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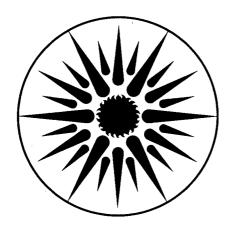
APPLIED SCIENCE DIVISION

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Air Infiltration Measurement Techniques

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Tracer gas techniques have become widely used to measure the ventilation rates in buildings. The basic principle involved is that of conservation of mass (of tracer gas) as expressed in the continuity equation; by monitoring the injection and concentration of the tracer, one can infer the exchange of air. Although there is only one continuity equation, there are many different experimental injection strategies and analytical approaches. These different techniques may result in different estimates of infiltration due to uncertainties and biases of the procedures. This report will summarize the techniques and the relevant error analyses.

As more detailed information is required for both energy and indoor air quality purposes, researchers are turning to complex, multizone tracer strategies. Both single gas and multiple gas techniques are being utilized, but only multigas are capable of uniquely determining the entire matrix of air flows. This report will also review the current effort in multizone infiltration measurement techniques.

Keywords: Ventilation, Infiltration, Tracer Gas, Multizone Measurement Techniques, Error Analysis, Uncertainty

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INTRODUCTION

The purpose of this report is to review the approaches to the use of tracer gases in the determination of infiltration (i.e. total outside air exchange). A decade ago the author presented work on this same topic; this report updates the earlier work. Other summaries of earlier work have been done as well. 2,3 Although the continuity equation has not changed, the tracer techniques now being used have developed in the interim. Both this report and the prior one consider both single-zone and multizone solutions, steady-state and transient methods, and potential sources of bias.

The issue of mixing (which is a rubric for phenomena such as short-circuiting, apparent flow delay, and effective volume,) was stressed heavily in the original work. Mixing must always be a serious concern in tracer gas experiments, The subject of non-tracer measurement techniques (i.e. modeling) was discussed in the original report, but, as modeling has become a topic of extensive effort, it is being reviewed by others, but will not be explicitly treated, herein.

Background

Tracer gases are used for a wide range of diagnostic techniques including leak detection^{5,6} and atmospheric tracing.⁷ One application which has had a resurgence in the last decade is the use of tracer gases to measure ventilation (i.e., air flow) in buildings.⁸ Ventilation is an important process in buildings because of its impact on both energy requirements and indoor air quality—both of which are topics of concern to society. Measurement of the tracer gas combined with conservation laws allows a quantitative determination of the tracer transport mechanism (i.e., a measurement of the air flow).

The vast majority of the ventilation measurements made to date have involved a single-tracer gas deployed in a single zone. This technique has proven very useful for building which may treated as a single zone (e.g., houses) and for more complex buildings in which there are isolatable sub-sections. However, as the need to understand more complex buildings has grown, tracer techniques that are able to treat multiple zones have been developed. Multizone techniques recognize that not only does air flow between the outside and the test space, but there are air flows between different parts (i.e., zones) of the test space and, in the complete case, they are able to measure these flows.

As in any experimental techniques, there are uncertainties associated with the fundamental measurements and these errors propagate to become uncertainties in the determination of air flows. Some work on the optimization of the single zone problem has been done. For example, Heidt¹⁰ has demonstrated that optimal precision in tracer decay measurement is on the order of the inverse air-change

rate; and D'Ottavio¹¹ has shown a decrease in precision when a two-zone building is treated as a single zone.

Mixing and Ventilation Efficiency

This report summarizes tracer gas methods for determining air flows between zones, but not within them. For our purposes, zones are statistically treated as having the same concentration of tracer anywhere within a zone. We call the time it takes a zone to reach this effectively homogeneous state the mixing time. As will be discussed in following sections, techniques in which the concentration changes quickly compared to the mixing time, may be subject to significant errors.

The zonal assumption is necessary when there is no knowledge about the size and location of the inlet and exhaust flows to each zone. In cases, (e.g. mechanically ventilated rooms) in which these quantities are known, tracer gases can be used to measure the ventilation efficiency within the zone. Age of air concepts are often used to describe spatial variation of ventilation. Sandberg ¹² summarizes the definitions and some of the tracer techniques for determining the efficiency (by seeding inlet streams or monitoring exhaust streams). In this language, our mixing assumption can be stated as "the mean age of air is equal to the nominal ventilation time."

Because they are usually concerned with the highly controlled environment of mechanically ventilated buildings, those looking at age of air concepts do not normally consider the time variation of the ventilation. Conversely, the simple zonal approach assumed herein, ignores spatial variations within nominal zones. There is some preliminary modeling work on combining these two approaches using more complex transfer functions for the zones. Further discussion of intrazonal air flows is beyond the scope of this report.

SINGLE ZONE VENTILATION

All tracer gas methods rely on the solution of the continuity equation to infer the ventilation from measurements of the concentrations and injected tracer flow rate. The continuity equation is as follows:

$$V \cdot C(t) + Q(t) \cdot C(t) = F(t)$$
(1.1)

which can be directly solved for the ventilation:

$$Q(t) = \frac{F(t) - V \cdot C(t)}{C(t)} \tag{1.2}$$

Equivalently, we can solve the continuity equation to predict the concentration as

a function of time given all of the other quantities:

$$C(t) = \int_{-\infty}^{t} \frac{F(t')}{V} e^{\int_{t}^{t'} \lambda(t'')dt''} dt'$$
(1.3)

The common definition of air change rate, λ , is used above:

$$\lambda(t) \equiv \frac{Q(t)}{V} \tag{2}$$

As Roulet¹⁴ points out, the continuity equation is a *mass* balance equation and serious errors can result if it is used as a *volume* balance equation unless proper precautions are taken. Accordingly, the concentrations are expressed in mass of tracer per unit volume to assure correctness even when the density of air varies from zone to zone (e.g., if the zones are at different temperatures).

If it were possible to solve eq. 1 on an instantaneous basis to find the ventilation, most of the rest of this report would be not be needed. However, because of measurement problem including mixing issues, the determination of an instantaneous ventilation value is practically impossible. Therefore it becomes necessary to use time-series data to reduce the uncertainties to make the analysis possible. The rest of this report details some of the approaches being used to overcome these measurement limitations.

The exact analytical technique for inverting the measured data to find the ventilation depends on both the experimental technique, the assumption made about the system, and the quantity of interest. The following sections develop the analysis for techniques currently in use. Virtually all of the widely used techniques assume that the ventilation is constant over the measurement period. Although this assumption is often violated, these techniques will be discussed in detail below.

GENERAL SOLUTIONS

The three versions of eq. 1 suggest three different approaches for the determination of the infiltration from the data. The term general here is used to indicated that there are no particular assumptions made about the injections or concentration other than the continuity equation itself. In following sections specific solutions which assume a particular control strategy (e.g. constant injection) will be considered.

Regression Techniques

Regression techniques use eq. 1.3 and assume that the infiltration is constant over the regression period $(0 \le t \le T)$ and then find the best set of parameters that fit the concentration and injection data to the following equation:

$$C(t) = C_R e^{-\lambda_R t} + \int_0^t \frac{F(t')}{V_R} e^{-\lambda_R (t-t')} dt'$$
(3)

The parameters C_R and λ_R are always treated as unknowns in the regression. The effective volume, V_R , may be treated as either a known or an unknown. The *precision* of the estimate of infiltration can usually be extracted from the details of the regression.

The regression will result in the best single estimate of infiltration to match the data. If the air change is constant, the regression estimate will result in an unbiased estimate of the true infiltration. If the infiltration is not constant, however, the regression will result in a biased estimate of the average air change.

Integral Techniques

Integral techniques make use of the fact that it is often easier to obtain integrals (or, equivalently, averages) of the measured data (i.e. injection and concentration) than it is to get a complete time history of both. In integral techniques the flows and concentrations are integrated from the *initial* time (i.e. t=0) to the final time (i.e. t=T). Thus, these techniques integrate eq. 1.1, assuming the infiltration is constant over the integral period:

$$Q_{I} = \frac{\int_{0}^{T} F(t)dt + V\left(C_{initial} - C_{final}\right)}{\int_{0}^{T} C(t)dt}$$

$$(4.1)$$

or, equivalently, in terms of averages:

$$Q_I = \frac{\overline{F} - V\overline{C}}{\overline{C}} \tag{4.2}$$

where the average concentration is given by

$$\overline{C} = \frac{1}{T} \int_{initial}^{final} C(t)dt$$
 (5.1)

the average accumulation is given by,

$$\overline{C} = \frac{C_{final} - C_{initial}}{T}$$
(5.2)

and the average injection is the total volume of injected tracer divided by the length of the measurement period:

$$\overline{F} = \frac{1}{T} \int_{0}^{T} F(t)dt \tag{5.3}$$

The uncertainty in this estimate of the infiltration is as follows:

$$\delta Q_I^2 = \frac{\delta \overline{F}^2 + \overline{C}^2 \delta V^2 + V^2 \delta \overline{C}^2 + Q_I^2 \delta \overline{C}^2}{\overline{C}^2}$$
(6.1)

$$\delta\lambda_I^2 = \frac{\delta \overline{F}^2}{\overline{C}^2 V^2} + \frac{\overline{F}^2}{\overline{C}^2 V^2} \frac{\delta V^2}{V^2} + \frac{\delta \overline{C}^2}{\overline{C}^2} + \lambda_I^2 \frac{\delta \overline{C}^2}{\overline{C}^2}$$
(6.2)

where

$$\delta \overline{C}^2 \equiv \frac{\delta C(0)^2 + \delta C(T)^2}{T^2} \tag{7}$$

In the absence of measurement error one can show, as Axley¹⁵ has done for the pulse technique, that the infiltration value calculated with an integral technique must have occurred sometime during the measurement period. Such an existance proof may be sufficient if it is exogenously determined that the ventilation moves within a narrow band. For most other purposes one wishes to know the average ventilation over the measurement period. If the measurement period, T, is known to be smaller than the characteristic time over which the infiltration changes, then the integral approach will give a good estimate of the average infiltration; otherwise, averaging techniques may be preferable.

Averaging Techniques

The techniques discussed so far all have assumed that the ventilation does not vary over the measurement period. The validity of this assumption will depend, of course, upon the physical situation being measured and the technique being used. If the ventilation is not actually constant over the measurement period it is not always clear what the previous techniques are actually calculating. This kind of error is qualitatively different from the types of error associated with uncertainties in the measured concentrations and flows. Average techniques are those which yield unbiased estimates of the average infiltration over the measurement period.

Although eq. 1.2 cannot practically be used to determine the instantaneous ventilation, it can be averaged over a measurement period in order to get the average ventilation:

$$\overline{Q} = \overline{\left(\frac{F}{C}\right)} - \frac{V}{T} \ln \left(\frac{C_{final}}{C_{initial}}\right)$$
 (8)

This general expression can now be used to determine the average ventilation and its uncertainty for the different experimental approaches previously discussed.

The uncertainty in the estimate of the average infiltration can be estimated the following equations:

$$\delta \overline{Q}^2 = \left(\overline{\left(\frac{1}{C}\right)}\right)^2 \delta \overline{F}^2 + \left(\overline{\left(\frac{F}{C^2}\right)}\right)^2 \delta \overline{C}^2 + \ln^2 \left(\frac{C_{final}}{C_{initial}}\right) \frac{\delta V^2}{\mathbf{T}^2} + \frac{V^2}{\mathbf{T}^2} \left(\frac{\delta C_{final}^2}{C_{final}^2} + \frac{\delta C_{initial}^2}{C_{inital}^2}\right) (9.1)$$

$$\delta \overline{\lambda}^2 = \left(\overline{\frac{1}{C}} \right)^2 \frac{\delta \overline{F}^2}{V^2} + \left(\overline{\frac{F}{C}} \right)^2 \frac{\delta \overline{C}^2}{V^2} + \left(\overline{\frac{F}{C}} \right)^2 \frac{\delta V^2}{V^2} + \frac{\delta C_{initial}^2}{T^2 C_{initial}^2} + \frac{\delta C_{final}^2}{T^2 C_{final}^2}$$
(9.2)

SPECIFIC TECHNIQUES

The general techniques cited above can be simplified by controlling the injection of tracer gas in certain well-behaved ways. Different groups have used different techniques depending on practical limitations. A rather complete listing of the groups using various techniques has been compiled by the Air Infiltration and Ventilation Centre ¹⁶ and will not be repeated herein.

Steady-State Techniques

Specific techniques tend to fall into two categories: steady-state techniques, which measure Q and tend to minimize the effect of the accumulation of tracer in the test volume. In all steady-state techniques the experiment is so arranged that

$$\overline{C} \ll \frac{\overline{F}}{V}$$
 (10)

and the accumulation can be ignored in all the steady-state techniques. which implies that

Since the steady-state techniques measure Q, an estimate of λ will have an increased error.

$$\frac{\delta\lambda^2}{\lambda^2} = \frac{\delta Q^2}{Q^2} + \frac{\delta V^2}{V^2} \tag{11}$$

Transient Techniques

Transient techniques tend to emphasize the change in tracer gas concentration in the room and measure λ ; inequality 10 is never true. Although Q and λ are simply related by the volume, the building volume the participates in air exchange is usually not known very precisely and the precision of the result may depend on which of the two types of approaches is used. Because the decay techniques are transient techniques and measure λ , the error in Q is increased by the uncertainty in the volume:

$$\frac{\delta Q^2}{Q^2} = \frac{\delta \lambda^2}{\lambda^2} + \frac{\delta V^2}{V^2} \tag{12}$$

TRACER DECAY

The most widely used type of tracer measurement is that of a tracer decay. In a tracer decay test the test space is initially charged up to a concentration of tracer gas appropriate to the instrumentation and then shut-off and allowed to decay. The analysis in all decay techniques is simplified because the terms in both the infiltration and uncertainty equations, involving the injection rate vanish.

The concentration is monitored during the period of zero injection and then used to infer the infiltration rate, λ . There are three different approaches to analyzing a tracer decay, all of which are transient approaches:

Decay Regression

If there is no tracer injection and the concentration is allowed to decay from some initial value, the integral for eq. 3 vanishes and we are left with the decay equation:

$$C(t) = C_R e^{-\lambda_R t} \tag{13}$$

This equation can be fit to the measured data using regression methods. The parameters of the fit are C_R and λ_R . An estimate of the precision can be made from the regression.

The normal method of analysis uses a linear regression on the log of the concentration. This technique is extremely sensitive to zero drift and tends to overweight the contribution of lowest concentration values to the fit. A detailed error analysis of this kind of regression is beyond the scope of this report, but the uncertainty analysis from the other decay techniques can be used to give a reasonable estimate of the *precision* of this technique.

Integral Decay

If over the measurement period there is no injection of tracer, eq. 4 simplifies to the following:

$$Q_I = -\frac{V\overline{C}}{\overline{C}} \tag{14.1}$$

or, equivalently,

$$\lambda_I = \frac{C_{initial} - C_{final}}{T \, \overline{C}} \tag{14.2}$$

The uncertainty in these quantities can be calculated from the uncertainties in the concentrations.

$$\frac{\delta \lambda_I^2}{\lambda_I^2} = \frac{\delta \overline{C}^2}{\overline{C}^2} + \frac{\delta C_{initial}^2 + \delta C_{final}^2}{\left(C_{initial} - C_{final}\right)^2} \tag{15}$$

Two-Point Decay

For a decay test the term involving the flow and concentration becomes identically zero and the average ventilation can be determined from the initial and final concentrations:

$$\overline{Q} = \frac{V}{T} \ln \left(\frac{C_{initial}}{C_{final}} \right) \tag{16}$$

This analysis technique has been used in what is known as the **container** method. ¹⁷ If the air change rate varies over the measurement period, this technique assures an unbiased estimate of the average; it is, however, a less precise technique than the others.

The uncertainty in the ventilation can be calculated from this expression:

$$\delta\lambda^2 = \frac{1}{T^2} \left(\frac{\delta C_{initial}^2}{C_{initial}^2} + \frac{\delta C_{final}^2}{C_{final}^2} \right)$$
 (17)

Assuming the initial and final concentration can be measured equally well,

$$\delta \overline{\lambda} \approx \frac{\sqrt{1 + e^{2\overline{\lambda}T}}}{T} \frac{\delta C}{C_{initial}} \tag{18}$$

the uncertainty is minimized by making the measurement time about the same at the decay time, $T_{opt} \approx \frac{1}{\overline{\lambda}}$. Thus

$$\frac{\delta \overline{\lambda}}{\overline{\lambda}} \ge 3 \frac{\delta C}{C_{initial}} \tag{19}$$

Note that this limit is inferior to the precision possible with the integral decay; such is the price of having an unbiased measurement.

PULSE TECHNIQUE

The **pulse** technique is a steady-state variant on the integral decay technique. ¹⁸ Unlike the decay techniques the measurement period begins before the tracer is injected and then waiting until the concentration decays. Also unlike decay techniques, the total volume of tracer gas injected into the space must be known. When the concentration has decayed sufficiently that the inequality of eq. 10 has been satisfied, the measurement can be terminated and the infiltration can be estimates as follows:

$$Q_I = \frac{\overline{F}}{\overline{C}} \tag{20}$$

The uncertainty in this estimate is as follows:

$$\frac{\delta Q_I^2}{Q_I^2} = \frac{\delta \overline{C}^2}{\overline{C}^2} + \frac{(\overline{C}^2 + \delta \overline{C}^2)V^2}{\overline{F}^2} + \frac{\delta V^2 \overline{C}^2}{\overline{F}^2} + \frac{\delta \overline{F}^2}{\overline{F}^2}$$
(21)

Since it takes infinitely long to decay to zero the accumulation term and its uncertainty must be considered, even though it does not contribute to the estimate of the infiltration. Because the injection is usually known quite precisely and the accumulation term is made small, the last two terms can usually be neglected.

Care must be taken not to wait too long or the concentration errors could increase because of the low average value.* The concentration-term error will increase as the square root of the measurement time and the accumulation-term error will decrease exponentially as does the instantaneous concentration. The optimum time will occur when these two terms are approximately equal.

$$e^{-\lambda T} << 1 \tag{22}$$

Since the total amount of tracer injected can be known quite well, the uncertainty only depends on the ability the measure the average concentration well and the measurement time:

$$\frac{\delta Q^2}{Q^2} \approx \frac{\delta \overline{C}^2}{\overline{C}^2} + e^{-2\lambda T} \tag{23}$$

^{*}A possible improvement on this technique would be to use a non-zero starting and stopping value by pre-injecting a small amount of tracer before beginning the experiment; thus allowing the accumulation term to be made smaller without needing such a long integration time. Such a technique approaches a modified constant concentration method, which is discussed in a later section.

Charge Up

If the pulse is injected quickly and the mixing in the test space if quite good. The rise in concentration due to the pulse of tracer gas can be used to determine the effective volume of the test space. However, care must be taken to insure that good mixing is present and that the air change rate is not too high, before this volume measurement technique should be considered.

CONSTANT INJECTION

If the tracer injection is non-zero, but constant the integral in eq. 3 can be simply evaluated:

$$C(t) = \frac{F}{Q_R} + \left(C_R - \frac{F}{Q_R}\right)e^{-\lambda_R t} \tag{24}$$

Although some researchers^{1,19} have fit this curve directly using non-linear methods to find the ventilation and the effective volume, most use of this method has been to wait until **steady state** has been reached and then the data is fit to the steady state equation:

$$Q_R = \frac{F}{C(t)} \quad \text{for } e^{-\lambda t} << 1 \tag{25}$$

Regression techniques in this instance are equivalent to an average. Multiple independent measurements of the ventilation can be used to increase the precision accordingly. Again, it should be noted that if the infiltration is not constant during the measurement period the result may be biased. The result will not be biased, however, if the following condition is met:

$$\left| \ln \left(\frac{C_{final}}{C_{initial}} \right) \right| \ll \overline{\lambda} T \tag{26}$$

Often, however, an average concentration is used in the denominator; such an approach is actually an integral approach (See the long-term average technique):

$$Q_I = \frac{F}{\overline{C}} \tag{27}$$

This approach will be biased, but not in the same way as above. As long the accumulation term remains small:

$$Q_I \le Q_R \approx \overline{Q} \tag{28}$$

Thus, Q_R is preferable to Q_I .

Charge Up

As noted above constant injection is usually done after steady-state has been achieved. However, if the mixing is good, it may be possible to use the period of tracer accumulation (i.e. $\lambda t \approx 1$) to determine λ (or, alternatively, the effective volume). Although the mixing constraints are not as severe as in the pulse charge-up technique, similar care must be taken.

LONG-TERM INTEGRAL

The long-term integral method—sometimes called the long-term (concentration) average method—is a steady-state approach in which the accumulation term is minimized by using a continuous injection and averaging long enough for the T in the denominator of the accumulation term to make the term arbitrarily small. Eq. 4.2 then becomes the following:

$$Q_I = \frac{\overline{F}}{\overline{C}} \qquad \text{for } \lambda T >> 1 \tag{29}$$

Note that although this technique is usually used with constant injection, any injection pattern could be used. (When done with constant injection, this technique is sometimes referred to as the *passive ventilation measurement* technique most typically used with PerFluorocarbon Tracer.)²⁰ Non-constant injection is a potential source of bias in the determination of the average infiltration.

If we assume that the accumulation term is sufficiently small so that errors attributable to the accumulation term are negligible, the uncertainty is this steady-state result is simply,

$$\frac{\delta Q^2}{Q^2} = \frac{\delta \overline{C}^2}{\overline{C}^2} + \frac{\delta \overline{F}^2}{\overline{F}^2} \tag{30}$$

One potential problem with this technique is that is requires a measurement time very long compared to the decay time, but it also assumes that ventilation is constant over that time. It has been shown.²¹ that even if the injection is constant, this technique always leads to a negative bias in the estimation of the ventilation unless the ventilation is quite constant. This technique is *not* recommended for use in determining the *average ventilation* in buildings.

The average ventilation is not always the quantity of interest in making tracer gas measurements. It has been shown²² that the quantity of interest for determining the amount of pollutant dilution provided by ventilation is not the average ventilation, but a quantity called the *effective ventilation*. The passive ventilation technique directly measures the effective ventilation and so may be the technique of choice for determining pollutant dilution.

CONSTANT CONCENTRATION

In the constant concentration technique is a steady-state technique in which an active control system is used to change the amount of injected tracer to maintain the concentration of tracer at some target level, C_T . With that assumption the accumulation term is identically zero and eq. 8 can used to calculate the average ventilation from the average injection:

$$\overline{Q} = \frac{\overline{F}}{C_T} \tag{31}$$

Although it does not enter the estimate of the ventilation the accumulation term can effect the uncertainty, if the control is not perfect:

$$\frac{\delta Q^2}{Q^2} = \frac{\delta \overline{F}^2}{\overline{F}^2} + \frac{\delta \overline{C}_T^2}{C_T^2} + \frac{V^2 \delta \overline{C}^2}{\overline{F}^2}$$
 (32)

The last term is due to the uncertainty in the accumulation term. The error in the second term comes from the difference between the average concentration during the period and the target.

EXAMPLE ERROR ANALYSES

In this section a few examples of how various techniques might be used in specific situations and what kind of uncertainties could be expected. For the purposes of these examples, it will be assumed that a single concentration measurement can be made with a 10% uncertainty scaled to the initial concentration. Although almost all analyzers have better accuracy than 10%, the vagaries of sampling and mixing make 10% a reasonable number. In any case, the calculated uncertainties will scale with this number. It will also be assumed that it is possible to make 10 measurements/hr.

The volume of the example zone will be assumed to be 100m^3 . For the decay methods only λ will be calculated and for the steady-state methods only Q will be calculated so that the uncertainty in the volume will not enter. However, (effective) it should be kept in mind that zone volumes are rarely known to better than 20% so that the additional error could be significant.

Average Decay

The last row should never be used in a real experiment, but it serves to demonstrate what can happen to the precision if the decay goes on too long.

	UNCERTAIN'	TIES FOR AN A	VERAGE DEC	AY
δλ	$\overline{\lambda}[h^{-1}]$			
Т	0.25	0.5	1	2
20 min	0.443 (177%)	0.464 (93%)	0.515 (52%)	0.657 (33%)
1 hour	0.163 (65%)	0.193 (39%)	0.290 (29%)	0.746 (37%)
3 hours	0.078 (31%)	0,153 (31%)	0.670 (67%)	13.4 (672%)
8 hours	0.093 (37%)	0.683 (137%)	37. (3726%)	∞

Integral Decay

As can be seen from the average-decay uncertainty analysis, the uncertainties begin to increase for long measurement times and high air exchange rates. This increase is due to the fact that the final concentration begins to get quite low and, hence, quite uncertain. The alternative decay procedures do not suffer from this and can give better precision in this range—provided the air change rate can be assumed to be constant.

UNCERTAINTIES FOR AN INTEGRAL DECAY				
δλ	$\lambda[h^{-1}]$			
${f T}$	0.25	0.5	1	2
20 min	0.442 (177%)	0.462 (92%)	0.503 (50%)	0.607 (30%)
1 hour	0.160 (64%)	0.181 (36%)	0.229 (23%)	0.356 (18%)
3 hours	0.067 (27%)	0.093 (19%)	0.160 (16%)	0.359 (18%)
8 hours	0.041 (17%)	0.076 (15%)	0.167 (17%)	0.456 (23%)

The last row of this table suggests that good precision can be achieved for longer measurement periods with an integral decay. However, an eight hour measurement period may only be justified if the infiltration were known to be constant. It should be noted that this technique is quite sensitive to errors caused by offsets in the concentration measurement. If there is no zero offset (and if the concentration measurement is still random at low values), the integral method has the potential for greater precision than the general decay method; the cost for this is potential bias if the infiltration is not constant.

If the weighting is done correctly, the regression decay has the potential for the same level of error as the integral decay. However, if the standard method is used, the regression decay will give unreliable results when the concentration is allowed to decay too far.

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Pulse Method

For the pulse method it is assumed that the amount of injected tracer is known to better than 1% and can be neglected in the error analysis.

	UNCERTAINTIES FOR THE PULSE METHOD			
δQ	$\overline{Q}[m^3h^{-1}]$			
T	25	50	100	200
20 min	23 (92%)	42 (85%)	72 (72%)	103 (51%)
1 hour	19 (78%)	30 (61%)	37 (37%)	30 (15%)
3 hours	11 (47%)	11 (23%)	8 (8%)	22 (11%)
8 hours	3 (14%)	2 (5%)	9 (9%)	36 (18%)

As in the decay methods, an eight hour measurement period may only be justified if the infiltration were known to be constant. Also like the decay methods, this technique is sensitive to zero offsets.

Because of the trade-off between additional measurement points, accumulation term, and low average concentration, the pulse technique's precision behavior is complex. However, there is an optimal measurement time for any given experimental set of conditions, which tends to be longer than for the decays.

Constant Concentration

The constant concentration method will be used as an example of the general steady-state method. It is assumed that the control is unbiased (i.e. that there is no difference between the *mean* concentration and the target. It is also assumed that the average flow can be determined to 1% precision; not all constant concentration systems can achieve this, but many can. With these assumptions and for the example values chosen, the error is dominated by the uncertainty in the accumulation term:

The absolute error in the table does not depend much on the infiltration, but on the measurement time. (This result is a consequence of the high measurement rate and the small injection error.) If the measurement time were extended, this trend would change.

UN	CERTAINTIES	FOR CONSTA	NT CONCENT	RATION
δQ	$\overline{Q}[m^3h^{-1}]$			
Т	25	50	100	200
20 min	42 (170%)	43 (85%)	43 (43%)	44 (22%)
1 hour	14 (57%)	14 (28%)	15 (15%)	16 (8%)
3 hours	5 (19%)	5 (10%)	5 (5%)	6 (3%)
8 hours	2 (7%)	2 (4%)	2 (2%)	3 (2%)

DISCUSSION OF SINGLE-ZONE METHODS

SUMMA	RY OF MEASU	RMENT METHODS		
NAME	MEASUREMENT	FLOW	BIASED	
	TYPE	CONTROL	AVERAGE?	
TRANSIENT METHOD	S (measure λ)			
(Simple) Decay	Regression	Decay	Yes	
Two-Point Decay	Average	\mathbf{Decay}	No	
Integral Decay	Integral	Decay	Yes	
Charge Up	Regression	Const. Inj./Pulse	*	
STEADY-STATE METHODS (measure Q)				
Pulse	Integral	Pulse	Yes	
Constant Injection	Regression	Constant Injection	No	
Long-Term Integral	Integral	Any	Yes	
Constant Concentration	Average	Constant Concentration	No	
* Usually used with other	r methods to ascert	tain effective volume.		

The choice of method to use in a given situation will depend on the practical details of the experiment as well as the reason for measuring the air change in the first place.

Decay and pulse methods require the least time and usually the least preparation. With the exception of the two-point method, all of these give biased estimates of the average air change rate. In most circumstances, however, the biases will be negligible as long as the measurement period is limited to times on the order of the air change time. Although unbiased, the two-point method may yield unacceptably large uncertainties in the air change rate, unless the precision in the measurement of those two concentrations can be made quite high.

Like the pulse technique, the long-term integral technique may yield a biased estimate of the average infiltration. Because of the much longer measurement period, however, the bias may not be negligible. Although the long-term technique may prove unacceptable for determining average infiltration, it can nevertheless provide an unbiased estimate of the effective air change rate.

The constant concentration technique can be both accurate and precise, but it requires the most equipment as well as sophisticated control systems and real-time data acquisition. The constant injecting technique (without charge-up) can be considered a somewhat simplier version of the constant concentration technique, in that no active control of the injection rate is needed. Rather segments of data which meet eq. 26 are used to determine the average infiltration. If this condition is not met eq. 8 must be used to analyze the data.

MULTIZONE MEASUREMENT TECHNIQUES

Single-zone (single-gas) measurement techniques have been used extensively in a wide variety of situations and by a large number of different people. Field experience has yielded a significant amount of practical knowledge about the limitations of the techniques. For complex buildings (i.e. those whose interiors are not a simple, well-mixed zone) cannot be easily described by a single-zone infiltration measurement. Thus, one might wish to determine flows to and from multiple zones in the building to better understand it. Although multizone techniques have been utilized and discussed for over a decade¹, ^{23,24} there has not been the same level of investigation or use.

To develop a multizone infiltration measurement technique, a matrix form of the continuity equation must be used:

$$\mathbf{V} \cdot \dot{\mathbf{C}}(t) + \mathbf{Q}(t) \cdot \mathbf{C}(t) = \dot{\mathbf{F}}(t) \tag{33.1}$$

For every zone of the system there will be a row in both the concentration and injection matrices. For every unique tracer there will be a column in those matrices. If there are N zones, the volume and air flow matrices will be square matrices of order N and the continuity equation can be rewritten with explicit indices:

$$\sum_{j=1}^{N} \left(V_{ij} C_{jk}(t) + Q_{ij}(t) C_{jk}(t) \right) = F_{ik}(t)$$
 (33.2)

If there are as many tracer species as there are zones, the problem is called *complete* and there will be an exact answer. Incomplete techniques, however, may still contain useful information and will be discussed at the end of the report.

The measured data are the flows and concentrations of each tracer gas in each zone. Specifically, C_{ij} , C_{ij} , and S_{ij} all represent the respective value of the jth tracer gas in the ith zone. The volume matrix can either assumed to be independently determined or derived from the measured data. For most practical purposes the volume matrix can be assumed diagonal with the individual zone volumes as the entries. If, however, there is short circuiting of the injected tracer from one zone to another, it can manifest itself as an off-diagonal volume element, but the sum of each column must be equal to the (effective) physical volume of the zone.

The interpretation of the air flow matrix requires a bit more explanation. The diagonal elements, Q_{ii} , represent the total flow out of that zone to all other zones and should have positive sign. The off-diagonal elements represent the flows between zones; specifically, $-Q_{ij}$ is flow from the *j*th zone to the *i*th zone. Since the flow from the *j*th zone to the *i*th zone can be different from the flow from the *i*th zone to the *j*th zone, this matrix will in general not be symmetric.

The flow matrix explicitly contains information about flows between measured zones and the total flow. If there are flows to zones other than those being measured (e.g., outside), the sum of some rows and columns of the flow matrix will be positive; and system is said to be *open*. If all zones of the building are monitored these flows to "elsewhere" are attributed to air exchange with the outside.

Conceptually, the three types of techniques (integral, regression, and averaging) can be applied for the multizone case as well as the single-zone case. For each approach, there are complications that arise due to the multizone nature of the problem The integral technique is the most straightforward generalization of the three approaches:

$$\mathbf{Q}_{I} = \left(\overline{\mathbf{F}} - \mathbf{V} \cdot \overline{\mathbf{C}}\right) \left(\overline{\mathbf{C}}\right)^{-1} \tag{34}$$

Like the single-zone case, there can be significant bias in the estimate—if the measurement period is too long. Because of the many air flows involved, the requirement is even more stringent in the multizone case. Furthermore, it is not necessary true that the result of this technique ever occurred during the measurement period.

The single-zone regression and averaging techniques rely on the integration of a simple first order differential equation in the concentration to get analytic solutions. The matrix nature of the multizone problem increases the practical difficulty of this significantly to the point where no general solution can be profitably presented. Specific solutions, however, will be presented below The uncertainty analysis associated with multizone ventilation is sufficiently detailed that it will not be repeated here. The reader is referred to the literature. 25

MULTIZONE DECAYS

For a decay technique the continuity equation becomes a set of coupled homogeneous, linear differential equations, whose solutions are linear combinations of decaying exponentials.

$$\lambda(t) \cdot \mathbf{C}(t) = -\dot{\mathbf{C}}(t) \tag{35}$$

Thus as the early work by Sinden²⁶ has demonstrated it can be reduced to an eigenvalue problem. From the eigenvalues and eigenvectors, the infiltration matrix can be recomposed.

Although this technique has been under academic study, ^{27,28} the constraints on the mixing time and data capture rate are so extreme that it may not be generally practicable. Because the coupled equations are solved by decoupling them into a set of higher order simple differential equations, uncertainties in the concentration can lead to very large uncertainties in the eigenvalues and thus in all of the terms of the matrix. Thus, the ability of the analysis to extract the flow rates will be limited as the number of zones increased.

In considering the analysis of their multizone decay data, Enai²⁹ has demonstrated that the optimal analysis period is between the mixing time and the longest time constant of the system. Eigenvalues far from the analysis time will, thus, be poorly determined. In this same report, mixed decay and constant injection strategies were employed, but no improvement in the analysis were found.

Integral Decay

Although it may contain an integral bias, the integral decay method is much more robust for the multizone decay than is an eigenvalue decomposition, and thus is usually superior:

$$\lambda_I = -\overline{\mathbf{C}} \cdot \overline{\mathbf{C}}^{-1} \tag{36}$$

If a unique tracer gas is injected into each zone both matrices on the right-hand side will tend to be well-conditioned.

MULTIZONE STEADY-STATE

The difficulty of the analysis and the high degree of potential uncertainty make transient multizone techniques less attractive. The steady-state techniques, however, do not suffer from the same problems. The condition of steady-state can be summarized as follows:

$$\overline{\left(\mathbf{F}(t)\cdot\mathbf{C}(t)^{-1}\right)} >> \overline{\left(\mathbf{V}\cdot\dot{\mathbf{C}}(t)\cdot\mathbf{C}(t)^{-1}\right)}$$
(37.1)

which is usually equivalent to

$$\overline{\mathbf{C}} \ll \overline{\lambda} \cdot \overline{\mathbf{C}}$$
 (37.2)

and will always be satisfied if

$$\mathbf{\bar{C}} \ll \mathbf{V}^{-1} \cdot \mathbf{\bar{F}}$$
 (37.3)

where the comparisons are done element-by-element and are independent of sign.

Thus the average infiltration is as follows:

$$\overline{\mathbf{Q}} = \overline{\left(\mathbf{F}(t) \cdot \mathbf{C}(t)^{-1}\right)} \tag{38}$$

Multizone Integral

In the multizone long-term integral method is analogous to the single-zone method:

$$\mathbf{Q}_I = \overline{\mathbf{F}} \cdot \left(\overline{\mathbf{C}}\right)^{-1} \tag{39}$$

in that it uses the average injections and concentrations and it has at least the same potential for bias. Thus, it might be more accurate to use a multizone pulse technique 15, than to use a long-term multizone concentration integral technique). 30

MULTITRACER MEASUREMENT SYSTEM

An intermediate technique currently in use by the author is called the multi-tracer measurement system (MTMS),³¹ in which the concentration of tracer gases are held in reasonably narrow ranges and the analysis is an integral analysis over a short time period (i.e. $T\approx 1/2h$):

$$\mathbf{Q} = \left(\overline{\mathbf{F}} - \mathbf{V} \cdot \overline{\mathbf{C}}\right) \left(\overline{\mathbf{C}}\right)^{-1} \tag{40}$$

The ostensibly large uncertainty reduced by using a high data rate and physical limits on the infiltration values. 32

MULTIZONE CONSTANT CONCENTRATION

The MTMS technique does not control the concentrations of all gases in all zones and, hence, could not be a true constant concentration system. There are currently no systems that do control all gases in all zones, and unless a reasonable method exists for effectively injecting *negative* amounts of tracer gas, there will be no *complete* multizone constant concentration system practical.

Although complete multizone constant concentration systems are not in use, there are single-gas multizone ones in operation. Such single-gas systems cannot measure any of the zone-to-zone air flows, but they attempt to measure the infiltration from outside to each zone. Without some knowledge of the interzonal air flows, however, it is impossible to estimate the uncertainty of these infiltrations. However, with reasonable control and low interzonal air flows, the precision can be acceptable.

Other Incomplete Approaches

If there are an insufficient number of tracer gases available for a complete analysis, there are various other approaches for determining the infiltration matrix (e.g. see 10th AIVC Conference proceedings). Almost all of these approaches use multiple sets of data taken at different times (e.g. using constant emission or multiple decay tests) Thus the columns in the concentration and flow matrices are not different tracers taken simultaneously, but some tracer taken at different times. Various control strategies such as charge-up, decay, rotating injection, etc. can be used in these approaches. For example, a general, regression, pulse technique is being used, to measure both the air flow rates and effective volumes using a single-tracer gas. All of these approaches must assume that the air flows do not change over the time of the entire measurements; regardless of the analysis approach used the measurement time will be significantly longer than if multiple tracers were used.

Another method of extracting useful information in an incomplete system is to use a second gas an an erstwhile single-gas constant concentration system.³⁸ In such a system one of the interzonal flows (and all of the infiltrations from outside) can be measured at a time.

DISCUSSION OF MULTIZONE METHODS

Single-gas constant-concentration systems are increasing in use and provide a bridge between single-zone techniques and a complete multizone technique. This technique is ideal for zones that are not well-connected and for which the outside air infiltration is the quantity of interest. As indoor air quality concerns increase,

multigas systems may become more popular.

The only multigas technique in common usage is the long-term integral technique using PFTs. However, care must be taken in interpreting these numbers as averages under varying conditions.

Incomplete techniques that require sequential measurements (e.g. repeated single-gas decays) like the long-term concentration average method suffer from sensitivity to variations in the air flows. These techniques should not be used if the air flow changes significantly (with respect to the uncertainty of the estimates) over the course of the complete experiment.

The short-term integral techniques (i.e. decay and pulse) show great promise for relatively simple determination of multizone air flows and may be the technique of choice for spot measurements.

Real-time systems such as MTMS offer a complete solution, although they are much more complex and cumbersome. Constant injection techniques with appropriate (i.e. inverse) concentration averaging is also a technique worth considering for continuous operation.

There are far fewer multizone measurement techniques in use compared with the single-zone ones, so it is far too early to do a complete summary, but there will undoubtedly be more to summarize in ten years.

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NOMENCLATURE

C	Instantaneous tracer gas concentration [kg/m ³]
\mathbf{C}	Multizone tracer gas concentration matrix [kg/m ³]
C_T	Target concentration of tracer [kg/m ³]
F_{-}	Instantaneous injection of tracer gas [kg/h]
\mathbf{F}	Multizone tracer injection matrix [kg/h]
λ	Air Change rate [h ⁻¹]
λ	Air change rate matrix [h ⁻¹]
N	Number of zones [-]
$oldsymbol{Q}^{+}$	Ventilation $[m^3/h]$
Q	Ventilation matrix [m ³ /h]
t_{\perp}	Time [h]
T	Length of measurement period [h]
V	Volume [m ³]
\mathbf{V}	Zone volume matrix [m ³]
ρ	Density of air in a zone [kg/m ³]
δX	Uncertainty in quantity X (e.g. concentration, injection, etc.)
$ar{X}$	Time average of quantity X over measurement period T
X	Time rate of change of quantity X
X_R	Indicates X is a parameter found from a regression.
X_I	Indicates X is found using an integral technique.

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