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LOW FREQUENCY NOISE IN JOSEPHSON JUNCTIONS*

John Clarke† and Gilbert Hawkins‡

ABSTRACT

Using a high resolution SQUID voltmeter, we have measured the spectrum of low frequency voltage fluctuations across a thin-film Josephson tunnel junction biased at a constant current I greater than the junction critical current I_c. We find that the frequency dependence of the voltage spectrum $V^2(f)$ may be accurately represented by the power law $V^2(f) \propto f^{-1}$ over the frequency range of our data: $10^{-2} < f < 10$ Hz. The dependence of the magnitude of the spectra at any single frequency upon the value of the bias current I and upon the sample temperature T supports our hypothesis that the observed voltage fluctuations arise from a modulation of the junction critical current I_c by equilibrium, thermodynamic temperature fluctuations in the active junction volume. We are able to interpret our measurements in terms of the semi-empirical theory[1] of Clarke and Voss for the low frequency fluctuation spectrum of systems obeying a diffusion equation. This interpretation provides design criteria which may prove useful in reducing the level of long-term drifts in systems employing Josephson tunnel junctions.

I. EXPERIMENTAL TECHNIQUES

The Josephson junctions investigated in our experiment are constructed by sputter deposition of Nb strips 2000Å thick and 180 μm wide followed by evaporation of Pb cross strips of similar dimensions. Prior to sputtering, the soda glass substrates are coated with 20 Å of Cr to insure mechanical adherence of the films. The junctions are shunted to a resistance of approximately 7 mΩ by the evaporation of disc-shaped Cu underlays a few millimeters wide and 7000 Å thick centered on the junction area. On some samples, the inductance of these shunts is reduced by the addition of a superconducting ground plane to insure that the I-V characteristic does not exhibit hysteresis. We estimate the hysteresis parameter $\beta_c = 2\pi I_c R^2 C/\Phi_0$ to be approximately 0.2, where C is the junction capacitance, R the shunt resistance, and Φ_0 the flux quantum. We have also constructed samples whose shunts are excluded from the immediate junction region to preclude the possible formation of small SNS junctions at the points of mutual contact of the Pb, Nb, and Cu.[2] The formation of such junctions might be expected to alter the temperature dependence of the total junction critical current, because of the exponential dependence of I_c upon T for SNS junctions.[3] However, we have as yet no evidence for the occurrence of this effect in samples employing the full disc geometry. The sample junctions are mounted in thermal contact with a Cu block suspended in a vacuum chamber. The thermal time constant of the block is chosen to be approximately 6 minutes in order to minimize the effects of temperature fluctuations of the He bath over the time scale of our measurements. All electrical connections to the junctions are made using 2 mil Nb leads spot welded to solder-coated brass tabs. Superconducting contact to the Nb strips cannot be achieved reliably by soldering and is accomplished by the use of evaporated Pb underlays.

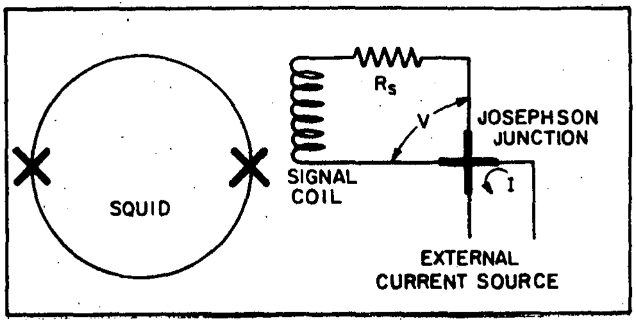

Fig. 1. Experimental apparatus for the measurement of voltage fluctuations across a thin film Josephson junction.

As shown in Fig. 1, an external current source provides the junction with a constant bias current $I \geq I_c$. The voltage V developed across the junction causes current to flow through standard resistor R_s to a superconducting signal coil whose field is coupled to a dc point-contact SQUID of toroidal geometry. In the feedback mode, the SQUID and its associated electronics act as a high gain (~ 10^8), low noise dc amplifier whose output is directly proportional to V for frequencies below the 300 Hz response of the SQUID electronics. The advantages of this amplifier configuration in our experiment are twofold. First, the amplifier may easily be impedance matched to the mΩ resistances of our samples by a suitable choice of the standard resistor R_s, typically 0.01 Ω. This enables us to measure junction voltages of as little as 10^{-13} V without amplifier noise limitation. Secondly, the input to the SQUID amplifier may be effectively dc offset for the measurement of small fluctuations in junction voltages which themselves are so large as to exceed the dynamic range of the SQUID. Offset is achieved with no loss of amplifier stability and with no deterioration of the dc frequency response by operating the SQUID with a large (but constant) number of flux quanta in the area enclosed by the point contacts.

II. RESULTS AND ANALYSIS

When the sample is biased with current $I > I_c$, the voltage V across the junction exhibits small fluctuations whose low frequency spectrum is analyzed by digitizing the output signal of the SQUID amplifier and calculating the relevant Fourier transforms on a PDP-11 computer. Data points for the frequency spectrum $V^2(f)$ for a typical junction are displayed logrithmically in Fig. 2 as a function of frequency f over the range $5 \times 10^{-3} < f < 10$ Hz. For this junction, I = 3.0 mA and I_c = 2.6 mA at T = 1.5K. In Fig. 3, smooth line fits to our data for the voltage spectra of a second junction are shown for several values of the bias current I in the range $I \approx I_c$ to $I = 2I_c$.

There are two important features displayed by the fluctuation spectra of Figs. 2 and 3. First, for all values of bias current I, the frequency dependence of the spectra is very nearly f^{-1} over a wide range of frequencies. The best straight line fits to our data for various sample junctions yield exponents in the range -0.9 to -1.15 for the frequency

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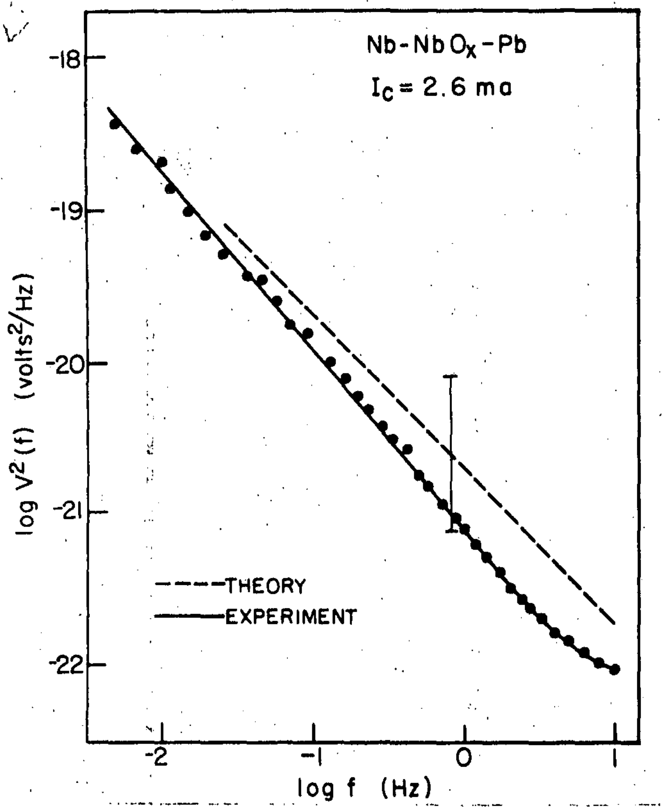

Fig. 2. Experimental measurements of the spectrum $V^2(f)$ of voltage fluctuations across a typical sample.

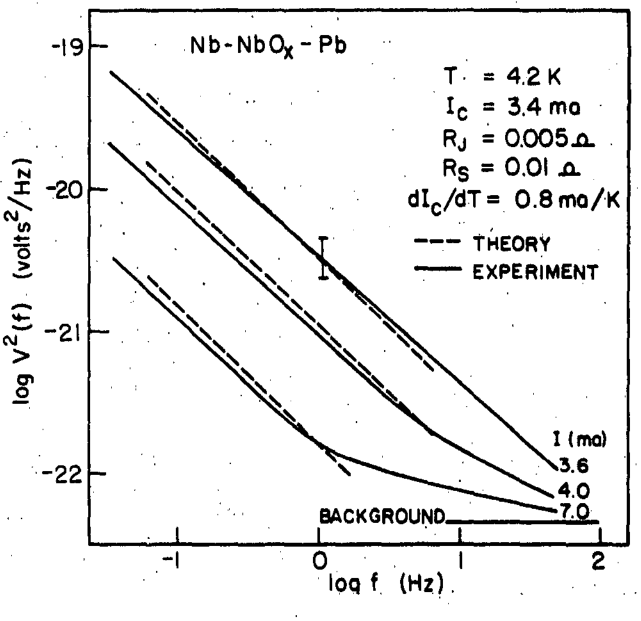

Fig. 3. Voltage spectra for three values of junction bias current I. Note that the fluctuations decrease with increasing I.

dependence of the spectra. (We have been careful to ascertain that, at low frequencies, the fluctuations associated with the detector, the standard resistor, and the bias current supply are well below the level of fluctuations associated with the junction, typically by a factor of 10^2. For frequencies greater than 10 Hz, the spectra begin to be dominated by the Johnson noise produced in the standard resistor R_S.) Secondly, at any <u>single</u> frequency, the magnitude of the fluctuations decreases with increasing bias current I.

These observations are extremely important in delineating the mechanism by which large, low frequency fluctuations are produced in thin film Josephson junctions. We propose that the f^{-1} frequency dependence of the spectra shown in Figs. 2 and 3 arises from thermodynamic temperature fluctuations in the junction volume, which modulate the junction critical current I_c through the derivative dI_c/dT and hence modulate the voltage observed across the biased junction. This proposed mechanism is closely related to the fluctuation process responsible for "1/f" noise in the voltage spectrum of thin metal films biased at constant current.[4] The experimental analysis presented in Ref. 1 provides strong evidence that, for thin film geometries, the spectrum of temperature fluctuations within the film obeys a 1/f power law over wide ranges of frequency. For metal films, these temperature fluctuations are coupled to the experimentally accessible variable (voltage) by a mechanism involving temperature modulation of the film resistance through the derivative dR/dT. Hence $\delta V = I(dR/dT)\delta T$ and the fluctuation spectrum scales as I^2. As in the case of thin metal films, the frequency dependence and the numerical magnitude of the power spectra of Josephson junctions provide strong evidence for the thermal origin of the fluctuations. However, for Josephson junctions, the analogous mechanism which couples thermal fluctuations to the experimentally observed junction voltages cannot be associated with thermal modulation of a resistive element, since the amplitudes of the spectra do not scale as I^2 but rather <u>decrease</u> with increasing I, as shown in Fig. 3. The observed dependence of $V^2(f)$ upon the bias current I instead supports the assertion that, for Josephson junctions, the primary coupling mechanism involves modulation of the critical current $I_c(T)$, which in turn alters the junction voltage in accordance with the Stewart-McCumber[5] relation

$$\left(\frac{\partial V}{\partial I_c}\right)_I^2 = \frac{R^2}{(I/I_c)^2-1} . \tag{1}$$

At high bias currents, the junction is more nearly an ideal resistive element whose value is increasingly insensitive to thermally induced fluctuations in I_c, thus accounting for the observed decrease in noise power with increasing I.

We may apply these qualitative observations to the theory of diffusive fluctuations developed by Clarke and Voss[1] to obtain the semi-empirical formula

$$V^2(f) = \left(\frac{\partial V}{\partial I_c}\right)_I^2 \left(\frac{dI_c}{dT}\right)^2 k_B T^2/C_V G f \tag{2}$$

as a prediction for the experimental voltage spectrum of thin film Josephson tunnel junctions. Here, G is a geometric factor of roughly 3, and C_V is the heat capacity of the active volume of the junction, which we take to be the junction area times the coherence length. Using our measured values of (dI_c/dT), and computing $(\partial V/\partial I_c)_I$ from Eq. (1), we find that the predictions of Eq. (2) are in remarkably good agreement with the experimentally determined fluctuation spectra, as indicated by the dashed lines in Figs. 2 and 3. Both the frequency dependence of the spectra and the dependence upon bias current I are in quantitative accord with our theoretical expectations. The excellent agreement between theory and experiment regarding the <u>magnitude</u> of the fluctuations is especially satisfying, since normalization of the theoretical spectra relies on the fundamental thermodynamic relation $\langle \delta T^2 \rangle = k_B T^2/C_V$. Because of our imprecise knowledge of several of the parameters required to evaluate Eq. (2) (such as the effective junction volume), our theoretical estimates are probably no more accurate than an order of magnitude in $V^2(f)$, although the spectra of most of the samples we have studied differ from the predictions of Eq. (2) by less than a factor of 5. We note that, as $I \to I_c$, the increase in the voltage fluctuations is not as rapid as that suggested by the Stewart-McCumber formula, Eq. (1), for $(\partial V/\partial I_c)_I$, because the value of the standard resistor R_s is only about a factor of two greater than the shunt resistance of the junction. Hence, account must be taken of the current flow through the SQUID voltmeter, which adds a term $(R/R_S)^2$ to the denominator of Eq. (1). In the extreme case, $R_S \ll R$, all the bias current in excess of I_c flows through the SQUID signal coil, and the observed fluctuation spectra become independent of I for $I > I_c$.

We have attempted to test the temperature dependence of the predictions of Eq. (2) for the voltage spectra by varying the temperature of the bath surrounding the vacuum can from 4.2K to 1.5K. Experimentally, we find that for most junctions the change in the magnitude of the noise spectra is not large, typically less than a factor of 5 over our experimentally accessible temperature range. This behavior may be understood from Eq. (2) by noting that, while the terms $T^2(dI_c/dT)^2$ in the numerator decrease roughly two orders of magnitude from 4.2K to 2.0K, the specific heat C_V in the denominator of Eq. (2) decreases over this temperature range by a factor of nearly 25, resulting in relatively little change in the noise power. At sufficiently low temperatures, the spectral power due to thermal modulation of the critical current should vanish as $(dI_c/dT)^2/T$. It would be important to investigate this temperature regime to ascertain the possible existence of other sources of low frequency noise not associated with thermal diffusion.

Although most of our sample junctions did not display marked variations in noise power as a function of temperature, we did encounter one anomalous junction whose noise power varied by nearly three orders of magnitude over a temperature range of 0.2K. This behavior was associated with, and could be explained by, the highly unusual temperature dependence of the critical current observed for this sample and shown in Fig. 4. The pronounced dip in the I_c vs T characteristic, centered about 1.7K, was quite reproducible and persisted when the sample was recooled after being warmed above T_c or to room temperature. Although the origin of this behavior is far from certain, we conjecture that the abnormal temperature dependence of $I_c(T)$ for this sample can be interpreted in terms of the McMillan theory of the proximity effect.[6] During construction of the junction, the sputtering current was decreased slowly near the end of the Nb deposition. As a result, the last layers of Nb were deposited at a rate lower than that known to be required to produce Nb films which are superconducting above 1K. Thus the junction formed was of the type Nb-N-Ox-Pb, where N is a normal layer. McMillan predicts that such a junction should in fact exhibit a minimum in the I_c vs T characteristic.

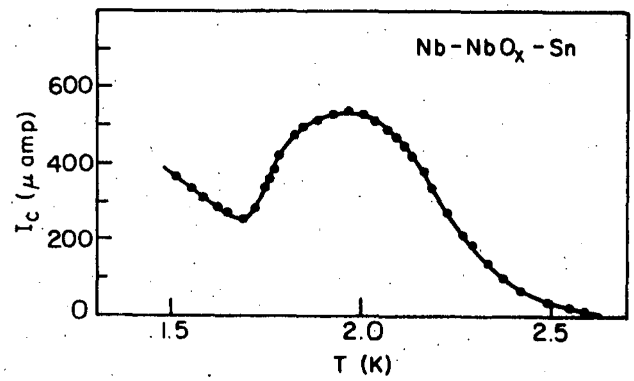

Fig. 4. Behavior of the junction critical current $I_c(T)$ for the anomalous sample discussed in the text.

Regardless of the source of the anomalous behavior, the regions for which $(dI_c/dT) \approx 0$ were of considerable use in our study of the thermal origin of 1/f noise in Josephson junctions. For this sample, we were able to alter the value of (dI_c/dT) from essentially zero at the local maximum of the I_c vs T characteristic to a relatively large value (2.2 mA/K) on the steepest edge of the dip by changing the temperature only 0.2K. Since this small temperature variation did not appreciably alter T^2 and C_V, we could directly observe the effect of the single term (dI_c/dT) in Eq. (2). As shown in Fig. 5, which compares the theoretical predictions of Eq. (2) with our experimental measurements of the fluctuation spectrum of the anomalous junction, the observed spectrum was indeed proportional to $(dI_c/dT)^2$ and in fact decreased by three orders of magnitude to a level below our amplifier noise at the temperature for which I_c vs T exhibited a local maximum. Clearly, it would be of importance to further investigate the temperature dependence of Eq. (2) by observing the fluctuations associated with other types of junctions, such as the SNS, for which the term (dI_c/dT) would dominate the temperature dependence of C_V.

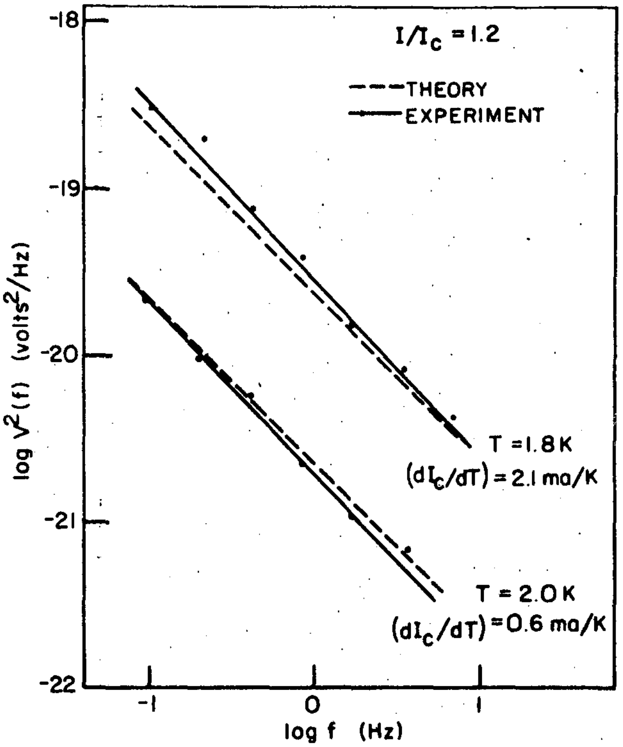

Fig. 5. Dependence of the fluctuation spectrum of the anomalous junction on the single parameter $(dI_c/dT)^2$.

Finally, we comment on two experimental complications associated with measurements of low frequency fluctuations. In general, to obtain reliable spectra, it is important that (1) the experimental probe used to observe the fluctuations not unduly perturb the system under investigation, and (2) that the system itself be in thermal equilibrium with a reservoir whose temperature does not fluctuate appreciably over the time scale of experimental interest. Regarding point (1), we have observed that, for $I \gtrsim 10$ mA, the effects of thermal heating produce additional noise and structure in the fluctuation spectra which cannot be described by a simple power law dependence upon frequency. It is not clear whether this additional noise is associated with thermal fluctuations in a sample far from equilibrium, or whether the distortion of the spectrum arises from thermal feedback, which introduces local heating in proportion to local temperature fluctuations. Regarding (2), we have investigated the effects of sample contact with a thermal reservoir which is in gross non-equilibrium; for example, with a He bath whose pressure is unregulated. For a wide variety of such non-equilibrium systems we find that the experimental spectra rise very rapidly at low frequencies, in approximate accordance with the power law $V^2(f) \propto f^{-2}$. For frequencies small compared to the reciprocal of the equilibration time of the sample and reservoir, we expect the voltage fluctuations across the junctions to mirror the temperature fluctuations of the reservoir. For frequencies higher than the frequencies at which the reservoir fluctuates, the observed spectra are consistent with that expected from equilibrium temperature fluctuations within the junctions themselves.

III. CONCLUSION

We have measured the low frequency voltage spectrum associated with the finite voltage state of thin film, oxide tunnel junctions and have identified the primary source of these fluctuations as modulation of the junction critical current by thermodynamic temperature fluctuations in the junction volume. This interpretation is in good numerical accord with our experimental results and correctly predicts the dependence of the spectrum on both frequency and junction bias current. Our results have important implications for such devices as SQUID voltmeters and magnetometers, since a lower limit to the long-term drift stability of systems employing Josephson junctions will be set by the behavior of the low frequency fluctuations inherent in the junctions themselves. We feel it should be possible to formulate practical design criteria, such as a minimization of $T^2(dI_c/dT)^2/C_V$, which could be used to achieve reduced levels of long-term drift in many systems relying on Josephson junctions to detect currents and magnetic fields.

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