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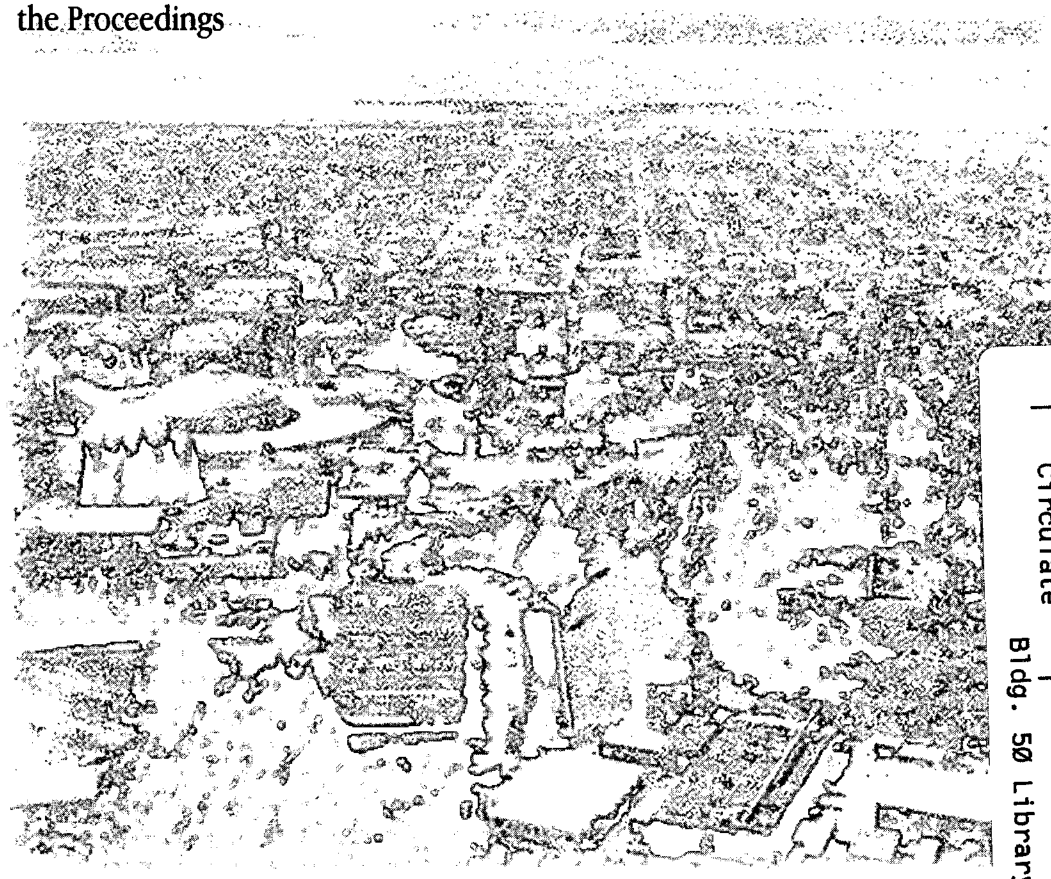

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# $(S_3)^3$ Theories of Flavor*

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**Abstract**

I present a supersymmetric theory of flavor based on the discrete flavor group $(S_3)^3$. The model can account for the masses and mixing angles of the standard model, while maintaining sufficient sfermion degeneracy to evade the supersymmetric flavor problem. I demonstrate that the model has a viable phenomenology and makes one very striking prediction: the nucleon decays predominantly to $Kl$ where $l$ is a *first* generation lepton. I show that the modes $n \to K^0\bar{\nu}_e$, $p \to K^+\bar{\nu}_e$, and $p \to K^0 e^+$ occur at comparable rates, and could well be discovered simultaneously at the SuperKamiokande experiment.

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# 1 Introduction

The origin of flavor has been a significant puzzle in particle physics since the discovery of the muon. The replication of fermion generations and the strongly hierarchical pattern of their masses and mixing angles is left unexplained in the Standard Model. In most theories of flavor, new symmetries are introduced at mass scales that are large compared to the electroweak scale. To stabilize this hierarchy of scales, it is natural to work in the framework of the supersymmetric standard model (SSM). However, this complicates the flavor problem by introducing a new sector of particles whose masses and mixing angles must also be understood. While no superpartner has yet been observed, the acceptable spectrum is constrained by low-energy processes. Most notably, a high degree of degeneracy is required among the light generation squarks to suppress dangerous flavor-changing effects [1], unless there is a strong alignment of quark and squark eigenstates [2, 3]. The challenge in a supersymmetric theory of flavor is to simultaneously explain both the suppression of flavor changing effects from the scalar sector and the hierarchical pattern of the quark Yukawa couplings.

In this talk, I will advocate imposing a discrete, gauged non-Abelian family symmetry on the SSM to obtain the desired degree of sfermion degeneracy [4]. This choice is reasonable given that global continuous symmetries may be broken by quantum gravitational effects [5], while gauged continuous symmetries may generate $D$-term contributions to the squark and slepton masses that are nonuniversal [6]. In Ref. [4], it was demonstrated that the non-Abelian discrete group $(S_3)^3$ is a promising choice for this flavor symmetry. The group $S_3$ has both a doublet ($\mathbf{2}$) and a non-trivial singlet representation ($\mathbf{1}_A$) into which the three generations of fermions can be embedded. In order to construct a viable model for the quarks, three separate $S_3$ factors are required, for the left-handed doublet fields $Q$, and the right-handed singlet fields $U$ and $D$. The first and second generation fields transform as doublets, which ensures the degeneracy among the light generation squarks. The third generation fields must then transform as $\mathbf{1}_A$s so that the theory is free of discrete gauge anomalies. While the group $S_3$ acts identically on three objects, the representation structure distinguishes between the generations. Thus, it is possible to choose the quantum numbers for the Higgs fields so that only the top Yukawa coupling is allowed in the symmetry limit. The hierarchical structure of Yukawa matrices can then be understood as a consequence of the sequential breaking of the flavor symmetry group.

After reviewing the basic scenario in the quark sector [4], I show how we may extend the $(S_3)^3$ model to the leptons [7]. This work was done in collaboration with Larry Hall and Hitoshi Murayama. I assume that the fundamental sources of flavor symmetry breaking are gauge singlet fields $\phi$ that transform in the same way as the irreducible "blocks" of the quark Yukawa matrices. I will call these fields 'flavons' below. I assume that the fermion Yukawa matrices arise from higher dimension operators involving the $\phi$ fields, that are present at the Planck scale. With flavor symmetry breaking originating only from the Yukawa matrices, I estimate the contributions to lepton flavor violation and proton decay. The latter originates from non-renormalizable operators that conserve $R$-parity, but violate baryon and lepton number, that are also presumably generated at the Planck scale. I show that the flavor symmetry is sufficient to suppress these operators to an acceptable level. In addition, I show that the dominant proton decay modes in the $(S_3)^3$ model are of the form $p \to Kl$, where $l$ is a *first* generation lepton. This is never the case in either supersymmetric or non-supersymmetric grand unified theories. The prediction of these rather unique modes is exciting since the total decay rate is likely to be within the reach of the SuperKamiokande experiment.

## 2 The Basic Model

In the $(S_3)^3$ model of Ref. [4], the quark chiral superfields $Q$, $U$, and $D$ are assigned to $\mathbf{2}+\mathbf{1_A}$ representations of $S_3^Q$, $S_3^U$ and $S_3^D$, respectively. The first two generation fields are embedded in the doublet, for the reasons described in the Introduction. The Higgs fields both transform as $(\mathbf{1}_A, \mathbf{1}_A, \mathbf{1}_S)$'s, so that the top quark Yukawa coupling is invariant under the flavor symmetry group. The transformation properties of the Yukawa matrices are:

$$Y_U \sim \left( \begin{array}{c|c} (\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S) & (\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S) \\ \hline (\mathbf{1}_S, \tilde{\mathbf{2}}, \mathbf{1}_S) & (\mathbf{1}_S, \mathbf{1}_S, \mathbf{1}_S) \end{array} \right) \tag{1}$$

$$Y_D \sim \left(\begin{array}{c|c} (\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2}) & (\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{1}_A) \\ \hline (\mathbf{1}_S, \mathbf{1}_A, \mathbf{2}) & (\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A) \end{array}\right) \tag{2}$$

where I use the notation $\tilde{\mathbf{2}} \equiv \mathbf{2} \otimes \mathbf{1}_A$,[‡]. Note that these matrices involve at most 7 irreducible multiplets of $(S_3)^3$. In Ref. [4], $(S_3)^3$ was broken by only four types of flavons: $\phi(\tilde{\mathbf{2}}, \mathbf{1}_S, \mathbf{1}_S)$, $\phi(\tilde{\mathbf{2}}, \tilde{\mathbf{2}}, \mathbf{1}_S)$, $\phi(\mathbf{1}_S, \mathbf{1}_A, \mathbf{1}_A)$, and $\phi(\tilde{\mathbf{2}}, \mathbf{1}_A, \mathbf{2})$, the minimal number which leads to realistic masses and mixings [8]:

$$Y_U = \left(\begin{array}{cc|c} h_u & h_c\lambda & -h_t V_{ub} \\ 0 & h_c & -h_t V_{cb} \\ \hline 0 & 0 & h_t \end{array}\right), \tag{3}$$

$$Y_D = \left(\begin{array}{cc|c} h_d & h_s\lambda & 0 \\ 0 & h_s & 0 \\ \hline 0 & 0 & h_b \end{array}\right), \tag{4}$$

with $\lambda \simeq 0.22$. The form of the Yukawa matrices presented above can be understood as a consequence of a sequential breaking of the flavor symmetry. I will assume that the $2 \times 2$ blocks of $Y_U$ and $Y_D$ are each generated by two flavon fields that acquire vevs at different stages of the symmetry breaking. Thus, $Y_D = Y_1 + Y_2$, and $Y_U = Y_1' + Y_2'$ where

$$Y_1 = \begin{pmatrix} 0 & a h_s \lambda \\ 0 & h_s \end{pmatrix}, \quad Y_2 = \begin{pmatrix} h_d & 0 \\ 0 & 0 \end{pmatrix}. \tag{5}$$

and

$$Y_1' = \begin{pmatrix} 0 & a' h_c \lambda \\ 0 & h_c \end{pmatrix}, \quad Y_2' = \begin{pmatrix} h_u & 0 \\ 0 & 0 \end{pmatrix}. \tag{6}$$

Note that $a$ and $a'$ are order one constants, with $a - a' = 1$.

# 3 Incorporating Lepton Sector

Three principles determine the precise transformation properties of the lepton fields:

1. There are no new flavor symmetries (e.g. new $S_3$ factors) that arise only in the lepton sector. The only flavor symmetry in the theory is $S_3^Q \times S_3^U \times S_3^D$.

[‡] $\tilde{\mathbf{2}} = (a, b)$ is equivalent to $\mathbf{2} = (b, -a)$.

2. The transformation properties of the lepton fields are chosen so that the charged lepton Yukawa matrix is similar to that of the down quarks.

3. The most dangerous dimension-five operator that contributes to proton decay, $(QQ)(QL)$, is forbidden in the $(S_3)^3$ symmetry limit.

As we will see below, these principles are sufficient to completely determine the transformation properties of the lepton fields.

Let us first consider the consequences of the first two conditions. The down-quark Yukawa matrix is a coupling between the left-handed quark fields $Q \sim (\mathbf{1}_A + \mathbf{2}, \mathbf{1}_S, \mathbf{1}_S)$ and the right-handed down quark fields $D \sim (\mathbf{1}_S, \mathbf{1}_S, \mathbf{1}_A + \mathbf{2})$. We know that the Yukawa matrix of the charged leptons is quite similar to that of the down quarks, up to factors of order three [9] at high scales:

$$m_b \simeq m_\tau, \qquad m_s \simeq \frac{1}{3} m_\mu, \qquad m_d \simeq 3 m_e. \tag{7}$$

Therefore, we look for an assignment of lepton transformation properties that leads automatically to this observed similarity. There are only two possibilities:

$$\begin{array}{c|cc} & S_3^Q & S_3^D \\ \hline L & \mathbf{1}_A + \mathbf{2} & \mathbf{1}_S \\ E & \mathbf{1}_S & \mathbf{1}_A + \mathbf{2} \end{array} \quad \text{or} \quad \begin{array}{cc} S_3^Q & S_3^D \\ \hline \mathbf{1}_S & \mathbf{1}_A + \mathbf{2} \\ \mathbf{1}_A + \mathbf{2} & \mathbf{1}_S \end{array} \tag{8}$$

The third condition above allows us to distinguish between these two alternatives. In the first assignment, the operator $(Q_i Q_i)(Q_j L_j)$ is allowed by the $(S_3)^3$ symmetry, and we have proton decay at an unacceptable rate. Therefore, only the second assignment in Eq. (8) satisfies all three criteria listed above.

The remaining question that we need to answer is how the factors of three in Eq. (7) enter in the Yukawa matrices. One plausible explanation is that they originate from fluctuations in the order one coefficients that multiply the $(S_3)^3$ breaking parameters. Thus, we assume $Y_D = Y_1 + Y_2$, while $Y_l = 3Y_1 + \frac{1}{3}Y_2$.

### 3.1 Lepton Flavor Violation

The strongest constraint on lepton flavor violation comes from the non-observation of the $\mu \to e\gamma$ decay mode. In our model, the contribution of the off-diagonal term in the purely left-handed slepton mass matrix (the LL matrix) is small enough ($\sim h_s^2\lambda \sim 1\times 10^{-7}$) to avoid the experimental constraint for any value of $m_{\tilde{l}}$ above the LEP bound. The stringent limits come from the purely right-handed slepton mass matrix (RR) and the left-right (LR) matrix, which we discuss in this section. For simplicity, we work in the approximation where the exchanged neutralino is a pure bino state.

The one-loop slepton and bino exchange diagram that picks up the off-diagonal (2,1) component in LR mass matrix generates the operator

$$\frac{e}{2}F_2(M_1^2,(m^2_{LR})_{21},m_R^2,m_L^2)\ \bar{e}_R i\sigma^{\mu\nu}\mu_L F_{\mu\nu}\,, \tag{9}$$

where $F_{\mu\nu}$ is the electromagnetic field strength and the function $F_2$ is defined in Ref. [7].

The decay width is given by

$$\Gamma(\mu\to e\gamma)=\frac{\alpha}{4}m_\mu^3|F_2|^2\,, \tag{10}$$

and the bound $\mathrm{Br}(\mu\to e\gamma)<4.9\times 10^{-11}$ implies $|F_2|<2.6\times 10^{-12}\ \mathrm{GeV}^{-1}$. In order to compare this bound to the prediction of our model, let us take $m_R=m_L=m=300$ GeV and $M_1=100$ GeV as a representative case. We obtain

$$\frac{(m^2_{LR})_{21}}{m^2}<1.0\times 10^{-5} \tag{11}$$

for this choice of parameters. In our model, the (2,2) and (1,2) elements in $Y_l$ belong to the same irreducible multiplet, and diagonalization of $Y_l$ also diagonalizes LR mass matrix at $O(h_s\lambda)$. The term which may not be simultaneously diagonalizable comes from the piece $Y_2\sim h_d$, and hence

$$(m^2_{LR})_{21}\sim m_d\lambda A, \tag{12}$$

where $m_d$ is the down quark mass evaluated at the Planck scale $m_d\simeq$ 10 MeV/3, and $A$ is a typical trilinear coupling. If we take $A\sim 100$ GeV, then $(m^2_{LR})_{21}/m^2\sim 0.8\times 10^{-6}$ and the constraint (11) is easily satisfied. The (1,2) element in the LR mass matrix contributes in exactly the same way as the (2,1) entry, except that the chiralities of the electron and muon in Eq. (9) are flipped. Hence, the (1,2) element is subject to the same constraint, which again is clearly satisfied in our model.

The RR mass matrix also contributes to the operator in (9). For the bino and slepton masses chosen earlier, we obtain the bound

$$\frac{(m^2_{RR})_{12}}{m^2} < 0.023, \tag{13}$$

while in our model

$$(m^2_{RR})_{12}/m^2 \simeq h_t V_{cb} \lambda \sim 0.009 \; . \tag{14}$$

Thus, the bound on the (1,2) element of the RR matrix also satisfied. Note that the contributions to $\mu \to e\gamma$ involving mixing to the third generation scalars are comparable to those in the minimal SO(10) model [10], and hence they are phenomenologically safe.

### 3.2 Proton Decay

Since we have assumed that all possible nonrenormalizable operators are generated at the Planck scale, the task of studying proton decay in our model is a simple one. We first write down all the possible dimension-five operators that contribute to proton decay and identify their transformation properties under $(S_3)^3$. The coefficients can be estimated as the product of Yukawa couplings that will produce the desired symmetry breaking effect. A list of possible operators and their coefficients is given in Ref. [7].

The leading contribution to proton decay comes from the operator

$$(Q_i Q_i)(Q_i L_i)/M_*,$$

where $M_* \equiv M_{Pl}/\sqrt{8\pi}$ is the reduced Planck mass, and where parentheses indicate a contraction of SU(2) indices. This operator transforms as a $(\mathbf{2}, \mathbf{1}_S, \mathbf{2})$ under $(S_3)^3$, and therefore has a coefficient of order $h_b h_s$. The coefficients for all four components of this operator are given by

$$\frac{c}{2}\frac{h_b}{M_*}[h_s(Q_2Q_2)(Q_1L_1) - h_s\lambda(Q_1Q_1)(Q_2L_1)$$
$$- \mathcal{O}(h_d)(Q_2Q_2)(Q_1L_2) + h_d(Q_1Q_1)(Q_2L_2)] \tag{15}$$

where $c$ is an unknown coefficient of $\mathcal{O}(1)$. The striking feature of this multiplet is that the operators involving first generation lepton fields $L_1$ have larger coefficients than those involving second generation fields $L_2$. Thus, our model favors proton decay to $\nu_e$ and $e$ over decay to $\nu_\mu$ and $\mu$. This result is in striking contrast to the situation in grand unified theories, where the amplitude is proportional to the Yukawa coupling of the final-state lepton. In our case, however, there is a residual $Z_2$ flavor symmetry in the limit where the first generation Yukawa couplings are set to zero. Under this symmetry, the $U$, $D$, and $L$ fields of the second and third generation are odd, while all other matter fields are even. Thus, when the first generation Yukawa couplings are set to zero, dimension-five operators of the form $QQQL$ containing $L_2$ or $L_3$ are forbidden by this $Z_2$, while those involving $L_1$ are invariant.

The predicted nucleon decay modes are obtained from Eq. (15) by 'dressing' the two-scalar-two-fermion operators with wino exchange. We obtain[§]

$$\begin{aligned} \mathcal{L} &= \frac{\alpha_W}{2\pi}\frac{c h_b h_s}{M_*}\left[-\lambda(su)(d\nu_e) + \lambda(su)(ue)\right] \\ &\times \ (f(c,e) + f(c,d)) \ , \end{aligned} \tag{16}$$

where parentheses now indicate the contraction of spinor indices. All the fields in (16) are in the mass eigenstate basis, and terms of higher orders in $\lambda$ have been neglected. The function $f$ is the "triangle diagram factor" [12], a function of the wino and scalar masses; above we have used the fact $f(c,d) = f(u,d)$ to good accuracy. The ratios of decay widths can be estimated using the chiral Lagrangian technique [13, 14, 11]. We find

$$\begin{aligned} &\Gamma(p \to K^+\bar{\nu}_e) : \Gamma(p \to K^0 e^+) : \Gamma(n \to K^0\bar{\nu}_e) \\ &= 0.4 : 1 : 2.7 \ . \end{aligned} \tag{17}$$

Note that the proton's charged lepton decay mode dominates over the neutrino mode. This is a consequence of the cancellation of the operators $(du)(s\nu_e)$ between the first two terms of Eq. (15). The dominance of $p \to K^0 e^+$ over $p \to K^+\bar{\nu}_e$ is rarely the case in SUSY-GUTs.

[§]In the following discussion, we assume that the Cabibbo mixing originates from the down sector, i.e. $a = 1$, $a' = 0$ in eq. (5) and (6). However we checked that all the results remain the same even when the Cabibbo mixing comes from both the down and the up sectors, or even solely from the up sector.

Finally, we come to the overall rate. We find

$$\tau(n \to K^0\bar{\nu}_e) = \frac{4\times 10^{31}\ \mathrm{yrs}}{c^2}\left(\frac{0.003\ \mathrm{GeV}^3}{\xi}\right.$$

$$\left.\frac{0.81}{A_S}\frac{5}{(1+\tan^2\beta)}\frac{\mathrm{TeV}^{-1}}{f(c,e)+f(c,d)}\right)^2 . \tag{18}$$

This result includes the effect of running the dimension-five operator between the Planck scale and $m_Z$, (a factor of $A_S = 0.81$ in the amplitude if $m_t = 175$ GeV, $\tan\beta = 2$, $\alpha_s(m_Z) = 0.12$) and between $m_Z$ and $m_n$ (a factor of 0.22 in the amplitude). In the expression above, $\xi$ is the hadronic matrix element of the four-fermion operator evaluated between nucleon and kaon states; its exact value is rather uncertain, but is estimated to be within the range $\xi = 0.003$–$0.03$ GeV$^3$. If we take that $M_2 \sim 100$ GeV and $m_{\tilde{q}} \sim 700$ GeV, and $m_{\tilde{l}} \sim 300$ GeV, then the triangle functions $f(c,e)+f(c,d) \sim (1.8\ \mathrm{TeV})^{-1}$. Thus, if $c = 1$ and $\xi = 0.003$ GeV$^3$, we obtain a mean lifetime $12.7 \times 10^{31}$ years, which can be compared to the experimental bound, $\tau(n \to K^0\bar{\nu}_e) > 8.6 \times 10^{31}$ years. It is interesting to note that the coefficient $4 \times 10^{31}$ in (18) would be the same in the minimal SU(5) SUSY-GUT with an extremely large color-triplet Higgs mass $M_{H_C} = 10^{17}$ GeV. Thus, the rate in our model is roughly comparable. Overall, the $(S_3)^3$ symmetry gives us just enough suppression of dimension-five operators to evade the current bounds, so the model is phenomenologically viable. Since the SuperKamiokande experiment is expected to extend Kamiokande's current reach by another factor or 30, there is a very good chance that the $n \to K^0\bar{\nu}_e$ mode may be seen. It is an exciting prediction of this model that the $p \to K^0e^+$ and $K^+\bar{\nu}_e$ modes are likely to be seen at the same time because their rates are close to each other, as we saw in eq. (17).

# 4 Conclusions

I have shown that the discrete flavor group $(S_3)^3$ can account for the masses and mixing angles of the standard model while simultaneously solving the supersymmetric flavor changing problem. The most striking prediction that emerged from the analysis is the dominance of proton decay to final states involving first generation lepton fields, unlike the case in SUSY GUTs. I showed that the ratios of decay widths for the largest modes $n \to K^0\bar{\nu}_e$, $p \to K^+\bar{\nu}_e$, and $p \to K^0e^+$ are approximately 0.4 :: 1 :: 2.7. Given the estimate of the total rate, I pointed out that all three modes may be within the reach of the SuperKamiokande experiment and could well be discovered simultaneously.

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