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Magnetoinductive Waves in 2D Periodic Arrays of Split Ring Resonators

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Abstract—Magnetoinductive waves in arrays of split ring resonators (SRRs) have been previously investigated. Here we characterize modes with real and complex wavenumber in two dimensional periodic arrays of SRRs. Each SRR is modeled as a single magnetic dipole, and the retrieval of the complex modal wavenumbers is performed by computing the complex zeroes of the homogeneous scalar equation characterizing the field in the array. The required periodic Green's function is analytically continued into the complex wavenumber space by using the Ewald method. The proposed method allows for the description of each complex mode when varying frequency, complementing previous investigations. In particular, we analyze proper and improper, bound and leaky, magnetoinductive waves.

I. INTRODUCTION

Magnetically coupled resonators can support propagating waves, which are generally referred to as magnetoinductive (MI) waves. The simplest set of resonators consists of a capacitor and an inductor coupled with other resonators of the same kind, which can be easily produced with capacitively loaded rings or split ring resonators (SRRs), as shown in the inset in Fig. 1. Magnetoinductive waves have been studied in arrays of SRRs periodic in one, two (2D) or three dimensions in [1] by using a circuit formulation taking into account the coupling between elements (showing that nearest neighbor approximation provides already with the salient propagation features; more coupling is needed for accuracy of the results). The propagation in linear arrays of SRRs in axial (i.e., wavevector is in the same direction of the magnetic dipole moment describing the SRR) or transverse (i.e., wavevector is in the transverse direction of the magnetic dipole moment describing the SRR) configuration has been studied experimentally in [2], and similar works (including theory) have been shown in [3-5]. The dispersion properties of a bi-periodic linear array of SRRs (contiguous loops are loaded with different capacitances) have been shown experimentally in [6] to support MI waves in two distinct frequency bands. The same phenomenon has been studied thoroughly in linear arrays in [7] by having also a unit cell with two SRRs whose relative distance is smaller than the period. The arranging in various orientations of the SRRs in two coupled linear arrays to tailor the dispersion diagrams has been shown in [8]. A comprehensive review of MI wave theory and applications has been also provided in [9-10]. The scope of this paper is the characterization of the bound (non radiating) and leaky

(radiating) modes with complex wavenumber in 2D arrays of SRRs (Fig. 1), as recently done in [11-12] for 1D and 2D arrays. We model each SRR as a single magnetic dipole through the single dipole approximation (SDA) [13-14] and use a periodic Green's function (GF) formulation based on the Ewald method to describe the field in the array as shown in [12, 15].

II. FORMULATION

The structure under analysis is the 2D periodic array of SRRs reported in Fig. 1, in a homogeneous background with relative permittivity ϵ_h , where each SRR is placed at position $\mathbf{r}_{mn} = ma\hat{x} + nb\hat{y}$, with $m, n = 0, \pm 1, \pm 2, \dots$, and a , and b are the periodicities along the x and y direction, respectively [12-13]. The $\exp(j\omega t)$ monochromatic time harmonic convention is assumed.

Figure 1. Schematic of the 2D periodic array of SRRs. For simplicity, only the case with SRRs laying in the x - y plane is shown.

We model each SRR as a single magnetic dipole. As such, the mn -th induced magnetic dipole moment is given by $\mathbf{m}_{mn}(\mathbf{k}_B) = \underline{\mathbf{a}}^{\text{mm}} \cdot \mathbf{H}_{mn}^{\text{loc}}(\mathbf{k}_B)$, where $\underline{\mathbf{a}}^{\text{mm}}$ is the magnetic polarizability tensor of the SRR, and $\mathbf{H}_{mn}^{\text{loc}} = \tilde{\mathbf{G}}^{\infty}(\mathbf{r}_{mn}, \mathbf{r}_{mn}, \mathbf{k}_B) \cdot \mathbf{m}_{mn}$ is the local magnetic field acting on it produced by all the SRRs but the one at position $\mathbf{r}_{mn}$, and $\tilde{\mathbf{G}}^{\infty}(\mathbf{r}_{mn}, \mathbf{r}_{mn}, \mathbf{k}_B)$ the regularized periodic dyadic GF (see [15]). For the SRR orientation in Fig. 1, as well as for the other two orthogonal orientations, the problem reduces to a scalar one, where only the component correspondent to the directions orthogonal to the ring plane, here the z direction, will be different from zero, as $\underline{\mathbf{a}}^{\text{mm}} = \alpha_{zz}^{\text{mm}} \hat{z}\hat{z}$, where

$\alpha_{zz}^{\text{mm}} = \mu_0(A^2/L)[1/(\omega_0^2/\omega^2 - 1 + jR/\omega L)]$ [16], with μ_0 the absolute permeability of free space, $A = \pi r^2$ the area of each ring, $L \approx \mu_0 r [\ln(16r/w) - 2]$ the self-inductance of the ring, with r and w ($w \ll r$) denoting the radius and the thickness of the ring as in the inset in Fig. 1, $\omega_0 = 1/\sqrt{LC}$ the resonance angular frequency of the ring, and $R \approx 0.5 \times R_s 2\pi r/w$ accounts for ohmic losses, with R_s the metal surface resistance. The quality factor of the ring series resonance is given by $Q = \sqrt{L/C}/R$. We use rings with the following characteristics: $L = 33$ nH, $C = 188$ pF, $Q = 200$, $\omega_0/2\pi = 63.87$ MHz, as in [9]; a practical design would be $r \approx 8.47$ mm, $w \approx 0.826$ mm, and $R = 66.24$ m Ω assuming SRR made of copper. Suppose that the array in Fig. 1 supports a mode with wavevector $\mathbf{k}_B = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}}$. Then, specifying previous equations at $\mathbf{r}_{00}$, one obtains the following dispersion equation $A_{zz}(\mathbf{k}_B) = 1 - \alpha_{zz}^{\text{mm}} \tilde{G}_{zz}^\infty(\mathbf{r}_{00}, \mathbf{r}_{00}, \mathbf{k}_B) = 0$ that can be solved for complex $\mathbf{k}_B$.

III. DISPERSION DIAGRAMS

Here we analyze modes with complex wavenumber traveling along the x direction, i.e., $\mathbf{k}_B = k_x \hat{\mathbf{x}}$, with $k_x = \beta_x - j\alpha_x$, embedded in free space (i.e., $\epsilon_h = 1$, $k = k_0$), with rectangular lattice $a = b = 23$ mm. The transverse-to-the-array wavenumber is $k_z = \sqrt{k^2 - \mathbf{k}_B \cdot \mathbf{k}_B}$. We distinguish between proper (i.e., decaying getting away from the array, $\text{Im}(k_z) < 0$) and improper (i.e., growing getting away from the array, $\text{Im}(k_z) > 0$) modes as previously done in [12], thus providing the knowledge on the physical bound and leaky modes excitable by a localized point source.

Figure 2. Example of dispersion diagram for the array in Fig. 1. Solid = proper, dashed = improper modes, dotted-dashed = $\beta_x = k$ (light line).

The dispersion diagrams of β_x and α_x for a dominant mode are shown in Fig. 2. At low frequencies, a physical backward ($\beta_x \alpha_x < 0$), bound ($|\beta_x| > k$), proper mode with large attenuation α_x is observed (see [12, 17] for a complete classification of physical modes). Increasing frequency, this mode becomes a physical backward leaky ($|\beta_x| < k$, note the dotted-dashed “light line” in the inset), proper, mode. Further increasing frequency, this mode transitions to a physical

forward ($\beta_x \alpha_x > 0$) leaky, improper, mode with very low attenuation α_x . Then, it becomes a non-physical (i.e., not excitable by a localized source [11-12]) bound improper mode. Analogous descriptions apply to 2D arrays of other SRR configurations laying in the x - z and y - z planes. Such modes can be used for guidance (based on magnetoinductive phenomena) or radiation (leaky modes, with $|\beta_x| < k$).

REFERENCES

- [1] E. Shamonina, V. A. Kalinin, K. H. Ringhofer, and L. Solymar, "Magnetoinductive waves in one, two, and three dimensions," *J. Appl. Phys.*, vol. 92, pp. 6252-6261, Aug 2002.
- [2] I. V. Shadrivov, A. N. Reznik, and Y. S. Kivshar, "Magnetoinductive waves in arrays of split-ring resonators," *Physica B: Condensed Matter*, vol. 394, pp. 180-183, May 2007.
- [3] R. R. A. Syms, I. R. Young, and L. Solymar, "Low-loss magnetoinductive waveguides," *J. Phys. D: Appl. Phys.*, vol. 39, p. 3945, Sep 2006.
- [4] M. C. K. Wiltshire, E. Shamonina, I. R. Young, and L. Solymar, "Experimental and theoretical study of magneto-inductive waves supported by one-dimensional arrays of "swiss rolls"," *J. Appl. Phys.*, vol. 95, pp. 4488-4493, Apr 2004.
- [5] M. C. K. Wiltshire, E. Shamonina, I. R. Young, and L. Solymar, "Dispersion characteristics of magneto-inductive waves: comparison between theory and experiment," *Electron. Lett.*, vol. 39, pp. 215-217, Jan 2003.
- [6] A. Radkovskaya, O. Sydoruk, M. Shamonin, E. Shamonina, C. J. Stevens, G. Faulkner, D. J. Edwards, and L. Solymar, "Experimental study of a bi-periodic magnetoinductive waveguide: comparison with theory," *IET Microw. Antennas Propag.*, vol. 1, pp. 80-83, Feb 2007.
- [7] O. Sydoruk, O. Zhuromskyy, E. Shamonina, and L. Solymar, "Phonon-like dispersion curves of magnetoinductive waves," *Appl. Phys. Lett.*, vol. 87, p. 072501, Aug 2005.
- [8] O. Sydoruk, A. Radkovskaya, O. Zhuromskyy, E. Shamonina, M. Shamonin, C. J. Stevens, G. Faulkner, D. J. Edwards, and L. Solymar, "Tailoring the near-field guiding properties of magnetic metamaterials with two resonant elements per unit cell," *Phys. Rev. B*, vol. 73, p. 224406, Jun 2006.
- [9] O. Sydoruk, O. Zhuromskyy, A. Radkovskaya, E. Shamonina, and L. Solymar, "Magnetoinductive waves I: Theory," in *Theory and Phenomena of Metamaterials*, F. Capolino, Ed., Boca Raton: CRC Press, 2009, p. 36.1.
- [10] O. Z. O. Sydoruk, A. Radkovskaya, E. Shamonina, and L. Solymar, "Magnetoinductive waves II: Application," in *Applications of Metamaterials*, F. Capolino, Ed., Boca Raton, FL: CRC Press, 2009, p. 14.1.
- [11] S. Campione, S. Steshenko, and F. Capolino, "Complex bound and leaky modes in chains of plasmonic nanospheres," *Opt. Express*, vol. 19, pp. 18345-18363, Sep 2011.
- [12] A. L. Fructos, S. Campione, F. Capolino, and F. Mesa, "Characterization of complex plasmonic modes in 2D-periodic arrays of metal nanospheres," *J. Opt. Soc. Am. B*, vol. 28, pp. 1446-1458, May 2011.
- [13] S. Steshenko and F. Capolino, "Single dipole approximation for modeling collection of nanoscatterers," in *Theory and Phenomena of Metamaterials*, F. Capolino, Ed., Boca Raton, FL: CRC Press, 2009, p. 8.1.
- [14] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, New York: Wiley, 1983.
- [15] S. Steshenko, F. Capolino, P. Alitalo, and S. Tretyakov, "Effective model and investigation of the near-field enhancement and subwavelength imaging properties of multilayer arrays of plasmonic nanospheres," *Phys. Rev. E*, vol. 84, p. 016607, Jul 2011.
- [16] R. Marques, F. Martin, and M. Sorolla, *Metamaterials with Negative Parameters*, Hoboken, NJ: John Wiley & Sons, 2008.
- [17] P. Baccarelli, S. Paulotto, and C. D. Nallo, "Full-wave analysis of bound and leaky modes propagating along 2D periodic printed structures with arbitrary metallisation in the unit cell," *IET Microw. Antennas Propag.*, vol. 1, pp. 217-225, Feb 2007.