Title

Generation of Laguerre-Gaussian 01 Mode for Use In Free Electron Lasers

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Paper Review of *Laguerre-gaussian Mode Laser Heater for Microbunching Instability Suppression in Free-electron Lasers*: Generation of Laguerre-gaussian 01 Mode for Use In Free Electron Lasers

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Abstract: A Laguerre-Gaussian mode laser, which exhibits a donut-shaped intensity profile, better suppresses Microbunching Instability in electron beams than standard Gaussian lasers. We posit that Laguerre-Gaussian modes can be generated with methods other than the spiral phase plates that are currently in use.

INTRODUCTION

Free Electron Lasers, which are important research tools in condensed matter physics, chemistry, and structural biology, require electron beams for operation. However, these electron beams are subject to the negative effects of Microbunching Instability (MBI). MBI is suppressed using a laser heater.

The laser heater at the Linac Coherent Light Source (LCLS) is a simple Gaussian beam, with maximum intensity at the center. However, this Gaussian laser creates a two-horned energy spread distribution in the electron beam at higher energy levels, which hurts MBI suppression [1].

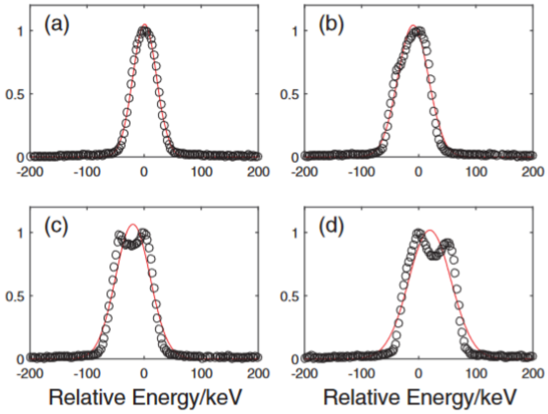

Figure 1: Electron beam energy distribution at laser heater energies (a) 20.5 (b) 26.7 (c) 30.1 and (d) 37.2 keV. Note the double-horn energy distribution as laser heater energy increases. [1]

Lasers can have Hermite-Gaussian or Laguerre-Gaussian modes. Hermite-Gaussian modes have rectilinear symmetry, which means they're symmetrical around both the x-axis and the y-axis, while Laguerre-Gaussian modes have radial symmetry [4].

A laser with the Laguerre-Gaussian mode LG_{01} has a donut-shaped intensity profile. Researchers at SLAC have shown that a laser heater with the LG_{01} mode avoids the two-horned electron beam energy distribution. This introduces a small, controlled energy spread early on in the pathway, which suppresses MBI instability further down [1].

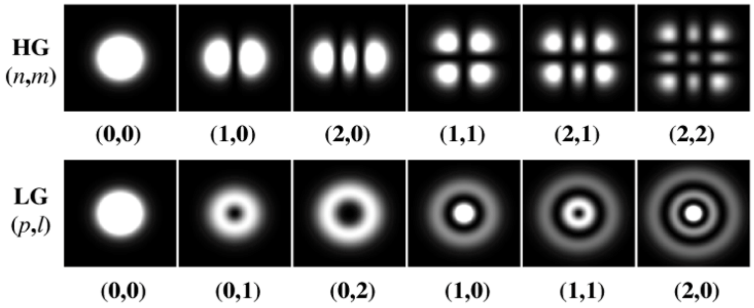

Figure 2: Hermite Gaussian and Laguerre Gaussian mode intensity cross-sections. Note that Laguerre-Gaussian intensity distributions tend to be “donut-shaped” [2].

The researchers at SLAC use a spiral phase plate to create an LG_{01} mode out of a simple Gaussian mode (LG_{00} or HG_{00}). The spiral phase plate is shaped like a spiral staircase with sixteen steps. It changes the phase of light depending on the azimuthal position of the light beam with regards to the axis of propagation [1].

However, spiral phase plates are not the only way to generate the desired LG_{01} mode. HG modes can be generated by inserting metal wires into the laser cavity where the desired HG modes have nodal lines. LG modes are linear compositions of HG modes. Using a pair of astigmatic lenses, LG_{01} can be generated from HG_{01} and HG_{10} [3].

METHODS

The HG modes necessary to generate the LG_{01} mode must be identified. We know that any LG mode can be decomposed into a linear composition of HG modes of the same order [3]. The LG_{01} mode, specifically, is created from the following HG modes [1]:

$$LG_{01} = HG_{10}n_x + iHG_{01}n_x \tag{1}$$

For first-order Hermite Gaussian modes [4]:

$$E_{10,HG}(x,y,z) = \frac{C_{10,HG}}{w(z)} \cdot \frac{2\sqrt{2}x}{w(z)} \cdot e^{-\frac{x^2+y^2}{w^2(z)}} \cdot e^{-i\frac{k}{2}\cdot\frac{x^2+y^2}{R(z)}} \cdot e^{i\zeta_{10}(z)} \tag{2}$$

$$E_{01,HG}(x,y,z) = \frac{C_{01,HG}}{w(z)} \cdot \frac{2\sqrt{2}y}{w(z)} \cdot e^{-\frac{x^2+y^2}{w^2(z)}} \cdot e^{-i\frac{k}{2}\cdot\frac{x^2+y^2}{R(z)}} \cdot e^{i\zeta_{01}(z)} \tag{3}$$

Note that:

$$\zeta_{mn}(z) = -(m+n+1)tan^{-1}\frac{z}{z_R} \tag{4}$$

$$\zeta_{10}(z) = \zeta_{01}(z) =- 2\,tan^{-1}\frac{z}{z_R} \tag{5}$$

$$C_{01} = C_{10} = \frac{1}{2}\sqrt{\frac{\omega\mu_0}{\pi k}} \tag{6}$$

Now:

$$E_{01,LG}(r,\phi,z) = \frac{1}{2w(z)}\sqrt{\frac{\omega\mu_0}{\pi k}} \cdot e^{-\frac{x^2+y^2}{w^2(z)}} \cdot e^{-i\frac{k}{2}\cdot\frac{x^2+y^2}{R(z)}} \cdot e^{-i2tan^{-1}\frac{z}{z_R}}\left(\frac{2\sqrt{2}x+i2\sqrt{2}y}{w(z)}\right) \tag{7}$$

Conversion to polar coordinates:

$$2\sqrt{2}x + i2\sqrt{2}y = 2\sqrt{2}re^{i\phi} \tag{8}$$

$$E_{01,LG}(r, \phi, z) = \frac{1}{2w(z)}\sqrt{\frac{\omega\mu_0}{\pi k}} \cdot e^{-\frac{r^2}{w^2(z)}} \cdot e^{-i\frac{kr^2}{2R(z)}} \cdot e^{-i2tan^{-1}\frac{z}{z_R}} \cdot \frac{2\sqrt{2}r}{w(z)} \cdot e^{i\phi} \tag{9}$$

From Beijersbergen, for the LG_{01} mode [3]:

$$E_{01,LG} = \frac{C_{01,LG}}{w(z)} \cdot \frac{r\sqrt{2}}{w(z)} \cdot e^{i\zeta_{01}(z)} \cdot e^{i\phi} \cdot e^{-\frac{-ikr^2}{2R(z)}} \cdot e^{-\frac{r^2}{w^2(z)}} \tag{10}$$

$$E_{01,LG} = \frac{C_{01,LG}}{w(z)} \cdot \frac{r\sqrt{2}}{w(z)} \cdot e^{i2tan^{-1}\frac{z}{z_R}} \cdot e^{i\phi} \cdot e^{-\frac{-ikr^2}{2R(z)}} \cdot e^{-\frac{r^2}{w^2(z)}} \tag{11}$$

A pair of two astigmatic lenses can create a phase shift between light on the two axes orthogonal to the direction of propagation [3].

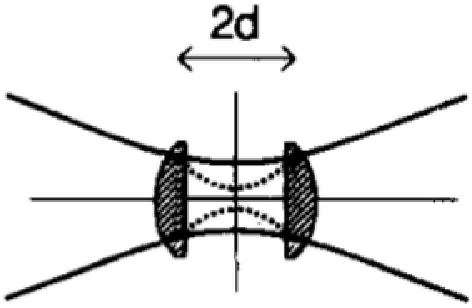

Figure 3: Gaussian beam envelope. The solid curve indicates the (y, z) plane and the dashed curve indicates the (x, z) plane. Both astigmatic mirrors are shown. [3]

Figure 3 shows that due to astigmatism, light on the (x, z) plane has a greater path length than light on the (y, z) plane, which introduces a relative phase shift. The lenses do not cause a change in polarization.

RESULTS AND INTERPRETATION

Besides the normalization constants, the expression for $E_{01,LG}$ calculated from the linear superposition of HG modes [Eq 9] is identical to the expression for $E_{01,LG}$ from Beijersbergen's paper [Eq 10]. Thus, it is shown that LG_{01} is a superposition of HG_{10} with iHG_{01}.

We demonstrate that a donut-shaped intensity beam with fixed polarization can be generated without the use of a spiral phase plate. Two metal wires, one for HG_{10} and one for HG_{01}, are inserted into the laser cavity where their nodal lines are. This causes the laser to output in-phase HG_{10} and HG_{01} modes. Then, a pair of astigmatic lenses introduces a shift in the relative path length of HG_{10} and HG_{01} (Figure 3). The lenses must be designed such that HG_{01} gains a 90-degree phase shift relative to HG_{10}.

The output of the two astigmatic lenses is $HG_{10} + iHG_{01}$ in the phasor domain. As shown in Eqs 9 and 10, this is equivalent to LG_{01}, which has a donut-shaped intensity profile (Figure 2).

CONCLUSION

Using the method outlined in this paper, the LG_{01} mode can be generated with two metal wires inserted into the laser cavity, two astigmatic lenses, and no spiral phase plate.

The benefits and drawbacks must be explored. One possibility is to analyze the relative efficiencies of these two methods. The thickness of spiral phase plates upon azimuthal progression are discrete in reality for ease of manufacturing, indicating the possibility of losses in conversion efficiency due to physical nonidealities. Furthermore, because astigmatic lenses are already manufactured in large quantities for prescription eyeglasses, they may be less expensive than spiral phase plates.

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