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Explicit Models of Willingness to Pay: A Monte Carlo Simulation

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Abstract

Explicit estimates of compensating or equivalent variation have replaced measures of consumer surplus in many applied settings.

This paper compares two widely used methods of making such estimates: so-called "hedonic price" or implicit markets approaches and discrete choice methods based upon stochastic utility functions. Both techniques are applied to identical data sets in an extensive monte carlo simulation. The results are compared in terms of their accuracy in estimating consumer "willingness to pay" for marginal and nonmarginal changes in consumption.

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Explicit Models of Willingness to Pay: A Monte Carlo Simulation*

by

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I. INTRODUCTION

Although the notion of income compensated demand curves dates from *Value and Capital* published almost fifty years ago, it has only been in the past two decades that economists have developed techniques to make the concept an operational tool. Increasingly, applied welfare economics has come to be based upon explicit estimates of consumers' "willingness to pay" for commodities rather than Marshallian notions of consumer surplus. Applications of these techniques to estimate willingness to pay have been quite diverse, including the demand for safer workplaces (Smith, 1983; Viscusi, 1981), for outdoor recreation (Portney, 1981), for improved air quality (Ridker and Henning, 1967; Harrison and Rubinfeld, 1978), for improved neighborhoods (Kain and Quigley, 1970; Blomquist, *et al*, 1988) and for urban transportation systems (Anas, 1982; Anas and Lee, 1982).

There are two different strands to the literature which relates the directly observable choices of households among commodity baskets with different prices to the unobservable income-compensated demands of those consumers. These formulations recognize the multi-dimensional attributes of economic commodities and the complexity of market prices. However, one strand of this literature views observable data as if they were generated by the tangencies of level sets of continuously differentiable utility functions with nonlinear but observable market prices (See Tinbergen, 1956 for an early formulation, Hausman, 1985, for a comprehensive review and Moffitt, 1986, for a selective review of this literature). The alternative approach views observable data as if they were generated by the choices of consumers made from well-defined discrete choice sets (See McFadden, 1973 for the original development, and Train, 1986, for a review of applications).

These approaches involve substantial differences in assumptions about the process by which data on consumer choices are generated, and they use very different statistical methodologies. Despite these differences and the increasing practical application of both techniques, there has been very little comparative analysis of these techniques for estimating consumers' willingness to pay for commodities.[1] This paper provides an extensive comparative analysis. It reports the results of a set of monte carlo experiments using these techniques to make inferences from the same data.

Section II below presents the details of two techniques, and Section III outlines the monte carlo comparisons. Preliminary results of these investigations are presented in Section IV, and the conclusions are noted in Section V.

II. EXPLICIT WELFARE COMPARISONS: DISCRETE CHOICE AND HEDONIC TECHNIQUES

Consider a set of observations on some composite good **X**, described by a vector of measurable characteristics, $\mathbf{X}_j = \{x_{1j}, x_{2j}, ..., x_{nj}\}$, which is observed to be traded at price $p_j = P(\mathbf{X}_j)$. X could be: a set of jobs with n different characteristics, such as manual difficulty, danger, etc; a set of recreational opportunities with different attributes; a set of dwellings; or various transportation or accessibility alternatives. Let I be a set of I consumers, and J be a set of composite goods with J elements; assume I $\leq$ J; suppose that consumers and goods are conveniently indexed so that each consumer with income y_i is observed to consume exactly one composite good with characteristics $\mathbf{X}_i$ at price p_i, leaving $y_i - p_i$ for consumption of z_i, "other goods," at a constant unit price

1 There have been a few limited attempts to reanalyze existing data sets using alternative methodologies, e.g. Galster (1977); Belsley, *et al* (1980); Brieman and Friedan (1985). The most systematic attempt to estimate alternative models using a body of real data is the work of Son (1987) analyzing the data generated by the U.S. Housing Allowance Supply Experiment. In contrast, there have been only a few attempts to infer the properties of these estimators by controlling the properties of the data. See Ohsfeldt (1988), Ohsfeldt and Smith (1985, 1988) and Quigley (1986).

of one. Observations on y, p, and **X** are inputs into the estimation of household willingness to pay for those characteristics which define the composite good.

We assume that households obtain utility from the composite good $\mathbf{X}_i$ and "other goods," yielding a direct and a "quasi" indirect utility function (Hausman, 1981),

1a) $U_{ii} = U(\mathbf{X}_i, z_i) = U(\mathbf{X}_i, y_i - p_i)$.

Households act to maximize utility over **X**, the set of composite goods, subject to a budget constraint,

2) $y_i = p_i + z_i = p_j + z_j$, for $j \in J$.

a. The Discrete Choice Model

Suppose the utility provided to household i by the choice of composite good i, U_{ii}, can be partitioned into a systematic component V_{ii} and a random or stochastic component ε_{ii}:

3) $U_{ii} = V_{ii}(\mathbf{X}_i, y_i - p_i) + \varepsilon_{ii} = V_{ii} + \varepsilon_{ii}$.

The probability that household i will choose good i corresponds to the probability that the choice of i yields a higher utility than any alternative,

4) $\text{Prob}\,[(V_{ii} + \varepsilon_{ii}) > (V_{ij} + \varepsilon_{ij})] \quad \forall\ j, \text{where } j \in J$.

Assumptions about the distribution of ε may make the probability statement tractable. As McFadden (1974) has shown, if the stochastic term is independently and identically distributed according to the extreme value distribution, then the choice probabilities and the resultant likelihood function are particularly convenient. The extreme value distribution has the cumulative probability density function

$$5)\quad \text{Prob}\,(\varepsilon_{ij} \leq \upsilon) = \exp\left[-\exp\{-\lambda(\upsilon + \omega/\lambda)\}\right],$$

with scale parameter λ and location parameter ω. If $\omega = 0.5772$ (Euler's constant), ε has mean zero. The form of the probability statement, using Equation 5, is

$$6)\qquad \pi_{ij} = \exp(V_{ij}) \Big/ \sum_{k \in J} \exp(V_{ik}),$$

where π_{ij} is the probability that household i will choose good j, and the likelihood function is the multinomial logit form. Again, goods and households are indexed such that household i is observed to choose good i. Thus π_{ii} is the probability that household i will choose the good which it has been observed to choose. Equation (6) is a well behaved probability function assigning probabilities between zero and one to all possible choices. One feature of this formulation (subsequently generalized, McFadden, 1978) is that the odds of choosing between any pair of alternatives is independent of the attributes of any other alternatives,

$$7)\qquad \pi_{iq} / \pi_{ir} = \exp\left[U_{iq} - U_{ir}\right].$$

If the systematic component of the utility function is of a given form with parameters **B**,

$$8)\quad V_{ij} = f(\mathbf{X}_j, y_i - p_j; \mathbf{B}),$$

then the vector of parameters **B** may be consistently estimated, by maximizing a likelihood function, L,

$$9)\quad L = \prod_{i \in I} \pi_{ii},$$

where π_{ii} is the estimated probability that the i th household would choose the good which it was observed to select out of the set *J*. It follows from the independence of irrelevant alternatives property that **B** can be consistently estimated from

$$10) \quad \pi_{ii} = \exp(V_{ii}) \Big/ \sum_{k \in T_i} \exp(V_{ik}) \quad ,$$

where T_i is a subset of possible alternatives *J* which includes the chosen alternative. As long as Equation (7) holds, consistent estimates of **B** can be obtained when the subsets T_i, which may differ for each household, are chosen to have the "uniform conditioning property". (This property is also generalized in McFadden, 1978.)

Estimation of the parameters of Equation (8) is sufficient to determine each consumer's compensating or equivalent variation associated with the composite commodities, be they jobs, houses, or transport modes.

b. Models with Nonlinear Budget Constraints: Continuous Prices

An alternative class of models for estimating willingness to pay concentrates on the information conveyed when agents optimize with respect to a nonlinear budget constraint or price function. In the most common formulation (Rosen, 1974), the price function is assumed to be continuous and twice differentiable,

$$11) \quad p_j = P\{x_{1j}, x_{2j}, \ldots, x_{nj}\} = P(X_j) \quad .$$

Limitations on arbitrage which generally follow from the nature of composite goods are sufficient to insure that this function is nonlinear.[2] Observations on the attributes of the

2 The applied literature based on this model is voluminous. See Witte, *et al* (1979) for an early application, and Follain and Jimenez (1985) for a review.

commodity and its market price are sufficient to estimate the parameters of Equation (11) by some "hedonic" method. The derivatives of this function represent the money costs of an additional unit of each attribute, at given levels of the other attributes. Maximizing (1) subject to the budget constraint in (2) yields an equilibrium such that each consumer's marginal rate of substitution between attributes of the composite commodity and all other goods will equal the marginal cost of that attribute. If $P(X_j)$ is continuous and differentiable, then for each household,

$$\text{12a)} \quad \frac{\partial U(X_i,z_i)/\partial x_{i1}}{\partial U(X_i,z_i)/\partial z} = \frac{\partial P(X_i)}{\partial x_{i1}}$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{\partial U(X_i,z_i)/\partial x_{in}}{\partial U(X_i,z_i)/\partial z} = \frac{\partial P(X_i)}{\partial x_{in}} \quad .$$

Estimates of this system of equations, the first order conditions, can be used to recover the parameters of the utility function **B** from which each consumer's compensating or equivalent variation can be computed. If there are only two goods (i.e., $n=1$), then the estimation of Equation (12a) is straightforward; if **X** is a composite good (i.e., $n > 1$), the estimation procedure may be somewhat more complex.

c. Market Equilibrium

Estimates of the utility function or the compensated demand function support rigorous welfare comparisons measuring consumer willingness to pay for x_1, -- say improved transport accessibility, or better air quality, or enhanced recreational opportunities. The foregoing discussion describes the equilibrium of the consumer, given observations on the composite goods and their exogenously determined market prices. Of course, in competitive equilibrium, the prices are themselves endogeneously determined.

To focus attention on the behavior of consumers, we choose a simple approach to price determination. We assume that suppliers are competitive profit maximizers, but instead of specifying production functions and factor markets (See Rosen, 1974), we assume that supplies are fixed and that the number of composite goods available J exceeds the number of consumers I. These assumptions,[3] together with the utility function and the assumption that each consumer chooses exactly one composite good, are sufficient to insure that competitive bidding will allocate each composite commodity $\mathbf{X}_i$ to a particular consumer with income y_i.

The particular set of prices which supports this allocation depends upon the distribution of the composite good, the distribution of income, and the preferences of consumers. The set of market clearing prices will have the following characteristics: the difference in price between adjacent goods along any dimension i must be large enough to dissuade any consumer from outbidding his next wealthier neighbor; the price difference must also be small enough so that the sacrifice in quality would outweigh the cost savings to any consumer considering a downward trade.

The set of prices with these characteristics arises from the competitive bids of consumers. Define $r_i = R_i(\mathbf{X}_i; y_i, U)$ as the maximum bid that a consumer with income y_i could pay for $\mathbf{X}_i$ and still achieve utility U. At the equilibrium of each consumer in the market,

1b) $U(\mathbf{X}_i, y_i - p_i) = U(\mathbf{X}_i, y_i - r_i)$.

3 A natural analog to these assumptions would be the allocation of existing dwellings with differing accessibility characteristics among consumers of varying incomes.

Assuming continuity and differentiability of the underlying functions, the market price function $P(X_i)$ will be the envelope of all consumers' bid functions, that is, $\partial P(X_i)/\partial x_i = \partial R(X_i;y_i;U)/\partial x_i$. As noted in equation (12a), the first order conditions are:

$$\text{12b)}\quad \frac{\partial U(X_i,z_i)/\partial x_{i1}}{\partial U(X_i,z_i)/\partial z} = \frac{\partial R(X_i;y_i;U)}{\partial x_{i1}} = \frac{\partial P(X_i)}{\partial x_{i1}}$$

$$\frac{\partial U(X_i,z_i)/\partial x_{in}}{\partial U(X_i,z_i)/\partial z} = \frac{\partial R(X_i;y_i;U)}{\partial x_{in}} = \frac{\partial P(X_i)}{\partial x_{in}}$$

Given the utility function, the market's inverse demand function, $P(X_i)$ is itself defined in terms of the system of differential equations in (12b) with $y_i = G(X_i)$, where $G(X_i)$ is the correspondence between composite goods and incomes defined above.

Alternatively, given the price function and its derivatives, estimates of equation (12b) can be used to recover the consumers' bid functions or the income compensated demand curve. If n>1, the solution may be impossible in closed form. However, the solution to this system of partial differential equations subject to the integrability conditions can generally be obtained by numerical methods. Of course, assumptions about the form of the direct or quasi indirect utility function in (1) may yield convenient forms for the system of equations in (12b); conversely assumptions about the form of (12) may yield closed form solutions for the compensated demand curve. (See Hausman, 1981, for examples.)

Ultimately, this means that a known utility function U and mapping G are sufficient to generate a body of "data" on discrete economic "agents" and their choices.[4] Each agent of income y_i is observed to consume some X_i and z_i and to make expenditures of $P(X_i)$ on X_i and y_i-$P(X_i)$ on z_i. This "data set" is consistent with utility maximization for each individual and with a competitive equilibrium in which each composite commodity is allocated to the highest bidder.

III. THE MONTE CARLO SIMULATIONS

a. Data Generation

To compare the continuous and discrete approaches, we require a data set consisting of observations on X_i, $G(X_i)$, and the market equilibrium price, $p_i = P(X_i)$, paid by each agent. Given consumers' utility functions and assuming a fixed and known supply of $\{x_j\}$ $j=1,...,J$, the specification of either $G(X_i)$ or $P(X_i)$ is sufficient to solve for the other. In this analysis, we select $G(X_i)$ and solve for $P(X_i)$.[5]

Assume that the true form of the utility function is GCES (See Murkerji, 1963, or Murray, 1975) and without loss of generality, that n = 1,

15) $U_{ij} = [\alpha x_j^{\beta} + (y_i - p_j)^{\gamma}]^{\delta}$,

4 We also need an initial condition. Let $P(X) = p_o$, where **X** is the composite commodity chosen by the consumer of lowest income and p_o is the opportunity cost of supplying composite commodities.

5 Stated somewhat more intuitively, in this analysis we select the utility functions, the income distribution and the commodity distribution and solve for the competitive equilibrium prices.

where α, β, γ, and δ are parameters.[6]

We then specify $y(x)=G(x)=(Ax)^{\eta}$ where A and η are arbitrarily chosen to produce a monotonically increasing function over the range of $\{x_j\}$. The consumer with the lowest income y_1 chooses x_1.

To solve for the equilibrium price function P(x) we differentiate (15) and substitute into (12b), yielding

$$\text{16a)} \quad \frac{dP(x_i)}{dx_i} = \frac{\alpha\beta x_i^{\beta-1}}{\gamma[G(x_i)-p_i]^{\gamma-1}},$$

or equivalently

$$\text{16b)} \quad \log\,[dP/dx_i] = \log[\alpha\beta/\gamma]+(\beta-1)\log x_i-(1-\gamma)\log[G(x_i)-p_i]\,,$$

and solve for $P(x_i)$ with initial condition $P(x_1)=p_o$.

The solution to (16) is obtained using a modified Runge-Kutta procedure. In this way a set of discrete points (x_i, y_i, p_i) is obtained. At each point or "observation" in this "data set," the first order conditions in (12) hold.

c. The Structure of the Simulations

The procedure outlined above yields a set of observations on the triple (x_i, y_i, p_i) consistent with a utility maximizing competitive equilibrium, given a utility function, a mapping from x to y, and an initial condition. Consider a set of $i=1,2,...,100$ such observations.

6 For $\beta, \gamma, \delta < 1$ the function has desireable properties. Further if $\beta=\gamma=1/\delta$, the function is CES; as β and γ approach zero, the function approaches Cobb-Douglas.

For each triple, select four (x, p) pairs from the rest of the data set using a simple random process. Use these pairs to form four triples $(x_{ij}, y_i, p_{ij}), j=2,3,4,5$ and designate the observed equilibrium triple as (x_{i1}, y_i, p_{i1}). Thus by construction, $V(x_{i1}, y_i-p_{i1}) > V(x_{ij}, y_i-p_{ij})$ for $j = 2,3,4,5$.

Next, for each of the 100 sets of five triples, select at random a deviate ε_{ij} distributed as in (5). Calculate $U_{ij}=V_{ij}+\varepsilon_{ij}$ $j=1,2,...,5$, and designate the triple corresponding to the maximum U_{ij} as the "chosen alternative."

This procedure yields a set of 100 observations on "chosen alternatives," and 100 corresponding sets of four "rejected alternatives" selected according to a sampling rule with the "uniform conditioning property." In each case, the chosen alternative is obtained by maximizing a stochastic utility function with an error drawn from the extreme value distribution.

This body of data is then used to estimate the income compensated demand for x. Estimates of the parameters of the underlying utility function are obtained for the "discrete choice" model by maximizing the likelihood function in Equation (9) using 100 sets of one chosen and four rejected alternatives. Estimates of the same parameters are also obtained by invoking the two step "hedonic" procedure suggested by Rosen and implemented by many researchers: Equation (11) is estimated directly and the derivatives are used to estimate Equation (16b). In the simulation exercise below, Equation (11) is estimated by a fifth order power series expansion.

The simulation analysis is undertaken using nine different types of data sets. These types correspond to three different sets of utility function parameters (and initial conditions) and three different scalings[7] of the disturbance term ε.

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7 Data sets were also created with $\varepsilon=0$. With zero error, all hedonic data sets are

The utility function parameters and initial conditions are reported in Table 1. These three sets of parameters were chosen to produce data sets with the three distinctly different Engel curves shown in Appendix Figure 1. The assumed mapping function G(x) is also reported in Appendix Figure 2. The parameters and initial conditions permit the computation of the equilibrium "hedonic" price relationship. Equilibrium price values are obtained by numerical integration of (16a) using the Runge-Kutta procedure. The market clearing hedonic price functions are shown in Appendix Figure 3.

The error scalings were chosen to produce "small," "medium," and "large" random components in the stochastic utility functions. For each of these nine categories (three sets of initial conditions by three error sizes), a total of 50 data sets were created. Each data set consists of 100 observations on one chosen alternative and four rejected alternatives.

Table 2 summarizes the effects of the errors drawn from the extreme value distribution on the data sets. For each of the nine categories, it reports the percentage of observations for which the "chosen" alternative (with the highest value of U) is not the alternative with the highest value of V, the systematic component of utility. When the stochastic error is chosen to be "small," that error is still sufficiently large to change the alternative which is selected in 13-19 percent of the cases. When the error is chosen to be "large," it changes the utility maximizing alternative more than half the time.

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identical. For the discrete choice model, randomly selected "rejected alternatives" will in general differ among data sets.

TABLE 1

Utility Functions And Initial Conditions

Engle Curve (See Appendix)	Income Elasticity of Expenditure on x	Utility Function	x_1	p_0
I	<1	$[.75x^{.25}+z^{.45}]^{\delta}$	1	20
II	>1	$[.75x^{.50}+z^{.55}]^{\delta}$	1	2
III	=1	$[.75x^{.25}+z^{.45}]^{\delta}$	1	10

TABLE 2

Percentage of Observations in which the Identity of the Utility Maximizing Alternative is Changed by Including a Stochastic Error*

Engle Curve (See Appendix)	Income Elasticity Expenditure on x	None	Size of Stochastic Error Small	Medium	Large
I	<1	0%	18.7%	34.7%	56.4%
II	>1	0	12.7	30.4	52.3
III	=1	0	17.4	35.8	57.4

* Each of the nine nonzero cell entries reports the percentage of the 5000 observations (50 data sets of 100 observations each) for that cell in which the identity of the utility maximizing alternative is changed by the inclusion of the stochastic error drawn from the extreme value distribution.

These data sets are then analyzed using the discrete choice and hedonic models.

IV. RESULTS

Each of the 450 separate data sets was analyzed using the two techniques described above. The coefficients estimated for each replication permit a comparison of the two methods used to estimate consumer willingness to pay for additional units of the composite good x. Because the underlying properties of the data are known, estimates from these statistical models can be compared quite easily to their actual values.

Table 3 summarizes the 3159 coefficients estimated by these procedures.[8] It presents the mean values of the coefficients, separately for the three Engel Curves and for each of three error scalings. The table also presents the standard deviation of the coefficient estimates. Each entry summarizes 50 replications of the estimation procedure. As the size of the error increases from zero to what we designate in Table 3 as "large," the means of most of the parameter estimates get both larger and further from the true specified values. Somewhat surprisingly, the dispersion of the estimates tends to fall as the error size increases. The standard deviations of all but one of the parameter estimates, using the hedonic method, are smaller when the error size is "large" then when it is "small." The pattern for the discrete choice estimates is not as clear as in the hedonic case, but it is apparently consistent. The standard deviations of the parameter estimates using the discrete choice method are in all cases smaller when

8 The estimates include: fifty replications of each of three coefficients for each of two statistical models for each of three basic markets (data sets) with three non-zero error sizes (50x3x2x3x3x3=2700); fifty replications of three coefficients for the discrete choice model for each of three basic markets (data sets) with zero error (50x3x3=450); the three coefficients for the hedonic model for each of the three basic markets (data sets) with zero error (3x3=9).

TABLE 3

Mean Values of Parameter Estimates (Standards Deviations in Parentheses)*

		Error Size Small	Medium	Large
I. Expenditure Elasticity<1				
1. Hedonic Method	$\alpha = 0.75$	0.499	2.456	3.616
		(0.172)	(0.529)	(0.250)
	$\beta = 0.25$	0.236	0.437	0.491
		(0.039)	(0.031)	(0.010)
	$\gamma = 0.45$	0.365	0.841	0.971
		(0.092)	(0.074)	(0.021)
2. Discrete Choice Method	$\alpha = 0.75$	0.273	1.816	3.669
		(0.734)	(2.122)	(2.564)
	$\beta = 0.25$	0.418	0.270	0.305
		(0.074)	(0.405)	(0.317)
	$\gamma = 0.45$	0.798	0.462	0.790
		(0.011)	(2.297)	(0.046)
II. Expenditure Elasticity>1				
1. Hedonic Method	$\alpha = 0.75$	0.609	4.320	5.114
		(0.614)	(1.227)	(0.431)
	$\beta = 0.50$	0.492	0.603	0.613
		(0.047)	(0.014)	(0.006)
	$\gamma = 0.55$	0.435	0.943	0.989
		(0.216)	(0.061)	(0.020)
2.Discrete Choice Method	$\alpha = 0.75$	2.045	1.097	2.532
		(0.869)	(8.147)	(0.924
	$\beta = 0.50$	1.755	0.518	0.521
		(8.522)	(0.384)	(0.235)
	$\gamma = 0.55$	0.745	0.795	0.799
		(0.164)	(0.068)	(0.056)
III. Expenditure Elasticity=1				
1. Hedonic Method	$\alpha = 0.75$	2.667	5.277	5.031
		(1.590)	(1.577)	(0.540)
	$\beta = 0.25$	0.382	0.458	0.460
		(0.066)	(0.028)	(0.011)
	$\gamma = 0.45$	0.761	0.991	0.990
		(0.197)	(0.081)	(0.030)
2. Discrete Choice Method	$\alpha = 0.75$	2.403	1.725	3.895
		(1.143)	(7.570)	(2.630)
	$\beta = 0.25$	1.524	0.324	0.370
		(9.652)	(0.483)	(0.134)
	$\gamma = 0.45$	0.789	0.795	0.806
		(0.085)	(0.062)	(0.002)

* Based upon 50 replications of each model for each of each of the 9 categories. Each replication is based upon 100 observations.

the error size is "large" than when it is "medium."

It is clear from Table 3 that neither procedure is particularly good at recovering the true parameters of the utility function. This however does not seem to inpede the ability of either model to predict consumer behavior. One indication of the ability of these two models to predict behavior in spite of their inability to recover the correct utility function parameters is noted in Table 4. The table presents the percent of observations in which consumers would make the same choice of x with either the true utility function parameters or those estimated by the statistical models. In other words, the table shows how well parameters estimated using the two methods predict the choices of the consumers in each sample of data.

Evidently, there is very little relationship between how well the estimation method recovers the true utility function parameters and how well it predicts consumers' choices among a discrete set of alternatives. For example, when the discrete choice method is used on data where the income elasticity of expenditure on x is less than 1, the estimated parameters are on average quite far from their specified values. (See Table 3.) With the medium sized error, the average estimate of β is actually negative.[9] In spite of this rather bad performance, however, Table 4 shows that those parameter estimates correctly forecast the choices of 42.6 percent of the consumers.

The results in Table 4 strongly suggest that the hedonic estimates are better than the discrete choice estimates in terms of forecasting the choices of consumers when the

9 The median of this set of parameter estimates (0.568) is considerably more reasonable.

TABLE 4

Percent of Consumer Choices Correctly Predicted*

	Error Size None	Small	Medium	Large
I. Expenditure Elasticity <1				
1. Hedonic Method	100.0%	80.4%	52.8%	28.2%
2. Discrete Choice Method	100.0	49.0	42.6	35.3
II. Expenditure Elasticity >1				
1. Hedonic Method	100.0	79.4	39.5	22.7
2. Discrete Choice Method	100.0	48.6	47.8	41.0
III. Expenditure Elasticity =1				
1. Hedonic Method	100.0	57.2	23.5	21.7
2. Discrete Choice Method	100.0	40.9	41.1	34.7

* Based upon 50 replications of each model for each of the 9 categories. Each replication is based upon 100 observations.

errors are small. Conversely, when the errors are large, the parameters estimated by the discrete choice method do much better than those estimated by the hedonic method.

The parameter estimates are sufficient to calculate compensating or equivalent variation for a consumer with any income and consumption pattern. To compare the relative performance of each estimation procedure under each simulated market, we calculate the slope of each income compensated demand curve at every point in (x,z) space which a consumer was observed to occupy. These slopes represent each consumer's willingness to pay for marginal units of x, that is, the consumer's compensating variation. Table 5 presents summary statistics on the mean values of these slopes, $(\partial U/\partial x)/(\partial U/\partial z)$, for each of the (9x50) simulated markets. It also presents the corresponding summary statistics on the same market simulations using the true parameter values. The table suggests that the hedonic procedure is somewhat more reliable at reproducing the average willingness to pay. In all nine cases, the parameter estimates from the hedonic technique predict average values of the compensating variation which are virtually identical to the true values. The mean of the average willingness to pay estimated by the hedonic method is never more than 1.7 percent different from the true mean in any of the nine categories. Further, the standard deviations reported in Table 5 indicate that, even when the error term, ε, is large, the average difference between the true and estimated compensating variations is seldom very large.

The discrete choice technique fares substantially worse in this comparison. With a small error size, the mean $(\partial U/\partial x)/(\partial U/\partial z)$ reported for the discrete choice

TABLE 5

True and Estimated Average Willingness to Pay For Marginal Unit of x
(Standard Deviations in Parentheses)*

	Error Size Small	Medium	Large
I. Expenditure Elasticity <1			
True Mean	0.670 (0.002)	0.672 (0.006)	0.675 (0.017)
1. Hedonic Method	0.666 (0.001)	0.666 (0.004)	0.665 (0.010)
2. Discrete Choice Method	0.675 (0.023)	0.679 (0.034)	0.693 (0.044)
II. Expenditure Elasticity >1			
True Mean	1.393 (0.002)	1.394 (0.008)	1.403 (0.028)
1. Hedonic Method	1.386 (0.001)	1.386 (0.005)	1.390 (0.018)
2. Discrete Choice Method	0.896 (1.791)	1.431 (0.161)	1.419 (0.066)
III. Income Elasticity =1			
True Mean	0.748 (0.002)	0.750 (0.007)	0.752 (0.022)
1. Hedonic Method	0.741 (0.001)	0.742 (0.004)	0.739 (0.014)
2. Discrete Choice Method	0.585 (0.790)	0.737 (0.108)	0.751 (0.027)

* Based upon 50 replications of each model for each of the 9 categories. Each replication is based upon 100 observations.

parameter estimates in markets II and III are substantially different from the "true" values reported in Table 5. (By 35.6 percent and 21.8 percent respectively.) The median values (not reported) do not reveal this discrepancy, however, suggesting that the divergence is limited to just a few trials. Surprisingly, this problem also seems to disappear when the error size is increased. This is quite similar to the pattern reported for the parameter estimates. (Table 3 above) Estimates of the compensating variation, using the discrete choice technique in markets II and III, become less dispersed as the size of the error increases.

Table A1 in the Appendix reports the correlation coefficients between the compensating variation measures whose means are reported in Table 5. With few exceptions, the average correlation is extremely high. Again, with the exception of the same two entries for the discrete choice model, the standard deviations are very small.[10] The average of consumers' marginal willingness to pay is well estimated by each of these methods. When the true values of the compensating variation are regressed on those estimated by each of the models, the intercept is always very close to zero, but is usually slightly negative (suggesting a tendency for the estimated compensating variations to be slightly higher than those derived from the true specified parameters). Conversely, the slope estimates tend to be slightly above one (suggesting that the estimated compensating variation rises less quickly with income than the true compensated variation).

10 Here again the median correlation coefficients are much closer to one than the mean values, suggesting that the divergences for these two entries are limited to a few of the 50 trials.

V. CONCLUSIONS

There has recently been an outpouring of research in applied welfare economics investigating consumers' willingness to pay for improved transport access, environmental amenities and a variety of public and privately produced services. This paper provides an extensive comparison of two methodologies widely employed to make such inferences.

These methodologies, so-called hedonic and discrete choice models, differ in their representation of the choices made by consumers and in their assumptions about the mathematical properties of preference sets and market price relationships.

Inferences about consumer willingness to pay based on these models are compared for three kinds of markets, (characterized by the expenditure elasticity on the commodity in question), under various error conditions (characterized as small, medium, and large errors). It is possible to compare the statistical results with the true compensated demands of consumers which were used to generate the experimental data.

The results of this analysis are summarzied in Figures 4 through 9. Figures 4, 5, and 6 summarize the marginal willingness to pay estimated by these models. Figures 7, 8, and 9 summarize the willingness to pay for a non marginal change predicted by these models.

Figure 4 presents the mean marginal willingness to pay estimated in the market where the income elasticity of expenditure is less than one. Panel A presents the distribution of estimates of willingness to pay when the error is small. Panels B and C indicate the distribution when the errors are medium and large respectively. Each panel presents the cumulative distribution of the 50 replications using the discrete choice (signified by d) and the hedonic choice model (signified by c). The panel also indicates the true average willingness to pay computed from each data set after the introduction of the stochastic error (signified by the dots.)[11] Finally, the straight line in each panel represents the mean willingness to pay for each data set in the absence of any stochastic terms. Figures 5 and 6 present analogous information for the other two markets.

When the error is small, panels A of Figure 4, 5, and 6 clearly reveal that the continuous model tends to yield underestimates of willingness to pay. (As indicated in Table 5, however, the average underestimate is quite small in magnitude). However, the distribution of the estimates is quite restricted in range. In contrast, there appears to be little bias in the willingness to pay estimated from the discrete choice model, but the range of mean estimates is much larger and the distribution is much more dispersed. When the error is increased, in Panels B and C of the figures, the same pattern persists. For roughly a fifth of the replications the "answers" provided by the discrete choice model are quite far off indeed.

Figures 7, 8, and 9 compare estimates of the willingness to pay for a non marginal change in the value of x, in this case the utility compensating income change associated with a reduction in x. In a typical applied setting, proposed changes are seldom really marginal; these figures may thus provide a more realistic comparison.

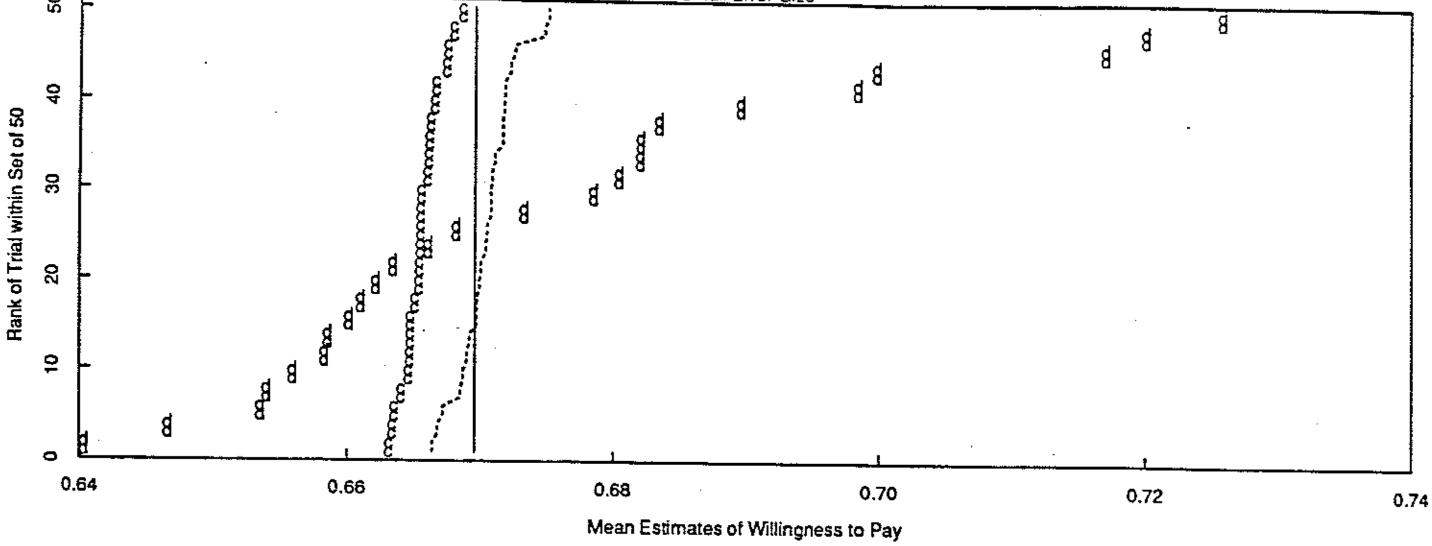
---

11 That is each dot represents $(1/100) \sum_{i=1}^{100} \left|(\partial u/\partial x)/(\partial u/\partial z)\right|_{x=x_i, z=z_i}$

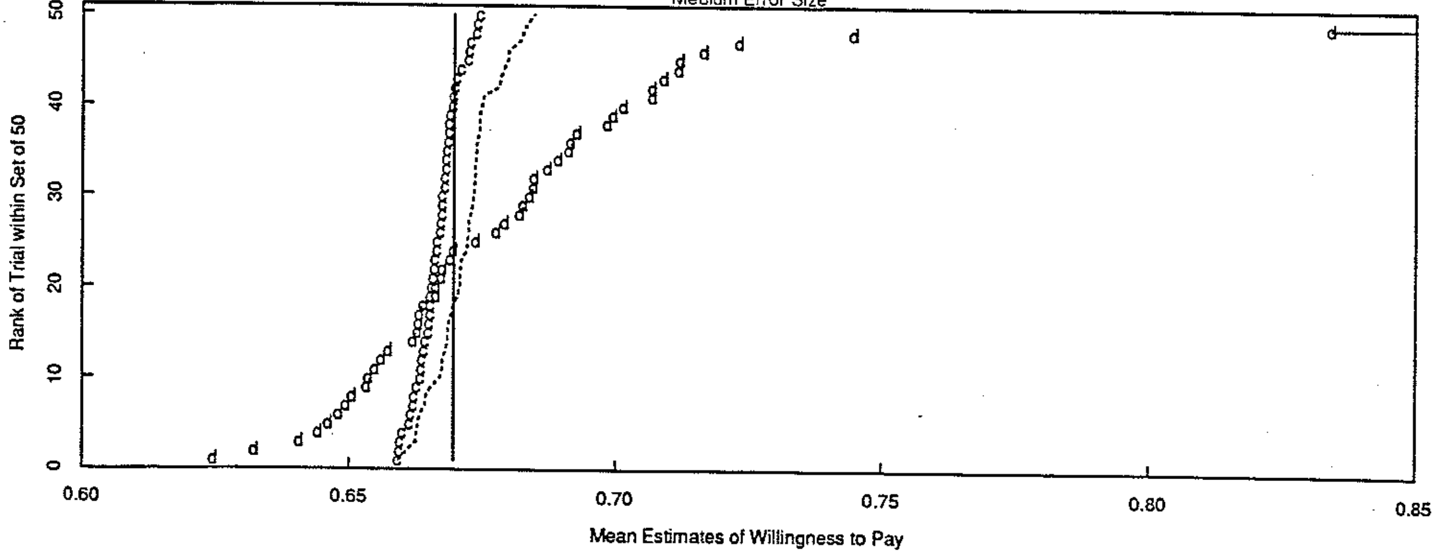
The mean willingness to pay estimated from each data set varies only because the error term induces some individuals to choose different commodities.

FIGURE 4

Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X <1
Small Error Size



Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X <1
Medium Error Size



Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X <1
Large Error Size

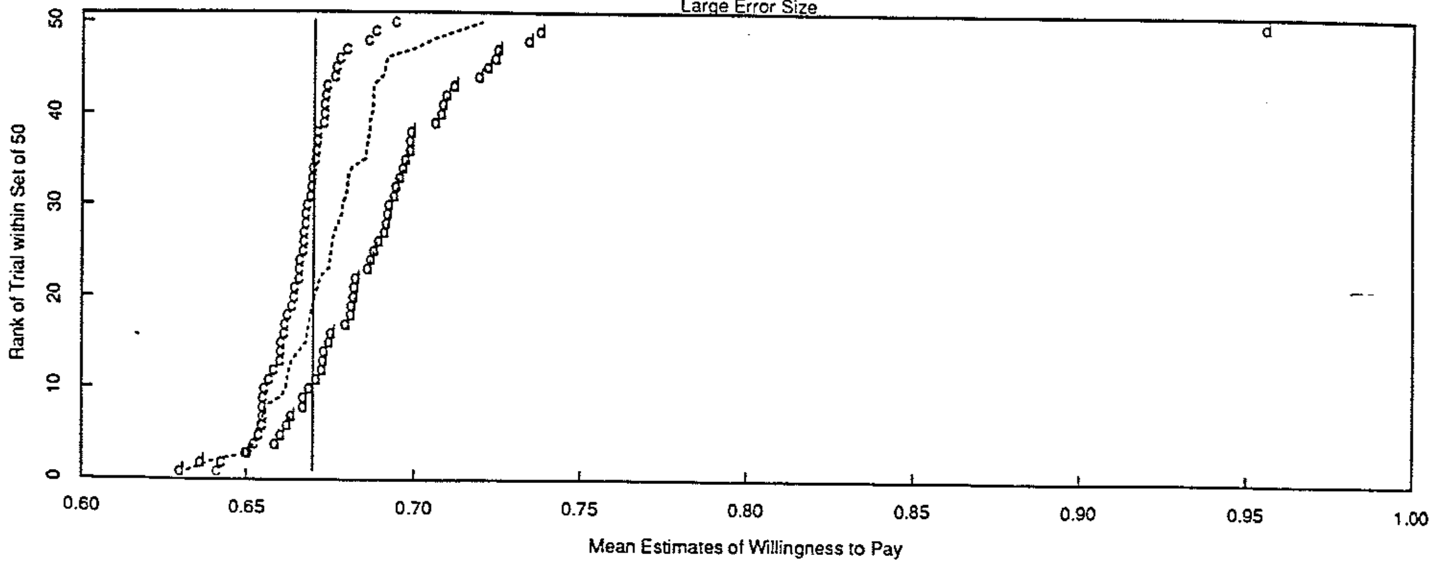
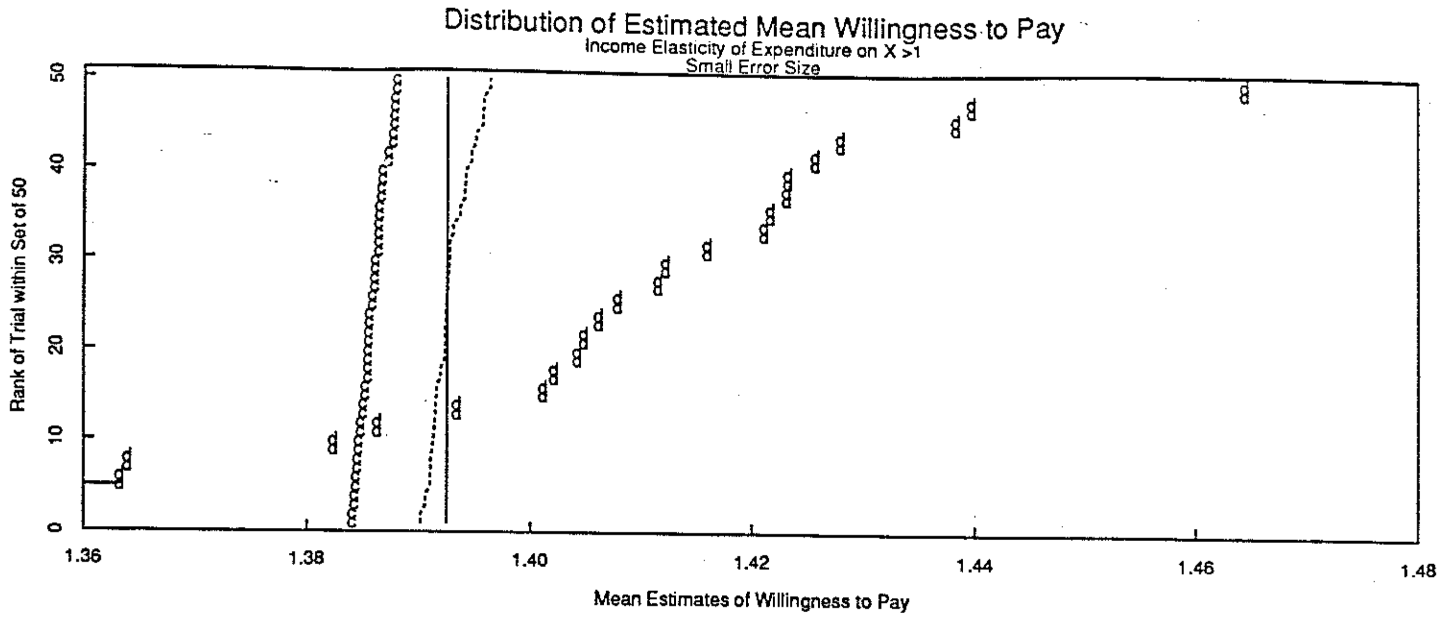
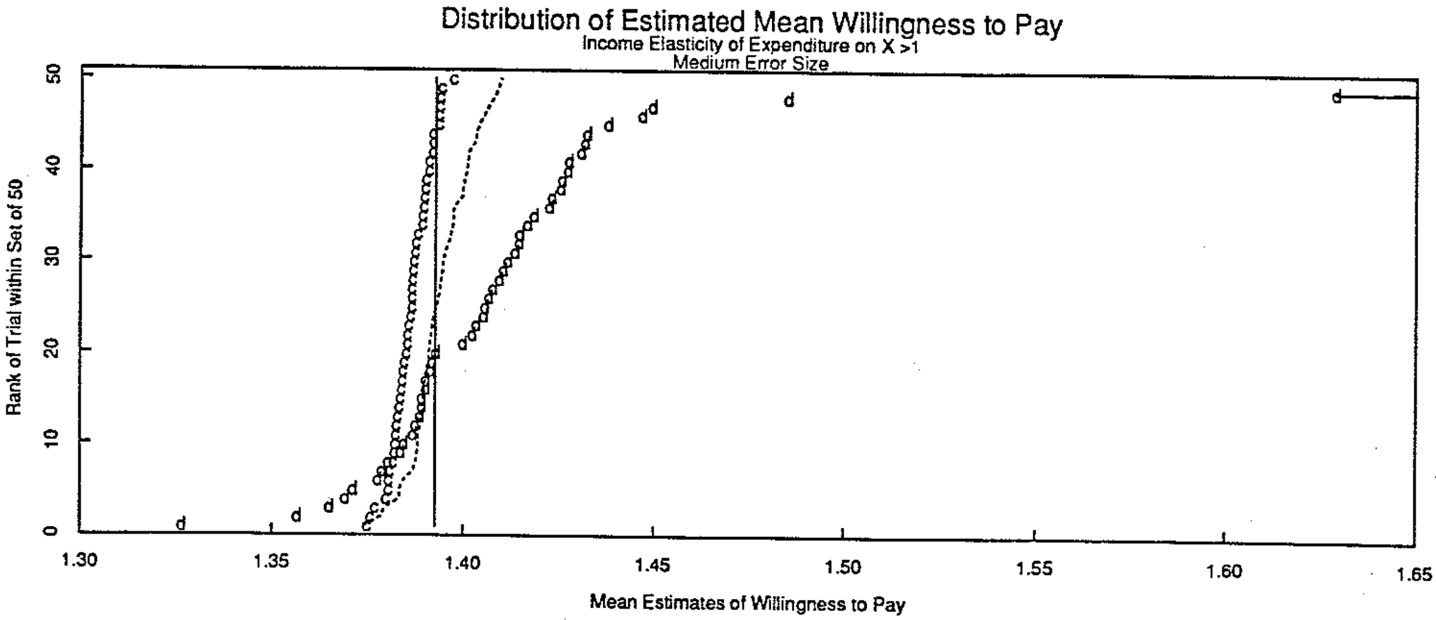


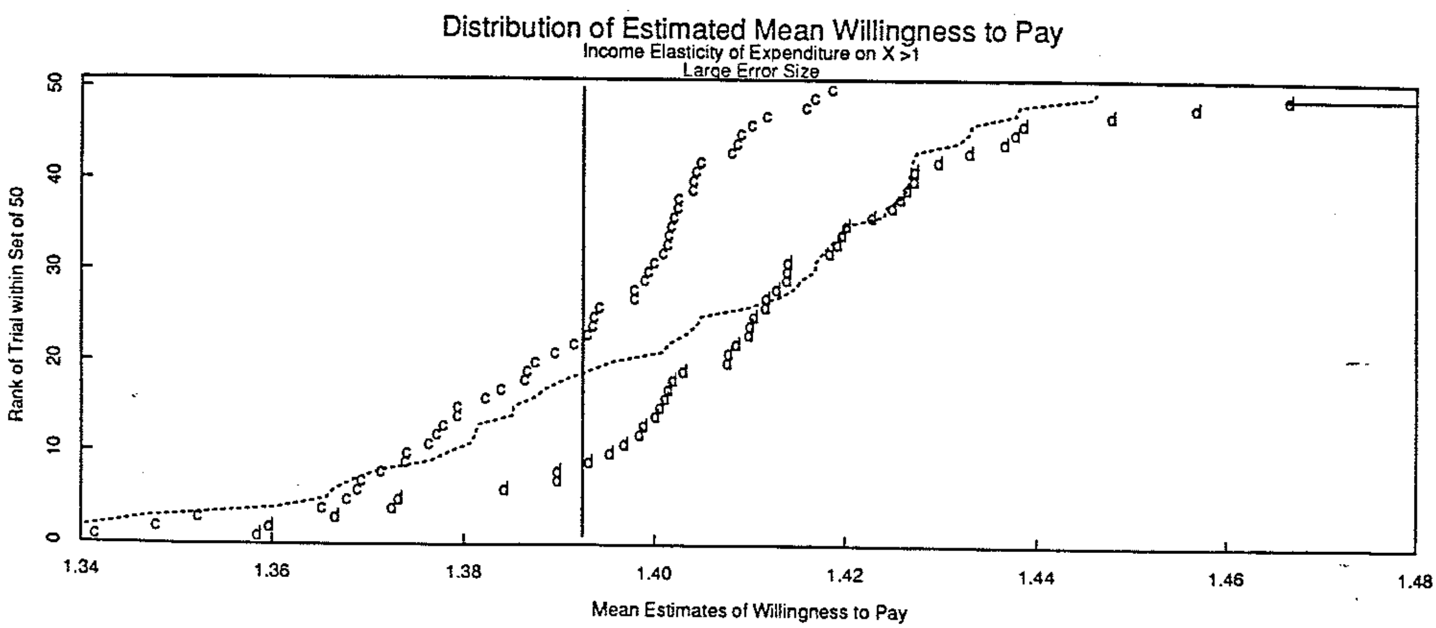
FIGURE 5



Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X >1
Small Error Size
Rank of Trial within Set of 50
Mean Estimates of Willingness to Pay
1.36
1.38
1.40
1.42
1.44
1.46
1.48



Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X >1
Medium Error Size
Rank of Trial within Set of 50
Mean Estimates of Willingness to Pay
1.30
1.35
1.40
1.45
1.50
1.55
1.60
1.65



Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X >1
Large Error Size
Rank of Trial within Set of 50
Mean Estimates of Willingness to Pay
1.34
1.36
1.38
1.40
1.42
1.44
1.46
1.48

FIGURE 6

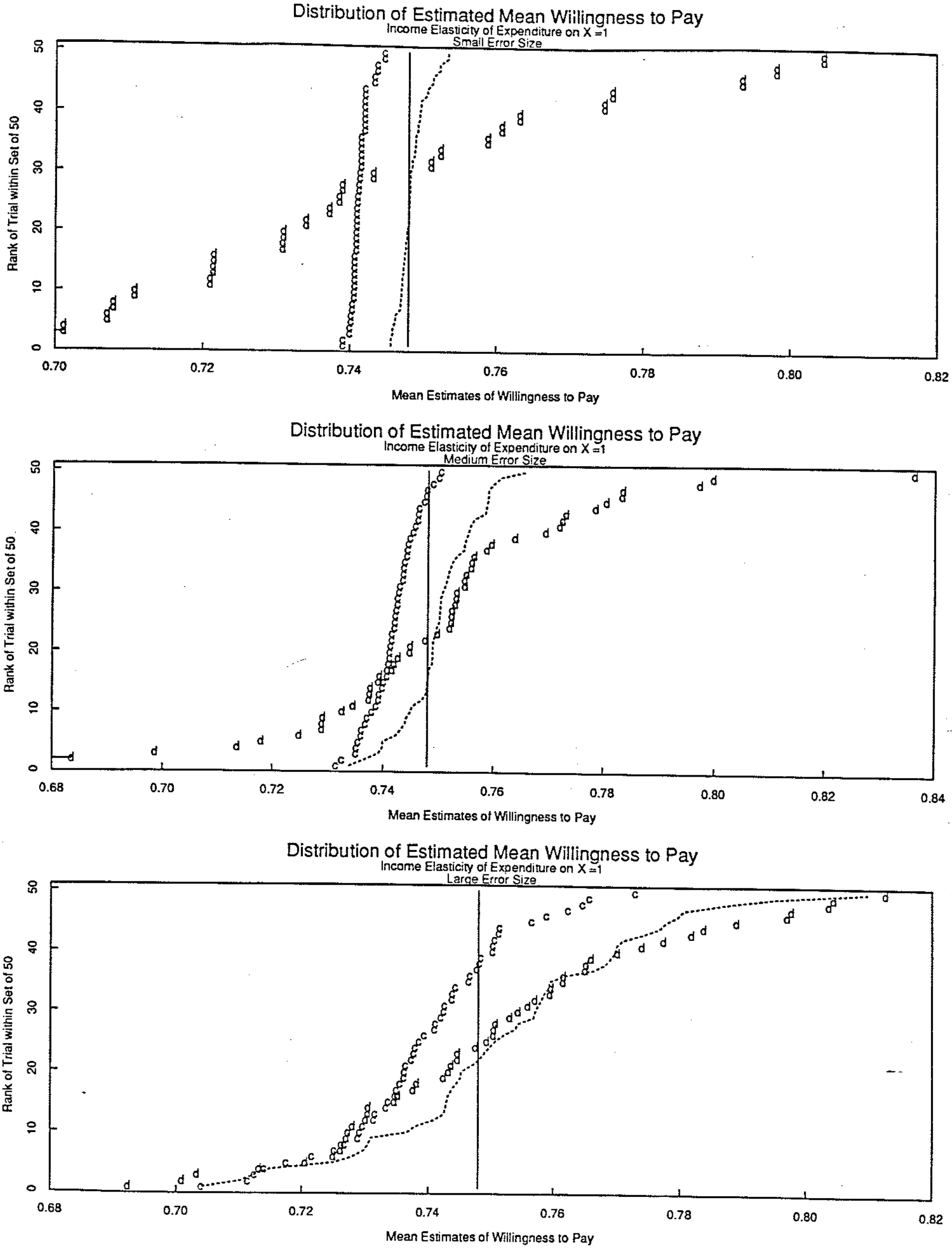

Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X =1
Small Error Size
Rank of Trial within Set of 50
0
10
20
30
40
50
0.70
0.72
0.74
0.76
0.78
0.80
0.82
Mean Estimates of Willingness to Pay
Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X =1
Medium Error Size
Rank of Trial within Set of 50
0
10
20
30
40
50
0.68
0.70
0.72
0.74
0.76
0.78
0.80
0.82
0.84
Mean Estimates of Willingness to Pay
Distribution of Estimated Mean Willingness to Pay
Income Elasticity of Expenditure on X =1
Large Error Size
Rank of Trial within Set of 50
0
10
20
30
40
50
0.68
0.70
0.72
0.74
0.76
0.78
0.80
0.82
Mean Estimates of Willingness to Pay

Figure 7

Utility Compensating Income Change for 10% decrease in X

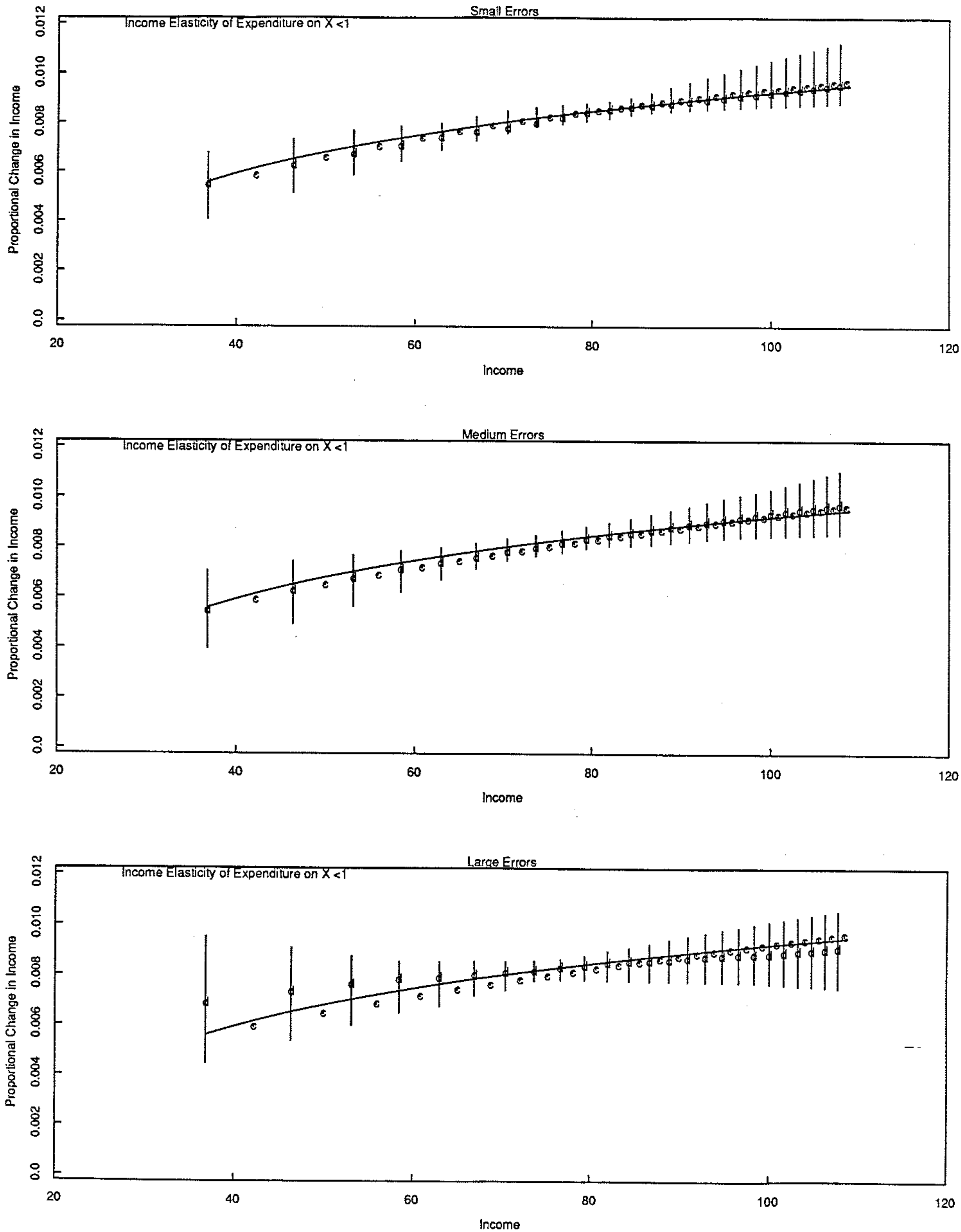


Figure 8

Utility Compensating Income Change for 10% decrease in X

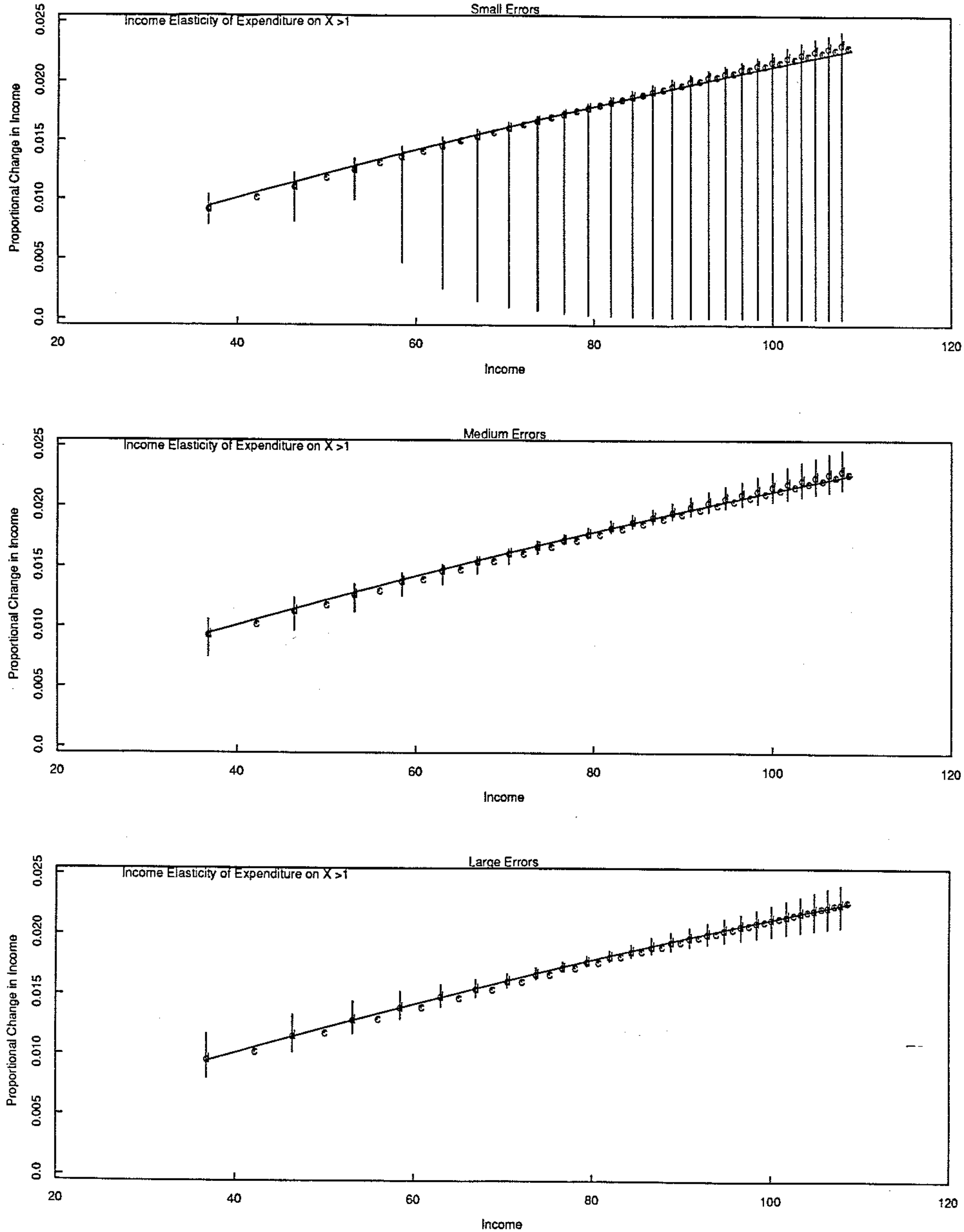
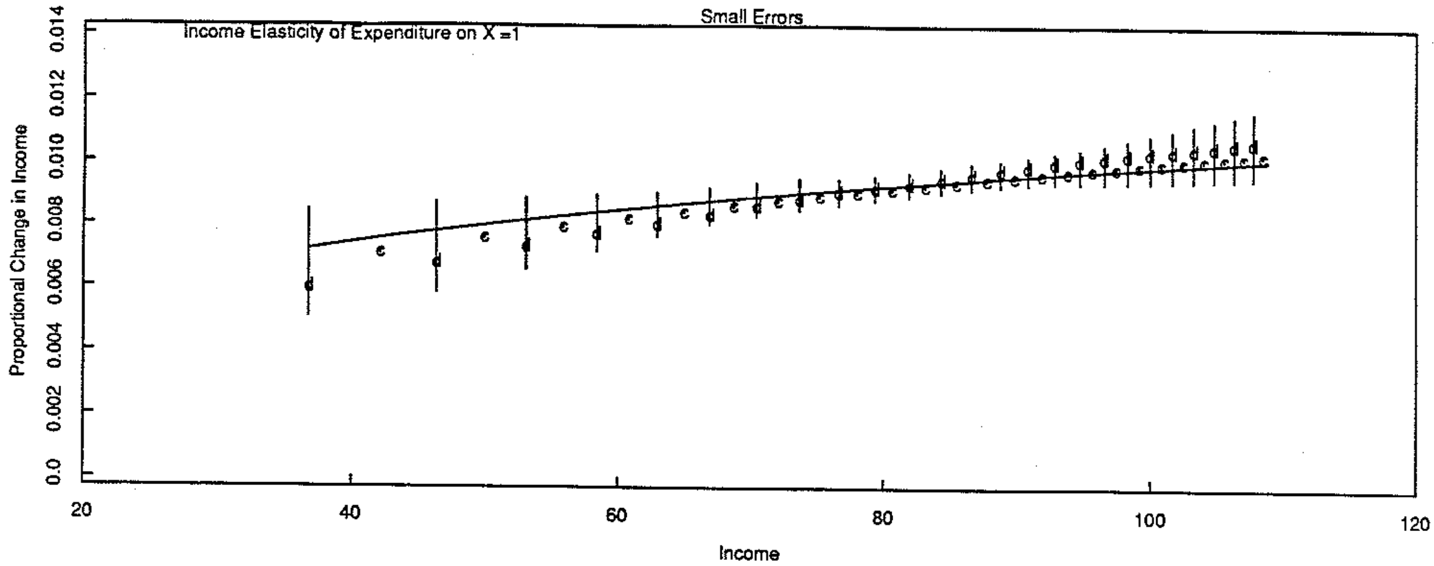
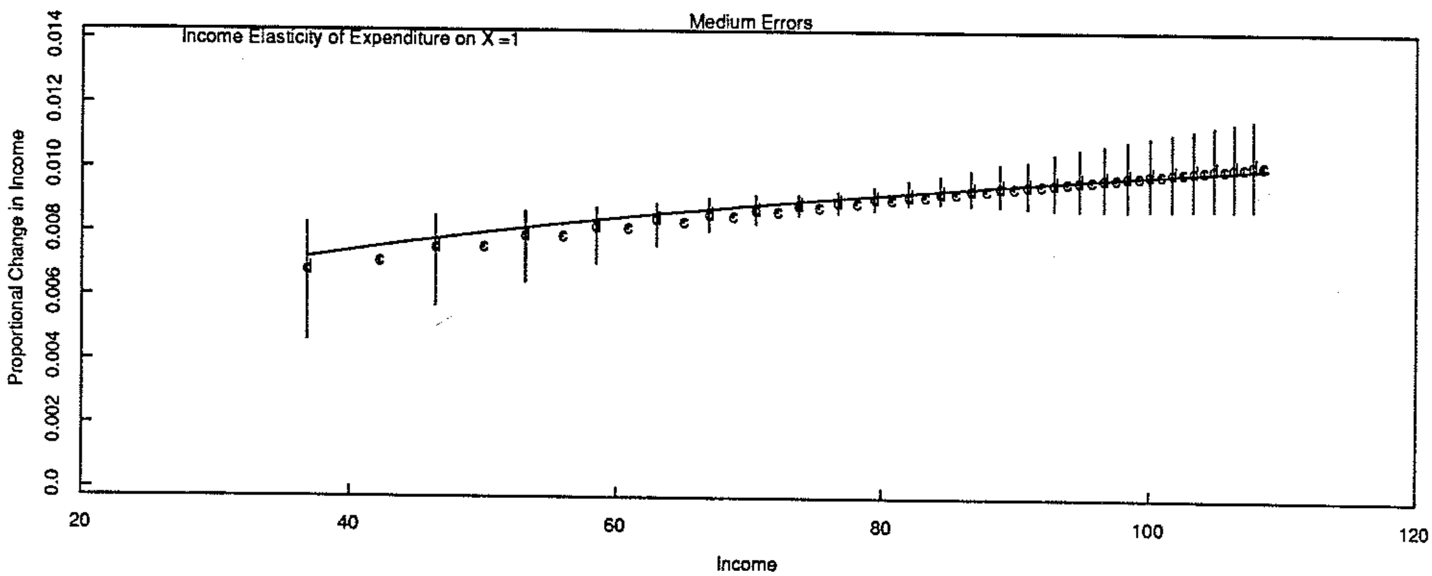


Figure 9

Utility Compensating Income Change for 10% decrease in X





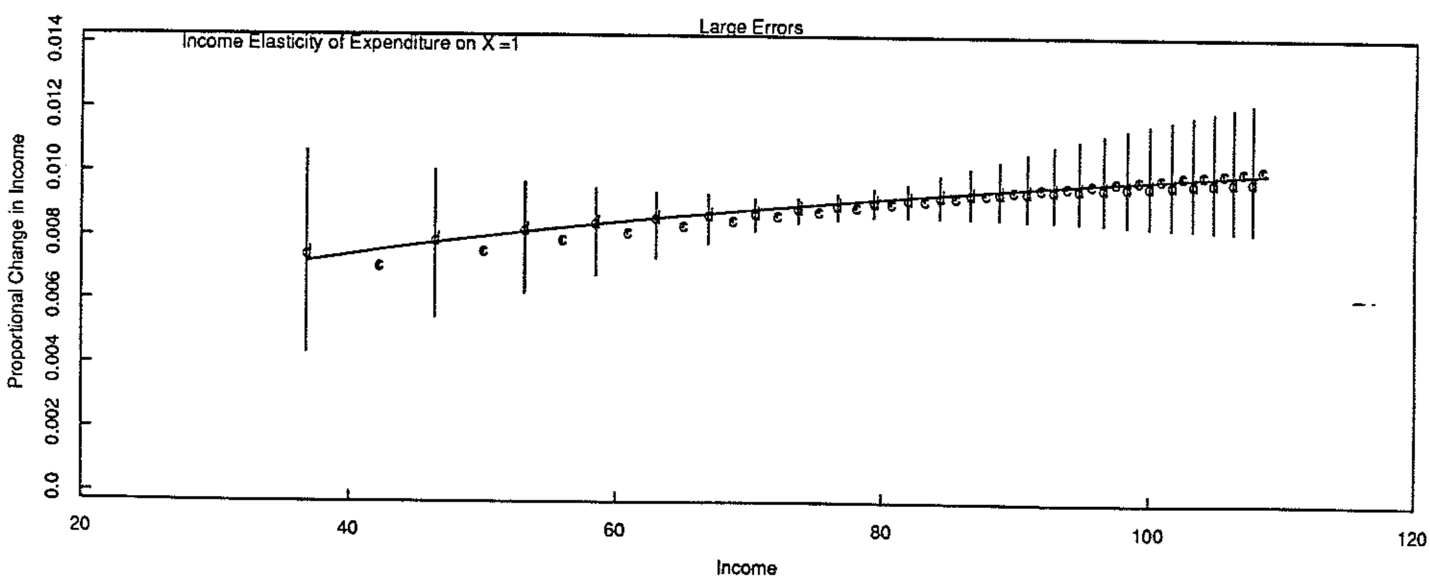


Figure 7 presents estimates of the income required to offset a ten percent decrease in x for the market in which the expenditure elasticity is less than one. The figure reports the mean estimate by income level for the discrete (again, signified by d) and continuous (signified by c) models as well as the true utility compensating income change. The figure also presents information on the range of estimates provided by the fifty replications. The solid vertical line represents the range of the estimates, trimmed to 80 percent (that is, by ignoring the five largest and the five smallest estimates).

Panel A presents the distribution of estimates when the errors are small; panels B and C present similar information when the errors are medium and large respectively.

The results confirm that each of the models does a good job of estimating the compensated demand for a non marginal change, at least on average, but that there are large deviations about that average. The deviations are, in almost all cases, smaller at average incomes than at the extremes.

In most cases, moreover, there is a tendency towards underestimation of the compensated demands at the low end of the income distribution and overestimation at the high end of the distribution. These tendencies are quite small, however.

It is of course quite dangerous to draw firm or general conclusions from an investigation using a restricted class of utility functions and very particular market settings. Nevertheless, it appears that the hedonic method yields estimates of consumer willingness to pay that are at least as good, from the view-point of applications, as those yielded by the discrete choice model.[12] Although both methods do remarkably well on average, the dispersion is much larger for the discrete model.

Whether this results from the specific simulations investigated, or the specific sampling rule, or is of more general application to similar problems in welfare economics, must await futher analysis. The results of this analysis do, however, increase the credibility of many empirical estimates presented in the applied literature about the value of air quality, workplace accessibility or alternative transport modes.

12 It should be noted that this finding arises even though the method of error generation does not insure that the errors for the hedonic regressions have zero mean.

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APPENDIX

TABLE A1

Average Correlation Coefficient between True and Estimated Willingness to Pay (Standard Deviations in Parentheses)*

	Error Size		
	Small	Medium	Large
I. . Expenditure Elasticity <1			
1. Hedonic Method	1.000 (0.000)	0.999 (0.001)	0.982 (0.007)
2. Discrete Choice Method	0.999 (0.001)	0.987 (0.073)	0.982 (0.040)
II.. Expenditure Elasticity >1			
1. Hedonic Method	1.000 (0.000)	0.999 (0.001)	0.989 (0.004)
2. Discrete Choice Method	0.873 (0.386)	0.992 (0.048)	0.992 (0.029)
III. Expenditure Elasticity =1			
1. Hedonic Method	1.000 (0.000)	0.999 (0.001)	0.988 (0.005)
2. Discrete Choice Method	0.942 (0.284)	0.992 0.046)	0.993 (0.003)

* Based upon 50 replications of each model for each of the 9 categories. Each replication is based upon 100 observations.

APPENDIX

Figure 1
Income Expansion Paths

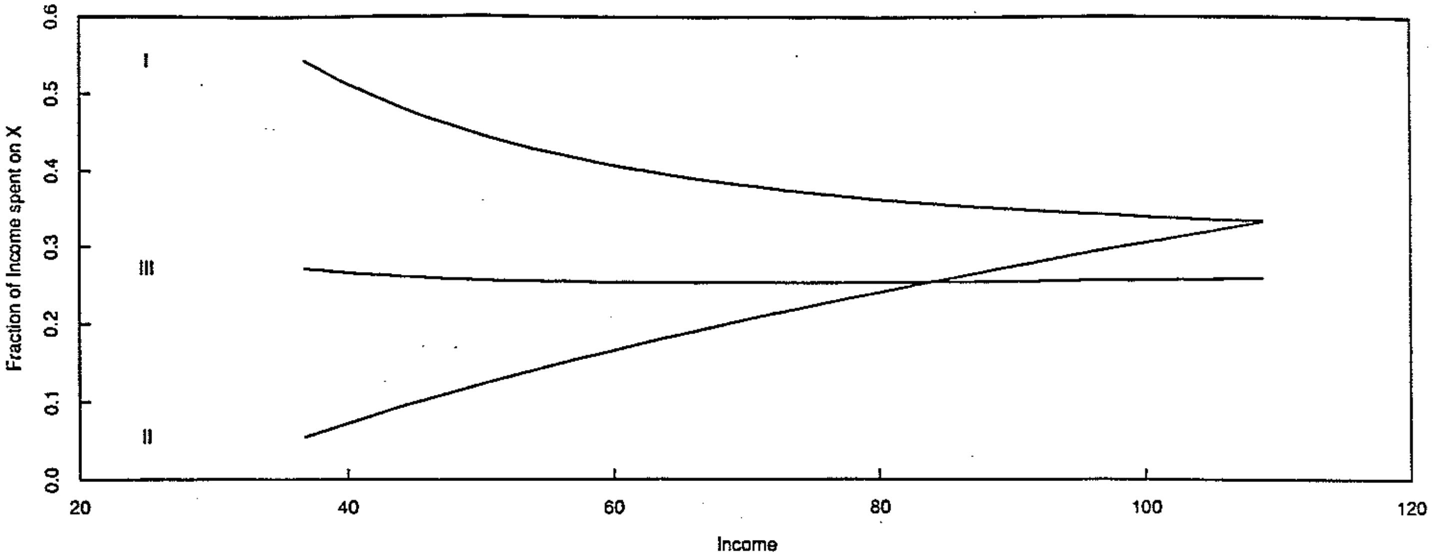


Figure 2
G(X) Mapping

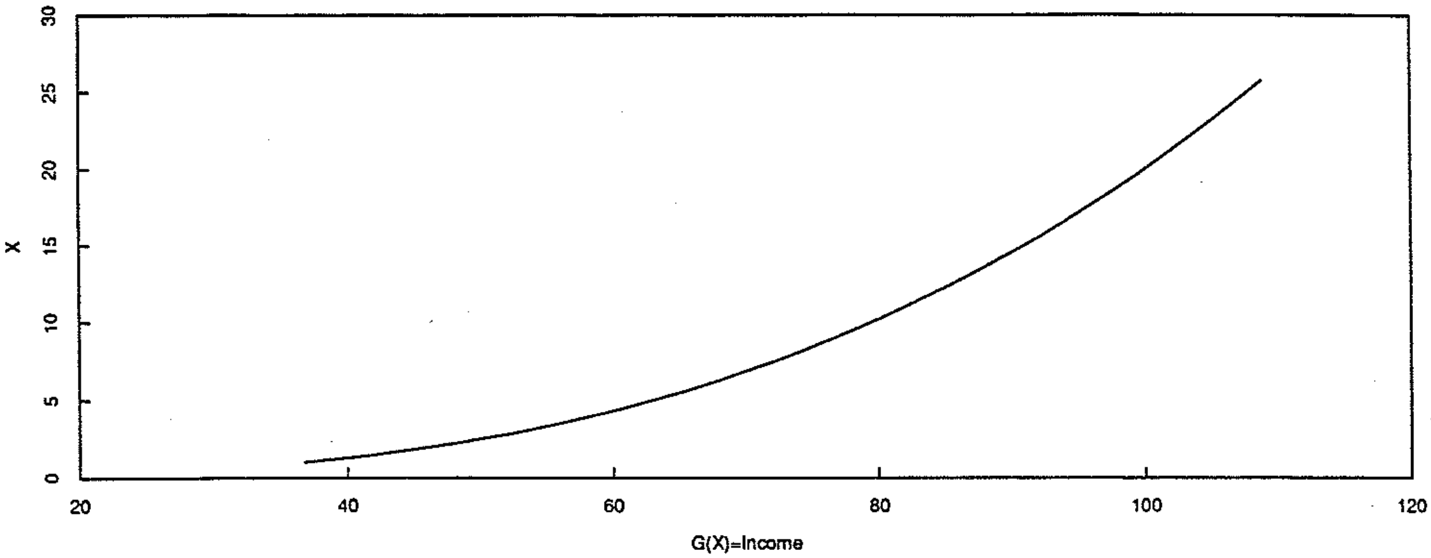
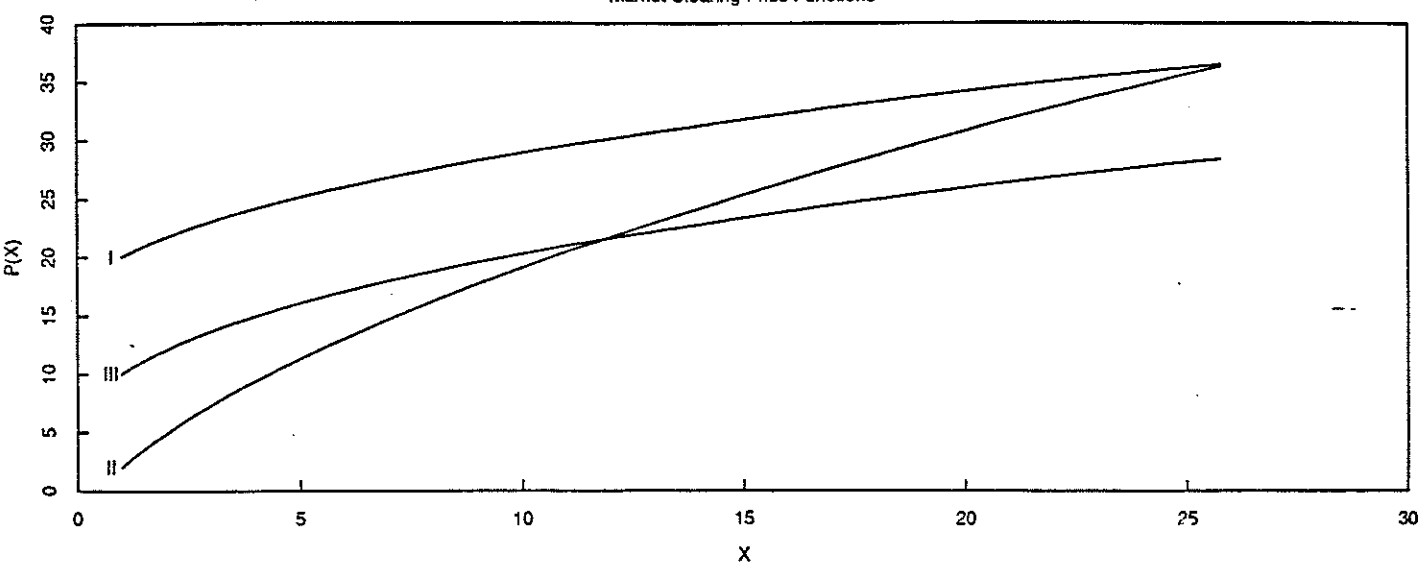


Figure 3
Market Clearing Price Functions



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