Title

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Journal

Physical Review Letters, 110(26)

ISSN

0031-9007

Authors

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Publication Date

2013-06-28

DOI

10.1103/physrevlett.110.265006

Gyrokinetic Theory of Turbulent Acceleration of Parallel Rotation in Tokamak Plasmas

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(Received 19 March 2013; published 27 June 2013)

A mechanism for turbulent acceleration of parallel rotation is discovered using gyrokinetic theory. This new turbulent acceleration term cannot be written as a divergence of parallel Reynolds stress. Therefore, turbulent acceleration acts as a local source or sink of parallel rotation. The physics of turbulent acceleration is intrinsically different from the Reynolds stress. For symmetry breaking by positive intensity gradient, a positive turbulent acceleration, i.e., cocurrent rotation, is predicted. The turbulent acceleration is independent of mean rotation and mean rotation gradient, and so constitutes a new candidate for the origin of spontaneous rotation. A quasilinear estimate for ion temperature gradient turbulence shows that the turbulent acceleration of parallel rotation is explicitly linked to the ion temperature gradient scale length and temperature ratio T_{i0}/T_{e0}. Methods for testing the effects of turbulent parallel acceleration by gyrokinetic simulation and experiment are proposed.

DOI: 10.1103/PhysRevLett.110.265006 PACS numbers: 52.55.−s, 52.30.Gz, 52.35.Ra

Problems of spontaneous flow generation, spin-up, and formation of differential rotation are ubiquitous in physics. Examples of these include, but are not limited to, the origin of the solar differential rotation [1], the formation of the atmospheric jet stream [2], the mechanism of spin up of a stratified fluid in a container [3], the formation of the solar tachocline [4], and the origin of intrinsic rotation in tokamak plasmas [5]. Many instances of spontaneous flow generation occur in a state of eddy or wave turbulence, and thus qualify as problems in the self-organization of flow patterns in turbulence [6]. This type of problem has considerable overlap with the classic paradigm of the turbulent magnetic dynamo [7]. Theoretical approaches to the question of flow self-organization are usually based on mean field theory methods. Examples include the anisotropic kinetic alpha effect [8] and the closely related lambda effect [9], both derived from mean field hydrodynamics.

Spontaneous (or, intrinsic) plasma rotation is an example of a spin-up process and is of great interest in magnetic fusion [10]. Plasma rotation is thought to play an important role in stabilizing macroscopic magnetohydrodynamic instabilities, such as resistive wall modes [11], and in reducing or regulating microturbulence and the associated losses. Intrinsic rotation is particularly important for International Thermonuclear Experimental Reactor, which cannot be adequately penetrated by conventional neutral beam injection, and so cannot achieve sufficient neutral beam injection driven rotation. Realization of this has driven intensive research in spontaneous rotation in the magnetic fusion energy community in recent years [12,13]. The associated theoretical research has also focused on mean field approaches to calculating the parallel rotation profile by Reynolds stress modeling, specialized to the complex geometry of tokamak plasmas [14]. Interestingly, just as the solar differential rotation is thought to arise from heat flux driven convective turbulence in a rotating system, spontaneous tokamak rotation is thought to arise from heat flux driven drift wave turbulence in a helical magnetic field [15].

In this Letter, we propose a new mechanism for the origin of spontaneous rotation in tokamaks. This mechanism is turbulent acceleration, and arises from the partially acoustic character of drift-ion temperature gradient turbulence. This mechanism does not arise from a Reynolds stress or from momentum transport, and thus has no antecedent in previous work on the mean field theory of rotation or flow generation.

Momentum transport can influence plasma rotation. The general parallel momentum transport equation per ion mass can be written as

$$\frac{\partial \langle n U_\parallel \rangle}{\partial t} + \nabla \cdot \Gamma_{r,\parallel} = M_\parallel, \tag{1}$$

where $\Gamma_{r,\parallel}$ is the radial flux of parallel momentum, and $M_\parallel$ is the turbulent momentum source or sink. The basic form of the momentum flux is given by $\Gamma_{r,\parallel} = \langle n \rangle \langle \tilde{v}_r \tilde{U}_\parallel \rangle + \langle \tilde{v}_r \tilde{n} \rangle \langle U_\parallel \rangle + \langle \tilde{v}_r \tilde{n} \tilde{U}_\parallel \rangle$ [16]. Here, the parallel Reynolds stress is $\Pi_{r,\parallel} = \langle \tilde{v}_r \tilde{U}_\parallel \rangle$, including diffusion, velocity pinch [17], and residual stress $\Pi^{\rm res}_{r,\parallel}$, which has been intensively investigated [14]. In particular, the residual stress is one component of the parallel Reynolds stress which is not proportional to either flow or flow gradient. It is thought to be the origin of intrinsic torque, since it contributes a term which is proportional to $\nabla \cdot \Pi^{\rm res}_{r,\parallel}$ to the parallel mean flow equation. This term contributes an intrinsic torque $-\nabla \cdot \Pi^{\rm res}_{r,\parallel}$, which can accelerate the plasma.

Although the most natural quantity for theoretical study is toroidal angular momentum density, the quantity measured and estimated from experimental observation is the toroidal ion velocity U_ϕ. The magnitude of U_ϕ can be approximated by $U_\parallel$ for tokamaks, since the toroidal field is much lager than the poloidal field. To explicitly link theoretical results to experimental observations, it is more convenient to investigate the evolution equation of parallel velocity rather than that of parallel momentum density. In general, the mean parallel velocity evolution equation can be written as

$$\frac{\partial \langle U_\parallel \rangle}{\partial t} + \nabla \cdot \Pi_{r,\parallel} = a_\parallel, \tag{2}$$

where $\Pi_{r,\parallel}$ is the parallel Reynolds stress, and $a_\parallel$ is the turbulent acceleration. In this Letter, we identify a turbulent acceleration term which cannot be written as a divergence of a Reynolds stress. It enters the rhs of the parallel rotation equation, and follows from gyrokinetic theory. The turbulent acceleration significantly affects parallel rotation, but its physics is fundamentally different from that of the residual stress. The residual stress is one component of the parallel Reynolds stress, so it enters into the rotation equation via its divergence, while turbulent acceleration is a local source or sink. We note that this difference is somewhat analogous to the difference between the turbulent energy pinch [18] (one component of the heat flux) and turbulent heating [19]. Just as the pinch and the turbulent heating, the turbulent acceleration and the turbulent residual stress also coexist and both are relevant to spontaneous parallel rotation. Therefore, investigation of the parallel turbulent acceleration is meaningful as well as potentially important.

We start from the nonlinear electrostatic gyrokinetic equation, in the continuity form [20]

$$\frac{\partial}{\partial t}(FB^*) + \nabla \cdot \left(\frac{d\mathbf{R}}{dt} FB^*\right) + \frac{\partial}{\partial v_\parallel}\left(\frac{dv_\parallel}{dt} FB^*\right) = 0, \tag{3}$$

for which gyrocenter equations of motion are

$$\frac{d\mathbf{R}}{dt} = v_\parallel \hat{\mathbf{b}} + \frac{c}{eB^*}\hat{\mathbf{b}} \times \left(e\nabla\langle\langle\delta\phi\rangle\rangle + \mu\nabla B + m_i v_\parallel^2 \hat{\mathbf{b}} \cdot \nabla\hat{\mathbf{b}}\right), \tag{4}$$

and

$$\frac{dv_\parallel}{dt} = -\frac{\mathbf{B}^*}{m_i B^*} \cdot (e\nabla\langle\langle\delta\phi\rangle\rangle + \mu\nabla B). \tag{5}$$

Here, $F = F(\mathbf{R}, \mu, v_\parallel, t)$ is the gyrocenter distribution function, μ is the gyrocenter magnetic moment, $\mathbf{B}^* = \mathbf{B} + v_\parallel \nabla \times \hat{\mathbf{b}}$, $B^* = \hat{\mathbf{b}} \cdot \mathbf{B}^*$ is the Jacobian of the transformation from the particle phase space to the gyrocenter phase space, and $\langle\langle\cdots\rangle\rangle$ denotes gyroaveraging.

By taking the moments of the nonlinear gyrokinetic equation, we obtain the equation for gyrocenter density, $n \equiv (2\pi/m_i)\int d\mu dv_\parallel FB^*$,

$$\frac{\partial}{\partial t} n + \nabla \cdot [(U_\parallel \hat{\mathbf{b}} + \mathbf{v}_{E\times B} + \mathbf{v}_{d\kappa} + \mathbf{v}_{d\nabla})n] = 0, \tag{6}$$

and the equation for gyrocenter parallel momentum per ion mass, $nU_\parallel \equiv (2\pi/m_i)\int d\mu dv_\parallel FB^* v_\parallel$,

$$\frac{\partial}{\partial t}(nU_\parallel) + \nabla \cdot \left[\frac{P_i}{m_i}\hat{\mathbf{b}} + (\mathbf{v}_{E\times B} + 3\mathbf{v}_{d\kappa} + \mathbf{v}_{d\nabla})nU_\parallel\right] = -\left[\frac{e}{m_i}\hat{\mathbf{b}} \cdot \nabla\delta\phi + \frac{c}{B}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla\hat{\mathbf{b}}) \cdot \nabla\delta\phi U_\parallel\right]n. \tag{7}$$

Here, a long wavelength approximation $k_\perp^2 \rho_i^2 \ll 1$ is used, $P_i = 2\pi\int d\mu dv_\parallel FB^*(v_\parallel - U_\parallel)^2 = (2\pi/m_i) \times \int d\mu dv_\parallel FB^* \mu B$ is the ion pressure, $\mathbf{v}_{E\times B} = c\hat{\mathbf{b}} \times \nabla\delta\phi/B$ is the fluctuating $\mathbf{E} \times \mathbf{B}$ drift velocity, $\mathbf{v}_{d\kappa} = cT_i/(eB)\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla\hat{\mathbf{b}})$ is the magnetic curvature drift velocity, and $\mathbf{v}_{d\nabla} = cT_i/(eB^2)\hat{\mathbf{b}} \times \nabla B$ is the magnetic gradient drift velocity. By summing Eq. (7) over all species and using the quasineutrality equation, a total momentum conservation equation can be obtained. We do not present it in this Letter, since gyrokinetic momentum conservation has already been discussed in detail in several recent works [21,22]. On one hand, the off-diagonal component of the electric part of the Maxwell stress tensor, $\Pi_{r,\parallel}^E \propto \tilde{E}_r \tilde{E}_\parallel$, coming from the polarization density [21], can easily be recovered. It plays a similar role to the usual residual stress, which also enters the total momentum equation, via its divergence. On the other hand, the time variation of toroidal momentum density due to $\mathbf{E} \times \mathbf{B}$ drifts [22] should be absent here, because the parallel component of $\mathbf{E} \times \mathbf{B}$ vanishes. The terms on the rhs of Eq. (7) are consistent with the turbulent toroidal—rather than parallel—momentum source in Ref. [23]. This comes from the parallel electric field, along with the effective magnetic field $\mathbf{B}^*/B^*$. It should be noted that spontaneous rotation does not contradict momentum conservation. The first is mainly carried by ions, while the second follows from the sum over the momenta of all species and the field momentum.

To obtain a more experimentally relevant quantity, the focus of this work is parallel ion rotation velocity, but not parallel momentum conservation or the ion parallel momentum. We subtract Eq. (6) from Eq. (7), and so obtain the ion parallel flow velocity equation

$$\frac{\partial}{\partial t}U_\parallel + \nabla \cdot [(\mathbf{v}_{E\times B} + 4\mathbf{v}_{d\nabla})U_\parallel] = -\left[2\mathbf{v}_{d\nabla} \cdot \frac{\nabla n}{n} - \frac{e}{T_i}\mathbf{v}_{d\nabla} \cdot \nabla\delta\phi - 2\mathbf{v}_{d\nabla} \cdot \frac{\nabla T_i}{T_i}\right]U_\parallel - \frac{1}{m_i}\hat{\mathbf{b}} \cdot \left(e\nabla\delta\phi + \frac{1}{n}\nabla P_i\right). \tag{8}$$

In low-β plasmas, $\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \nabla\hat{\mathbf{b}}) \simeq (1/B)\hat{\mathbf{b}} \times \nabla B$, so the magnetic curvature drift can be approximated as the magnetic gradient drift, i.e., $\mathbf{v}_{d\kappa} \simeq \mathbf{v}_{d\nabla}$. Note that the drift velocities are compressible in toroidal geometry. $\nabla \cdot \mathbf{v}_{E\times B} \simeq 2(e/T_i)\mathbf{v}_{d\nabla} \cdot \nabla\delta\phi$ and $\nabla \cdot \mathbf{v}_{d\nabla} = \mathbf{v}_{d\nabla} \cdot (\nabla T_i)/T_i$ are used when deriving the preceding equation. The parallel (toroidal) velocity evolution equation was also studied in [24,25] from reduced magnetohydrodynamic equations and Braginski fluid equations, respectively. However, ion pressure gradient along the toroidal direction was ignored in [25]. Both these works focused on magnetic geometric effects on rotation, which is different from the emphasis of our work. In particular, the key effect of our study is not a toroidal effect, but rather is related to parallel pressure gradients (i.e., acoustics).

The mean parallel velocity equation can be derived by taking a flux surface average of Eq. (8), i.e.,

$$\frac{\partial}{\partial t}\langle U_\parallel\rangle + \nabla\cdot\Pi_{r,\parallel} = a_\parallel. \tag{9}$$

As mentioned before, $\Pi_{r,\parallel}$ is the parallel Reynolds stress, which has been intensively studied in previous work. In this Letter, we focus on the rhs of Eq. (9), $a_\parallel$, the parallel turbulent acceleration. It can be written as

$$a_\parallel = \frac{1}{m_i n_0}\langle\delta n\hat{\mathbf{b}}\cdot\nabla\delta T_i\rangle - 2\left\langle\frac{\delta T_i}{T_i}\mathbf{v}_{d\nabla}\cdot\nabla\frac{\delta n}{n_0}\right\rangle\langle U_\parallel\rangle + \left\langle\delta U_\parallel\mathbf{v}_{d\nabla}\cdot\nabla\left(\frac{e\delta\phi}{T_i} - 2\frac{\delta n}{n_0} - 2\frac{\delta T_i}{T_i}\right)\right\rangle. \tag{10}$$

Note that this turbulent acceleration term cannot be written as a divergence of a parallel Reynolds stress. It plays the role of local source or sink of parallel rotation, and so is significant for parallel rotation. In particular, the first term in the turbulent acceleration is related to gyrocenter density fluctuations and ion temperature fluctuations, but is independent of the parallel velocity. Therefore, it can provide a net drive for spontaneous parallel rotation without any external momentum input. However, the physics is fundamentally different from the intrinsic torque induced by residual stress which enters the parallel rotation equation via the term $\sim\nabla\cdot\Pi_{r,\parallel}^{\rm res}$. We also note that the first term results from the parallel ion pressure gradient, and so is related to ion acoustic dynamics. The origin of this turbulent acceleration is different from that of the turbulent momentum source in Ref. [23], which resulted from the toroidal electric field. The first term always exists, whether the geometry is toroidal or cylindrical. This is different from the toroidal effects discussed in [24,25]. However, the other two terms come from the correction to the parallel direction along the background magnetic field, due to toroidal geometry effects. These are subdominant to the first term. Thus, we focus only on the ion acoustic related turbulent acceleration in the following discussion.

We present a quasilinear estimation of the parallel acceleration, $a_\parallel \approx v_{\rm thi}^2\langle\delta\hat{n}\hat{\mathbf{b}}\cdot\nabla\delta\hat{T}_i\rangle$, where $v_{\rm thi} = \sqrt{T_{i0}/m_i}$ is the ion thermal velocity, $\delta\hat{n} = \delta n/n_0$, and $\delta\hat{T}_i = \delta T_i/T_{i0}$. For simplicity, an adiabatic electron response is assumed, i.e., $\delta\hat{n} = e\delta\phi/T_{e0} = \tau\delta\hat{\phi}$, with $\tau = T_{i0}/T_{e0}$ and $\delta\hat{\phi} = e\delta\phi/T_{i0}$. Therefore, the temperature ratio dependence will be introduced in the quasilinear expression of the turbulent acceleration. The ion temperature fluctuations can be obtained by linearizing the ion temperature evolution equation [16] as follows:

$$-i\left(\omega_k - \frac{14}{3}\omega_{di,k} + i|\triangle\omega_k|\right)\delta\hat{T}_i = i\omega_{*Ti}\delta\hat{\phi}_k - i\frac{4}{3}\omega_{di,k}(\delta\hat{n}_k + \delta\hat{\phi}_k), \tag{11}$$

where $\omega_{*Ti} = -k_\theta\rho_i v_{\rm thi}/L_{Ti}$ is the ion diamagnetic drift frequency, with $L_{Ti} = -(\partial\ln T_i/\partial r)^{-1}$ as the ion temperature gradient scale length, $\omega_{di} \approx -k_\theta\rho_i v_{\rm thi}/R_0$ as the ion magnetic drift frequency, $\triangle\omega_k$ as the $\mathbf{E}\times\mathbf{B}$ nonlinearity-induced self-decorrelation rate, and the absolute value of $\triangle\omega_k$ is required by causality. Combining the density fluctuations and the ion temperature fluctuations, the turbulent acceleration can be written as

$$a_\parallel \approx \tau v_{\rm thi}^3\frac{\rho_i}{L_{Ti}}\sum_k(\Re\tau_{ck})\left[1 - \frac{4}{3}(1+\tau)\frac{L_{Ti}}{R_0}\right]k_\parallel k_\theta\langle|\delta\hat{\phi}_k|^2\rangle, \tag{12}$$

where $\tau_{ck} = [-i(\omega_k - (14/3)\omega_{di,k} + i|\triangle\omega_k|)]^{-1}$ is inverse of the ion propagator, and $\Re$ means real part. Since L_{Ti} is small in comparison with the major radius R_0, the second term in Eq. (12) is subdominant. In the following, we keep only the first term, which comes from the ion diamagnetic drift.

Note that nonzero turbulent acceleration also requires parallel symmetry breaking as does the residual stress in the parallel Reynolds stress. Various cases of parallel symmetry breaking mechanisms for the residual stress, such as $\mathbf{E}\times\mathbf{B}$ shear [26], intensity gradient [27], etc., have been studied. In toroidal geometry, $k_\theta = m/r$ and $k_\parallel = k_\theta x\hat{s}/(qR_0)$, where $\hat{s}$ is the magnetic shear, $x = r_{m,n} - r$, and $r_{m,n}$ is the radial location of the resonant surface. Proceeding as in the study of the residual stress caused by intensity gradient, i.e., $I_k(x) = |\delta\hat{\phi}_k|^2(x) = I_k(0) + x(\partial I_k/\partial x)$ [27], it follows that the turbulent acceleration can be written as

$$a_\parallel \approx \tau v_{\rm thi}^3\frac{\rho_i}{L_{Ti}}\frac{\hat{s}}{qR_0}\sum_k(\Re\tau_{ck})k_\theta^2 x^2\frac{\partial I_k}{\partial x}. \tag{13}$$

For $\hat{s} > 0$, a positive (negative) intensity gradient results in a positive (negative) parallel turbulent acceleration, and a cocurrent (countercurrent) rotation is thus driven by this effect.

To elucidate the magnitude of the turbulent acceleration, we compare it to the divergence of the residual stress $\nabla\cdot\Pi_\parallel^{\rm res}$ and to the divergence of the diffusive velocity flux $\nabla\cdot(\chi_\parallel\nabla\langle U_\parallel\rangle)$. We claim there are the appropriate comparisons, as the effects are all dimensionally similar, and scale as the rate of change of velocity. For symmetry breaking by intensity gradient, the residual stress is given by $\Pi_\parallel^{\rm res} = v_{\rm thi}^3\frac{\hat{s}}{qR_0}\sum_k(\Re\tau_{ck})k_\theta^2\rho_i x^2\frac{\partial I_k}{\partial x}$ [27]. Thus, the ratio of the turbulent acceleration to the divergence of the residual stress is

$$a_\| / \nabla \cdot \Pi_\|^{\rm res} \sim \tau L / L_{T_i}. \quad (14)$$

Here, L is the length scale of variation of the residual stress, which can vary between L_{T_i} and L_I, with $L_I = ((\partial I_k/\partial x)/I_k)^{-1}$ being the intensity gradient scale length. We see that the two contributions to the intrinsic torque are roughly comparable, depending upon τ and L/L_{T_i}. Comparing $a_\|$ to the turbulent diffusive velocity decay rate $\chi_\| \langle U_\| \rangle / L_v^2$ (corresponding to diffusive velocity confinement rate $\langle U_\| \rangle / \tau_\|$, where $1/\tau_\| \sim \chi_\| / L_v^2$) gives

$$\frac{a_\| L_v^2}{\chi_\| \langle U_\| \rangle} = \frac{L_v^2 \triangle^2}{\rho_i L_{T_i} L_s L_I} \tau \frac{v_{\rm thi}}{\langle U_\| \rangle}. \quad (15)$$

Here, L_v is the scale length of the parallel flow gradient, L_s is the magnetic shear scale length, and $\chi_\| = \sum_k (\Re \tau_{ck}) k_\theta^2 \rho_i^2 v_{\rm thi}^2 I_k$ is the turbulent diffusivity of parallel flow. For the particular case $v_{\rm thi}/\langle U_\| \rangle \sim a/\rho_i$ and $L_v \sim L_{T_i} \sim a$, Eq. (15) reduces to $(a_\| L_v^2)/(\chi_\| \langle U_\| \rangle) \sim \tau(\triangle^2/L_s L_I)(a^2/\rho_i^2)$. This ratio is roughly an order of unity. We have shown that the turbulent acceleration is qualitatively different from, but is quantitatively comparable to, the divergence of the residual stress and the divergence of the diffusive flux. Therefore, it is necessary to include the turbulent acceleration for the study of parallel rotation.

One important question concerning turbulent acceleration is how the theory can be tested by numerical simulations. The crux of this issue is that while the total local intrinsic torque density τ_I can be measured directly [28]—by numerical cancellation experiments [29] or other means—it is not so clear how to distinguish residual stress and acceleration contributions, since $\tau_I = -\nabla \cdot \Pi_\|^{\rm res}(r) + a_\|(r)$. To this end, we propose a comparison between integrated intrinsic torque, as a measure for the cases of (i) vanishing fluctuations on the boundary [i.e., $\delta\phi(\pm a) = 0$] and (ii) finite boundary fluctuations [$\delta\phi(\pm a) \neq 0$]. This choice follows from the observation that the radially integrated intrinsic torque is given by

$$T_I = \int_{-a}^{a} \tau_I(r) dr = -\Pi_\|^{\rm res}(r)|_{-a}^{a} + \int_{-a}^{a} a_\|(r) dr. \quad (16)$$

Note that $T_I \neq 0$ is required for a net spin-up. For the case of vanishing turbulence on the boundary, $T_I = \int_{-a}^{a} a_\|(r) dr$, so a finite value can result only from turbulent acceleration, and thus T_I constitutes a direct measure of the integrated turbulent acceleration. However, for a corresponding case with $\delta\phi(\pm a) \neq 0$, but other quantities (i.e., parameters, profiles, etc.) the same, T_I has contributions from both the residual stress on the boundary and the radially integrated $a_\|$. Subtracting the results for $\int_{-a}^{a} \tau_I(r) dr$ for the two cases could suggest a trend which reveals the residual stress contribution. Note that $a_\|$ and $\Pi_\|^{\rm res}$ both depend upon the same spectral cross-correlator and the same symmetry breaking mechanism. This comparison should separate at least the radially integrated contributions to the intrinsic torque from $\Pi_\|^{\rm res}$. Since it is a comparison of radially integrated quantities, it should not be very sensitive to turbulence spreading near the boundary and related phenomena. Finally, truth in advertising compels us to say that this test will elucidate only the radially integrated $a_\|$, but not the local profile of $a_\|$. Further consideration is required to address that.

Of course, a second important question is how to measure $a_\|$ and thus test the theory in a physical experiment. To this end, we note that the presence of $a_\|$ necessarily breaks the condition of zero total velocity flux (i.e., Reynolds stress) in a steady state of intrinsic rotation. Thus, $\partial \langle U_\| \rangle / \partial t = -\partial_r \langle \tilde{v}_r \tilde{U}_\| \rangle + a_\|$ and stationarity imply

$$\langle \tilde{v}_r \tilde{U}_\| \rangle = \int_0^r dr' a_\|(r'), \quad (17)$$

so that a finite value of $\langle \tilde{v}_r \tilde{U}_\| \rangle(r)$ implies $\int_0^r dr' a_\|(r') \neq 0$, i.e., a finite value of the radially integrated turbulent acceleration. The parallel Reynolds stress $\langle \tilde{v}_r \tilde{U}_\| \rangle$ could be measured directly by a number of means, such as Mach probes (at the edge) [30] or beam emission spectroscopy velocimetry [31] using an image in the r-parallel plane. We note that this would be a challenging new application of beam emission spectroscopy, which so far has been used only for velocimetry measurement of $\langle \tilde{v}_r \tilde{v}_\theta \rangle$ [32]. Given the daunting prospect of measuring $\langle \tilde{v}_r \tilde{U}_\| \rangle$, we offer a second, purely macroscopic test. Using the condition of stationarity and the total velocity balance condition, we have the jump condition

$$\nabla \langle U_\| \rangle |_r^{r+\triangle r} = \left(\frac{\Pi_\|^{\rm res}}{\chi_\|} + \frac{V_{\rm pinch} \langle U_\| \rangle}{\chi_\|} - \frac{\int_0^r dr' a_\|(r')}{\chi_\|} \right)_r^{r+\triangle r}.$$

For smoothly varying profiles and $\chi_\|$, $V_{\rm pinch}$, and $\Pi_\|^{\rm res}$, it follows that $\nabla \langle U_\| \rangle |_r^{r+\triangle r} \cong -\int_r^{r+\triangle r} dr' a_\|(r')/\chi_\| \sim -\triangle r \times a_\|/\chi_\|$. Here, $\chi_\|$ could be approximated by χ_i, determined independently. To assure smoothness, $\triangle r$ must be smaller than the scale of possible variation of $\Pi_\|^{\rm res}$, $V_{\rm pinch}$, etc. In practice, this surely is satisfied if $\triangle r < L_I$, a typical mesoscale. Taking $L_I \sim (\rho_i L_{T_i})^{1/2}$, for $\rho_i \sim 0.1$ cm, $L_{T_i} \sim 50$ cm, one can have $\triangle r \lesssim 2.24$ cm. Thus, present day high resolution beam blip charge exchange recombination spectroscopy measurements [33] should have sufficient accuracy for this. Note that the key point here is that the fastest varying contribution to $\nabla \langle U_\| \rangle$ on mesoscales comes from $\int_0^r dr' a_\|(r')$. Observe that for sufficiently small $\triangle r$, this method gives an effectively local measurement of $a_\|$.

In summary, we discovered a turbulent acceleration term in the parallel rotation equation by a calculation based on the gyrokinetic equation. The turbulent acceleration cannot be written as a divergence of the parallel Reynolds stress, which is similar to the turbulent momentum source found independently in Ref. [23]. It has different physics from the residual stress, which enters the rotation equation as a divergence. In particular, the fact that the residual stress contributes a divergence term to the rotation equation means that its effect on net rotation enters via its value at the edge. In contrast, parallel acceleration can be distributed throughout the entire cross section and is not particularly edge sensitive. A new candidate mechanism for the origin of spontaneous rotation is thus revealed. We proposed a method for testing the effects of turbulent parallel acceleration by gyrokinetic simulation based on the difference between the turbulent acceleration and residual stress. We also proposed a direct experimental test to determine the relative contributions from $\Pi_{\parallel}^{res}$ and $a_{\parallel}$.

We are grateful to X. Garbet, T. S. Hahm, L. Chen, J. M. Kwon, X. W. Hu, and Z. X. Lu for useful discussions. This work was supported by the MOST of China, under Contracts No. 2013GB112002 and No. 2011GB109001, the NSFC Grants No. 10990214 and No. 10935004, the MEST of Korea via the WCI Project No. 2009-001, the U.S. DOE Grant No. DE-FG02-04ER54738.

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