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Neutralino dark matter in focus point supersymmetry

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Abstract

In recent work, it has been argued that multi-TeV masses for scalar superpartners are not unnatural. Indeed, they appear to have significant phenomenological virtues. Here we explore the implications of such 'focus point' supersymmetry for the dark matter problem. We find that constraints on relic densities do not place upper bounds on neutralino or scalar masses. We demonstrate that, in the specific context of minimal supergravity, a cosmologically stable mixed gaugino-Higgsino state emerges as an excellent, robust dark matter candidate. We estimate that, over a wide range of the unknown parameters, the spin-independent proton-neutralino cross sections fall in the range accessible to planned search experiments.

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1. Introduction

The idea that supersymmetry is an approximate symmetry of Nature, broken at a 'low' scale, is best and most concretely motivated by two general arguments. First, it addresses the question of why radiative corrections to the electroweak symmetry breaking scale, due to the exchange of virtual particles at some much larger unification or Planck scale, do not pull the electroweak scale to a much higher value (the gauge hierarchy problem [1]). Second, it brings the unification of couplings calculation, which does not quite work if we use only the particles of the Standard Model, into adequate agreement with experiment [2]. However neither of these arguments, on the face of it, yields very specific constraints on the spectrum of masses of the supersymmetric (R-parity odd) partners of ordinary (R-parity even) particles: the first because it is inherently semi-quantitative, the second because it depends on these masses only logarithmically.

On the other hand, it is becoming notorious that the existence of superpartners with masses below 1 TeV is difficult to reconcile with established limits on various flavor violating effects, unless one assumes accurate degeneracy of squarks and of sleptons. Even if one assumes such degeneracy, supersymmetric contributions to CP violation and (in unified theories) proton decay become uncomfortably large, unless the squarks and sleptons are heavy.

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Therefore one is motivated to examine supersymmetric models with large scalar superpartner masses. The issue immediately arises: To what extent can such models, when critically scrutinized, embody the motivating features we mentioned in our opening paragraph?

Recently it has been argued, using a seemingly reasonable objective definition of naturalness, that all squark and slepton masses can be taken well above 1 TeV with no loss of naturalness [3,4]. The mathematical basis of this result is the existence of focus points in renormalization group trajectories, which render the weak scale (i.e., the Higgs potential) largely insensitive to variations in unknown supersymmetry parameters [3–5]. We should note that while in these models the squark and slepton masses are unusually large (compared to conventional wisdom), the electroweak gaugino and Higgsino particle masses are generically well below 1 TeV.

In the focus point models, moreover, the quantitative aspect of the unification of coupling constants is actually slightly improved relative to more traditional supersymmetry models, reducing the need for significant high scale threshold corrections [6].

Besides preserving and sharpening the original motivations for supersymmetry, the focus point models with heavy scalars ameliorate the problems noted above [6]. Specifically, constraints on CP violation from electric dipole moments can be satisfied even with $\mathcal{O}(1)$ phases in the fundamental parameters, and the predicted rate for nucleon decay – being proportional to the square of gaugino and inversely proportional to the fourth power of squark/slepton masses – is suppressed and less dangerous, though still perhaps accessible.

In this note we will consider the implications of focus point supersymmetry for neutralino dark matter. We will concentrate on the focus point mechanism in its simplest 'default' incarnation, assuming a universal scalar mass within minimal supergravity. The focus point mechanism can also be realized in alternative models featuring gravity- [3,4], gauge- [7], and anomaly-mediated [8] supersymmetry breaking. In addition, the requirement of a universal scalar mass, though sufficient, is not necessary. For the precise set of requirements, see Ref. [4].

Of course, the cosmology of minimal supergravity (with universal scalar mass $m_0 \leq 1$ TeV) has been studied extensively. Prominent among conclusions one finds in the literature are the following:

(i) In essentially all of parameter space, the lightest neutralino χ is Bino-like, with gaugino fraction $R_\chi > 0.9$ [9]. (See below for the definition of R_χ.)

(ii) The neutralino relic abundance $\Omega_\chi h^2$ increases as m_0 increases. The requirement that neutralinos not overclose the universe, along with the requirement that the lightest supersymmetric particle (LSP) be neutral, then typically leads to stringent upper bounds on scalar and/or gaugino masses, independent of naturalness considerations [10–19].

(iii) Dark matter detection rates decrease as m_0 increases. The rates predicted range widely over parameter space, but the prospects for discovery at ongoing experiments grow increasingly dim as m_0 increases [20].

In our analysis, we confirm these conclusions – for $m_0 \leq 1$ TeV. However, we find that they fail for focus point models with $m_0 > 1$ TeV. Instead, we find that in such models:

(i) The lightest neutralino is a gaugino-Higgsino mixture over much of parameter space.

(ii) Neutralino relic abundances do not overclose the universe. Rather, they tend to lie in the cosmologically interesting range. Therefore, in particular, cosmological considerations alone do not place stringent upper limits on superpartner masses, even apart from possible conspiracies involving poles [21,22] or co-annihilation [16,23].

(iii) Direct detection rates may be fairly large, with proton-neutralino cross sections plausibly falling within the range 10^{-6}–10^{-8} pb. (See the more detailed discussion and plots below.)

These results are especially interesting since ongoing and planned dark matter detection experiments promise to cover the indicated range.

2. Neutralinos in focus point supersymmetry

The models we consider, like many supersymmetric models, contain a natural cold dark matter candidate [24]. Assuming that $R \equiv (-)^{B+L+2S}$ is accurately conserved, the lightest supersymmetric (R-odd) particle is stable. We will be considering models where, with no special adjustment, this is a neutral weakly interacting massive particle (WIMP), which is commonly called the neutralino.

The relic densities and detection rates of neutralino dark matter can be calculated, assuming straightforward extrapolation of Big Bang cosmology to $T \sim 10$ GeV [25]. In principle they depend on details of the entire supersymmetric spectrum, but in practice, especially in focus point models, they are mainly determined by the properties of the neutralino LSP itself.

Assuming R-parity conservation and minimal field content, at tree level the neutralino mass matrix is

$$\begin{pmatrix} M_1 & 0 & -m_Z\cos\beta\sin\theta_W & m_Z\sin\beta\sin\theta_W \\ 0 & M_2 & m_Z\cos\beta\cos\theta_W & -m_Z\sin\beta\cos\theta_W \\ -m_Z\cos\beta\sin\theta_W & m_Z\cos\beta\cos\theta_W & 0 & -\mu \\ m_Z\sin\beta\sin\theta_W & -m_Z\sin\beta\cos\theta_W & -\mu & 0 \end{pmatrix} \tag{1}$$

in the basis $(-i\tilde{B}, -i\tilde{W}^3, \tilde{H}_u^0, \tilde{H}_d^0)$. Here M_1 and M_2 are the soft Bino and Wino masses, μ is the Higgsino mass parameter, and $\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$ is the ratio of Higgs vacuum expectation values. We parametrize the gaugino/Higgsino content of the lightest neutralino according to

$$\chi = a_1(-i\tilde{B}) + a_2(-i\tilde{W}^3) + a_3\tilde{H}_u^0 + a_4\tilde{H}_d^0 , \tag{2}$$

and define the gaugino fraction of χ to be

$$R_\chi \equiv |a_1|^2 + |a_2|^2 . \tag{3}$$

In the framework of minimal supergravity, there are 4 continuous parameters and 1 binary choice:

$$m_0, M_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu) . \tag{4}$$

Here, m_0, $M_{1/2}$, and A_0 are, respectively, the universal scalar mass, gaugino mass, and trilinear scalar coupling at the grand unified theory (GUT) scale $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV. Given values for these input parameters, all the couplings and masses of the weak scale Lagrangian are determined through renormalization group (RG) evolution. In our work we use two-loop RG equations [26] and include one-loop threshold corrections from supersymmetric particles to the gauge and Yukawa coupling constants [27,28]. We minimize the Higgs potential after including all one-loop effects, and include one-loop corrections in all superpartner masses [28]. This framework imposes several specific relations among the weak scale supersymmetry parameters. In particular, $M_1 \simeq M_2/2 \simeq 0.4 M_{1/2}$, and $|\mu|$ is fixed by the condition of electroweak symmetry breaking, which, at tree-level, is

$$\tfrac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \approx -m_{H_u}^2 - \mu^2 , \tag{5}$$

where the last relation is valid for moderate and large $\tan\beta$.

The essence of the focus point mechanism is the observation that the weak scale value of $m_{H_u}^2$ is remarkably insensitive to variations in the fundamental GUT scale supersymmetry parameters, even for multi-TeV m_0. By Eq. (5), therefore, for $\tan\beta \gtrsim 5$ [4], the electroweak scale is insensitive to variations in these parameters, and in this sense multi-TeV values of m_0 are natural.

Contours of gaugino fraction R_χ are presented in Fig. 1. (For reference, contours for the fine-tuning parameter c, as defined in Ref. [3], are also given in Fig. 1. While large $M_{1/2}$ leads to large fine-tuning, large m_0 does not, as a result of the focus point discussed above.) For $m_0 \leq 1$ TeV, the lightest neutralino is nearly pure Bino, with $R_\chi \gtrsim 0.9$. This well-known result arises from the circumstance that RG evolution typically drives $m_{H_u}^2$ large (relative to the gaugino masses) and negative, which by Eq. (5) implies $|\mu|$ much larger than M_1 and M_2.

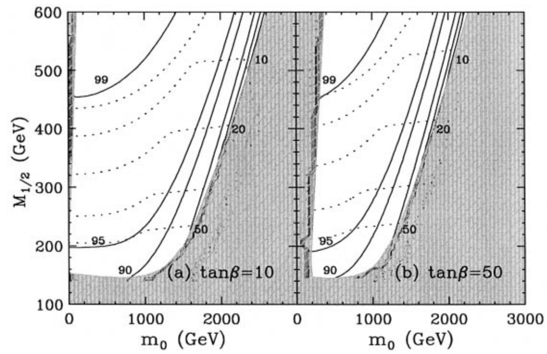

Fig. 1. Contours of constant LSP gaugino fraction R_χ (in percent) in the $(m_0, M_{1/2})$ plane for $A_0 = 0$, $\mu > 0$, and two representative values of $\tan\beta$. The shaded regions are excluded by the requirement that the LSP be neutral (top left) and by the chargino mass limit of 95 GeV (bottom and right). Dashed contours are for the fine tuning parameter $c = 20$, 30, 50, 75 and 100, from below (see text).

For $m_0 > 1$ TeV, however, χ contains significant amounts of Higgsino over much of the parameter space. For large and moderate values of $\tan\beta$, large values of m_0 generate positive corrections to $m^2_{H_u}$, rendering it less negative. Thus if all other parameters are fixed, we find that as m_0 increases, $|\mu|$ decreases, and eventually $|\mu| \sim M_1, M_2$, which leads to significant mixing between Higgsino and gaugino states. As m_0 increases further, the Higgsino content of χ increases until ultimately one enters the domain $|\mu| < 95$ GeV, which is excluded by limits from chargino searches at LEP II [29]. At that point, we have entered the shaded region of Fig. 1.

Note that the above discussion holds for $\tan\beta \gtrsim 5$ where the focus point mechanism is operative. For small $\tan\beta$, μ becomes sensitive to $m^2_{H_d}$, and large m_0 gives *negative* contributions to $m^2_{H_u}$. Both of these effects imply that, as m_0 increases, $|\mu|$ also increases, and so there is no mixed gaugino-Higgsino region for small $\tan\beta$.

3. Relic abundance

There is ample and increasingly precise evidence that the energy density of the universe is not dominated by the observed luminous matter [30]. The evidence is conveniently expressed in terms of ratios $\Omega_i \equiv \rho_i/\rho_c$, where the ρ_i are energy densities and ρ_c is the critical density, and $h \approx 0.65 \pm 0.1$, the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}. The luminous matter density is roughly $\Omega_{\text{lum}} \sim 0.005$, and the successful predictions of Big Bang nucleosynthesis require baryon density $\Omega_b h^2 \approx 0.02$. At the same time, rotation curves of spiral galaxies require matter density $\Omega_m \gtrsim 0.1$, and a variety of observations, ranging from the velocities of galaxies within galactic clusters to the luminosities of Type Ia supernovae, favor values in the range $0.2 \lesssim \Omega_m \lesssim 0.6$. There is therefore a need for both baryonic and non-baryonic dark matter. For non-baryonic dark matter, which is of interest here, galactic rotation curves require $\Omega_{\text{DM}} h^2 \gtrsim 0.025$, and large scale measurements suggest $0.1 \lesssim \Omega_{\text{DM}} h^2 \lesssim 0.3$.

Neutralinos remain in thermal equilibrium until their pair annihilation rate drops below the Hubble expansion rate. The relic density of neutralinos is therefore determined primarily by their cross section for pair annihilation in the early universe. Neutralinos annihilate to fermion pairs through the t-channel exchange of sfermions $\tilde{f}$, and via $\chi\chi \to Z, A, h, H \to f\bar{f}$, where A is the CP-odd Higgs boson, and h and H are the lighter and heavier CP-even Higgs bosons. When kinematically accessible, annihilation to two body states of gauge and Higgs bosons are also relevant.

The calculation of neutralino relic density has been discussed at length in the literature. We use the results of Drees and Nojiri [21,31], as implemented by Jungman et al. [32]. *S*- and *P*-wave contributions to all tree-level processes with two-body final states are included.

In Fig. 2 we display the neutralino relic density $\Omega_\chi h^2$ in the $(m_0, M_{1/2})$ plane for two representative values of $\tan\beta$, $A_0 = 0$, and $\mu > 0$. These plots exhibit the most important parameter dependencies. Variations in A_0 within its natural range have little effect, as the A parameter flows to a fixed point at the weak scale. Similarly, changing the sign of μ does not produce qualitatively new results. We have chosen the sign less constrained by $b \to s\gamma$ (see below).

In most of parameter space, the expected accuracy in Fig. 2 is $\mathcal{O}(10\%)$ [32]. More significant errors are possible in special regions. First, a more refined treatment is necessary just below annihilation thresholds [33,34]. Such a treatment will smooth out the kinks in the contours of Fig. 2, which are caused by the opening of annihilation channels such as *WW* and *ZZ*. Second, co-annihilation is important very near the left and right borders of the allowed region, where the LSP is nearly degenerate with staus [16,35] and Higgsinos [36], respectively. Finally, *s*-channel poles also require a more careful analysis [33,34,37,14]. The Z pole possibility, $2m_\chi = m_Z$, is now essentially eliminated by the chargino mass bound of 95 GeV [29], but a h pole is possible near the bottom of the allowed region, as indicated in Fig. 2. H and A poles, though possibly significant for high $\tan\beta$ in the $m_0 \leq 1$ TeV region [22,38], are absent for $m_0 > 1$ TeV for all $\tan\beta \lesssim 50$ and $\mu > 0$. (The sign of μ enters through the finite supersymmetric threshold corrections to the bottom Yukawa coupling [39].) Thus, although special effects warrant more sophisticated analysis in limited regions of parameter space, they are absent in the bulk of the focus point region. They do not affect our main conclusions.

For sub-TeV m_0, we see that $\Omega_\chi h^2$ is a monotonically increasing function of m_0. In this region, the LSP is nearly Bino-like, and so the dominant process is $\chi\chi \to f\bar{f}$ through *t*-channel $\tilde{f}$ exchange. As m_0 and $m_{\tilde{f}}$ increase, this annihilation process is suppressed, and the relic density grows. This behavior, along with the requirement of a neutral LSP, has led many authors to conclude that upper bounds on the relic density typically impose stringent upper limits on superpartner masses [10–17]. As these results are independent of, and less subjective than, considerations based on naturalness or the fine structure of gauge coupling unification, they have often been taken, by the authors themselves or others, as robust phenomenological upper bounds. If correct, this conclusion would have important implications for supersymmetry searches at the Tevatron and the LHC, and in planning for future linear colliders [14,18,19].

But when we consider $m_0 > 1$ TeV, motivated by the focus point or otherwise, we find that the behavior of this function reverses. The reason for this behavior is not difficult to locate. For although as m_0 increases the *t*-channel sfermion exchange process is more and more suppressed, as noted in Section 2, at the same time the LSP gradually acquires a significant Higgsino component. Because of this, other diagrams become unsup-

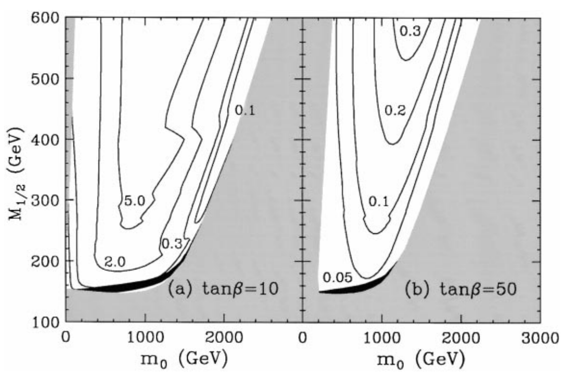

Fig. 2. Contours of constant relic density $\Omega_\chi h^2$ in the $(m_0, M_{1/2})$ plane for $A_0 = 0$, $\mu > 0$, and two representative values of $\tan\beta$. In the black shaded region, $|2m_\chi - m_h| < 5$ GeV, and the h pole becomes important. The light shaded regions are as in Fig. 1.

pressed, and pair annihilation again becomes efficient. The cosmologically interesting region, with a mixed gaugino-Higgsino LSP and $0.1 \lesssim \Omega_\chi h^2 \lesssim 0.3$, extends at least to $m_0 \sim 10$ TeV, $M_{1/2} \sim 6$ TeV and $m_\chi \sim 2.5$ TeV. Thus, for all collider applications, cosmological considerations do *not* provide upper bounds on superpartner masses, even in the constrained framework of minimal supergravity. It has been noted previously that cosmological bounds might be weakened or eliminated by the effects of co-annihilation [16,23] or poles [21,22]. Here, we see that even without appealing to such conspiratorial effects, sweeping claims that cosmology provides upper bounds on superpartner masses are unfounded. Note that the cosmologically interesting region of parameter space with $m_0 > 1$ TeV is in no sense small. For fixed $M_{1/2}$, this region extends over one to several hundred GeV in m_0, depending on $\tan\beta$. It is comparable in area to the allowed region with $m_0 \leq 1$ TeV.

For all $\tan\beta \gtrsim 5$, we find relic densities in the cosmologically interesting range $0.025 < \Omega_\chi h^2 < 0.3$ over most of the focus point regime with $m_0 > 1.5$ TeV. In fact, as can be seen from Fig. 2, for $\tan\beta = 50$, $\Omega_\chi h^2$ never saturates the upper bound, even for $m_0 \sim 1$ TeV. This additional suppression of $\Omega_\chi h^2$ is not due to any enhancement of the LSP's Higgsino component from large $\tan\beta$. As can be seen in Fig. 1, the gaugino purity of the LSP is largely independent of $\tan\beta$. Rather it results from two enhancements to the process $\chi\chi \to A \to f\bar{f}$. First, the $Af\bar{f}$ coupling is proportional to $\tan\beta$ for isospin $-\frac{1}{2}$ fermions. Second, for large $\tan\beta$, the bottom Yukawa coupling h_b becomes significant and lowers m_A. This can be seen by noting that, for $h_t = h_b$, symmetry under interchange $t \leftrightarrow b$ implies that $m^2_{H_d}$ also has a weak scale focus point, and so $m_A \sim \mathcal{O}(100\ \text{GeV}) \ll m_0$.

For small $\tan\beta$, as noted in Section 2, there is no mixed gaugino-Higgsino region for large m_0. Thus, $\Omega_\chi h^2$ grows monotonically as m_0 increases. Low values of $\tan\beta$, such as $\tan\beta = 2$, have been widely considered in the literature. They are, however, increasingly disfavored by current null results in Higgs searches, and, in any case, are less natural than moderate and large values, as low $\tan\beta$ typically leads to large values of $|\mu|$ [4].

4. Direct detection

Having shown that focus point models predict cosmologically interesting densities of dark matter, we now consider their predictions for dark matter detection rates. The interactions of neutralinos with matter are dominated by scalar (spin-independent) couplings [31,32]. These interactions are mediated either by t-channel h and H exchange, or via $\chi q \to \tilde{q} \to \chi q$. The former diagrams contain $h\chi\chi$ and $H\chi\chi$ vertices, which are suppressed for Bino-like χ. They also contain h,H-nucleon vertices. These are dominated by contributions arising from fundamental couplings of h,H to the strange quark and indirectly to gluons through heavy quark loops.

In the calculation of dark matter detection rates, we assume a Maxwellian χ velocity distribution with velocity dispersion $\bar{v} = 270\,\text{km/s}$, and a local dark matter density of $\rho_0 = 0.3\,\text{GeV/cm}^3$.

A major uncertainty in the estimate of dark matter detection rates arises from the poorly determined value of the nucleon matrix element

$$f_{T_s} \equiv \langle N | m_s \bar{s}s | N \rangle / m_N \,. \qquad (6)$$

If it is correct to treat the effect of the strange quark mass term $m_s\bar{s}s$ in first order perturbation theory, then we can identify f_{T_s} as the fractional change in the nucleon mass from what it would be in a world with massless s quarks. Values as high as $f_{T_s} = 0.62$ and as low as $f_{T_s} = 0.08$ have been considered in the literature [40].

The traditional strategy for evaluating f_{T_s} is to combine an 'experimental' determination of the σ-term $\langle N | \frac{m_u + m_d}{2} (\bar{u}u + \bar{d}d) | N \rangle$ with a fit of the flavor $SU(3)$-breaking term $\langle N | \frac{1}{3} m_s (2\bar{s}s - \bar{u}u - \bar{d}d) | N \rangle$ to the baryon octet splitting, and finally the ratios of light quark masses from the π/K mass ratio. This procedure is problematic, however, in several ways. First, theory relates the value of the σ term to the value of the πN

scattering amplitude at an unphysical point, and a rather elaborate and numerically unstable extrapolation is necessary (hence the quotation marks). Second, in getting to the quantity of interest, the uncertainty in the σ term is amplified by the large factor $2m_s/(m_u+m_d)$. Third, the nucleon matrix element of the $SU(3)$ breaking term is presumably dominated by the valence quark terms, so that the term of interest is subdominant. Fourth, and exacerbating all these, the use of first-order perturbation theory for the baryon splittings is questionable to begin with.

Clearly it would be very desirable to have a first-principles estimate of f_{T_s}. Straightforward evaluation in lattice gauge theory appears very difficult, because three-point amplitudes with disconnected components are noisy and computationally demanding. Here we would like to suggest an alternative strategy that avoids any measurement of three-point amplitudes. The basic idea is to vary the strange quark mass, starting with a very large value and bringing it down by small increments. In the heavy quark limit we have a universal answer by integrating out the heavy quark to obtain a gluon operator, which is (essentially) the trace of the energy-momentum tensor with a definite coefficient [41]. And if we change the strange quark mass by a small amount Δm_s, we have for the induced change in the nucleon mass

$$\Delta m_N = \langle N|\Delta m_s \bar{s}s|N\rangle , \tag{7}$$

by first-order perturbation theory. Of course, it is implicit in this procedure that one is doing a fully unquenched calculation, with dynamical quarks (including the strange quark). Since the error is at least quadratic in the step size, one should achieve good accuracy by taking the steps sufficiently small. Note that, since the nucleon state is changing at each step of the calculation, it is not correct to do the whole thing in one jump, despite the apparent linearity of Eq. (7). In any case, Eq. (7) allows one to extract the matrix element of interest by measuring masses only, without ever involving three-point functions.

No result of this type is currently available, and so at present one must rely on informed guesswork for a numerical estimate of f_{T_s}. There is a very extensive literature on the subject, which we will not review here. We will confine ourselves to two brief observations.

If one treats the strange quark as heavy, rather than light, then one obtains the universal value $f_{T_{\rm Heavy}} = 0.074$. This is presumably a lower limit. But of course if one takes the strange quark mass to zero, then f_{T_s} will likewise vanish. Thus, in considering values of f_{T_s} that are many times the heavy quark value, one is implicitly postulating that this quantity varies very rapidly as a function of m_s, which may be implausible.

Quenched calculations of the nucleon to rho mass ratio, and even more so calculations with two dynamical quarks, agree remarkably well with experiment, down to the few percent level. This fact can only be consistent with the idea that the strange quark is significantly perturbing the nucleon mass if the strange quark contributes an accurately equal proportion of the rho mass. In general, that would seem to require a conspiracy. Just such a ‘‘conspiracy’’ does take place for heavy quarks, since they can be integrated out in favor of a charge renormalization, whose primary effect is just to change the overall mass scale (dimensional transmutation). However, as we have seen, if it is valid to treat the strange quark as heavy, then its contribution is small.

While these considerations are far from definitive, they do seem to us to make the larger end of the range of values discussed for f_{T_s} appear dubious. In preparing our plots, we have adopted the conventional value $f_{T_s} = 0.14$. For extreme values in the range quoted above, the proton-neutralino cross sections plotted may decrease by roughly a factor of 2, or increase by a factor of 5.

In Fig. 3, we present proton-neutralino cross sections σ_P. For $m_0 \lesssim 1$ TeV, the dominant contribution is through squarks, and so the interaction rate decreases monotonically for increasing m_0 [20]. A naive extrapolation would then suggest very small detection rates in the focus point region. However, for $m_0 \gtrsim 1$ TeV, this behavior is reversed – in this region, as m_0 increases, the LSP’s Higgsino content increases, and so the Higgs boson diagrams become less and less suppressed. (This feature was also noticed in Ref. [42], where the region of large m_0 was singled out on the basis of Yukawa coupling unification in SO(10) GUT models.) Detection rates may therefore be large in the focus point region. This is especially true for large $\tan\beta$, where the

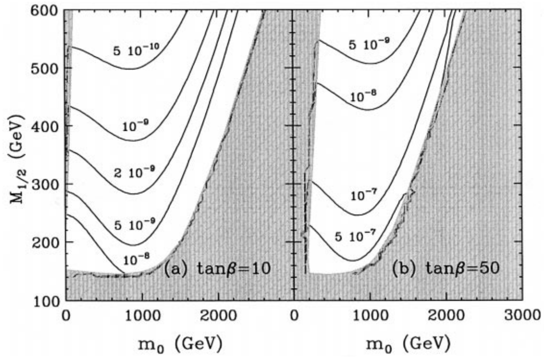

Fig. 3. Contours of proton-χ cross section σ_P in pb in the $(m_0, M_{1/2})$ plane for $A_0 = 0$, $\mu > 0$, and two representative values of $\tan\beta$. The shaded regions are as in Fig. 1.

effect noted in Section 3, a relatively light H boson with $\tan\beta$ enhanced $Hf\bar{f}$ couplings, leads to large and dominant heavy Higgs amplitudes [43].

In Fig. 4, we show the correlation between relic density and proton-neutralino cross section for a representative sample of points in parameter space. As usual, there is a strong anti-correlation, with large σ_P corresponding to low $\Omega_\chi h^2$. Nevertheless, we find that, for a given $\tan\beta$ and $\Omega_\chi h^2$, points with large m_0 have proton-neutralino cross sections comparable to those with conventional sub-TeV m_0. For the optimal case of large $\tan\beta$, points with $m_0 > 1$ TeV and $\Omega_\chi h^2 > 0.1$ $(\Omega_\chi h^2 > 0.025)$ have cross sections as large as $\sigma_P \sim 2 \times 10^{-7}$ pb $(\sigma_P \sim 10^{-6}$ pb).

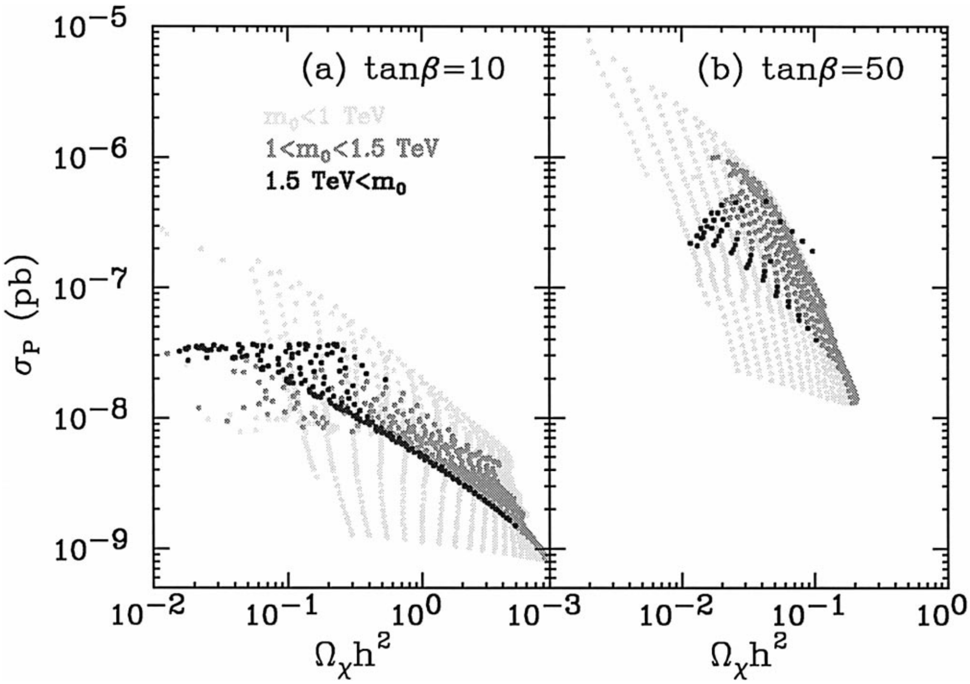

Fig. 4. Regions of the $(\Omega_\chi h^2, \sigma_P)$ plane populated by minimal supergravity models with $m_0 \leq 1$ TeV (light shaded), 1 TeV $< m_0 \leq 1.5$ TeV (medium shaded), and 1.5 TeV $< m_0$ (dark shaded). The parameters scanned are those given in Fig. 1, subject to the additional naturalness constraint $M_{1/2} \leq 400$ GeV. We assume neutralino velocity dispersion $\bar{v} = 270\,\mathrm{km/s}$ and local density $\rho_0 = 0.3\,\mathrm{GeV/cm^3}$, and $f_{T_s} = 0.14$ (see text).

5. Discussion

Several ongoing and planned experiments, using a variety of techniques, are devoted to the search for WIMP dark matter [44]. Assuming that the matter-WIMP interactions are dominated by spin-independent couplings, these experiments may be compared by scaling the matter-WIMP cross sections for each detector material to the proton-WIMP cross section σ_P and displaying the sensitivity in the (m_χ, σ_P) plane.

In Fig. 5 we plot predictions of focus point supersymmetry in the (m_χ, σ_P) plane. We find that, for a given m_χ, cross sections are typically maximized for large m_0, and large cross sections near current sensitivities are possible. At present, the DAMA Collaboration has reported data favoring the existence of a WIMP signal in their search for annual modulation [45]. When WIMP velocity uncertainties are included [46], the preferred range of parameters is 10^{-6} pb $\lesssim \sigma_P \lesssim 10^{-5}$ pb and 30 GeV $\lesssim m_\chi \lesssim$ 200 GeV (3σ CL). The DAMA result has been criticized [47], and a recent analysis of the CDMS Collaboration excludes the 3σ DAMA region at $>84\%$ CL [48]. While the situation is at present unclear, in the near future both experiments will improve their sensitivities significantly. It is noteworthy, in any case, that current experiments are on the verge of probing the region of parameter space arising in focus point models.

It is also interesting to compare the discovery potentials of dark matter searches and future colliders in focus point models. In the focus point scenario, all squarks and sleptons are beyond the reach of the Tevatron and may present significant challenges even for the LHC. However, the lightest Higgs boson has mass $m_h \lesssim 120$ GeV, and so may be discovered in Run II of the Tevatron. The correlation between m_h and σ_P is presented in Fig. 6. We note that focus point models provide a natural setting for Higgs masses above the current bound, as the large squark masses raise the Higgs mass through their radiative corrections. At the same time, both Higgs and dark matter searches are promising, and will confront the focus point scenario over the next few years.

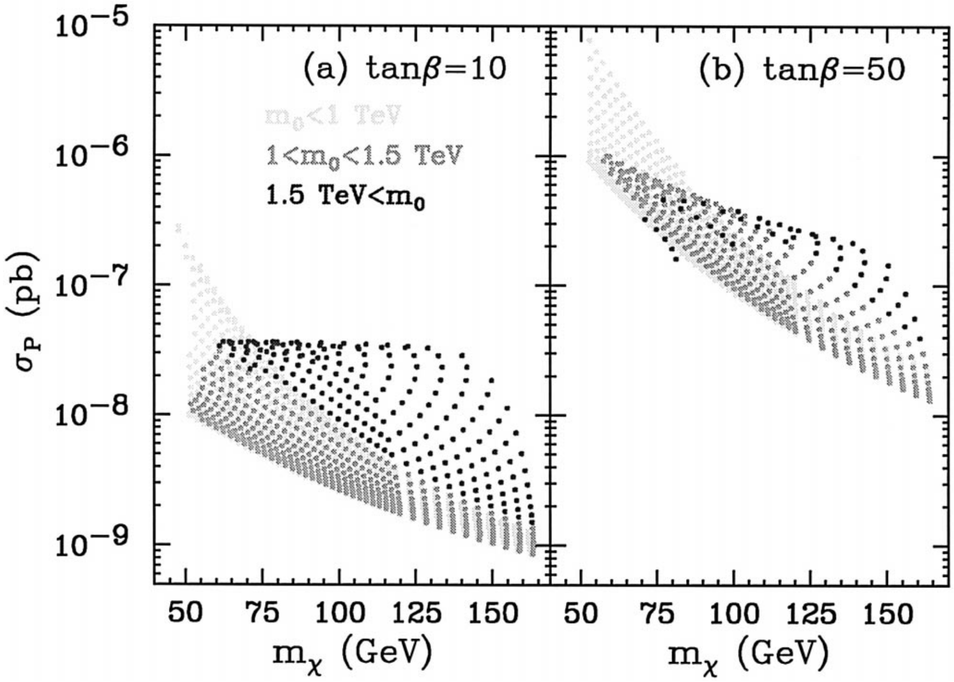

Fig. 5. Points in minimal supergravity parameter space in the (m_χ, σ_P) plane. The parameters scanned, symbols, and assumptions are as in Fig. 4.

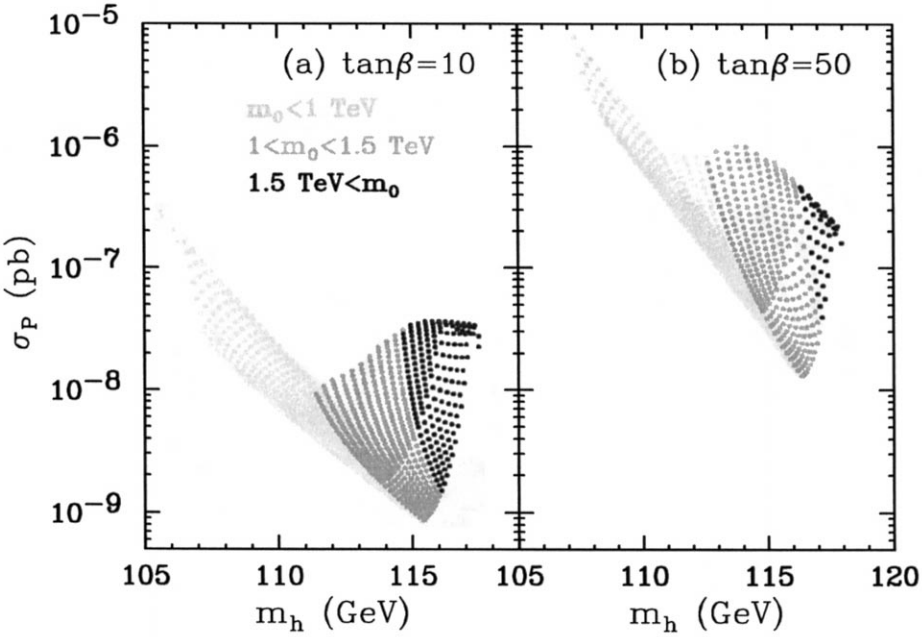

Fig. 6. Points in minimal supergravity parameter space in the (m_h, σ_P) plane. The parameters scanned, symbols, and assumptions are as in Fig. 4.

It has been argued that the regions of parameter space which give large dark matter detection rates may also predict large rates for other processes, such as proton decay [49] and $b \to s\gamma$ [50]. Conversely, present bounds from such processes may exclude the largest possible rates for dark matter detection. The proton decay rate, as noted in the introduction, scales as $m_{\tilde{q}}^{-4}$, so it is relatively suppressed in the focus point models. The process $b \to s\gamma$ provides a stringent constraint on large $\tan\beta$ only for one sign of μ. For the positive sign (the one adopted in our plots), destructive interference between the chargino and charged Higgs diagrams typically leads to predictions within the experimental bounds.

Focus point scenarios can also imply interesting rates for anti-matter detectors and experiments sensitive to dark matter annihilation in the cores of the earth or sun [44].

Finally, of course, neutralinos and charginos themselves are targets for discovery at colliders. If they are found, detailed measurements of their masses and compositions could then be made, and might even be used to exclude neutralinos as dark matter candidates [51]. However, colliders will never be able to distinguish unstable particles with long decay lengths from those stable on cosmological time scales. Since some well-motivated models with low energy supersymmetry breaking predict LSPs which are stable on accelerator detector, but unstable on cosmological, scales, in principle identification of neutralinos as the dark matter of our universe cannot be made at accelerators. Ultimately this identification requires detection, either directly or indirectly, of neutralinos permeating space.

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