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Observability of Quasiparticle-Pair Interference Current In Superconducting Weak Links

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ABSTRACT

Calculations of static I_o-V_o curves, and of the heights of steps induced by rf radiation, have been carried out for current biased Josephson junctions with both a phase-independent quasiparticle conductance G_o and a phase-dependent quasiparticle pair "interference" conductance $G_1\cos\phi$. Little or no effect of the term in $G_1\cos\phi$ is found for junctions with zero capacitance. It thus appears that this term can be

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neglected in many practical applications of weak links. Substantial effects are found for the case of finite junction capacitance. The term in $G_1\cos\phi$ then enhances the hysteresis of the static I_o-V_o curve by an amount which roughly corresponds to a factor of 2-5 increase in capacitance, and causes the static voltage to increase above the value expected from the shunt conductance alone. The most easily observable effect of the term in $G_1 \cos\phi$ on the rf induced step heights is a pronounced shift in the values of rf current for which maxima occur for low values of normalized rf frequency.

According to the original work of Josephson[1] as developed by Josephson[2,3] and by Nam,[4] the current in a superconducting tunnel junction is given by

$$I(t) = I_c \sin\phi(t) + [G_o(V) + G_1(V) \cos\phi(t)]V(t). \tag{1}$$

The first term on the right is the usual phase dependent Josephson, or pair tunneling, current. The second term, G_oV is the dissipative quasiparticle tunneling current neglecting coherence.[2] The third term,which is both dissipative and phase dependent, has been described[5,6] as an interference term between the pair and quasiparticle currents when the effects of coherence on the quasiparticle distribution are included.[7] Because the quasiparticle current in a tunnel junction biased well below the gap is small, the interference term has usually been neglected. Recent experiments on the Josephson plasma resonance in Pb-Pb tunnel junctions by Pedersen _et al_.[6] have shown evidence for the existence of the term in $G_1\cos\phi$ and obtained a value for the ratio $\gamma = G_1/G_o$ of -0.9 ± 0.1. A subsequent calculation by Poulson[5] from the microscopic theory of tunnel junctions has shown that, for small voltages, the ratio is essentially independent of V, and is equal to -0.93 for the junctions used by Pedersen _et al_.

It is of interest to explore whether a term in $G_1\cos\phi$ with γ of this order is present in the proper description of weak links other than tunnel junctions. Since the shunt conductance G_o is large in point contacts, Dayem bridges, and proximity effect bridges, the existence of such a term might be of considerable practical importance for device applications. The theory of such weak links is not developed sufficiently to answer this question reliably. In order to obtain an experimental answer, we have calculated such experimentally observable quantities as the static I_o-V_o characteristic and the height of rf induced steps in the presence of voltage-independent G_1 and G_o with a constant negative ratio. In the absence of a shunt capacitance, little or no effect due to the $\gamma G_o V \cos\phi$ term is obtained. When finite capacitance is introduced, observable effects are predicted. None of the predicted effects, however, are dramatic enough for the existence of the term in $\gamma G_o \cos\phi$ to be verified from published data.

We introduce the junction shunt capacitance into Eq. (1) in the usual way,[8,9] letting $I = C\frac{dV}{dt}$ for the capacitor, and replacing V with $\frac{\hbar}{2e}\frac{d\phi}{dt}$. Using convenient dimensionless units[8] we obtain, for the time evolution of the phase in a junction biased with a constant I_o,

$$\beta_c \frac{d^2\phi}{d\tau^2} = i_o - \sin\phi - \frac{d\phi}{d\tau}(1+\gamma\cos\phi), \tag{2}$$

where $i_o \equiv I_o/I_c$, $\tau \equiv 2e\, I_c t/\hbar G_o$, and $\beta_c = 2eI_c\, C/\hbar G_o^2$. Taking G_o and γ to be constant we have integrated Eq. (2) numerically to find the periodicity of $d\phi/d\tau$ and used this to determine the dependence of the time averaged dc voltage $V_o \equiv \frac{I_c}{G_o}\langle\frac{d\phi}{d\tau}\rangle_\tau$ across the junction on the applied dc current I_o for given values of γ and β_c.

Static I_o-V_o characteristics corresponding to earlier calculations for $\gamma = 0$[8,9] and also for $\gamma = -0.95$ are shown in Fig. 1 for several values of the capacitance parameter β_c. These static characteristics are independent of γ for zero junction capacitance, as can be shown by analytic integration of Eq. (2) with $\beta_c = 0$.[10] For a given $\beta_c > 0$, and a given bias current, the dc voltage increases as γ goes from 0 to -0.95. Note that for β_c and γ both finite, the I_o-V_o characteristic drops below the asymptotic line $I_o = V_oG_o$. This corresponds to a power dissipation greater than I_o^2/G_o over a wide range of current. The voltage at which $I_o = V_oG_o$ scales as $\beta_c^{-1/2}$ for $\beta_c \gtrsim 0.1$.[11] Similar calculations for $\gamma = -0.5$ show smaller deviations, with the crossover moving towards higher currents; for the (probably unphysical) case of $\gamma > 0$, the deviation has the opposite sense and the I_o -V_o curve bulges toward lower voltages.

It is difficult to establish the existence of the $G_o \cos\phi$ conductance by fitting data for various weak links to the curves in Fig. 1, since β_c and G_o must be determined as well as γ.

The most pronounced effect of including the $\gamma\, G_o \cos\phi$ conductance is the increase in the hysteresis of the I_o-V_o curve at finite β_c. Defining a hysteresis parameter $\alpha \equiv i_{min}/i_o$ in the usual way,[8,9] we plot β_c vs α in Fig. 2 for $\gamma = 0$, -0.5 and -0.95. For purposes of comparison, we have also shifted the curve for $\gamma = 0$ down by an amount corresponding to a reduction in β_c by a factor 0.45 to show that the shape of the curve is most strongly affected in the range of small hysteresis ($\alpha \lesssim 1$).

The deviation of the static I_o-V_o curves for finite γ can be most easily understood by examining Fig. 3, in which we plot a single period of the time evolution of the phase for various values of γ and β_c at a fixed value of $i_o = 1.2$. First consider the case $\gamma = \beta_c = 0$. For ϕ near $\pi/2$, the bias current flows primarily through the superconducting junction, so there is little dissipation in the shunt resistor. When ϕ is near $3\pi/2$, the junction current is opposite to the bias current, so the current (and dissipation) in the shunt resistor is large. As the phase spends more time near $\pi/2$ during a cycle than near $3\pi/2$, the average dissipation is less than the value I_o^2/G_o for the shunt alone. This corresponds to a time-averaged forward supercurrent, and an average voltage $V_o < I_o/G_o$.

As β_c is increased for $\gamma = 0$, the phase-time characteristics change as shown in Fig. 3(a). The inertial effect of the capacitance smooths out the ac supercurrent, equalizing the time spent near $\pi/2$ and near $3\pi/2$. As $\beta_c \rightarrow \infty$ at constant i_o, the voltage approaches the value $V_o = I_o/G_o$ and the time-averaged supercurrent vanishes.

When we choose $\gamma = -0.95$, we obtain the phase-time characteristics shown in Fig. 3(b). A careful comparison of Fig. 3(a) with Fig. 3(b) shows that the ratio of the time spent near $\pi/2$ to that spent near $3\pi/2$ is independent of γ for $\beta_c = 0$. As we increase β_c from zero, the decreased conductance near $\phi = 0$ shortens the junction time constant and decreases the time spent near $\phi = \pi/2$ while the increased time constant near $\phi = \pi$ increases the time spent near $\phi = 3\pi/2$. There is a regime where the phase actually spends more time near $3\pi/2$ than near $\pi/2$, giving a time-averaged <u>reverse</u> supercurrent, so that the dissipation and the time-averaged voltage are greater than for the conductance alone, $V_o > I_o/G_o$. As $\beta_c \rightarrow \infty$ the capacitor again smooths out the time evolution of the phase so that the I_o-V_o characteristic approaches the asymptotic value, $V_o = I_o/G_o$.

Because of the present interest in ac Josephson effect devices, the possible influence of a finite γ on the ac response of various types of weak links is of practical importance. In order to explore this question we include an rf current source $i_{rf} \cos\Omega_{rf}\tau$ in the right-hand side of Eq. (2), where $\Omega_{rf} = \hbar\omega_{rf}G_o/2eI_c$ and $i_{rf} = I_{rf}/I_c$. As a measure of the rf response of the junction we have computed the dependence of the height of the radiation induced steps with n = 0, 1 and 2 on i_{rf}. For $\beta_c = 0$, no effects of finite γ were observed for values of reduced frequency $\Omega_{rf} \lesssim 0.16$. The effects of a finite γ increase with Ω_{rf}, but remain quite small even for Ω_{rf} as large as 1.7. They include a small shift in the positions of the minima, a lifting of the minima away from zero step height, and a small decrease in the height of the subsidiary maxima. All of these effects would be difficult to observe. Since junctions for applications generally have $\alpha = 1$, step heights were also computed for $\beta_c = 0.15$, the largest value giving no hysteresis. The largest effects then occur for small values of the product $\Omega_{rf}\beta_c = \omega_{rf}\, C/G_o$. Neither $\beta_c = 0.15$, nor $\gamma = -0.95$ alone has an appreciable effect on the step heights, but in combination they produce a marked shift in the positions of the subsidiary maxima, as shown in Fig. 4 for $\Omega_{rf} = 0.16$.

Even though a finite γ increases the harmonic content of the phase-time curve, the heights of integral order steps are not very sensitive to γ. For these steps, the rf period averages over n periods of the phase oscillation. Although we have not explored the γ-dependence of subharmonic steps, these might have a different sensitivity, as several rf cycles occur during one phase period.

We conclude that a phase dependent quasiparticle-pair conductance term $\gamma G_o \cos\phi$ produces potentially observable effects on the static

$I_o - V_o$ curves and the radiation induced steps of Josephson junctions shunted by both resistance and capacitance. In no case, however, are the effects large enough to be recognized in published data on weak links, or to require the inclusion of this term in an equivalent circuit used for device design.

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10. We thank Y. Taur for first pointing out this result.

11. Note that $\beta_c^{1/2} = \omega_p C/G_o = \omega_p \tau_o$ where ω_p is the zero-phase plasma frequency. For $\omega_p \tau_o \gtrsim 1$, the modulation of the phase in a cycle depends on the ratio $\langle d\phi/dt \rangle / \omega_p$, becoming appreciable for $\langle d\phi/dt \rangle / \omega_p \lesssim 1$. Since $(V_o G_o / I_o)\beta_c^{1/2} = \omega_p^{-1} \langle d\phi/dt \rangle$, the voltage at which crossing the asymptote $V_o = I_o/G_o$ occurs will scale as $\beta_c^{-1/2}$ for all $\beta_c \gtrsim 1$.

FIGURE CAPTIONS

Fig. 1. Normalized static I_o-V_o curves for various values of the junction capacitance parameter β_c and pair quasiparticle interference parameter, γ.

Fig. 2. Plot of the junction capacitance parameter β_c against the junction hysteresis parameter α for various values of the pair-quasiparticle/interference conductance $\gamma G_o \cos\phi$. The dashed line is the $\gamma = 0$ curve shifted downward by an amount corresponding to a reduction in β_c by a factor of 0.45.

Fig. 3. Influence of a finite pair-quasiparticle interference conductance $\gamma G_o \cos\phi$ on the time evolution of the junction phase ϕ.

(a) Zero interference current $\gamma = 0$, but various values of β_c.

(b) Finite interference current $\gamma = -0.95$, and various values of β_c.

Fig. 4. Plot of the height of the steps for n = 0, 1, and 2 as a function of the rf current at the reduced frequency $\Omega_{rf} = 0.16$. The locations of the subsidiary maxima are shifted by the combined effects of a small junction capacitance $\beta_c = 0.15$ and a finite quasiparticle-pair interference term $\gamma = -0.95$.

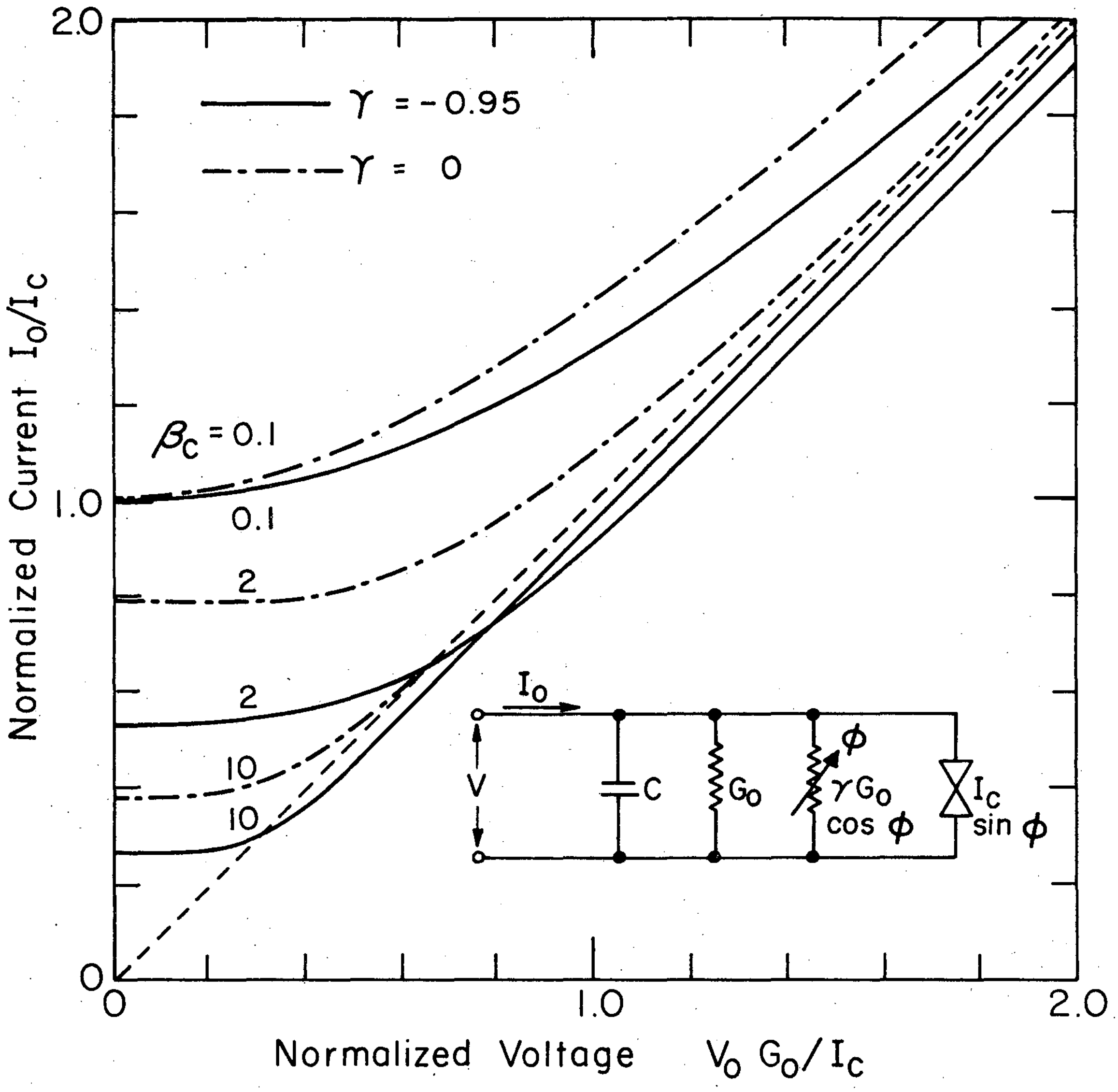

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Fig. 1

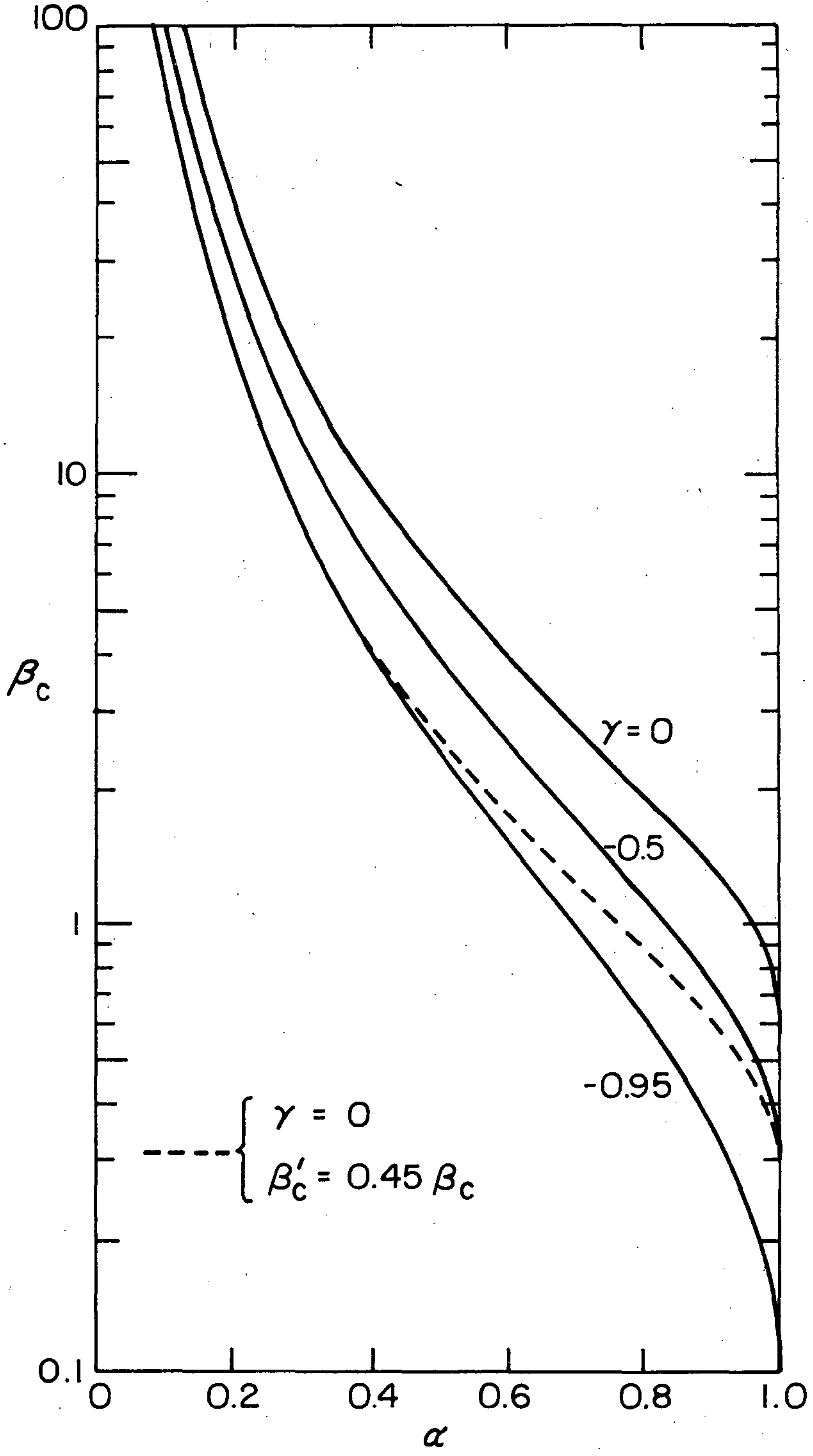

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Fig. 2

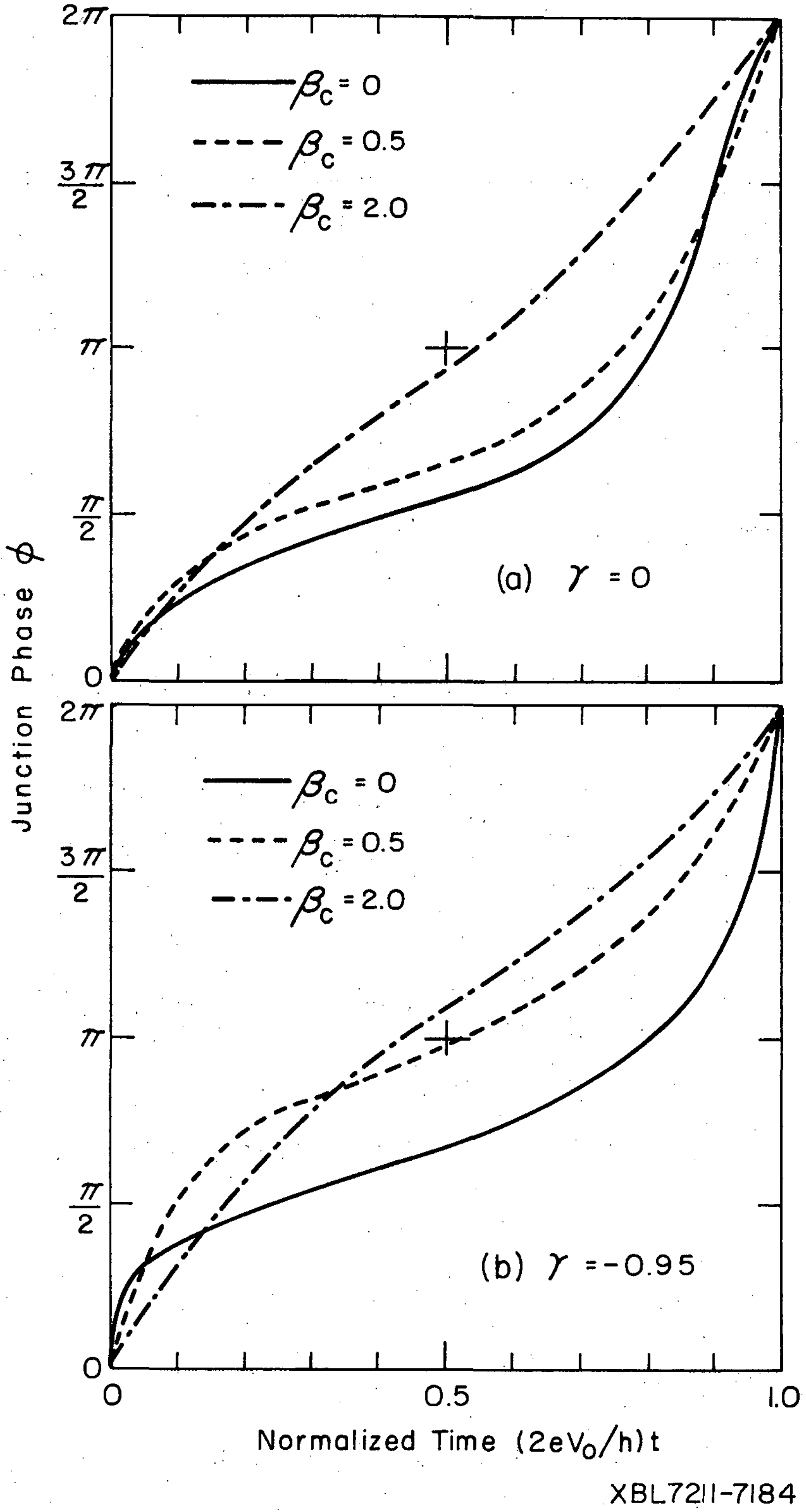
Junction Phase ϕ
2π
$\frac{3\pi}{2}$
π
$\frac{\pi}{2}$
0
$\beta_c = 0$
$\beta_c = 0.5$
$\beta_c = 2.0$
(a) $\gamma = 0$
(b) $\gamma = -0.95$
0
0.5
1.0
Normalized Time $(2eV_0/h)t$
XBL7211-7184

Fig. 3

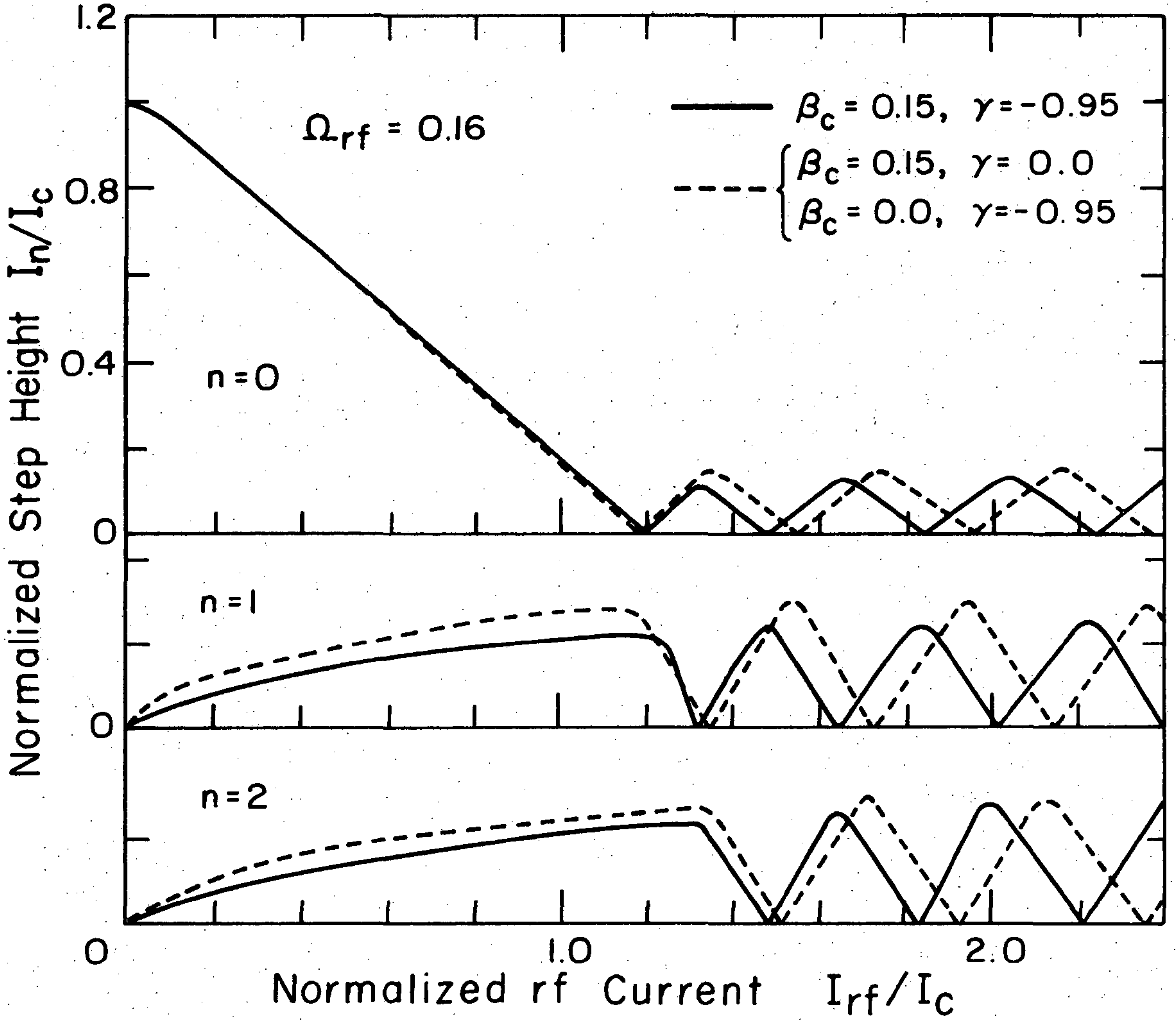

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Fig. 4

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