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### Author

Garcia-Luna-Aceves, J.J.

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# Embracing Interference in Ad Hoc Networks Using Joint Routing and Scheduling with Multiple Packet Reception <sup>★</sup>

Xin Wang <sup>a,\*</sup> J.J. Garcia-Luna-Aceves <sup>a,b</sup>

<sup>a</sup>*Department of Computer Engineering, University of California, Santa Cruz  
1156 high street, Santa Cruz, CA 95064, U.S.A.  
Phone: 1-831-4595436, Fax: 1-831-4594829*

<sup>b</sup>*Palo Alto Research Center (PARC)  
3333 Coyote Hill Road, Palo Alto, CA 94304, U.S.A.*

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## Abstract

We present an approach that takes advantage of multi-packet reception (MPR) to reduce the negative effects of multiple access interference and therefore increase the capacity of an ad hoc network. We analyze the performance upper bound of joint routing and scheduling for ad hoc networks that embrace interference by using MPR. We formulate the optimization problem under a deterministic model and seek to maximize the aggregate network throughput subject to minimum rate requirements. We then propose a polynomial-time heuristic algorithm aimed at approximating the optimal solution to the joint routing and channel access problem under MPR. We show the effectiveness of our heuristic algorithm by comparing its performance with the upper bound.

*Key words:* Routing, Scheduling, Multiple packet reception

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## 1 Introduction

The protocol stacks used in ad hoc networks today are based on the

not necessarily reflect the views of the funding agencies.

\* Corresponding author

*Email addresses:*

wangxin@soe.ucsc.edu (Xin Wang ),

jj@soe.ucsc.edu (J.J.

Garcia-Luna-Aceves).

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premise that a given receiver is able to decode at most one transmission correctly. Accordingly, transmitters and receivers orchestrate transmissions trying to avoid multiple access interference (MAI). However, the seminal work by Gupta and Kumar [1] demonstrated that the per source-destination throughput of a wireless network with a protocol stack that avoids MAI sharply decreases as number of nodes increases in the network.

Fortunately, such multi-packet reception (MPR) techniques as multiuser detection (*MUD*), successive interference cancellation (*SIC*), directional antennas (*DA*), and multiple input multiple output (*MIMO*) [2–4] have become a reality. For example, one of the basic premises of MUD in code-division multiple access (CDMA) system is that signals from different users should be estimated jointly, which makes it possible for the nodes to receive multiple packets simultaneously; employing transmitting and receiving antenna arrays with properly designed space-time codes can significantly increase the rate of reliable communications, allowing the separation of multiple users transmitting at the same time; the adaptive antenna arrays also allow receivers to receive multiple packets simultaneously, which is known as space division multiple access (*SDMA*). With MPR techniques, multiple nodes around a receiver can transmit concurrently with the receiver being able to decode the concurrent transmissions. Toumpis and Goldsmith [5] have shown that the capacity re-

gions for ad hoc networks are significantly increased when multiple access schemes are combined with spatial reuse (i.e., multiple simultaneous transmissions), multi-hop routing (i.e., packet relaying), and *SIC*. Furthermore, Garcia-Luna-Aceves et. al [6] demonstrated recently that protocol architectures that exploit MPR increase the order capacity of random wireless ad hoc networks by a factor  $\Theta(\log n)$  under the protocol model, where  $n$  is the number of nodes in the network. They also showed that MPR provides a better capacity improvement for ad hoc networks than network coding when the network experiences a single-source multicast and multi-pair unicasts.

From the above, it is clear that using MPR is an attractive approach for making ad hoc networks scale. However, as promising as the above theoretical recent results on the use of MPR in ad hoc networks are, there is much work to be done before MPR-based ad hoc networks can be reduced to practice. The transmissions that are to be decoded at a receiver need to be sent synchronously, and the number of concurrent transmissions allowed around a receiver cannot exceed the number of concurrent transmissions that the receiver can decode, which may be smaller than the number of neighbors near the receiver if the network is densely connected. Furthermore, the communication protocols used to date in ad hoc networks have been designed to avoid MAI, and are derivatives of protocols and architectures originally designed for wired networks based on point-to-point links. For example,

the IEEE 802.11 DCF adopts a similar back-off strategy than Ethernet when more than one transmission occurs around a receiver. Similarly, the IETF MANET routing protocols work independently of the channel access method, even though it is *not* true that routing in ad hoc networks occurs over a pre-existing network topology and the transmission over one link does not impact the transmissions over other links, as it can be done in a wired network.

The motivation for the work presented in this paper is that, for MPR to really help ad hoc networks scale, the protocols used in such networks have to be redesigned from the ground up to *embrace*, not *combat*, MAI. We present an approach to the joint routing and scheduling problem in ad hoc networks in which nodes are endowed with MPR. Our joint routing and scheduling solution takes advantage of multiple concurrent transmissions around receivers that are part of routes from sources to destinations. The main contribution of this paper consists of deriving the formulation of the joint routing scheduling optimization in ad hoc networks using MPR, and proposing a novel heuristic that approximates the upper bound on performance.

The rest of the paper is organized as follows. First, Section 2 summarizes prior work related to our joint route-scheduling problem. Our survey of prior work reveals that, while much work has been reported on joint routing and channel access scheduling for multi-channel networks, the joint optimization of routing and

channel access under MPR is a new problem. We investigate the achievable performance gain offered by MPR given such system information as network topology, traffic pattern, and bandwidth requirements. Section 3 presents the performance upper bound for joint routing and scheduling using MPR. We introduce an heuristic approach in Section 4 aimed at approximating this upper bound. Section 5 shows the numerical results comparing our heuristic with the upper bound using different scenarios. Section 6 concludes the paper.

## 2 Related work

Considerable work exists on joint routing, scheduling, and channel assignment in ad hoc networks in which each node is endowed with a single radio capable of accessing multiple channels, as well as networks in which each node is endowed with multiple radios, each capable of accessing multiple channels [7][8][9][10][11][12][13][14][15].

Raniwala et al. [7] propose a centralized channel assignment and routing algorithm, which uses an heuristic approach to obtain a static channel assignment. An improved distributed channel assignment algorithm is proposed in [8]. Kyasanur et al. [9] propose an interface assignment strategy where the number of available interfaces is less than the number of available channels. It fixes a channel on one radio and switches channels on other radios. Nodes can commu-

nicate with each other through the fixed common radio without requiring specialized coordination algorithms.

The efforts most closely related to ours are [10] [11] [12] [13] [14] [15].

Kodialam et al. [10] consider the problem of jointly routing and scheduling transmissions to achieve a given rate vector. They develop tight necessary and sufficient conditions for the achievability of the rate vector. They also propose polynomial time approximation schemes for solving the routing problem. They use a simple interference model, which is derived from the CDMA based multi-hop networks to map the scheduling problem to edge coloring problem. They show that their solution is within  $\frac{2}{3}$  of the optimal solution. Zhang et al. [11] formulate the problem for joint routing and channel switching in wireless mesh networks under a deterministic model, and seek to minimize overall system activation time in use to satisfy given end-to-end traffic demands subjected to the multi-access interference among neighboring transmissions and the radio interface constraint at each node. They adapt a column generation based approach to solve this problem, which decomposes the original problem into sub-problems and solves them iteratively. Alicherry et al. [12] mathematically formulate the joint channel assignment and routing problem for infrastructure wireless mesh networks. They aim to maximize the bandwidth allocated to each traffic aggregation point subjected to fairness constraint and

propose a constant approximation algorithm for this NP-hard problem. Their algorithm takes interference constraint into account and is based on flow transformation. Performance evaluation shows that the algorithm performs much better than the worst case bounds. Kodialam et al. [13] develop a network model that characterizes the channel, radio and interference constraint in a fixed broadband wireless network, which provides necessary and sufficient conditions for a feasible channel assignment and schedule. They use necessary conditions to derive upper bounds on the capacity in terms of achievable throughput and propose two link channel assignment algorithms, one static and the other dynamic, to achieve a performance that is close to optimal. Simulation results demonstrate that the dynamic link channel assignment scheme performs close to optimal on the average, while the static link channel assignment algorithm also performs very well. Meng et al. [14] propose a mathematical model of multi-radio wireless mesh networks based on radio and radio-to-radio link. They provide a unified way to characterize the node-radio constraint as well as the wireless interference constraint. Then they develop a linear sufficient condition for a feasible schedule in multi-channel multi-radio wireless. With this sufficient condition, they formulate the joint routing and channel assignment problem as a linear programming problem that will compute the optimal routes for the given set of flows. They obtain flow schedule on wireless links with vertex coloring algorithms from the scheduling

graph. They prove that the optimality gap is above a constant factor. Bhatia et al. [15] formulate a mathematical framework where routing, link scheduling and stream control of MIMO links can be jointly optimized for throughput maximization in the presence of interference. An efficient approximation algorithm is proposed to solve the throughput optimization problem subject to fairness constraints.

From the above summary of prior work, we can observe that the joint routing and scheduling problem has not been solved for the case of ad hoc networks in which nodes employ MPR radios. Similar to the previous works [10] [11] [12] [13] [14] [15], we also formulate the joint routing and scheduling problem as an optimization problem. However, each radio is capable of decoding multiple transmissions simultaneously in our formulation, and we further propose an heuristic algorithm to efficiently approximate the system throughput upper bound.

### 3 Formulation of Joint Routing and Scheduling Problem

#### 3.1 Assumptions

We assume that each node is synchronized on slot systems and nodes access the channel based on slotted time boundaries. Each time slot is numbered relative to a consensus starting point. We assume that nodes are endowed with a single radio, and

hence nodes cannot transmit and receive at the same time. In order to maximize the system throughput, multipath routing is used.

Table 1 lists the notation used throughout this paper.

#### 3.2 Wireless Transmission and Interference Model

Gupta et al. [1] used two interference models to study the capacity of wireless networks. The first is a *protocol model* that assumes interference is an all-or-nothing phenomenon. The second is a *physical model* that considers the impact of interfering transmissions on the signal-to-noise ratio. In this paper, we use the protocol model to investigate the performance of upper layer (MAC and routing) protocols.

In the protocol model [1], two nodes can communicate directly if they are within a distance  $d(n)$ , and the transmission from node  $X_i$  to node  $X_j$  is successful only if there is no other transmitters within distance  $(1 + \Delta)d(n)$  to node  $X_j$ , where  $\Delta$  is a parameter that depends on the characteristics of the physical layer. The protocol model inherently implies that the disks of different concurrent receivers with radius  $d(n)$  are disjoint.

Applying the same protocol model to wireless networks with MPR capability means that nodes are able to receive successfully multiple packets concurrently, as long as the trans-

Table 1  
Symbol Table

$t$	time slot	$T$	total number of time slots
$S(t)$	transmission schedule for time slot $t$	$N_f$	total number of traffic flows
$I_{ij}$	interference link set for link $(i, j)$	$u_m$	sender of the $m$ th flow
$v_m$	receiver of the $m$ th flow	$r_{ij}^m$	the $m$ th flow rate on link $ij$
$v_{ij}$	time fraction of the slot scheduled to link $(i, j)$ to satisfy the bandwidth requirements	$b_{ij}$	bandwidth of link $(i, j)$
$R$	MPR receive range	$M$	the maximum number of simultaneous transmissions a node can decode
$G$	connectivity graph	$G_T$	transmission graph
$G_c$	reduced conflict graph	$N$	total number of nodes
$N_p$	number of multipaths	$E_r$	previous selected relay link set
$E_a$	set of active links which can co-exist with $E_r$	$E_c(i)$	the subset of active links that interfere with $i$
$F(i)$	fraction of influenced links for link $i$	$F(p)$	total fraction of influenced links for path $p$
$D(p)$	distance for path $p$	$C(p)$	cost function of path $p$
$E_r(p)$	relay link set when path $p$ is selected	$B_p(i)$	bandwidth for link $i$ if path $p$ is chosen
$R_f$	bandwidth requirements for a specific traffic flow $f$	$B(i)$	available bandwidth for link $i$

mitters are within a radius of  $r(n)$  from the receiver and all other transmitting nodes have a distance larger than  $(1 + \Delta)r(n)$ . The key difference is that MPR allows the receiver node to receive multiple packets from different nodes within its disk of radius  $r(n)$  simultaneously. Note that  $d(n)$  in point-to-point communication is a random variable while  $r(n)$  in MPR is a predefined value.

The MPR capabilities of nodes in a network can be illustrated by a receiver matrix  $E$ . Each element of this matrix,  $\epsilon_{nk}$ , is defined as  $\Pr[k \text{ packets are correctly received} \mid n \text{ packets are transmitted in the transmission radius}]$ .

$$E = \begin{bmatrix} \epsilon_{10} & \epsilon_{11} & 0 & \cdots & 0 \\ \epsilon_{20} & \epsilon_{21} & \epsilon_{22} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \epsilon_{n0} & \epsilon_{n1} & \epsilon_{n2} & \cdots & \epsilon_{nn} \end{bmatrix} \quad (1)$$

We use a strong MPR protocol model ( $E_1$ ) in this paper, which assumes a receiver can decode all the data streams if and only if the number of simultaneous transmissions in the receive range ( $R$ ) is less than  $M$ , as shown in Equation 2 and 3.

$$E_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \quad (2)$$

$$\epsilon_{ij} = \begin{cases} \epsilon_{ii} = 1 & i \leq M \\ \epsilon_{i0} = 1 & i > M \\ 0 & \text{all others} \end{cases} \quad (3)$$

### 3.3 Problem Formulation

We represent an ad hoc network with a undirected *connectivity graph*  $G(V, E)$ , where  $V$  represents the set of nodes in the network, and  $E$  is the

set of undirected links. If node  $u$  is in the receive range of node  $v$ , then there is a link  $(u, v)$  in  $E$ . We assume that each node can decode up to  $M$  simultaneous transmissions and all edges in  $G$  have the unit channel capacity ( $b_{ij} = 1$ ).

The inputs of the problem include the connectivity graph  $G(V, E)$ , the MPR capability  $M$  and the channel capacity  $b_{ij}$ . The outputs of the optimization problem are: 1). the link set selected for transmissions, it is the route selection result; 2). the time fraction each link  $(i, j)$  could obtain to transmit/receive ( $v_{ij}$ ). We use a simple example to illustrate this, as Figure 1 shows.

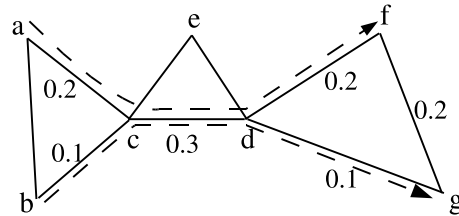


Fig. 1. Optimization problem input/output example

There are two traffic flows:  $\{a \rightarrow f, b \rightarrow g\}$ . Through solving the optimization problem, we get the selected link set  $\{(a, c), (b, c), (c, d), (d, f), (d, g)\}$ , then we know the routes  $\{(a, c, d, f); (b, c, d, g)\}$  are selected. The time fraction of each link is shown in Figure 1, how to transfer it into the corresponding transmission scheduling will be further discussed in Section 3.4.

We define two distinct links  $(i, j)$  and  $(u, v)$  ( $j$  and  $v$  are receivers) are *interfere* if one the transmitters  $i(u)$  is in the receive range of the other



link's receiver  $v(j)$ . We define a transmission schedule  $S(t)$  for time slot  $t$  ( $t \in 1, \dots, T$ ) to be *feasible* if and only if, for any scheduled link  $i$  in  $S(t)$ , the number of interfering links is less than  $M$ .

For  $N_f$  traffic flows, let the source and destination of the  $m$ th flow be  $u_m$  and  $v_m$ . We use  $r_{ij}^m$  to denote the  $m$ th flow rate on link  $(i, j)$ . We denote by  $v_{ij}$  the time fraction assigned to link  $(i, j)$  to meet the bandwidth requirements of flows. Note that  $v_{ij} \in [0, 1]$  and is schedulable.  $I_{ij}$  represents the set of links that interfere with link  $(i, j)$ .

Given the above definitions and notations, we formulate the joint routing and scheduling problem as follows:

Maximize the sum of flow rate reaching all receivers

$$\text{Maximize } \sum_{m=1}^{N_f} \sum_i r_{iv_m}^m$$

subject to the following four constraints:

(1) *Flow conservation constraint:*

$$\sum_i r_{ij}^m = \sum_i r_{ji}^m, \forall j \in V - \{u_m, v_m\} \quad (4)$$

$$\sum_i r_{u_m i}^m = \sum_i r_{i v_m}^m \quad (5)$$

$$\sum_i r_{i u_m}^m = \sum_i r_{v_m i}^m = 0 \quad (6)$$

(2) *MPR capability constraint:*

$$\sum_i v_{ij} + \sum_{\forall (k,l) \in I_{ij}} v_{kl} \leq MC(q), \quad \forall j \in V - \{u_m, v_m\} \quad (7)$$

(3) *Node radio constraint:*

$$\sum_i v_{u_m i} \leq 1 \quad (8)$$

$$\sum_i v_{i v_m} \leq 1 \quad (9)$$

$$\frac{1}{M} \sum_i v_{ij} + \sum_i v_{ji} \leq 1, \quad \forall j \in V - \{u_m, v_m\} \quad (10)$$

(4) *Bandwidth constraint:*

$$\sum_m r_{ij}^m \leq b_{ij} v_{ij} \quad (11)$$

The above four constraints can be explained as follows.

*Flow conservation constraint:* Constraint 4 states that the amount of incoming flow is equal to the amount of outgoing flow for all nodes, except the traffic source and destination. Constraint 5 holds that the total amount of outgoing traffic equals to the total amount of incoming traffic for the  $m$ th flow. Equation 6 restricts the problem by stating that there is no flows on any incoming links to source  $u_m$  and there is no flows on any outgoing links from destination  $v_m$ .

*MPR capability constraint:* Alicherry et al. [12] proved through geometry arguments that for any feasible time fraction vector,

$$v_{ij} + \sum_{\forall (k,l) \in I_{ij}} v_{kl} \leq C(q) \quad (12)$$

where  $q = R_I/R_T$ , with  $R_I$  denoting the interference range and  $R_T$ , the transmission range.

The term  $C(q)$  depends only on  $q$ . For example  $C(q) = 4, 8, 12$  for  $q = 1, 2, 2.5$ , respectively (more discussion about  $C(q)$  can be found in [16]). For link  $(i, j)$ , we denote by  $RC$  the region formed by the union of two circles  $C_i, C_j$  of radius  $R_I$  each, centered at node  $i$  and  $j$ .

Because each receiver using MPR can decode up to  $M$  simultaneous transmissions in the receive range  $R$ , the number of links that do not pair-wise interfere with each other can be obtained by computing the maximum number of circles of radius  $R$  that can be packed in  $RC$ . Similar to [12], we can re-formulate this problem to a cycle packing problem. The only difference is, for MPR, now in each cycle of radius  $R$ , there are up to  $M$  links. Then we can obtain the MPR capability constraint shown in Equation 7. The first part represents the total transmission fractions to receiver  $j$ , and the second part represents the transmission fractions to other receivers that are in the interference range.

*Node radio constraint:* Constraints 8 restricts that the total fractions of transmission at the traffic source is not greater than 1. Constraints 9 restricts that the total fractions of receiving time at the traffic destination is not greater than 1. For the rest of the relay nodes, since each node can decode up to  $M$  simultaneous transmissions, in other words, the receive time of each node can be

maximally combined by a factor of  $\frac{1}{M}$ , and given each node has a single transceiver, the radio constraint is shown in Equation 10.

*Bandwidth constraint:* Equation 11 guarantees that the total bandwidth allocated to different flows on a specific link will not exceed the link capacity.

We note that although the simultaneous transmissions can be maximally combined by a factor of  $\frac{1}{M}$  using MPR, the time fractions obtained may not be schedulable. Hence, Constraints 7 and 10 are only necessary conditions.

By solving the above optimization problem, we can obtain a performance upper bound of the joint routing and scheduling problem using MPR. Jain et al. [17] have shown that even the derivation of all feasible scheduling in a single-channel wireless network is an NP-hard. The size of our optimization problem increases exponentially with the number of links. However, we can use a mixed linear programming solver [18] to obtain numerical results against which we can compare different heuristics.

### 3.4 Link Scheduling

Once the transmission fraction of each link is computed, it can be represented into a transmission schedule by a sequence of four steps: directed graph transformation, transmission time fraction reduction, conflict

graph transformation, and vertex coloring. We describe each step in the following paragraphs.

### 3.4.1 Directed Graph Transformation

The output of the problem formulation in Section 3.3 includes two parts: 1). which links are selected for transmissions; 2). the transmission time fraction of each selected link. The first part is the result of route selection, and the second part forms the transmission scheduling. However, in order to transfer the transmission fractions into the corresponding link scheduling, we need to know the direction of each link. This is because the link directions impose different constraints on the transmission scheduling, which further influences the final scheduling results. For example, in Figure 2(a), link  $(i, j)$  cannot transmit concurrently with link  $(j, k)$  due to the node-radio constraint. However, in Figure 2(b), links  $(i, j)$ ,  $(j, k)$  can be scheduled for transmissions at the same time through using MPR.

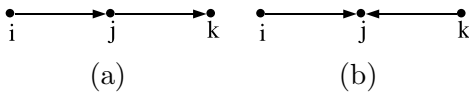


Fig. 2. Influence of link directions on the transmission scheduling

Given that the connectivity graph  $G$  is undirected, we need to transfer  $G$  to a directed transmission graph  $G_T$  using the output of the optimization problem. First, we remove the links that have zero transmission time fraction from  $G$ . Then we use an extended Dijkstra algorithm to

search all the possible paths between each source and destination pair, as Algorithm 1 shows.

We use Figure 3 as an example to illustrate the process of directed graph transformation. In Figure 3(a), there are three traffic flows  $\{a \rightarrow c\}$ ,  $\{b \rightarrow d\}$  and  $\{c \rightarrow d\}$ . There are six links selected and the corresponding transmission fractions are shown besides the links. We first remove the links with zero transmission fraction in  $G$ , then through using Algorithm 1, we can get the corresponding transmission graph, as Figure 3(b) shows.

### 3.4.2 Transmission Time Fraction Reduction

Because MPR allows multiple links to be used simultaneously for transmissions, links are scheduled in groups rather than individually as in traditional link scheduling for point-to-point communications. We propose the following three steps to form the correct group scheduling: 1). First we group the links according to their destinations; 2). then we sort the links targeted to the same destination according to the ascending order of the time fraction required. We denote there are  $n_k$  links to destination  $k$  and  $v_{1k} < v_{2k} < \dots < v_{n_k k}$ , as Table 2 shows.

3). Finally we combine different transmission fractions into transmission groups. As table 3 shows, the link set  $\{(i, k), (i+1, k), \dots, (\min(i+M-1, n_k), k)\}$  forms a transmission group  $g_m$ . The transmission fraction of  $g_m$  is  $v_{ik}$  and subtracted from

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**Algorithm 1** Directed graph transformation algorithm
 

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**Notations:**
 $G(V, E)$ : connectivity graph;

 $v_{ij}$ : transmission time fraction for every link  $(i, j) \in E$ ;

 $AB_{ij}$ : available bandwidth of every link  $(i, j) \in E$ , at the beginning,  $AB_{ij} = v_{ij}$ ;

 $(s_i, d_i)$ : the  $i$ th traffic source and destination pair;

 $Dijkstra(G, s_i, d_i)$ : return a path  $p_i$  of  $(s_i, d_i)$ , which bandwidth is

 $BW_i = \min_{ij} \{AB_{ij}\}, \quad \forall (i, j) \in p_i$ ;

- 1: **for each**  $s_i$  **do**
  - 2:     **while**  $Dijkstra(G, s_i, d_i) \neq \emptyset$  **do**
  - 3:         **for each** link  $(i, j) \in p_i$  **do**
  - 4:              $AB_{ij} = AB_{ij} - BW_i$ ;
  - 5:         **end for**
  - 6:     **end while**
  - 7: **end for**
- 

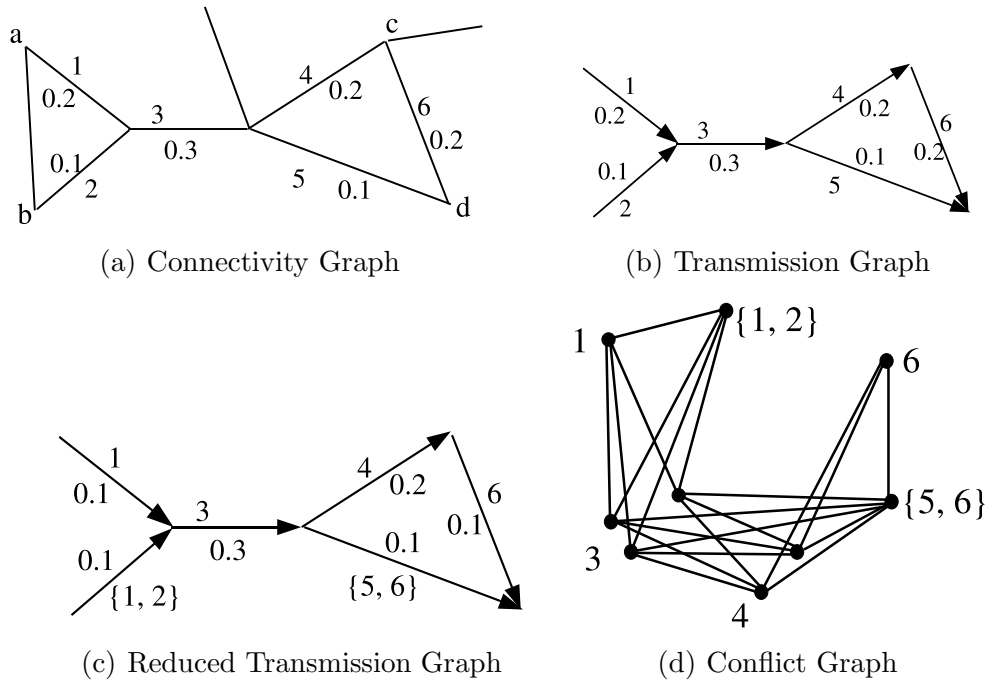


Fig. 3. Graph Transformation

Table 2

Transmission time fraction list

Link	$(1, k)$	$(2, k)$	$\dots$	$(n_k, k)$
Time fraction	$v_{1k}$	$v_{2k}$	$\dots$	$v_{n_k k}$

the transmission fractions of  $\{(i + 1, k), \dots, (\min(i + M - 1, n_k), k)\}$ , respectively. This process continues until there are no more links that can be combined. The total transmission time fraction reduction process is shown in Algorithm 2.

### 3.4.3 Conflict Graph Transformation

After obtaining the transmission fraction groups, we transform the connectivity graph  $G$  to a *reduced conflict graph*  $G_c$ .

Each link  $(i, j)$ /transmission group  $g$  in  $G$  needs  $\lceil v_{ij}T \rceil / \lceil v_gT \rceil$  slots, where  $T$  is the total number of slots to be scheduled. We represent each link/transmission group in  $G$  by  $\lceil v_{ij}T \rceil / \lceil v_gT \rceil$  nodes in  $G_c$ . If two links/transmission groups are in conflict with each other in  $G$ , there are edges between the corresponding vertexes in  $G_c$ .

Fig 3(c) and Fig 3(d) illustrate the process of transmission time fraction reduction and conflict graph transformation, respectively. We assume  $T = 10$ . As shown in the figure, for the two links 1 and 2, which have the same destination, after the transmission fraction reduction, link 1 has transmission fraction 0.1 and transmission group  $\{1, 2\}$  has transmission fraction 0.1. We represent transmission group  $\{1, 2\}$  as one node in  $G_c$ , and given that  $\{1, 2\}$  conflicts with link 1, there are links between these two nodes in  $G_c$ . Since link 3 has transmission fraction 0.3, it corresponds to three nodes in  $G_c$ . We

note that Wu et al. [19] used a similar approach, which is called *usage conflict graph* to bound the power-rate function. Meng et al. [14] adapted a similar graph transformation approach to generate the scheduling graph.

### 3.4.4 Vertex Coloring:

After the prior three steps, the link flow scheduling problem is equivalent to a vertex coloring problem of  $G_c$ . By coloring adjacent nodes in  $G_c$  with different colors, we can obtain a conflict free link scheduling in  $G$ . We adapt an existing vertex coloring algorithm to obtain a conflict free scheduling. In this paper, we use the DSATUR algorithm by Breaz [20]. In DSATUR, the vertex to be colored next is the one that has the maximum *saturation degree*, i.e. the vertex whose colored neighbors contain the largest number of different colors. DSATUR will color a vertex with minimal feasible color and is a sequential coloring algorithm with polynomial complexity.

## 4 Heuristic Approach

In this section, we present an heuristic approach to approximate the system throughput upper bound that can be computed based on the formulation stated in the previous section. The heuristic consists of route selection followed by link scheduling.

As indicated in the problem formulation of Section 3.3, multipath routing

Table 3

Transmission group list

Link	$(i, k), \dots, \min(i + M - 1, n_k)$	$(i + 1, k), \dots$	$\dots$
Time fraction	$v_{ik}$	$v_{i+1k} - v_{ik}$	$\dots$

**Algorithm 2** Transmission time fraction reduction algorithm**Notations:** $n_k$ : the number of links which have the same destination  $k$ ; $v_{ik}$ : transmission time fraction,  $v_{1k} < v_{2k} < \dots < v_{n_k k}$ ; $m$ : transmission group index; $v_{g_m}$ : transmission time fraction of transmission group  $g_m$ ;

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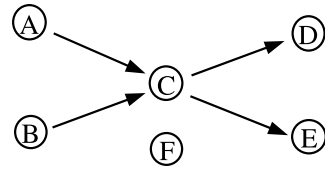
1: for  $i = 1$  to  $n_k - 1$  do
2:   if  $v_{ik} > 0$  then
3:      $g_m = (i, k)$ ;
4:     for  $j = i + 1$  to  $\min(j + M - 1, n_k)$  do
5:        $v_{jk} = v_{jk} - v_{ik}$ ;
6:        $g_m = g_m \cup (j, k)$ ;
7:     end for
8:      $v_{g_m} = v_{ik}$ ;
9:      $m = m + 1$ ;
10:  end if
11: end for

```

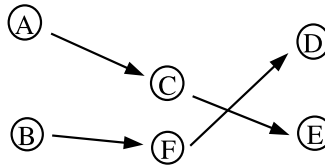
protocol is used to increase the system throughput. The existing multipath routing protocols can be classified according to the type of paths they use:

- (1) Node-disjoint paths [21], which are paths such that a node appears in exactly one path.
- (2) Link-disjoint paths [22] [23], which are paths such that a node may appear in more than one path; however, the same pair of nodes can appear in exactly one path.
- (3) Minimum-cost paths [24], which are paths such that they have the minimum cost amongst all available paths. These paths may be link or node disjoint or may have no such constraints imposed on them.

Compared with the other two alternatives, we conjecture that node-disjoint paths can better approximate the performance upper bound. We give a simple example to illustrate this in Fig 4.



(a) Non-disjoint multipath



(b) Node-disjoint multipath

Fig. 4. Influence of node-disjoint routing on link scheduling

We assume each node can decode

up to 2 simultaneous transmissions ( $M = 2$ ). When link-disjoint or non-disjoint paths are used, the routes selected are  $\{A \rightarrow C \rightarrow E\}$  and  $\{B \rightarrow C \rightarrow D\}$ . Since node  $C$  can not transmit and receive at the same time, the optimal scheduling for MPR transmissions is letting  $AC$  and  $BC$  transmit simultaneously, then node  $C$  transmit to node  $D$  and  $E$ , respectively. While for the node-disjoint paths, which select paths  $\{A \rightarrow C \rightarrow E\}$  and  $\{B \rightarrow F \rightarrow D\}$ , it can schedule two links to transmit in each time slot and fully exploit the MPR capability, as shown in Table 4. Based on this consideration, we choose node-disjoint paths in the heuristic approach.

We assume that traffic flows are served according to their arrival sequence. Through route selection, we aim to not only select the paths with large bandwidth, but also balance the traffic throughout the network so that the MPR capability of each node is fully exploited. In order to achieve this, we introduce an unified cost function for each possible path and use an extended Prim's minimum spanning tree (MST) algorithm to select up to  $N_p$  minimum cost paths. At each step of the heuristic approach, one relay link will be selected, and it will be added to the selected relay link set  $E_r$ . For each  $E_r$ , there will be a candidate link set  $E_a$  that can coexist with all links in  $E_r$ . For a specific relay link  $i$ ,  $E_c(i)$  is the subset of active links that interfere with  $i$ ,  $E_c(i) \subseteq E_a$ .

If link  $i$  is chosen, we define the frac-

tion of influenced links of  $i$  as:

$$F(i) = \frac{|E_c(i)|}{|E_a|} \quad (13)$$

where  $|E_c(i)|$  and  $|E_a|$  are the cardinalities of  $E_c(i)$ ,  $E_a$ , respectively.

$F(i)$  reflects the impact of link  $i$  to the congestion of the network. Through using MPR, relay links that interfere with the previous selected links can still be scheduled at the same time. But the more links it interfere with, it will reduce more total available MPR capabilities from the system. A relay link  $i$  with a smaller  $F(i)$  is preferred because small  $F(i)$  indicates that we can form a new relay link set with a larger available MPR capability, which can admit more potential traffic flows. We modify Prim's MST algorithm to greedily select the links with the minimum  $F(i)$ . The selected links also need to be node-disjoint with the previous selected paths. Then we update the relay link set for path  $p$  and the available bandwidth for link  $i$ :

$$\begin{aligned} E_r(p) &= E_r(p) \cup \{i\} \\ B_p(i) &= B_p(i) - R_f \end{aligned} \quad (14)$$

where  $E_r(p)$  is the relay link set when path  $p$  is selected,  $B_p(i)$  is the bandwidth for link  $i$  if path  $p$  is chosen,  $R_f$  is the bandwidth requirement for the flow  $f$ .

The total fraction of influenced links for path  $p$  is:

$$F(p) = \sum_i F(i) \quad (15)$$

Table 4  
Joint multipath routing and schedule example

Time slot number	$i$	$i + 1$	$i + 1$
Transmission schedule for non-disjoint multipath	$AC, BC$	$CD$	$CE$
Transmission schedule for node-disjoint multipath	$AC, BF$	$CE, FD$	$AC, BF$

The distance for path  $p$  is:

$$D(p) = |E_r(p)| \quad (16)$$

where  $|E_r(p)|$  is the cardinality of  $E_r(p)$ .

The cost function of node disjoint path  $p$  is defined as:

$$C(p) = (1 - \alpha) \frac{1}{\min_i \{B_p(i)\}} + \alpha F(p) \quad (17)$$

where  $\alpha$  is an impact factor. The first part of Equation 17 represents the bandwidth bottleneck along the path  $p$ , while the second part represents the potential MPR capability consumption of path  $p$ . By selecting the path with the minimum  $C(p)$ , we try to select a path with large available bandwidth and MPR capability.

After the cost metric of all the possible node-disjoint paths is obtained, if there are paths that can satisfy the bandwidth requirements of the traffic flow, the path with the minimum cost is selected; otherwise, the traffic flow is rejected. If there are multiple paths with the same cost metric, the path with the shortest distance is chosen.

Finally the relay link set and the available bandwidth for the selected links are updated:

$$\begin{aligned} E_r &= E_r \cup \{i\}, & \forall i \in p \\ B(i) &= B(i) - R_f, & \forall i \in p \end{aligned} \quad (18)$$

where  $E_r$  is the relay link set and  $B(i)$  is the available bandwidth for link  $i$ .

The heuristic continues, until  $N_p$  paths are selected or there are no more node-disjoint paths available, as Algorithm 3 shows.

After the total relay link set  $E_r$  is obtained, we propose Algorithm 4 to generate a conflict free transmission schedule, it will assign each link in the relay link set  $E_r$  with the smallest slot  $t$  that has less than  $M$  interference transmissions.

## 5 Performance Analysis

### 5.1 Complexity analysis

Compared with the optimal scheme, the heuristic approach serves the traffic flows according to the flow arrival sequence and reduces the computation complexity.

The computation complexity for calculating the fraction of influenced links  $F(i)$  of each relay link  $i$  is at most  $O(N^2)$ , where  $N$  is the total number of nodes in the network. The worst case complexity for initializa-



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**Algorithm 3** Route selection algorithm

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**Notations:**  
 $N_p$ : the number of multipaths;  
 $N$ : total number of nodes;  
 $p_i$ : the  $i$ th selected path,  $i \in 1, \dots, N_p$

- 1: **for each** traffic flow  $f$  **do**
- 2:     **for**  $i = 1$  to  $N_p$  **do**
- 3:         primMST( $G, N$ )
- 4:         {
- 5:              $F(i) = \frac{|E_c(i)|}{|E_a|}$ ;
- 6:              $E_r(p) = E_r(p) \cup \{i\}$ ;
- 7:              $B_p(i) = B_p(i) - R_f$ ;
- 8:         }
- 9:          $F(p) = \sum_i F(i)$ ;
- 10:          $D(p) = |E_r(p)|$ ;
- 11:          $C(p) = (1 - \alpha) \frac{1}{\min_i \{B_p(i)\}} + \alpha F(p)$ ;
- 12:         **if**  $\exists p_m, p_n; C(p_m) == C(p_n)$  **then**
- 13:              $p_i = \operatorname{argmin}_p D(p)$ ;
- 14:         **else**
- 15:              $p_i = \operatorname{argmin}_p C(p)$ ;
- 16:         **end if**
- 17:          $E_r = E_r \cup \{i\}, \quad \forall i \in p_i$
- 18:          $B(i) = B(i) - R_f, \quad \forall i \in p_i$
- 19:     **end for**
- 20: **end for**

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**Algorithm 4** Link scheduling algorithm of the heuristic approach

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**Notations:**  
 $t$ : time slot,  $t \in \{1, 2, \dots, T\}$ ;  
 $S(t)$ : transmission schedule for time slot  $t$ ;

- 1: **for each** link  $(i, j) \in E_r$  **do**
- 2:     Assign  $(i, j)$  to the smallest slot  $t$  for which
- 3:      $\forall (u, v) \in S(t), |I_{uv}| < M$
- 4: **end for**

---

tion is  $O(N^4)$ . The computation complexity of Prim's MST is  $O(N^2)$  and the cost for node-disjoint path check is  $O(N)$ . Hence, the overall complexity of the approach is  $O(N_p N^4)$ .

## 5.2 Numerical Results

In this section, we try to illustrate how much performance improvement is due to multipaths, how much performance improvement is due to using MPR, and how close is the performance of the heuristic approach compared with the upper bound. We compare the upper bounds and

the related heuristic approaches for the following scenarios under different topologies: (a) single shortest path with optimal scheduling; (b) node-disjoint paths (without MPR,  $M = 1$ ) with optimal scheduling; (c) node-disjoint paths (with MPR) with optimal scheduling; (d) heuristic approach. For the single shortest path routing, we adapt the Dijkstra’s algorithm. For multipath routing with/without MPR, we extended Dijkstra’s algorithm to select up to  $N_p$  node-disjoint paths, as stated in Section 4. In the heuristic approach, we set the maximum number of paths to be maintained ( $N_p$ ) as  $M$ ,  $\alpha = 0.4$  and  $q = 1$ . We define the system throughput as the sum of the total receive time fractions at all the receivers. The system throughput is in terms of time slots (each receiver can receive up to one time slot, as defined in Constraint 9).

### 5.2.1 Grid Topology

We first investigate the performance under a  $4 \times 4$  regular grid. As Fig 5 shows, each node is  $D$  away from each other. The receive range  $R = \sqrt{2}D$  and there is a flow from 1 to 16. The route selection result is shown in Table 5 and the system throughput result is shown in Table 6.

From Table 5 and Table 6, we observe when there is no MPR ( $M = 1$ ), in order to increase the system throughput, five paths are used to increase the spatial reuse of the system and each link is scheduled to transmit for only a small fraction of time. With MPR, we not only increase the sys-

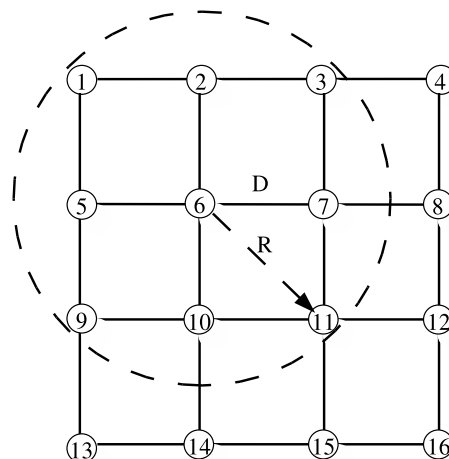


Fig. 5. Grid Topology

tem throughput, but also reduce the number of paths used and the average path distance. Given that longer paths imply more link transmissions, this indicates that fewer system resources are consumed using MPR and traffic flows tend to have smaller end to end delays.

### 5.2.2 Random topology

We use a scenario with 50 nodes uniformly distributed across a  $300 \times 300$  square meters area. We vary the average node degree by changing the receiving range ( $R$ ) of each node, as Fig 6 shows. We set up 10 traffic flows in each topology. The senders and destinations are more than two hops away from each other to ensure that the metrics measured are reflective of multi-hop traffic.

The results obtained for system throughput comparison for different node degrees is shown in Table 7 and Fig 7. We observe that the system throughput does not increase linearly with the MPR complexity. This

Table 5  
Grid topology path selection comparison

	Selected route
Single shortest path	$P : (1 \rightarrow 6 \rightarrow 11 \rightarrow 16);$
Multipath without MPR ( $M = 1$ )	$P_1 : (1 \rightarrow 6 \rightarrow 11 \rightarrow 16);$ $P_2 : (1 \rightarrow 2 \rightarrow 7 \rightarrow 12 \rightarrow 16);$ $P_3 : (1 \rightarrow 5 \rightarrow 10 \rightarrow 15 \rightarrow 16)$ $P_4 : (1 \rightarrow 2 \rightarrow 3 \rightarrow 8 \rightarrow 12 \rightarrow 16)$ $P_5 : (1 \rightarrow 5 \rightarrow 9 \rightarrow 14 \rightarrow 15 \rightarrow 16)$
Multipath with MPR $M = 2, 3, \dots$	$P_1 : (1 \rightarrow 6 \rightarrow 11 \rightarrow 16);$ $P_2 : (1 \rightarrow 2 \rightarrow 7 \rightarrow 12 \rightarrow 16);$
Heuristic ( $M = 2$ )	$P_1 : (1 \rightarrow 6 \rightarrow 11 \rightarrow 16);$ $P_2 : (1 \rightarrow 2 \rightarrow 7 \rightarrow 12 \rightarrow 16);$
Heuristic ( $M = 3$ )	$P_1 : (1 \rightarrow 6 \rightarrow 11 \rightarrow 16);$ $P_2 : (1 \rightarrow 2 \rightarrow 7 \rightarrow 12 \rightarrow 16);$ $P_3 : (1 \rightarrow 5 \rightarrow 10 \rightarrow 15 \rightarrow 16);$

Table 6  
Grid topology throughput comparison

	System throughput (time slots)
Single shortest path	0.333
Multipath without MPR ( $M = 1$ )	0.833
Multipath with MPR, $M = 2, 3, \dots$	1.0
Heuristic ( $M = 2$ )	0.6667
Heuristic ( $M = 3$ )	0.75

Table 7  
Throughput of single shortest path routing with different average node degree

Average node degree	4	8	12
System throughput (time slots)	2.8333	4.1000	3.35

is because the system bottleneck is the fact that nodes cannot transmit and receive at the same time. If the total system resources (time can be used to transmit and receive over all wireless spectrums) are dedicated to traffic delivery to or from some bottleneck nodes, performance cannot be further improved by the increase of MPR capability.

As Fig 7 shows, the increase of the

node degree leads to conflicting effects on the system throughput. The system throughput increases if the node degree is increased from 4 to 8. This is because the influence of node degree increase on the spatial reuse are as follows: 1). in the two-hop range, with the increase of node degree, less concurrent transmissions can be accommodated to transmit at the same time-slot; 2). while from the system point of view, since there

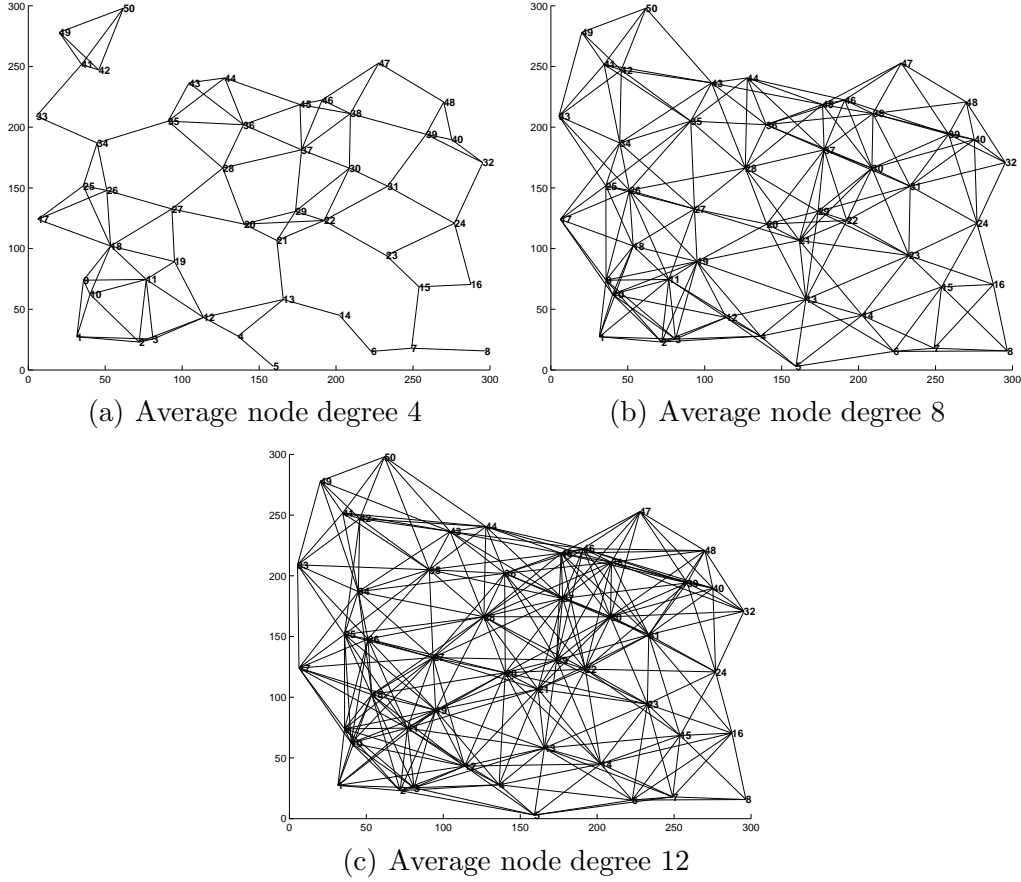


Fig. 6. Random topology

are more paths that can be used, the traffics can be balanced throughout the network, the spatial reuse of the system increases. The former will dominate at the beginning (the extreme case is the network is disconnected), the latter will dominate when both the optimal approach and the heuristic can find enough transmission links to increase the spatial reuse of the system.

However, if the node degree is further increased (from 8 to 12), the system throughput decreases. This is because the number of hops becomes smaller and more links interfere with each other. Consider the extreme case that all nodes are in the receive

range of each other, then the problem reduces to the link scheduling problem and the upper bound of the system throughput will be  $M \times \frac{M}{N_f}$  (there are at most  $M$  simultaneous transmissions and there are  $N_f$  transmission pairs).

Now for the topology with average node degree 8, we vary the number of traffic flows from 10 to 30 and compare the system throughput. The results are shown in Table 8 and Fig 8. When we add more traffic flows, the number of interference links also increases and it is more difficult to find multiple paths to distribute the traffic load across the network. Accordingly, the performance improvement

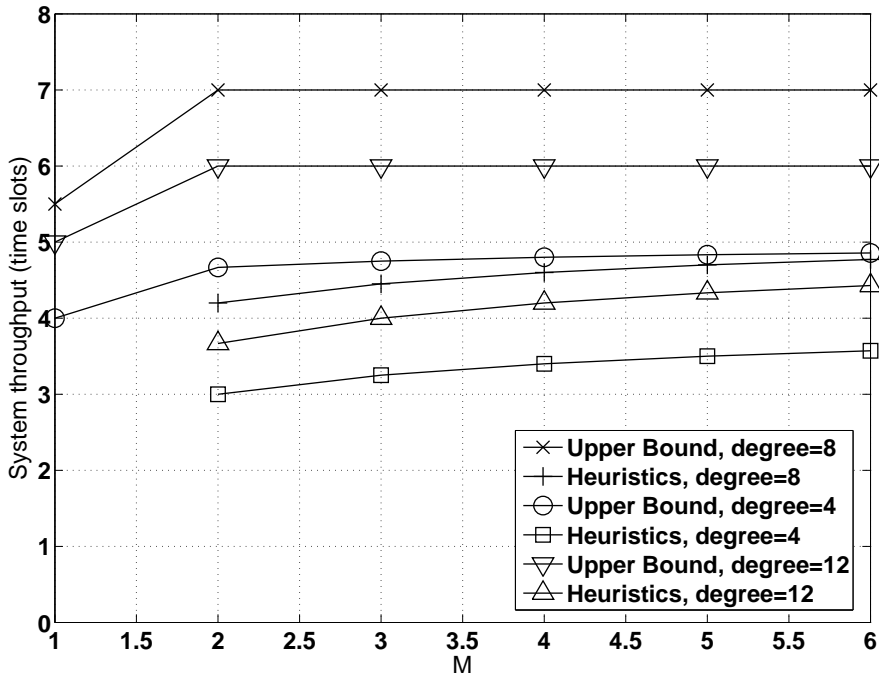


Fig. 7. Throughput of multipath routing with different average node degree and different MPR capability

due to MPR decreases. Compared with the upper bound, the heuristic becomes more and more tight.

From all the numerical results shown in Section 5.2.1 and Section 5.2.2, we find that joint routing and scheduling with multipath MPR achieves a much better performance than the single shortest-path routing approach. By using node disjoint multipath routing, we can make full use of the spatial separation of the links and schedule the concurrent transmissions to the largest extent. Furthermore given that there are always bottleneck nodes that are the destinations of multiple transmissions, an increased MPR ability alleviates the traffic scheduling problem at the bottleneck nodes. The combination of multipath routing and transmis-

sion scheduling can fully leverage the MPR capability. The proposed heuristic algorithm is within the 60% of the performance upper bound, which indicates the need for further investigation of better approximations to the optimal solution.

## 6 Conclusions

In this paper, we formulated the first heuristic approach for joint routing and scheduling in ad hoc networks in which nodes are endowed with MPR capabilities. Given that no prior approaches exist, we formulated a model for the joint routing and scheduling problem for ad hoc networks using MPR, and derived the necessary condition for the opti-

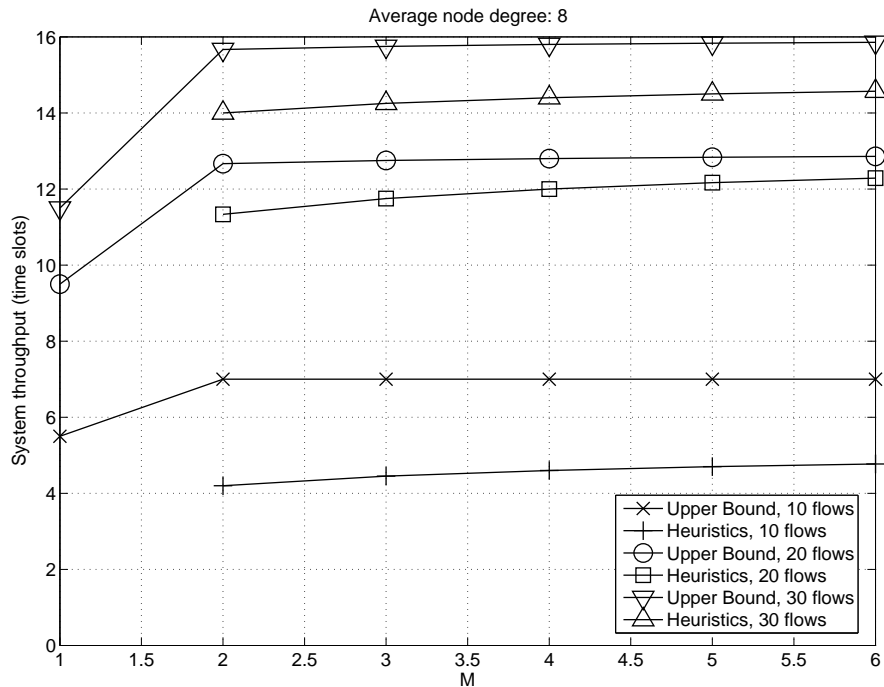


Fig. 8. Throughput of multipath routing with different flow number and different MPR capability

Table 8

Throughput of single shortest path routing with different flow number

Flow number	10	20	30
System throughput (time slots)	4.10	7.65	8.6667

mization problem and analyzed the performance upper bound. Numerical results demonstrate that our heuristic approach can effectively exploit the MPR capability.

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