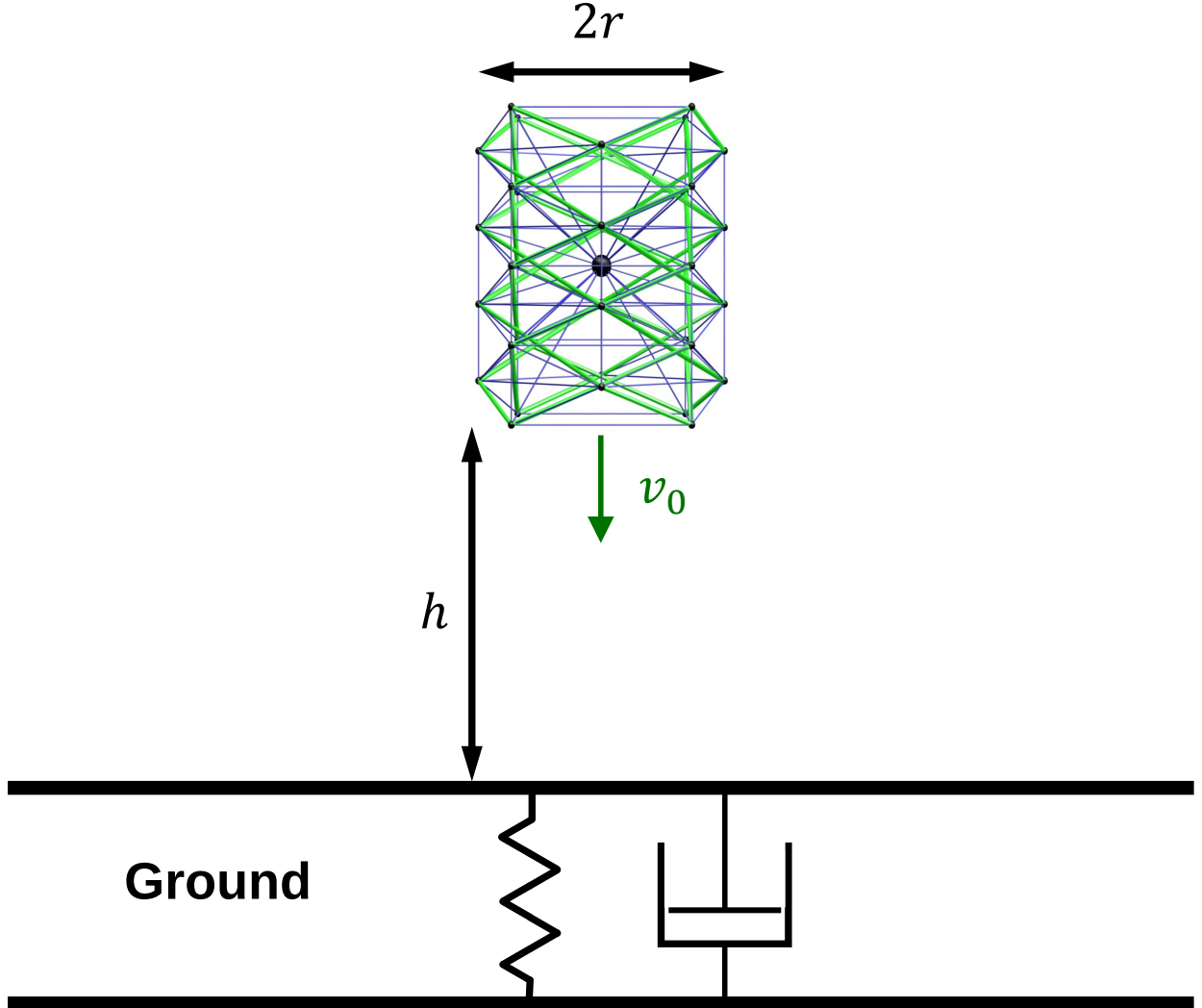


Figure 3.1: Initial conditions of each DHT carrier. The ground is modeled as a spring damper system where the associated ground stiffness spring constant is p_c and the associated ground damping coefficient is c_c . The lander falls from an initial height, h , which represents the distance between the ground plane and the bottom of the carrier. The initial velocity, v_0 , is zero in this paper.

since the carrier is dropped from a static condition:

$$\dot{\mathbf{n}}_i = \begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix}. \quad (3.3)$$

The node velocity vectors are tracked by column vector $\dot{\mathbf{n}} \in \mathbb{R}^{3n_n}$ and node acceleration vectors are tracked by column vector $\ddot{\mathbf{n}} \in \mathbb{R}^{3n_n}$:

$$\dot{\mathbf{n}} = \begin{bmatrix} \dot{\mathbf{n}}_1 \\ \dot{\mathbf{n}}_2 \\ \vdots \\ \dot{\mathbf{n}}_{n_n} \end{bmatrix}, \ddot{\mathbf{n}} = \begin{bmatrix} \ddot{\mathbf{n}}_1 \\ \ddot{\mathbf{n}}_2 \\ \vdots \\ \ddot{\mathbf{n}}_{n_n} \end{bmatrix}, \quad (3.4)$$

the acceleration vector, $\ddot{\mathbf{n}}_{n_n}$, is calculated as the product of the inverse mass matrix, $\mathbf{M} \in \mathbb{R}^{3n_n \times 3n_n}$, and the net force experienced by the DHT carrier:

$$\ddot{\mathbf{n}} = \mathbf{M}^{-1} (\mathbf{f}_E - \mathbf{f}_I), \quad (3.5)$$

where vector $\mathbf{f}_E \in \mathbb{R}^{3n_n}$ is the external force and vector $\mathbf{f}_I \in \mathbb{R}^{3n_n}$ is the internal force experienced by the DHT carrier.

3.3 Carrier Forces

The external forces on each node are separated into three components, gravitational, normal, and frictional forces as follows:

$$\mathbf{f}_{E_i} = \mathbf{f}_{G_i} + \mathbf{f}_{N_i} + \mathbf{f}_{F_i} = m_i \begin{bmatrix} 0 \\ g_E \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ f_{N_i} \\ 0 \end{bmatrix} + \mathbf{f}_{F_i}, \quad (3.6)$$

where g_E is Earth gravity. The frictional force component, $\mathbf{f}_{F_i}$, and the normal force component, $\mathbf{f}_{N_i}$, are zero when the vertical node positions are above the $x - z$ ground plane:

$$f_{N_i} = \begin{cases} 0; & n_{iy} > 0, \\ p_c n_{iy}^2 + c_c 2n_{iy} \dot{n}_{iy}; & n_{iy} \leq 0, \end{cases} \quad (3.7)$$

$$\mathbf{f}_{F_i} = \begin{cases} 0; & \|\mathbf{v}_{G_i}\| = 0, \\ -\eta f_{N_i} \frac{\mathbf{v}_{G_i}}{\|\mathbf{v}_{G_i}\|}; & \|\mathbf{v}_{G_i}\| \neq 0. \end{cases} \quad (3.8)$$

where n_{iy} is the vertical component of the positional node of the i^{th} node. A value of $n_{iy} > 0$ indicates that the node is above the ground plane. Figure 3.2 showcases the placement of the various external forces experienced by the carrier. The ground stiffness constant and damping coefficient are denoted by p_c and c_c , respectively. The frictional forces act parallel to the ground plane and pointed towards the $x - z$ node velocity, $\mathbf{v}_{G_i} = [\dot{n}_{ix} \ 0 \ \dot{n}_{iz}]^T$. The

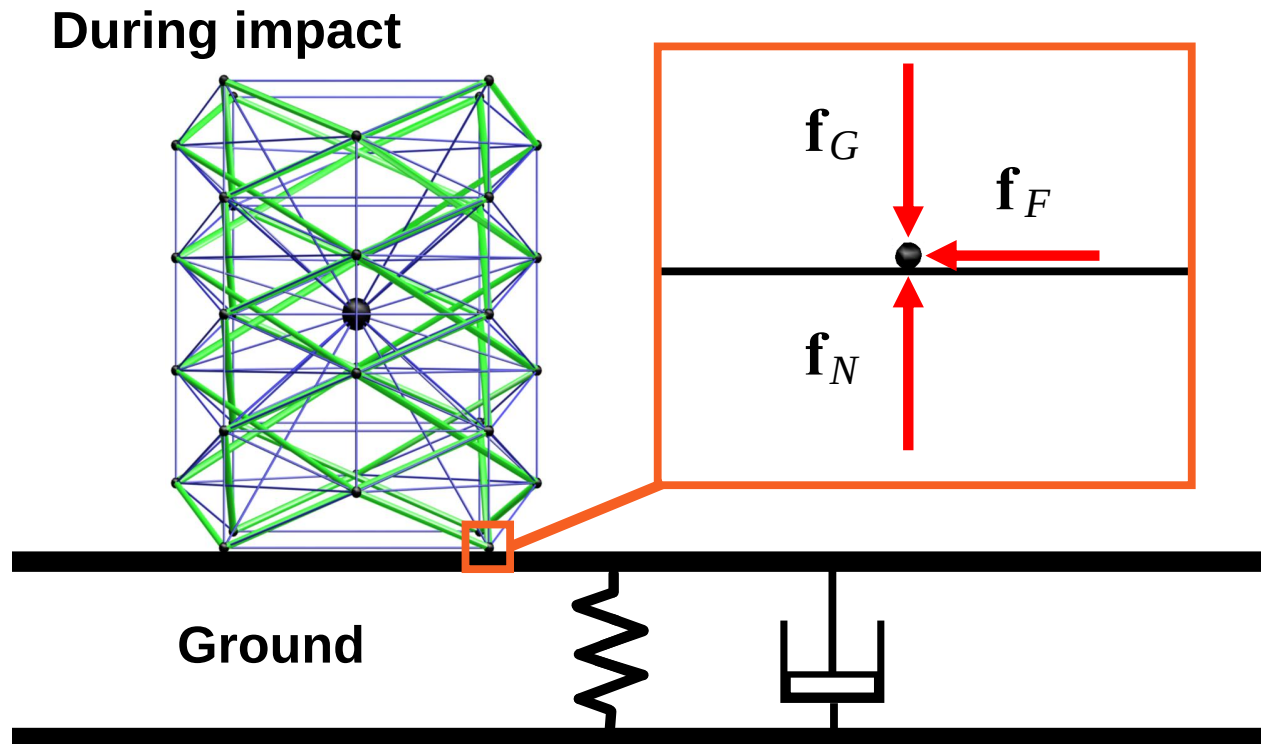


Figure 3.2: External forces acting on DHT carrier as it impacts the ground plane where $\mathbf{f}_F$ represents frictional forces, $\mathbf{f}_N$ represents normal forces, and $\mathbf{f}_{G_i}$ represents gravitational forces.

internal forces on each node have components of force that form string and bar members, $\mathbf{f}_s$ and $\mathbf{f}_b$, respectively. The internal force on the nodes are calculated as follows:

$$\mathbf{f}_I = (\mathbf{C}_s^\top \otimes \mathbf{I}_3)\mathbf{f}_s + (\mathbf{C}_b^\top \otimes \mathbf{I}_3)\mathbf{f}_b, \quad (3.9)$$

where the connectivity matrices of the strings and bars (derived in [37]) are represented as $\mathbf{C}_s \in \mathbb{R}^{n_s \times n_n}$ and $\mathbf{C}_b \in \mathbb{R}^{n_b \times n_n}$. The Kronecker product, $\otimes$, of the transpose of the connectivity matrix and the identity matrix, $\mathbf{I}_3 \in \mathbb{R}^{3 \times 3}$, is multiplied by the forces on each respective member type. The force component for strings is denoted by $\mathbf{f}_s$ and the force component for bars is denoted by $\mathbf{f}_b$. The summation of these two force components enables $\mathbf{f}_I$ to capture the net force experienced by each node.

The forces experienced by each string and bar member are placed in $\mathbf{f}_s \in \mathbb{R}^{3n_s}$ and $\mathbf{f}_b \in \mathbb{R}^{3n_b}$:

$$\mathbf{f}_s = \begin{bmatrix} \mathbf{f}_{s_1} \\ \mathbf{f}_{s_2} \\ \vdots \\ \mathbf{f}_{s_{n_s}} \end{bmatrix}, \quad \mathbf{f}_b = \begin{bmatrix} \mathbf{f}_{b_1} \\ \mathbf{f}_{b_2} \\ \vdots \\ \mathbf{f}_{b_{n_b}} \end{bmatrix}. \quad (3.10)$$

where n_s and n_b are the number of strings and bars in the DHT carrier, respectively. These forces are a function of the cross-sectional area of the member, internal stresses, and member lengths:

$$\mathbf{f}_{s_i} = \sigma_{s_i} A_{s_i} \frac{\mathbf{s}_i}{\|\mathbf{s}_i\|}, \quad \mathbf{f}_{b_i} = \sigma_{b_i} A_{b_i} \frac{\mathbf{b}_i}{\|\mathbf{b}_i\|}. \quad (3.11)$$

where A_i is the cross-sectional area, σ_i is the internal stress, and $\mathbf{s} \in \mathbb{R}^{3n_s}$ and $\mathbf{b} \in \mathbb{R}^{3n_b}$ are the length vectors for strings and bars, respectively. The length vectors are constructed using the corresponding connectivity matrix and node position vectors:

$$\begin{aligned} \mathbf{s} &= (\mathbf{C}_s \otimes \mathbf{I}_3) \mathbf{n} = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_{n_s} \end{bmatrix}, & \mathbf{s}_i &= \begin{bmatrix} s_{ix} \\ s_{iy} \\ s_{iz} \end{bmatrix}, \\ \mathbf{b} &= (\mathbf{C}_b \otimes \mathbf{I}_3) \mathbf{n} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_{n_b} \end{bmatrix}, & \mathbf{b}_i &= \begin{bmatrix} b_{ix} \\ b_{iy} \\ b_{iz} \end{bmatrix}. \end{aligned} \tag{3.12}$$

The following equation shows the formulation of stresses in the string, σ_s , and bar, σ_b , members:

$$\begin{aligned} \sigma_{s_i} &= \begin{cases} E_{s_i} \varepsilon_{s_i} + c_{s_i} \dot{\varepsilon}_{s_i}; & \text{if } E_{s_i} \varepsilon_{s_i} + c_{s_i} \dot{\varepsilon}_{s_i} \geq -\sigma_{scr_i} \\ 0; & \text{if } E_{s_i} \varepsilon_{s_i} + c_{s_i} \dot{\varepsilon}_{s_i} < -\sigma_{scr_i} \end{cases}, \\ \sigma_{b_i} &= E_{b_i} \varepsilon_{b_i}, \end{aligned} \tag{3.13}$$

where ε_{s_i} and ε_{b_i} represent the strains in the string and bar members. The strain rate is indicated by $\dot{\varepsilon}_{s_i}$. Strain rate is only calculated for string members as they are modeled viscoelastically. A strain rate term and damping constant c_{s_i} are required in viscoelastic members to accurately model internal stress. A branched form is used to represent stresses

in strings in Eq. (3.13) because the strings experience zero stress once they buckle past their critical buckling stress σ_{s_i} . To calculate critical buckling stress, a form of Euler's critical buckling equation is used where the moment of inertia and initial string length for the i^{th} string are indicated by I_{s_i} and s_{i_0} , given as follows:

$$\sigma_{s_{cr_i}} = \frac{\pi^2 E_{s_i} I_{s_i}}{A_{s_i} s_{i_0}^2}. \quad (3.14)$$

Members in the DHT payload carriers are modeled as solid cylinders with a circular cross-sectional area. To calculate the internal stress in Eq. (3.13), strain must first be calculated. The current lengths for strings and bars are denoted by $\|\mathbf{s}_i\|$ and $\|\mathbf{b}_i\|$ and initial length for bars are denoted by b_{i_0} . The strain in these members is calculated as follows:

$$\varepsilon_{s_i} = \frac{\|\mathbf{s}_i\| - s_{i_0}}{s_{i_0}}, \quad \varepsilon_{b_i} = \frac{\|\mathbf{b}_i\| - b_{i_0}}{b_{i_0}}. \quad (3.15)$$

The rate of strain for string members, $\dot{\varepsilon}_{s_i}$, is derived using chain rule:

$$\dot{\varepsilon}_{s_i} = \frac{1}{s_{i_0}} \frac{d\|\mathbf{s}_i\|}{dt} = \frac{1}{s_{i_0}} \frac{d\|\mathbf{s}_i\|}{ds_i} \frac{ds_i}{dt} = \frac{1}{s_{i_0}} \frac{\mathbf{s}_i^\top}{\|\mathbf{s}_i\|} \dot{\mathbf{s}}_i, \quad (3.16)$$

where the change in string length is stored in a column vector, $\dot{\mathbf{s}}_i$, and composed of changes

in string length for each Cartesian coordinate, defined as:

$$\dot{\mathbf{s}} = (\mathbf{C}_s \otimes \mathbf{I}_3) \dot{\mathbf{n}} = \begin{bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \vdots \\ \dot{\mathbf{s}}_{n_s} \end{bmatrix}, \quad \dot{\mathbf{s}}_i = \begin{bmatrix} \dot{s}_{ix} \\ \dot{s}_{iy} \\ \dot{s}_{iz} \end{bmatrix}. \quad (3.17)$$

3.4 Carrier Mass

The mass matrix, $\mathbf{M}$, contains the lumped node masses, m_i , that are composed of the node mass itself plus half of any bar or string that connects to that node. The mass matrix is derived as follows:

$$\begin{aligned} \mathbf{m} &= \frac{1}{2} |\mathbf{C}_s^\top| \mathbf{m}_s + \frac{1}{2} |\mathbf{C}_b^\top| \mathbf{m}_b + \mathbf{m}_p \\ &= \frac{1}{2} |\mathbf{C}_s^\top| \begin{bmatrix} m_{s_1} \\ m_{s_2} \\ \vdots \\ m_{s_{n_s}} \end{bmatrix} + \frac{1}{2} |\mathbf{C}_b^\top| \begin{bmatrix} m_{b_1} \\ m_{b_2} \\ \vdots \\ m_{s_{n_b}} \end{bmatrix} + \begin{bmatrix} m_{p_1} \\ m_{p_2} \\ \vdots \\ m_{p_{n_n}} \end{bmatrix} \\ &= \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_{n_n} \end{bmatrix}, \end{aligned} \quad (3.18)$$

where $\mathbf{m}$ is a vector containing the total mass of each node, $\mathbf{m}_P \in \mathbb{R}^{n_n}$ is a matrix containing

the mass of each free-moving joint, and $|\cdot|$ is the element-wise absolute value of the matrix. Each free-moving joint is assumed to have the same mass, m_{p_i} , as seen in Table 3.1.

Each string and bar member may have a different mass, depending on the length of the member. The length of each string, m_{s_i} , or bar, m_{b_i} , is defined by the following:

$$\begin{aligned} m_{s_i} &= \rho_{s_i} A_{s_i} s_{i_0}, \\ m_{b_i} &= \rho_{b_i} A_{b_i} b_{i_0}, \end{aligned} \tag{3.19}$$

where ρ_{s_i} and ρ_{b_i} are densities of the string and bar members, respectively. The completed mass matrix is used to calculate the acceleration of the nodes in the DHT carrier using the following equation:

$$\mathbf{M} = \hat{\mathbf{m}} \otimes \mathbf{I}_3, \tag{3.20}$$

where the $\hat{\mathbf{m}} \in \mathbb{R}^{n_n \times n_n}$ is the diagonalized form of the total mass vector $\mathbf{m}$. A Taylor series expansion is used to find the DHT node position, velocity, and accelerations during the simulation.

3.5 Taylor Series Expansion

A Taylor expansion is employed to obtain the position, velocity, and acceleration matrix for the nodes within the DHT carriers for every time step as follows:

$$\mathbf{n}_{i+1} = \mathbf{n}_i + \dot{\mathbf{n}}_i \Delta t + \frac{1}{2} \ddot{\mathbf{n}}_i \Delta t^2 + \frac{1}{6} \ddot{\mathbf{n}}_i \Delta t^3 + \mathcal{O}(\Delta t^4), \tag{3.21}$$

$$\mathbf{n}_{i-1} = \mathbf{n}_i - \dot{\mathbf{n}}_i \Delta t + \frac{1}{2} \ddot{\mathbf{n}}_i \Delta t^2 - \frac{1}{6} \ddot{\mathbf{n}}_i \Delta t^3 + \mathcal{O}(\Delta t^4). \quad (3.22)$$

Equations (3.22) and (3.21) are combined to find the node position matrix for next time step, $\mathbf{n}_{i+1}$, as follows:

$$\mathbf{n}_{i+1} = 2\mathbf{n}_i - \mathbf{n}_{i-1} + \ddot{\mathbf{n}}_i \Delta t^2, \quad i \geq 1. \quad (3.23)$$

Equation (3.23) is not valid until the second iteration in the loop. $\mathbf{n}_{-1}$ needs to be defined in the first loop, which is obtained when $i = 0$ in equation (3.22):

$$\mathbf{n}_{-1} = \mathbf{n}_0 - \dot{\mathbf{n}}_0 \Delta t + \frac{1}{2} \ddot{\mathbf{n}}_0 \Delta t^2, \quad i = 0. \quad (3.24)$$

The equations above find the node positions for the duration of the simulation. The following two expansions detail the derivation for the velocity matrices:

$$\mathbf{n}_i = \mathbf{n}_{i+1} - \dot{\mathbf{n}}_{i+1} \Delta t + \frac{1}{2} \ddot{\mathbf{n}}_{i+1} \Delta t^2 + o(\Delta t^3), \quad (3.25)$$

$$\mathbf{n}_{i-1} = \mathbf{n}_{i+1} - \dot{\mathbf{n}}_{i+1} (2\Delta t) + \frac{1}{2} \ddot{\mathbf{n}}_{i+1} (2\Delta t)^2 + o(\Delta t^3). \quad (3.26)$$

Equations (3.25) and (3.26) are then combined to eliminate the acceleration term as follows:

$$4\mathbf{n}_i - \mathbf{n}_{i-1} = 3\mathbf{n}_{i+1} - 2\dot{\mathbf{n}}_{i+1}\Delta t, \quad (3.27)$$

$$\dot{\mathbf{n}}_{i+1} = \frac{1}{2\Delta t}(3\mathbf{n}_{i+1} - 4\mathbf{n}_i + \mathbf{n}_{i-1}). \quad (3.28)$$

Equation (3.23) is substituted into (3.28) to obtain the velocity matrix for every loops:

$$\dot{\mathbf{n}}_{i+1} = \frac{1}{2\Delta t}(2\mathbf{n}_i - 2\mathbf{n}_{i-1} + 3\ddot{\mathbf{n}}_i\Delta t^2). \quad (3.29)$$

3.6 Carrier and Environment Properties

Material properties of strings and bars are outlined in Table 3.2. The chosen material are considered for their high strength-to-density ratios that enhance the performance of the DHT carriers while maintaining a low structure mass. The starting condition for each simulation assumes a DHT carrier dropping from rest with a distance of 2 m between the ground and the bottom-most location on the DHT carrier. Thus, the initial velocity of the nodes in the carrier is 0 m/s. The complete environment parameters are found in Table 3.1.

Table 3.1: Initial conditions and environmental parameters assumed for the DHT carrier simulations.

Input	Value	Explanation
h	0.02 m	Initial distance between bottom node and ground
v	-6.23 m/s	Initial vertical velocity
m_{load}	5 kg	Mass of central payload
g_E	-9.81 m/s ²	Earth gravity
m_{p_i}	0.010 kg	Mass of each connectivity node
S	1.1	Factor of safety
p_c	5×10^5 N/m ²	Ground stiffness constant
c_c	1×10^4 N s/m ²	Ground damping coefficient
η	0.5	Ground friction coefficient

Table 3.2: Material properties of the strings and bars assumed for the DHT carriers.

Property	Strings: Nylon	Bars: Titanium
Density, ρ	1123 kg/m ³	4705 kg/m ³
Young's Modulus	1×10^8 Pa	1.15×10^{11} Pa
Damping Coefficient	5×10^6 Pa s	0 (assumed)
Strength	9.4×10^7 Pa	1×10^9 Pa

Chapter 4

Double-helix Tensegrity Optimization Analysis

¹ The dynamic simulation outlined in Section 4.1 is able to calculate and output a matrix of stresses in the DHT carrier at each time step. This chapter outlines the bounds of optimization and describes methods used to restrict the optimizer to find favorable results. This chapter also details the results of an optimization study with a requirement of delivering a 5 kg payload mass. The maximum stress and payload acceleration are important factors in determining the effectiveness of a DHT carrier configuration. An optimal DHT carrier is able to protect the payload from an assumed maximum of $10g_E$ while having a low mass. Manually finding the optimal DHT carrier parameters for each carrier shape is tedious. There are a total of nine variable parameters that introduce changes in carrier performance, all of which with large ranges as see in Table 4.1.

¹Portions of this section are reprinted or adapted from [Shyam A. Patel and Edwin A. Peraza Hernandez, 2021, “Design of Double-Helix Tensegrity Payload Carriers for Impact Load Attenuation,” AIAA Scitech 2021 Forum. <https://doi.org/10.2514/6.2021-0563>]; reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.

4.1 Optimization Environment

The MATLAB[®] genetic algorithm function `ga` is used to iterate through the DHT carrier variables and converge on efficient solutions to the three carrier shapes. A population size, maximum generation limit, and maximum stall generation are defined to guide the optimization. The population size defines the number of parent solutions in each subsequent generation and is set to the square of the number of input variables. Thus, the nine input parameters found in Table 4.1 define the population size to be 540 parent solutions in each generation. The max stall generation defines the number of generations where the minimization values fall below a predefined stall criteria. When the max stall generation has passed, the optimization is triggered to end. Finally, the max generation size is the number of generations the optimization can run before the optimization is complete, given that the max stall generation limit has not been triggered. To compare the relationship between shape and carrier topology, three optimization studies are conducted; one study is conducted for each shape.

The ranges of values within which the optimizer is able to perform its study is found through means of trial and error. The goal of the optimizer is to minimize a defined error function within these bounds of operation. This error function is constructed such that within a group of *effective* DHT carriers, a carrier with a smaller mass obtains a lower error score. An *effective* carrier is simply defined as a carrier that does not experience material or structural failure during the impact sequence. Additionally, an effective carrier can attenuate impact load such that the payload experiences accelerations lower than $10g_E$.

!

Table 4.1: Design variables and boundary conditions of the optimization study, found through trial and error.

Input	Type	Range
p	Integer	[4, 10]
q	Integer	[4, 10]
L	Continuous	[0.5, 1.2] m
$r_{\bar{s}}$	Continuous	[0.5, 8.0] mm
$r_{\underline{s}}$	Continuous	[0.5, 8.0] mm
r_b	Continuous	[0.5, 8.0] mm
c	Continuous	[-0.1, 0.1]
n_P	Continuous	[-0.45, 0.45]
R/L	Continuous	[0.25, 2.0]

4.2 Optimization Penalties

The error function the optimizer attempts to minimize, referenced as the *fitness function*, is noted by f and defined as follows:

$$f = m + p_g + p_{\sigma_{\bar{s}}} + p_{\sigma_{\underline{s}}} + p_{\sigma_{b_c}} + p_{\sigma_{b_t}} + p_{V_c} + N, \quad (4.1)$$

where the m is mass of the structure, p is the normalized scalar penalty of each design constraint. Additionally, the subscript $\underline{s}$ denotes penalty values associated with internal string members and subscript $\bar{s}$ denotes penalty values associated with the surface string members. These two penalties are separated because the two string types have separate definable radii, as seen in Table 4.1.

The normalized penalty is defined as follows:

$$p_i = \lambda_i \gamma_i, \quad (4.2)$$

where γ_i is the penalty constant applied to each non-dimensionalized fitness function error, λ_i , to normalize the magnitude of the penalties across the fitness function. This step is needed to normalize the values between stress, which can be in the order of 10^6 and the other errors such as payload acceleration, g_E , where the error is often less than the magnitude of 10^2 . Normalizing penalties across the fitness function allows the optimizer to gather meaningful data from different DHT configurations. The values of the penalty constants, γ_i , are as follows:

$$\begin{aligned} \gamma_g &= 1^{15}, \quad \gamma_{\sigma_{\bar{s}}} = 1, \quad \gamma_{\sigma_{\underline{s}}} = 1, \\ \gamma_{\sigma_{b_c}} &= 1, \quad \gamma_{\sigma_{b_t}} = 1, \quad \gamma_{V_c} = 1^{14}, \end{aligned} \tag{4.3}$$

and the values of the error, λ_i , are given as follows:

$$\begin{aligned}
\lambda_g &= \begin{cases} 0; & \text{if } g_P \leq g_{max} \\ ((g_{max} - g_P)^2); & \text{if } g_P > g_{max} \end{cases} \\
\lambda_{\sigma_{\bar{s}}} &= \begin{cases} 0; & \text{if } \sigma_{\bar{s}} \leq \frac{\sigma_{\bar{s}max}}{S} \\ (\frac{\sigma_{\bar{s}max}}{S} - \sigma_{\bar{s}})^2; & \text{if } \sigma_{\bar{s}} > \frac{\sigma_{\bar{s}max}}{S} \end{cases} \\
\lambda_{\sigma_{\underline{s}}} &= \begin{cases} 0; & \text{if } \sigma_{\underline{s}} \leq \frac{\sigma_{\underline{s}max}}{S} \\ (\frac{\sigma_{\underline{s}max}}{S} - \sigma_{\underline{s}})^2; & \text{if } \sigma_{\underline{s}} > \frac{\sigma_{\underline{s}max}}{S} \end{cases} \\
\lambda_{\sigma_{b_c}} &= \begin{cases} 0; & \text{if } \sigma_b \geq -\frac{\sigma_{b_{cr_i}}}{S} \\ (\frac{\sigma_{b_{cr_i}}}{S} - \sigma_{b_c})^2; & \text{if } \sigma_{b_c} < -\frac{\sigma_{b_{cr_i}}}{S} \end{cases} \\
\lambda_{\sigma_{b_t}} &= \begin{cases} 0; & \text{if } \sigma_{b_t} \leq \frac{\sigma_{b_{tmax}}}{S} \\ (\frac{\sigma_{b_{tmax}}}{S} - \sigma_{b_t})^2; & \text{if } \sigma_{b_t} > \frac{\sigma_{b_{tmax}}}{S} \end{cases} \\
V_c &= \begin{cases} 0; & \text{if } V_c \leq V_{cmax} \\ (V_{cmax} - V_c)^2; & \text{if } V_c > V_{cmax} \end{cases} \\
N &= \begin{cases} 0; & \text{if } N_f = 0 \\ 1000; & \text{if } N_f = 1 \end{cases}
\end{aligned} \tag{4.4}$$

where g_P is the maximum acceleration that the payload experiences and g_{max} is the maximum allowable payload acceleration without error. The stresses in the DHT carrier members are compared to their maximum yield and buckling stresses divided by a factor of safety, S . The material yield failure for bars and strings are $\sigma_{b_{tmax}}$ and $\sigma_{\underline{s}max}$ and $\sigma_{\bar{s}max}$, respectively and

their values are stated in Table 3.2. The tensile stress of every member is calculated at each time step, as in Eq. (3.14). The material yield strength is compared to the maximum tensile stress in the members of the DHT. The tensile of the internal and surface strings are noted as $\sigma_{\underline{s}}$ and $\sigma_{\overline{s}}$, respectively. The non-dimensionalized fitness function error for tensile forces are based off of the largest difference in tensile failure across all members of a member type throughout the entire simulation.

$$\begin{aligned}
&\text{Find: } p, q, L, r_{\overline{s}}, r_{\underline{s}}, r_b, c, n_P, R/L \\
&\text{That minimize: } m \\
&\text{Subject to: } \text{Constraints found in Eqn. 4.4}
\end{aligned} \tag{4.5}$$

For bars, a buckling constraint must also be considered. Euler's critical load formula is used to calculate the critical buckling stress for each bar member:

$$\sigma_{b_{cr_i}} = \frac{\pi^2 E_{b_i} I_{b_i}}{A_{b_i} b_{i_0}^2}, \tag{4.6}$$

where I_{b_i} is the area moment of inertia for the i^{th} bar in the bar matrix. The critical buckling stress changes with each bar because each bar can have a different length, b_{i_0} . The non-dimensionalized fitness function error for buckling, $\lambda_{\sigma_{b_c}}$, is calculated based off of the largest difference in buckling failure out of all the bars in a DHT carrier and across the entire simulation.

The volumetric constraint, V_c , penalizes values of the second-order axial node concentration (defined in Chapter 3) that generate free-moving joints outside of the volumetric constraints bound by the input length, l , and radius-to-length ratio, R/L . If the second order concen-

tration is too high or too low, the placement of the end nodes exceed the bounds of the structure. The highest structural node is compared to the total length of the carrier to find whether the the DHT carrier has been generated with a node out of its defined volume. If the height of the tallest node is greater than the length of the structure, a penalty must be applied. This penalty condition is defined by the following:

$$V_c = \|n_{t_y} - L\|, \quad (4.7)$$

where n_{t_y} is the y -axis position of the highest node in the DHT carrier.

The final penalty in Eq. (4.4) is triggered if the tensegrity carrier flips inwardly on itself. When a carrier flips inwards on itself, the bottom most nodes obtain an axial position coordinate that is higher than that of the central payload. This condition is impossible in the real world as the internal strings that connect to the payload would stop the nodes from flipping upwards. To track this failure condition, the axial position of the bottom-most nodes is compared to the axial position of the central payload. Once the bottom-most payload rises past the central payload, the node flip constraint, N , is triggered and the following penalty is added to the fitness function:

$$N = \begin{cases} 0, & \text{if } n_{b_y} \leq n_{p_y} \\ 1, & \text{if } n_{b_y} > n_{p_y} \end{cases} \quad (4.8)$$

where the axial node position of the bottom-most node is indicated by n_{b_y} and the axial node position of the payload is indicated by n_{p_y} . If the node flip constraint is triggered, a

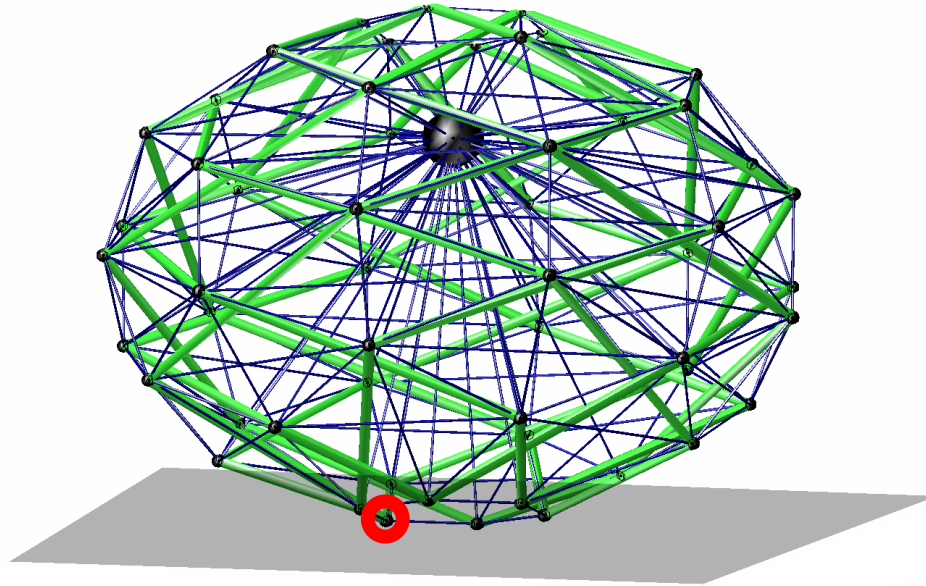
constant penalty of 1000 is added to the fitness function. A visual representation of a node flip condition is found in Fig. 4.1.

4.3 Optimization Results - 5 kg Payload

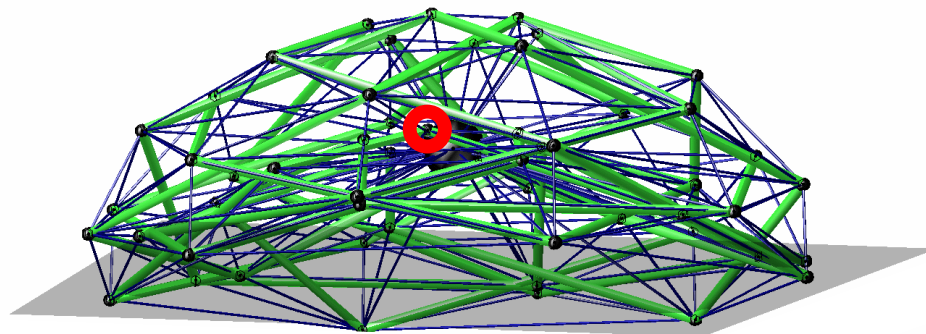
The MATLAB[®] genetic algorithm `ga` function is used to continuously run the dynamic simulation while varying the carrier topologies and noting trends. The optimizer iterates through to a defined maximum number of generations unless a converged result is found. The reduction of the fitness score by the optimization study is seen 4.2.

The genetic algorithm solver was able to converge on a design for each of the three DHT carrier shapes without penalty. Summarized results are found in Table 4.2 below. The parabolic DHT carrier had the lowest mass of the three carriers and thus achieved the lowest fitness score with a carrier mass of 2.33 kg. The ellipsoidal DHT carrier had the second lowest fitness score with a carrier mass of 3.88 kg. Finally, the cylindrical DHT carrier had the highest fitness score with a carrier mass of 5.44 kg.

The results indicate that a lower axial and radial complexity reduce mass of the DHT carriers. Each of the different DHT carrier shapes converged to different string and bar member lengths to account for differences in stress distributions during impact. The converged cylindrical DHT carrier design was the heaviest of the three carriers and had the thickest string and bar members. Higher bar and string member radii illustrates higher overall stresses experienced in the carrier. The cylindrical carrier had the lowest radius to length ratio among the three carriers. A low radius to length ratio indicates a slenderer shape as seen in Fig 4.7. The linear axial node distribution may be a result of an even stress distribution across the structure. The payload is situated slightly higher than the axial center of the structure. As seen in Figure 4.3, the structure compresses axially but not enough to require a payload



Before impact



After impact

Figure 4.1: Example of a DHT carrier exhibiting a node flip. The highlighted node in red flips upwards to a height above the central payload after impact.

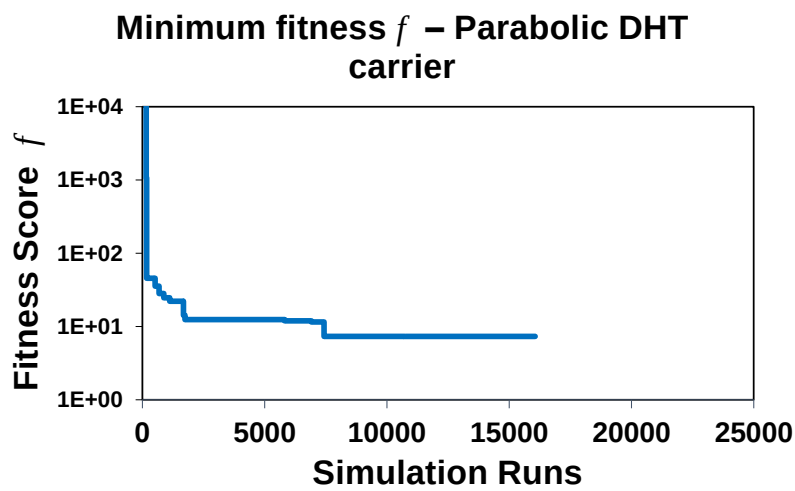
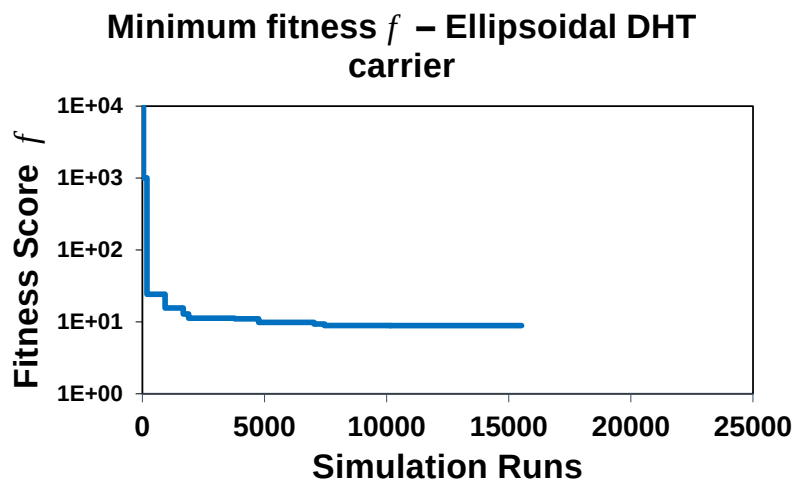
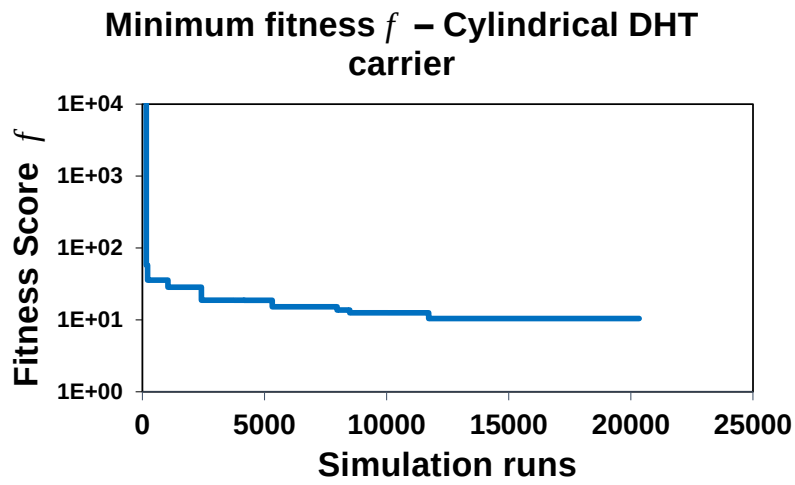


Figure 4.2: Best fitness score as progression of optimization.

height much higher than the origin.

The converged ellipsoidal DHT carrier design required thinner string and bar members than the cylindrical carrier. The radius to length ratio for the ellipsoidal carrier was greater than the cylinder and was closer to a spherical configuration. The ellipsoidal carrier achieved the lowest payload acceleration of the three DHT carriers. Although the payload acceleration was the lowest, the fitness score of the ellipsoidal carrier was not the lowest. As explained in Section 4.2, the optimizer is focused solely on minimizing mass once the deceleration threshold of $10g_E$ is met. The axial height of the ellipsoidal carrier is high to account for the large axial deformation the carrier goes through during impact as seen in Figure 4.5.

The parabolic DHT carrier continues the trend of having low complexity in the axial and radial directions. The surface strings of the converged parabolic DHT carrier design are exceptionally thin at just 1.3 mm in radius. The parabolic DHT carrier also has thinner bar and internal string members than the other two carrier shapes. The lower member radii can be attributed to larger structure deformations as seen in figure 4.6 which led to lower maximum stresses experienced by each member.

Although the structure shows large deformation, the parabolic shape of the structure enables even deformation without circumferential interference as in the cylindrical carrier. The even deformation enables the payload height to be situated lower in the parabolic DHT carrier compared to the cylindrical and ellipsoidal carriers. An inadvertent benefit to a lower center of mass is that there is less likelihood that the DHT carrier will flip during its drop because of weather conditions.

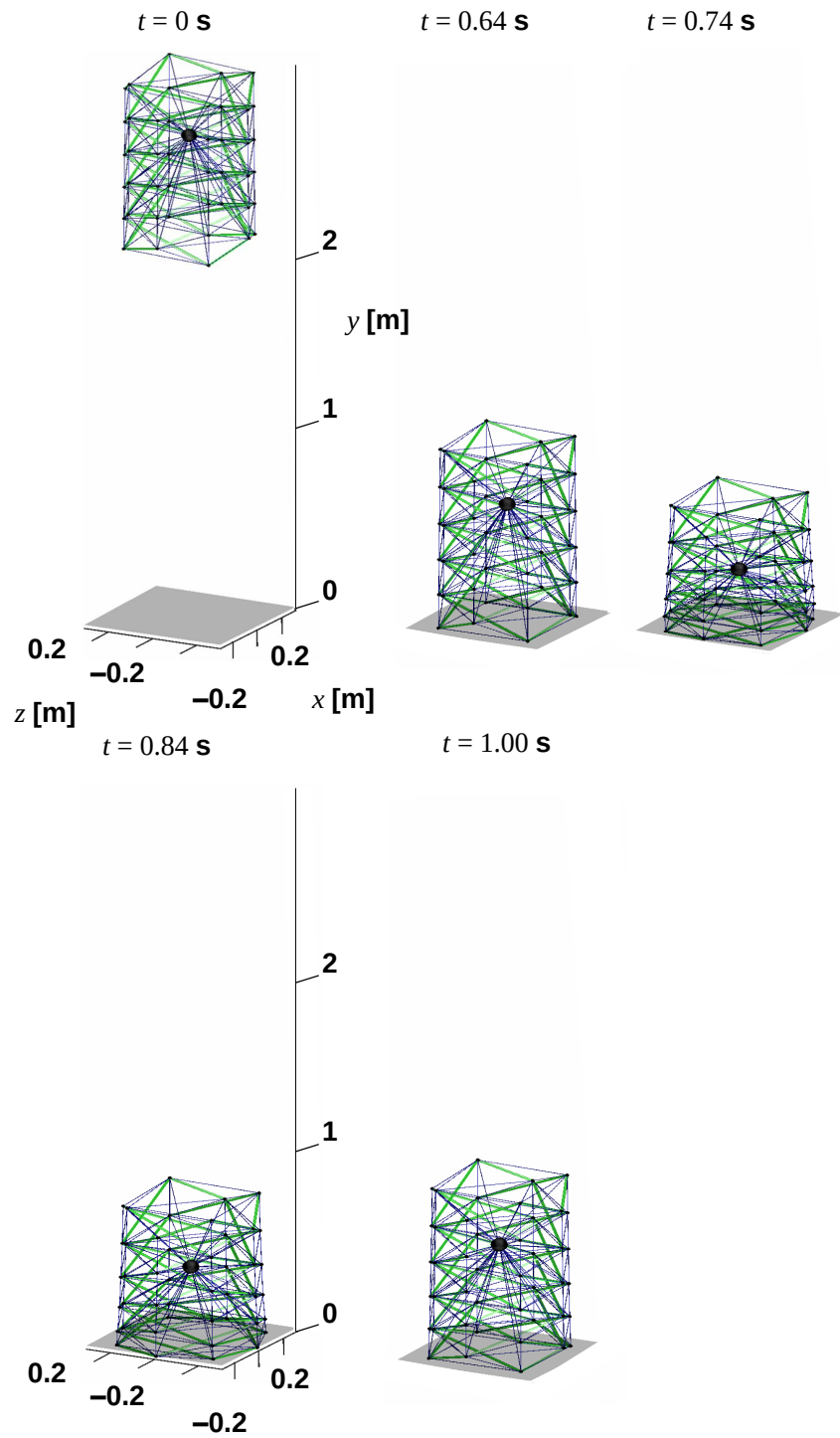


Figure 4.3: Time frames from a cylindrical DHT carrier impacting the ground with a 5 kg payload.

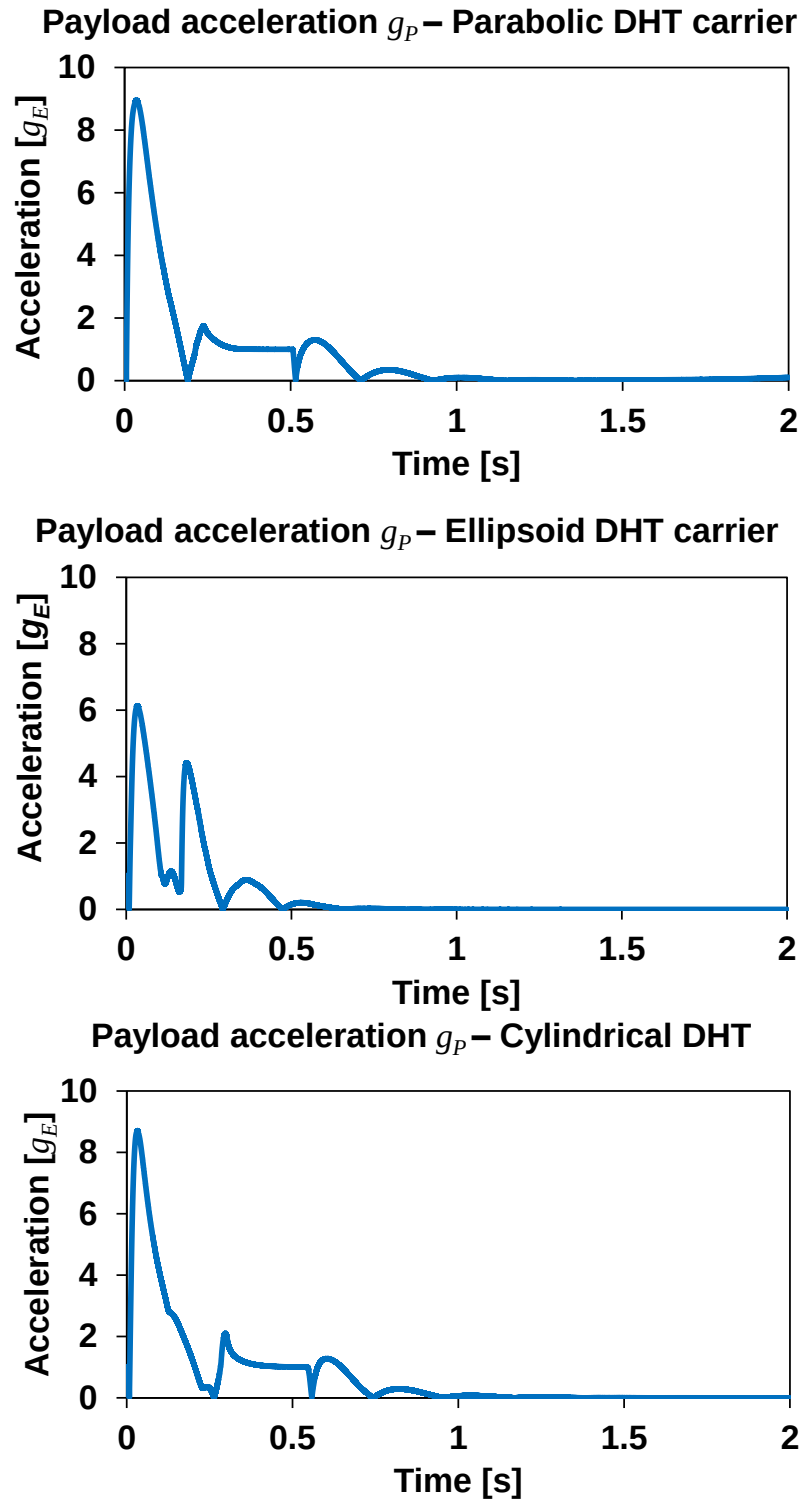


Figure 4.4: Acceleration of payload inside of the optimized parabolic DHT carriers during impact.

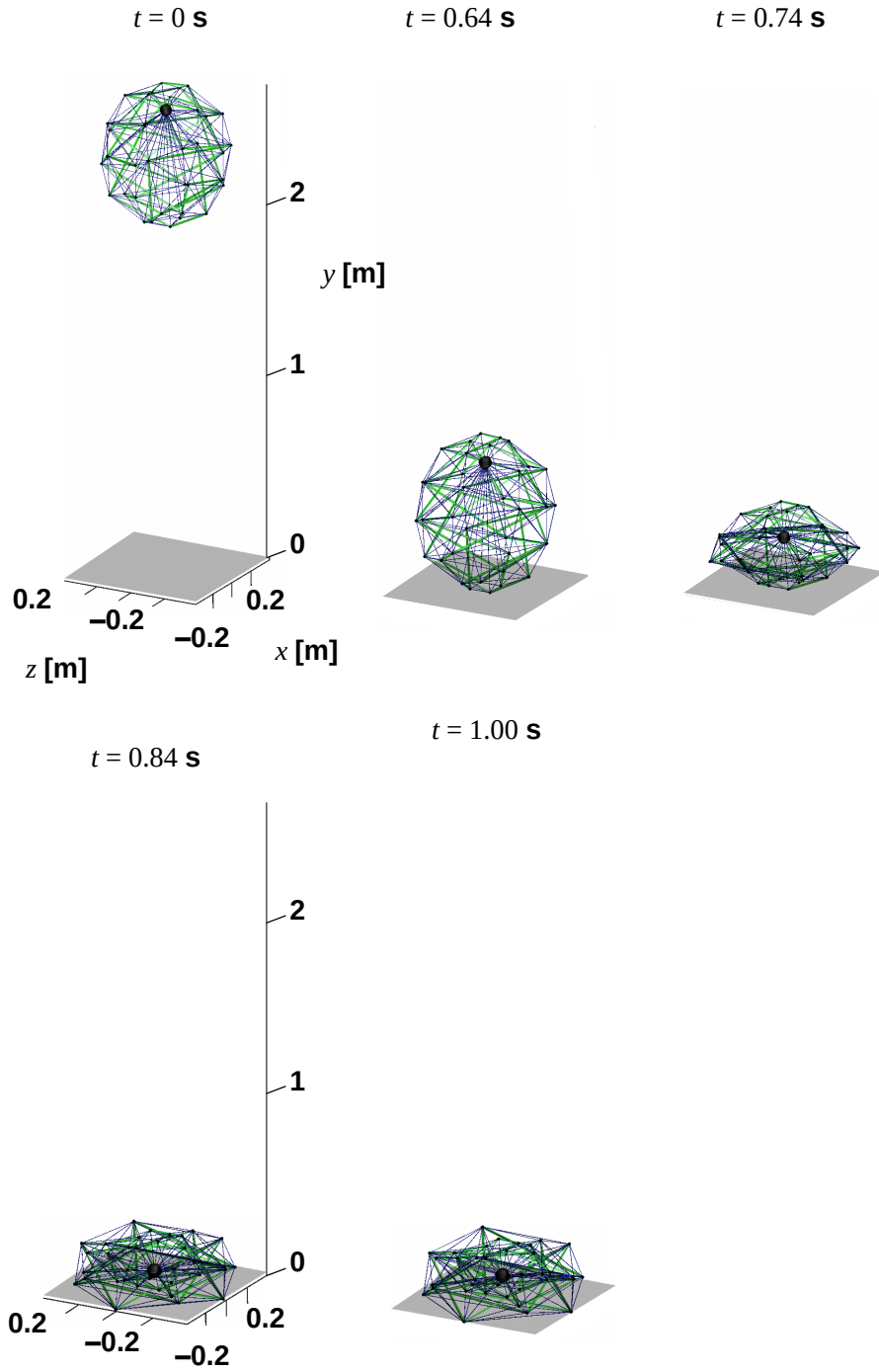


Figure 4.5: Time frames from a ellipsoidal DHT carrier impacting the ground with a 5 kg payload.

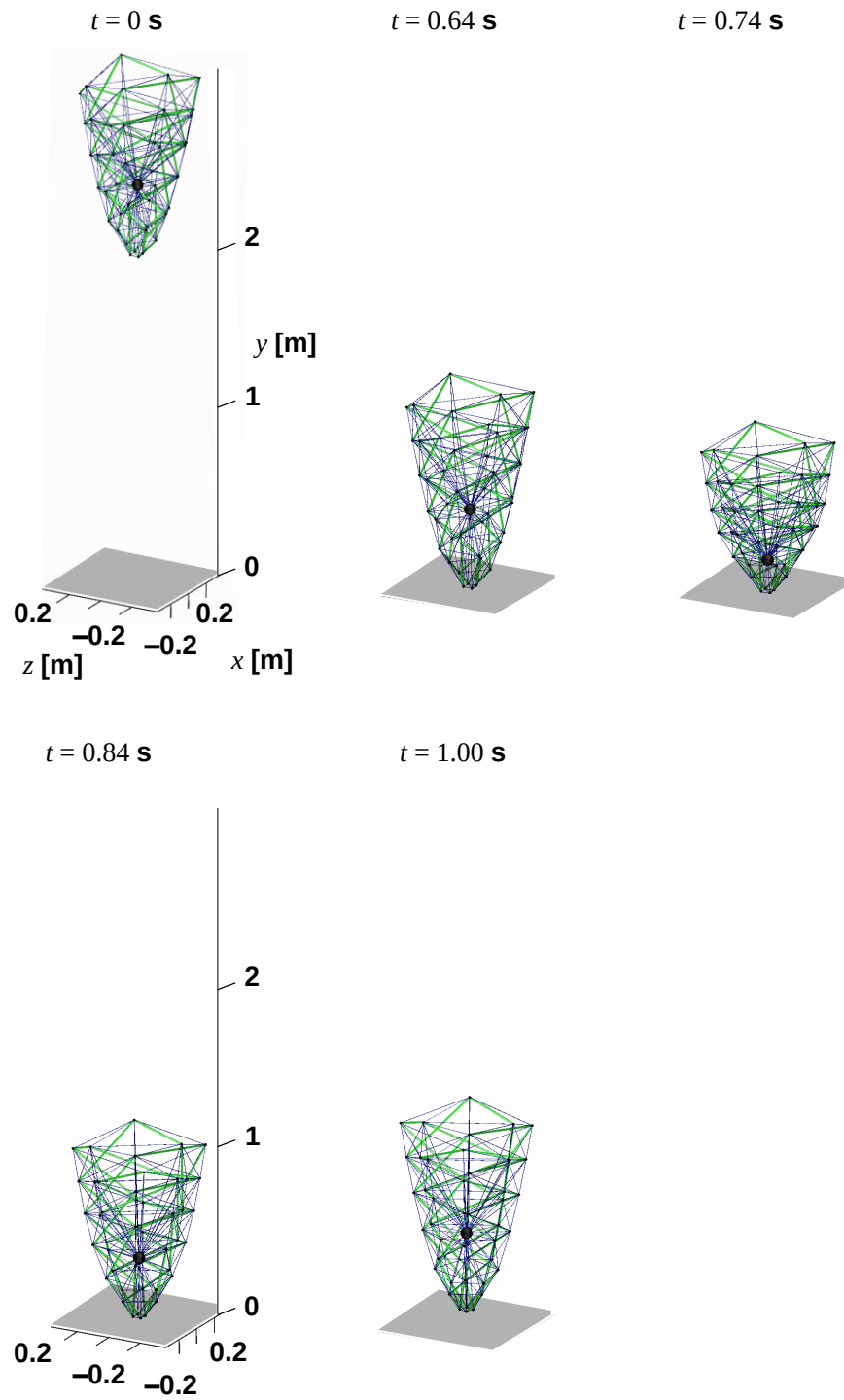


Figure 4.6: Time frames from a parabolic DHT carrier impacting the ground with a 5 kg payload.

Table 4.2: Optimized DHT carrier parameters.

Output	Cylindrical DHT	Ellipsoidal DHT	Parabolic DHT
p	4	4	4
q	5	4	5
L	0.93 m	0.81 m	1.12 m
$r_{\bar{s}}$	3.0 mm	2.5 mm	1.3 mm
$r_{\underline{s}}$	1.1 mm	0.8 mm	0.6 mm
r_b	3.7 mm	3.4 mm	2.5 mm
c	0.0091	0.0015	-0.0076
n_P	0.1526	0.3412	-0.0936
R/L	0.304	0.444	0.556
f	10.44	8.88	7.33
m	10.44 kg	8.88 kg	7.33 kg
$m - m_{load}$	5.44 kg	3.88 kg	2.33 kg
g_P	$8.71g_E$	$6.15g_E$	$8.99g_E$

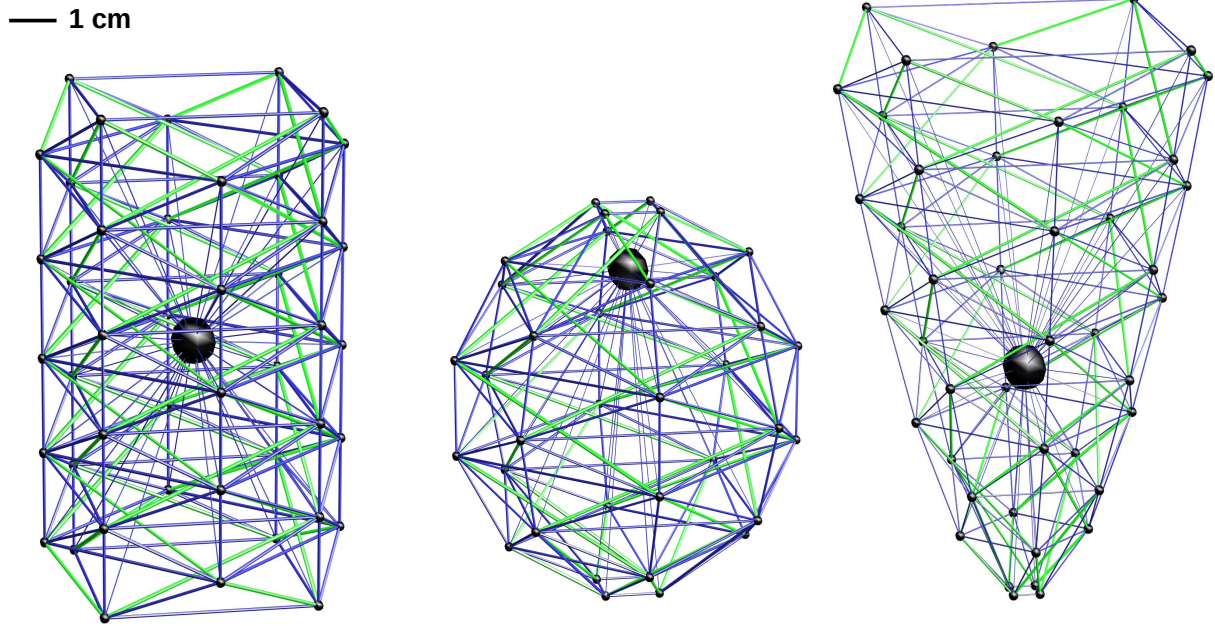


Figure 4.7: Converged optimization designs for the three DHT payload carriers. Left to right: Cylindrical, ellipsoidal, and parabolic DHT carriers.

Chapter 5

Tensegrity Morphing Wing Optimization Analysis

This chapter presents a multi-objective optimization study conducted on a tensegrity morphing wing. The tensegrity torsional mechanism used in the wing is described in section 5.1. Parametrization of wing topology, finite element analysis, and material choices for the tensegrity morphing wing are detailed in sections 5.2–5.4. The multi-objective optimization and final results are presented in sections 5.5 and 5.6.

5.1 Tensegrity Torsional Wing Mechanism

The underlying twisting tensegrity mechanism is composed of a cylindrical spar, ribs, and a network of wires. The spar is oriented from the wing root to the wing tip, stretching the entire wingspan and is the primary bending load bearer. The ribs are located in an equidistant manner along the wing in the spanwise direction, acting as the primary airfoil shape holders and separate the wing into tensegrity cells—the region between each pair of

ribs. Each rib has a circular hole where the stationary spar resides through, the spar axis was designed to be the center of torsion. Each tensegrity cell contains a uniform, evenly distributed pattern of wires around the spar to hold the system together. The wires are the tensile members of the tensegrity structure and play a role in both stabilizing and providing a means to actuate the twisting motion the ribs. A side profile of the wing twist is provided in Fig. 5.1.

Figure 5.2(a) is a simplified illustration of the internal torsional mechanism. Here, only the bounding rings and a pair of the actuator and longitudinal wires are shown. When activated, the actuator wires contract to generate a torque that twists the torsional mechanism, which in turn, rotates the adjacent ribs. During this process, the longitudinal wires extend and develop an internal biased restoring torque. Similar to a torsional spring, the tensegrity column returns to its initial state once the actuator wires are deactivated and returned to their initial length. The tensegrity cell in its entirety with all components involved is shown in Fig. 5.2(b).

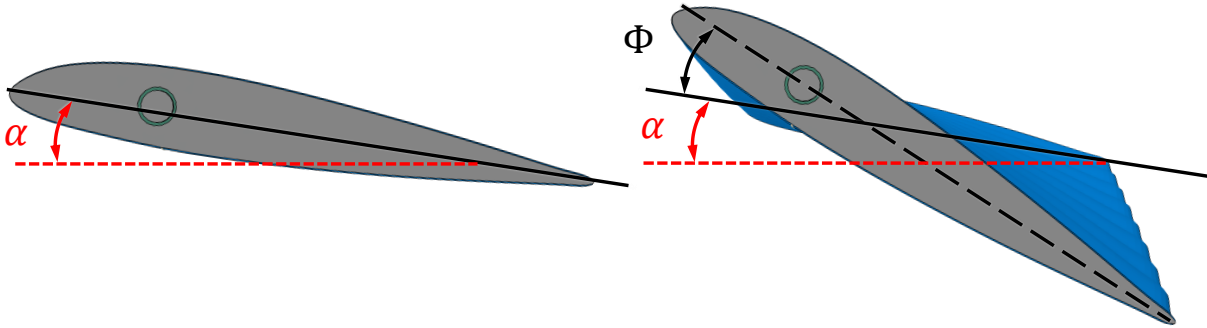


Figure 5.1: Diagram showing twisting action and deflection angle of the morphing tensegrity wing where α represents the angle of attack of the wing while ϕ denotes the wing tip deflection angle with respect to the root of the wing.

A coordinate system with the x -direction pointing towards chord direction from the leading to the trailing edge and the z -direction pointing along in the span wise direction from root to tip is used to define the wire positions. The origin of the coordinate system is located in the center of the face of the spar at the root of the wing. Figure 5.3 shows a tensegrity cell

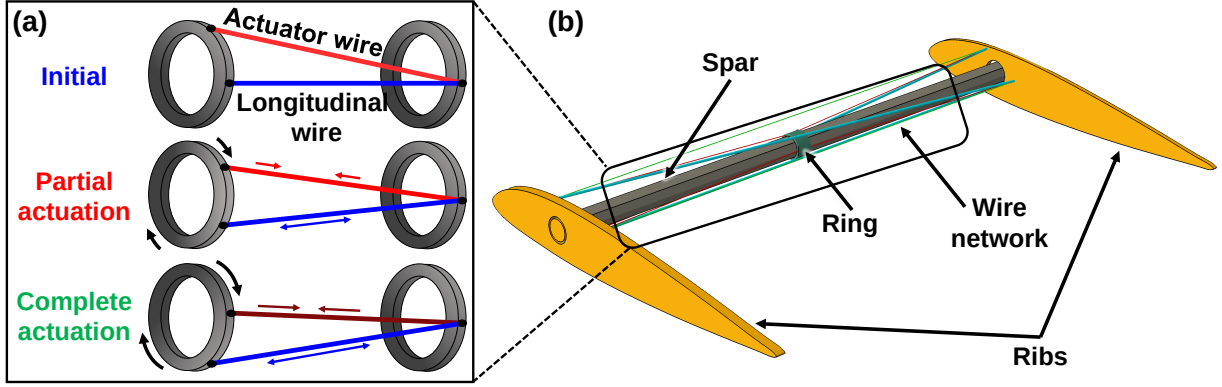


Figure 5.2: Torsional tensegrity mechanism. (a) Simplified schematic of a cylindrical cell exhibiting torsional motion enabled by contraction of an actuator wire. (b) Corresponding placement of the cell within the tensegrity twisting wing.

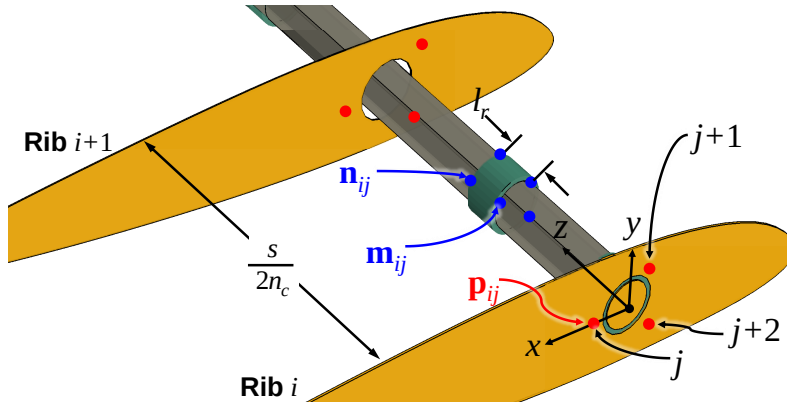


Figure 5.3: Schematic showing index convention and wire end points $\mathbf{n}_{ij}$, $\mathbf{m}_{ij}$, and $\mathbf{p}_{ij}$. The rib number and the wire set number are represented by the indices i and j , respectively.

along with the coordinate system, associated dimensions, and points that are used to define the end points of each wire in the torsional tensegrity mechanism.

Indices i and j denote the position vectors at the end of each wire in the torsional mechanism at 0° wing twist. The number of wire sets found in the mechanism are represented by the index $j \in \{0, \dots, n_w - 1\}$. The index starts at the wire set that is oriented in the direction along the x -axis and increments in the counterclockwise direction around the z -axis on the x - y plane. The wires are distributed evenly around the center of the spar and their radial distance from the center of the spar is defined by R . The even distribution of the wires around the spar defines the angle between each wire as $2\pi/n_w$ where n_w is the total number

of wire sets. The axial length of the rings around the spar, l_r , is fixed while R and r are proportional to the radius of the spar, r_{spar} . The radius of the spar is discretely varied during the optimization process presented in Section 5.5. The index i starts from the wing root ($i = 0$), denoting the rib number, and increases along the span for every rib until the wing tip is reached ($i = n_c - 1$). Here, it is important to keep note that n_c denotes the number of cells and not the number of ribs. The space between each rib is given by $s/(2n_c)$, as seen in Fig. 5.3. The total span of the wing in this study, $s/2$, is half of the length of the total wingspan of the model aircraft. Half of the wingspan is used as only one side of the wing is studied.

The wire end point locations are defined by vectors $\mathbf{p}_{ij}, \mathbf{m}_{ij}, \mathbf{n}_{ij} \in \mathbb{R}^3$ as shown in Fig. 5.3. The position vectors for the wire end points, based on previous work in [43], are as follows:

$$\mathbf{p}_{ij} = \begin{bmatrix} R \cos\left(\frac{2\pi j}{n_w}\right) \\ R \sin\left(\frac{2\pi j}{n_w}\right) \\ \frac{si}{2n_c} \end{bmatrix}, \quad \mathbf{m}_{ij} = \begin{bmatrix} r \cos\left(\frac{2\pi j}{n_w}\right) \\ r \sin\left(\frac{2\pi j}{n_w}\right) \\ \frac{s}{2n_c} \left(i + \frac{1}{2}\right) - \frac{l_r}{2} \end{bmatrix}, \quad \mathbf{n}_{ij} = \begin{bmatrix} r \cos\left(\frac{2\pi j}{n_w}\right) \\ r \sin\left(\frac{2\pi j}{n_w}\right) \\ \frac{s}{2n_c} \left(i + \frac{1}{2}\right) + \frac{l_r}{2} \end{bmatrix}. \quad (5.1)$$

Each type of the three wires are depicted in Fig. 5.4. The start and end points of each of these wires is described in Table 5.1. The end point index j for the longitudinal wires are the same as the start point. This configuration ensures that the longitudinal wire stretches once the actuator torque is applied to twist the wing. The longitudinal wire stretch provides restores the wing to its original configuration once the wing actuators are relaxed. The actuator wires have end points, $j + 1$, for each iteration of rib, $i + 1$, so that once they are contracted they are able to cause a radial torque and twist the rib. The stabilizer wires are similar to the longitudinal wires in that they share the same start and end indices for the j index. A key difference is that the stabilizer wire connects to the ring around the spar in order to provide stability to the torsional mechanism. The connection to the ring also

Table 5.1: Start and end nodes of the longitudinal, actuator, and stabilizer wires forming the torsional tensegrity mechanism; $i \in \{0, \dots, n_c - 1\}, j \in \{0, \dots, n_w - 1\}$.

Wire type	Start node	Finish node
Longitudinal	$\mathbf{p}_{ij}$	$\mathbf{p}_{i+1\ j}$
Actuator	$\mathbf{p}_{ij}$	$\mathbf{p}_{i+1\ j+1(\text{mod } n_w)}$
Stabilizer	$\mathbf{p}_{ij}$	$\mathbf{m}_{ij}$
	$\mathbf{n}_{ij}$	$\mathbf{p}_{i+1\ j}$

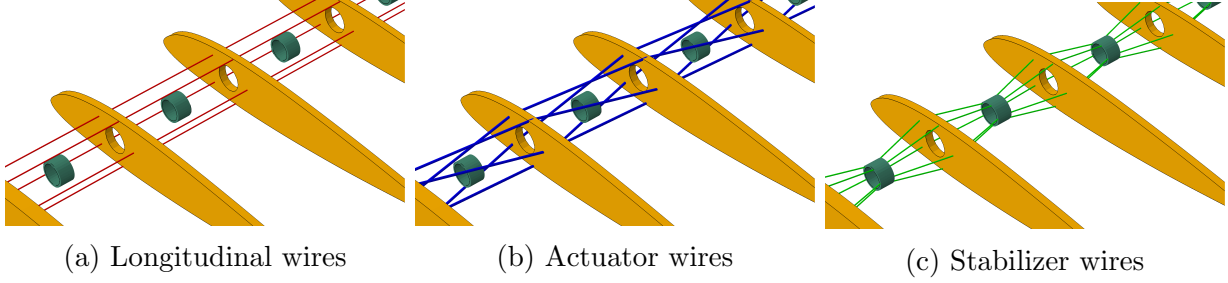


Figure 5.4: Illustration of the position of the longitudinal, actuator, and stabilizer wires in two cells obtained employing the wire end point connectivity provided in Table 5.1.

enables concentric movement around the spar for the ribs. The outer ribs of the wing are equipped with additional stabilizer rings to maintain the stability and alignment of the spar and ribs along the span of the wing. These rings contact the spar but allow the spar to rotate freely.

5.2 Parameterization of Morphing Wing

To expand the design space for the optimization of the tensegrity morphing wing, its design parameters must be characterized. Certain design parameters such as the actuator wire diameter are held constant during the optimization process as their effects were previously explored in a previously conducted design study [20]. Additionally, material sourcing constraints limit the actuator wire dimensions. The diameter of the actuator wires is held constant at 3.25 mm. The end goal of the optimization is to create a configuration that can be replicated in the real world.

The tensegrity wing design has two topological parameters corresponding to the number of cells, n_c , and the number of wire sets, n_w . Both n_c and n_w are explored in the optimization presented in Section 5.5. Each set of wires is composed of one actuator wire, one longitudinal wire, and two stabilizer wires. Examples showing wing designs with 5, 7, and 9 cells and a single cell with different numbers of wire sets are shown in Figs. 5.7 and 5.8, respectively.

The longitudinal and stabilizer wires, illustrated in Fig. 5.4, are both defined to have the same thickness in this optimization study. The thickness of these wires, d_{wires} is parameterized to find the optimal wire thicknesses. The skin thickness, t_{sk} , is parameterized to obtain a thickness that provides aerodynamic stability while not contributing enough additional stress on the wing twist mechanism to cause failure or impede maximum achievable twist. The thickness of the ribs, determined by t_{rib} provides strength at the cost of mass. The optimizer balances these two between a given set of bounds to find the optimal thickness. An illustration of the wire and rib thickness is found in Fig. 5.5. Additional parameters include the location of the spar rod along the chord of the wing and the radius of the spar rod itself, as seen in Fig. 5.6. The location of the spar along the chord of the wing, x_{loc} , changes the concentration of stresses on the wing as the center of rotation changes. Changing the center of rotation in turn affects the moments and forces on the wing from air effects. The radius of the spar rod, r_{spar} , changes the strength of the spar and mass of the overall wing.

The final two parameters relate to the physical topology of the wing topology. The number of ribs are controlled by the parameter, n_c , which indicates how many cells, or spaces between ribs, are generated in the wing structure, as seen in Fig. 5.7. A higher number of cells in the wing enables for greater maximum twist angle in the wing since there are more actuator sections. A drawback to a high number of wing cells is the increased mass from adding additional components with every generated cell. Additionally, all configurations have the same wingspan. This makes it so that the wire lengths in wing configurations with lower cell counts have longer actuator, stabilizer, and longitudinal wire sets. Similarly, the number

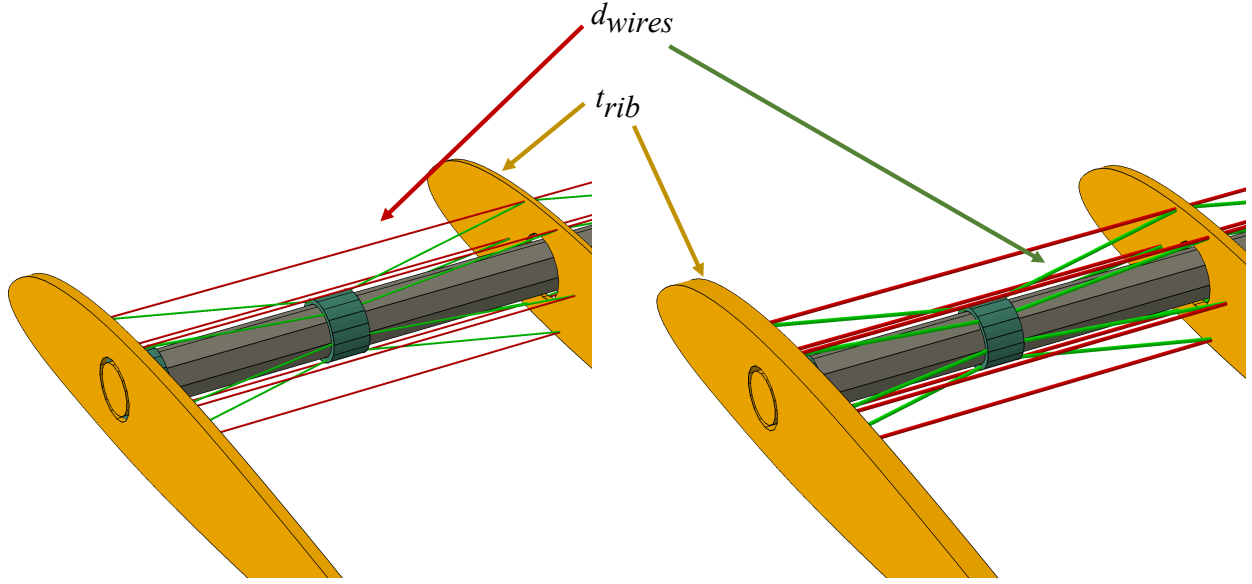


Figure 5.5: Demonstration of change in thickness of passive wires and wing ribs. The longitudinal and stabilizing wires are both varied via d_{wires} and the rib thickness is varied via t_{rib} . Not pictured in this figure are the actuator wires which are given a constant thickness.

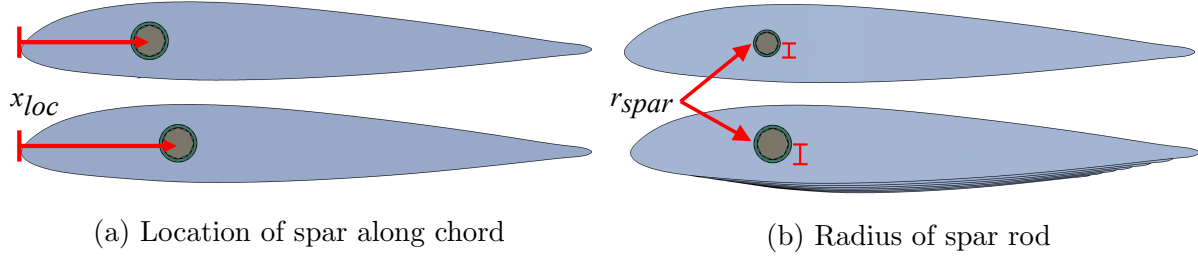


Figure 5.6: Illustration of the position of the longitudinal, actuator, and stabilizer wires in cells obtained employing the wire end point connectivity provided in Table 5.1.

of wire sets in each cell is parameterized. The number of sets in each of the cells is the same, and the number of sets is controlled by n_w . A diagram showing the range of wire sets available to the optimizer is found in Fig. 5.8. The spar is left out of this figure to better showcase the wire configurations. Increasing the number of wire sets distributes more of the stresses between the actuator and passive wires. Increasing the number of wire sets to a point of failure would be a result of permanent deformations in the wing caused from an overabundance of twist torque from the actuating wires.

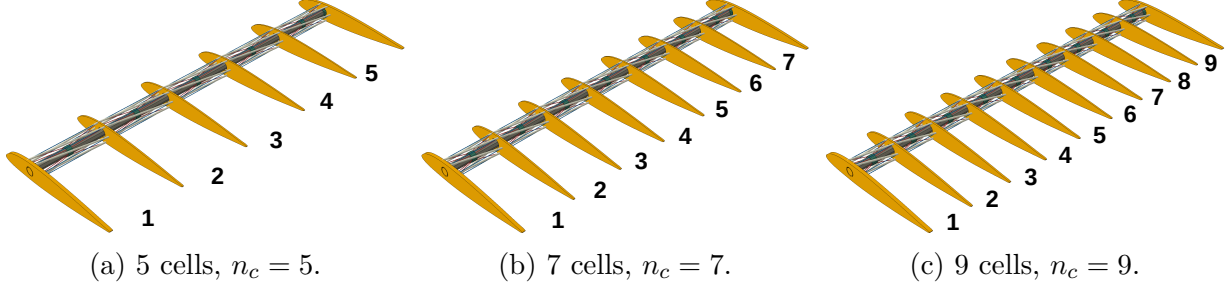


Figure 5.7: Torsional tensegrity mechanisms with different number of cells bounded by ribs (n_c).

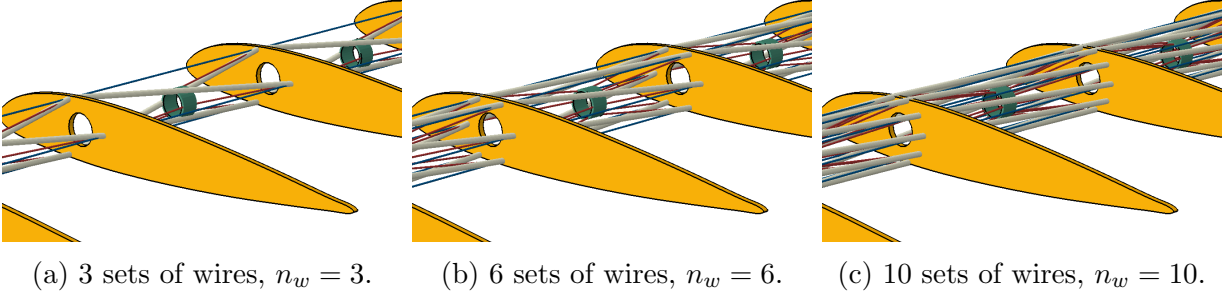


Figure 5.8: Torsional tensegrity mechanisms with different number of wire sets (n_w). The central spar is omitted to enable a better display of the wire members. The value of n_w applies to all three types of wires.

5.3 Finite Element Model and Boundary Conditions

Abaqus CAETM is used to create a finite element model of the tensegrity wing. **Abaqus CAETM** is able to assess the stresses and deformations of the wing caused by both aerodynamic and internal mechanism-related load. Section 5.2 describes the wire set configuration and section 5.1 describes the tensegrity torsional mechanism that were used to create the finite element model. Additionally, the model size and shape is modeled after the Carl Goldberg Falcon Mark II UAV seen in Fig. 5.9. The model sizing accuracy is verified by comparing the measured sizing of the aircraft to the model sizing. The **Python** console within **Abaqus CAETM** is used to run a script which generates the tensegrity mechanism topology within the software. The script also parametrizes the variables within the wing so that the topology of the finite element components can be easily modified. The shell elements for the finite element model are defined within the scope of the **Abaqus CAETM** shell elements library. The

ribs and skin are thin and thus the **Abaqus S4R** shell elements (linear 4-node shell elements with reduced integration) are used to represent them since are much thinner compared to their planform area. The stabilizer rings found in the start and end ribs along with the wing spar are both modeled using the three-dimensional continuum elements **C3D8R** (eight-node brick elements with reduced integration). The wires are represented by **T3D2** elements (2-node linear 3-D truss) since the forces applied to them are unidirectional tension and compression. Thus, the wires have a very small cross section compared to their lengths. Finally, the rings withing the cells are modeled using the **C3D10** elements (10-node tetrahedral elements). The components inside of the mechanism (i.e. wires, ribs, rings) are connected together at various node locations to ensure an accurate study can be performed. A mesh convergence study is then conducted in **Abaqus CAE™** to understand the suitable mesh resolution to minimize computational expense while maintaining accuracy in the simulation. The final converged mesh is presented in Fig. 5.10 where the wing mesh is shown both with and without the skin.



Figure 5.9: Remote controlled model aircraft used for this study and its half span and wing chord dimensions. This aircraft is the Carl Goldberg Falcon Mark II Remote Controlled Aircraft.

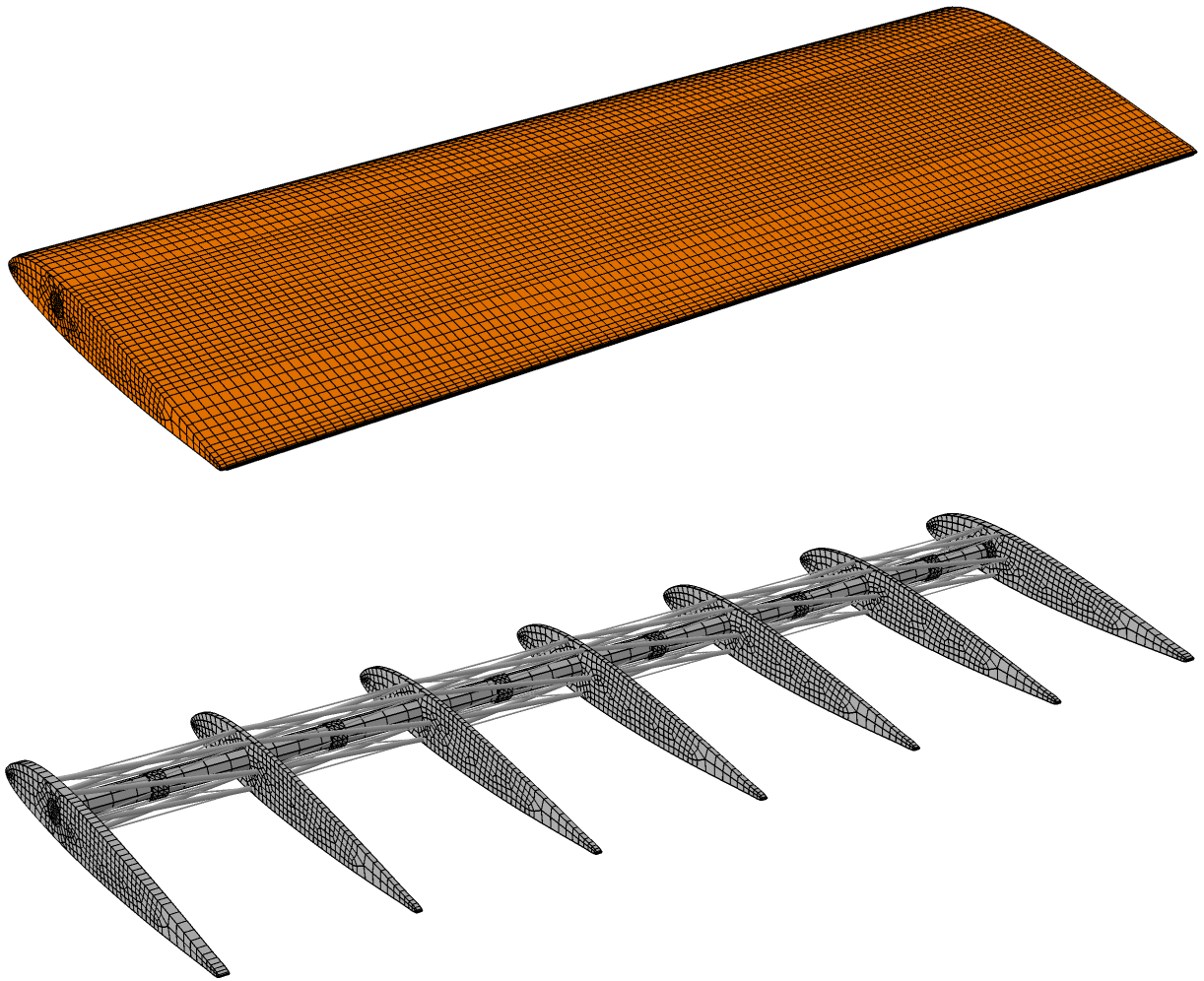


Figure 5.10: Finite element meshes of the wing skin and the internal wing structure obtained from a mesh convergence study.

Boundary conditions must be defined for the finite element model. One edge of the wing is fixed, as seen in Fig. 5.11, representing the wing root that is attached to the fuselage. Figure 5.12 represents the outer boundary conditions of skin of the wing. In each cell, the contact between the spar, its ring, and tip rib are modeled by a tied constraint to enable the free movement of the spar around the rings as well as the ability for it to rotate about other axes. The contact for the rings themselves are modeled as non-penetrable and frictionless so that they can rotate freely around the spar. The stabilizer rings are also modeled in this way and are set in the gap between the start and end ribs and the spar. The material properties found in Table 5.2 are used in the finite element model.

To assess performance under external aerodynamic loads, an additional boundary condition is applied to the finite element model. This boundary condition, shown in Fig. 5.12, is created from a pressure coefficient (c_p) distribution from the leading edge to the trailing edge of the top and bottom surfaces of the wing at a 0° angle of attack. This distribution is calculated using methods described in previous work [20].

The pressure coefficient is defined as:

$$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho v^2}, \quad (5.2)$$

where the dynamic pressure, $\frac{1}{2}\rho v^2$, is calculated based on standard conditions (sea level air density $\rho = 1.225 \text{ kg/m}^3$). An air speed of $v = 20 \text{ m/s}$ is chosen based on the typical air velocity of the reference vehicle. The pressure differential values, $\Delta p = p - p_\infty$, are calculated from the known pressure coefficient, c_p , and the other known variables described above in Eq. 5.2.

The dynamic-implicit and non-linear geometry options are chosen for the **Abaqus** simulation.

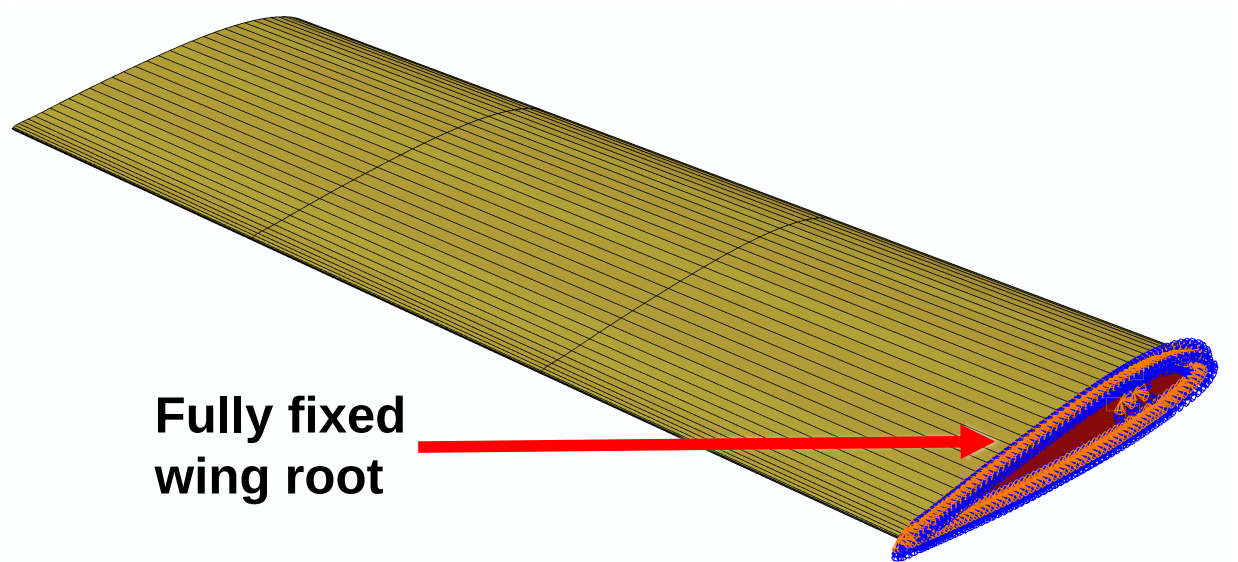


Figure 5.11: Isometric view of the root boundary condition for the morphing wing. Arrow indicates attachment between wing root and aircraft body.

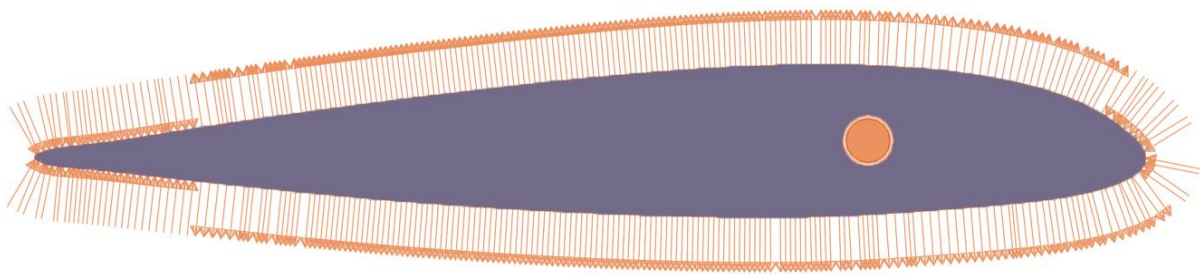


Figure 5.12: Pressure coefficient distribution across the top and bottom surfaces of the wing.

Table 5.2: Material parameters of the components of the tensegrity torsional wing.

Property	Al 6061-T6, Fig.5.13	Grade 5 Ti, Fig.5.14	Desmopan 790 TPU, Fig.5.15
Density, ρ [kg/m ³]	2700	4430	1210
Elastic Modulus, E [GPa]	68.9	113.8	0.1
Poisson's Ratio	0.30	0.33	0.48
Strength [MPa]	276.0	880.0	54.0

The dynamic implicit option considers quasi-static deformation analysis as step type which reduces the computational expense while still providing accuracy. The high quantity of deformation that occurs at different locations during the simulation makes the non-linear geometry (*i.e.*, NLGEOM in Abaqus) option highly suitable for this simulation.

5.4 Wing Material Choices

The materials used for the morphing wing are shown in Table 5.2. Aluminum was chosen for its high strength to weight ratio as well as its relatively low cost. The spar, ribs, and rings are all made of aluminum 6061-T6 as seen in Fig. 5.13. Grade 5 titanium was used for the stabilizer and longitudinal wires as it has a relatively high elastic modulus at 113.8 MPa. Additionally, its high tensile strength makes grade 5 titanium very suitable for the uniaxial loading that occurs on the wire sets. Figure 5.14 shows the portions of the wing made of titanium. Finally, the skin of the wing was chosen to be made of a thermoplastic polyurethane (TPU) as the material needs to be able to perform well when subject to high strain during the torsional actuation of the wing. Figure 5.15 shows the Desmopan 790 TPU used and its placement around the torsional mechanism.

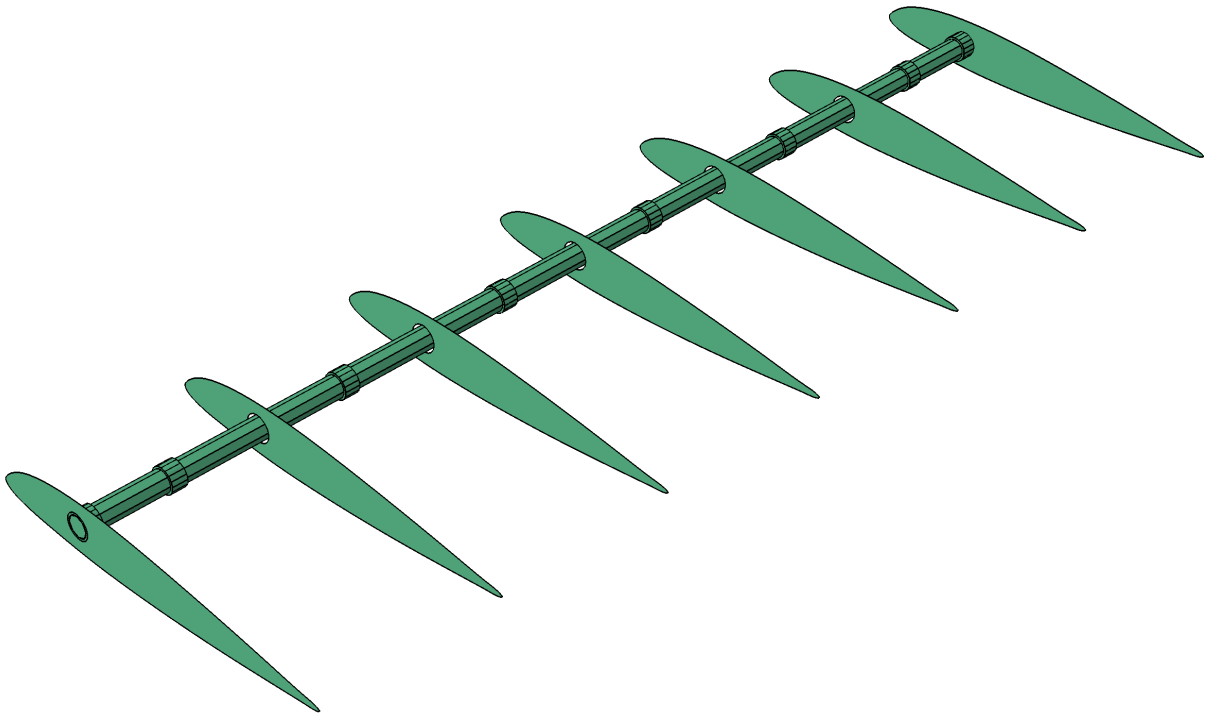


Figure 5.13: Al 6061-T6.

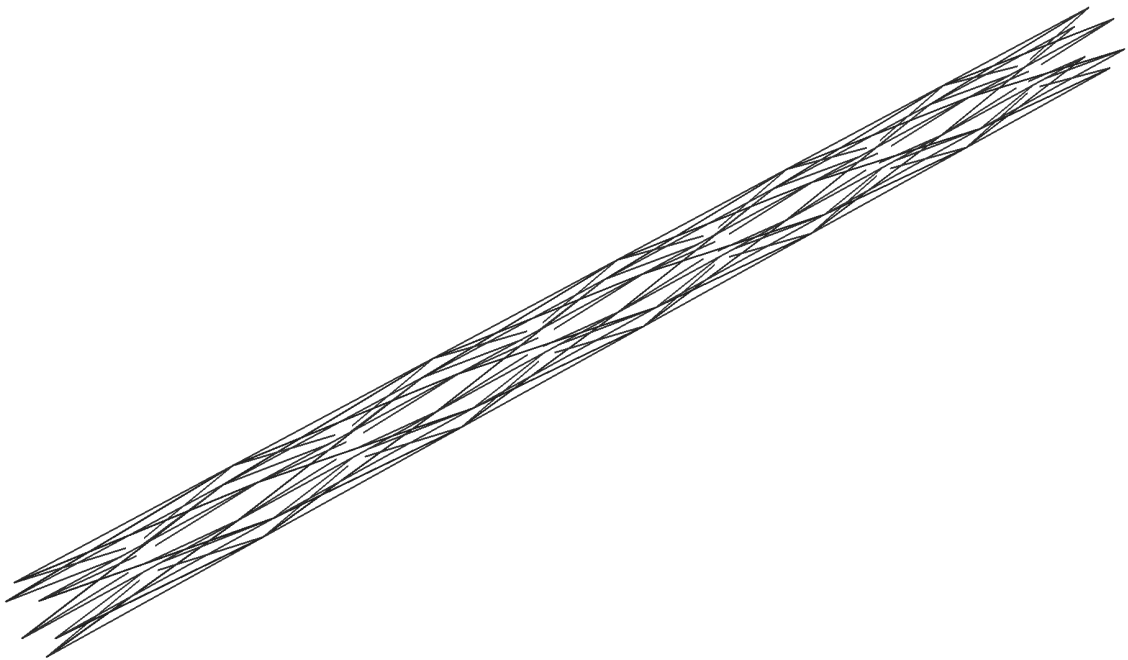


Figure 5.14: Grade 5 Titanium.

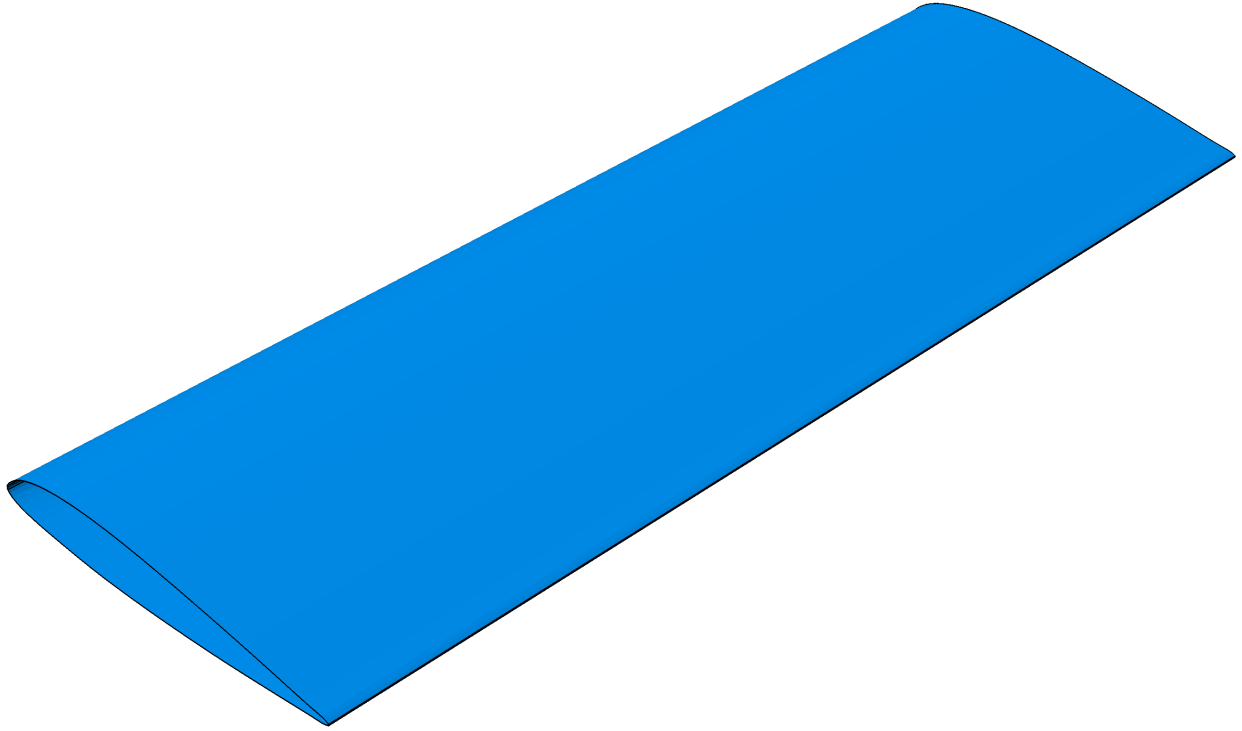


Figure 5.15: Desmopan 790 TPU.

5.5 Multi-objective Optimization

The topological properties of optimal wings are found using the MATLAB[®] multi-objective genetic algorithm framework, `gamultiobj`. The multi-objective genetic algorithm iterates through a range of discrete variables and chooses an optimal set of configurations based off the given objectives. This method generates a Pareto front with a range of optimal configurations based on optimization objectives. For the smooth twisting wing, the two main objectives to optimize were maximum twist angle and wing mass. The maximum twist angle of the wing is most impacted by the number of twisting sections and number of actuator wires. Increasing these parameters increases the number of component pieces and therefore may significantly increase mass. The multi-objective genetic algorithm enables the discovery of optimization trends that lead to a Pareto front containing a range of configurations on a scale between each optimization parameter.

Table 5.3: Lower and upper bounds for the parameters that control the topology of the smooth twisting wing.

Design variable	Description	Discrete values
n_c	Number of cells	5, 6, 7, 8, 9
n_w	Number of wires	4, 5, 6, 7, 8, 9, 10
t_{sk}	Thickness of skin	1.25, 1.50, 1.75, 2.00, 2.25 mm
r_{spar}	Radius of spar	3.00, 4.00, 5.00, 6.00, 7.00 mm
t_{rib}	Thickness of ribs	3.00, 4.00, 5.00, 8.00, 10.00 mm
d_{wires}	Diameter of passive wire	0.813, 1.016, 1.829, 2.337, 3.251 mm
x_{loc}	Spar location along chord length	0.225, 0.25, 0.275
l_r	Thickness of ring	8, 10, 12 mm

The optimization occurs over a discrete range of values as seen in Table 5.3. These bounds are defined based on a previous design of experiments study [43]. Discrete variables are chosen to improve the manufacturability of the optimized wing topologies. The optimization occurs across eight discrete variables with a population size of 40 and the number of generations set to 30. The population size defines the number of parent solutions in every genetic mutation cohort. The limiting factor for the population size was the computational resources required for each finite element simulation. Each simulation takes between two and four hours to run. With a generation size of 40 and the number of generations set to 30, the maximum number of possible simulations equates to 1200. To reduce redundant computation, a maximum stall limit is set to 10. The maximum stall limit defines the number of generations the optimization can compute without significant decrease in the fitness score. Once the optimization notices no significant improvement over the maximum stall limit, the optimization is ended. Overall, the multi-objective optimization ran 534 finite element simulations. Thus, taking thirteen generations to converge to an optimal range of solutions.

The multi-objective optimization function must be given constraints to operate within. Table 5.3 below describes the input to the multi-objective optimization function. The eight parameters discussed above are the target parameters for the optimization. The optimization is to find an optimal set of these eight variables that minimize both the mass, m , and the negative of the wing twist angle, $-\phi$. A negative value must be applied to the twist

angle as the optimizer is only able to minimize or maximize constraints. The bounds of these target variables that the optimizer operates within are the discrete bounds found in Table 5.3 above.

$$\begin{aligned}
&\text{Find: } n_c, n_w, t_{sk}, r_{spar}, t_{ribs}, d_{act}, x_{loc}, l_r \\
&\text{That minimize: } m, -\phi \\
&\text{Subject to: } \text{Discrete values seen in Table 5.3}
\end{aligned} \tag{5.3}$$

During the finite element simulation, stresses on the morphing wing are continuously measured. The output from the finite element simulation is triggered once there is an onset of material or structural failure in the morphing wing. Thus, the results from the optimization study indicate maximum theoretical values of wing twist achievable by each configuration.

5.6 Results of Multi-objective Optimization

The MATLAB[®] multi-objective optimization function `multiobj` continually runs the FEA simulation described in Section 5.3 while varying the design parameters described in Section 5.2. The function keeps track of configurations relating to desirable twist angle and mass while penalizing configurations that fail during the FEA simulation. The optimization continues to run and generates an output of results along a mass and twist angle Pareto front. The final Pareto front in Fig. 5.16 showcases the distribution of variations and their relationship to the mass and twist angle achieved by each variation.

From the Pareto front, three designs are chosen to represent minimum mass, maximum twist angle, and an intermediate design. The input parameters for these three designs are found in Table 5.4. The output response from these three designs are presented in Table 5.5. Design

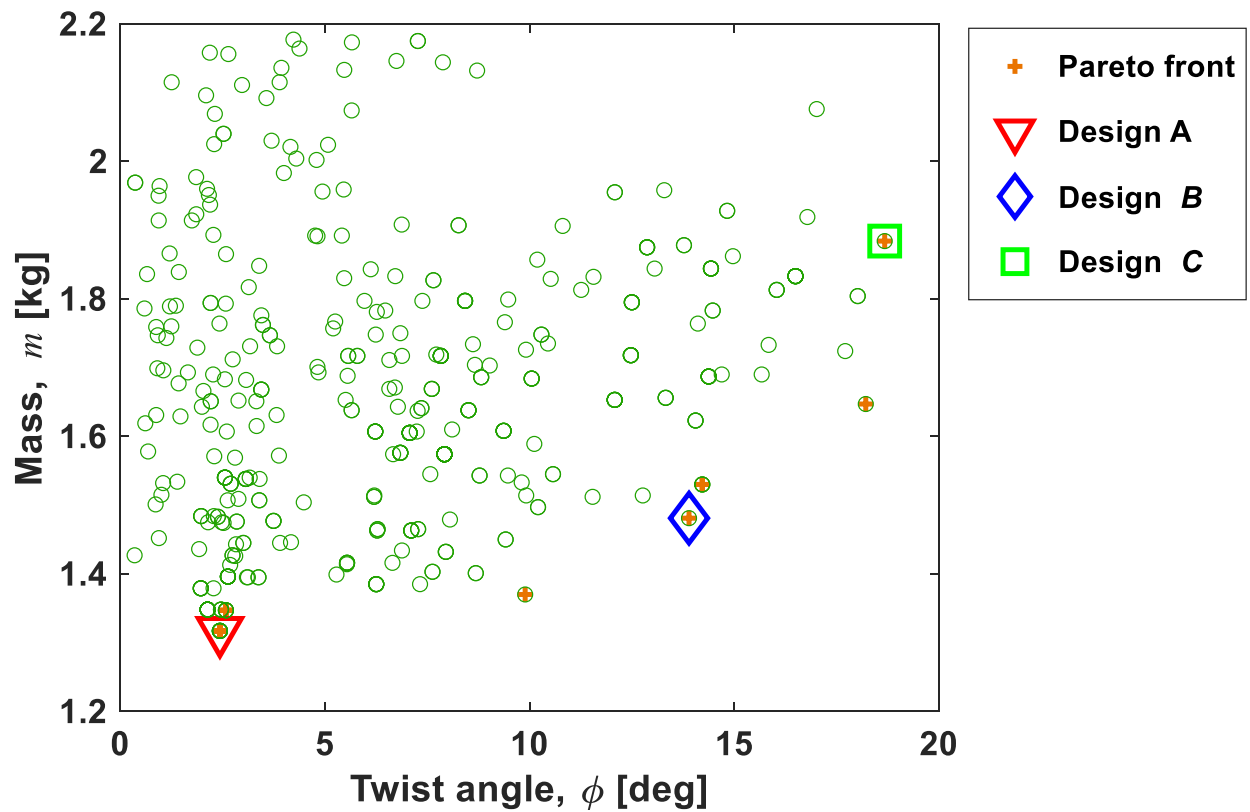


Figure 5.16: Pareto front of the optimized tensegrity column twisting wing optimization. Three designs of interest are chosen ranging from minimum mass to maximum twist angle. Design $\mathcal{A}$ represents the minimal mass configuration, design $\mathcal{B}$ represents an intermediate configuration, and design $\mathcal{C}$ represents a maximum twist angle configuration.

$\mathcal{A}$ has the lowest twist angle of the three designs at just 2.34 degrees. However, design $\mathcal{A}$ also has the lowest mass at 1.317 kg. Design $\mathcal{A}$ has the least amount of ribs which is one factor contributing to its low twist angle and low mass. This design also has the lowest spar radius at just 5.00 mm and the largest ring thickness at 12.00 mm. Output visualization of design $\mathcal{A}$ is presented in Fig. 5.17.

Design $\mathcal{B}$ has a twist angle of 13.90 degrees and a mass of 1.481 kg. The twist angle is greater than design $\mathcal{A}$ but lower than design $\mathcal{C}$. Similarly, its mass sits between the two other designs. Design $\mathcal{B}$ has six ribs, one more than design $\mathcal{A}$ and two less than design $\mathcal{C}$. Its number of wire sets also sits between the other two designs at six wire sets. The twist angle of design $\mathcal{B}$ is much closer to design $\mathcal{C}$ than it is to design $\mathcal{A}$ and shares some common design inputs such as spar radius, r_{spar} , and ring radius, l_r . Although design $\mathcal{B}$ has only one more rib than design $\mathcal{A}$, $\mathcal{B}$ has more than 11 degrees of additional twist. Another difference is the increased spar radius which could provide greater strength in the mechanism before failure. Design $\mathcal{B}$ has one less wire set than $\mathcal{A}$, most likely due to the additional cell in the design. Each additional cell in the design increases the maximum achievable twist angle as the actuator wires gain an additional end point position – as described in Section 5.1. The final render of design $\mathcal{B}$ is shown in Fig. 5.18.

Finally, design $\mathcal{C}$ has the highest twist angle and mass at 18.68 degrees and 1.884 kg, respectively. Design $\mathcal{C}$ has the highest number of ribs at eight and the lowest number of wire sets at five. The gain in twist angle from design $\mathcal{B}$ to $\mathcal{C}$ is significant and can be attributed mainly to the increased number of cells, similar to the gain found from design $\mathcal{A}$ to design $\mathcal{B}$. This design also has the thickest rib thickness at 5.00 mm. Thicker ribs may be attributed to higher stresses within the mechanism. A visual representation of design $\mathcal{C}$ is presented in Fig. 5.19. The difference in mass between designs $\mathcal{A}$ and $\mathcal{C}$ is 0.567 kg while the difference in maximum twist angle is 16.25 degrees. The changes in the design inputs from Table 5.4 show that the largest impact when it comes to mass is the number of cells in the morphing wing.

Table 5.4: Three Pareto front designs are chosen and listed below. Design $\mathcal{A}$ for minimum mass, design $\mathcal{B}$ with an intermediate mass and twist angle, and design $\mathcal{C}$ with the maximum twist angle.

Design parameter	Design $\mathcal{A}$	Design $\mathcal{B}$	Design $\mathcal{C}$
n_c	5	6	8
n_w	7	6	5
t_{sk} [mm]	1.25	1.25	1.25
r_{spar} [mm]	5.00	7.00	7.00
t_{rib} [mm]	3.00	3.00	5.00
d_{wires} [mm]	3.251	3.251	3.251
x_{loc}	0.25	0.25	0.25
l_r [mm]	12	10	10

Table 5.5: Design response from optimized Pareto front designs.

Response	Design $\mathcal{A}$	Design $\mathcal{B}$	Design $\mathcal{C}$
ϕ [°]	2.43	13.90	18.68
m [kg]	1.317	1.481	1.884

The number of cells and number of wire-sets also seem to have the biggest contributions to the twist angle. Among the three designs, the thickness of the skin, t_{sk} , diameter of passive wires, d_{wires} , and location of the spar along the chord of the wing, x_{loc} , converged to the same values. A previous design of experiments study using the same twisting wing topology found a twist angle for a mass of 1.80 kg to be 8.56 degrees. The multi-objective optimization was able to find a design with the same mass but more than double the twist angle.

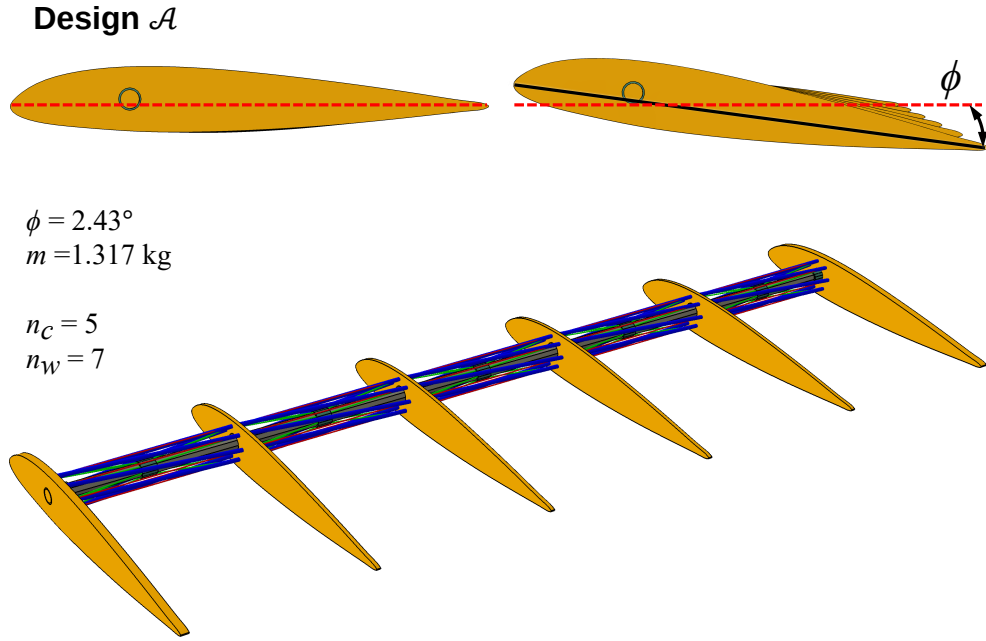


Figure 5.17: Deflection of design $\mathcal{A}$. Design $\mathcal{A}$ has five cells and seven wire sets. Design $\mathcal{A}$ can twist to a maximum deflection angle of 2.43° and has a mass of 1.317 kg.

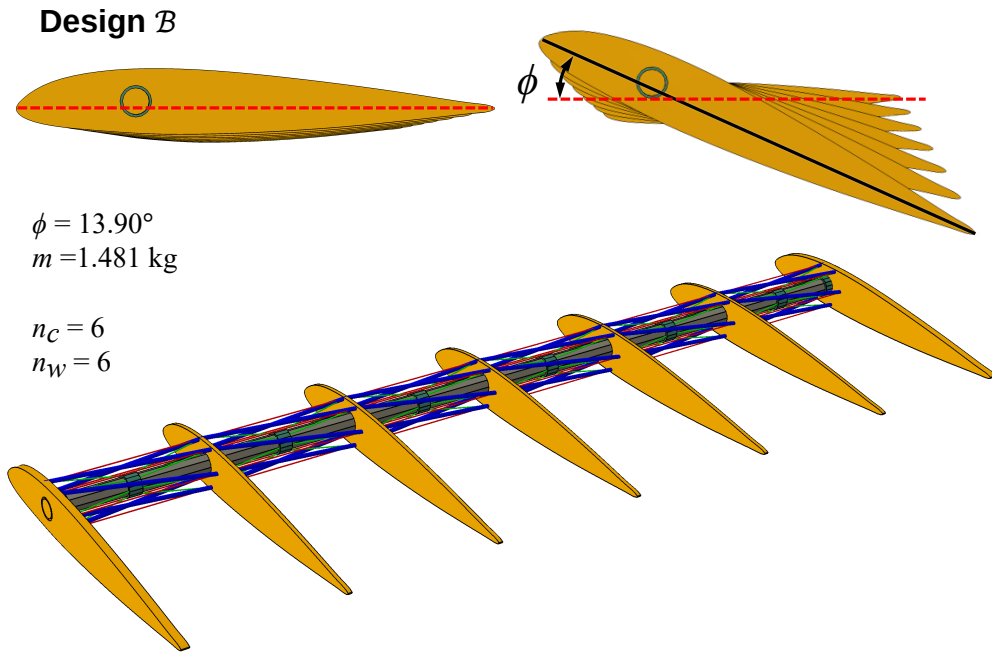


Figure 5.18: Deflection of design $\mathcal{B}$. Design $\mathcal{B}$ has six cells and six wire sets. Design $\mathcal{B}$ can twist to a maximum deflection angle of 13.90° and has a mass of 1.481 kg.

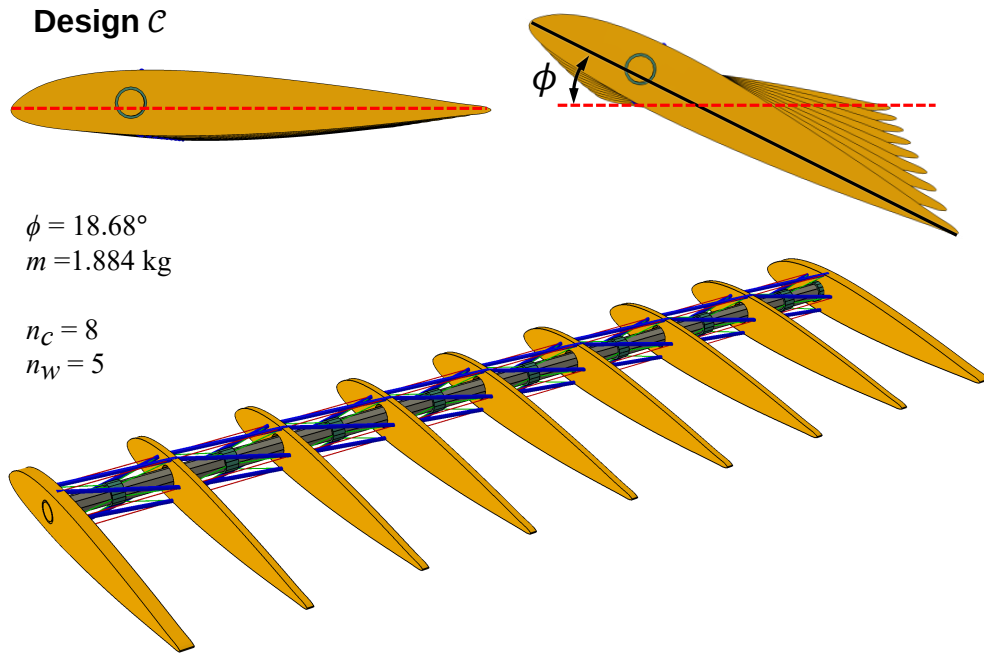


Figure 5.19: Deflection of design $\mathcal{C}$. Design $\mathcal{C}$ has eight cells and five wire sets. Design $\mathcal{C}$ can twist to a maximum deflection angle of 18.68° , the highest of the three chosen designs. Design $\mathcal{C}$ has the highest mass at 1.884 kg.

Chapter 6

Conclusion and Suggested Future Exploration

This thesis presents the use of optimization to find favorable tensegrity topologies in two aerospace applications. Section 6.1 presents conclusions of the findings in the optimization analysis of the double-helix tensegrity payload carrier and Section 6.2 presents conclusions of the findings in a multi-objective optimization analysis of a tensegrity morphing wing.

6.1 Double-helix Tensegrity Payload Carrier

The optimization analysis resulted in three successfully converged double-helix tensegrity payload carriers that were able to meet the mission requirements of dropping a 5 kg payload from a height of 2 m while keeping the payload deceleration under $10g_E$. These mission requirements were based off of acceptable drone operating conditions and typical loads experienced by payloads during shipment. A reduced-order dynamic simulation was employed to retrieve carrier member stresses and payload acceleration during impact. This reduced-order

dynamic simulation represents a package drop with a height measured from the bottom of the payload carrier to the ground surface. This drop occurs under Earth gravity conditions and a zero initial velocity condition. Three DHT payload carrier shapes are explored in this thesis: Cylindrical, ellipsoidal, and parabolic. An optimization study that is constrained to minimize mass is performed on each shape. Additional constraints in this study penalize payload accelerations greater than $10g_E$ and any material and structural failures that occur during the drop sequence. The optimization studies converged well within the constraint boundaries for each of the three shapes, presenting minimal mass DHT carriers that can absorb impact loads through structural and material deformations. Of the three shapes, the converged design for the parabolic DHT carrier presented the lowest structure mass at just 2.33 kg (47% of the mass of the payload). The parabolic DHT carrier did not surpass any of the optimization constraints and maintained structural integrity while its payload experienced an acceleration of $8.99g_E$. The cylindrical and ellipsoidal DHT carriers had structure masses at 5.44 kg and 3.88 kg, respectively, with payload accelerations of $8.72g_E$ and $6.15g_E$. All three of the converged carrier designs are reusable as no material or structural failure occurred during the impact sequence. This provides great insight into the effectiveness of tensegrity payload carriers as a more sustainable delivery solution when compared to alternative drone-based package delivery methods that use single-use polymer parachutes. One trade-off from single-use delivery methods is the increased weight of the tensegrity payload carriers. The increased weight would limit the flight time of the delivery drone. However, the additional benefits of more accurate deliveries may offset the flight time limit for delivery companies. Additionally, the tensegrity payload carriers may have added benefits for applications outside of residential package delivery where flight time requirements are less critical such as in critical medicine delivery in rural areas. Overall, the tensegrity payload carriers optimized well to the given task and show potential for application in missions with varied objectives.

Additionally, the functionality of these DHT carriers in package delivery applications enables

similar applicability in other fields such as space-bound landers. For such an application, a similar model to the one described in this thesis could be used with adjusted environment parameters to optimize double-helix tensegrity landers. Optimization of DHT structures using eco-friendly, easy-to-source, and cheaper materials is an additional suggestion for future work. This would increase feasibility for the use of these re-usable, lightweight tensegrity payload carriers. Another suggestion is the examination of aerodynamic impact on carrier drop trajectory and the influence of weather and winds on the DHT payload carriers.

6.2 Tensegrity morphing wing

A tensegrity column-based morphing wing based on the dimensions of a remote controlled UAV was derived and parameterized. An FEA model was created in ABAQUS and boundary conditions are defined for the morphing wing. A previous design of experiments study showed a reduction in drag coefficient and increased lift-to-drag ratio in the tensegrity column-based morphing wing design in comparison to wings with traditional control surface designs. The morphing wing was parameterized so that the physical parameters of the wing can be modified in the optimizer. These parameters affect the maximum twisting angle of the morphing wing which has been shown to be a key indicator of performance. The bounds of optimization are based on a previous design study and were chosen with a wide range so that the optimizer was able to work with a larger pool of variations. This design of experiments study also provided a baseline to compare the multi-objective optimization study to. The optimization was set to reduce mass and increase the maximum flight twist angle. The final optimization showed great promise in development of a viable configuration for a tensegrity column-based twisting wing. The final Pareto front generated by the optimization study was used to obtain three optimal designs with varying bias towards either reduced mass or increased twist angle. Of the three chosen designs, the maximum wing twist achieved was

18.68°. The high twist achieved by the wing under stresses in the finite element simulation provides validation into the usefulness of optimization techniques for tensegrity systems. The morphing wing was able to optimize well to its given objectives, demonstrating the adaptability of the tensegrity morphing wing concept. The low mass and high twist angle of the converged solutions demonstrates that the tensegrity mechanism is able withstand the external and internal loads associated with flight at low speeds on a model aircraft. The success in this study opens the door to additional future studies where the tensegrity morphing wing concept is scaled up to aircraft that have larger wingspans. Future work involves characterization of skin wrinkling penalties to further optimize drag reduction. Additionally, building a full scale model of the optimized wings from this study would determine the feasibility of the optimization with respect to real world performance.

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Appendix A

List of Symbols for Double-helix Tensegrity Payload Carrier

C_b	=	Bar connectivity matrix
C_s	=	String connectivity matrix
c	=	Axial concentration of structural nodes
c_i	=	$i \in \{0, 1, 2\}$; Coefficients of quadratic axial distribution of nodes in DHT carrier
c_c	=	Ground damping coefficient, N/m ²
c_s	=	Viscoelastic damping coefficient, Pa s
E	=	Young's modulus, Pa
f	=	Fitness function
$\mathbf{f}_{E_i}$	=	External forces acting on nodes of DHT payload carrier, N
f_{s_i}	=	Internal force acting on string members
f_{b_i}	=	Internal force acting on bar members
g	=	Earth gravity, m/s ²
g_{max}	=	Maximum allowed acceleration non-dimensionalized by gravity, m/s ²
g_s	=	Maximum acceleration in g 's experienced by the payload during impact

h	=	Initial distance between bottom of the cylinders and the ground, m
L	=	Axial length of the structure, m
m	=	Total system mass, kg
m_{load}	=	Mass of central payload, kg
m_{p_i}	=	Lumped mass of each node, kg
n_i	=	Node location
n_P	=	Position vector defining axial payload position
$\mathbf{n}_P$	=	Payload coordinates
p	=	Number of units in the circumferential direction
p_c	=	Ground stiffness coefficient, N/m ²
q	=	Number of units in the axial direction
R/L	=	Ratio of radius to length of DHT structures
r_{ss}	=	Radius of surface strings, m
r_{si}	=	Radius of internal strings, m
r_b	=	Radius of the bars, m
s	=	Factor of safety
s_i	=	Length of string members
b_i	=	Length of bar members
V_c	=	Volume constraint magnitude
v_0	=	Initial vertical velocity, m/s
Z_{pos}	=	Non-dimensional position of central payload along the axial direction
ϵ_{s_i}	=	Strain experienced by string members
ϵ_{b_i}	=	Strain experienced by bar members
γ_i	=	Scalar penalty multipliers
λ_i	=	Scalar penalties
ν	=	Ground friction coefficient
ρ_s	=	Density of the string members, kg/m ³

ρ_b	=	Density of the bar members, kg/m ³
σ_{s_i}	=	Stress incurred by string members
σ_{b_i}	=	Stress incurred by bar members
σ_{bc_d}	=	Maximum stress exceeding bar compression stress limit, Pa
σ_{bt_d}	=	Maximum stress exceeding bar tension stress limit, Pa
σ_{si_d}	=	Maximum stress exceeding internal string limit, Pa
σ_{ss_d}	=	Maximum stress exceeding surface string limit, Pa

Appendix B

List of Symbols for Tensegrity Morphing Wing

c_p	=	Pressure coefficient
d_{act}	=	Diameter of actuator wire, mm
d_{wires}	=	Diameter of longitudinal and stabilizer wires, mm
l_r	=	Axial length of stabilizer rings, mm
m	=	Mass of wing, kg
$\mathbf{m}_{ij}$	=	Position vector of end points of wires, applying to stabilizer type wires
$\mathbf{n}_{ij}$	=	Position vector of start points of wires, applying to stabilizer type wires
n_c	=	Number of cells
n_w	=	Number of wire sets in each cell
$\mathbf{p}_{ij}$	=	Position vector of end points of wire, applying to all wire sets
R	=	Radial distance between wire end points and central axis of the spar, mm
r	=	Radius of rings, mm
r_{spar}	=	Radius of spar, mm
s	=	Wing span of Carl Goldberg Falcon Mark II UAV, cm

t_{rib}	=	Thickness of ribs, mm
t_{sk}	=	Thickness of wing skin, mm
v	=	Velocity of air during finite element analysis of twisting wing, m/s
x_{loc}	=	Spar location along chord length
α	=	Angle of attack of wing, °
ϕ	=	Maximum twist angle of wing, °
ρ	=	Density of air, kg/m ³