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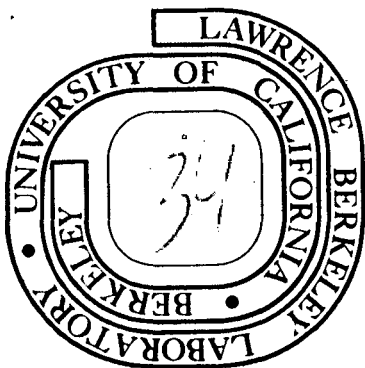
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FUNDAMENTALS OF THE PARTICULATE PHASE  
IN A GAS-SOLID MIXTURE

Woon-Shing Yeung\*

ABSTRACT

Some fundamental behaviors of a gas-solid mixture were studied from a microscopic point of view. For a dilute suspension, one can look upon it as a mixture of a continuum gas, the carrier fluid, and a semi-rarefied gas, the solid particles. The boundary conditions of such a suspension were examined in detail and finally, the general Eulerian formulation of the "particle gas" was given and specialized to a curved pipe geometry.

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## Introduction

There are two major methods to handle the particle phase in a dilute suspension: the Lagrangian description and the Eulerian description. The Lagrangian description fixes attention on a single particle throughout its whole trajectory in the region of interest. Under simplified assumptions, it involves only solving a set of ordinary differential equations subject to given initial conditions<sup>1</sup>. The Eulerian formulation, on the other hand, assumes the particle phase as a continuum and the resulting governing equations for the particle phase are quite similar to the well-known Navier-Stokes equations in fluid mechanics (see Section II). Even under simplified assumptions, it requires the solution of a set of partial differential equations subject to both initial and boundary conditions. Of most importance, the continuum assumption must be justified in using the Eulerian formulation of the particulate phase. Fortunately, most practical physical systems involving gas-particle mixtures satisfy the continuum assumption. For a discussion of the continuum assumption, see, for example, Soo<sup>2</sup>. In those systems where particle-particle interaction is insignificant, the Lagrangian technique readily generates the particle trajectories. On the other hand, if particle-particle interaction cannot be neglected, as in a recirculating zone or turbulence regions, the solution of the Lagrangian formulation is a quite

formidable task. For such systems, the use of the Eulerian formulation is conceptually more correct since the randomness of the particulate phase is accounted for by way of the formulation. Many authors, in particular Soo<sup>3,4,5,6</sup>, Singleton<sup>7</sup> and Marble<sup>8</sup>, have successfully applied the Eulerian formulation to solve the particulate phase of a dilute suspension of some simple physical systems, such as pipe flow and flow over a flat plate. In fact, some of the papers are so general that even electromagnetic effect and deposition of the particles are included. Unfortunately, there is scarce literature dealing with more complicated systems than those mentioned above. Also, little effort is devoted to the fundamental behavior of the particulates in a suspension, especially near the physical boundary of the system. It is the purpose of the present paper to discuss some fundamental aspects of a dilute suspension from the molecular point of view and to give a rather general Eulerian formulation and appropriate boundary conditions as applied to the gas-particle flow through a curved pipe with circular cross-section in a coal gasification environment. No attempt has been made to solve the resulting equations.

## II. Molecular aspects of a gas-particle mixture.

The fundamentals of a gas-particle mixture must be understood before the formulation of the general governing equations for each phase. Of particular importance are the transport properties of each individual phase and phase-phase interactions. One way to calculate these quantities is by use of kinetic theory of a mixture of gases, assuming, of course, the validity of the continuum approximation of the particle phase. The major task is to solve the Boltzmann equation of transport. However, it is not the purpose of this section to adopt the mathematical methods of kinetic theory to the gas-particle system; but rather, to bring out some basic features of the gas-particle system by looking upon it from a molecular point of view.

A dilute mixture of solid particles and gas is very similar to a mixture of gas-molecules and electrons, respectively, as far as the size distribution is concerned. Since particles in a dilute suspension are much larger than the gas molecules, their random intensity of motion, or thermal agitation, is very much less than that of the gas molecules. Thus a dilute suspension is similar to a Lorentzian gas<sup>9</sup> and it is logical to follow the treatment of Lorentz<sup>10</sup> if one tries to attempt the problem from the Boltzmann equation. For a further discussion, the interested reader is referred to Culick<sup>11</sup>.

Suppose a gas-solid mixture is at rest in a container. Because of the random motion of the gas molecules, the solid particles would undergo Brownian motion. The R.M.S. (root-mean-square) speed of a particle will be given by the Einstein<sup>12</sup> formula

$$\overline{(C_p^2)}^{\frac{1}{2}} \sim \left\{ \frac{9kT}{4\pi a^3 \rho_p} \right\}^{\frac{1}{2}} \quad (1)$$

with

- $\overline{(C_p^2)}^{\frac{1}{2}}$  the R.M.S. speed of a particle,
- k Boltzmann constant,
- T body temperature of the particle,
- a particle radius,
- $\rho_p$  the material density of the particle.

As an example, we take  $T = 1255.5^\circ\text{K}$ ,  $\rho_p = 3.89 \times 10^{-6} \text{ gm/c.c}$  and  $a = 10^{-2} \text{ cm}$ , which are the conditions found inside a coal gasifier. From equation (1), we can easily calculate  $\overline{(C_p^2)}^{\frac{1}{2}}$  as 0.2 cm/sec. It is seen that the R.M.S. speed of a particle is much less than that of a gas molecule, which has a R.M.S. molecular speed comparable to sound speed of the gas phase. Hence, if there is a macroscopic motion of the gas-particle mixture, say about 1000 cm/sec, one can generally neglect the Brownian motion of the particles. On the other hand, one may not generally neglect the collisions between particles themselves. It is

quite misleading to say that when the interparticle distance is large compared with the particle size, one can neglect the collisions between particles. As an example, the intermolecular distance of hydrogen gas at N.T.P. condition is about 10 times the molecular size, which is large compared with the molecular size; yet the collision frequency is as high as  $10^{29}$  between all hydrogen molecules. Although the collision frequency between particles themselves is very much less than  $10^{29}$  as will be shown below, collision may be significant even in a dilute suspension constituting 10% by weight of particles.

We now proceed to estimate the various interaction lengths (mean free path) and collision frequency for a dilute suspension. The results only apply to the equilibrium case, i.e., the suspension at rest in a container. However, they serve as good comparisons even when the suspension undergoes a macroscopic motion. The following formulae can be found in Chapman and Cowling<sup>13</sup>. We shall assume that the molecules and particles are smooth, rigid, elastic spheres. Let

$$\sigma_{12} = \frac{1}{2}(\sigma_1 + \sigma_2) \quad (2)$$

where

$\sigma_1$  : diameter of a particle

$\sigma_2$  : diameter of a molecule.

The total number of collisions between pairs of particle and molecule per unit time is

$$N_{12} = N_{21} = 2n_1 n_2 \sigma_{12}^2 \left( \frac{2\pi k T m_0}{m_1 m_2} \right)^{\frac{1}{2}} \quad (3)$$

where

$n_1$  : number density of particles per unit volume of mixture

$n_2$  : number density of molecules per unit volume of mixture

$m_1$  : mass of a particle

$m_2$  : mass of a molecule



and

$$m_0 = m_1 + m_2 \quad (4)$$

It follows that the collision frequencies between molecules themselves and particles themselves are

$$N_{22} = 4n_2^2 \sigma_2^2 \left(\frac{\pi kT}{m_2}\right)^{\frac{1}{2}} \quad (5)$$

$$N_{11} = 4n_1^2 \sigma_1^2 \left(\frac{\pi kT}{m_1}\right)^{\frac{1}{2}} \quad (6)$$

The collision frequency for a particle with a particle is

$$f_{11} = \frac{N_{11}}{n_1} \quad (7)$$

for a molecule with a molecule

$$f_{22} = \frac{N_{22}}{n_2} \quad (8)$$

for a particle with a molecule

$$f_{12} = \frac{N_{12}}{n_1} \quad (9)$$

and for a molecule with a particle

$$f_{21} = \frac{N_{21}}{n_2} \quad (10)$$

The mean free path between particles and particles is given by

$$l_{11} = \frac{\sqrt{2}}{\pi n_1 \sigma_1^2} \quad (11)$$

Similarly, between molecules and molecules

$$l_{22} = \frac{\sqrt{2}}{\pi n_2 \sigma_2^2} \quad (12)$$

between particle and all molecules

$$l_{12} = \sqrt{\frac{m_2}{m_0}} \frac{1}{\pi \sigma_{12}^2 n_2} \quad (13)$$

and between a molecule and all particles

$$l_{21} = \sqrt{\frac{m_1}{m_0}} \frac{1}{\pi \sigma_{12}^2 n_1} \quad (14)$$

For the same condition used above to calculate the R.M.S. speed of the particles, it is found that

$$f_{11} \sim 0.3 \text{ collisions/sec,}$$

$$f_{21} \sim 2 \times 10^3 \text{ collisions/sec,}$$

$$N_{11} \sim 450 \text{ collisions/sec/cm}^3,$$

and  $N_{21} \sim 2 \times 10^{22} \text{ collisions/sec/cm}^3$

for  $n_1 = 1.5 \times 10^3 \text{ particles/cm}^3$  and  $n_2 = 10^{19} \text{ molecules/cm}^3$ , which correspond to about 10% by weight of particles typical inside the coal gasifier.

It is evident that particle-particle collision is negligible in comparison with particle-molecule collision. Since collision is the main mechanism of exchanging momentum, one can neglect the interaction between particles themselves and retain only the interaction term between particle and gas phase, in a macroscopic formulation of the system, such as the Eulerian formulation. The word macroscopic is crucial in justification of neglecting particle-particle interaction, since microscopically there are still 450 collisions/sec/cm<sup>3</sup> between particles, however negligible as compared to particle-molecule collisions. If we are to formulate the motion of each particle, as is done in Lagrangian method, these collisions must be taken into account. Thus, to be rigorous and practical, some sort of statistical averaging method and assumptions of the particle-particle interaction should be introduced in conjunction with the Lagrangian technique for systems where particle-particle collision is significant, at least in a microscopic scale. No such difficulty arises when using the Eulerian

approach because only average quantities are formulated in this approach. We can also estimate the relative effect of each phase upon the other from comparing the interaction lengths  $l_{21}$  and  $l_{12}$ . From equations (13) and (14), since  $m_1 \gg m_2$  and  $n_1 \ll n_2$  in most dilute suspensions, it follows that

$$l_{21} \gg l_{12} \quad (15)$$

The longer the interaction length, the weaker the interaction of one phase upon the other. Thus the effect of particle phase on the gas phase, as suggested by  $l_{21}$ , is less prominent than the effect of the gas phase on the particle phase, as suggested by  $l_{12}$ .

### III. Boundary conditions of the particle phase.

The derivation of appropriate conditions at a physical boundary can be very complicated and difficult. As mentioned in the Introduction, there is scarce literature discussing the boundary conditions of a suspension at a solid surface. In this section, we shall discuss further the fundamental aspects of a dilute suspension with the guidance of the results obtained in Section II and the boundary conditions of the particle phase used hitherto by other authors.

It is well known that the phenomenological no-slip condition on a solid wall in conventional continuum fluid mechanics governed by the Navier-Stokes equations is only approximately true. The actual boundary conditions must be obtained from the solution of Boltzmann equation and suitable reflection laws for the molecular impact with the wall. It also turns out that the usual boundary conditions for use with the Navier-Stokes equation are not the actual macroscopic value of the gas velocity at the wall, because of the invariable presence of the Knudsen layer (see Kogan<sup>14</sup>). In the regime of semi-rarefied gas dynamics, the solution of the Navier-Stokes equation together with the

so-called slip condition at the boundary has been shown to be adequate in most cases. In the case of a gas mixture, some components may be in the slip flow regime and others in the no-slip regime. The exact derivation of boundary condition becomes very complicated because of the interaction among components.

The determination of which regime a flow belongs to depends on the magnitude of the appropriate mean free path of each component. In a dilute suspension Section II shows that different magnitudes of free paths exist and one may expect that different flow regimes exist for different phases. Consequently, the particle and gas phases do not satisfy the same boundary conditions. In order to derive the proper boundary conditions for the particle phase, details of the interaction between the particle and the solid boundary must be known. The possibilities of attrition upon impact, embedding into the wall and different shapes of the particles render an exact derivation almost impossible. However, from the estimation of the mean free path of the particle phase (equation (11)) and comparison with the mean free path of the gas phase (equation (12)), there is reason to believe that the particulate phase is similar to a semi-rarefied gas in the slip regime. Thus, some of the important results derived for the semi-rarefied gas may be applied to the particle phase.

Before discussing this further, we shall mention that the particles that constitute the particulate phase are very much different from the molecules that constitute the gas phase. For a simple gas in the subsonic region, the gas macroscopically can always adjust its streamline to align with the boundary surface. Hence we only speak of slip velocity tangential to the wall of the gas. For a 'particle gas,' however, the streamlines, or particle trajectories, often do not align with the physical boundaries. Thus, in addition to the slip velocity tangential to the wall, there is also a nonvanishing normal velocity component of the particle phase at the wall. Macroscopically, this nonvanishing normal velocity component causes deposition of the particles on the boundaries.

It should be noted deposition is caused by the presence of field forces, such as gravity force, electrostatic force, etc. Once deposition occurs, a layer of particles at the boundary may build up to a point such that a sliding bed will proceed downstream<sup>6</sup>. The appropriate boundary condition for this case is given by the conservation of the total particle flow:

$$\frac{\partial}{\partial s} \iint_A \rho_p w_p dA = \sigma'_L P \rho_p v_L - \sigma \iint_A u_{pw} \rho_p dA - \sigma_a P \rho_p v_a \quad (16)$$

where

$\sigma'_L$  : lift probability due to lift force

$\sigma$  : sticking probability depending on material properties

$\sigma_a$  : sticking probability due to adhesion

$v_L$  : lift velocity

$v_a$  : adhesion velocity

$P$  : perimeter of the cross-section A (Figure 1)

$\rho_b$  : density of particles at packed bed condition

$\rho_p$  : local particle phase density

$w_p$  : streamwise particle velocity in the direction s

and subscript w refers to the value at the wall unless otherwise stated.

The sticking velocity  $u_{pw}$  can be approximated as<sup>6</sup>,

$$U_{pw} \cong (fp)_u \tau_m \quad (17)$$

where  $(f_p)_u$  is the field force acting on the particle phase in the normal, or u, direction and  $\tau_m$  is the momentum equilibration time. If the particle motion belongs to the Stokes flow regime,  $\tau_m$  is given by

$$\tau_m = \frac{2}{9} \frac{\bar{\rho}_p a^2}{\mu} \quad (18)$$

where  $\bar{\rho}_p$  is the material density of the particles. In general,  $\tau_m$  depends on both the gas velocity and particle velocity. When there is no net deposition

of the particles, the normal velocity component at the physical boundary vanishes, even when field forces are present. Care must be taken not to regard  $u_{pw}$ , or any other variables, as the value individual particle attains, since an individual particle does bombard the wall with nonvanishing velocity components normal to the wall. It is only the average value of the normal velocity components of all the particles at a particular location on the wall that vanishes.

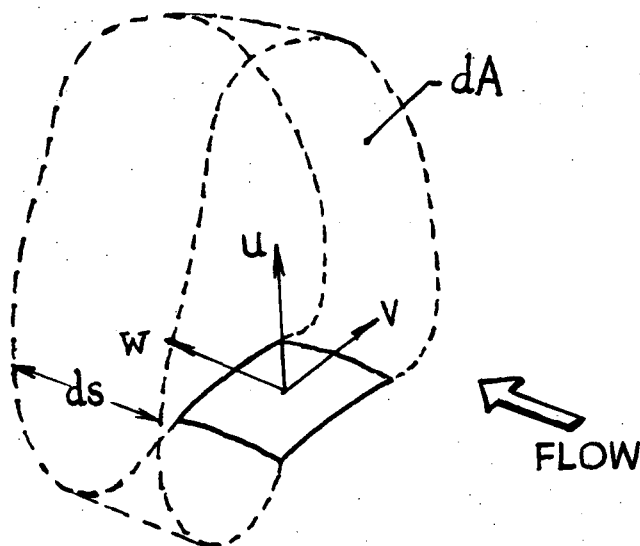


Figure 1

Several constants were introduced in (16). They are  $\sigma_L$ ,  $\sigma$ ,  $\sigma_a$ ,  $v_L$ ,  $v_w$ ,  $\rho_b$ . Theoretically, these can be found by a detailed consideration of the statistical mechanics of the particle phase; however, it is more practical to resort to experiments. For the slip velocity components, we extend the corresponding formula for one-dimensional rarefied gas dynamics to our general three-dimensional case:

$$v_{pw} = C_1 (\tau_w)_v / \mu, \quad (19a)$$

$$w_{pw} = C_2 (\tau_w)_w / \mu, \quad (19b)$$

where  $(\tau_w)_v$  is the wall shear stress of the particle phase in the v direction and  $(\tau_w)_w$  is the wall shear stress of the particle phase in the w direction.  $C_1$  and  $C_2$  are two phenomenological constants of proportionality and  $\mu$  is the viscosity of the suspension. It is very important to realize that equation (19a) and (19b) are the only plausible generalizations to those formulas obtained in rarefied gas dynamics. There is no rigorous proof of their applicability to a dilute suspension. Soo and Tung<sup>6</sup> have proposed the following:

$$\vec{v}_{p_w} = L_p \Delta_{p \approx w}, \quad (20)$$

where  $\vec{v}_{p_w}$  is the slip velocity vector at the wall,  $\Delta_{p \approx w}$  is the deformation tensor of the particle phase at the wall, and  $L_p$  is identified as an interaction length between the fluid phase and the particle phase due to wall effect, wake effect, turbulence and molecular diffusivity. This definition of  $L_p$  is not completely justified by Soo. Furthermore,  $L_p$  was calculated using equation (20) rather than a quantity known in advance of the calculation. It is therefore more appropriate to use a proportionality constant in place of  $L_p$ , thereby avoiding any ad hoc definition of  $L_p$ , as is done in equation (19a) and (19b). We shall now give the general governing equations with boundary conditions for the particulate phase of a dilute suspension flowing through a curved pipe.

#### IV. Eulerian formulation of the particle phase

The following assumptions are made:

(1) The inertial coupling in momentum due to the change of frame of reference is neglected<sup>15</sup>. Also the effect due to unsteady flow field in the viscous range is not taken into account. The force acting on the particulate phase due to pressure gradient of the fluid phase can be shown to be negligible<sup>16</sup>.

Finally, the volume fraction of the particulate phase is very small so that the force due to change in volume fraction is not important.

(2) The transport properties of the suspension can be approximated, to the first order, by those of the fluid phase alone.

(3) The particle-particle interaction is negligible compared to the particle-fluid interaction. Hence there is little momentum transfer among particles by the process of particle-particle collision. Under such an assumption, the 'particle gas' possesses no (or very insignificant) viscosity, diffusivity, or conductivity and there is no shear stress term due to particle-particle interaction appearing in the equation of motion. However, particles can transport momentum to other particles by diffusion through the gas phase. The stress term resulting from this diffusion process is known as diffusional stress for the particulate cloud. As the term suggests, it is important only when the diffusivity of the particles through the fluid is not small. For molecular diffusion, Einstein<sup>12</sup> has found that

$$D_p = \frac{2kT}{6\pi\bar{\mu}a} \quad (21)$$

where

$D_p$  : molecular diffusivity of particles in the fluid

$T$  : temperature of the particles

$\bar{\mu}$  : fluid viscosity

$a$  : particle diameter.

For most situations, the molecular diffusivity is very small, especially for particle sizes of above 100 micron. On the other hand, if the fluid phase is turbulent, the diffusivity of the particles through the turbulent fluid, which is no longer given by equation (21), may be significant.

(4) Since the R.M.S. speed of the particles is very much less than that of the gas molecules (Section II), the static pressure of a dilute suspension



is due solely to the fluid phase.

For the present example, a set of toroidal coordinates is used. The velocity components are denoted by  $u, v$  and  $w$  in the directions of  $r$  increasing,  $\psi$  increasing and  $\phi$  increasing, respectively. The orientation of the curved elbow is such that the mid-plane of the elbow is parallel to the vertical plane (Figure 2).

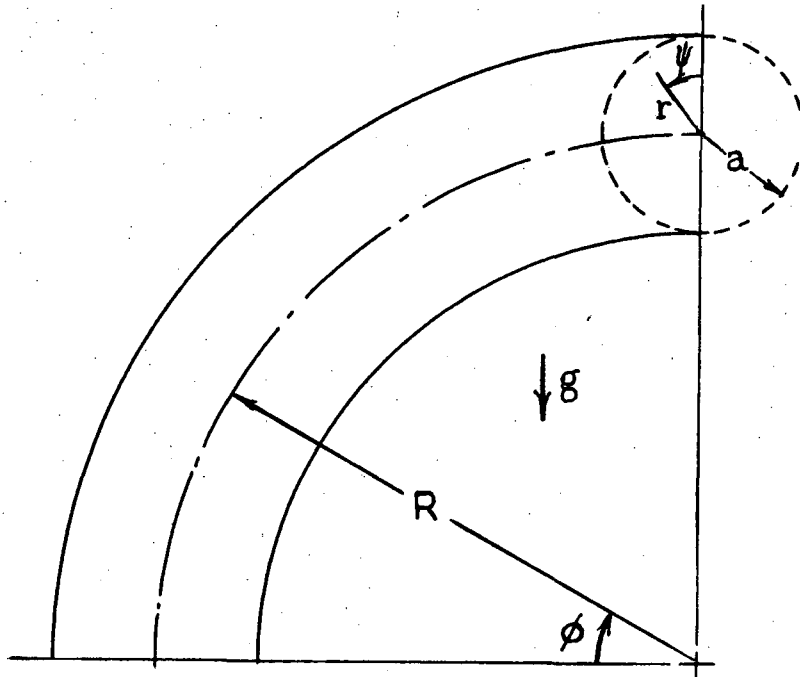


Figure 2. Orientation of curved pipe and toroidal coordinates

Assuming the particulate phase is a continuum, the momentum equation can be written as

$$\rho_p \frac{D\vec{u}_p}{Dt} = \vec{\nabla} \cdot (\underline{\underline{\tau}}_p) + \rho_p \vec{f}_p - \rho_p \frac{\vec{u} - \vec{u}_p}{\tau_m} \quad (22)$$

with

$\rho_p$  as the particle phase density

$\vec{u}_p$  the particle velocity vector

$\underline{\underline{\tau}}_p$  the diffusional stress tensor of the particle phase

$\vec{f}_p$  any field forces acting on the particles

and  $\vec{u}$  the gas velocity vector.

In deriving equations (22), assumptions (1), (3) and (4) have been used. As is done in fluid mechanics,  $\underline{\tau}_p$  is assumed to take on the Newtonian stress tensor form:

$$\underline{\tau}_p = (\lambda_p \operatorname{div} \vec{u}_p) \underline{\underline{I}} + \mu_p \underline{\underline{\Delta}}_p \quad (23)$$

$\lambda_p$  and  $\mu_p$  are two coefficients of viscosity due solely to the interaction between the fluid and the particle phase.  $\underline{\underline{\Delta}}_p$  is the deformation tensor,

$$\underline{\underline{\Delta}}_p = (\vec{\nabla} \vec{v}_p + (\vec{\nabla} \vec{v}_p)^{-1}) \quad (24)$$

and  $\underline{\underline{I}}$  is the unitary tensor.

Assuming Stokes' hypothesis here, we have

$$\lambda_p = -\frac{2}{3} \mu_p \quad (25)$$

For a dilute suspension, the viscosity of the particulate phase due to diffusion through the fluid phase can be approximated as<sup>6</sup>

$$\mu_p \approx \rho_p D_p \quad (26)$$

Equation (23) becomes

$$\underline{\tau}_p = \rho_p D_p [\vec{\nabla} \vec{v}_p + (\vec{\nabla} \vec{v}_p)^{-1}] - \frac{2}{3} \rho_p D_p (\vec{\nabla} \cdot \vec{v}_p) \underline{\underline{I}} \quad (27)$$

Thus, when only molecular diffusion is present,  $D_p$  will be very small (equation (20)) and  $\underline{\tau}_p$  can be neglected. For this case, equation (21) is reduced to

$$\frac{D\vec{u}_p}{Dt} = \vec{f}_p - \frac{\vec{u} - \vec{u}_p}{\tau_m} \quad (28)$$

We shall only consider gravity force and lift force. Written in toroidal coordinates and denoting  $\bar{\rho}$  as the material density of the fluid, Eq. (28) becomes

$$\begin{aligned} \frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial r} + \frac{v_p}{r} \frac{\partial u_p}{\partial \psi} - \frac{v_p^2}{r} - \frac{w_p^2 \cos \psi}{R+r \cos \psi} + \frac{w_p}{R+r \cos \psi} \frac{\partial u_p}{\partial \phi} \\ = -g \cos \phi \cos \psi \left(1 - \frac{\bar{\rho}}{\rho_p}\right) - f_L - \frac{u-u_p}{\tau_m} \end{aligned} \quad (29a)$$

$$\begin{aligned} \frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial r} + \frac{v_p}{r} \frac{\partial v_p}{\partial \psi} + \frac{w_p}{R+r \cos \psi} \frac{\partial v_p}{\partial \phi} + \frac{u v_p}{r} + \frac{w_p^2 \sin \psi}{R+r \cos \psi} \\ = g \sin \phi \sin \psi \left(1 - \frac{\bar{\rho}}{\rho_p}\right) - \frac{v-v_p}{\tau_m} \end{aligned} \quad (29b)$$

$$\begin{aligned} \frac{\partial w_p}{\partial t} + u_p \frac{\partial w_p}{\partial r} + \frac{v_p}{r} \frac{\partial w_p}{\partial \psi} + \frac{w_p}{R+r \cos \psi} \frac{\partial w_p}{\partial \phi} + \frac{u w_p \cos \psi}{R+r \cos \psi} - \frac{v w_p \sin \psi}{R+r \cos \psi} \\ = -g \cos \phi \left(1 - \frac{\bar{\rho}}{\rho_p}\right) - \frac{w-w_p}{\tau_m} \end{aligned} \quad (29c)$$

where  $f_L$  is the lift force near the wall due to fluid shear. (See, for example, Scaffman<sup>17</sup>).

The species continuity equation is

$$\frac{\partial \rho_p}{\partial t} + \text{div}(\rho_p \vec{v}_p) = -\text{div}(\vec{j}_p) + \Gamma \quad (30)$$

$\Gamma$  accounts for the creation or annihilation of particles in the flow. Assuming no chemical reaction,  $\Gamma \equiv 0$ , and Fick's law of diffusion holds for the diffusional flux  $\vec{j}_p$ , we have

$$\vec{j}_p = -\rho D_p \vec{\nabla} \left(\frac{\rho_p}{\rho}\right) \quad (31)$$

where  $\rho$  is the density of the suspension. It can be readily shown that

$$\rho = \rho_p + \rho_f \approx \rho_f \quad (32)$$

for a dilute suspension, and if the fluid phase is incompressible,  $\rho_f$  is constant and we have

$$\vec{j}_p \cong -D_p \vec{\nabla}(\rho_p) \quad (33)$$

Substituting (33) into (30), we have

$$\frac{\partial \rho_p}{\partial t} + \vec{\nabla} \cdot (\rho_p \vec{v}_p) = \vec{\nabla} \cdot (D_p \vec{\nabla} \rho_p) \quad (34)$$

If  $D_p$  is very small, we recover the usual continuity equation for a single phase

$$\frac{\partial \rho_p}{\partial t} + \vec{\nabla} \cdot (\rho_p \vec{v}_p) = 0 \quad (35)$$

In toroidal coordinates, equation (35) becomes

$$\begin{aligned} \frac{\partial \rho_p}{\partial t} + \frac{\partial \rho_p u_p}{\partial r} + \frac{1}{r} \frac{\partial \rho_p v_p}{\partial \psi} + \frac{1}{R+r \cos \psi} \frac{\partial \rho_p w_p}{\partial \phi} \\ + \left( \frac{u_p}{r} + \frac{u_p \cos \psi}{R+r \cos \psi} - \frac{v_p \sin \psi}{R+r \cos \psi} \right) \rho_p = 0 \quad (36) \end{aligned}$$

Equations (29a-c) and (36) are the necessary governing equations under the assumptions used. For the boundary conditions, equation (16), when written in toroidal coordinates, gives

$$\begin{aligned} \frac{\partial}{\partial \phi} \int_0^{2\pi} \int_0^a \rho_p w_p r \, dr d\psi = \sigma_L' a \int_0^{2\pi} (R+r \cos \psi) v_L \rho_b \, d\psi \\ - \sigma_a \int_0^{2\pi} (R+r \cos \psi) (u_{pw} \rho_{pw}) \, d\psi - \sigma_a a \int_0^{2\pi} (R+r \cos \psi) v_a \rho_{pw} \, d\psi \quad (37a) \end{aligned}$$

where  $u_{pw}$  is the velocity component normal to the pipe surface at the wall, given as

$$u_{pw} = \left[ -g \cos \phi \cos \psi \left( 1 - \frac{\bar{\rho}}{\rho_p} \right) - f_L \right] \tau_m \quad (37b)$$

For the slip velocity components, equations (19a) and (19b) give

$$v_{pw} = C_1 (\tau_{pw})_{\psi\psi} / \mu \quad (38)$$

$$w_{pw} = C_2 (\tau_{pw})_{\phi\phi} / \mu \quad (39)$$

From equation (27)

$$(\tau_{pw})_{\psi\psi} = \frac{2}{a} \frac{\partial v_p}{\partial \psi} \Big|_w \rho_{pw} D_p - \frac{2}{3} \rho_{pw} D_p \left[ \frac{1}{a} \frac{\partial v_p}{\partial \psi} + \frac{1}{R+a \cos \psi} \frac{\partial w_p}{\partial \phi} - \frac{v_p \sin \psi}{R+a \cos \psi} \right]_w \quad (40)$$

Solving for  $v_p$  in (38), we have

$$v_p \left[ 1 + \frac{2}{3} \rho_{pw} D_p \frac{C_1}{\mu} \frac{\sin \psi}{R+a \cos \psi} \right] = \frac{C_1}{\mu} \left\{ \frac{4}{a} \rho_{pw} D_p \frac{\partial v_p}{\partial \psi} \Big|_w - \frac{2}{3a} \frac{\rho_{pw} D_p}{R+a \cos \psi} \frac{\partial w_p}{\partial \phi} \Big|_w \right\} \quad (41)$$

Similarly,

$$(\tau_{pw})_{\phi\phi} = \frac{4}{3} \frac{\rho_{pw} D_p}{R+a \cos \psi} \frac{\partial w_p}{\partial \phi} \Big|_w - \frac{4}{3} \frac{v_p \sin \psi}{R+a \cos \psi} \rho_{pw} D_p - \frac{2}{3a} \rho_{pw} D_p \frac{\partial v_p}{\partial \psi} \Big|_w \quad (42)$$

and equation (39) becomes

$$w_p = \frac{C_2}{\mu} \left\{ \frac{4}{3} \frac{\rho_{pw} D_p}{R+a \cos \psi} \left[ \frac{\partial w_p}{\partial \phi} \Big|_w - v_p \sin \psi \right] - \frac{2}{3a} \rho_{pw} D_p \frac{\partial v_p}{\partial \psi} \Big|_w \right\} \quad (43)$$

It should be mentioned that  $\tau_{pw}$  consists of only velocity components tangential to the pipe surface. In deriving the boundary condition for one-dimensional rarefied gas dynamics, only the tangential velocity component is involved. Since equation (19a) and (19b) are direct extensions of the one-dimensional rarefied gas dynamics theory, it is consistent to use only tangential velocity components in  $\tau_{pw}$ . In this way, the normal and tangential velocity components at the wall are uncoupled from each other. Its validity has to be verified by experiments or numerical results, or both. Since  $C_1$  and  $C_2$  in equations (41) and (43) are of unknown magnitudes, terms containing  $D_p$  may not be neglected in deriving the boundary conditions.

Equations (29a-c) and (36), together with boundary conditions (37a-b), (41) and (43) can be solved for  $\rho_p$  and  $\vec{u}_p$ . Since equations (29a-c) do not contain  $\rho_p$ , one can assume a form of  $\rho_{p_w}$  and solve the momentum equations (29a-c) subject to boundary conditions (37b), (41) and (43) and initial condition at the pipe entry. Then the continuity equation (36) can be solved with the assumed form for  $\rho_{p_w}$ . The correctness of  $\rho_{p_w}$  is then checked using equation (37a). The process is repeated until (37a) is satisfied.

#### V. Conclusion

Some molecular aspects of a dilute suspension were studied. Although the results derived in section II apply strictly to an equilibrium gas-particle mixture, the major conclusions remain valid when the mixture undergoes a macroscopic motion. In many dilute gas-particle mixtures, the particulate phase behaves like a semi-rarefied gas in the slip regime. This is not surprising since the interaction length among particles is relatively long. It is therefore expected that the particulate phase slips along the wall with velocity components proportional to the deformation tensor, a result extended directly from the theory of one-dimensional rarefied gas dynamics. There is little experimental or numerical data supporting this derivation, in particular when it is applied to complicated geometries such as a curved tube.

In this paper only the Eulerian formulation was considered. As mentioned previously, particle-particle collisions, although insignificant compared to particle-molecule or molecule-molecule collisions, are present even in a dilute suspension. Any attempt to formulate the equation of motion of an individual

particle must account for these collisions. Since the Eulerian formulation uses the continuum concept, the particle-particle interaction is automatically included in the analysis. It is neglected on the basis that it is small compared to particle-fluid interaction. Since it accounts for the randomness of the particles, the Eulerian formulation is conceptually more correct. Moreover, the solutions to the Eulerian formulation, such as density and velocity, do not apply to individual particles. Instead, they are the average values of all particles confined in a volume large enough to contain a great number of particles yet small enough when compared with the characteristic dimension of the region of interest. The main disadvantage using the Eulerian approach is that the governing equations are first-order, elliptic partial differential equations. Except for some simple geometries, they are quite difficult to solve. No numerical results have been obtained at the present stage.

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