Title

Effect of Stiffness Degradation on Earthquake Ductility Requirements

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By
Ray W. Clough

October 1966

COLLEGE OF ENGINEERING
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EFFECT OF STIFFNESS DEGRADATION ON EARTHQUAKE DUCTILITY REQUIREMENTS

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Ray W. Clough

Prepared under the sponsorship of
Structural Engineers Association of California

University of California
Berkeley, California

October, 1966

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FOREWORD

The research study which is described in this report was made possible by a grant in aid to the University of California, College of Engineering, from the Structural Engineers Association of California, Robert B. Dalton, Jr., President.

The work was carried out under the general supervision and technical responsibility of Professor R. W. Clough, Department of Civil Engineering. The computer programs were written and analytical results obtained by Mr. Sterling B. Johnston, Graduate Student in the Division of Structural Engineering and Structural Mechanics. All computer work was performed by the University of California Computer Center.

I. INTRODUCTION

Background

It has been recognized for many years that a strong earthquake will induce forces and displacements in typical building structures which greatly exceed the effects resulting from the application of the earthquake loading specified in standard building codes. On this basis, it is clear that buildings designed for normal code lateral forces will be stressed beyond the elastic limit by a major earthquake, and the need for ductility in the design becomes evident.

The Structural Engineers Association of California recognized this need and made ductility a specific design requirement for tall buildings in the famous Section 2313, Paragraph "j" of the Recommended Lateral Force Provisions first proposed in 1958.(1) In the original proposal, the ductility necessary for the satisfactory performance of tall buildings was considered to be that provided by a steel frame with moment-resistant connections. Subsequently, with the aim of modifying these restrictions, the Seismology Committee of the Structural Engineers Association asked the Portland Cement Association to conduct a series of tests of reinforced concrete frames which would indicate the ductility capability of this form of construction. In prescribing the test program for the concrete frames, the Seismology Committee noted that ductility factors on the order of 4 to 6 were to be expected in the members of typical multistory buildings under earthquake excitation(2) and specified that the test frames be subjected to a cyclic loading sequence which would ultimately develop deformations of this order of magnitude.

Results of the PCA concrete frame ductility investigation[3] demonstrated that properly designed frame members and connections could develop significant ductile deformations during repeated loadings with no loss of strength. The force-deflection history recorded during the PCA test of a well designed beam-column assembly, shown in Fig. 1, is evidence of this fact. However, although the test specimen shows no loss of strength with repeated loadings, it is clear that its effective stiffness (represented by the slope of the load-deflection curve) is reduced as the amplitude of the deformation increases. This "degrading stiffness" property contrasts sharply with behavior of an ordinary elasto-plastic material, shown in Fig. 2, in which it is assumed that the stiffness retains its initial value so long as the applied load is less than the yield level loading.

Because practically all of the theoretical analyses on which the predicted earthquake ductility requirements in simple structures have been based had been carried out for ordinary elasto-plastic materials, a question was raised as to the relative earthquake resistance of structures having the observed degrading stiffness property. For given amplitudes of deformation it is evident that less energy is absorbed per cycle by a system with degrading stiffness as compared with an ordinary elasto-plastic system, and it was thought that the earthquake ductility requirements might be increased proportionately in such materials.

In order to make possible the thorough investigation of this question, the Structural Engineers Association of California in Feb-1966 provided a research grant of $2500 to the College of Engineering,

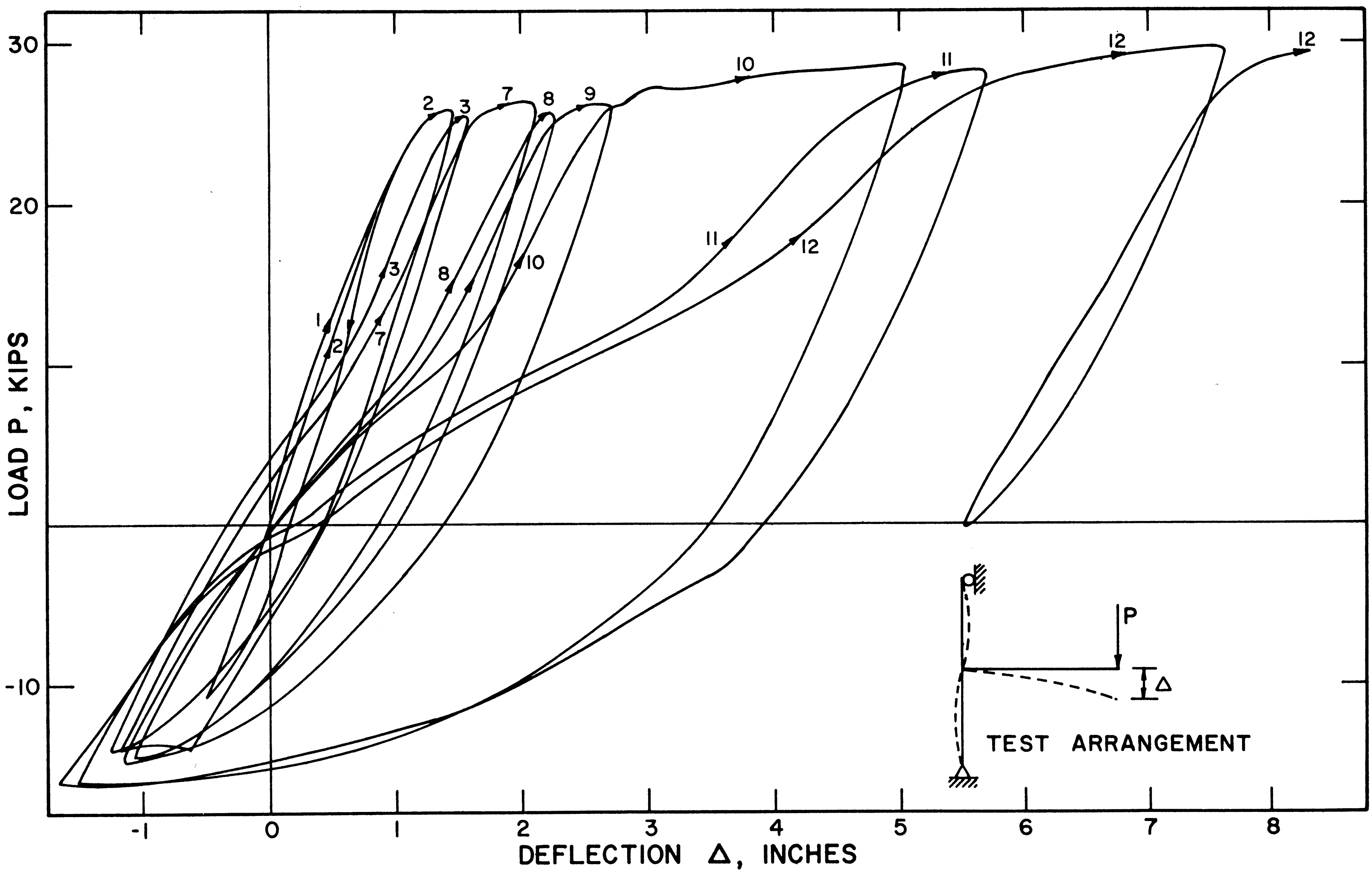


FIG. I TYPICAL CYCLICAL LOAD TEST - PCA REINFORCED CONCRETE SPECIMEN VA

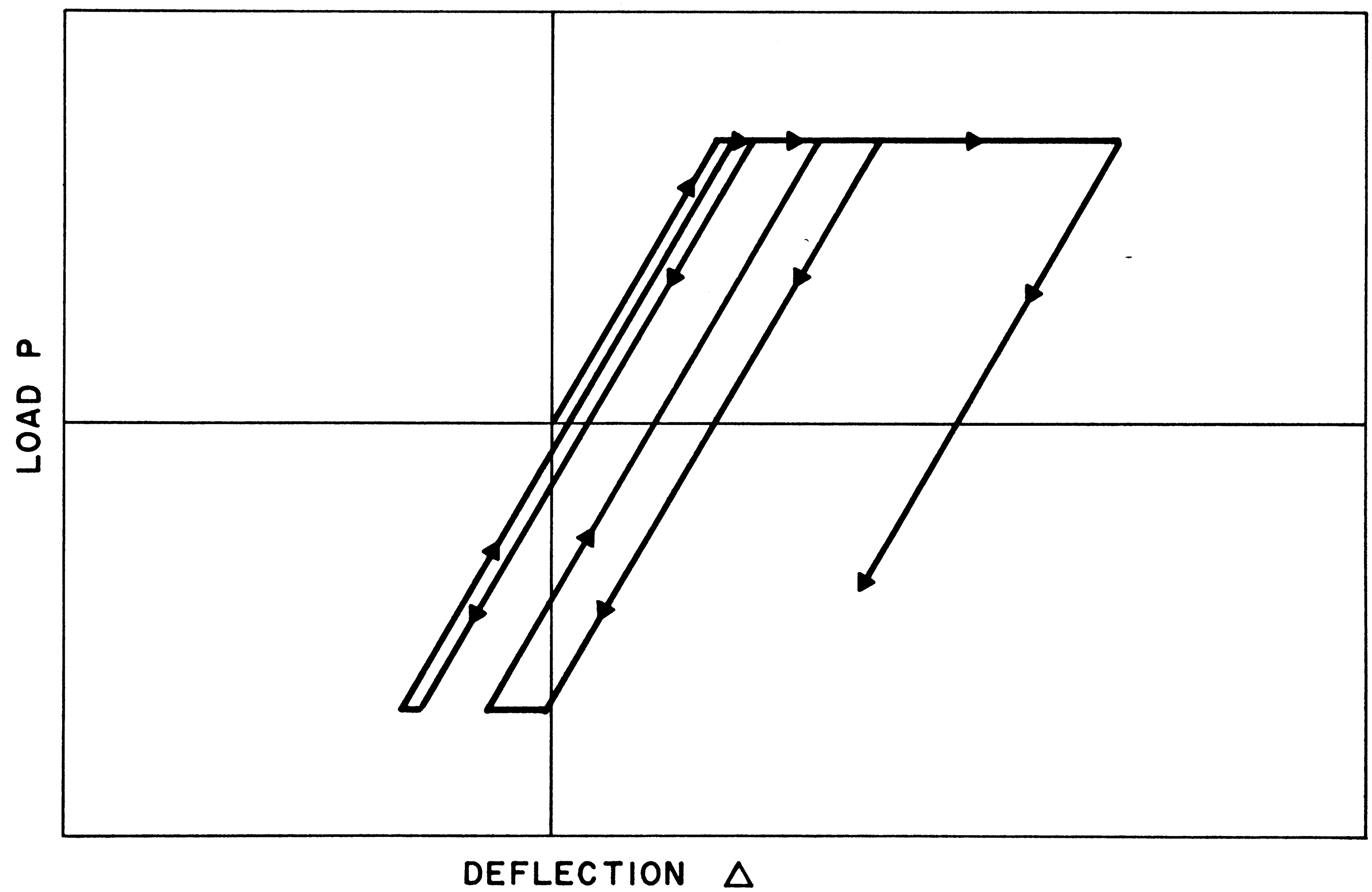


FIG. 2 TYPICAL CYCLICAL LOAD BEHAVIOR - IDEALIZED ELASTO PLASTIC SPECIMEN

University of California, Berkeley. The investigation was carried out by Dr. R. W. Clough, Professor of Civil Engineering, with the assistance of Mr. Sterling B. Johnston, Graduate Student in Structural Engineering and Structural Mechanics, and the principal results of the investigation are presented in this report.

Objective and Scope of the Investigation

The purpose of this study was to determine the earthquake resistance of structures exhibiting a degrading stiffness property similar to that shown in Fig. 1 as compared with the performance of equivalent ordinary elasto-plastic structures as characterized by Fig. 2. For this purpose only simple, single-degree-of-freedom structures were considered, and the analyses were carried out numerically by digital computer. In each analysis, the magnitude of inelastic deformations produced by a given earthquake ground motion in equivalent elasto-plastic and degrading stiffness systems were compared. Also, the displacement amplitude generated in a similar fully elastic structure was determined in each case. Variables considered in the investigation included the strength, damping, and period of vibration of the structure; four different earthquake ground motion records were used as the exciting mechanisms. A total of 192 cases were studied involving different combinations of these basic parameters, considering both the ordinary elasto-plastic and the degrading stiffness property for each case. The principal results of all these analyses are presented graphically in this report.

II. IDEALIZED NON-LINEAR STRUCTURAL MODELS

Basic Single-Degree-of-Freedom System

The structural system which was considered in this investigation is shown in Fig. 3. It consists of a rigid girder supported by weightless columns which are assumed fixed against rotation at the base. The physical properties of the system, for loads less than the yield level, are characterized by the girder mass M and the elastic column stiffness, k, or more fundamentally by the period of vibration $T = 2\pi\sqrt{M/k}$. The non-linear characteristics are represented by the relationship between lateral load V and deflection u, which may be expressed in the form of a force-deflection diagram. The types of non-linear properties considered in this study are described in the following paragraphs.

Ordinary Elasto-Plastic Model

The ordinary elasto-plastic system is characterized by the force-deflection relationship shown in Fig. 4a. Two properties of the structure may be identified in this figure: (1) the yield strength V_y, i.e., the load at which yielding occurs, and (2) the elastic stiffness $k_e = V_y/u_y$. It will be noted that increases of deflection beyond the yield level, u_y, take place with no increase of load; also, the unloading stiffness is identical with the initial elastic stiffness. In general it is assumed that the structure is symmetrical, so that the negative yield load is the same as the positive.

This is the type of material property which has been assumed in most previous studies of the non-linear earthquake response of structures.

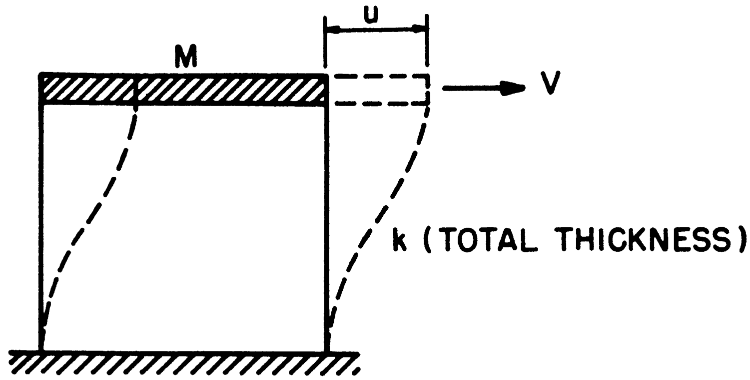
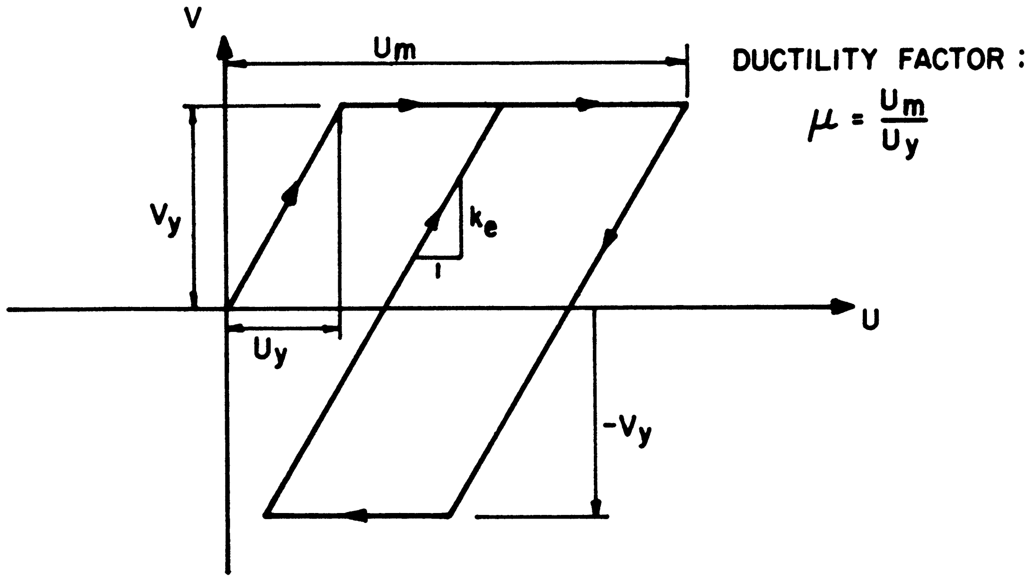
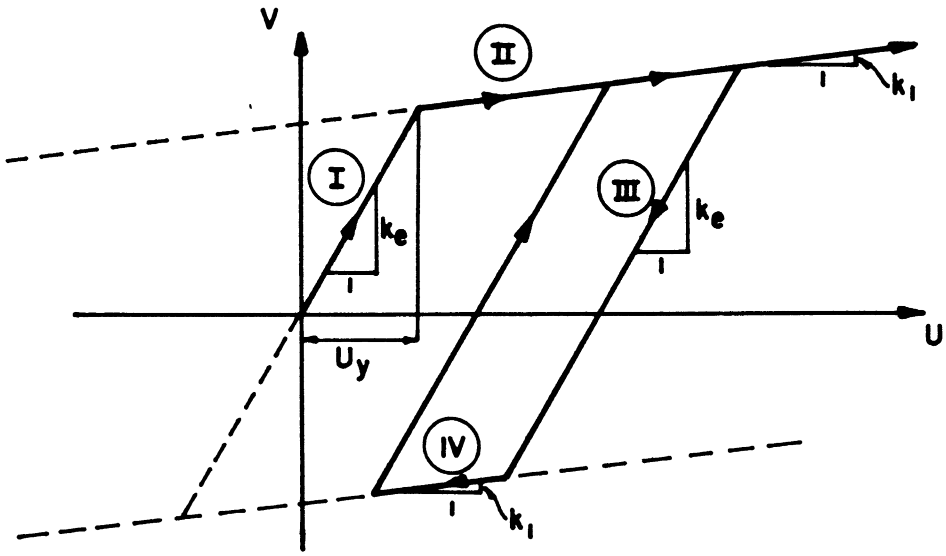


FIG. 3 BASIC SINGLE DEGREE OF FREEDOM SYSTEM



a. ORDINARY ELASTO-PLASTIC PROPERTY



b. BI-LINEAR ELASTO-PLASTIC PROPERTY

FIG. 4 IDEALIZED ELASTO-PLASTIC BEHAVIOR

The amount of non-linear deformation which is developed may be represented by the ductility factor, μ, which is defined as the ratio of the maximum deflection (elastic plus plastic) to the elastic limit deflection:

$$\mu = \frac{u}{u_y}$$

Clearly both positive and negative ductility factors may be evaluated.

Bi-Linear Elasto-Plastic Model

A more general version of the elasto-plastic system is the bi-linear model represented in Fig. 4b. This differs from the idealized elasto-plastic system only in that the stiffness is not zero for deflections increasing beyond the yield limit; the force continues to change with deflection after yielding, but at a different rate. An additional parameter is required to define the bi-linear system: the post-yield stiffness, k_1. An alternative measure of the bi-linear stiffness is the stiffness ratio $\gamma = k_1/k_e$. In the present investigation this stiffness ratio was varied between $\pm$ 5 per cent.

It is of interest to note that the behavior of the bi-linear model may be categorized by four regimes, as indicated in Fig. 4b:

I. Positive velocity, elastic: stiffness = k_e

II. Positive velocity, yielding: stiffness = k_1

III. Negative velocity, elastic: stiffness = k_e

IV. Negative velocity, yielding: stiffness = k_1

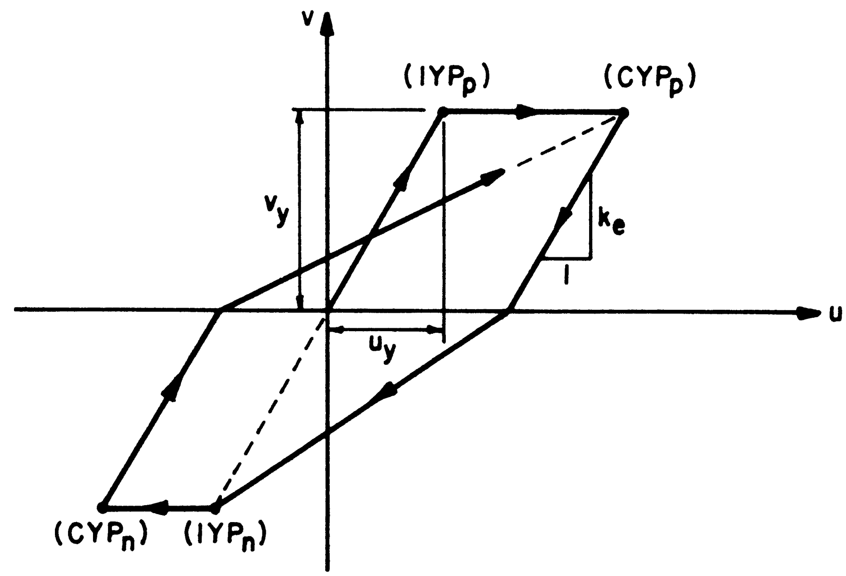
It should be recognized that reversals of velocity during the elastic regimes I or III do not cause changes of stiffness, whereas a reversal of velocity during the plastic regimes II or IV results in return to the elastic stiffness.

Basic Degrading Stiffness Model

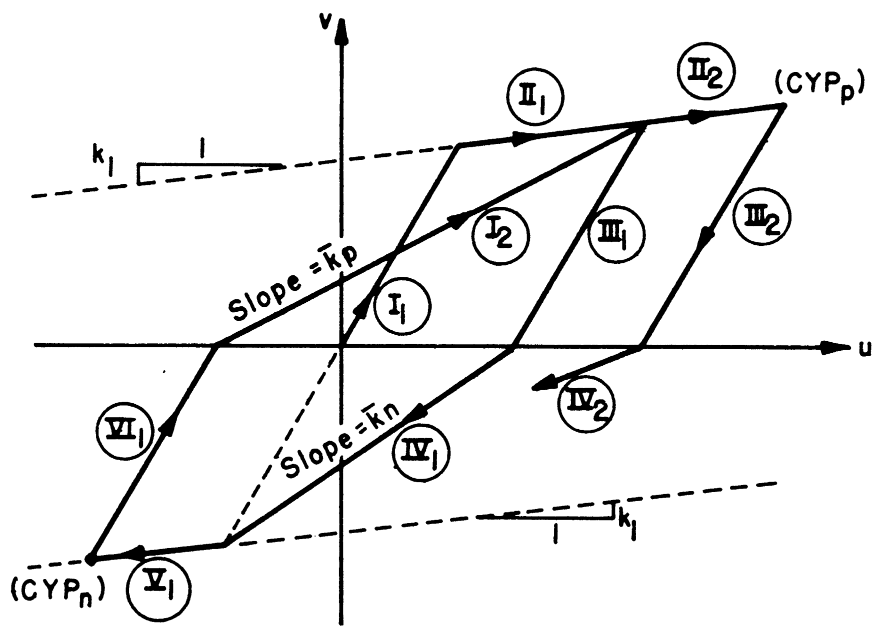
For the purpose of this investigation, it was assumed that the degrading stiffness property evidenced in the PCA test data of Fig. 1 might be represented by the type of force-deflection relationships shown in Fig. 5. Two different classes of system were considered, corresponding respectively to the elasto-plastic and bi-linear models of the ordinary non-linear material shown in Fig. 4.

The ideally plastic system is depicted in Fig. 5a. In this case, the initial behavior is identical to that shown in Fig. 4a, and is characterized by the same parameters: yield strength, V_y, and initial stiffness, k_e. However, after loading, yielding, and unloading, the negative loading stiffness is assumed to be defined by two points on the load-deflection diagram: (1) the force-deflection condition at which positive unloading terminated, and (2) the current negative yield point, CYP_n. For the initial negative loading cycle, the current negative yield point is called the initial negative yield point (IYP_n) and is defined by the initial (virgin) negative yield force-displacement condition. However, after negative yielding has taken place, the CYP_n is defined by the maximum negative displacement which has occurred at any previous time and the corresponding negative yield force.

Unloading from the negative loading regime takes place with the initial elastic stiffness, as in an ideal elasto-plastic material, but the degrading stiffness property then controls the subsequent positive loading phase. The degrading stiffness during positive loading also is defined by two points on the load-deflection diagram, in exact analogy with the negative loading case: (1) the force-deflection



a ORDINARY DEGRADING STIFFNESS PROPERTY



b BILINEAR DEGRADING STIFFNESS PROPERTY

FIG. 5 ASSUMED DEGRADING STIFFNESS BEHAVIOR condition at which the previous regime terminated, and (2) the current positive yield point, CYP_p. The CYP_p is given by the maximum positive displacement which has occurred at any previous time, and the corresponding positive yield force. If no positive yielding has yet occurred, the CYP_p is defined by the initial positive yield force-displacement condition (IYP_p).

<u>Bi-linear Degrading Stiffness Model</u>

A more general version of the degrading stiffness system is represented by Fig. 5b. This corresponds to the bi-linear system of Fig. 4b in that the stiffness after yield, k_1, is not zero but may be specified arbitrarily. Alternatively, the yield stiffness may be specified by the stiffness ratio $\gamma = k_1/k_e$, and as for the previous case, this ratio was varied within the range $\gamma = \pm 5$ per cent in the present investigation. The degrading stiffness property for the bi-linear case is defined exactly as for the special case ($k_1 = 0$) which was described above, on the basis of the force-deflection condition at the end of the preceding regime and the appropriate current yield point (CYP).

The degrading stiffness behavior may be divided into six regimes as illustrated in Fig. 5b:

I.	Positive velocity, loading:	initial stiffness = k_e or degrading stiffness = $\bar{k}_p$
II.	Positive velocity, yielding:	stiffness = k_1
III.	Negative velocity, unloading:	stiffness = k_e
IV.	Negative velocity, loading:	degrading stiffness = $\bar{k}_n$
V.	Negative velocity, yielding:	stiffness = k_1

VI. Positive velocity, unloading: stiffness = k_e

The first deformation cycle of a possible loading history is identified in Fig. 5b by the subscript 1 associated with each regime number, while the second cycle history is indicated with the subscript 2.

The hypothesis on which this degrading stiffness behavior was established is that the yield mechanism involves cracking of the concrete and ductile deformation of the reinforcing bars. Unloading is assumed to involve recovery of the elastic deformations in the steel and thus results in changes of deflection at the initial slope. However, loading in the reverse direction requires that the crack be closed before negative yield can take place. The degrading stiffness property is assumed to be associated with the crack-closing phenomenon. It is of interest to note that after any amount of yielding has occurred this degrading stiffness mechanism gives rise to a hysteresis loop for all cycles of loading and unloading, whether yielding takes place during the cycle or not. This is in distinct contrast with the elasto-plastic system of Fig. 4 in which hysteretic energy losses result only from yielding during that cycle; the ordinary elasto-plastic model behaves as a typical elastic system for loads less than the yield level.

III. ANALYSIS OF DYNAMIC RESPONSE

Equations of Dynamic Equilibrium

The structural system which was subjected to earthquake excitation is shown in Fig. 6. It consists of a rigid girder of mass M, supported by weightless columns having a total lateral displacement stiffness, k. In addition, it is assumed that a viscous damper, c, represents the energy loss mechanism for elastic response.

The response of the system is characterized by the displacement of the mass relative to the ground, u. During motions which exceed the elastic limit of the system, it is assumed that the stiffness k varies in accordance with the response regime which prevails, as discussed in Chapter II.

The forces acting on the mass, M, during its earthquake response include the inertia force F_I, the spring force of the columns V, and the viscous damping force F_D; thus the equation of dynamic equilibrium is:

$$F_I + F_D + V = 0 \tag{1}$$

The inertia force is given by the product of the mass and the total acceleration:

$$F_I = M\ddot{u}_t \tag{2}$$

where the dots represent differentiation with respect to time and u_t represents the absolute motion of the mass measured from a fixed reference axis. The spring force for elastic motions is given by the product of the column stiffness and the displacement of the mass relative to the ground:

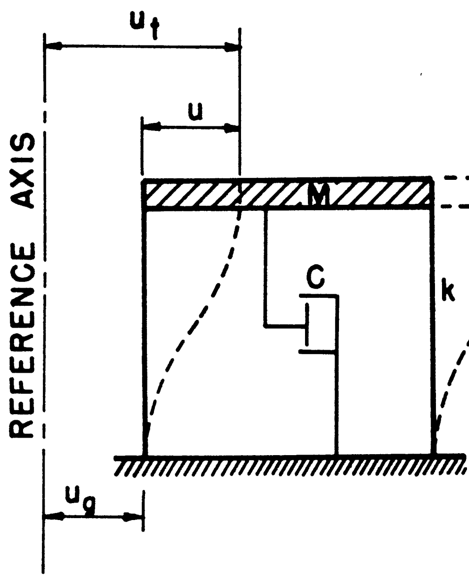


FIG. 6 DYNAMIC MODEL WITH EARTHQUAKE EXCITATION

$$V = k_e u \tag{3}$$

while the damping force is the product of the damping coefficient c and the velocity of the mass relative to the ground:

$$F_D = c\dot{u} \tag{4}$$

Introducing these expressions into Eq. 1 yields:

$$M\ddot{u}_t + c\dot{u} + k_e u = 0 \tag{5}$$

Finally, noting that the total displacement can be expressed as the sum of the ground motion, u_g, plus the relative displacement, u:

$$u_t = u_g + u \tag{6}$$

or correspondingly for the accelerations

$$\ddot{u}_t = \ddot{u}_g + \ddot{u} \tag{7}$$

it is possible to represent the total inertia force of Eq. 7 as the sum of the force associated with ground accelerations plus that associated with the relative accelerations

$$F_I = M\ddot{u}_t = M\ddot{u}_g + M\ddot{u} \tag{2a}$$

Introducing Eq. 2a into Eq. 5 and transferring the term associated with the ground accelerations to the right hand side leads to the equilibrium equations for earthquake response:

$$M\ddot{u} + c\dot{u} + k_e u = -M\ddot{u}_g \tag{8}$$

Dividing Eq. 8 by M:

$$\ddot{u} + \frac{c}{M}\dot{u} + \frac{k_e}{M}u = -\ddot{u}_g$$

and introducing the relationships

$$\frac{k_e}{M} = \left(\frac{2\pi}{T}\right)^2$$

$$\frac{c}{M} = \frac{4\pi}{T}\lambda$$

where $\lambda = c/\bar{c}$ is the damping ratio, i.e., the ratio of the actual damping coefficient to the critical damping coefficient, $\bar{c}$; and the critical damping coefficient is given by

$$\bar{c} = \frac{4\pi}{T} M$$

the equation of motion can be written in the form:

$$\ddot{u} + \frac{4\pi}{T} \lambda \dot{u} + \frac{4\pi^2}{T^2} u = -\ddot{u}_g \qquad (9)$$

Here it is clear that the response, u, to any given ground motion depends only on the period of vibration, T, and the damping ratio, λ, of the structure, i.e., these parameters define the essential structural characteristics of a linear system.

When considering the non-linear response of the structure, it is necessary that the spring resistance to displacement, V, be expressed appropriately in terms of the stiffness pertinent to each regime of the response, as indicated by the force-deflection relationships of Figs. 4 and 5. In this case, the yield strength of the structure, V_y, is a significant physical parameter. It is convenient to specify this quantity as a ratio, β, to the total weight of the system:

$$\beta = \frac{V_y}{W} = \frac{V_y g}{M} \qquad (10)$$

The other significant measure of the non-linear behavior is the post-yield stiffness, k_1, which is defined by its ratio to the initial elastic stiffness as explained in Chapter II:

$$\gamma = \frac{k_1}{k_e} \qquad (11)$$

Thus the four parameters which completely define the physical characteristics of the non-linear system are T, λ, β and γ.

Numerical Analysis Procedure

Because the lateral stiffness of the structure varies from regime to regime during the dynamic response, its behavior is non-linear and the equations of motion can only be integrated by numerical procedures. In the present investigation, the response history was divided into very short, equal time increments and the motion determined by assuming that the acceleration varied linearily during each increment.[4] As the response analysis proceeds, the structure passes from one regime to another, in accordance with Fig. 4 or 5, and the appropriate stiffness property, k, is introduced into Eq. 8 as required for the current regime. In general, the transitions from one regime to another do not occur at the end of a time increment. Thus these transition conditions were accounted for by establishing the time at which the transition occurred and then computing the response to the end of one regime by means of a partial time increment and evaluating the behavior during beginning of the next regime using the complementary short time increment.

Digital Computer Program

The numerical analysis procedure described above was carried out automatically by means of a Fortran program written for the IBM 7094 computer operated by the University of California Computer Center. Input information required by the program included:

(1) The basic structural parameters: T, λ, β, γ

(2) The type of non-linear property: ordinary or degrading stiffness

(3) The acceleration history of the desired earthquake motion.

The output of each computer analysis included a listing of:

(1) The number of times negative and positive yield occurred

(2) The maximum negative and positive displacements and ductility factors

(3) The total earthquake energy input to the structure, as well as the total hysteretic and damping energy losses during

the earthquake. In addition, an option was provided for printing out the complete time-history of the response including:

(1) Base shear force, V

(2) Displacements of the ground, u_g, and of the structure relative to the ground, u.

(3) Components of work and energy, summed from the beginning of the earthquake.

In all cases, the computation time increment was 0.005 seconds and the earthquake response was computed for approximately 30 seconds of excitation, plus an additional 5 seconds of free vibration which allowed the system to approach a rest condition.

IV. SCHEDULE OF CASES STUDIED

Ranges of Significant Parameters

In order to establish the relative earthquake response characteristics of structures exhibiting the degrading stiffness behavior as compared with those having the idealized elasto-plastic behavior, it was necessary to compare their performances for a wide range of structural properties. The significant structural parameters were defined in Chapter 3; the ranges considered for each of these basic variables are described below.

(1) Period of Vibration: T

Initially it was intended only to study the behavior of long period structures which might be considered equivalent to tall buildings, and the period range of 0.9 to 3.0 seconds was selected for study, considering 0.3 second period increments. However, initial calculations indicated that the behavior of the longer period structures were quite similar, so it was decided that 0.6 second period increments would be sufficient in long period range. Also, it became evident that the short period structures behaved quite differently from the long period systems, so two shorter period systems were considered. The complete range of structural periods finally considered were as follows: T = 0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, and 2.7 seconds.

(2) Damping Ratio: λ

The damping ratios considered in these analyses were intended to encompass the range of conditions which might be expected in typical structures in the field, and included the following values:

$\lambda = 0$, 2%, 5%, and 10%

(3) Yield Strength Ratio: β

In order that the results of this study might be correlated most easily with current design practice the yield strength ratio of the structure was established by reference to SEAOC code design strength requirements. The design base shear force V_d (for a structure located in Seismic Zone 3) is specified by the code as follows:

$$V_d = KCW \tag{12}$$

wherein the coefficient K depends on the type of framing system and varies between 0.67 and 1.33, W is the total dead weight of the structure, and the seismic coefficient C is related to the period of vibration as follows:

$$C = \frac{0.05}{\sqrt[3]{T}} \tag{13}$$

For the purpose of this investigation, it was assumed that the yield load might be taken as twice the design load, to account for the differential between yield and design stresses as well as for the strengthening influence of non-structural components. On this basis, the yield strength ratio is given by

$$\beta = \frac{V_y}{W} = \frac{2V_d}{W} = \frac{K(0.10)}{\sqrt[3]{T}} \tag{14}$$

Thus, as is shown by Eq. 14, the strength ratio used in the present investigation is dependent only on the period of vibration T, and on the framing type parameter K. Strength ratios used in this research program are listed in Table I. Also shown as a matter of interest, is the elastic limit deflection, u_y, for each case, which may be computed from the expression:

$$u_y = \frac{g}{4\pi^2} \beta T^2 \tag{15}$$

in which g is the acceleration of gravity.

TABLE I: YIELD PROPERTIES OF TEST STRUCTURES

a. Strength Ratio $\rho = V_y/W$

Period - T	Framing Coefficient - K			
	0.67	0.80	1.00	1.33
0.3	0.100	0.119	0.149	0.199
0.6	0.079	0.095	0.118	0.158
0.9	0.070	0.083	0.104	0.138
1.2	0.063	0.075	0.094	0.125
1.5	0.059	0.070	0.088	0.116
1.8	0.055	0.066	0.082	0.110
2.1	0.052	0.063	0.078	0.104
2.7	0.048	0.058	0.072	0.096

b. Yield Displacement $\overset{u_y}{V_y}$ - inches

Period - T	Framing Coefficient - K			
	0.67	0.80	1.00	1.33
0.3	0.088	0.105	0.131	0.175
0.6	0.279	0.335	0.414	0.556
0.9	0.555	0.659	0.824	1.11
1.2	0.887	1.06	1.32	1.76
1.5	1.30	1.54	1.94	2.55
1.8	1.74	2.09	2.60	3.48
2.1	2.24	2.72	3.37	4.48
2.7	3.41	4.10	5.12	6.81

(4) Bilinear Stiffness Ratio: γ

In most analyses, only the idealized plastic yielding condition was considered, i.e., the bilinear stiffness ratio was set at zero. However, in some supplementary analyses, the effect of strain hardening and of strength degradation was evaluated. In these cases the bilinear ratio was taken to be $\gamma = +5\%$ and $\gamma = -5\%$, respectively.

Earthquake Ground Motions

Because each recorded earthquake motion has its own individual characteristics, the results obtained by applying any single excitation cannot be considered representative of an "average" earthquake condition. Thus for the purposes of the present investigation it was necessary to consider several ground motions in order to obtain a valid comparison of the response behavior of the two types of material properties.

The ground motions which were employed in this study were:

(1) El Centro 1940, N-S component (EC 40 NS)

(2) Taft 1952, S-W component (TA 52 SW)

(3) El Centro 1934, N-S component (EC 34 NS)

(4) Olympia 1949, E-W component (OL 49 EW)

The El Centro 1940 N-S component, which is the strongest earthquake motion yet recorded, was used as the excitation with the full range of structural parameters. In order to reduce the computer time requirements, the other earthquake excitations were applied only to selected limiting combinations of these parameters. The complete schedule of analyses which were performed during this investigation is given in Table II. For the principal phase of the investigation,

TABLE II: SCHEDULE OF TEST CASES

a. No Bi-linear Stiffness $\gamma = 0$

Note: All combinations of the listed values were used

Earthquake	Period - T	Damping -	Framing - K
	seconds	percent	
EC40NS	0.3 0.6 0.9 1.2 1.5 1.8 2.1 2.7	0 2 5 10	0.67 0.80 1.00 1.33
EC34NS TA52SW OL49EW	0.3 0.9 1.5 2.1 2.7	0 10	0.67 1.33

b. Bi-linear Analyses $\gamma \neq 0$

Note: Only the four cases shown were considered

Earthquake	Period - T	Damping -	Framing - K	Bi-linear Ratio-
	seconds	percent		percent
EC40NS	2.7	5	0.67	+5
EC40NS	2.7	5	0.67	-5
EC40NS	0.3	10	0.67	+5
EC40NS	0.3	10	0.67	-5

the bi-linear stiffness parameter, γ, was set at zero so that yielding occurred at constant force. Cases considered in this phase of the work are shown in Table 2a; all combinations of the various parameters listed with each earthquake were used.

In a supplementary phase of the investigation, the behavior of systems exhibiting bi=linear force-deflection properties was studied; cases considered in this work are listed in Table 2b. The earthquake excitation considered in each of these bi-linear analyses was EC 40 NS.

V. RESULTS OF ANALYSES

Displacement-Time History

The most fundamental result of the analyses carried out in this investigation is the time-history of displacement response, i.e., the history of motion of the mass of the structure relative to the ground during the course of the earthquake. However, it is not practicable to present graphs of these displacement histories for each of the 384 non-linear analyses which were made. Only two typical cases will be shown to illustrate the type of behavior which was observed, and to demonstrate the relative response characteristics of the two types of non-linear material properties. The ground motion used in each of these example analyses was El Centro 1940, N-S component.

Case A:

The first case which will be considered is a long period structure having the following properties: $T = 2.7$ secs, $\lambda = 5\%$, $K = 0.67$. The displacement response history of this system, considering the ordinary elasto-plastic material (bi-linear stiffness, $\gamma = 0$), is shown in Fig. 7a. The intervals during which yielding takes place in this system are identified by arrows (yielding occurs only while strains are increasing, of course). As may be seen in Table I, the elastic limit deflection for this structure is 6.81 inches. The ground motion ceased at about 30 seconds in this earthquake, so the last 5 seconds of the response history represents merely the residual damped free vibration response.

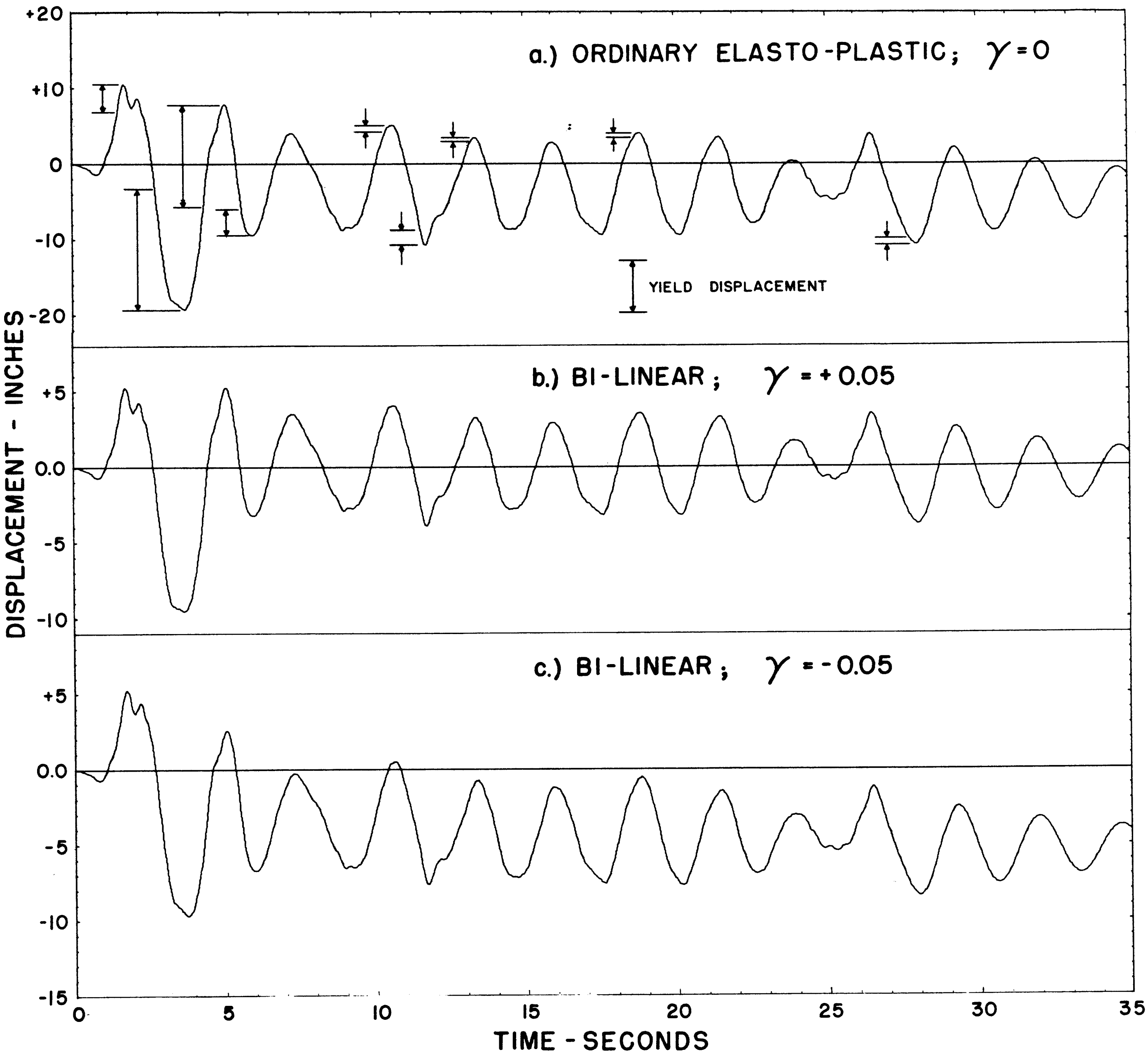


FIG. 7 DISPLACEMENT RESPONSE OF ELASTO-PLASTIC STRUCTURE

STRUCTURE PROPERTIES: T=2.7 SECONDS, K=0.67, λ= 5%
EL CENTRO 1940 N-S EARTHQUAKE

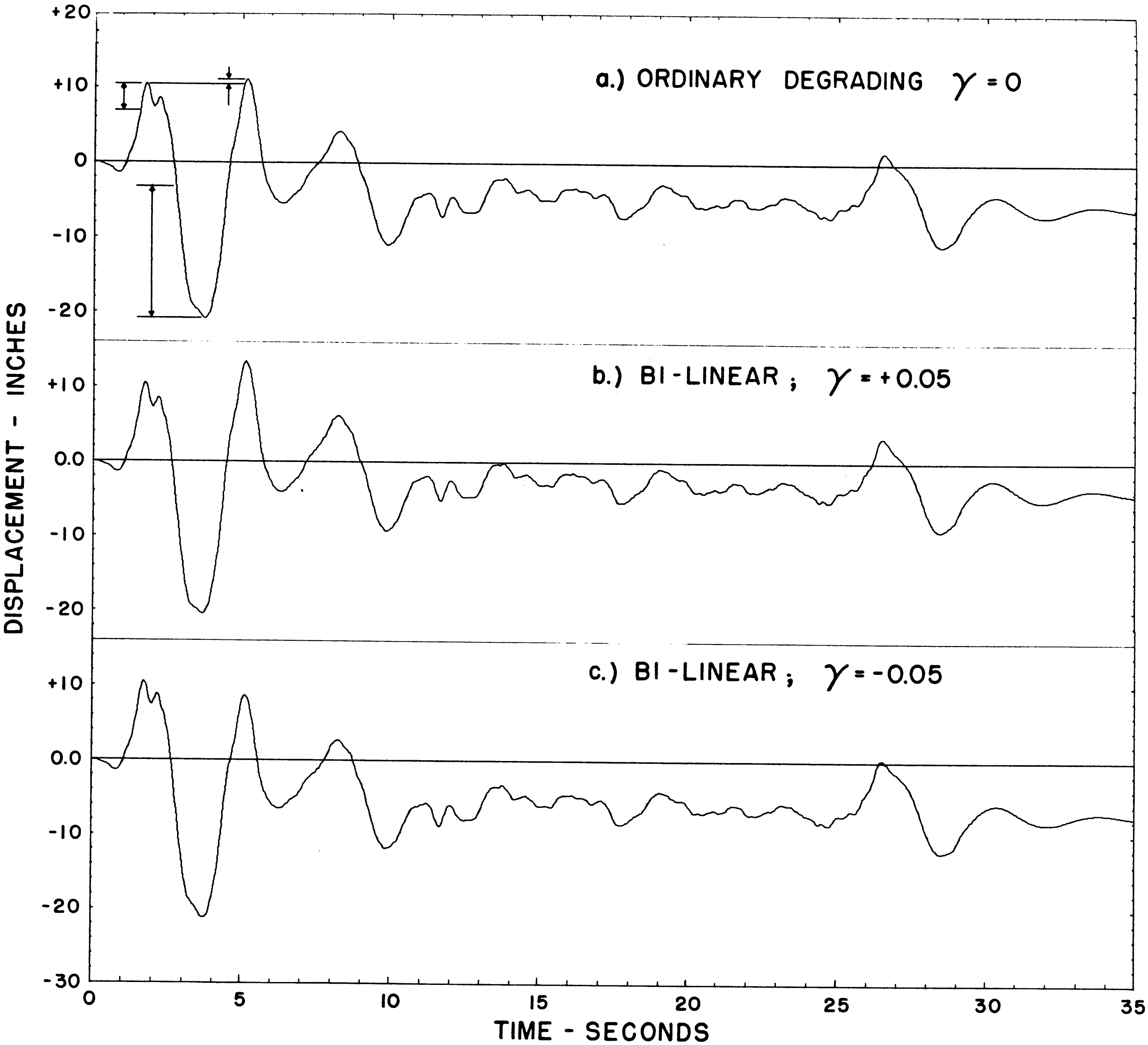


FIG. 8 DISPLACEMENT RESPONSE OF DEGRADING STIFFNESS STRUCTURE

STRUCTURE PROPERTIES : T=2.7 SECONDS, K=0.67, λ = 10 %

EL CENTRO 1940 N-S EARTHQUAKE

The earthquake response of a corresponding system having the degrading stiffness property is shown in Fig 8a. The essential differences in behavior of the two material properties is clearly evident in a comparison of Figs. 7 and 8. The elasto-plastic system remains essentially a harmonic oscillator, with typical vibratory response behavior. Although yielding occurs in many of the vibration cycles, the principal effect of the yielding is to limit the amplification of response--the motion continues to take place at essentially the free vibration frequency. The degrading stiffness system, on the other hand, shows no real vibratory response after the initial large amplitude motions. The initial response is nearly the same in Figs. 7a and 8a, but the loss of stiffness resulting from these large yield deformations greatly reduces the subsequent response in the degrading system.

Also shown on Figs. 7 and 8 are the responses for bi-linear systems of the ordinary and degrading types, respectively. Figs. 7b and 8b represent the response of systems having a post-yield stiffness of +5 per cent ($\gamma = +0.05$), while Figs. 7c and 8c show the behavior for the corresponding negative post yield stiffness ($\gamma = -0.05$). Comparison of the response of the bi-linear systems with those computed in the ordinary case ($\gamma = 0$, Figs. 7a and 8a) reveals that this small bi-linear characteristic is of little importance in the earthquake response of a long period, flexible structure.

Another view of the response of these long period, non-linear systems is presented in the force-deflection diagrams shown in Fig. 9. The assumed force-deflection properties are clearly evident in the

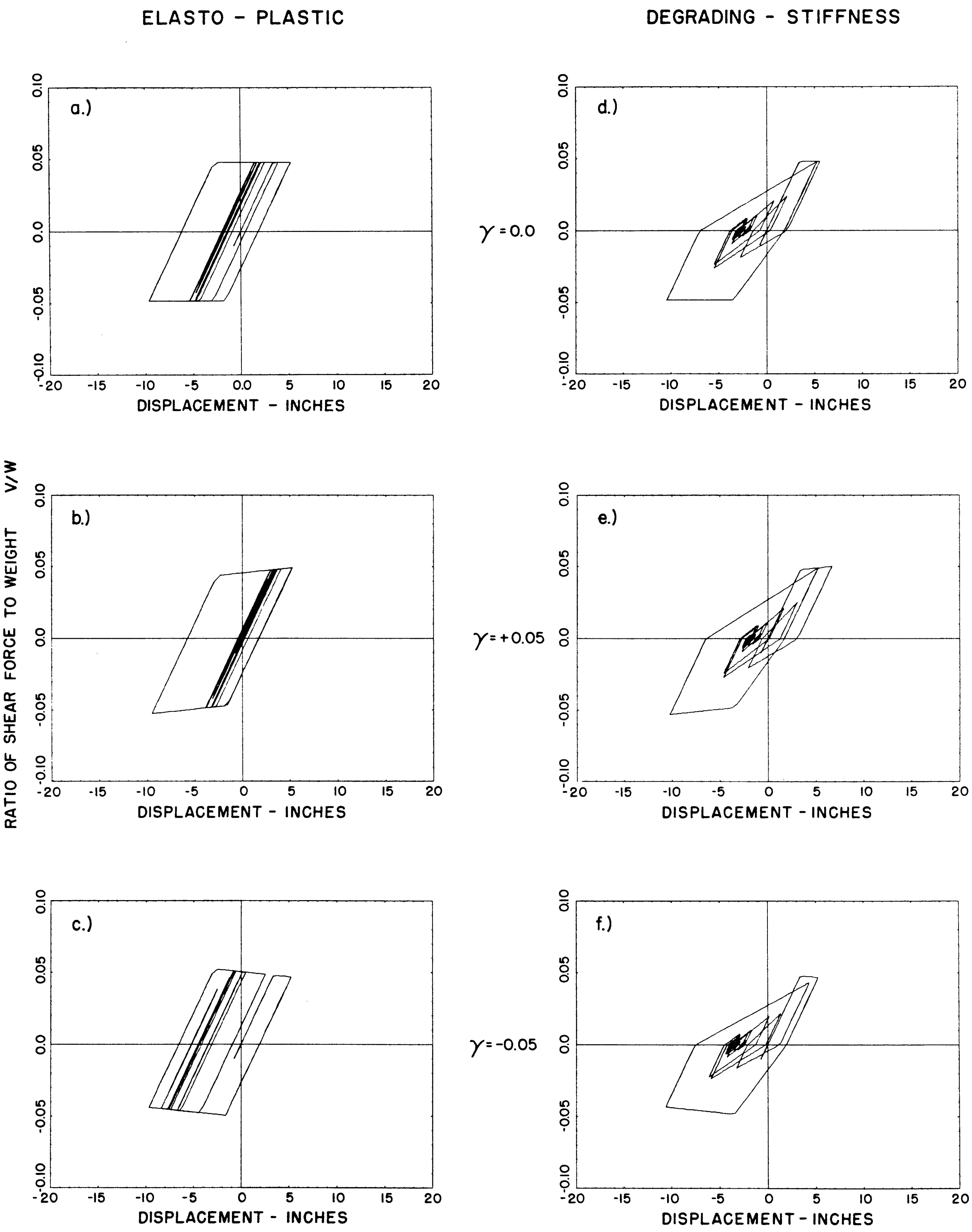


FIG. 9 FORCE DISPLACEMENT DIAGRAMS - EL CENTRO 1940 N-S EARTHQUAKE

STRUCTURE PROPERTIES T = 2.7 SECONDS, K = 0.67, λ = 10%

form of the diagrams; the ordinary elastic-plastic systems are shown at the left while the corresponding degrading stiffness systems are at the right. The generally smaller amplitudes of the motions developed by the degrading stiffness material after yielding are clearly evident in this figure. Also, the minor differences result- from the different bi-linear stiffness ratios are of interest. The numbers shown on the graphs represent the times (after initiation of the earthquake) at which various points on the force-deflection diagrams were developed.

Case B:

The period of vibration of a structure is the most important single factor in controlling its earthquake response behavior. For this reason a short period structure will be considered in order to show its basically different response mechanism. The properties of this structure are as follows: $T = 0.3$ secs., $\lambda = 10\%$, $K = 0.67$; its elastic limit deflection is 0.088 inches. The displacement history of the ordinary elasto-plastic structure, $\gamma = 0$, is presented in Fig. 10a while the corresponding degrading stiffness case is shown in Fig. 10b. These figures demonstrate that the short period structure is much more responsive to the earthquake excitation than was the longer period system. Also, it is clear that the degrading stiffness structure responds more actively than the ordinary elasto-plastic model. The response frequency is reduced, of course, by the loss of stiffness, but the displacement amplitudes are significantly increased.

The most significant parts of these displacement history

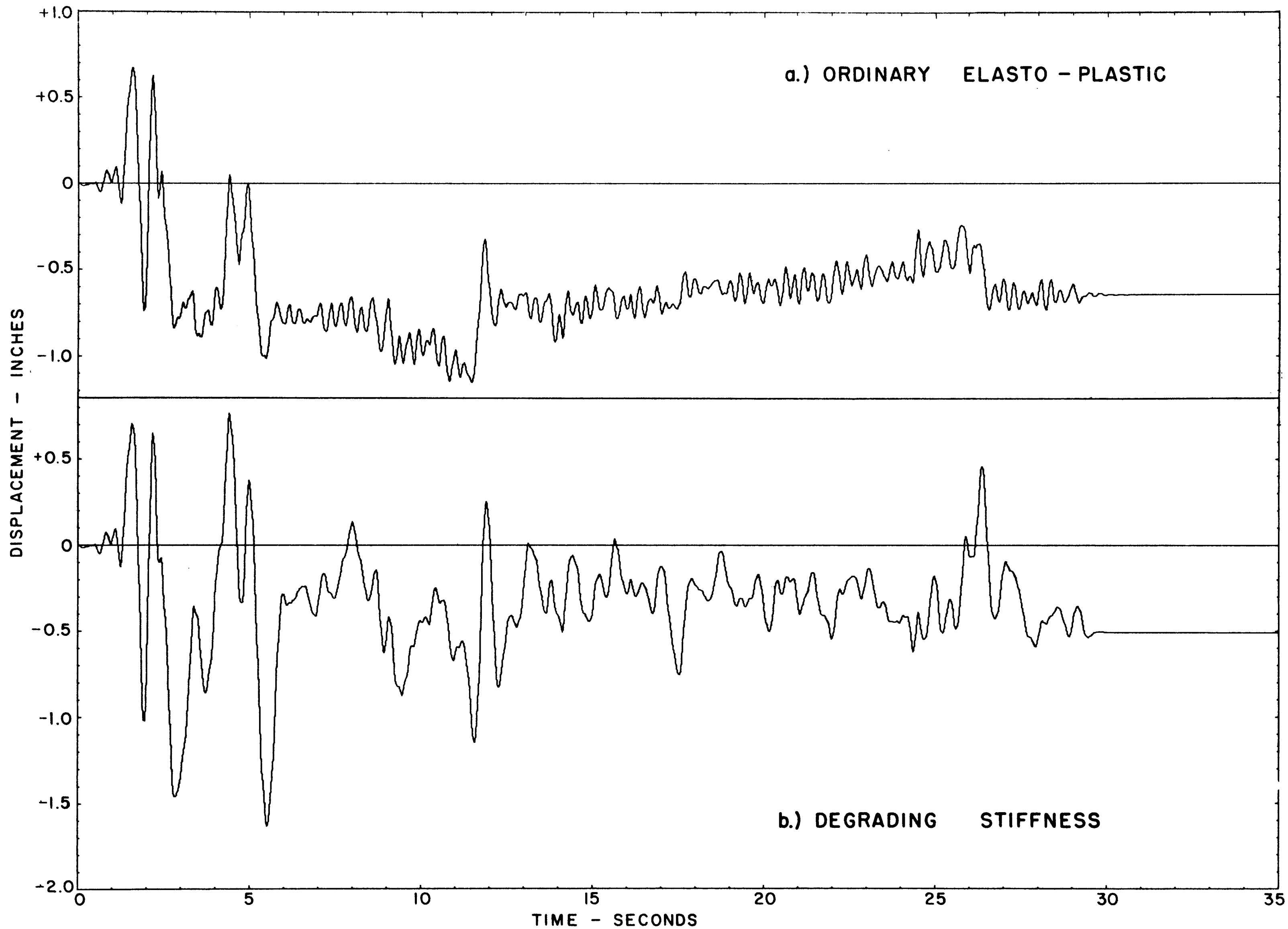


FIG. 10 DISPLACEMENT RESPONSE — EL CENTRO 1940 N-S EARTHQUAKE

STRUCTURE PROPERTIES: $K = 0.67$, $\lambda = 10\%$, $T = 0.3$ SECONDS graphs, the first six seconds, are shown to an expanded time scale in Fig. 11 in order that their details might be examined more closely. The ordinary elasto-plastic material property response is shown in the graphs at the left, while the corresponding degrading stiffness results are at the right. The top graph in each case is for a zero bi-linear stiffness ($\gamma = 0$); i. e., these duplicate the first part of the graphs shown in Fig. 10. The positive bi-linear stiffness case ($\gamma = 0.05$) which is shown immediately below, is seen to cause a noticeable reduction of response, but the effect is not dominant. On the other hand, the negative bi-linear stiffness property ($\gamma = -0.05$), which is illustrated at the bottom, has a disastrous effect on these short period structures. For both the ordinary and the degrading stiffness materials the yielding continues to increase until a zero strength condition is reached--at which time the structure collapses. This is a very significant contrast to the behavior of the long period structure with the negative bi-linear property, where the effect was negligible.

The force-deflection graphs for the first portion of the responses shown in Fig. 11 are presented in the same sequence in Fig. 12. The essentially different character of the behavior demonstrated by the ordinary and the degrading stiffness materials is evident in these graphs. Also, the failure mechanism which develops in the negative bi-linear materials is clearly shown in the two lower graphs.

Maximum Ductility Factors

Although the displacement-time history plots are of great interest

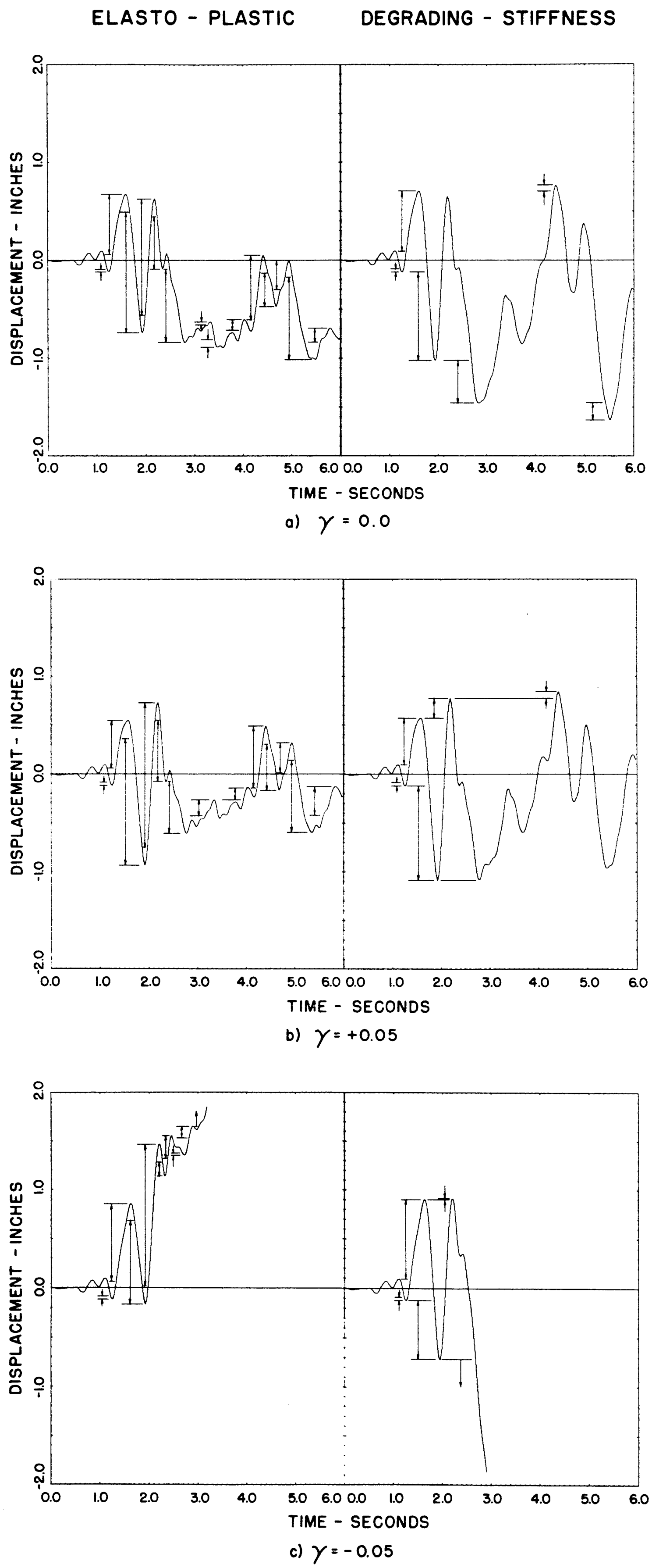


FIG. 11 DISPLACEMENT RESPONSE - EL CENTRO 1940 N-S EARTHQUAKE

STRUCTURE PROPERTIES T=0.3 SECONDS, K=0.67, λ=10%

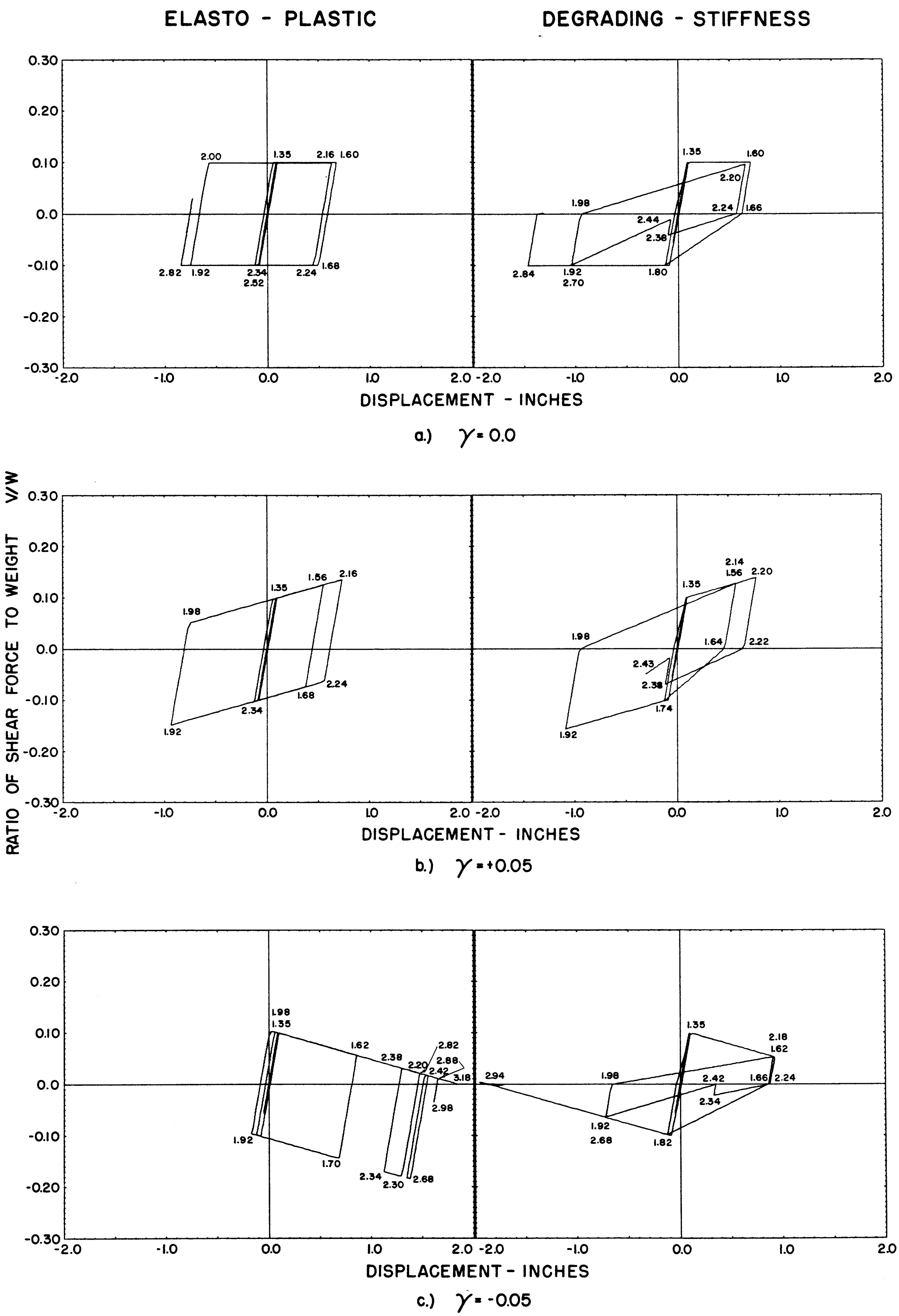


FIG. 12 FORCE-DISPLACEMENT DIAGRAMS - EL CENTRO 1940 N-S EARTHQUAKE

STRUCTURE PROPERTIES: T=0.3 SECONDS, K=0.67, λ=10%

in evaluating the earthquake response behavior mechanism, the most significant behavior characteristic from a practical standpoint is the maximum deflection achieved at any time during the earthquake. Moreover, the most valuable measure of the maximum deflection is its ratio to the elastic limit deflection. This ratio, which is designated the ductility factor, μ, has been determined for each of the analysis cases considered in this investigation. Plots showing the variation of maximum ductility factor, μ, with period of vibration of the structure are shown in Figs. 13 through 22. Each figure represents the results of a particular earthquake excitation applied to a given strength of structure. The effects of different damping ratios are shown by the separate curves on each graph.

As was mentioned in Chapter IV, only for the EC 40 NS excitation was the complete range of structural strengths, periods of vibration and damping ratios considered. Results for these cases are presented in Figs. 13-16, plotted to a semi-log scale because of the wide variation in ductility requirements resulting from changes in period of vibration. For the weakest structure at a period of vibration of 0.3 seconds, the computed ductility factors are as great as 25 or 50 in the undamped ordinary and degrading stiffness cases, respectively, whereas the 2.7 second period structures have corresponding ductility requirements of less than two. Results for the other earthquake excitations are presented in Figs. 17-22; these involve only the $K = 0.67$ and $K = 1.33$ structural systems, and damping ratios of $\lambda = 0$ and $\lambda = 10\%$, as may be seen in Table 2.

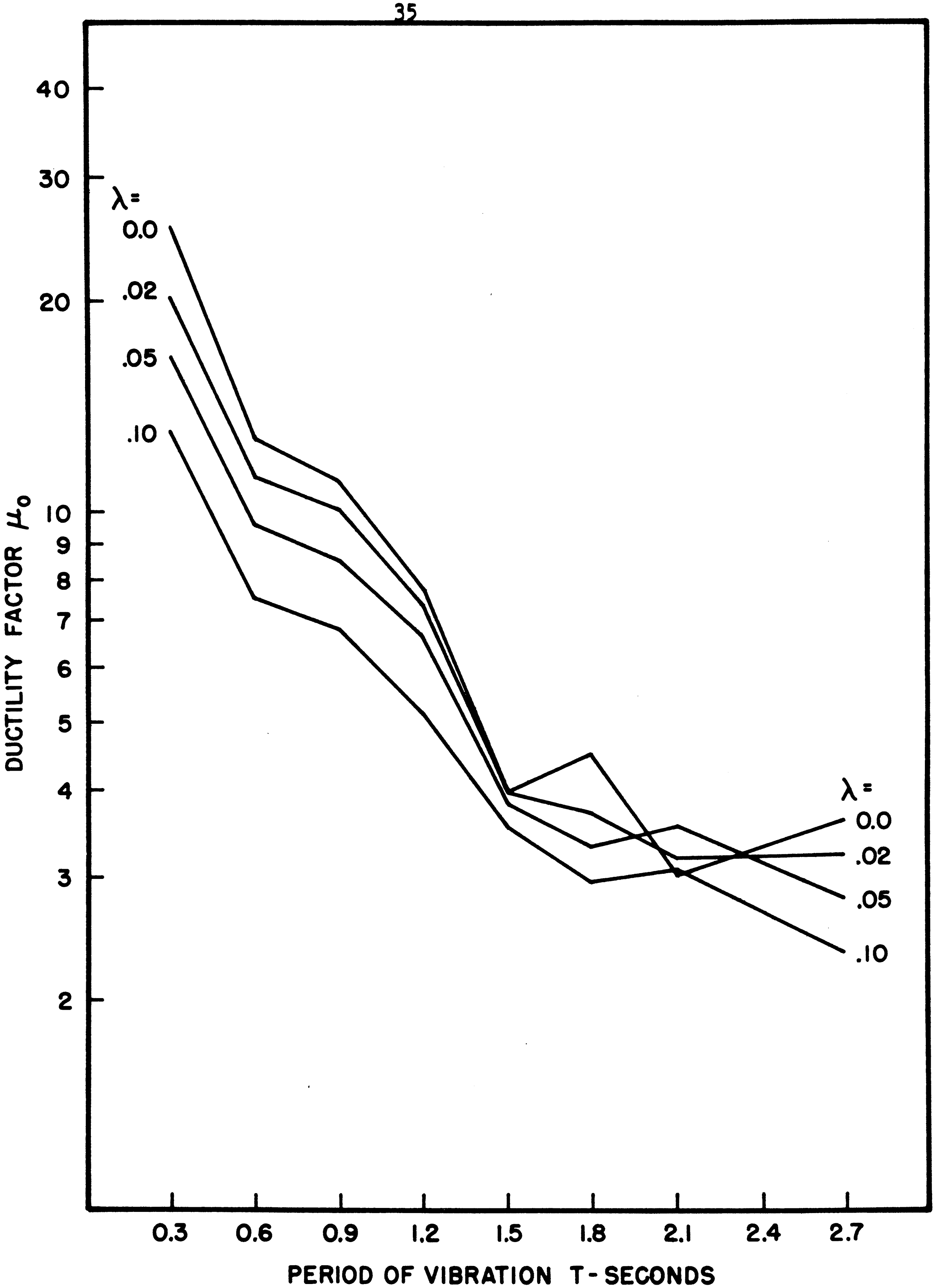


FIG. 13 MAXIMUM DUCTILITY FACTOR : EL CENTRO 1940 N-S EARTHQUAKE

a.) ORDINARY ELASTO-PLASTIC ; k = 0.67

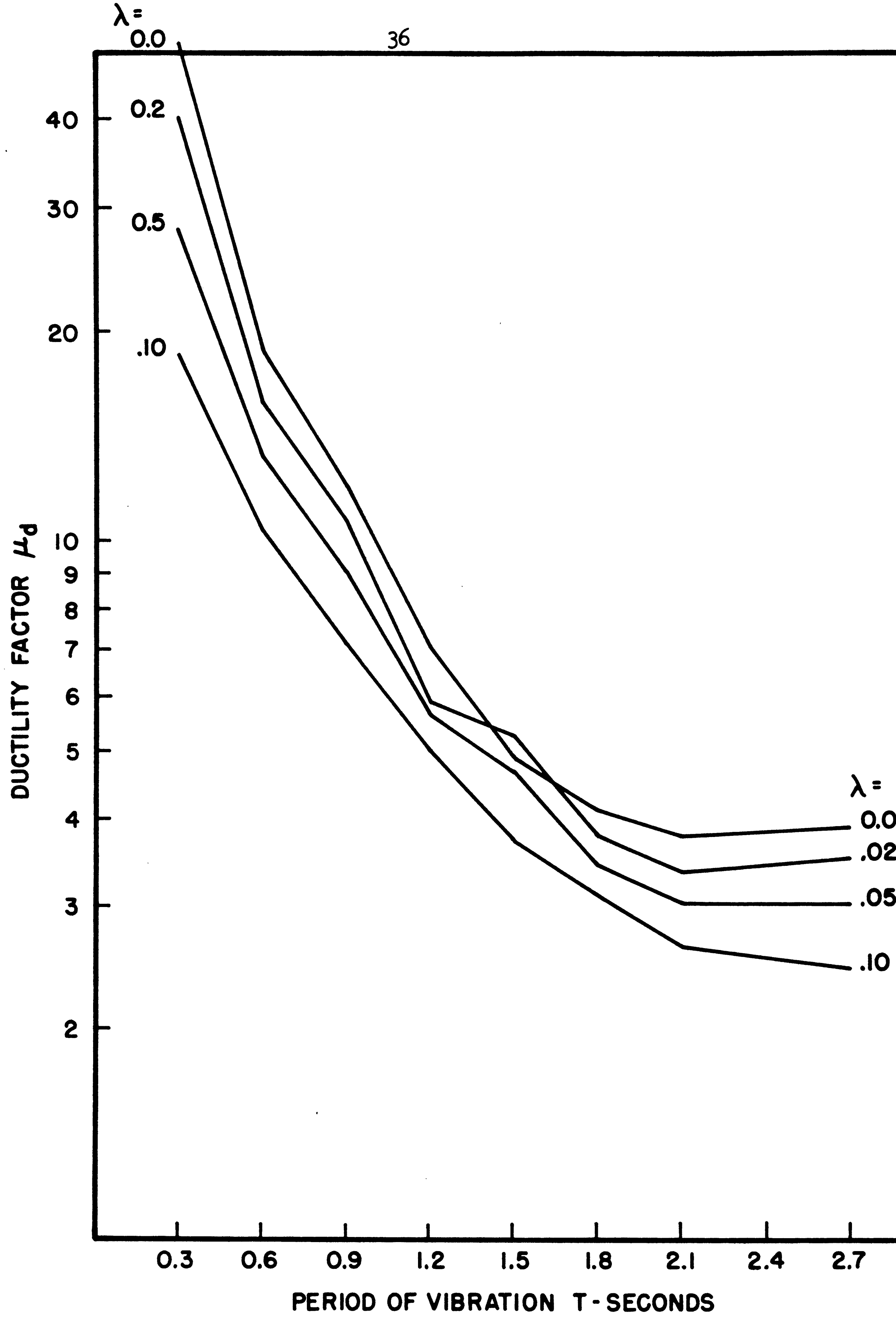


FIG. 13 MAXIMUM DUCTILITY FACTOR: EL CENTRO 1940 N·S EARTHQUAKE

b.) DEGRADING STIFFNESS, k = 0.67

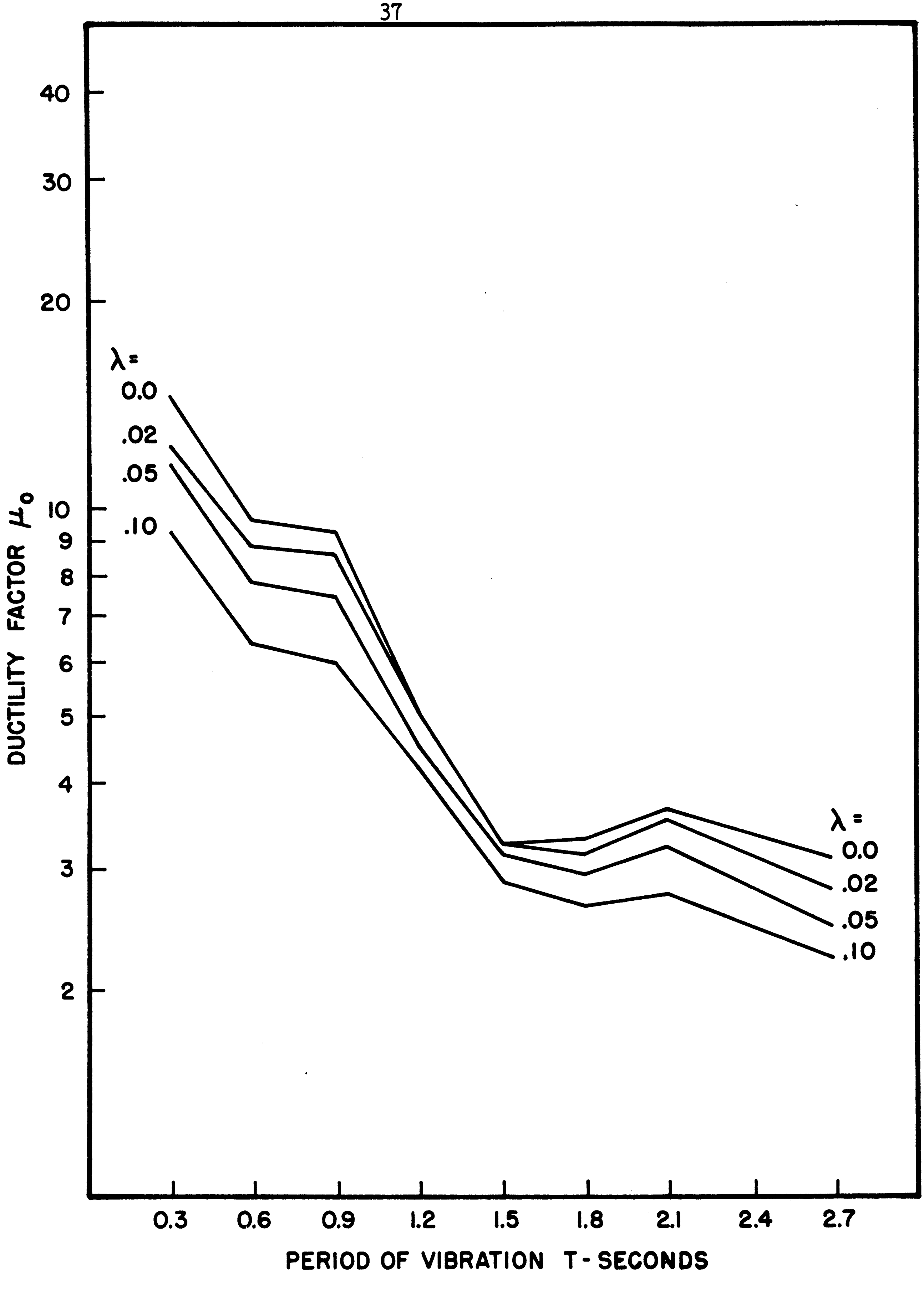


FIG. 14 MAXIMUM DUCTILITY FACTOR: EL CENTRO 1940 N-S EARTHQUAKE

a.) ORDINARY ELASTO-PLASTIC ; k = 0.80

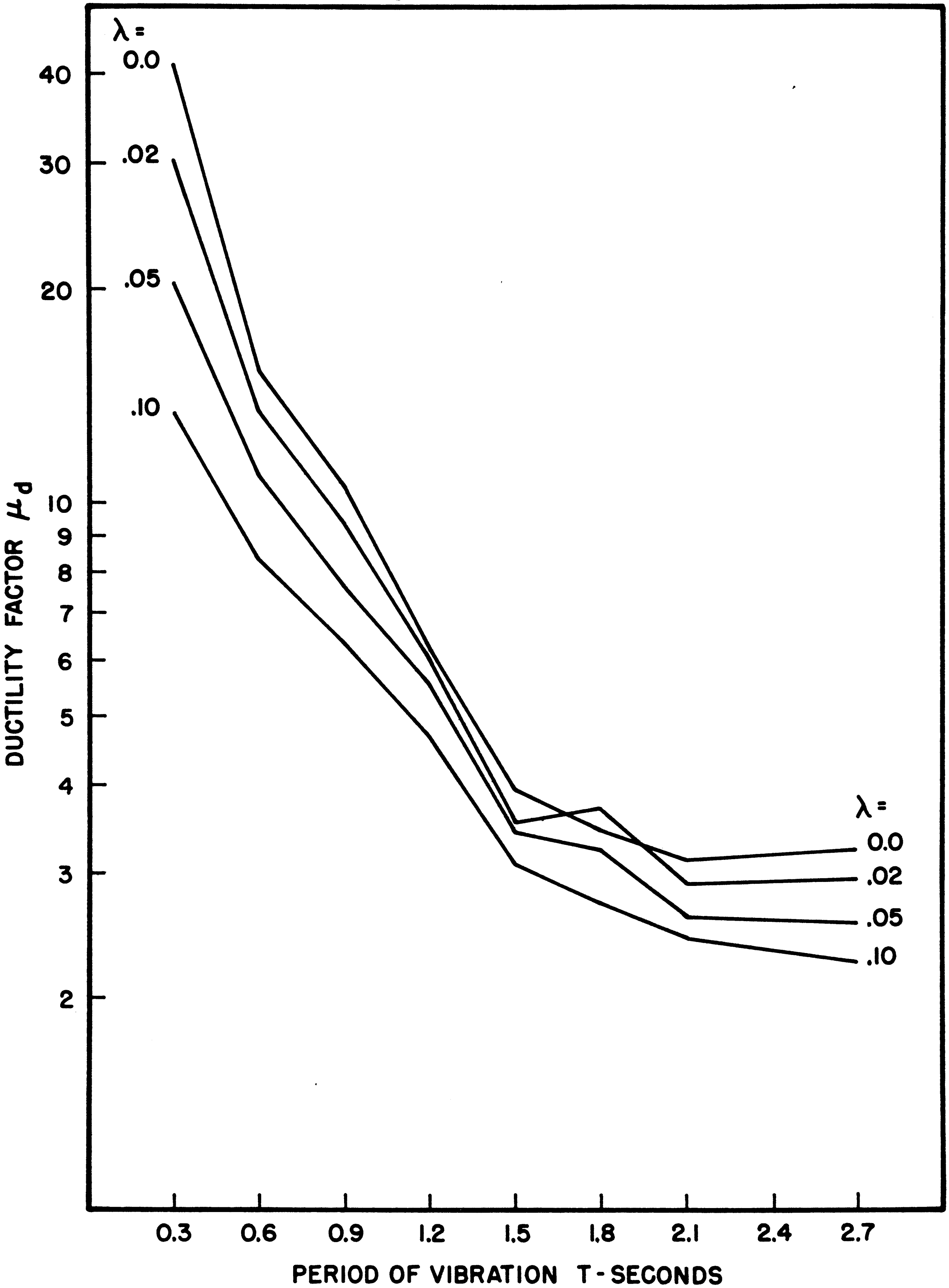


FIG. 14 MAXIMUM DUCTILITY FACTOR:
EL CENTRO 1940 N·S EARTHQUAKE

b.) DEGRADING STIFFNESS: k = 0.80

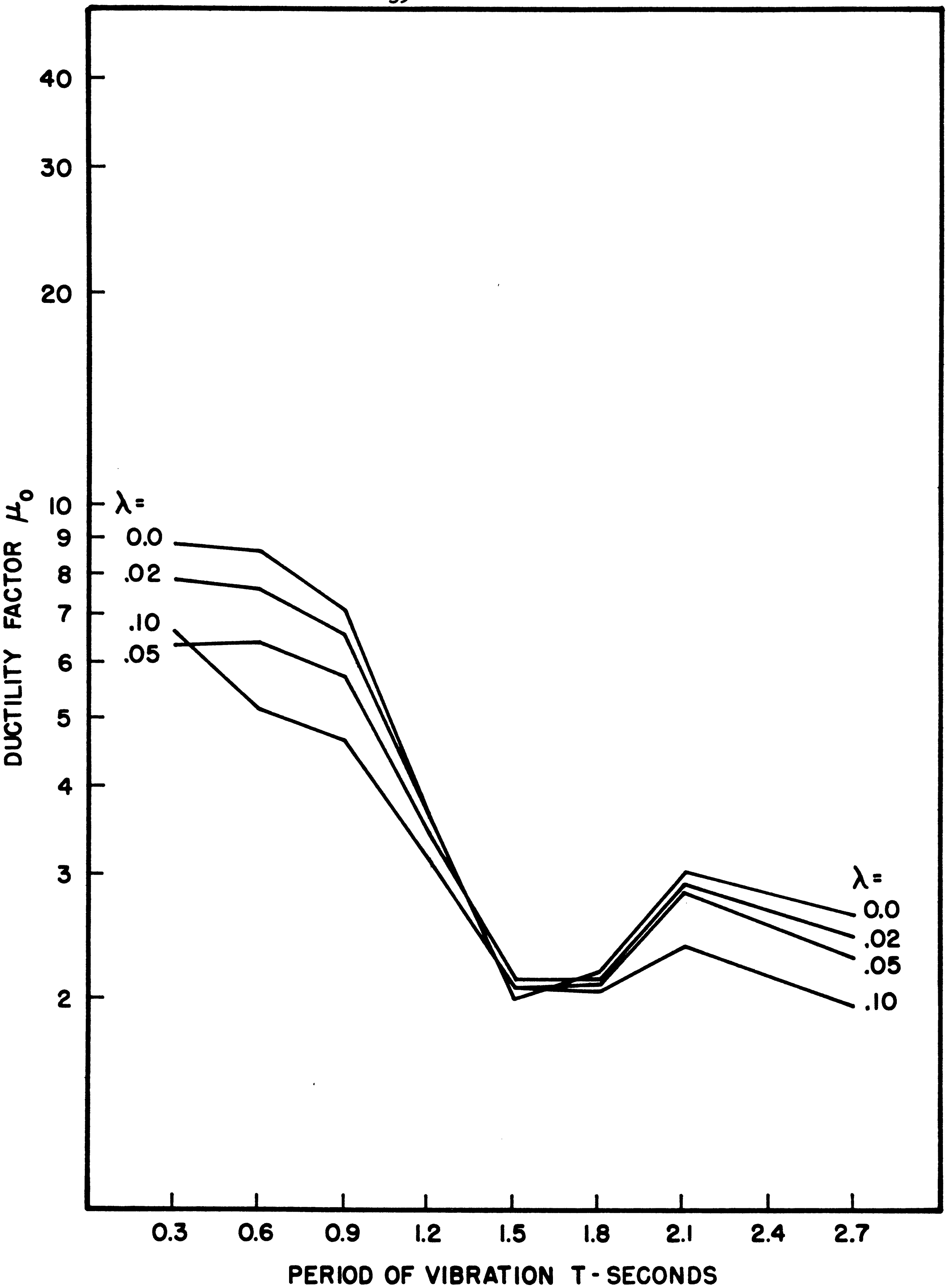


FIG. 15 MAXIMUM DUCTILITY FACTOR:
EL CENTRO 1940 N·S EARTHQUAKE

a.) ORDINARY ELASTO-PLASTIC; k = 1.00

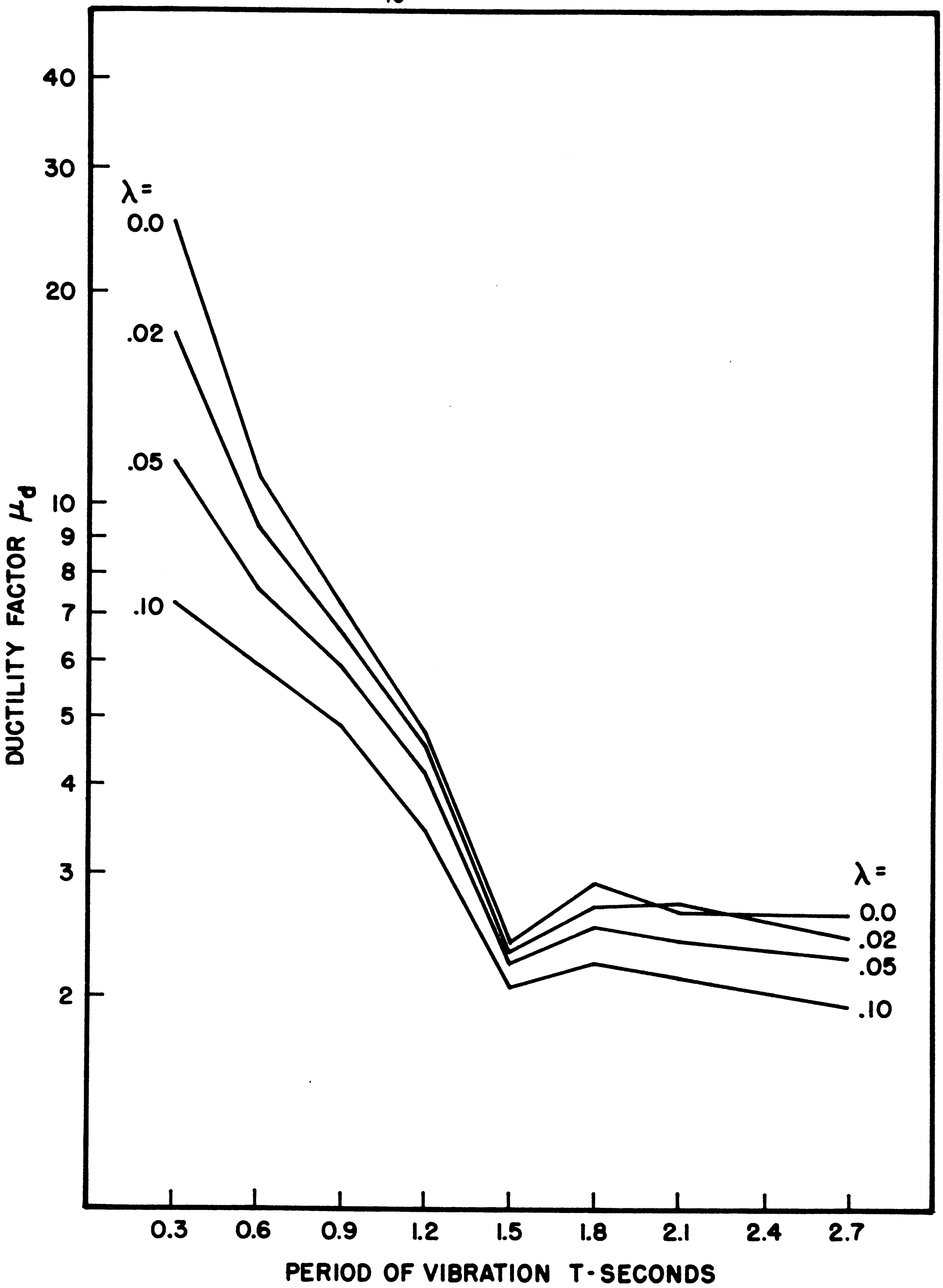


FIG. 15 MAXIMUM DUCTILITY FACTOR: EL CENTRO 1940 N·S EARTHQUAKE

b.) DEGRADING STIFFNESS; k = 1.00

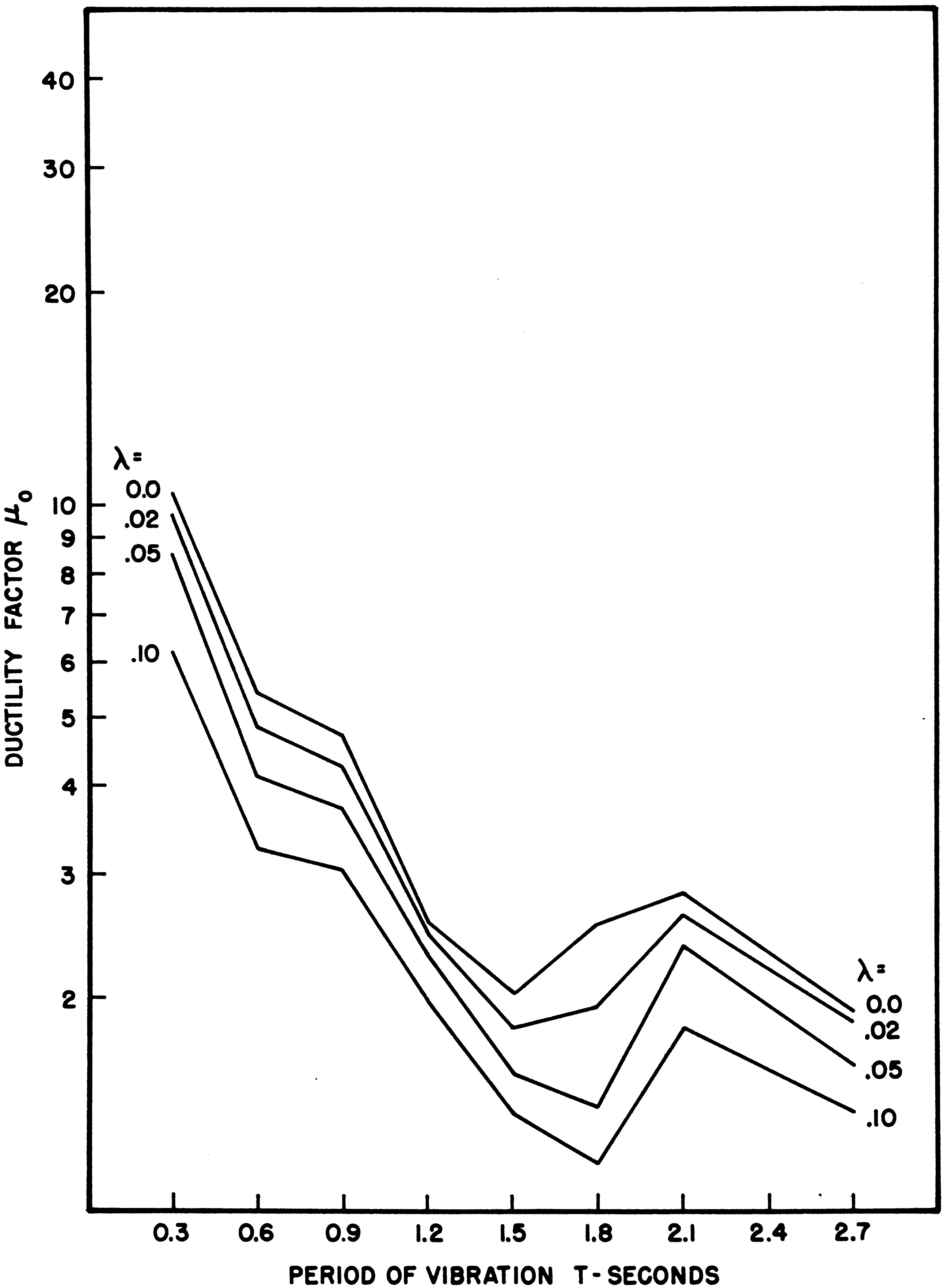


FIG. 16 MAXIMUM DUCTILITY FACTOR:
EL CENTRO 1940 N·S EARTHQUAKE
a.) ORDINARY ELASTO-PLASTIC; k = 1.33

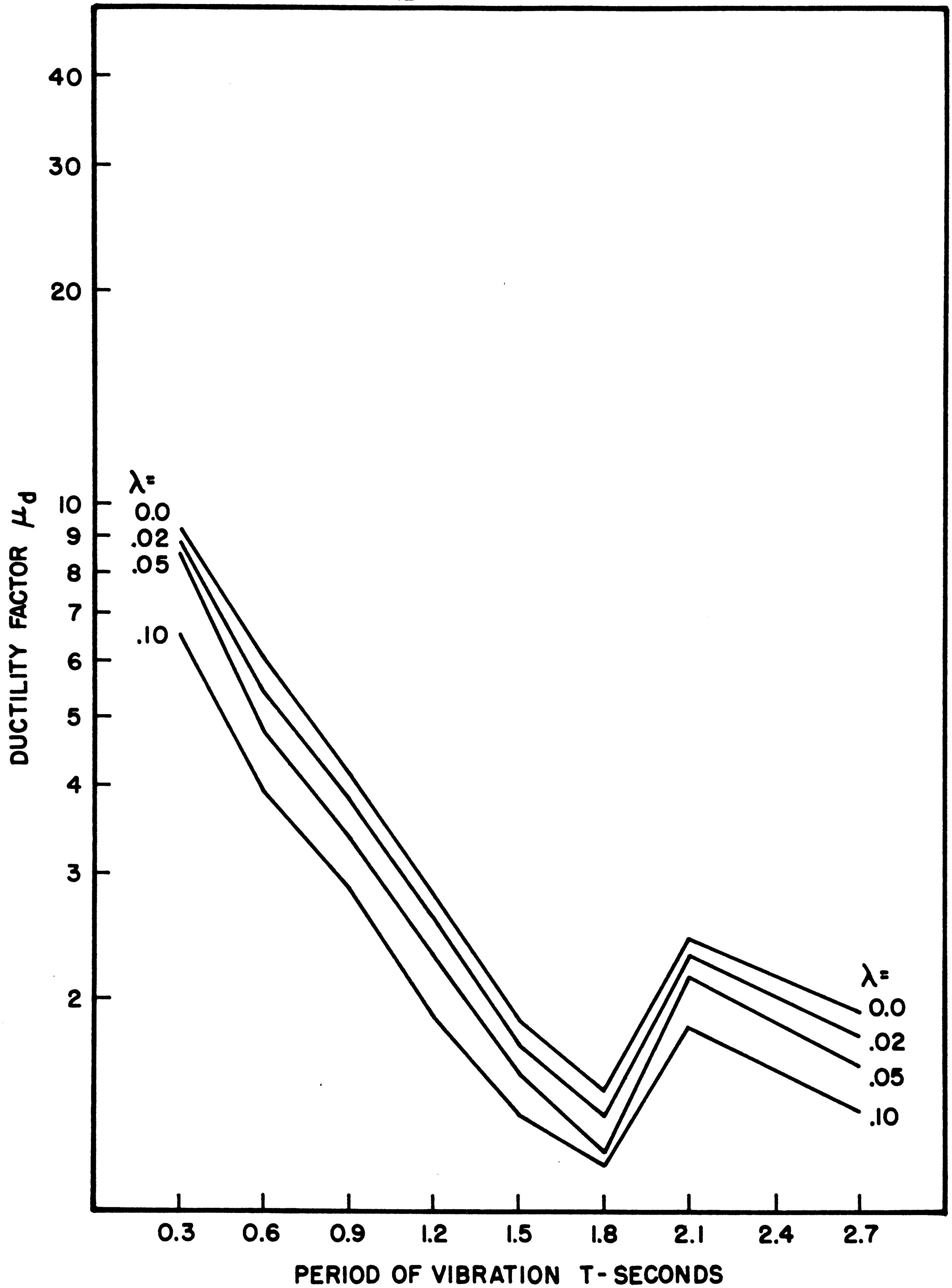


FIG. 16 MAXIMUM DUCTILITY FACTOR : EL CENTRO 1940 N·S EARTHQUAKE

b.) DEGRADING STIFFNESS ; k = 1.33

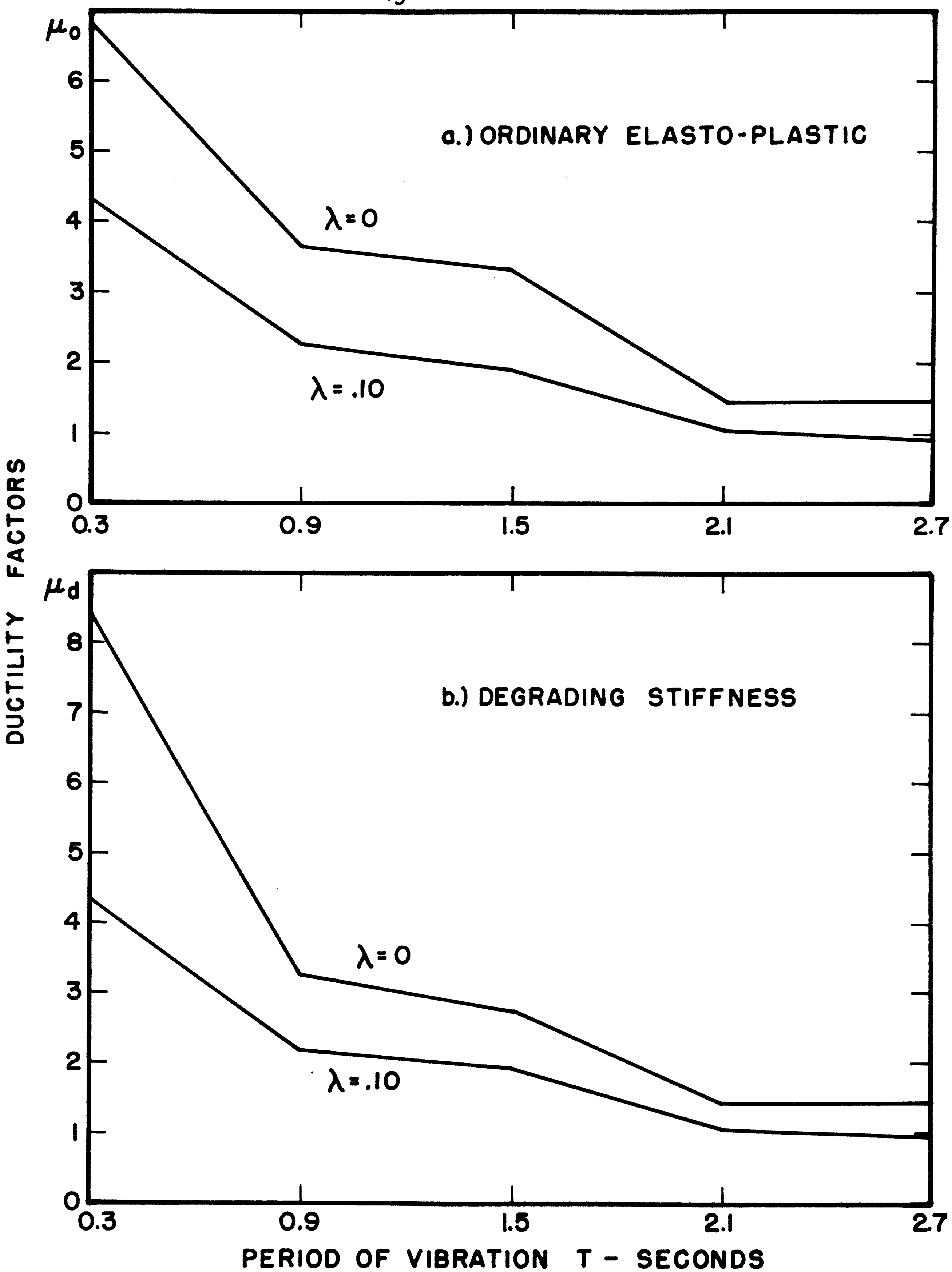


FIG.17 MAXIMUM DUCTILITY FACTOR: TAFT 1952 S-W EARTHQUAKE

STRENGTH FACTOR: K=0.67

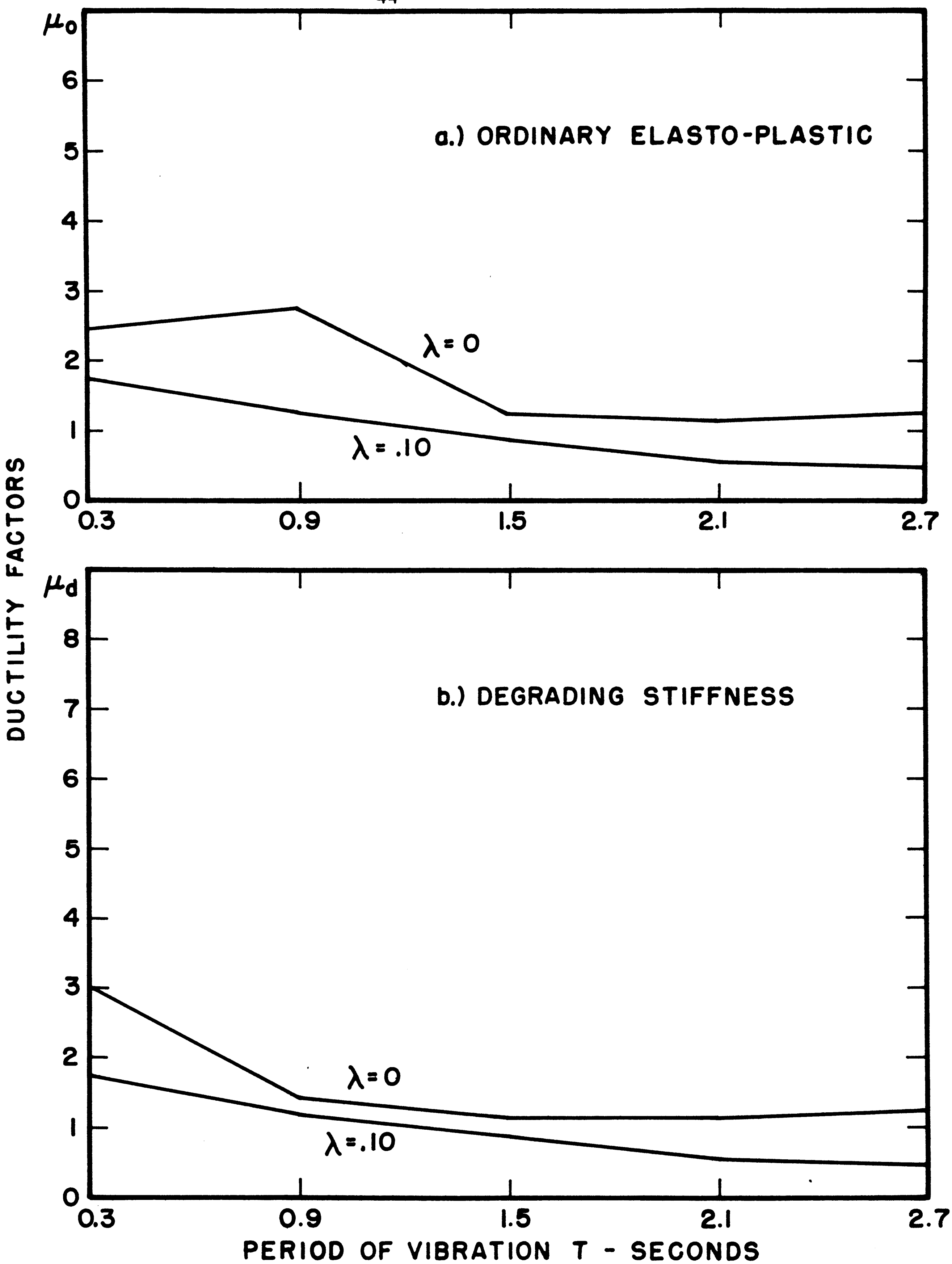


FIG. 18 MAXIMUM DUCTILITY FACTOR: TAFT 1952 S-W EARTHQUAKE

STRENGTH FACTOR: K = 1.33

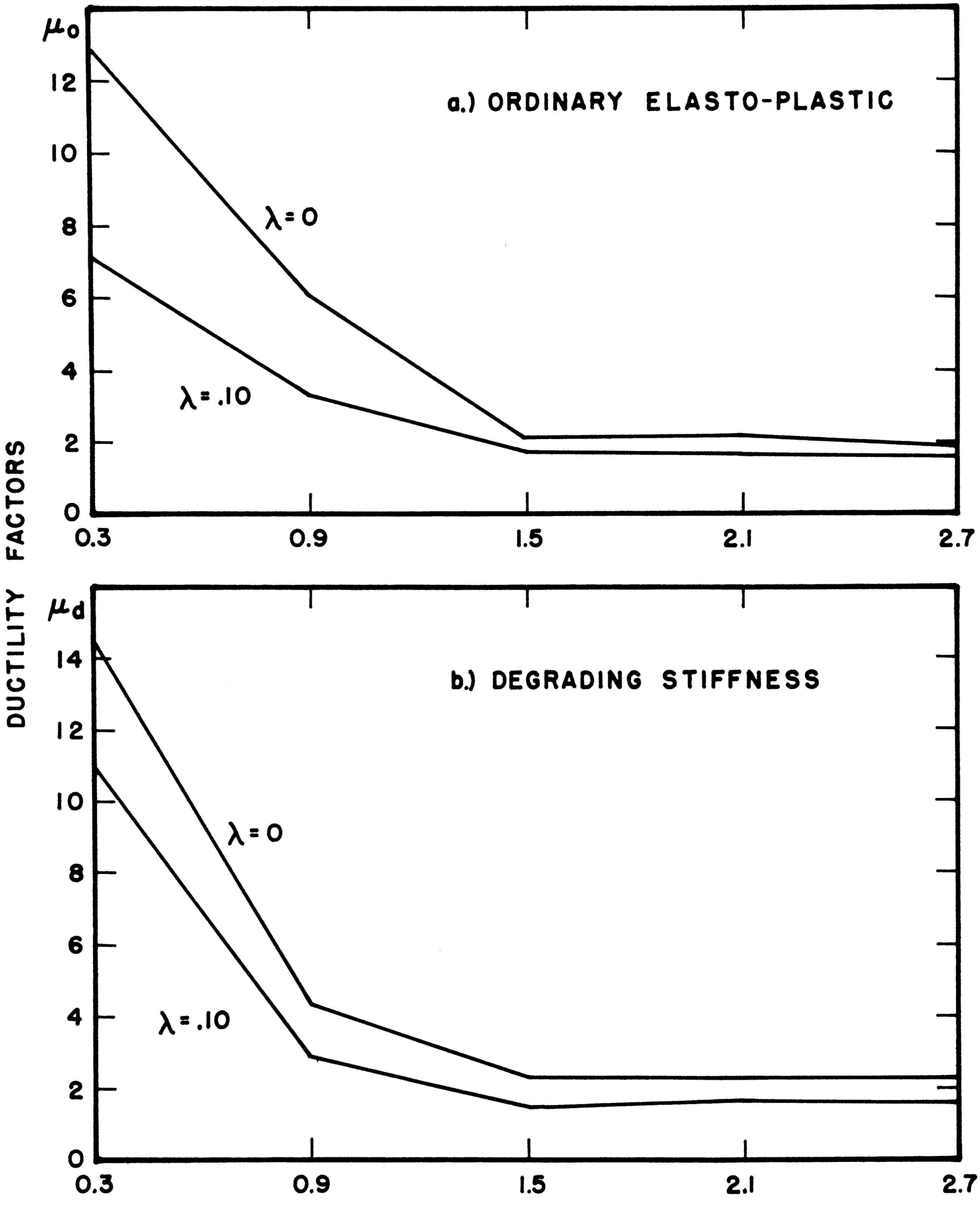


FIG. 19 MAXIMUM DUCTILITY FACTOR
OLYMPIA 1949 E-W EARTHQUAKE

STRENGTH FACTOR : k = 0.67

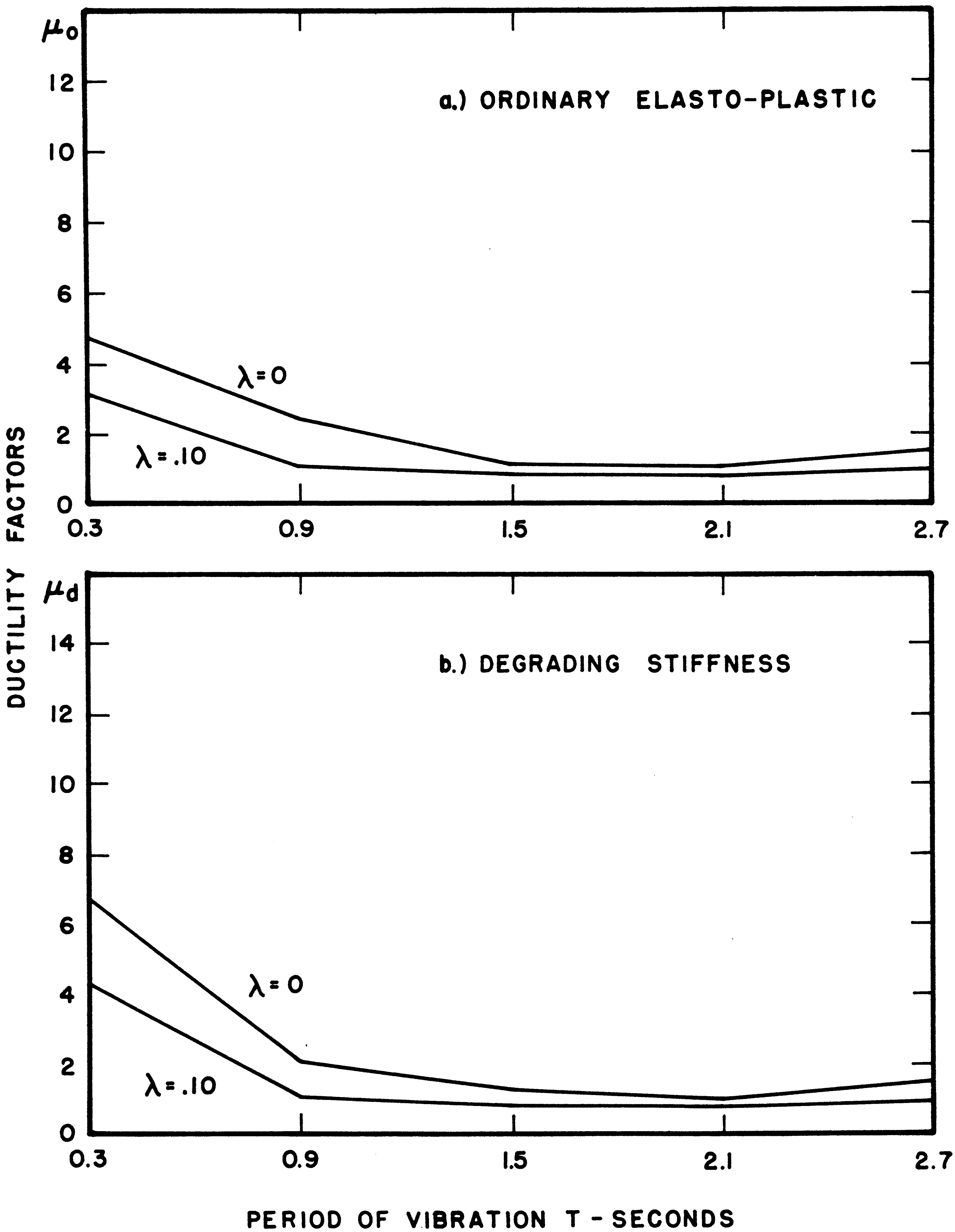


FIG. 20 MAXIMUM DUCTILITY FACTOR
OLYMPIA 1949 E-W EARTHQUAKE
STRENGTH FACTOR : K = 1.33

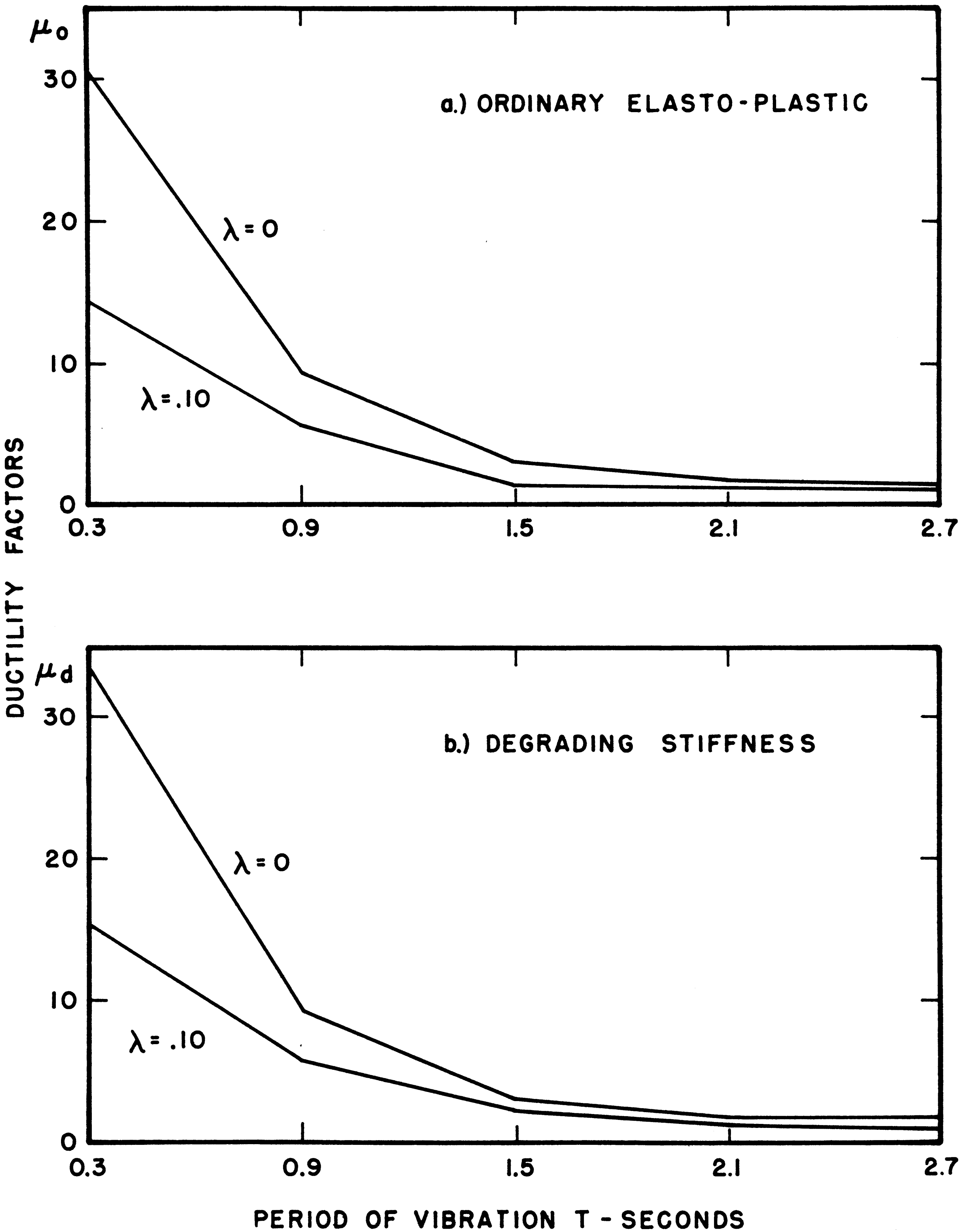


FIG. 21 MAXIMUM DUCTILITY FACTOR
EL CENTRO 1934 N-S EARTHQUAKE
STRENGTH FACTOR: K = 0.67

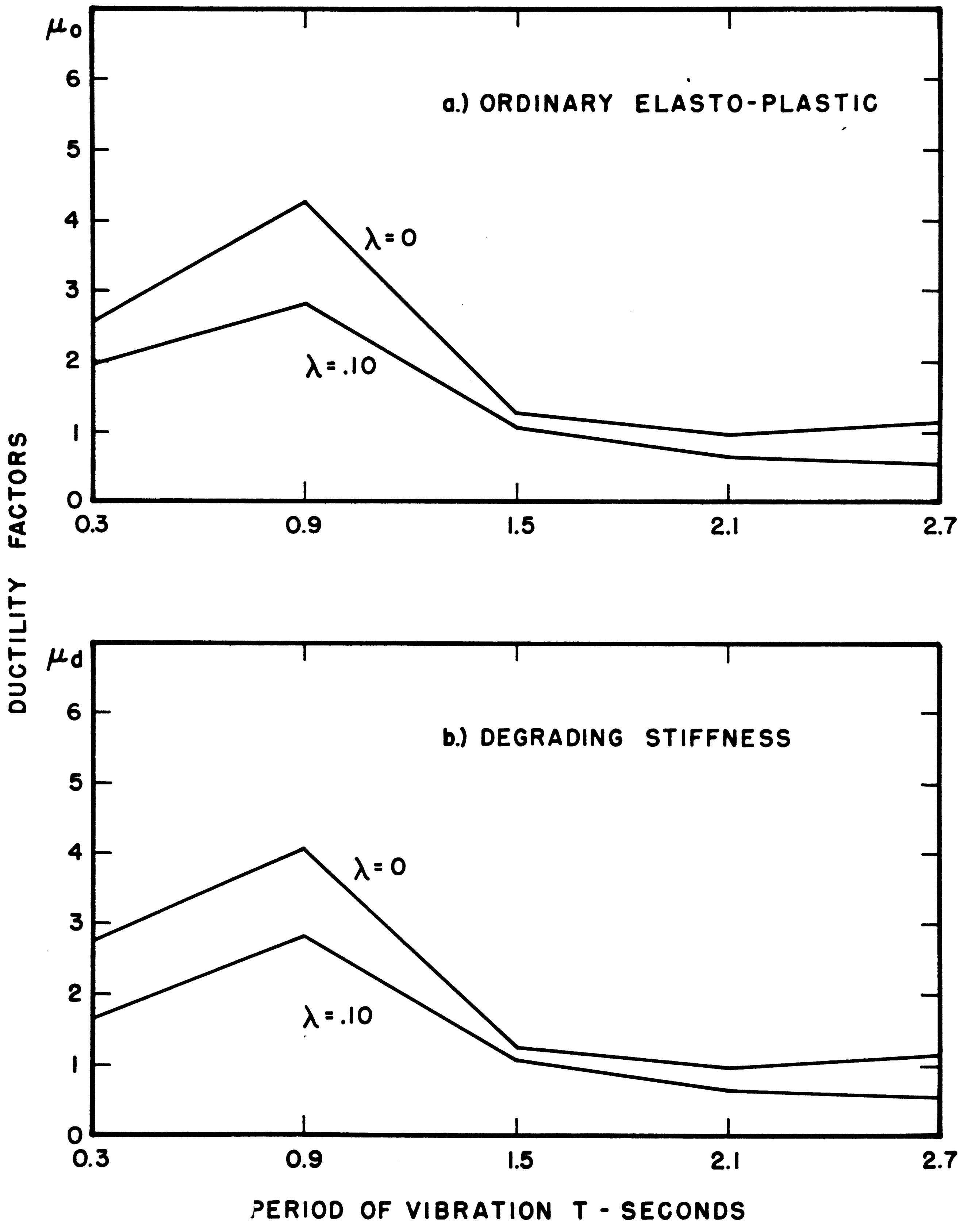


FIG. 22 MAXIMUM DUCTILITY FACTOR EL CENTRO 1934 N-S EARTHQUAKE

STRENGTH FACTOR: K = 1.33

Relative Ductility Requirements

For the purposes of this investigation, the most significant information to be gleaned from the ductility factor curves discussed above is the relative performance of the two types of materials. In other words, the principal concern is with the extent to which the degrading stiffness property alters the deformation developed in the structure from that which would have occurred with the ordinary elasto-plastic material. Graphs showing the ratio of the degrading stiffness ductility factor to the ordinary elasto-plastic ductility factor (μ_d/μ_o) for each of the various cases are shown in Figs. 23 to 26. It is significant to note that this relative ductility ratio is fairly constant; it varies only between 0.6 and 1.4 over practically the entire range of test conditions considered--except for the short period (T = 0.3 sec) case subjected to the strongest earthquake (EC 40 N-S). For this stiff structure, the degrading stiffness material demonstrates a significant increase in ductility requirement, with factors nearly 3 times as great as the corresponding ordinary elasto-plastic requirement for two structural strengths. However, for the longer period structures, which correspond to typical multi-story buildings, the degrading stiffness property does not appear to be responsible for any significant increase in ductility requirements.

Response Intensity Factors

Study of the ductility requirements described above reveals that these results vary greatly with each of the four basic parameters considered in the analyses. Therefore an attempt was made to establish the relative importance of two of these factors,

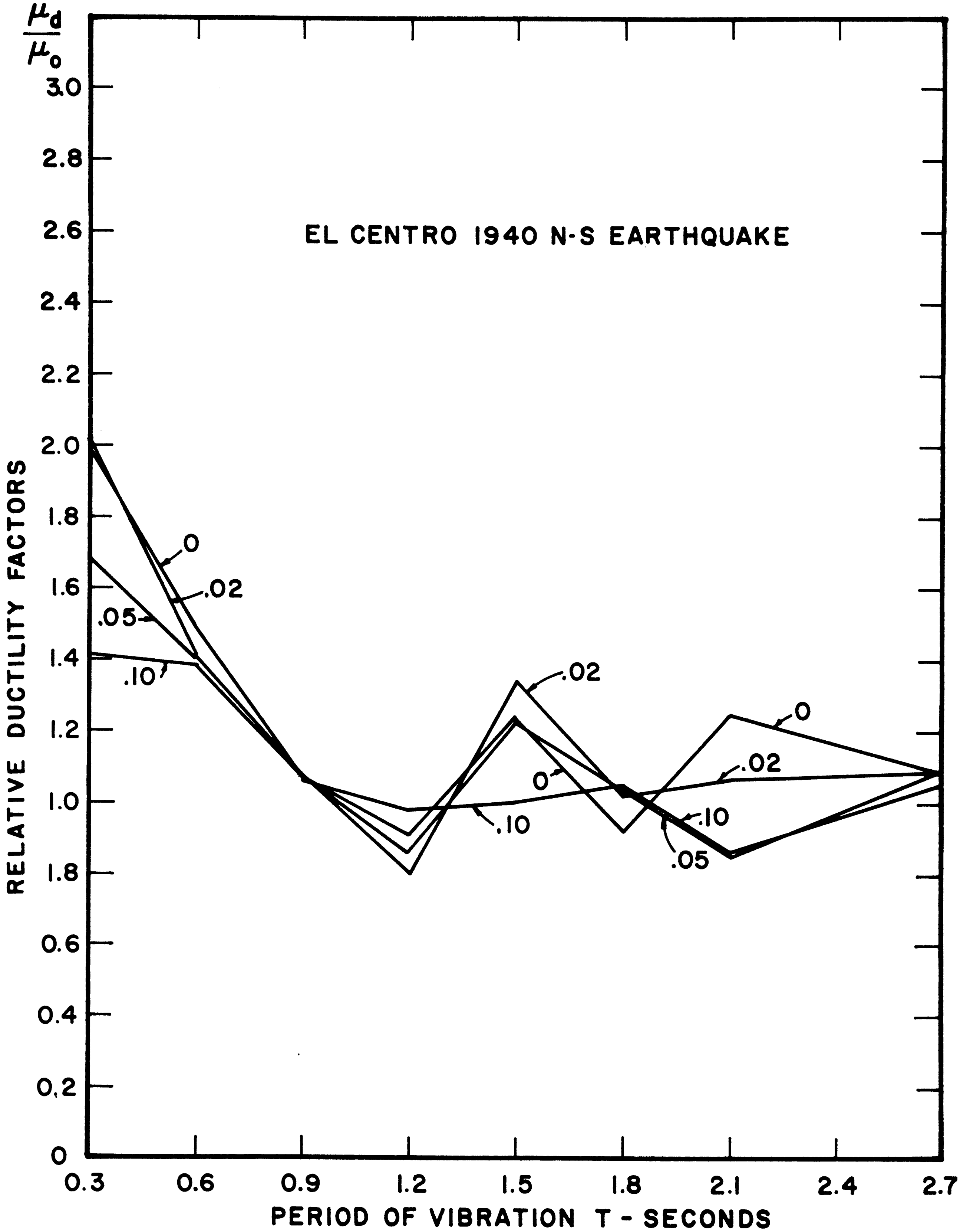


FIG. 23 RELATIVE DUCTILITY REQUIREMENTS — DEGRADING / ORDINARY

a.) STRENGTH FACTOR: K = 0.67

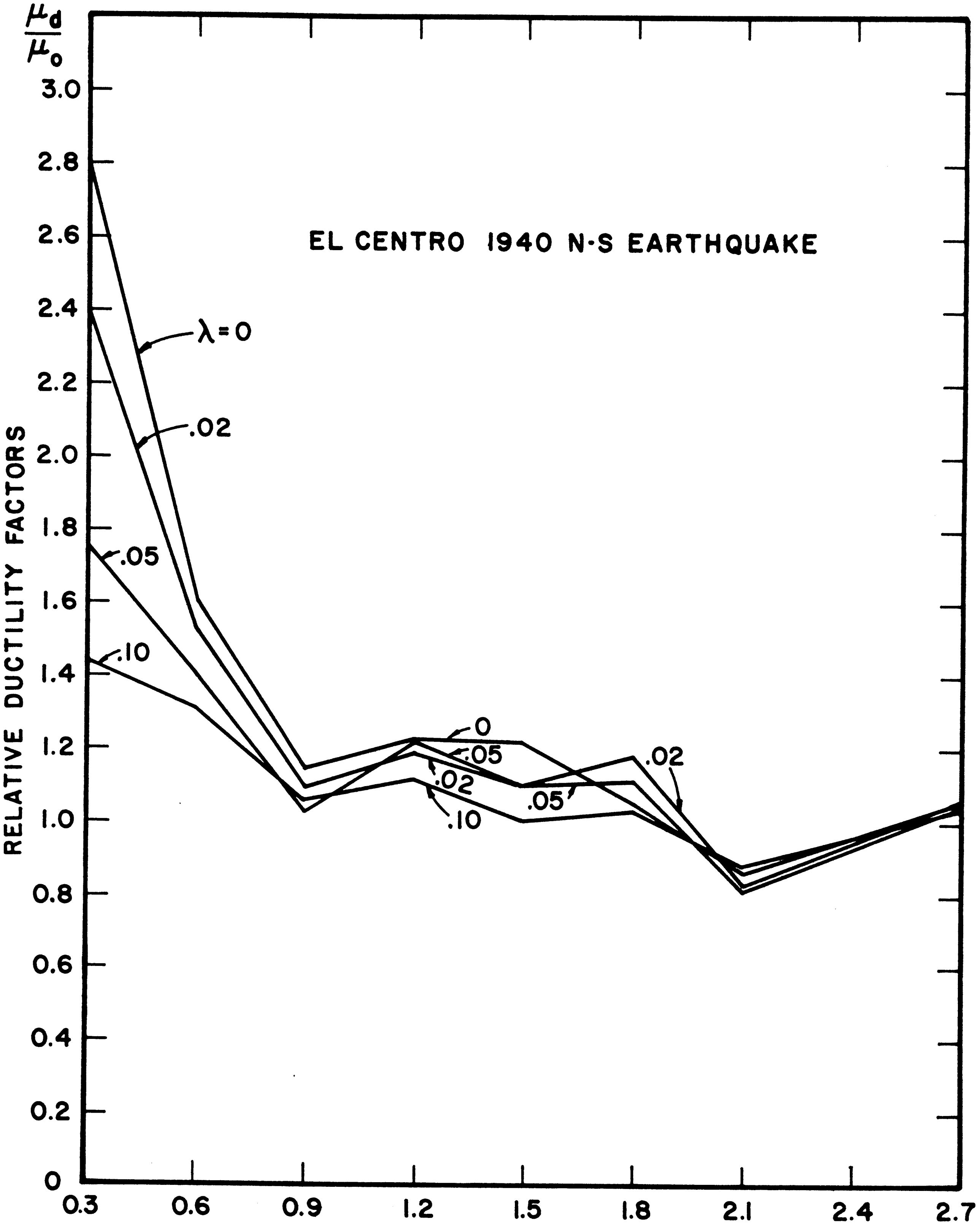


FIG. 23 RELATIVE DUCTILITY REQUIREMENTS — DEGRADING / ORDINARY

b.) STRENGTH FACTOR: K = 0.80

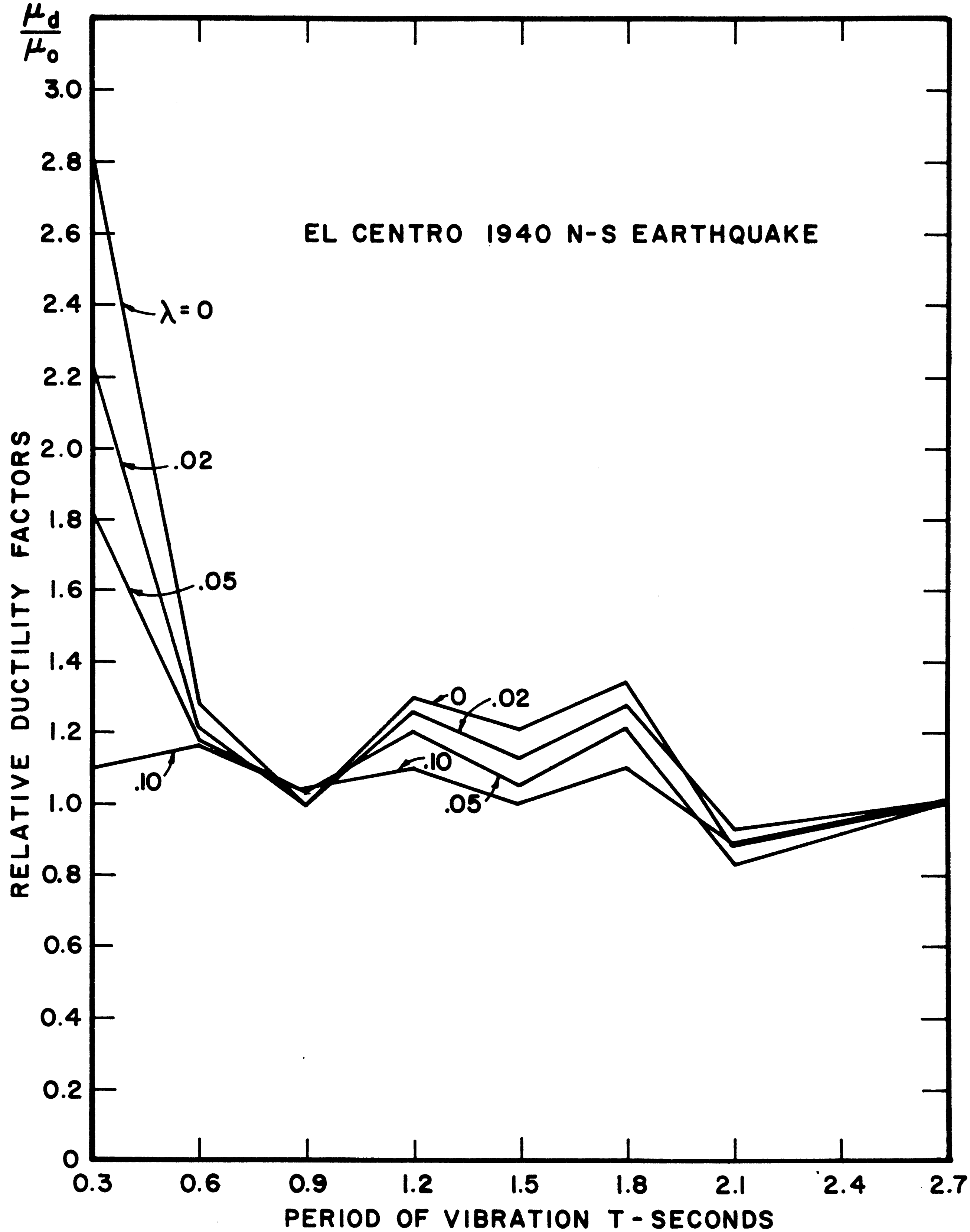


FIG. 23 RELATIVE DUCTILITY REQUIREMENTS — DEGRADING / ORDINARY

c.) STRENGTH FACTOR : K = 1.00

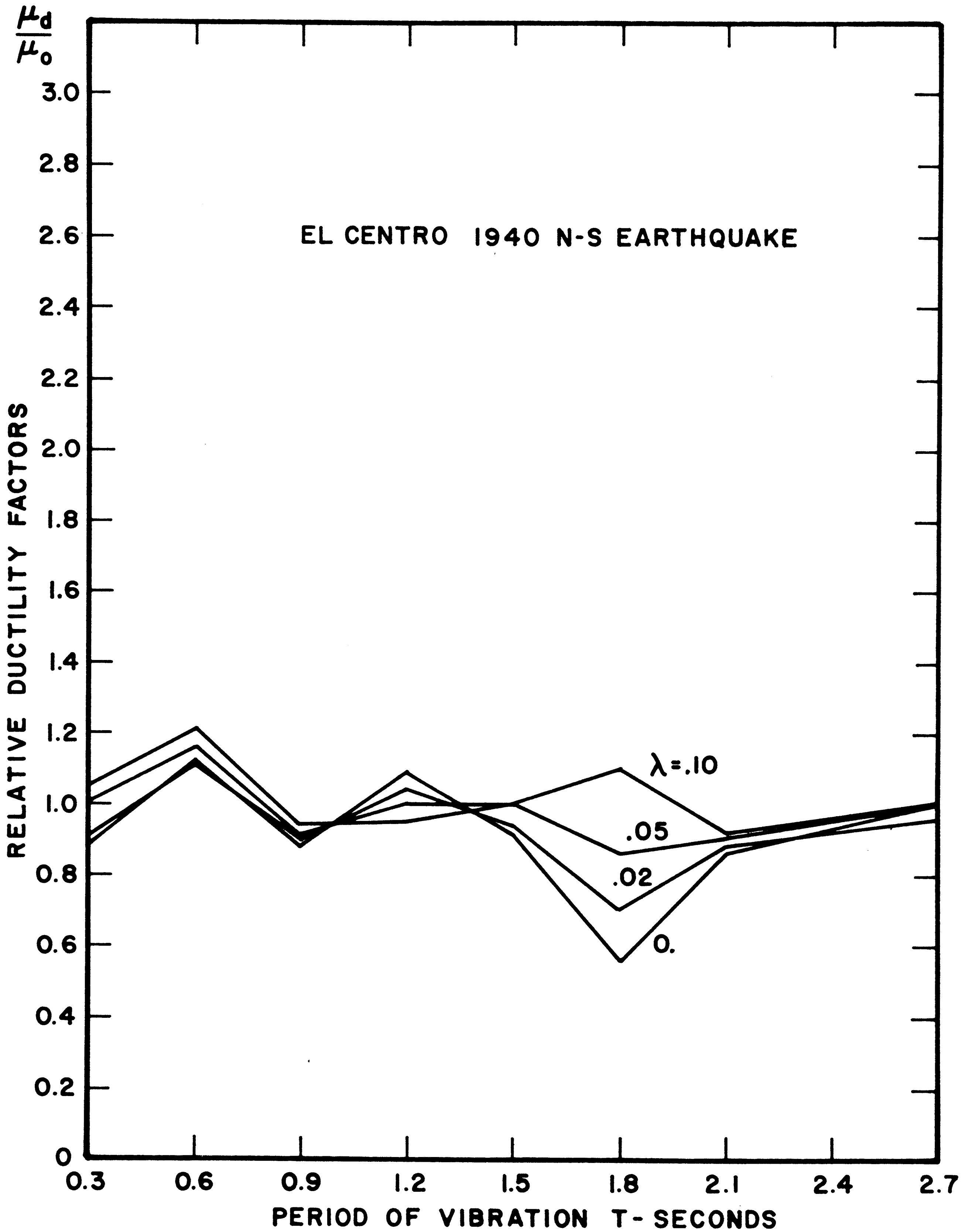


FIG. 23 RELATIVE DUCTILITY REQUIREMENTS — DEGRADING / ORDINARY

d.) STRENGTH FACTOR: K = 1.33

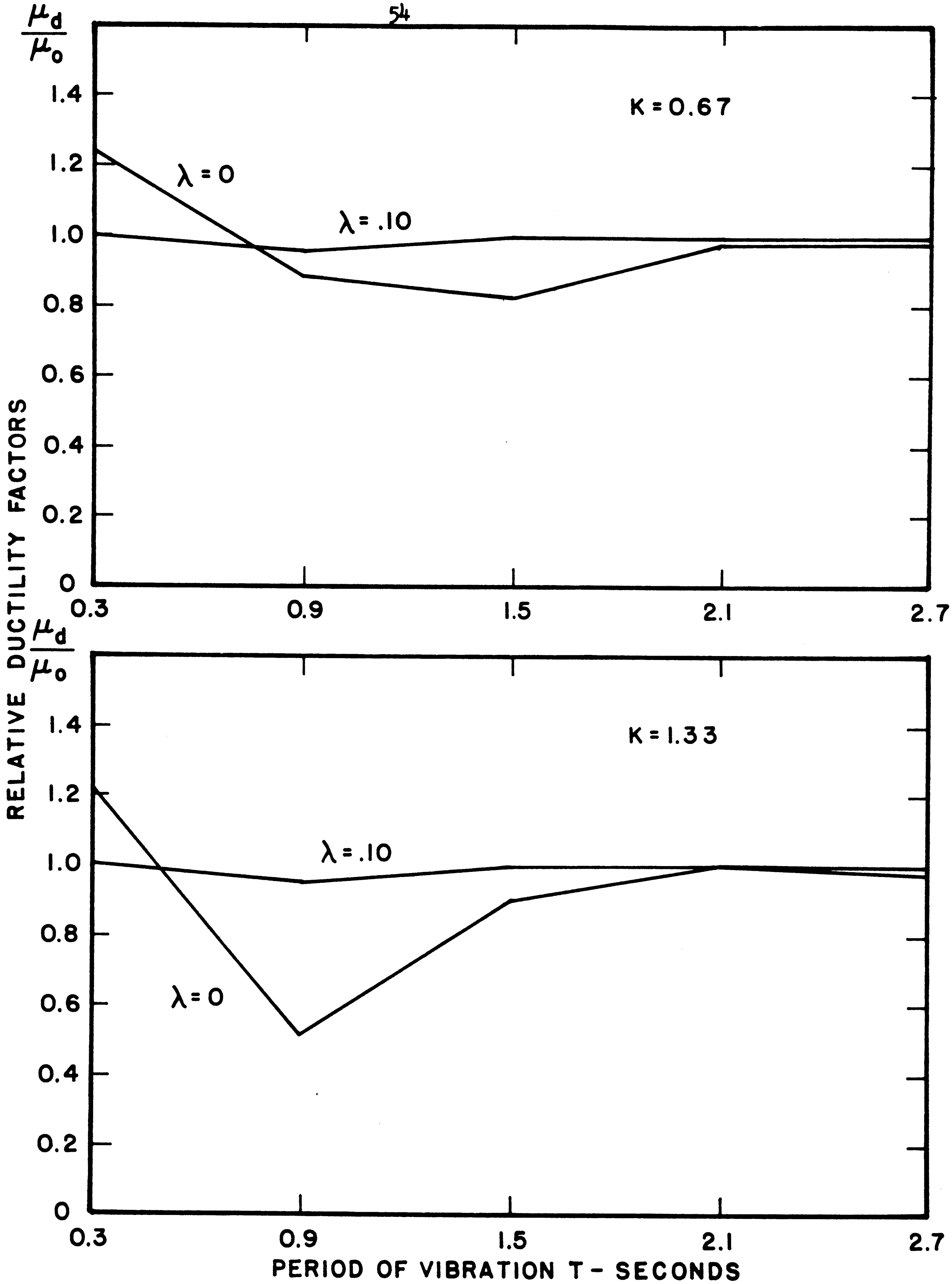


FIG. 24 RELATIVE DUCTILITY REQUIREMENTS – DEGRADING / ORDINARY

TAFT 1952 S-W EARTHQUAKE

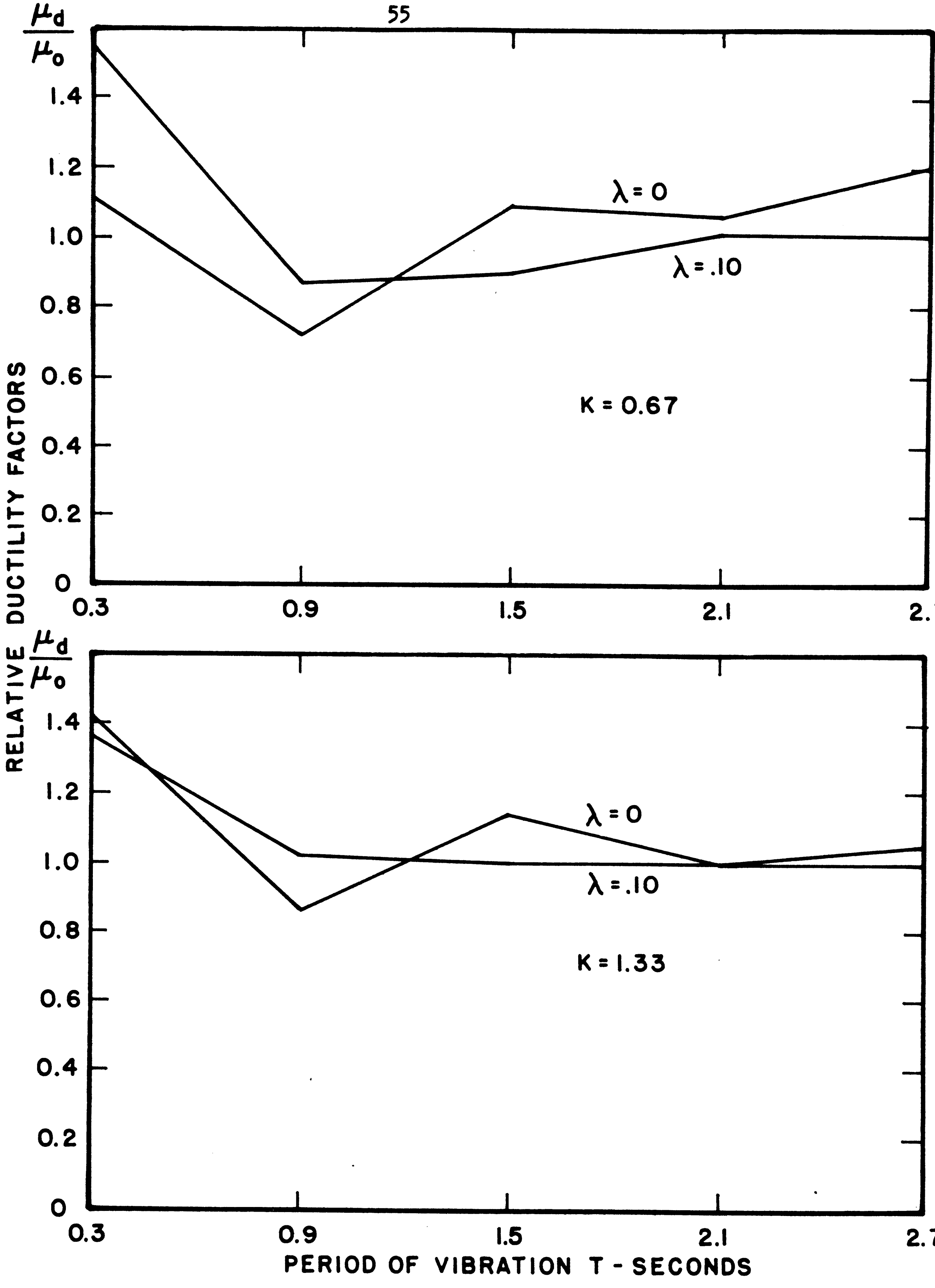


FIG. 25 RELATIVE DUCTILITY REQUIREMENTS – DEGRADING / ORDINARY

OLYMPIA 1949 E-W EARTHQUAKE

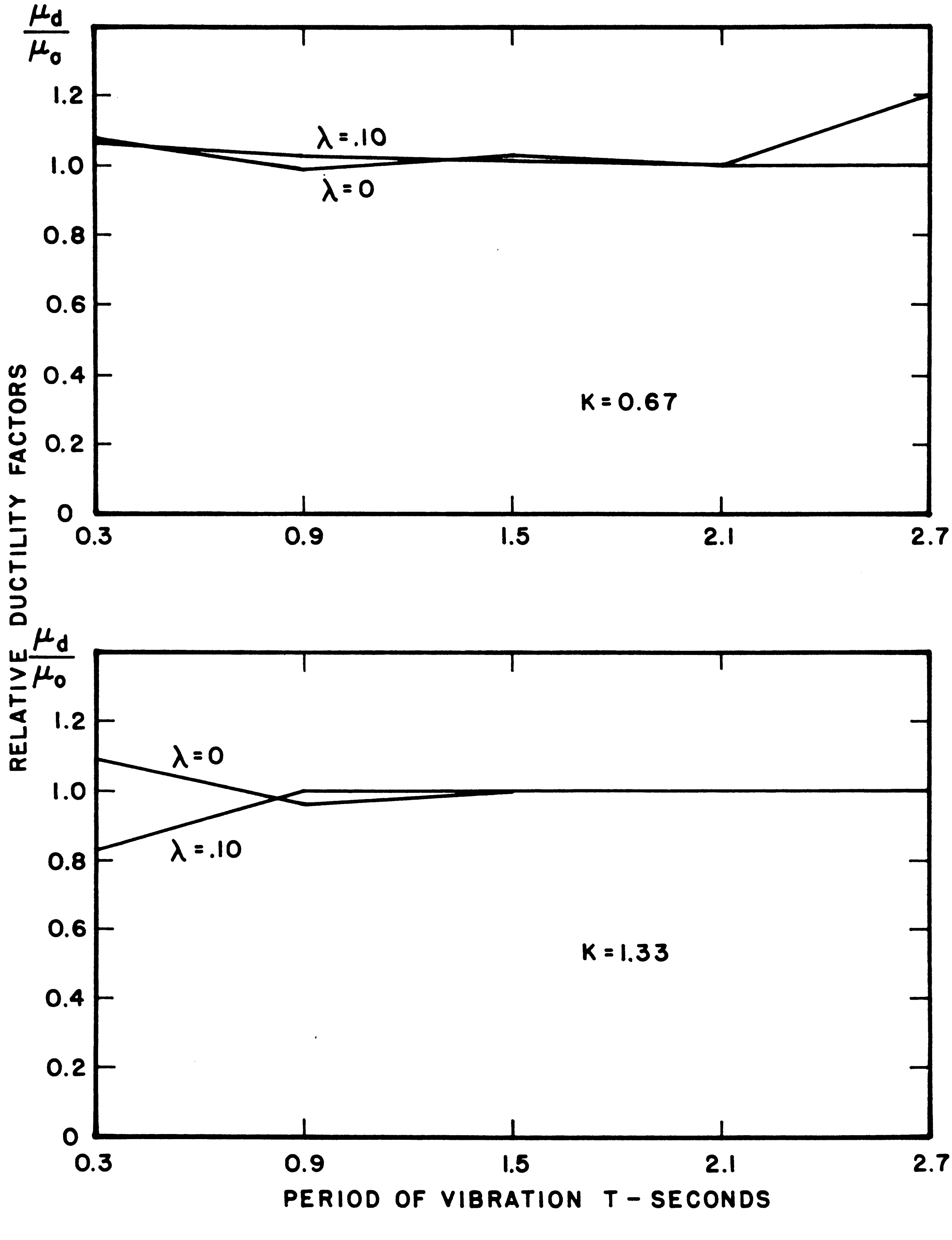


FIG. 26 RELATIVE DUCTILITY REQUIREMENTS – DEGRADING / ORDINARY

EL CENTRO 1934 N-S EARTHQUAKE

earthquake input and structural strength, by establishing response ratios for corresponding values of the other parameters--damping ratio and period of vibration. For this purpose, the weakest earthquake excitation, TA 52 S W, and weakest structural parameter, $K = 0.67$, were taken as the reference response case. The ratio of the response obtained for each of the other earthquake and structural strength combinations to the response in the Taft 52, $K = 0.67$ case was calculated for each period of vibration and damping ratio. Response ratios of this type determined for the EC 40 NS earthquake, $K = 0.67$ case are presented in Fig. 27. Areas under each of the response ratio curves were then computed to obtain an average effect over the complete period range, and these areas were averaged to account for the range of damping ratios. The resulting average response ratio area factors, plotted as functions of structural strength, are shown in Fig. 28. Clearly the strength and the earthquake excitation each have a systematic effect on the response intensity.

To determine the earthquake intensity factor, the response ratio area factor for the given earthquake in Fig. 28 was divided by the Taft earthquake response ratio area factor, for both $K = 0.67$ and $K = 1.33$. The average of these two ratios was then taken to be the earthquake intensity factor, I_E, with the following results:

EC 40 NS: $I_E = 2.55$

EC 34 NS: $I_E = 1.63$

OL 49 EW: $I_E = 1.32$

TA 52 SW: $I_E = 1.00$

The strength intensity factor was then determined by dividing

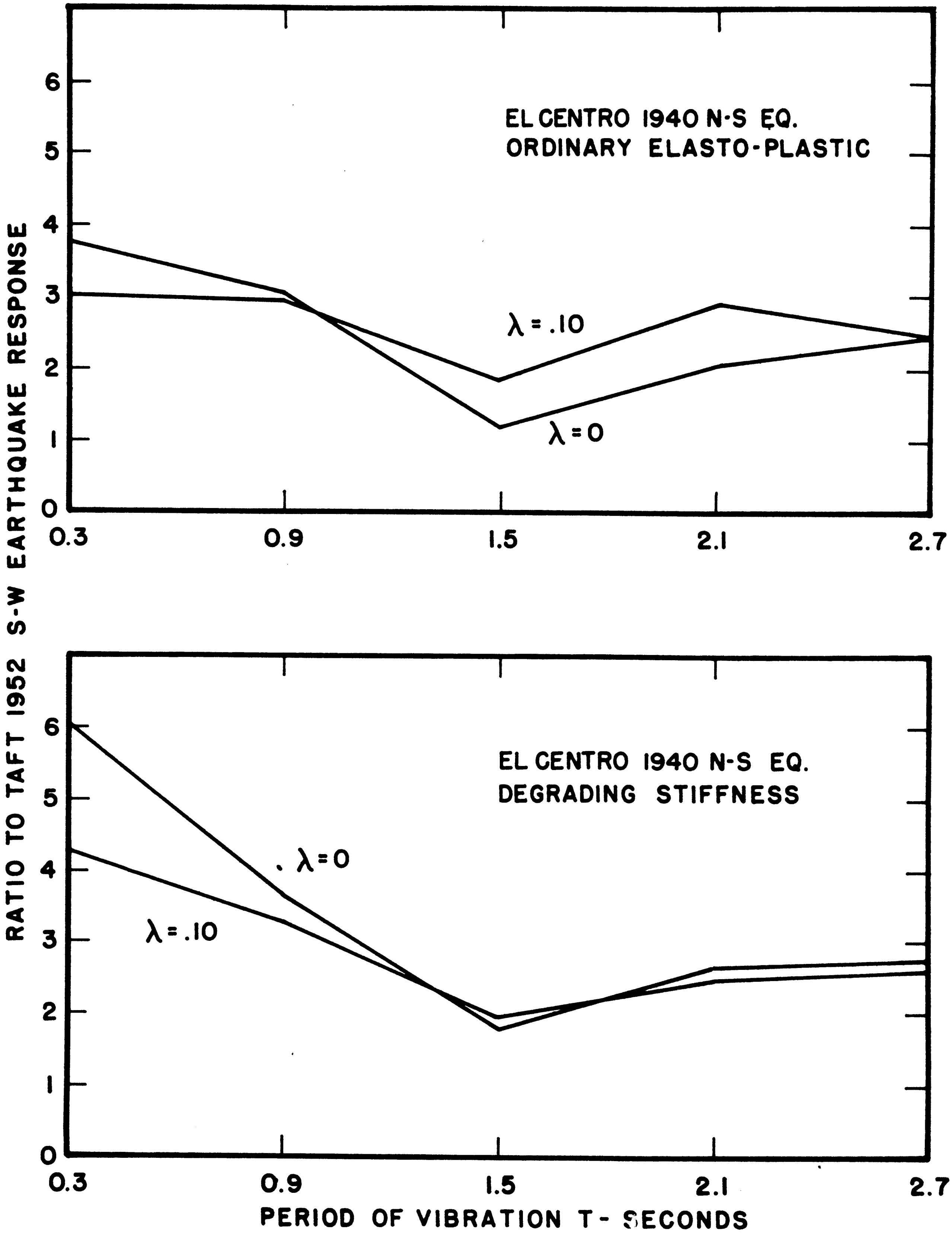


FIG. 27 EARTHQUAKE RESPONSE INTENSITY RATIOS
STRENGTH FACTOR: K = 0.67

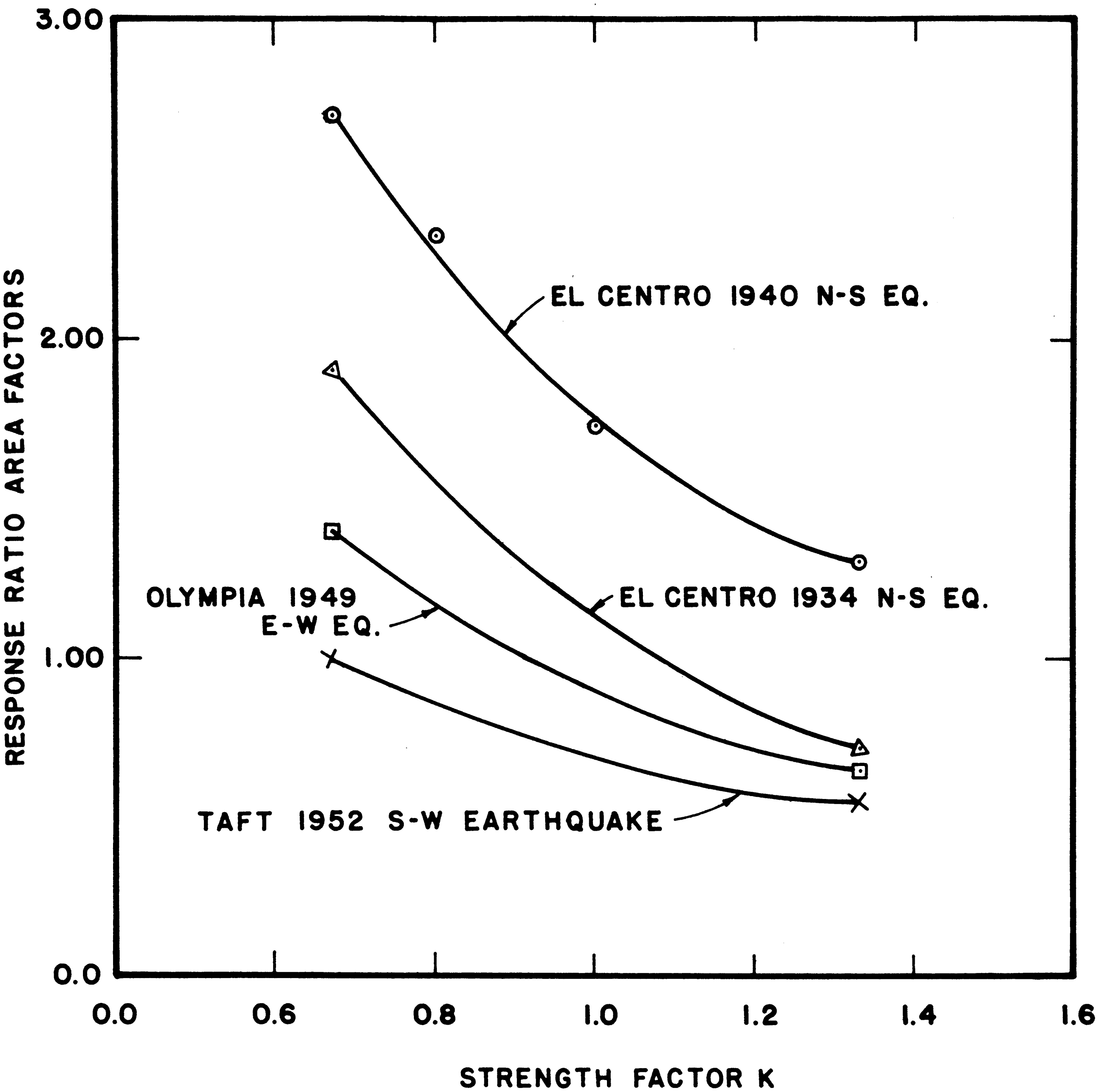


FIG. 28 AVERAGE RESPONSE RATIO AREA FACTORS

the response ratio area factor shown in Fig. 28 for a given strength by the K = 0.67 response ratio area factor, for each of the four earthquakes. The results are shown plotted in Fig. 29. It is of interest to note that the response intensity computed in this fashion varies essentially inversely with the strength factor: the plot of the inverse relationship $I_S = \frac{0.67}{K}$ is indicated for comparison.

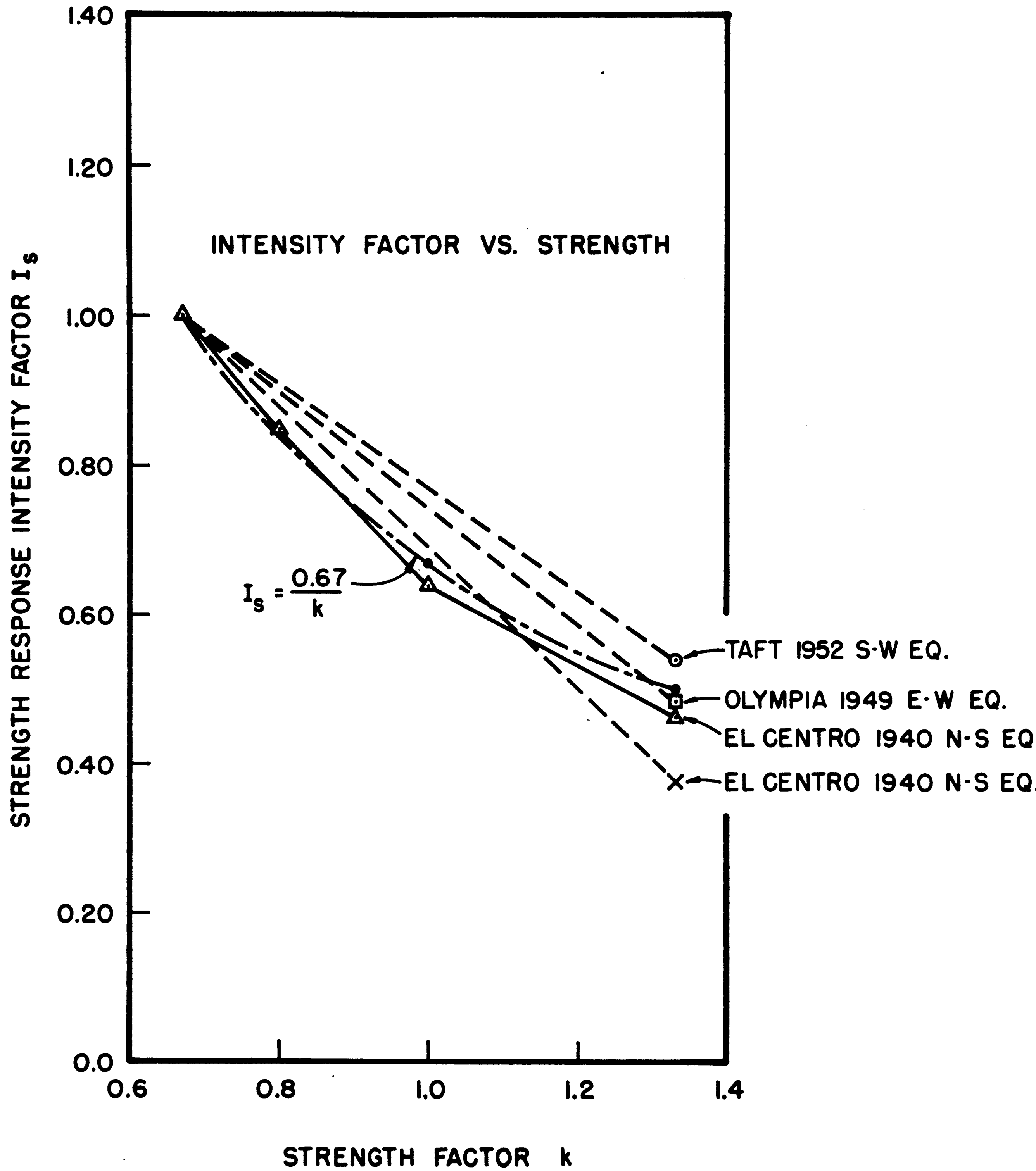


FIG. 29 EFFECT OF STRENGTH FACTOR ON RESPONSE INTENSITY

VI. DISCUSSION AND CONCLUSIONS

Results of this investigation show that the earthquake response of a single degree of freedom structure having a degrading stiffness property similar to that demonstrated by the concrete frames tested by the PCA[3] will be distinctly different from an equivalent system having the ordinary idealized elasto-plastic property. The principal difference in these two response mechanisms results from the fact that an ordinary elasto-plastic system reverts to its original elastic properties as soon as the lateral shear force is reduced below the yield level, while the degrading system is permanently affected by yielding with its properties being changed in proportion to the amount of yielding.

Thus the elasto-plastic system returns to its original vibration period during all intervals when it is not actually yielding, and behaves exactly like an undamaged structure during such intervals. This type of behavior may be considered reasonably representative of a mild structural steel frame. On the other hand, the degrading stiffness structure is significantly less resistant to deflection after it has undergone yield deformations, and thus responds to later phases of the earthquake in a fashion completely different from its initial response behavior. It seems reasonable to assign this type of behavior to a concrete frame in which the cracking associated with large amplitude yielding provides visible evidence of a physical change in the structural properties.

Contrary to some advance expectations, however, the analyses carried out during this investigation have demonstrated that the reduction of stiffness which occurs in the degrading stiffness model does not cause any significant change in the ductility factors which are developed during earthquakes, at least for long period structures such as might be considered typical of multi-story buildings. This seems to be due primarily to the fact that the buildup to the maximum response deformation is essentially an elastic phenomenon resulting ultimately in oscillations which exceed the elastic limit. The response tends to stabilize very quickly after yielding occurs, thus the post-yield behavior (which distinguishes the ordinary and degrading stiffness models) is not of great consequence in controlling the maximum yield amplitudes.

Specific conclusions which may be drawn from various aspects of the results of this investigation follow.

Ductility Requirements

The ductility requirements imposed by earthquake excitations on structures with strengths defined by the SEAOC Code provisions were found to vary strongly with period of vibration: large ductility factors are developed in short period structures and much less ductility is required in the flexible, long period structures. Also, the ductility requirements were found to vary directly with earthquake intensity, and inversely with the design strength of the structure. On the basis of comparisons of ductility requirements, it was concluded that relative intensity factors of 2.55 (EC 4C NS), 1.63 (EC 34 NS), 1.32 (OL 49 EW), and 1.00 (TA 52 SW) might be assigned to the earthquake components considered in this investigation. For the short period structures, ($T < 1$ sec), an earthquake of EC 40 NS intensity may be expected to produce ductility factors in excess of 6 except in the strongest structures: $K = 1.33$), and factors in excess of 20 were computed in several cases. On the other hand for long period structures ($T > 1$ sec), this same excitation produced ductility factors less than 6 in practically all cases.

Ductility Factor Ratios

The degrading stiffness material generally showed slightly higher ductility requirements than the ordinary elasto-plastic material for equivalent cases. However, for the longer period structure, ($T > 0.6$ sec) the ductility factor ratio almost always was found to be in the range $0.8 < \mu_d/\mu_o < 1.2$, and in many cases the ratio was essentially unity. Thus it may be concluded that the degrading stiffness property does not materially affect the yield amplitudes developed in long period, simple structures during earthquakes. On the other hand, the short period structure ($T = 0.3$ sec.) with degrading stiffness properties was found to have significantly larger ductility requirements than the corresponding system composed of ordinary elasto-plastic material. Ductility factor ratios of 2 or 3 were computed in some short period structures subjected to the EC 40 NS excitation.

Maximum Deflection Ratios

Although the results have not been discussed in this report due to space limitations, it was found in this investigation that the maximum deflections developed during the non-linear responses of nearly all cases were quite similar to the maximum deflections developed during a fully elastic response. This conclusion was found to be equally valid for the degrading stiffness material as for the ordinary elasto-plastic material. (The fact that non-linear and elastic responses were essentially equal had been noted earlier for the elasto-plastic material property[2]). The only instance where the non-linear system showed appreciably different (greater) response than the elastic structure was in the shortest period case, T = 0.3 seconds It is this independence of the amplitude of displacement from the strength of the structure which causes the ductility factor to vary inversely with the strength.

Bi-linear Stiffness Characteristics

For long period structures, it is apparent that a reasonable bi-linear stiffness property ($\gamma = \pm 0.05$) has very little effect on the earthquake response of the system. Thus, slight strain hardening effects or degradation of strength with yielding should not be important factors in the earthquake performance of tall buildings. On the other hand, even slight strength degradation characteristics can be disastrous in the earthquake resistance of stiff, short-period structures. Obviously, the product of the ductility factor and the negative bi-linear stiffness ratio cannot be permitted to exceed unity ($\mu\gamma > -1$) if the structure is to retain any strength.

Recommendations for Further Study

Although it has been concluded on the basis of the results of the present investigation that the degrading stiffness property does not materially affect the earthquake resistance of long period structures such as multi-story buildings, it appears that the resistance of short period structures may be seriously reduced by this type of behavior. Thus it seems highly desirable to initiate a further investigation on the earthquake resistance of degrading stiffness structures in the short period range: $0 < T < 0.6$ seconds. Ductility requirements in such structures appear to be critical, and the effects of various forms on non-linearity may by significant. Another factor which may be of importance is the influence of the degrading stiffness property on the higher modes of vibration of multi-story buildings. The higher mode behavior may be somewhat similar to the response of short period structures, in which case the degrading stiffness property could have a very detrimental effect on the performance of the structure. This effect can be studied only by a more comprehensive program of investigation considering systems with many degrees of freedom.

Finally, the influence of vertical loads on ductility requirements should be considered for both single and multiple degree of freedom systems. Some preliminary studies of vertical load effects on single story structures, conducted during the present investigation, indicated that these effects might be significant in the short period range, although they seem to be of little consequence in longer period systems. Conceivably, the effects could be very important in tall buildings, and the relative behavior of ordinary elasto-plastic materials, as compared with the degrading stiffness materials, should be evaluated.

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