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An analytical and numerical study of a vertically discretized multi-paddle wavemaker for generating free surface and internal waves

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ABSTRACT

A vertically discretized multi-paddle wavemaker is proposed that generates various wave types without the depth-invariant flow condition limitation associated with conventional piston type wavemakers. A first-order mathematical formulation for the paddle stroke is derived and used to selectively generate free surface waves similar to either piston type or flap type wavemakers, depending on the number of utilized paddles. The mathematical formulation is further extended to a multi-phase internal wave case. Paddle discretization performance is evaluated using OpenFOAM, a numerical model that solves the Navier-Stokes equations for two (or more) immiscible and incompressible fluids. Dynamic meshing capability is incorporated in the model. Model results show **multiple** discrete paddles efficiently generate deep water waves under laminar flow conditions. In intermediate wave conditions, the multi-paddle discretization is less advantageous. Internal waves and resulting free surface waves among air, oil, and water phases are simulated using only two paddles and imposing a fast mode. Model results agree with flow characteristics for both internal and locked free surface waves.

1. Introduction

Wave generation theory has been actively studied since early 1950s. Biésel and Suquet (1951) presented a linear theory for the piston and flap type wavemakers succeeding the classical wavemaker history of Havelock (1929). Ursell et al. (1960) experimentally verified their piston type solution while investigating energy dissipation and nonlinear effects. Madsen (1970) demonstrated various effects that arise when the piston type wavemaker is utilized based on the linear wave theory (e.g., leakage around the wavemaker), excluding the re-reflection of waves. These studies have provided a reasonable performance for given wave conditions, however, their usability is limited. For example, a piston type wavemaker does not accurately represent the incipient velocity profile near the wave paddle under deep water wave conditions ($kh \gg 1$) due to the depth-invariant motion. Similarly, a flap type wavemaker is unable to force a uniform velocity profile when generating shallow water waves ($kh \ll 1$). Although wavemaker induced velocity profile impacts decrease as the waves propagate approximately two wavelengths ($2L$), in practice this requires either narrow observations zones or long flumes to accurately represent both the waveform and its hydrodynamics.

Extensive work has focused on deriving analytical wavemaker solutions (e.g. Flick and Guza, 1980; Schäffer, 1996; Aknin and Spinneken, 2017; Khait and Shemer, 2019; Veloso Lima et al., 2019). Flick and Guza (1980) developed the solution applicable to the piston and flap type wavemakers by dividing the solution into harmonic free wave and harmonic Stokes (forced) wave components to exclude spurious errors originated from sub- and super-harmonic wave components. In the solution, only the first-order trapped modes are considered since the second-order harmonic modes quickly disappear. Schäffer (1996) further derived the second-order solution without assumptions applied to Flick and Guza (1980), such as the first-order trapped modes, shallow water depth, and narrow banded wave spectra. Compared to experimental data, however, notable deviations in amplitude and phase were observed (Aknin and Spinneken, 2017). Recently, Khait and Shemer (2019) presented the nonlinear formulation using higher-order spectral method

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(e.g. Houtani et al., 2018) for improved predictions. In addition, the first-order analytical formulation for the generation of N-waves using a piston type wavemaker was derived and successfully verified in the physical experiments (Veloso Lima et al., 2019).

A variety of different wavemaker shapes modified from the ‘classical’ piston or flap type forms have been proposed to improve performance (e.g. Taniguchi and Kasai, 1972; Hyun, 1976; Chwang, 1983; Chwang and Li, 1983). Taniguchi and Kasai (1972) and Hyun (1976) introduced a modified flap type wavemaker with their lower ends huddled in the wall that minimizes disturbance on the back side of wave tank. Chwang (1983) and Chwang and Li (1983) studied porous piston type wavemakers to reduce wave paddle pressure. Hybrid type wavemakers were also developed to eliminate the second harmonic waves induced by the aforementioned dissimilarity between the ‘classical’ wavemaker motions and resulting velocity profiles (Fontanet, 1961; Flick and Guza, 1980; Schäffer, 1996; Khait and Shemer, 2019). This wavemaker engages both piston and flap types by changing the bottom position of the paddle in the ‘classical’ flap type wavemaker and functions as a piston type if paddle is infinitely long. To generate extremely long waves in the laboratory, various types of new wavemaker system were developed (Raichlen and Lee, 1978; Wu, 1988; Goseberg et al., 2013; Lu et al., 2017; McGovern et al., 2018). Raichlen and Lee (1978) and Wu (1988) proposed an inclined-plate wavemaker to study the characteristics of wave generation and propagation by the tectonic sea-floor motion on a planar slope. A pump-driven wave generation technique was suggested for efficient generation of extremely long waves in the laboratory (Goseberg et al., 2013). Lu et al. (2017) proposed a bottom-tilting wavemaker that can generate waves significantly longer than a solitary wave to study tsunami-induced long waves. An experimental study further showed that a pneumatic long-wave generator can produce scaled tsunami waveforms in a relatively short flume (McGovern et al., 2018).

Internal waves and corresponding free surface motions are commonly observed in estuaries and river mouths in a layered-fluid system due to suspended muddy sediments or fluid density gradients. Maxeiner and Dalrymple (2011) studied the mixing of mud and resonant wave interactions driven by clear fluid waves utilizing a piston type wavemaker where a layer of mud was recessed 3.5 m away from the wavemaker. Hill and Foda (1998) investigated the generation of internal gravity waves between oil and fresh water under progressive surface waves using a flap type wavemaker, thus the flow condition is strictly limited to the barotropic mode. To independently generate surface and internal waves, non-classical wavemaker systems were also developed. Lewis et al. (1974) generated the internal waves between freon-kreosene and water using a specially designed u-shaped piston type wavemaker. A vertically gated wavemaker has been another option to generate an internal solitary wave in a two-fluid system (e.g. Kao et al., 1985; Michallet and Barthelémy, 1998). Umeyama (2002) proposed two different wavemaker systems, i.e., an oil-pressure type wave generating pump and a horizontal flap type board to generate short and long internal waves, respectively. Most recently, Ko and Lynett (2014) has used a hydraulic-control wavemaker where separated jets force the flow in two different fluid layers.

For the practical perspective, a unified wavemaker design solution for both free surface and internal waves generation is desirable for a wide range of applications. A vertically discretized multi-paddle wavemaker proposed by Naito (2006) and Keaney et al. (2014) is ideal, since it is capable of reproducing the controlled flow hydrodynamics for multiple fluid layers while retaining velocity profiles similarity. In this study, a complete solution is derived for the vertically discretized multi-paddle wavemaker that can selectively generate both free surface and internal waves. Novel analytical solutions based on the linear wave theory and boundary depth conditions are developed and presented for paddle stroke length. The analytical solutions and wave performance are validated for a selected number of paddle (N) for free surface and internal waves using numerical modeling.

2. Mathematical formulation

The mathematical formulation is fundamentally based on the linear potential theory (e.g. Ursell et al., 1960; Dean and Dalrymple, 1991). **Thus, the obtained solution is only applicable to waves with small amplitudes.** Paddle strokes for each paddle are analytically developed for both free surface and internal waves generation.

2.1. Free surface waves

The vertically discretized multi-paddles are defined in the wavemaker system as depicted in Figure 1. The stroke of each paddle $S(z)$ is piecewise-defined as

$$S(z) = S_i \quad \text{for} \quad -h_i \leq z < -h_{i-1}, \quad (1)$$

Figure 1: A schematic diagram of the vertically discretized multi-paddle wavemaker system.

where the subscript $i (= 1 \sim N)$ indicates each layer corresponding to the paddle. Over this system, the boundary value problem can be posed assuming irrotational flow for incompressible fluids. The governing equations can be expressed as follows using the velocity potential (Φ) (Dean and Dalrymple, 1991).

$$\nabla^2 \Phi = 0, \quad (2)$$

where Φ satisfies $(u, w) = (\partial\Phi/\partial x, \partial\Phi/\partial z)$, in which u and w are the horizontal and vertical velocities of the fluid particle, respectively. Linearized free surface and bottom boundary conditions are defined as follows (Dean and Dalrymple, 1991):

$$\eta = \frac{1}{g} \frac{\partial\Phi}{\partial t} \quad \text{at } z = 0, \quad (3)$$

$$\frac{\partial\Phi}{\partial z} = \frac{\partial\eta}{\partial t} \quad \text{at } z = 0, \quad (4)$$

$$\frac{\partial\Phi}{\partial z} = 0 \quad \text{at } z = -h_N, \quad (5)$$

where η is the surface elevation and g is gravitational acceleration.

The kinematic lateral boundary condition initiating paddle and therefore, water particle motion is given as:

$$u(x, z, t) = \frac{S(z)}{2} \sigma \cos \sigma t \quad \text{at } x = 0, \quad (6)$$

where $\sigma = 2\pi/T$ is the wave angular frequency. It should be noted that equation 6 is specified at the time-mean position of each paddle, $x = 0$, not at the instantaneous location of wave paddle, $x = \frac{S(z)}{2} \sin \sigma t$. Since the velocity potential obtained from the Laplace equation (equation 2) is composed of both progressive and non-progressive (i.e., evanescent) components (see (Dean and Dalrymple, 1991)), $S(z)$ also contains the progressive, $S_p(z)$, and evanescent, $S_e(z)$, components, and is rearranged as:

$$S(z) = S_p(z) + S_e(z) = \frac{H \cosh k(h_N + z)}{\sinh kh_N} + S_e(z), \quad (7)$$

Figure 2: Definition sketch of two-layer fluid system having a multi-paddle wavemaker.

where H and $k = 2\pi/L$ denote wave height and wave number, respectively. To eliminate the evanescent mode in the solution, the weight function, $f(z)$, is further adopted utilizing the orthogonality of solution as used in Dean and Dalrymple (1991). The vertical integration of equation 7 from $-h_N$ to 0 with $f(z) = \cosh k(h_N + z)$ produces:

$$\begin{aligned} \int_{-h_N}^0 S(z)f(z)dz &= \sum_{i=1}^N S_i [\sinh k(h_N - h_{i-1}) - \sinh k(h_N - h_i)] \\ &= \frac{2kh_N + \sinh 2kh_N}{4 \sinh kh_N} H, \end{aligned} \quad (8)$$

where the summation is obtained from the depth integration of vertical structure of waves. Since S_i inherently follows the profile of $S(z)$ in equation 7, the ratio between two discrete strokes can be evaluated for constant paddle span (i.e., $h_i - h_{i-1} = \text{const.}$) as follows:

$$\frac{S_i}{S_1} = \frac{\sinh k(h_N - h_{i-1}) - \sinh k(h_N - h_i)}{\sinh k(h_N - h_0) - \sinh k(h_N - h_1)}. \quad (9)$$

The stroke of each paddle is now obtained by combining equations 8 and 9:

$$S_i = \frac{\sinh k(h_N - h_{i-1}) - \sinh k(h_N - h_i)}{\sum_{n=1}^N [\sinh k(h_N - h_{n-1}) - \sinh k(h_N - h_n)]^2} \frac{2kh_N + \sinh 2kh_N}{4 \sinh kh_N} H. \quad (10)$$

This solution includes two reference cases, i.e., flap type and piston type solutions (e.g. Ursell et al., 1960). When $N \rightarrow \infty$ and the strokes increase linearly, equation 10 recovers a well-known flap type solution while the piston type solution is obtained for $N = 1$. Moreover, the solution can be further simplified depending on the wave dispersion property (i.e., kh_N). For example, if $kh_N \rightarrow 0$ and $N = 1$, equation 10 converges to the simplified solution derived in Galvin.

2.2. Internal waves

The vertically discretized multi-paddle wavemaker theory is uniquely applicable for any fluid having a depth-varying flow structure owing to the flexibility in the vertical arrangement. Internal wave generation between fluids of different densities is demonstrated and the analytical solutions for paddle motion are derived.

A two-layer fluid system is shown in Figure 2. The water depth, density, and velocity potential in the upper layer are denoted as h_u , ρ_u , Φ_u , respectively, while those for the lower layer are h_l , ρ_l , Φ_l , respectively. Assuming two fluids are stably stratified, the density ratio $\rho_r (= \rho_u/\rho_l)$ is less than 1. The number of paddles installed at each layer is set to be N and M for the upper and lower layer, respectively. Wave motions at the free surface and the fluid interface are denoted as η_s and η_i , respectively. It should be noted that both motions share the identical phase function because the free surface wave is locked to the internal motion (e.g. Son and Lynett, 2014b).

Figure 3: The ratio between H_i and H_s for (a) fast mode and (b) slow mode as a function of relative water depth, where $\rho_r = 0.881$.

The internal and free surface wave motions under the two-layer fluid system have been analytically obtained based on linear potential wave theory in Liu et al. (2013), which presented the progressive component of velocity potential at each layer as follows:

$$\Phi_u = (A \cosh kz + B \sinh kz) \sin(kx - \sigma t) \quad \text{for } 0 \leq z < h_u, \quad (11)$$

$$\Phi_l = C \cosh k(z + h_l) \sin(kx - \sigma t) \quad \text{for } -h_l \leq z < 0, \quad (12)$$

where coefficients included are defined as:

$$A = \frac{H_i}{2\sigma} \left[\left(1 - \frac{1}{\rho_r} \right) g + \frac{\sigma^2}{\rho_r k \tanh(kh_l)} \right], \quad (13)$$

$$B = \frac{\sigma H_i}{2k}, \quad (14)$$

$$C = \frac{\sigma H_i}{2k \sinh(kh_l)}, \quad (15)$$

where H_i is the internal wave height. Of note, the dispersion relationship of the internal wave is drawn out to be:

$$\sigma^4 [\coth kh_u \coth kh_l + \rho_r] - \sigma^2 gk [\coth kh_u + \coth kh_l] + (1 - \rho_r)gk^2 = 0. \quad (16)$$

In particular, two distinct modes exist, fast (k^+) and slow (k^-) mode (see Figure 3). The relation between free surface (H_s) and internal wave heights is expressed as:

$$H_i = \frac{\sigma^2}{\sigma^2 \cosh kh_u - gk \sinh kh_u} H_s. \quad (17)$$

Since the velocity profile at each layer is of a different function as described in equations 11 and 12, the derivation for each layer proceeds separately. The upper layer stroke, S^u , is presented first. Similar to equation 7, the following condition is derived at the lateral boundary of the upper layer:

$$S^u(z) = S_p^u(z) + S_e^u(z) = \frac{2k}{\sigma} (A \cosh kz + B \sinh kz) + S_e^u(z). \quad (18)$$

Integration over the depth of upper layer with multiplying the weight function, $f(z) = (A \cosh kz + B \sinh kz)$, produces,

$$\begin{aligned} \int_0^{h_u} S^u(z) f(z) dz &= \sum_{i=1}^N S_i^u \left[\frac{A}{k} (\sinh kh_{u,i} - \sinh kh_{u,i-1}) + \frac{B}{k} (\cosh kh_{u,i} - \cosh kh_{u,i-1}) \right] \\ &= A^2 \frac{k}{\sigma} \left(\frac{\sinh 2kh_u}{2k} + h_u \right) + AB \frac{1}{\sigma} (\cosh 2kh_u - 1) + B^2 \frac{k}{\sigma} \left(\frac{\sinh 2kh_u}{2k} - h_u \right), \end{aligned} \quad (19)$$

where S_i^u is the stroke of i -th paddle in the upper layer. Following the same steps applied to the free surface wave generation (see equation 9), S_i^u can be obtained for a constant paddle span (i.e., $h_{u,i} - h_{u,i-1} = \text{const.}$) as follows:

$$S_i^u = \frac{k}{2\sigma} \left[A^2 (\sinh 2kh_u + 2kh_u) + 2AB(\cosh 2kh_u - 1) + B^2 (\sinh 2kh_u - 2kh_u) \right] \cdot \frac{[A(\sinh kh_{u,i} - \sinh kh_{u,i-1}) + B(\cosh kh_{u,i} - \cosh kh_{u,i-1})]}{\sum_{n=1}^N [A(\sinh kh_{u,n} - \sinh kh_{u,n-1}) + B(\cosh kh_{u,n} - \cosh kh_{u,n-1})]^2}. \quad (20)$$

In the same manner, the stroke at j -th paddle in the lower layer S_j^l can be also obtained as,

$$S_j^l = \frac{k}{2\sigma} (\sinh 2kh_l + 2kh_l) \frac{[(\sinh k(h_l - h_{l,j-1}) - \sinh k(h_l - h_{l,j-1}))]}{\sum_{m=1}^M [(\sinh k(h_l - h_{l,m-1}) - \sinh k(h_l - h_{l,m-1}))]^2}. \quad (21)$$

In the case of a single paddle for each fluid system (i.e., $N = M = 1$), the strokes of upper and lower paddles are simplified as:

$$S^u = \frac{k}{2\sigma} \frac{A^2 k (\sinh 2kh_u + 2kh_u) + 2AB(\cosh 2kh_u - 1) + B^2 k (\sinh 2kh_u - 2kh_u)}{A \sinh kh_u + B(\cosh kh_u - 1)}, \quad (22)$$

$$S^l = \frac{k}{2\sigma} \frac{\sinh 2kh_l + 2kh_l}{\sinh kh_l}, \quad (23)$$

in which, further approximation of $kh \rightarrow 0$ can be applied for the generation of long internal waves.

3. Numerical model

Two depth-resolving numerical models for multi-phase flows are utilized to validate the mathematical formulation derived in this study (section 2). Based on the open-source OpenFOAM CFD toolbox, olaDyMFlow (e.g. Higuera et al., 2013, 2015) and multiphaseInterDyMFOam (e.g. Vakhrushev et al., 2014) solvers are adopted. The capabilities of olaDyMFlow and multiphaseInterDyMFOam include the interface tracking scheme while minimizing the numerical diffusion at the interfaces (Berberović et al., 2009; Klostermann et al., 2012), automatic mesh motion and data mapping to mimic the wavemaker motions (Jasak and Tuković, 2010), and the wave absorption technique at the outlet boundary for the free surface waves (Higuera et al., 2013). The multiphaseInterDyMFOam is capable of resolving more than two phases, hence it is used for the internal wave case (see section 5).

3.1. Governing equations

The mass conservation and momentum equations are based on volume of fluid method (VOF). In a multi-phase flow system, mass conservation equations for incompressible fluids can be written as (e.g. Drew, 1983):

$$\frac{\partial \phi^k}{\partial t} + \frac{\partial \phi^k u_i^k}{\partial x_i} = 0, \quad (24)$$

where $i = 1, 2$ for two-dimensional (2D) flow, i.e., streamwise ($i = 1$) and vertical ($i = 2$) components. The variables u^k denotes the velocity of each phase where the superscript k denotes air (a), oil (o), water (w) phases, respectively. The summation of volumetric concentration (ϕ^k) leads to the global mass conservation equation as $\phi^a + \phi^o + \phi^w = 1$. The oil phase is considered only in internal wave cases (section 5), and only air and water phases are utilized for the rest of the paper (i.e., $\phi^o = 0$). The each phase is considered to be immiscible with other phases and combined as a single mixture phase (superscript m). The mass conservation equation for the mixture phase is expressed as

$$\frac{\partial u_i^m}{\partial x_i} = 0, \quad (25)$$

in which, $u_i^m = \sum_k \phi^k u_i^k$.

The momentum equations for the mixture phase are written as

$$\frac{\partial \rho^m u_i^m}{\partial t} + \frac{\partial \rho^m u_i^m u_j^m}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu^m \left(\frac{\partial u_i^m}{\partial x_j} + \frac{\partial u_j^m}{\partial x_i} \right) \right] + \rho^m g \delta_{i2} + T_i, \quad (26)$$

in which, ρ^m is the mixture density, satisfying $\rho^m = \sum_k \rho^k \phi^k$. Here, we specify $\rho^a = 1 \text{ kg/m}^3$, $\rho^o = 881 \text{ kg/m}^3$, and $\rho^w = 1000 \text{ kg/m}^3$. Likewise, the dynamic viscosity for the mixture phase, μ^m , is defined as $\mu^m = \sum_k \rho^k \nu^k \phi^k$ where $\nu^a = 1.48 \times 10^{-5} \text{ m}^2/\text{s}$, $\nu^o = 2.32 \times 10^{-5} \text{ m}^2/\text{s}$, and $\nu^w = 10^{-6} \text{ m}^2/\text{s}$. The variable p represents fluid pressure. **In this study, laminar flow is considered and no turbulence closure is incorporated in the model.** In the momentum equation, therefore, no turbulence modeling is applied. The last term on the right-hand-side (RHS) of equation 26 represents interfacial tension, obtained as (Vakhrushev et al., 2014):

$$T_i = \sum_{j,k} \left[\sigma^{jk} \gamma^{jk} \left(\phi^k \frac{\partial \phi^j}{\partial x_i} - \phi^j \frac{\partial \phi^k}{\partial x_i} \right) \right], \quad (27)$$

with phase pairs $(j, k = ao, aw, ow)$. The interfacial tension coefficients for 20°C are given as $\sigma^{ao} = 0.032 \text{ kg/s}^2$, $\sigma^{aw} = 0.072 \text{ kg/s}^2$, $\sigma^{ow} = 0.022 \text{ kg/s}^2$ for at air-oil (ao), air-water (aw), and oil-water (ow) interfaces. It should be noted that σ^{ow} is lower than σ^{ao} and σ^{aw} due to the different molecular structures of water and oil reducing attractive forces (Hollebone, 2017). The variable, γ^k , denotes the curvatures for each interface. In general, T_i has a minor effect for mildly varying interfaces (e.g. Kim et al., 2017) where $H/L < 0.03$ in this study.

3.2. Numerical implementations

3.2.1. Interface compression

The interfaces among the phases are tracked by the VOF method. The continuity equation is solved using an interface compression method (Berberović et al., 2009; Klostermann et al., 2012) to minimize diffusion at the interfaces. The equation 25 can be re-written as (Kulju et al., 2015):

$$\frac{\partial \phi^k}{\partial t} + \frac{\partial \phi^k u_i^m}{\partial x_i} + \frac{\partial (1 - \phi^k) \phi^k u_i^{rk}}{\partial x_i} = 0, \quad (28)$$

in which, u_i^{rk} is the relative velocity between phases. The cell face flux of u_i^{rk} , denoted as $()_f$, is calculated as

$$(u^{rk})_f = \min \left(c_\gamma \cdot \frac{(u^m)_f}{\gamma^k}, \frac{(u^m)_f}{\gamma^k_{\max}} \right) \cdot \frac{\nabla \gamma^k}{\nabla \gamma^k + b}, \quad (29)$$

where $c_\gamma = 1$ is the weight of compression flux. The parameter b is a small number preventing a zero denominator, set as $10^{-8} / \sqrt[3]{V}$, in which V is the averaged cell volume. To solve the equations 28 and 29 while assuring boundedness, a multi-dimensional universal limiter for explicit solution (MULES) method is adopted (Rudman, 1997).

3.2.2. Dynamic mesh

Dynamic meshing is utilized to directly simulate wavemaker motion (Higuera et al., 2013; Jasak and Tuković, 2010). From section 2, the time series of wavemaker velocity, U_i^P , is obtained for each boundary of the paddle (see equation 6), in which $i = 1$ (top), $2, \dots, N$ (bottom). Then, the streamwise paddle location is calculated as $X_i^P = \int_0^t U_i^P dt$ where the instantaneous paddle location for the paddle motion is derived as $X_i^P = \frac{S_i}{2} \sin(\sigma t)$. Following the newly defined boundaries at each time step, the numerical domain is deformed with an automatically re-constructed mesh. The deformation velocity of the mesh points, U_i , satisfies

$$\nabla \cdot (d \nabla U_i) = 0 \quad (30)$$

where d is the diffusivity distributing the boundary motion to the mesh, set to be uniform in this study. In these numerical experiments, the paddles move only in x direction, hence the vertical grid points stay static. For $N > 1$, X_i^P may differ for each i , particularly for the deep water wave condition. Near the wavemaker boundary, therefore, discrepancy of X_i^P between paddles leads to a sharp transition at the edges. In this study, two connected cells adjacent

Case ID	phases	kh	N	h [m]	T [s]	L [m]	H [m]	H/L
<i>Fkh8N1</i>	air, water	8.05	1	0.5	0.5	0.39	0.01	0.026
<i>Fkh8N2</i>	air, water	8.05	2	0.5	0.5	0.39	0.01	0.026
<i>Fkh8N4</i>	air, water	8.05	4	0.5	0.5	0.39	0.01	0.026
<i>Fkh8N20</i>	air, water	8.05	20	0.5	0.5	0.39	0.01	0.026
<i>Fkh6N1</i>	air, water	6.19	1	0.5	0.57	0.51	0.01	0.02
<i>Fkh6N20</i>	air, water	6.19	20	0.5	0.57	0.51	0.01	0.02
<i>Fkh3N1</i>	air, water	3.06	1	0.3	0.63	0.62	0.015	0.024
<i>Fkh3N20</i>	air, water	3.06	20	0.3	0.63	0.62	0.015	0.024
<i>Fkh1N1</i>	air, water	1.06	1	0.3	1.2	1.77	0.02	0.011
<i>Fkh1N2</i>	air, water	1.06	2	0.3	1.2	1.77	0.02	0.011
<i>Fkh1N4</i>	air, water	1.06	4	0.3	1.2	1.77	0.02	0.011
<i>Fkh1N20</i>	air, water	1.06	20	0.3	1.2	1.77	0.02	0.011

Table 1

A summary of free surface wave cases with selected wave conditions

to the another paddle are linearly deformed while preserving the mass. To maintain the mesh quality during the simulation, $X_d = X_i^P - X_{i-1}^P$ is limited to a grid width in x direction.

The paddle and bottom boundaries are specified as a no-slip for u^m components parallel to the wall and no-flux boundary for wall-normal u^m components and scalar quantities (i.e., ϕ^k and p). The top boundary is specified as an open boundary controlling the inflow flux based on p and the two spanwise lateral boundaries are treated as empty condition in OpenFOAM.

3.2.3. Numerical schemes

OpenFOAM is based on the Finite Volume Method (FVM), in which FVM has an advantage in evaluating partial differential equations on a complex meshed geometry (e.g. McDonald, 1971). Gauss's theorem is applied to convert the advection and diffusion terms into surface integrals for each cell. An upwind scheme is applied to the advection term. A central difference scheme with a non-orthogonal correction is used for the diffusion term (Jasak, 1996). An explicit Euler scheme is adopted for time integration. The model time step is controlled by Courant-Friedrichs-Lewy condition:

$$C_0 = \frac{|u^m| \Delta t}{\Delta} \leq C_{max}, \quad (31)$$

where C_0 is the Courant number, Δ is the characteristic length of grid size, and Δt is an automatically adjusted time step. $C_{max} = 0.2$ is used to balance numerical stability and computational cost. The Pressure Implicit with Splitting of Operator (PISO) algorithm (e.g. Rusche, 2002) is then applied to solve u^m where u^m and p are coupled based on Rhie and Chow method (Rhie and Chow, 1983)

4. Free surface wave

Free surface waves (i.e., $\phi^o = 0$) are simulated using OlaDyMFlow. In addition to the dynamic mesh capability, a 2D active wave absorption technique is available in olaDyMFlow. Outlet boundary velocity is adjusted to minimize reflected waves using a digital filter (Higuera et al., 2013; Schäffer and Klopman, 2000). It should be reiterated here that the 2D active wave absorption method works better for shallow water waves, and a weak wave reflection has been reported for other wave conditions (Higuera et al., 2013). Hence, the model results are analyzed prior to reflected waves affect the domain. Free surface waves test conditions range from deep water ($kh \gg 1$) to intermediate depth ($kh \sim O(1)$) conditions. Each simulation case is assigned an ID. For the free surface waves, F is prepended to the case ID, and kh values are indicated in the middle, and the number of utilized paddles follows. The number of paddles is selected as $N = 1, 2, 4, 20$ for the comparison.

4.1. Deep water condition

Deep water waves are tested in the initial still water depth (h_N) of 0.5 m. A monochromatic wave train with the wave period (T) of 0.5 s and length (L) of 0.39 m is generated using the vertically discretized multi-paddle wavemakers

Figure 4: Time series of modelled X_i^P under deep water wave condition at (a) top paddle ($i = 1$), (b) middle paddle ($i = 10$), (c) bottom paddle ($i = 20$), and (d) $S_i/2$ for $Fkh8N1$ (blue solid curves), $Fkh8N2$ (red dashed curves), $Fkh8N4$ (green dash-dotted curves), $Fkh8N20$ (orange solid curves) cases, respectively.

with $N = 1, 2, 4, 20$, where $kh = 8.05$ in this case. The 2D numerical domain of 8 m long ($> 20L$) and 0.625 m deep (i.e., $1.25h_N$) was constructed with uniform grids of 2.5 mm width (dx) and 1.0 mm height (dz). A total number of two million computational grid points were used for the simulations. The wave height is selected as 10 mm, hence wave steepness is set to be $H/L = 0.026$ and 10 cells are used to resolve free surface fluctuations.

The paddle stroke, X_i^P (equation 10), is calculated as shown in Figure 4. A ramp up period of $3T$ is applied during the initial paddle motion using a hyperbolic tangent function to improve numerical stability. The initial location of the wavemaker is set to be $X_i^P(t = 0)$. The horizontal displacement of top paddle is similar for all cases (Figure 4a). However, the paddle movement of $Fkh8N20$ rapidly approaches to 0 towards the bottom. The total water volume in unit width pushed by the wavemaker are $2.5 \times 10^{-3} \text{ m}^3/\text{m}$, $1.3 \times 10^{-3} \text{ m}^3/\text{m}$, $8.2 \times 10^{-4} \text{ m}^3/\text{m}$, $6.3 \times 10^{-4} \text{ m}^3/\text{m}$, for $N = 1, 2, 4, 20$, respectively (Figure 4d). Thus, $Fkh8N20$ generated deep water waves efficiently, utilizing much smaller water volumes compared to other cases. (25.2% of $Fkh8N1$, 48.6% of $Fkh8N2$, and 77.0% of $Fkh8N4$).

Modelled surface elevations, η_m , are compared with the prescribed wave amplitude (height), a_p (H_p), at different cross-shore locations (Figure 5). In the numerical simulation results, the instantaneous water depth was calculated using the vertically-integrated volumetric concentration of ϕ^w , and the free surface elevation is then calculated as $\eta_m(x, t) = \int \phi^w(x, z, t) dz - h_N$. Near the wavemaker, η_m is deviated with reference to a_p , especially with less N (Figure 5a). For example, in the case with $N = 1$, enhanced η_m is observed close to the wave maker ($x/L = 0/2$). Such a discrepancy is most likely attributed to evanescent components existing near the wavemaker when the wavemaker motion is dissimilar to the target wave orbital motion (Dean and Dalrymple, 1991). Figure 5e shows the ratio of modelled wave height (H_m) to H_p where H_m was obtained by averaging over last five waves in the quasi-steady state (i.e., 16th to 20th waves). Maximum error in H_m/H_p was shown to be around $x/L = 0.2$ where 50.0% for $Fkh8N1$, 22.4% for $Fkh8N2$, 6.4% for $Fkh8N4$, 0.6% for $Fkh8N20$. The discrepancy decreases with increasing N . Strictly speaking, $N = \infty$ is needed to optimize the simulation under deep water wave conditions. As expected, sufficiently

Figure 5: Time series of η_m with respect to a_p under deep water condition at (a) $x/L = 0.2$, (b) $x/L = 0.7$, (c) $x/L = 2$, (d) $x/L = 5$, and (e) H_m/H_p as a function of x/L for *Fkh8N1* (blue solid curves), *Fkh8N2* (red dashed curves), *Fkh8N4* (green dash-dotted curves), *Fkh8N20* (orange solid curves) cases, respectively.

away from the wavemaker ($x/L > 2$), η_m is well captured regardless of number of paddles used (Figures 5c and 5d).

In every case, η_m is slightly underpredicted, roughly up to 5% even in the quasi-homogeneous region ($x/L > 2$). This is due to the wave nonlinearity, proven with a smaller H_p test case inducing less error (not shown here). The error is reduced for longer wave length cases (see section 4.2). Furthermore, dynamic mesh-induced discretization errors may play a role in this study (e.g. Dunbar et al., 2015; Bruinsma et al., 2018).

Waves generated by wavemakers may contain evanescent modes, particularly when wavemaker motion deviates from the wave orbital velocity profile. Usually, only the progressive wave component propagates and evanescent modes

remain close to the wavemaker. The wave energy spectra $E_{\eta\eta}(f)$ are analyzed to investigate the impact of N on model results. Using the Fourier transform of η_m auto-correlation function, $R_{\eta\eta}(\zeta)$, $E_{\eta\eta}(f)$ is calculated as

$$E_{\eta\eta}(f) = 2 \int_{-\infty}^{\infty} R_{\eta\eta}(\zeta) \exp(-i2\pi f \zeta) d\zeta, \quad (32)$$

where $R_{\eta\eta}(\zeta) = \int_0^{T^* - |\zeta|} \eta_m(t) \eta_m(t + \zeta) dt$, in which T^* was set to be equal to the simulation duration, i.e. $T^* = 20T$ s. The inverse of equation 32 can be also written as:

$$R_{\eta\eta}(\zeta) = \frac{1}{2} \int_{-\infty}^{\infty} E_{\eta\eta}(f) \exp(i2\pi f \zeta) df. \quad (33)$$

Major oscillation modes at each frequency can be identified through the wave energy spectrum (Figure 6). Generally, all cases have a single peak at $f_p = 1/T$ where the magnitude of the peak is $\sim O(10^4)$ larger than that of the secondary super-harmonic mode at $2f_p$. The observed super-harmonic components are known to originate not only from the second-order interaction between the first-order bichromatic waves but also from the self-interaction of a single monochromatic wave (Kennedy et al., 2001; Son and Lynett, 2014a; Wunsch, 2017). These tendencies are consistent for all cases at every cross-shore location. Major discrepancies are low frequency, close to the wavemaker, where *Fkh8N1* has two or three times stronger $E_{\eta\eta}$ at $x/L = 0.2$, compared to other cases (Figure 6a). At $x/L = 0.7$ where the minimum H_m/H_p obtained, the tendency is reversed. Away from the wavemaker, $x/L > 2$, $E_{\eta\eta}$ is consistent with others. It should be noted that wave dispersion due to the aforementioned numerical diffusion is shown at $x = 2L$ (Figure 6d), where the sharp peak of $E_{\eta\eta}$ at f_p tends to be smoothed out.

Figure 7 shows the x - z snapshots of kinetic energy (KE) per unit mass (dm) at a given time, estimated as $KE/dm = \frac{u^2 + w^2}{2}$. Away from the wavemaker ($x/L > 2$), KE/dm in all cases are similar, recovering the wave shape in deep water wave condition (namely, KE/dm due to the wave orbital motion is much stronger close to the free surface). However, strong near-bed KE/dm are observed in *Fkh8N1* up to $x/L = 2$ due to the piston type wavemaker motion (Figure 7a). The streamwise distance recovering KE/dm for the deep water wave condition gets shorter with applying more paddles (i.e., $x/L = 1$ for *Fkh8N2*, $x/L < 0.2$ for *Fkh8N4*, and $x/L \sim 0$ for *Fkh8N4*).

An index of agreement (Willmott and Wicks, 1980; Willmott, 1981), IA , is used to evaluate model results:

$$IA = 1 - \frac{\sum_j (M_j - O_j)^2}{\sum_j (M_j - \bar{O} + O_j - \bar{O})^2}, \quad (34)$$

where the subscript j denotes j th data point in space, O_j represents the mathematical solution, and M_j represents the model results. Zero represents complete disagreement while a value of 1 is perfect agreement. The variable $\bar{O}$ represents the mean value of O_j .

Based on potential theory, velocity profiles for free surface waves are given as (Dean and Dalrymple, 1991):

$$u(x, z, t) = \frac{H_p}{2} \sigma \frac{\cosh[k(h+z)]}{\sinh(kh)} \cos(kx - \sigma t), \quad (35)$$

$$w(x, z, t) = \frac{H_p}{2} \sigma \frac{\sinh[k(h+z)]}{\sinh(kh)} \sin(kx - \sigma t). \quad (36)$$

The vertical profiles of modelled velocity components are compared with equations (35) and (36) at three different cross-shore locations (Figure 8). The time steps are selected for the comparison coincide with the wave crest and trough, u and w , respectively (i.e., $\max(u)$ and $\max(w)$). The modelled wave velocity fields reached quasi-steady state after the ramping up period, hence u and w are averaged over last five waves, consistent with Figure 5e. For *Fkh8N20*, the overall agreement of u (w) is good throughout the numerical domain where $IA > 0.99$ are obtained right next to the wavemaker. At $x/L > 0.3$, $IA > 0.995$ is obtained for both u and w from *Fkh8N4* (Figure 8g). On the other hand, the agreement of u (w) is less satisfactory near the wavemaker with *Fkh8N1* (blue solid curves in Figure 8), where $IA = 0.927$ (0.713), 0.976 (0.852), 0.995 (0.999), at $x/L = 0.2$, $x/L = 0.7$, $x/L = 2$, respectively. We conclude a distance of $2L$ is required to achieve the orbital velocities corresponding to the deep water wave condition when $N = 1$ is applied. This distance decreases with increasing N , and only $x/L \sim 0.1$ is required for $N \geq 4$.

Figure 6: Wave energy spectra of *Fkh8N1* (blue solid curves), *Fkh8N2* (red dashed curves), *Fkh8N4* (green dash-dotted curves), *Fkh8N20* (orange solid curves) at (a) $x/L = 0.1$, (a) $x/L = 0.5$, (a) $x/L = 1$, and (a) $x/L = 5$. The black dotted vertical lines denote $f_p = 1/T$

4.2. Intermediate depth condition

Wave orbital motions are nearly vertically uniform for shallower waves, thus the advantage of additional paddles disappears. An intermediate wave condition ($kh = 1.06$) is tested with h_N of 0.3 m, T of 1.2 s, L of 1.77 m (*Fkh1N* cases in table 1). Similar to section 4.1, a 2DV numerical domain of 30 m long ($\sim 20L$) and 0.375 m deep ($1.25h_N$) was constructed with uniform grids of 4 mm width (dx) and 1.5 mm height (dz) using 1.87 million computational grid points. Wave height and wave steepness are selected as 20 mm and $H/L = 0.011$, respectively. For each intermediate case, X_i^P is also obtained using equation 10. Unlike the deep water wave case (Figure 4), the discrepancy in X_i^P is much smaller in intermediate wave conditions (Figure 9). The utilized water volume in unit width to generate waves is relatively equivalent where *Fkh1N1* ($5.8 \times 10^{-3} \text{ m}^3/\text{m}$), *Fkh1N2* ($5.7 \times 10^{-3} \text{ m}^3/\text{m}$), *Fkh1N4* ($5.6 \times 10^{-3} \text{ m}^3/\text{m}$), *Fkh1N20* ($5.6 \times 10^{-3} \text{ m}^3/\text{m}$).

Figure 10 shows η_m compared at different cross-shore locations for each N . Under the intermediate wave condition, η_m is nearly identical regardless of N , and $H_m/H_p = 1$ is obtained for the last five waves throughout the entire flume. As briefly mentioned in section 4.1, the underprediction of η_m is not shown in this wave condition with less wave steepness.

Likewise, the velocity profiles evolve into the target profile more rapidly and are slightly more accurate with additional paddles (Figure 11). For instance, *Fkh1N4* and *Fkh1N20* have $IA > 0.99$ for u profile from $x/L < 0.01$ to the end of the flume where the distances to achieve $IA > 0.99$ for u are $0.1L$ and $0.04L$ for *Fkh1N1* and *Fkh1N2*, respectively. For w profiles, the agreements are good with all cases where $IA > 0.997$ are obtained. To conclude, utilizing more N is desirable for efficient deep water wave generation while the single piston type wavemaker reason-

Figure 7: The x - z snapshots of KE/dm at $t/T = 20$ (*Fkh8N1*, (a); *Fkh8N2*, (b); *Fkh8N4*, (c); *Fkh8N20*, (d)). The white dashed curves represent $\phi^w = 0.5$.

ably represents intermediate wave conditions, and presumably for shallow water conditions. It should be reiterated here that the limitation of flap type wavemaker in generating shallow water waves is considered to be similar to that of *Fkh8N1*.

4.3. Effect of the number of paddles

The conventional piston type and flap type wavemakers have been widely adopted in the laboratories and in the numerical models (e.g. Higuera et al., 2015). The piston (flap) type wavemaker is more suitable for shallow (deep) water wave generation since the flow feature induced by the paddle movement mimics the wave orbital velocity under each wave condition. It should be noted that applying a piston type wavemaker for deep water wave generation (or the other way around) is possible though somewhat undesirable since additional streamwise direction is required to converge target wave height and velocity profiles (see Section 4.1). The primary advantage of the vertically discretized multi-paddle wavemaker is that each paddle stroke can be adjusted independently to efficiently achieve quasi-homogenous

Vertically discretized multi-paddle wavemaker

Figure 8: Vertical profiles of (a,b,c) u under the wave crest and (d,e,f) w under the wave trough for deep water waves at (a,d) $x/L = 0.2$, (b,e) $x/L = 0.7$, (c,f) $x/L = 2$, IAs of (g) u and (h) w as a function of x/L (blue solid curves, $Fkh8N1$; red dashed curves, $Fkh8N2$; green dash-dotted curves, $Fkh8N4$; orange solid curves, $Fkh8N20$). Symbols represent the mathematical solution.

298 waveforms for any given wave condition. This minimizes streamwise length requirements and maximizes the available
 299 laboratory observation zone. Additionally, the vertical flexibility applies to internal wave generation (see Section
 300 5). Notably, horizontally discretized multi-paddle wavemakers have been widely used in 3D wave basins to generate
 301 directional waves. Control technology and mechanical operation are similar and would likely minimize challenges in
 302 adopting vertically discretized multi-paddle wavemakers.

303 Figure 12 shows the streamwise distance (x/L) required to obtain the converged flow characteristics for a range
 304 of kh values with different N applied. Additional numerical simulations were conducted at depths of $kh = 6.19$ and
 305 3.06 using $N = 1$ and 20 paddles (table 1) to consider range of normalized streamwise distance required for desired
 306 converged flow characteristics. It is expected that the $N2$ and $N4$ cases at $kh = 6.19$ and 3.06 locate in between $N1$
 307 and $N20$ results. The selected thresholds represent the location where $\partial(H_m/H_p)/\partial(x/L)$ becomes less than 0.1
 308 (also see Figures 5e and 10d) and IA gets larger than 0.99 for the vertical profiles of u (also see Figures 8g and 11g)
 309 and w (also see Figures 8h and 11h), comparing with the analytical solutions.

310 In general, as kh increases (i.e., approaching the deep water condition) the conventional piston type wavemaker
 311 (blue circles in Figure 12) requires longer streamwise distance to achieve stabilized flow characteristics. Near im-
 312 mediate convergence ($x/L < 0/1$) is observed with $N = 20$ in all four kh value locations (orange crosses in Figure 12).

Figure 9: Time series of modelled X_i^P under intermediate wave condition at (a) top paddle ($i = 1$), (b) middle paddle ($i = 10$), (c) bottom paddle ($i = 20$), and (d) $S_i/2$ for $Fkh1N1$ (blue solid curves), $Fkh1N2$ (red dashed curves), $Fkh1N4$ (green dash-dotted curves), $Fkh1N20$ (orange solid curves) cases, respectively.

It should be noted that a slight underprediction in η_m from numerical diffusion is observed in Figure 12a. Although increasing N significantly reduces the convergence distance, it may be impractical to have large numbers of physical paddles (i.e., $N \geq 20$) in the laboratory environment. Notably, using 2 or 4 paddles (red crosses and green squares in Figure 12, respectively) reasonably reproduce u and w profiles in the deep water condition.

5. Multi-phase internal wave

Conventional-type wavemakers have not been fully successful in reproducing internal waves, hence newly devised wavemakers have been proposed (e.g. Lewis et al., 1974; Kao et al., 1985; Michallet and Barthelemy, 1998; Umeyama, 2002; Ko and Lynett, 2014). Owing to its vertical flexibility to any given flow condition, the wavemaker system proposed in this study can also function as an internal wave generator. Based on the newly developed theory (see section 2.2), internal waves are simulated using multiphaseInterDyMFOam (e.g. Vakhruşev et al., 2014). Unlike the free surface waves, OpenFOAM wave absorbing boundary condition is unavailable for internal wave applications necessitating a sufficiently long numerical domain ($> 10L$) to exclude reflected waves. The grid size is identical to section 4.1. Using a single paddle ($N = M = 1$) for each layer of fluid (i.e., water and oil phases), internal waves are generated with a wave period of $T = 2$ s, still water depths of $h_u = 0.15$ m and of $h_l = 0.15$ m for the oil and water layers, respectively. **Internal waves are common in the shallow coastal zone (e.g., stratified flows in a river mouth). Hence, only the shallow water wave condition is simulated in this study, however the extension to the deep water waves is available directly from the analytical solution.** The wave height at the water-oil interface (H_l) is selected as 10 mm, leading to the wave height at the oil-air interface (H_s) as 21.5 mm following equation 17. Based on equations 22 and 23, the S for each layer is calculated for a given wave condition (Figure 13). It should be noted that only the fast mode (k^+) was considered in the input (i.e., $\sigma^2 = gk^+ \tanh[k^+(h_u + h_l)]$), resulted in $k^+h_u = k^+h_l = 0.30$, where the

Figure 10: Time series of η_m with respect to a_p under intermediate wave condition at (a) $x/L = 0.2$, (b) $x/L = 0.7$, (c) $x/L = 2$, and (d) H_m/H_p as a function of x/L for *Fkh1N1* (blue solid curves), *Fkh1N2* (red dashed curves), *Fkh1N4* (green dash-dotted curves), *Fkh1N20* (orange solid curves) cases, respectively.

slow mode motion is ignored (i.e., $\sigma^2 = gk^{-\frac{1-\rho_r}{\coth(k^-h_u)+\coth(k^-h_l)}}$). In general, slow mode motion becomes active when $\rho_r \approx 1$ and H_s is of minor compared to that of the fast mode motion (Liu et al., 2008, 2013). In this study, free surface fluctuation is significant, larger than the interface wave height (see Figure 3a). It is well-known that the η_i and η_s tend to be in phase for the fast mode (Liu et al., 2008, 2013), hence the calculated displacement of both paddles are also shown to be in phase (Figure 13).

Surface elevations of water (lower) and oil (upper) phases are calculated as $\eta_i(x, t) = \int \phi^w(x, z, t) dz - h_l$ and $\eta_s(x, t) = \int [\phi^w(x, z, t) + \phi^o(x, z, t)] dz - (h_u + h_l)$. The modelled η_i (η_s) with reference to a_i (a_s) at selected cross-shore locations are shown in Figures 14a - 14c (Figures 14d - 14f). In general, modelled results converge to a_i and a_s well. The ratio of modelled to prescribed wave heights, averaged over the last five waves in the entire flume shows the discrepancies less than 3.5% (2.6%) for H_i (H_s) in $x/L > 1$ (Figure 14g). However, small-scale spatial oscillations are observed in the modelled results, especially in H_s (Figure 14g) which may be caused by excluding the slow mode of the system.

The wave energy spectrum in a spatial extent $E_{\eta\eta}(k)$ is analyzed similar to section 4.1, corresponding to the auto-correlation of modelled η_i and η_s at $t/T = 15$ (Figure 15). Only the first half of the numerical domain is used to exclude reflected wave effects. Resonant characteristic lengths of the internal wave for both interfaces are dominated

Figure 11: Vertical profiles of (a,b,c) u under the wave crest and (d,e,f) w under the wave trough for intermediate waves at (a,d) $x/L = 0.2$, (b,e) $x/L = 0.7$, (c,f) $x/L = 2$, IAs of (g) u and (h) w as a function of x/L (blue solid curves, $Fkh1N1$; red dashed curves, $Fkh1N2$; green dash-dotted curves, $Fkh1N4$; orange solid curves, $Fkh1N20$). Symbols represent the mathematical solution

by the fast mode (k^+). Since the two modes coexist by the nature (see equation 16), wave energy is transferred to the slow mode (k^-) although the magnitude of $E_{\eta\eta}(k^-)$ is significantly lower than $E_{\eta\eta}(k^+)$. Transfer to the slow mode is not apparent in $E_{\eta\eta}(k^-)$ using η_s , due to the fact that η_s gets much smaller in the slow mode (e.g. Figure 3).

The velocity profiles for each layer of fluid are compared with the mathematical solution (Liu et al., 2008, 2013) (figure 16). In general, modelled u under the wave crest and w under the wave trough profiles oscillate slightly, presumably due to the slow mode. Generally, however, reasonable IA are obtained for u and w in the lower layer where $\min(IA)$ is 0.989 (0.96) for u (w). For u (w) in the upper layer, IA is slightly lower where $\min(IA)$ is 0.959 (0.787). In particular, figure 16h shows a significant spatial variance for the IA of w in the upper layer. It is expected that the interplay between fast and slow modes may be altered for different wave conditions (e.g., different densities and depth). Therefore, a comprehensive validation of newly developed wavemaker theory for the generation of internal waves under a variety of wave conditions should be considered as future work.

Figure 12: Normalized streamwise distance (x/L) to obtain converged (a) H_m , (b) u , and (c) w (blue circles, $N = 1$; red pluses, $N = 2$; green squares, $N = 4$; orange crosses, $N = 20$). Dashed lines represent $kh = \pi$.

6. Conclusion

A vertically discretized multi-paddle wavemaker is proposed for free surface and internal waves generation. Complete solutions for each paddle stroke based on the first-order linear wave theory were derived using a weight function following the velocity profiles. The notable advantage of complete solutions is the vertical flexibility of wavemaker motion following given velocity profiles. Thus, the vertically discretized multi-paddle wavemaker ensures a fast convergence to intended waveforms, particularly when wave orbital velocity is vertically non-uniform such as deep water waves. In addition, the independent movement of each paddle can force multi-layer fluids separately, hence new wavemaker can be also applied to internal wave generation.

A numerical investigation was conducted for the free surface waves using a two-phase mixture solver (olaDyM-Flow), based on OpenFOAM. The numerical model was compared with the mathematical solution for the free surface elevation and velocity profiles. In the deep water wave condition, more paddles are desirable. $Fkh8N20$ contains fewer evanescent modes compared to $Fkh8N1 - 4$, and rapidly converges to the target free surface elevation and velocity profiles for a given wave condition. Sufficiently far away from the wavemaker ($x/L > 2$), $Fkh8N1$ also reached reasonable agreement. For intermediate wave conditions, model results yielded similarly good agreement with the mathematical formulation (Dean and Dalrymple, 1991), regardless of the number of paddles used in the system.

Internal wave numerical simulation was conducted using multiphaseInterDyMFOam that considers a multi-phase

Figure 13: Time series of modelled X^P for the internal wave case for upper (thick grey curve) and lower (thin black solid curve) paddles, respectively.

mixture, i.e., air, oil, and water phases. For the internal wave case, shallow water wave condition was used while only imposing the fast mode. The model results were compared with the mathematical solution (Liu et al., 2008, 2013) and showed reasonable agreement, suggesting a potential for the newly proposed method in the internal wave generation. Model results also indicated a transition to the slow mode induces slight oscillations although such a transition in the surface wave component is of minor.

This analysis presented here represents a first step toward realizing and implementing a vertically discretized multi-paddle waveamaker for free surface wave and internal wave generation. Extensive numerical or physical experiments under many different wave conditions should be followed to achieve an ideal setup.

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Figure 14: Time series of (a,b,c) η_i/a_i and (d,e,f) η_s/a_s at (a,d) $x/L = 1$, (b,e) $x/L = 2$, (c,f) $x/L = 3$, and (g) H_m/H_p as a function of x/L for H_i (thick grey curves) and H_s (thin black solid curves) cases, respectively.

Figure 15: Wave energy spectra of η_i (thick grey curves) and η_s (thin black dashed curves) at $t/T = 15$. Two black dotted vertical lines denote fast (k^+) and slow (k^-) modes, as indicated.

Figure 16: Vertical profiles of (a,b,c) u under the wave crest and (d,e,f) w under the wave trough for the lower (mathematical solution: circles; model results, thick gray solid curves) and the upper layer (mathematical solution: crosses; model results, thin black dashed curves) at (a,d) $x/L = 1$, (b,e) $x/L = 2$, (c,f) $x/L = 3$. IAs of (g) u and (h) w as a function of x/L . The black dotted lines represent h_l .

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