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POLICY-ORIENTED INTERREGIONAL DEMOGRAPHIC ACCOUNTING AND A GENERALIZATION OF POPULATION FLOW MODELS

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In recent years interest has been growing in demographic accounts analogous to such economic accounts as input-output. Stolnitz suggested in 1964 [8] an expansion of the labor inputs in interregional input-output accounts, which would specify labor inputs by their characteristics and region of residence. Coefficients would be calculated in the usual manner based on current ratios so that, for instance, expansion of an industry in one region would draw (import) female workers from another region. Rogers [7] investigated a related approach. More recently, Stone [9, 10] has explored the design and use of deomgraphic accounts for forecasting and various areas of policy planning, but without interregional detail. Rees and Wilson have been publishing a series of papers [6] on interregional demographic accounts which carefully lay out the full framework of transformations among categories over a segment of time. Indeed, the interest is such that in Vol. 5, No. 1 (1973) Environment and Planning devoted practically the entire issue to demographic accounts.

This paper has two purposes. The first is to discuss generally some of the uses of interregional demographic accounts for forecasting and policy evaluation. The second, more formal and technical, is to show that such accounts, by placing the phenomenon of migration in the context of a system, lead to some necessary but generally neglected considerations of considerable importance both for theory and for practical applications.

An overview of one model of interregional demographic accounts and its uses

The discussion will be aided by a brief, impressionistic overview of such a model, which I have developed [1]. It is, in many ways, similar to the Wilson-Rees model, but stripped down from the full complement of transitional categories to make it operational. A prototype of this model has been built for the United States, for 243 regions, and it has been run by 5-year increments to the year 2000 for a variety of forecasting and policy analysis projections.

The nation is divided into some set of regions, and data is gathered on the population of the region, its birth and death rates, a square matrix of gross migratory flows among regions,[1] and such other information as may be needed for behavioral analysis. This information may include such variables as local climates, geographic coordinates, local incomes and levels of education, and so on and so forth. Needless to say, the practical difficulties are great in gathering, cleaning up, and making consistent this body of data, but this is not the topic of this paper.

The population of each region is advanced from period to period by adding births, subtracting deaths, adding inmigrants and subtracting outmigrants. For projection it is necessary to have some functions or constant relations to generate these components of change. By analogy, these constant relations are in an input-output system the technical coefficients, which state that the proportion of each input per unit of output for each industry is constant. In the case of demographic accounts there are several such relations, and they are considerably more complicated.

In our model we collapsed birth and death rates into a local rate of natural increase for simplicity. This rate varies widely among regions, from zero to rates typical of developing countries. Local rates of natural increase were projected into later periods proportionately to alternative projections of national natural increase, adjusted for the net migration history of the region. This adjustment, which I call a "demographic multiplier," is based on the fact that the overwhelming majority of migrants are young and fertile, so that migratory gains or losses strongly affect the age composition of regions and consequently their rate of natural increase. This relation, as were all others, was calibrated by multiple regression to past experience.

Migration received the most attention in the model, on two grounds. First, migration is the form of demographic interaction among regions in the national system, and is thus the primary reason for thinking in a systems framework. Second, the United States has experienced a sharp decline in birth rates, so that interregional migration assumes the greatest potential for variations in local growth or decline.

Gross (that is to say, directional) migrations from each region to every other were projected for each period by an elaborate form of the gravity model (discussed in the second part of this paper) that considered incomes, population sizes, local climate, and lagged natural increase and net migration. These lagged variables are of great importance. Natural increase lagged by some 20 years serves to indicate the proportion of the population which is at an age with a high propensity to migrate, while similarly lagged (by about 5 years) past net migration indicates the proportion of the population who have been migrants in the past and who therefore have a high propensity to move. I believe it is such lagged variables, representing the effect of past history on present behavior, that account for much of the hysteresis or continuing momentum in population phenomena. But I shall refrain from a full presentation and interpretation of the variables used in my model because my purpose in this paper is to raise issues about the general logic of such models rather than the particulars of one of them.

If such a model is to be used for projection or for policy, all of the variables for each locality must somehow be produced for each period, either endogenously (as in the moving forward of population stocks by adding and subtracting flows) or exogenously. Exogenously generated variables might be state variables, such as national crude birth rate projections, or policy variables, such as variations in local income through taxes or subsidies, policy-set limits to local populations, or any other according to the ingenuity of the designer. In our model, various demographic variables were endogenously generated after setting the initial conditions, local climate was exogenous (assumed constant), and income was generated endogenously through an equation which was highly significant statistically but admittedly had a large error term. Income at each locality for each period was needed both for projection and because modifications on local income through taxes and subsidies were our principal policy or control variables.

With this model we explored for the national system of regions (in our case, the 200-odd metropolitan areas and the non-metropolitan residue) the consequences of alternative national projections of birth rates, and of a dozen alternative policies, such as the favoring of small cities, alternative national growth centers policies, the favoring of certain regions, the favoring of the poorest regions, the interregional equalization of incomes, the maximization of total income. We also considered, but did not carry out, analysis of the optimal pattern of subsidies to growth centers to self-sustaining growth, the effects of growth centers upon the surrounding regions, and the effects of direct controls upon population movements or growth or decline of particular regions.

Quite obviously, such a model may be quite inaccurate, but still it may be better than any other way of exploring the future. Overall, my sense is that for a large demographic system, such as the United States or for the European Common Market, in which substantial population movements are present or possible, this type of analysis is useful for what may be called normal effects. This means that for broad classes of regions or cities, the conclusions of this type of model will be better than other projective methods. But that for unusual circumstances (such as the Irish potato famine, massive depressions, radical sectoral shifts in the economy, cultural or religious paroxisms, and the like) the model is very limited, especially for particular localities. This, in other terms, is a reflection of the tendency of statistically-formulated simulation or policy models to contain less variance within their range than is the case in reality. Most generally, it is an aspect of statistically calibrated models to concentrate on central rather than extreme values. Which is to say, in the end, that such models are evolutionary rather than revolutionary. But this is a general characteristic, with its accompanying failings, rather than a particular failing of such models.

A final comment is needed in this general overview. This is that any such model will produce thousands or tens of thousands of numbers. They amount, in their output, to future censuses of the endogenously produced variables. And anyone who has tried to summarize past experience from the censuses will understand that the richness of the data and the permutations of combinations of that data can result in utter confusion in the interpretation of the output of any such model. From this it is obvious that it is necessary to reduce the output of such models to intelligible proportions, if they are to be of any use.

This takes two forms for practical purposes. First, the initial disaggregation into regions will need to be recombined into categories (rich-big, Southern-small, and so forth) which are relevant for political or other purposes. Secondly, the model can only produce information in terms of variables which are endogenously produced. If the model does not produce, for instance, information about suicide rates or alienation, it is useless to ask it about them. On the other hand, combinations of variables and categories within the model can produce a great deal of information, such as national indices of population concentration or interregional income distribution (including the direct effect of such distributions and their secondary effects). A broad range of social indicators (to use that much abused term) can be generated from intelligently chosen permutations of the endogenous variables. In very simple terms, for classes of regions, such a model can produce data which is organizable to give insight on a number of matters. Such insight is based on information endogenously generated by the model (with the assumption of endogenous outputs), and consists in the intelligent reduction of thousands of numbers to a manageable few. The congruence of the dimensionality of the behavioral model to the dimensionality of the social purposes (or social indicators) is the key here. If the congruence is high, the model will be very useful within its reliability; if the congruence is low, whatever the statistical reliability of the model, it will be of little use.

Opportunity and competition in a migratory system*

Demographic accounting, by viewing population as a system of stocks and flows, permits a generalization of migratory relations which is of considerable theoretical interest as well as important for policy. I will show that, unless the interdependence of the system is explicitly considered, important variables will be omitted or assumptions will be made implicitly which would not be readily granted if made explicit. In other words, viewing migrations as a system does make a difference, and various approaches which can be found in the literature can be shown to be extreme special cases of a generalized formulation. For instance, I will show that those who study the relation of outmigration to local characteristics unknowingly assume a unit elasticity in the supply of jobs at the destination with respect to prospective migrants, while those who study inmigration as a function of local characteristics assume that outmigration in the rest of the system is unit elastic to the opportunities

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* Since writing the following section, I believe I have improved the interpretation and derivation of the model, and I hope to present these advances in later papers.

provided.[2]

Let us start with the push model, which is currently the most active form of migration research in the United States. This model seeks to relate gross outmigration to a function v_i of local characteristics. This function may include the size and demographic characteristics of the population at i, local climate, variables dealing with economics, with education, with culture, with past migration, and with any other of a vast number of considerations. Quite properly, the literature has concerned itself with the variables and form of the function v_i, but for our argument what goes into v_i is not important.

Consider now that every migrant leaving i must arrive at some destination. Call w_j the attractiveness or pulling power of a destination j. Again, a substantial literature addresses itself to the variables and functional form of w_j, but for our argument the specification does not matter. However w_j is constructed, the outmigrants from i will distribute themselves among their possible destinations in proportion to their respective pulling powers. That is to say, the migration from i to j will be

$$M_{ij} = v_i(w_j/\sum_k w_k), \qquad (1)$$

where M_{ij} is the number of migrants from i to j, w_j is the pulling power of j, and v_i is the total number of gross outmigrants from i. However, the reader should keep in mind that, as we generalize the model, we shall refine the definition of these variables.

It may be that a relational term or function, t_{ij}, is relevant to link the attractive characteristics at j to the characteristics at i. This would refine eq. (1) into

$$M_{ij} = v_i(w_j t_{ij} / \sum_k w_k t_{ik}). \qquad (2)$$

In the literature, the most common form of this relational function is some negative power of distance between i and j, in which case we have a conventional gravity model, with a Huff normalization adjustment [3]. But, of course, this relational function may take into account many other relations between origin and destination, such as functional distance, number of ex-residents of i now living at j, differences or commonalities in language, religion, ethnicity, or industrial composition, and so forth. Again, we are not concerned here with the insides of the relational function since our argument is general. Indeed, if we chose to ignore it, we would only implicitly set $t_{ij} = 1$ in all cases. But it is worth noting that, whereas v_i consists of variables measured in i, and w_j of variables measured in j, t_{ij} must be constructed of variables measured in both i and j.

Continuing our exploration of the push model, by summing equations such as (2) for migrants from all origins into j, we obtain the gross inmigration into j:

$$\sum_i M_{ij} = \sum_i v_i(w_j t_{ij} / \sum_k w_k t_{ik}) = w_j \sum_i (v_i t_{ij} / \sum_k w_k t_{ik}). \qquad (3)$$

This is the inmigration relation implied by push models. We will save its detailed examination until we have converted it to the general notation, but suffice it to say here that it states that gross inmigration will be proportional to the attraction of j and to the pool of migrants after taking account of their alternative opportunities. In other words, that j will receive migrants in simple proportion to their availability, or that the absorptive capacity of j is unit elastic to the supply of migrants.

Consider now the implications of the gravity model approach. This begins not with the total departures from one place, or the total arrivals at another, but with the number of migrants between them, so that its basic relation is

$$M_{ij} = v_i w_j t_{ij}. \tag{4}$$

If we ask what will be the total arrivals at a destination j, we find

$$\sum_i M_{ij} = w_j \sum_i v_i t_{ij}, \tag{5}$$

which is visibly different from what is implied by the push model as shown in eq. (3).

Equally, if we ask what the gravity model assumes gross outmigration from i to be, we find

$$\sum_j M_{ij} = v_i \sum_j w_j t_{ij}, \tag{6}$$

which contrasts with the push models $\sum_j M_{ij} = v_i$. That is to say, the push model holds that outmigration is totally determined by local conditions at the origin, while the gravity model says that, given local conditions v_i, total departures will be unit elastic to the attraction of outside opportunities. This will be seen more clearly in the generalized notation, but it is of particular policy interest because of the lively debate between Lowry [4] and a multitude of critics. I will not enter into the specifics of this argument, which has centered on the variables and structure of the push function, v_i. But it is worth noting that two different models are involved. Lowry's gravity model assumes full elasticity for push, while his critics use push models that assume zero elasticity.

A third common migration model, which may be called the pull model, focuses on the arrival of migrants at j, and attributes them solely to characteristics at j; that is, its point of departure is $\sum_i M_{ij} = w_j$. From this it follows that total departures from i will be $\sum_j M_{ij} = v_i \sum_j (w_j t_{ij} / \sum_k v_k t_{kj})$, which obviously differs from what is assumed in the push model (that $\sum_j M_{ij} = v_i$) or in the gravity model as shown in eq. (6). Similarly, the implicit form of the flows, $M_{ij} = v_i w_j t_{ij} / \sum_k v_k t_{kj}$, is obviously different from the other flow formulations.

A comparison with a fourth model, devised by Wilson [11] in reference to traffic and frequently called the entropy model, will be postponed because of its complexity until we have presented the general model.

These different models, then, imply very different things, as can be seen once the system consequences of their various points of departure are spelled out. But they can be put in a common framework which shows them to be special cases of a general model. Their differences then reduce to whether they assume values of 0 or -1 for two key parameters which stand for elasticities, while in reality the values of these parameters are most probably intermediate between 0 and -1. The various models in the common notation are shown in Table 1.

Two variables must be defined, which I shall call competition, C and opportunity, O. These variables, it must be stressed, bring in no new measurements but are merely combinations of the v's, w's, and t's. Two new parameters, α and β, are now raised as matters for empirical estimation, whereas their value has been assumed in existing models.

The definition of these variables is made difficult because they are functions of each other, and their definition and explication will have to be simultaneous.

Let us refine the definition of v_i to be the internally determined outmigration from i, or its internal propensity to produce migrants. Without yet defining opportunity O_i operationally, let us call α the elasticity of outmigration to opportunities. Thus, the total number of outmigrants from i will be $v_i O_i^\alpha$. The share of prospective migrants from i to j will depend on the attractive pull of j on i, $w_j t_{ij}$, and on the alternative opportunities available to migrants from i, resulting in $w_j t_{ij}/O_i$. Applying this share to the total migrants from i to all places, and summing over i, we obtain $\sum_i (v_i O_i^\alpha)(w_j t_{ij}/O_i) = w_j \sum_i v_i t_{ij} O_i^{\alpha-1}$. Dividing through by w_j to convert to prospective arrivals per unit of attraction at j, we arrive at our definition of competition

$$C_j = \sum_i v_i t_{ij} O_i^{\alpha-1}. \tag{8}$$

The interpretation of C_j, then, is that it is the potential number of migrants that might arrive at j per unit of its attractiveness, w_j, having taken account of the special relation of j with migrants from every source and their alternative opportunities. Most simply, it is the <u>ex ante</u> number of migrants per unit of pull at j; which is to say the number which would arrive if competition did not matter. Competition may also be interpreted as the pool of migrants per opportunity at j.

The definition of opportunity, O_i, is similarly arrived at. We refine the definition of w_j to be the internally determined attractions or opportunities available to migrants at j. This basic attraction is modified by the propensity of opportunities at j to expand in response to additional supplies of (or demands by) prospective migrants, which is to say competition raised to an exponent (elasticity) β: C_j^{β}. Thus, the actual number of migrants who will arrive at j will be $w_jC_j^{\beta}$. From the point of view of a resident at i, these opportunities at j must be weighted by the relational function t_{ij} which may obtain between i and j, and discounted by the total number of migrants competing per unit of opportunity at j, which is to say C_j. In brief, from the point of view of a resident at i, the opportunities at j are $w_jC_j^{\beta}t_{ij}/C_j = w_jt_{ij}C_j^{\beta-1}$. Summing over all the possible destinations, we obtain the total outside opportunities available to a resident of i:

$$O_i = \sum_j w_j t_{ij} C_j^{\beta-1}. \tag{9}$$

In our construction of O_i and C_j we stated in passing that total departures from i will be $v_iO_i^{\alpha}$, and that total arrivals at j will be $w_jC_j^{\beta}$. Without deriving it, I will merely state here that the flow equation for migrants between i and j will be

$$M_{ij} = v_i w_j t_{ij} O_i^{\alpha-1} C_j^{\beta-1}. \tag{10}$$

In an appendix I will present in greater detail the structure of eq. (10), but here I merely want to show its consistency with the total arrivals and departures. For departures, we merely sum over all possible destinations j:

$$\sum_j M_{ij} = \sum_j v_i w_j t_{ij} O_i^{\alpha-1}\ C_j^{\beta-1} = v_i O_i^{\alpha-1}\left(\sum_j w_j t_{ij} C_j^{\beta-1}\right).$$

But from eq. (9) we see that the expression in the parenthesis is precisely the definition of O_i, so that we have

$$\sum_j M_{ij} = v_i O_i^{\alpha-1} O_i = v_i O_i^{\alpha}.$$

In a parallel fashion, summing over the origins to obtain total arrivals at j, we have:

$$\sum_i M_{ij} = \sum_i v_i w_j t_{ij} O_i^{\alpha-1} C_j^{\beta-1} = w_j C_j^{\beta-1} \left(\sum_i v_i t_{ij} O_i^{\alpha-1} \right).$$

From eq. (8) we see that the expression in the parenthesis is the definition of C_j, so that we have

$$\sum_i M_{ij} = w_j C_j^{\beta-1} C_j = w_j C_j^{\beta}.$$

I fear that the reader will by now be confused to some degree, and somewhat uncertain as to what he has been made to swallow. What I have presented is a highly circular or consistent set of very general relations, and it is possible to enter it from many points, to derive things otherwise, and indeed to explicate them quite differently. My greatest difficulty in writing this has been to choose the mode of presentation, where to break into the circle. Now it may be best to move on to the examination of the various models in the literature to see how they appear in this general framework. Table 1 is a summary of the discussion which follows, and the last column shows that the standard models differ from one another by assuming all the permutations of values of 0 or 1 for α or β; I shall argue and cite some evidence that the values are likely to be intermediate.

The push model assumes that outmigration is totally determined by local conditions at i and unaffected by opportunities, so that it assumes that $\alpha = 0$. Of necessity this implies that in the flow equation O_i will have an exponent of $(\alpha-1) = -1$; and, by following the algebra, that $\beta = 1$, which is to say that jobs at destinations expand as necessary to absorb migrants. In effect this assumes that jobs follow people. On

Table 1. General model of interregional gross migration and special cases

	Flow from i to j M_{ij}	Departures $\sum_j M_{ij}$	Arrivals $\sum_i M_{ij}$	Opportunity O_i	Competition C_j	Elasticities α	β
general model	$v_i w_j t_{ij} O_i^{\alpha-1} C_j^{\beta-1}$	$v_i O_i^{\alpha}$	$w_j C_j^{\beta}$	$\sum_j w_j t_{ij} C_j^{\beta-1}$	$\sum_i v_i t_{ij} O_i^{\alpha-1}$	$0 \leq \alpha \leq 1$	$0 \leq \beta \leq 1$
push model	$v_i w_j t_{ij} O_i^{-1} C_j^{0}$	$v_i O_i^{0}$	$w_j C_j^{1}$	$\sum_i w_j t_{ij} C_j^{0}$	$\sum_i v_i t_{ij} O_i^{-1}$	0	1
pull model	$v_i w_j t_{ij} O_j^{0} C_j^{-1}$	$v_i O_i^{1}$	$w_j C_j^{0}$	$\sum_j w_j t_{ij} C_j^{-1}$	$\sum_i v_i t_{ij} O_i^{0}$	1	0
elastic gravity model (Lowry)	$v_i w_j t_{ij} O_i^{0} C_j^{0}$	$v_i O_i^{1}$	$w_j C_j^{1}$	$\sum_j w_j t_{ij} C_j^{0}$	$\sum_i v_i t_{ij} O_i^{0}$	1	1
inelastic gravity model (Wilson)	$v_i w_j t_{ij} O_i^{-1} C_j^{-1}$	$v_i O_i^{0}$	$w_j C_j^{0}$	$\sum_j w_j t_{ij} C_j^{-1}$	$\sum_j v_i t_{ij} O_i^{-1}$	0	0

Note: The point of departure of the various special case models is indicated by a frame around the expression.

the other hand, the pull model assumes precisely the opposite: that opportunities or jobs at destination are determined exclusively by local characteristics and are totally inelastic with respect to the flow of inmigrants. That is to say, that opportunities are inelastic to the pool of migrants, or $\beta = 0$. This is typical of economic base (and, more generally, of economic) approaches, which assume that people follow jobs, typically projecting $w_j(C_j^0)$ through some sectoral estimates in the growth of employment, and assuming that the flows of people will adjust exactly to these changes in employment through migration. Implicitly this assumes that outmigration is exactly proportional or unit elastic at the origin to the demand for migrants at destination; which is to say they assume that $\alpha = 1$. In reality jobs chase people and people chase jobs.

The gravity model assumes both that outmigration is unit elastic to outside opportunities, and that jobs at destination are unit elastic to prospective migrants; which is to say it assumes $\alpha = \beta = 1$. By contrast, Wilson's entropy model, which was designed initially for modelling of intraurban traffic, assumes that both the rates of departure and the rates of arrival are fixed, or that they are both totally inelastic ($\alpha = \beta = 0$). This model is also, ultimately a gravity model, but with very strong double proportionality controls. Hence, in Table 1 we have called it inelastic gravity model, whereas the Lowry-type of model is called elastic gravity model. The push and the pull models might be called one-sided elastic gravity models.

Having now reduced these various models to a common framework, we see that they vary only in making diverse *a priori* assumptions about the elasticities α and β. Yet the values of their elasticities are, ultimately, an empirical matter, and may even be expected to vary from time to time and place to place. Thus, in periods of general labor surplus, one may expect that people will tend to chase jobs more than the reverse; that is to say that α will be relatively larger and β relatively smaller than in periods of full employment, when jobs will chase after workers. But in general, I would find it surprising if either were unit or zero elastic. Rather, it is as if these four existing approaches delimited the square with vertices at (0, 0), (0, 1), (1, 1), and (1, 0) in a space of coordinates α and β. In reality, under most circumstances, we would expect α and β to be a point inside this square. We would expect outmigration to respond somewhat to outside opportunities $(\alpha > 0)$, but probably less than with full proportionality $(\alpha < 1)$. It is possible that there may even be cases, such as gold rushes or other instances of exaggerated expectations where α might be greater than 1. Similarly, we would expect the rate of arrivals to vary somewhat with the available pool of migrants $(\beta > 0)$ but for competition among them to result in less than full proportionality $(\beta < 1)$.

Indeed, in my empirical work [1] I estimate $\alpha = .3$ and $\beta = .1$ for the United States in the period 1955-1960. It should be noted that this was a period of relative economic stagnation. The only other comparable figures of which I am aware are Muth's [5] who found a substantial elasticity of job formation to net migration, and Greenwood [2] who found a similar elasticity for gross inmigration. Their figures cannot be directly compared, however, because they deal with actual migration rather than potential, as in C_j. I am not aware of any literature on estimates that would relate to the elasticity of outmigration.

If the case I have developed for including consideration of opportunity and competition is accepted, some practical considerations come to the fore, principally because it is laborious, expensive, and technically difficult to estimate them. From the perspective of scientific research, the question is whether omission of these variables is very damaging to traditional approaches. My judgment is that it is reasonably so. In the first place, competition and opportunity have substantial correlation with some of the variables, such as income, normally included in the specification of the functions v and w. Hence, there will be bias as some of their effect will be attributed to such correlated variables as may be included, or missed in the case of negatively correlated variables. Secondly, because variables with such elasticities will have important second-order effects, given the range of values of C and O, there will be substantial loss of accuracy for projection.

For policy, these considerations are amplified by others. The inclusion of these systemic variables and estimation of their parameters will clearly be relevant to such matters as the determination of the number and location of growth centers undertaken at one time, to the choice among policies of regional development versus policies of aid to migration, to an estimation of the effects of a growth center upon its surrounding region, and so forth. Government intervention may take the form of affecting variables within the functions v and w, such as the number of jobs, local incomes, tax rates, levels of education, provision of housing, or any others. They may also take the form of action upon the relational function t_{ij}, by improving transportation or lowering its cost, by programs of helping find jobs and resettlement grants, by travel and residence permits, and so forth. Whatever the policy variables acted upon, they will be aspects of the functions of local characteristics, v and w, or aspects of the relational functions, t_{ij}. These, in turn, will change the values of the systemic functions of opportunity and competition at all other locations to a lesser or greater extent. Thus, as systemic characteristics have local consequences, local actions in turn change the web of systemic relations.

<u>Appendix: A parsing of the general flow relation</u>

In the general formulation, the equation for the flow of migrants from i to j is

$$M_{ij} = v_i w_j t_{ij} O_i^{\alpha-1} C_j^{\beta-1}. \tag{10}$$

The structure of this relation is more intelligible if we rewrite it as follows:

$$M_{ij} = (v_i O_i^{\alpha}) \; [w_j (w_j C_j^{\beta} / w_j C_j) t_{ij} O_i^{-1}],$$

where the expression within the first parenthesis is the total number of migrants to leave i, and the expression within the bracket is the share who will arrive at j. Within the bracket:

- w_j : basic attraction of j;
- $w_j C_j^{\beta}$: total number of migrants who will arrive at j from all origins;
- $w_j C_j$: ex ante number of migrants who would arrive at j from all origins, i.e., the number who would arrive if they could all be accommodated;
- $w_j C_j^{\beta} / w_j C_j$: the number of actual places at j per seeker; this, of course, reduces to $C_j^{\beta-1}$; thus the initial attraction w_j is weighted by the ratio of places to seekers;
- t_{ij} : a further weighting taking account of any special relations from i to j.

Thus far, the expression within the bracket reduces to $w_j C_j^{\beta-1} t_{ij}$, which is the opportunities at j from the point of view of a migrant from i. Thus, we divide this by O_i, the opportunities at all other places in the system, similarly defined from the point of view of i, to obtain the proportion of migrants from i who will go to j.

Similarly, the terms of the flow equation can be regrouped so as to be interpreted as the total arrivals at j times the share of these arrivals coming from i.

Footnotes

1. In the United States this matrix was produced by the Census by asking a 25% sample of respondents where they lived five years previously.

2. My discussion is limited here to gross migration. Net migration which is the difference of gross outmigration from gross inmigration, has been much studied, but net migration is a statistical abstraction, not a form of human behavior: people either come or go.

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