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UNIVERSITY OF CALIFORNIA, SAN DIEGO

Simulations of Two-Stream Instability in Opposite Polarity
Dusty Plasmas

A Thesis submitted in partial satisfaction of the requirements

for the degree Master of Science

in

Electrical Engineering (Applied Physics)

by

Stephen Eric Clark

Committee in Charge:

Kevin Quest, Chair
William Coles
Barnaby Rickett
Marlene Rosenberg

2011

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Chair

University of California, San Diego

2011

DEDICATION

I would like to dedicate this thesis to Elizabeth Hoyt for sticking with me throughout the years and pushing me to strive for more. I would not have accomplished everything I have today without her love and support.

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ABSTRACT OF THE THESIS

Simulations of Two-Stream Instability in Opposite Polarity
Dusty Plasmas

by

Stephen Eric Clark

Master of Science in Electrical Engineering (Applied Physics)

University of California, San Diego, 2011

Kevin Quest, Chair

One dimensional Particle in Cell simulations of a dust-dust counter-streaming instability in a plasma containing dust grains of opposite charge polarity are presented. This dust-dust instability has potentially the lowest threshold drift for a dust wave instability in an unmagnetized dusty plasma. The linear and nonlinear development of the dusty plasma instability is investigated, including the effects of collisions with background neutrals, and a background electric field which acts as a driver to impart the drift velocities of the counter-streaming charged dust particles. The saturation of the linear instability appears to be dust heating related to dust trapping. Potential double layer formation from dust-dust turbulence is observed

in cases with a high neutral collision rate. A comparative study is done with varying collision rates and background electric fields to explore the nonlinear development as a function of collision rate and background electric field. These simulation results could help guide future laboratory and micro-gravity dusty plasma experiments on this instability. The results could also relate to similar counter-streaming instabilities in pair plasmas and negative ion plasmas.

Chapter 1

Introduction

Dusty plasmas are plasmas containing small (micron to sub-micron) solid particulates, or dust grains, that get electrically charged in the plasma. In low temperature laboratory plasmas, the grains are generally charged due to the collection of plasma particles. Since the electrons are more mobile, they collide with the dust grains more frequently than ions and the grain becomes negatively charged. The charge-to-mass ratio of the dust grains is much smaller than that of the plasma ions. For example, in an argon rf or dc glow discharge with an electron temperature of 2 eV, a micron sized dust grain would have a negative charge of about 3000 electron charges, and a mass of about $10^{12} - 10^{13}$ proton masses [1]. The presence of the charged dust leads to the appearance of very low frequency waves associated with the motion of the massive charged dust. An example is the dust acoustic wave (DAW) [2] which is the very low frequency analog of the ion acoustic wave. In the DAW, the dust provides the inertia while the restoring force arises from pressures of the background electrons and ions. The phase speed ($v_{ph} = \omega/k$, where ω and k are the frequency and the wave number, respectively) of the DAW is larger than the dust thermal speed but much smaller than the ion thermal speed. There have

been studies on the excitation of dust acoustic waves by the ion-dust streaming instability, in which DAWs are excited by ions streaming relative to dust with a speed greater than or equal to the ion thermal speed ([3, 4]). There have also been several numerical simulation studies of the nonlinear development of ion-dust streaming instabilities including collisions with neutrals, which are important in laboratory dusty plasmas since they are typically weakly ionized, as well as an external electric field which induces the ion-dust streaming [5, 6]. However, there has been scarce work done on dust wave instabilities driven by relative streaming of dust grains, particularly when the dust grains have opposite charge polarity.

Although dust grains immersed in low temperature laboratory plasmas are often negatively charged, under certain conditions dust grains can also become positively charged. This is due to processes such as photoelectric emission if there is an ultraviolet radiation source, thermionic emission if the grain is sufficiently heated, secondary emission if there is a source of high energy electrons, etc. (see e.g. [1]). There could even be grains of opposite polarity in a plasma if the grains have different electrical properties. For example, suppose there are two types of dust grains in a plasma, one type with a high photoelectric yield Y and the other type with a low Y , then the grain with a high Y could be positively charged while the grain with low Y could be negatively charged in a plasma irradiated by an ultraviolet source [7]. Dusty plasmas containing grains of opposite polarity have analogs in negative ion plasmas (see e.g. [15]), which contain two species of ions, one positively charged and one negatively charged, which may also contain a background of lighter electrons.

There may be some naturally occurring dusty plasmas in space where the dust grains can have opposite polarity. This may be due, for example, to the grain size dependence of charging mechanisms such as secondary electron emission, or due to the different photoelectric emission properties of grains under solar ultra-

violet (UV) radiation. Examples of regions where there may be dust of negative and positive charge polarity include the Earth's upper mesosphere [34, 35, 36], cometary tails [37], Jupiter's magnetosphere [37, 38], as well as dust-devils on Mars, where the dust grains could be charged by triboelectricity [39].

Recently, there have been several theoretical works, mainly using fluid theory, on the nonlinear behavior of dust acoustic and related waves in dusty plasmas containing dust grains of opposite polarity (e.g. [8, 9, 10, 11, 12, 13]). There have also been several studies on the linear behavior of streaming instabilities in such plasmas using fluid theory [7, 14].

In this thesis, we present numerical simulation studies of the linear and nonlinear development of a counter-streaming dust-dust instability in a collisional plasma containing dust of opposite polarity, immersed in an external electric field. Although there have been previous simulation studies of ion-dust streaming instabilities [5, 6], our simulation studies are of a dust-dust streaming instability, which has a much lower critical drift speed than the ion-dust streaming instability. A fluid theory analysis of a counter-streaming instability in such a system was reported by D'Angelo [7]. In contrast to the latter study, we focus on a regime where the dust drift speeds are smaller than the dust acoustic speeds, so that slow waves, with phase speed smaller than dust acoustic speeds, could be excited. This slow wave instability has potentially the lowest threshold drift considered thus far for a dust wave instability in an unmagnetized dusty plasma. This regime is analogous to the regime of two-stream instability in electronegative plasmas discussed by Tuszewski and Gary [16] (see also [17]), in which positive and negative ions counter-stream with drift speeds larger than their respective thermal speeds but smaller than the phase speed of fast ion-acoustic waves in the system. Our simulation results may be relevant to these types of two-stream instabilities in negative ion plasmas, as well as electron-positron plasmas [40], and pair plasmas such as fullerene plasmas

which have recently been produced in the laboratory [41].

In laboratory or space dusty plasma environments, the counter streaming of opposite polarity dust grains could be induced by an electric field. In addition, dusty plasmas are often weakly ionized plasmas, so collisions of charged particles with neutrals can be an important aspect of the system. It is for these reasons that we include collisions and a background electric field to drive instability in our simulation. We are motivated to choose baseline parameters for the simulation such that it may guide any future laboratory or micro-gravity dusty plasma experiments. In addition to this baseline case we perform a comparative study to explore the effects of varying the collision rate and background electric field on the nonlinear development of this instability.

This thesis is organized as follows. In Chapter 2, a fluid analysis of marginal stability and a linear kinetic analysis of the dust-dust counter-streaming instability are given. Possible laboratory parameters are considered under which other instabilities, such as the ion-dust streaming instability and the higher frequency ion acoustic instability are stable, yet the dust-dust instability grows. Numerical simulations of the linear and nonlinear development of the dust-dust counter-streaming instability are presented in Chapter 3 for these parameters. Chapter 4 gives a summary and discussion of applications. Appendix A describes certain details regarding the simulation used in this thesis.

Chapter 2

Linear Theory

2.1 Model system

The model consists of a weakly ionized plasma comprised of electrons, singly charged ions, and two species of multiply charged dust grains. One dust species is positively charged and the other dust species is negatively charged. The condition of overall charge neutrality is

$$n_e + Z_- n_- = n_i + Z_+ n_+ . \quad (2.1)$$

Here, n denotes the species density, where the subscripts $e, i, +, -$ refer to electrons, ions, positively charged dust, and negatively charged dust, respectively. In addition, Z_- and Z_+ are the charge states of the negatively and positively charged dust species. The ions are assumed to be singly charged, or rather $Z_i = 1$. There is a background neutral gas, argon, which collides with the plasma. There is also an external constant electric field E_0 in the x-direction. We consider a one-dimensional (1D) system, in which the electric field induces steady-state drifts in

all the particle species, given by

$$\mathbf{V}_j = \frac{q_j \mathbf{E}_0}{m_j \nu_j}. \quad (2.2)$$

Here q_j and m_j are the charge and mass of each particle species, and ν_j is the collision frequency of each charged species with the neutral background gas.

2.2 Fluid analysis

The equations of continuity and momentum for each charged species are

$$\frac{\partial n_j}{\partial t} + \frac{\partial(n_j v_j)}{\partial x} = 0, \quad (2.3)$$

$$n_j m_j \left(\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} \right) + T_j \frac{\partial n_j}{\partial x} - q_j n_j E = -\nu_j n_j m_j v_j, \quad (2.4)$$

where T_j is the temperature of charged particle species $j = (+, -, e, i)$ (in energy units). Here and in the following we will assume that the electron and ion drift velocities, V_e and V_i , are much smaller than their respective thermal speeds, that the phase speeds of the unstable waves are much smaller than the ion and electron thermal speeds, and that mean free paths for ion and electron collisions with neutrals are much larger than the unstable wavelengths. In this case, we can assume Boltzmann distributed ions and electrons, so that the expressions for the ion and electron species in (2.4) become

$$\frac{\partial n_i}{\partial x} = \frac{n_i e E}{T_i}, \quad (2.5)$$

$$\frac{\partial n_e}{\partial x} = -\frac{n_e e E}{T_e}. \quad (2.6)$$

Along with equations (2.1), (2.3), (2.4), (2.4), (2.5), and (2.6), we also need to solve Poisson's equation given by:

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_j n_j q_j, \quad (2.7)$$

where ϕ is the self-consistent electrostatic potential of the system. Now if we rewrite the variables $v_j = V_j + \delta v_j$, $n_j = n_{j0} + \delta n_j$, and $E = E_0 + \delta E$, we can linearize the above equations assuming that the perturbed electric field $\delta E = -\partial\phi/\partial x$, and that ϕ as well as the perturbed density δn_j and velocity δv_j vary as $\exp[-i(\omega t - kx)]$. Assuming quasi-neutrality, we obtain the following fluid dispersion relation (see [16] for negative ion plasma):

$$1 - \frac{C_-^2}{(v_{ph} - V_-)(v_{ph} - V_- + \frac{i\nu_-}{k}) - v_{t-}^2} - \frac{C_+^2}{(v_{ph} - V_+)(v_{ph} - V_+ + \frac{i\nu_+}{k}) - v_{t+}^2} = 0. \quad (2.8)$$

Here, $v_{ph} = \omega/k$ is the phase velocity of the wave, and $v_{tj} = (T_j/m_j)^{1/2}$ is the thermal speed of species j . In addition, the dust acoustic speeds are given by $C_{\pm} = \omega_{p\pm}\lambda_D$ where $\omega_{p\pm} = [4\pi n_{\pm}(Z_{\pm}e)^2/m_{\pm}]^{1/2}$ is the plasma frequency of the dust species, and λ_D is the linearized Debye screening length in the background plasma, given by

$$\frac{1}{\lambda_D^2} = \frac{1}{\lambda_{Di}^2} + \frac{1}{\lambda_{De}^2} = \frac{4\pi n_i e^2}{T_i} + \frac{4\pi n_e e^2}{T_e}. \quad (2.9)$$

Following [16], in the regime where $C_{\pm} \gg v_{t\pm}$ and $V_{\pm}$, two solutions of (2.8) are two stable fast waves with $v_{ph} = \pm(C_+^2 + C_-^2)^{1/2} = C_f$. The other two solutions are slow waves with $v_{ph} \sim v_{t\pm}$ and $V_{\pm}$, which can be obtained by retaining terms of order $V_{\pm}^2$ in (2.8). To proceed further we neglect collisions, which is roughly valid in the regime where the mean free path for dust-neutral collisions is much larger than the unstable wavelengths. For the fluid equations to be valid, this then implies that the dust drift speeds are larger than their respective thermal speeds. We then obtain from (2.8)

$$v_{ph}^2 C_f^2 - 2v_{ph}(V_+ C_-^2 + V_- C_+^2) + C_+^2(V_-^2 - v_{t-}^2) + C_-^2(V_+^2 - v_{t+}^2) \approx 0. \quad (2.10)$$

The solution of (2.10) yields

$$v_{ph} \approx \frac{V_- C_+^2 + V_+ C_-^2}{C_f^2} \pm \left[\frac{v_{t-}^2 C_+^2 + v_{t+}^2 C_-^2}{C_f^2} - \frac{C_+^2 C_-^2}{C_f^4} V_{rel}^2 \right]^{1/2}, \quad (2.11)$$

where $V_{rel} = V_+ - V_-$. From (2.11), we see that instability requires that the value under the square root must be negative to obtain a positive imaginary part of ω . This condition requires a relative dust drift

$$\frac{|V_{rel}|}{v_{t+}} > \left[\left(\frac{T_-}{T_+} \frac{m_+}{m_-} + \frac{C_-^2}{C_+^2} \right) \left(\frac{C_+^2}{C_-^2} + 1 \right) \right]^{1/2}. \quad (2.12)$$

This mechanism of the dust-dust instability may be due to the coupling of the slow waves associated with the dust flows in opposite directions, in analogy with the ion-ion instability in negative ion plasmas (see [18]). There can be an energy exchange between a positive energy slow wave and a negative energy slow wave, with the negative energy wave giving up its energy to the positive energy wave so both waves grow [18].

2.3 Linear kinetic analysis

Although the linear fluid analysis given in the previous section can provide guidance for the threshold V_{rel} , a full kinetic analysis provides a more accurate linear growth rate in regimes where the dust drift speeds are not much larger than their respective thermal speeds. In addition, the effects of collisions can easily be included, and we can also investigate whether other dust or ion wave instabilities can be excited for a particular choice of parameters. We assume the particle distribution functions for all species are given by drifting Maxwellians in our model 1D plasma. The linear kinetic dispersion relation (see [33]) is

$$1 + \sum_j [1 + \zeta_j Z(\zeta_j)] \left[1 + \frac{i\nu_j}{\sqrt{2}kv_{tj}} Z(\zeta_j) \right]^{-1} = 0. \quad (2.13)$$

Here

$$\zeta_j = \frac{\omega - \mathbf{k} \cdot \mathbf{V}_j + i\nu_j}{\sqrt{2}kv_j}, \quad (2.14)$$

and $Z(\zeta)$ is the plasma dispersion function [19].

We will now discuss the parameters for a dusty plasma containing opposite polarity dust grains. D'Angelo [7] considered theoretically a plasma irradiated by a UV source that contains two types of dust grains, one with a high photoemission efficiency and one with a low photoemission efficiency. The dust with high photoemission efficiency gets charged positively by the photoelectric emission of electrons. D'Angelo [7] solved for the surface potential ϕ_s of two different types of grains, with photoemission efficiencies of 0.1 and 1, in a plasma with ion mass roughly 40 proton masses, $T_e \sim 2$ eV and $T_i \sim 0.2$ eV, irradiated by a UV light with photon flux P . It was shown that grains of opposite polarity could exist under certain ratios of P/n where $n \sim n_i \sim n_e$. Using the results presented in Figure 5 of [7], at $P/n \sim 2 \times 10^8$ cm/s, and assuming that $P \sim 4 \times 10^{16}$ cm⁻² s⁻¹ (so that $n \sim 2 \times 10^8$ cm⁻³) we find that $\phi_s \sim -T_e/e$ for the negatively charged grains and $\phi_s \sim 2T_e/e$ for the positively charged grains. We assume that both types of grains have uniform radius $a = 0.1$ micron. In this case, we estimate that $Z_+ \sim 280$ and $Z_- \sim 140$. Further, we assume that the positively charged dust is composed of material with a higher specific gravity ~ 7 g/cm³ (e.g., cerium oxide which has a low work function) while the negatively charged dust has a lower specific gravity of about 3.5 g/cm³ (e.g., carbonaceous material). Thus $m_- \sim 8.5 \times 10^9 m_p$, $m_+/m_p \sim 1.7 \times 10^{10}$, and we take Z/m to be the same for both dust species. We also assume that $T_- = T_+ = T_i$, so that the dust thermal speeds are $v_{t-} \sim 4.9$ cm/s and $v_{t+} \sim 3.5$ cm/s.

For the plasma described in the previous paragraph, we assume the ratios $n_-/n_i = n_+/n_i = 5 \times 10^{-4}$, so that both Z_-n_-/n_i and Z_+n_+/n_i are $\ll 1$, in order for the analysis leading to Figure 5 in D'Angelo [7] to be roughly valid. With these values, the dust plasma frequencies are $\omega_{p+} \sim 900$ rad/s and $\omega_{p-} \sim 635$ rad/s. The ratios of the dust acoustic speed, $C_{\pm} \sim \lambda_{Di}\omega_{p\pm}$ to the dust thermal speed $v_{t\pm}$

is $C_-/v_{t-} \sim Z_-(n_-T_i/n_iT_-)^{1/2} \sim 3.1$ and $C_+/v_{t+} \sim 6.3$. Thus if each dust species has a drift on the order of its dust thermal speed, one can obtain a situation like the two-stream situation in an electronegative plasma considered by Tuszewski and Gary [16], where the positive and negative ion drift speeds are less than the ion acoustic speeds in the system.

We consider a background argon gas with pressure ~ 4 Pa (neutral density $n_n \sim 10^{15} \text{ cm}^{-3}$). Taking the ion-neutral cross section as $\sigma_i \sim 4 \times 10^{-15} \text{ cm}^2$ and the electron-neutral cross section as $\sigma_e \sim 5 \times 10^{-16} \text{ cm}^2$, we have that $\nu_i/\omega_{pi} \sim 0.1$ and $\nu_e/\omega_{pe} \sim 10$. For the dust neutral collisions, $\sim 4a^2n_nv_nm_n/m_{\pm}$ (here v_n and m_n are the neutral thermal speed and mass, respectively), we have that $\nu_- \sim 60 \text{ s}^{-1}$ and $\nu_+ \sim 30 \text{ s}^{-1}$.

We estimate the electric field E_0 required to achieve dust drift speeds that are larger than their thermal speeds yet much smaller than their dust acoustic speeds. Using (2.2) and assuming an external electric field $E_0 \sim 2 \text{ V/m}$ (in the positive x-direction), we find that $V_- = -Z_e e E_0 / m_- \nu_- \sim -5.2 \text{ cm/s}$ and $V_+ = Z_+ e E_0 / m_+ \nu_+ \sim 10.4 \text{ cm/s}$. Thus the dust drifts are less than the corresponding dust acoustic speeds, but may still be large enough for a two-stream instability. According to (2.12), the critical relative dust drift for this system is on the order of $V_{rel} \sim 2.8$, while for the above parameters we have $V_{rel} \sim 4.5v_{t+}$, so it appears the dust-dust instability should occur.

On the other hand, with this value of electric field, the ion drift is $V_i = eE_0/m_i\nu_i \sim 0.02v_i$ and the electron drift is $V_e = -eE_0/m_e\nu_e \sim -0.017v_e$. The ion drift is too small to drive a dust acoustic instability, since the ratio V_i/v_{ti} would have to be larger than $\nu_+/\omega_{p+} \sim 0.03$ or $\nu_-/\omega_{p-} \sim 0.1$. The electron drift is too small to drive an ion acoustic instability since V_e/v_{te} would have to be larger than $\nu_i/\omega_{pi} \sim 0.1$. Thus it appears that only the dust-dust instability persists for these parameters.

Figure 2.1 shows the frequency and growth rate of the dust-dust instability driven by the relative streaming of the positively and negatively charged dust grains. This was obtained by solving (2.13), using the dimensionless parameters: $m_i/m_p = 40$, $m_-/m_p = 8.5 \times 10^9$, $m_+/m_p = 2m_-/m_p$, $T_e/T_i = T_e/T_+ = T_e/T_- = 10$, singly charged ions, $Z_- = 140$, $Z_+ = 280$, $n_-/n_i = n_+/n_i = 5 \times 10^{-4}$, $\nu_e/\omega_{p+} = 3.3 \times 10^4$, $\nu_i/\omega_{p+} = 3.3 \times 10^2$, $\nu_+/\omega_{p+} = 1/30$, $\nu_-/\omega_{p+} = 1/15$, $V_e/v_{te} = -0.017$, $V_i/v_{ti} = 0.02$, $V_+/v_{t+} = 3$, and $V_-/v_{t+} = -1.5$, which correspond to the parameters described earlier in this chapter. The growth rates and frequencies in the figures are normalized to the positive dust plasma frequency $\omega_{p+} = (4\pi n_+(Z_+e)^2/m_+)^{1/2}$ and the wavenumber is normalized to the positive dust Debye length $\lambda_{D+} = (T_+/4\pi n_+(Z_+e)^2)^{1/2}$. These values were chosen such that these results could be compared to the simulation results in Chapter 3 without having to convert to different normalizations. Figure 2.2 shows that with these parameters, the excitation of dust acoustic waves by ion streaming, i.e., the ion-dust streaming instability, does not occur.

Figure 2.1: Real, ω_r and imaginary, γ parts of ω obtained by solving (2.13) using the parameters given in the text.

Figure 2.2: Dust acoustic waves are not excited by ion streaming for these parameters, which are the same as in Figure 2.1.

Chapter 3

Simulation Results

The following sections will show results that were obtained by running a Particle in Cell (PIC) simulation of a dust-dust instability in a plasma containing counter-streaming opposite-polarity dust grains. The plasma in the simulation consist of two species of dust grains each with opposite charge. The two species of dust are counter-streaming in the simulation reference frame. The dust grains collide with a neutral background gas, which slows and thermalizes the plasma, and there is a background electric field which imparts a constant relative drift to both species of dust.

The simulation uses a Langevin scattering operator to model collisional effects [21]. The Langevin scattering operator uses a constant collision frequency for each species that interacts with the neutral background gas. The gas is modeled such that it is isothermal and maintains a constant temperature. It effectively immerses each species of the plasma in a constant temperature bath and will either heat or cool the species to the neutral gas temperature. The neutral gas can also have a drift velocity that each species tends to. In the cases examined the neutral gas has zero drift in the simulation frame of reference. Since the collisions of the

positive and negative dust grains with the background neutral gas have a tendency to slow the particles down to the drift velocity of the neutral gas, a background electric field is imposed to counteract the deceleration. The electric field is set such that it exactly counteracts the force imposed by the collisions to maintain a particular drift velocity during the linear growth phase of the simulation.

$$E_0 = \frac{m_j \nu_j V_j}{q_j} \quad (3.1)$$

In this equation m_j is the mass of the j^{th} species, ν_j is the collision frequency, V_j is the drift velocity, and q_j is the charge of the species.

The simulation is in a regime where the drift imposed on the ions and electrons is such that it is small relative to their thermal velocities i.e. $V_e \ll v_e$ and $V_i \ll v_i$. In the case described in Chapter 2, $V_i/v_{ti} \sim 0.02$ and $V_e/v_{te} \sim 0.017$. Since their streaming velocities are so much less than their thermal velocities, thermal effects dominate. Since this is the case we can treat the ions and electrons as Boltzmann distributed, i.e.

$$n_{e,i} = n_{e0,i0} \exp \frac{-q_{e,i} \phi}{T_{e,i}}. \quad (3.2)$$

The ions and electrons merely provide mobile charge for shielding the dust grains.

3.1 Unit normalizations

The simulation units are dimensionless parameters that are used so that the simulations can be run using numbers that are not very big or small. The scaling of the problem is what is important, for example it is the ratio of the relative drift velocity to the thermal speed v_{t+} that is the main factor in driving this linear dust-dust instability. It is the scaling of the numbers to important parameters in the system that determine the behavior of the system. The normalizations and

Table 3.1: Unit normalizations used in the simulation, where r denotes the reference values and j denotes the species.

Description	Symbol	Definition
Reference Frequency	ω_r	$\sqrt{4\pi n_r q_r^2 / m_r}$
Reference Length	x_r	$\sqrt{T_r / 4\pi n_r q_r^2}$
Reference Velocity	v_r	$\sqrt{T_r / m_r}$
Wave Electrostatic Potential	ϕ	$q_r \phi / T_r$
Wave Electric Field	E	$q_r x_r E / T_r$
Time	t	$t \omega_r$

reference units are shown in Table 3.1. All of the parameters that are not listed in Table 3.1 are simply their respective parameter values divided by the reference value, for example $T_j/T_r \rightarrow T_j$. In the following simulation cases the reference values are chosen to be that of the positive dust species at the beginning of the simulation, for example $T_r = T_{+0}$, $\omega_r = \omega_{p+}$, $x_r = \lambda_{D+}$, $v_r = v_{t+}$, etc.

3.2 Simulation parameters

Table 3.2 contains the parameters that were used in the simulations. The baseline simulation parameters, case I, were analyzed in Chapter 2 using fluid and kinetic theory. Simulations with lower collision frequencies and background electric fields are presented to explore how the nonlinear development of the system changes as a function of collision frequency and background electric field. The cases with lower collisionality may give rise to other instabilities, such as ion acoustic waves, but they are not considered in this analysis since the electrons and ions are considered to be Boltzmann distributed.

Table 3.2: Normalized simulation parameters are listed in the tables. The left table contains parameters that are shared amongst all of the simulation cases, whereas the right table contains the parameters that are specific to each particular simulation case. T_n denotes the temperature of the neutral gas.

Parameter	Value
m_+/m_r	1
m_-/m_r	1/2
Z_+/Z_r	1
Z_-/Z_r	1/2
$Z_{i,e}/Z_r$	1/280
$T_{+,-,n,i}/T_r$	1
T_e/T_r	10
$n_{+,-}/n_r$	1
n_i/n_r	2000
n_e/n_r	2140
V_+/v_r	3
V_-/v_r	-1.5

Case	I	II	III	IV
ν_+/ω_r	1/30	1/120	1/480	1/1920
ν_-/ω_r	1/15	1/60	1/240	1/960
$\frac{q_r x_r}{T_r} E_0$	0.1	2.5e-2	6.3e-3	1.6e-3

Figure 3.1: Average energy density in the wave electric field for simulation case I versus time.

3.3 Comparison of case I with linear theory

The first step in analyzing the simulation results is to calculate the growth rate of the system during the linear phase of the instability. Figure 3.1 shows the average electric field energy density as a function of time. It can be seen that the system is in a phase of linear growth up until $t\omega_r \sim 50$, which is roughly 5 growth times since $\gamma/\omega_r \sim 0.1$. At this point the instability saturates. We will explore the saturation and the mechanism for shutting off the instability in the next section.

The energy plotted in Figure 3.1 is the energy density in the electric field summed over all of the modes. If the electric field is Fourier transformed into k space and the energy is calculated in each mode one obtains Figure 3.2. Upon inspection we find that the mode with the peak energy is at $kx_r \sim 0.28$, which

Figure 3.2: Energy spectral density of the wave electric field at $t\omega_r = 35$.

Table 3.3: Calculated simulation growth rates for selected modes in simulation case I.

Mode	kx_r	γ/ω_r
3	0.094	0.062
4	0.126	0.091
5	0.157	0.110
6	0.188	0.114
7	0.220	0.123
8	0.251	0.130
9	0.283	0.121
10	0.314	0.103

Figure 3.3: Calculated simulation growth rates from Table 3.3 compared to kinetic theory.

matches well with the peak growth wavenumber in Figure 2.1. Now we can calculate the growth rate of that particular mode by taking the time history of the Energy Spectral Density and calculating the growth in the energy of the peak mode. The growth is calculated by measuring the slope of the curve in a log scale. The result of the calculation is that the growth rate of the peak mode at $kx_r \sim 0.28$ is $\gamma/\omega_r \sim 0.13$, which also matches well with the linear kinetic theory calculation in Figure 2.1. The growth rates in the modes near the peak fall off as they do in Figure 2.1. The curve in Figure 3.2 has modes that fall off at about the same width as the curve in Figure 2.1.

The growth rates in Table 3.3 are calculated from the simulation results. The modes in the table were selected because they had enough energy to come out of the noise and the linear growth slope could be spotted manually and calculated. The values in Table 3.3 are plotted in Figure 3.3 against the growth rate predicted from the kinetic linear dispersion relationship, Equation 2.13. It can be seen from Figure 3.3 that the growth rates calculated from the simulation agree reasonably well with kinetic theory.

Cases II through IV have growth rates that are comparable to that of case I since $\nu_{+,-}/v_{t+}k_{max} \ll 1$ for all cases, and will not be explicitly compared against linear kinetic theory.

3.4 Nonlinear development

3.4.1 Baseline simulation: case I

In this section we will discuss the mechanism for shutting off the linear instability as well as discuss the long time nonlinear development of the simulation.

Figure 3.4 through Figure 3.8 show the progression of the simulation in phase space as well as the corresponding velocity distribution function for several times selected at key points in the simulation, such as linear growth, linear saturation, nonlinear development, and the end state of the simulation.

There will be many figures that follow showing snapshots of data at particular points in time for the following simulations. Figure 3.4 shows a snapshot of the initial state of simulation case I. There are four plots in each figure. The first plot, in the top left denoted by an (a) is the phase space plot. The shade depicted by the colorbar corresponds to the number of simulation "macro-particles" that reside are in each position and velocity bin. The electric field plot, denoted by (b), on the top right has two curves. The blue dashed line is the value of the background electric field that is imposed in the simulation and the solid black line is the calculated wave electric field. The distribution plot, denoted by (c), is the summation of the phase space plot across the simulation domain, and again is measured in "macro-particles." The wave potential, denoted by (d), is plotted on the bottom right. This style of figure will be used throughout the text to illustrate the state of the simulations.

There are other useful figures that illustrate the state of the simulation across time. One is Figure 3.1, which was shown earlier in the linear theory comparison to calculate the growth rate of the simulation. It shows the average wave electric field energy density in dimensionless units. It is a measure of energy in the wave field throughout the simulation. The important part of the figure is the relative energy to that at the beginning of the simulation. It gives clues regarding the amount of energy in the system. A useful value in a collisionless system without an electric field would be the ratio of wave energy to the total energy in the system, since the total energy of the system would be conserved throughout the course of the simulation. In the simulations presented there is a flux of energy being put

Figure 3.4: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case I at $t\omega_r = 0$

into the system via the background electric field, and energy being removed from the system as particles are cooled due to collisions, so the ratio of wave energy to the total energy at the start of the system does not mean much. For this reason, the graph is presented in dimensionless energy density units.

The temperature profile and average particle velocity profiles, Figures 3.9 and 3.10 respectively, are also good indicators of the state of the simulation. The temperature is a measure of the standard deviation of the particle velocities multiplied by the species mass. The temperature is only defined for a Maxwellian distribution, but it still gives useful information about the width of the distribution. If the temperature increases, the velocity distribution of the respective species broadens.

One thing to note regarding Figure 3.4 is that the velocity distribution of

Figure 3.5: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case I at $t\omega_r \sim 40$

the negative dust species has a lower peak magnitude and is broader even though the initial temperatures listed in Table 3.2 state that they are the same. Since the thermal velocity $v_{tj}^2 = T_j/m_j$ and the mass of the negative dust species is half that of the positive dust species, the thermal velocity will be larger by a factor of $\sqrt{2}$, as will the width of the distribution function. The peak magnitude is then lower since the density of the positive and negative dust species is equal and the area under the distribution curve is equal for both the positive and negative species.

It can be seen in Figure 3.1, Figure 3.9, and Figure 3.10 that the linear instability saturates at $t\omega_r \sim 50$. One can calculate the growth rate using fluid or kinetic theory referencing the temperature and drift velocities at $t\omega_r \sim 50$ from Figure 3.9 and Figure 3.10. This can be used as a rough guide for determining if the linear instability shuts off under the conditions that are present in the simulation at that time. The values selected assume that the velocity distributions are drifting

Figure 3.6: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case I at $t\omega_r \sim 60$

Maxwellian distributions, but in actuality it can be seen from Figure 3.5 that the peak of the positive dust species distribution isn't shifting but the velocity distribution is broadening on the low velocity side of the curve, which makes the average velocity look lower than it did in the initial state. The values used in the calculation are $T_+/T_r \sim 1.75$, $T_-/T_r \sim 1.4$, $V_+/v_r \sim 2.75$, and $V_-/v_r \sim -1.2$. Calculating the fluid threshold drift in (2.12) can be used as a very rough guide to estimate the threshold drift. It will not be a very good estimate since we are in a regime where the drift velocities are only a few times the dust thermal velocities, but one can calculate the threshold drift for a comparison to kinetic theory. Carrying out the calculation, one obtains $|V_{rel}| \sim 2.5$. We see that fluid theory predicts that there will still be an instability. We now use kinetic theory, which includes thermal effects to calculate the linear growth rate. The solution of the kinetic dispersion relation (2.1) is plotted in Figure 3.11. Indeed the linear

Figure 3.7: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case I at $t\omega_r \sim 128$

instability shuts off at that point in the simulation. The growth rate γ/ω_r is not positive for any wavelength. This shows that if there were two drifting Maxwellian distributions in the state specified above that it would be a stable system. This can be used as a rough guide to determine when the instability shuts off.

The distribution function of the negative dust species initially broadens, but once the linear instability saturates the negative dust species then stabilizes at a temperature that is slightly warmer than its initial state. Since the negative dust has collision rate with the neutral gas that is twice that of the positive species, it has a tendency to stay cooler as the Langevin operator will be cooling it at a higher rate.

In systems where there are large amplitude wave electric fields, particle trapping can occur. Particle trapping is a condition where there are large potential

Figure 3.8: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case I at $t\omega_r \sim 312$

wells produced by the waves in the system which can give energy to the particles in the system. The particle trapping condition stated in [6] is

$$eZ_{\pm}\phi \approx \frac{m_{\pm}}{2} (v_{ph} - V_{\pm})^2, \quad (3.3)$$

where v_{ph} is the phase velocity of the most unstable wave mode. If we cast equation (3.3) it in terms of the simulation dimensionless units we get:

$$\frac{Z_{\pm}}{Z_r} \left(\frac{q_r \phi}{T_r} \right) \approx \frac{1}{2} \frac{m_{\pm}}{m_r} \left(\frac{v_{ph} - V_j}{v_r} \right)^2. \quad (3.4)$$

Now we can simplify expression (3.4) further by recognizing that for our parameter set the charge to mass ratio is unity for both the positive and negative dust species. We can approximate the phase velocity by looking at Figure 2.1 and finding $k_{max}x_r \sim 0.25$, which is the wavenumber that corresponds to the most unstable mode. We can also determine an estimate to $\omega_{max}/\omega_r \sim 0.05$. Now using $v_{ph} = \omega/k$ we can determine that for our system $v_{ph}/v_r \sim 0.2 \ll V_j/v_r$. So we can

Figure 3.9: Positive and negative dust species temperature profile versus time for simulation case I.

approximate the trapping condition as:

$$\frac{q_r \phi}{T_r} \approx \frac{1}{2} \left(\frac{V_{\pm}}{v_r} \right)^2. \quad (3.5)$$

Using $V_+/v_r = 3$ and $V_-/v_r = -1.5$, we can see that we need $\frac{q_r \phi}{T_r} \sim 1.1$ to trap the majority of negative dust particles and $\frac{q_r \phi}{T_r} \sim 4.5$ to trap the majority of the positive dust particles.

Upon first inspection it would seem that we are mostly trapping negative dust particles, but not the positive species. Looking at Figure 3.5, we can see that the magnitude of the potential is getting large enough for significant trapping to be occurring for the negative dust species. If we look at Figure 3.6 we might expect the negative dust velocity distribution to be widening more, but that is not the case. It seems that the Langevin operator is thermalizing and cooling the negative dust species, while the background electric field is maintaining its negative drift

Figure 3.10: Average particle velocity for positive and negative dust species versus time for simulation case I.

velocity. We can calculate an approximate trapping rate in the wave field, denoted by $\nu_{T\pm}$, by using the formula listed in [23]:

$$\nu_{T\pm} = \sqrt{\frac{Z_{\pm} e k_{max} E}{m_{\pm}}}. \quad (3.6)$$

After normalizing the equation and again using the fact that the charge to mass ratio for both positive and negative dust species is one, we obtain the fairly simply expression:

$$\frac{\nu_{T\pm}}{\omega_r} = \sqrt{k_{max} x_r \frac{q_r x_r E}{T_r}}. \quad (3.7)$$

Now if we use $k_{max} x_r \sim 0.25$, use a value for $\frac{q_r x_r E}{T_r} \sim 0.3$, then divide by ν_-/ω_r , we get a ratio of $\nu_{T-}/\nu_- \sim 4$. This means that after some number of bounces the negative dust particles may be getting detrapped by collisions.

The end state of the simulation is tending toward shielding of the background electric field, as seen in Figure 3.8, with high energy positive dust particles

Figure 3.11: Growth rate and real frequency of the dust-dust instability at $t\omega_r \sim 50$. The values used for the calculation are listed in Table 3.2, except for the temperature and drift Velocities, which are $T_+/T_r \sim 1.75$, $T_-/T_r \sim 1.4$, $V_+/v_r \sim 2.75$, and $V_-/v_r \sim -1.2$.

zipping between the large structures across a significant potential drop. Looking at the velocity distributions in Figure 3.8, we see that the large collision rate seems to be thermalizing the positive and negative dust species, which in turn makes them Maxwellian distributed. The Maxwellian distribution of the dust species facilitates the shielding which occurs in the high density regions. The collision rate of the system could potentially be contributing to this particular end state of the system. This looks like it may be the formation of a double layer, though more work is needed to determine this for sure.

There are a number of papers which discuss double layers and their formation. Verheest discusses solitons and double layers in opposite polarity dusty plasma [29]. It has been shown that double layers form in plasmas with two stream

Figure 3.12: Absolute value of charge density of positive and negative dust species for simulation case I at $t\omega_r \sim 312$.

instabilities [27, 28]. It has also been shown in one and two dimensional simulations that ion acoustic turbulence can lead to double layer formation [30, 31, 32].

One paper [26] discusses the formation of double layers that are initially formed from a density dip in a current carrying plasma, which in turn accelerates the plasma in the lower density region to conserve the current across the domain. The potential double layer in case I could possibly form in part due to the periodic boundary conditions of the simulation and the current that the plasma is carrying. The initial state of simulation case I has two species drifting, and there is a net current in the positive x direction. Since the electrons and ions are Boltzmann distributed they are assumed to have zero net drift and do not carry current. The state seems to form initially by means of the linear instability which generates density perturbations in the plasma. The double layer structure could be forming due to the background electric field accelerating the dust particles out of the low

density regions to conserve the net current. The particles then enter a region where the background electric field gets shielded out. This in turn lowers the density of the dip region further, which increases the potential drop and the electric field in the low density region. Since the simulation performed here has periodic boundary conditions, the state can only evolve to a certain extent, where the physical size of the plasma structure is on the order of the size of the domain. This means that the size of the structure near the end of the simulation is defined by an artificial domain limitation. This can be seen in Figure 3.8 and Figure 3.12. The double layer in these one dimensional simulations potentially depends on the boundary conditions and the strength of the background electric field, but more work needs to be done to analyze the development of the structure in this case. It would be useful to run large aperiodic simulations to see how the nonlinear state of the system evolves with different boundary conditions.

We will now examine systems with lower collisionality and lower background electric fields. This comparative study is done to determine what factors contribute to the nonlinear development of this baseline simulation case. These simulation studies may help guide future experimental work, though the cases with very low collision rates may not be realistic as other instabilities may be excited, which are neglected due to the treatment of the ions and electrons as Boltzmann distributed.

3.4.2 Comparison of the baseline simulation with case II

Simulation Case II is similar to case I except that the collision frequencies and background electric field have been reduced by a factor of four. The background electric field is being reduced in proportion to collision rate to yield the same drift velocity as in simulation case I. The initial state of the simulation and the linear development of the system are very similar to that of case I. The dif-

Figure 3.13: Average energy density in the wave electric field for simulation case II versus time.

ferences lie in the nonlinear development of the system. The initial state of the system has not been pictured since it is identical to the initial state of simulation case I shown in Figure 3.4, except for the fact that the background electric field is one fourth that of simulation case I.

Comparing Figure 3.13 to Figure 3.1, one immediately notices a couple of things. The linear part of the curve from $t\omega_r = 0$ to $t\omega_r \sim 50$ appears very similar to simulation case I. The differences become apparent after linear saturation occurs. This system appears to have cyclical behavior. This is most likely a consequence of having a background electric field and collisions in the system. There is a constant influx of energy into the system due to the background electric field and energy being removed from the particles via cooling that comes from collisions with the background neutral gas.

Figure 3.14: Temperature of positive and negative dust species in simulation case II versus time.

We will use the same trapping condition that was used in simulation case I, since lowering the collisionality shouldn't affect the linear dispersion dramatically. Linear theory predicts similar real frequency and growth rate to that of simulation case I since $\gamma/\omega_r \gg \nu/\omega_r$. We see in Figure 3.16 that the negative dust velocity distribution is shifting some particles toward the phase velocity of the wave. It becomes much more apparent in Figure 3.17. Using a value for $\frac{q_r x_r E}{T_r} \sim 0.5$, with $k_{max} x_r \sim 0.25$, and plugging into (3.7), and taking the ratio of this value with the collision frequency listed in Table 3.2, one gets $\nu_{T-}/\nu_- \sim 21$. This value is larger than it was in case I. This means that negative particles are not being detrapped so quickly.

Eventually the system comes to a pseudo-equilibrium that would normally look like a thermalized plasma due to mixing in phase space, but since there is a background electric field present, the plasma separates back into two beams as

Figure 3.15: Average particle velocity of positive and negative dust species in simulation case II versus time.

shown in Figure 3.18. The plasma then becomes unstable again and there is linear growth as depicted in Figure 3.19.

We can see here that since the background electric field is weaker and the collision rate is lower, the large scale structure that was seen in simulation case I doesn't appear. Since the collision rate is lower, the plasma does not thermalize due to collisions and shielding of the background electric field does not occur. This allows the background electric field to accelerate the positive and negative dust particles, which forms beams out of the positive and negative dust species which then excites linear growth cyclically. The cycle of linear excitation seems to be a function of the electric field strength and collisions. A weaker background electric field requires more time to accelerate the particles back into a beam.

Figure 3.16: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case II at $t\omega_r \sim 40$

Figure 3.17: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case II at $t\omega_r \sim 60$

Figure 3.18: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case II at $t\omega_r \sim 197$

Figure 3.19: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case II at $t\omega_r \sim 253$

Figure 3.20: Average energy density in the wave electric field for simulation case III versus time.

3.4.3 Simulation case III

This case has an even lower collision rate and background electric field. The collision rates and the background electric field are one fourth those of simulation case II. It should begin to asymptote to the collisionless case as the collisionality and background electric field are now getting small.

The energy density in the electric field portrayed in Figure 3.20 again has the familiar linear growth slope and saturation. It then looks like the plasma begins to mix in phase space due to particle trapping and that is what is causing saturation to occur. The trapping condition is again going to be the same at case I, and since the collision frequency is continuing to get smaller it is safe to say that $\nu_{T\pm}/\nu_{\pm} \gg 1$. This means that the conditions for trapping as the mechanism for saturation are all there. If we look at Figure 3.23 and Figure 3.24 we can see

Figure 3.21: Temperature of positive and negative dust species in simulation case III versus time.

that trapping is occurring for the negative dust species, and even some of the low energy positive dust particles are being trapped.

The plasma thermalizes from phase space mixing and has the help of a small amount of collision with the neutral gas background. The end state of the simulation looks to be a thermalized plasma with phase space holes.

One peculiarity to note is that the average velocity of the negative dust species actually becomes positive for a time. This is exaggerated in case IV and will be discussed in more detail in the following section.

Figure 3.22: Average particle velocity of positive and negative dust species in simulation case III versus time.

Figure 3.23: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case III at $t\omega_r \sim 40$

Figure 3.24: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case III at $t\omega_r \sim 60$

Figure 3.25: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case III at $t\omega_r \sim 260$

Figure 3.26: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case III at $t\omega_r \sim 312$

Figure 3.27: Average energy density in the wave electric field for simulation case IV versus time.

3.4.4 Simulation case IV

This case is the final simulation run that will be examined. The collision rates and the background electric field are one fourth those of case III, or rather the baseline collision rates and electric field divided by 64.

We start by again looking at the average energy density of the simulation as a function of time. The system again shows linear growth and saturation at the same times as the systems before.

If we look at Figure 3.30 and Figure 3.31 we definitely see particle trapping. We can use the same analysis used in the previous cases for the trapping condition and the trapping frequency. We can use the same reasoning used in case III to state that $\nu_{T\pm}/\nu_{\pm} \gg 1$, and most of the trapping is occurring on the negative dust

Figure 3.28: Temperature of positive and negative dust species in simulation case IV versus time.

species. Looking at the distribution plot in Figure 3.32 we see that the negative dust species is spreading throughout velocity space and is becoming very chopped up. There are phase space vortices that appear. They eventually coalesce into a single phase space hole that can be seen in Figure 3.33. Similar behavior can be seen in the simulation done in [24], which is a simulation of a collisionless pair plasma. Pair plasmas are two species plasmas where the species have opposite charge and the same mass. There are no background species to provide shielding in a pair plasma. Pair plasmas exhibit some of the same behavior as the system we are examining. The simulation in [24] shows a similar end state to this case, where the plasma looks thermalized with phase space holes.

Upon inspection of Figure 3.29, we see that the average particle velocity of the negative dust species changes direction. This is due to the fact that the negative dust species velocity distribution is broadening, as can be seen in Figure 3.32 and

Figure 3.29: Average particle velocity of positive and negative dust species in simulation case IV versus time.

Figure 3.33. The broadening happens more on the side of the velocity distribution where the wave action is occurring. This tends to shift the mean of the distribution more toward the phase velocity of the most unstable wave mode. Since the electric field is so weak it does not accelerate the particles back to their original drift velocities. If we look at the electric field plot in Figure 3.33, we see that the wave electric field is much stronger than the background electric field, so particle motion associated with the wave field dominates over the acceleration due to the background electric field.

Figure 3.30: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case IV at $t\omega_r \sim 40$

Figure 3.31: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case IV at $t\omega_r \sim 60$

Figure 3.32: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case IV at $t\omega_r \sim 122$

Figure 3.33: Phase space (a), electric field (b), velocity distributions (c), and wave potential (d) of simulation case IV at $t\omega_r \sim 312$

Chapter 4

Summary and Discussion

In this thesis we explore the linear and nonlinear behavior of a dust-dust instability in a plasma containing dust grains of opposite charge polarity, taking into account collisions with neutrals and a background electric field. The model dusty plasma contains four species of charged particles: electrons, ions, positively charged dust grains, and negatively charged dust grains. The dust grains do not have equal mass, but their charge to mass ratio is the same. The plasma is weakly ionized and embedded in a neutral background gas; this means that collisions of charged particles with the background neutral gas can be important and are included in the model. There is also a background electric field which imparts a drift velocity to all of the charged species in the plasma. It was shown in Chapter 2 that the drift that is imparted to the positively and negatively charged dust grains by the background electric field is sufficient to drive the dust-dust instability, but the ion and electron drift speeds are not sufficient to excite instabilities. Since the electron and ion streaming velocities are much less than their respective thermal velocities, and since we consider waves with phase speed much less than the ion thermal speed, the ion and electron species are modeled as Boltzmann distributed.

Linear theory was developed in chapter 2, first using fluid theory to predict the critical drift velocity of this dust-dust instability. Since fluid theory uses approximations which are not valid for the parameters that were chosen, as the drift speeds for the positively and negatively charged dust grains are only a few times their respective thermal speeds, thermal effects can be important. Thus, we then numerically solved the fully kinetic dispersion relationship to predict the linear growth rate and frequency of the instability in our system.

In chapter 3, a Particle in Cell (PIC) simulation of the plasma was presented where there was a constant background electric field in the system, as well as collisions with neutrals modeled using the Langevin operator. The Langevin operator models collisions using a constant collision rate, and it has a tendency to either cool or heat the dust grains to the temperature of the background gas, as well as slow the dust grains since the background gas had zero drift in the simulation reference frame. The first simulation presented was of the set of parameters discussed in chapter 2. It was seen that the linear instability grows as predicted by linear kinetic theory. The saturation mechanism appears to be a form of effective dust heating that is related to dust particle trapping. The dust trapping widens the distribution and reduces the average particle velocity of the distribution enough to shut off the linear instability. It was seen that the negative dust species is trapped by the wave, and some of the low energy positive dust grains are trapped as well.

The nonlinear development of the baseline simulation looks to develop into a potential double layer due to the strong electric field that accelerates the dust particles out of the low density areas to maintain the current that is passing through the plasma in the low density regions. This result seems to depend on the background electric field, the background collision rate, and the periodic boundary conditions used in the simulation. It should be noted that this is a one dimensional periodic simulation. It would be interesting to consider this counter streaming dust-dust

instability in an aperiodic one dimensional system, or even in a two or three dimensional system to see if these density voids form under other conditions. There is more work that needs to be done to completely understand this phenomenon.

A comparative study was done in which a number of simulations were performed with reduced collision rates and reduced background electric fields to attempt to understand the nonlinear development of the system. It was seen that the simulations with reduced background electric fields and collision rates did not form the large scale structures that were seen in simulation case I. We saw that in simulation case II there was a periodic re-excitation of the linear instability due to cooling of the positive and negative dust species by the Langevin operator, which models background neutral collisions, and the acceleration of the positive and negative dust species by the background electric field driver.

In cases III and IV the background neutral collisions were relatively weak and the system developed in a similar fashion to collisionless one dimensional simulations. It looked like the saturation mechanism for linear instability shutoff was dust heating via trapping. Then depending on the collision rate the particles could be detrapped and thermalized by the Langevin collision operator.

In the future it could be interesting to analyze single particle trajectories and calculate statistics on how often particles detrap and compare these statistical results to theory. This could be related to dusty plasma experiments that use Particle Imaging Velocimetry (see [42, 43, 44]) to measure dust grain velocity distribution functions by tracking single particle motion in a laboratory system. The results from this set of simulations could also be compared against similar instabilities in pair plasmas and negative ion plasmas. There needs to be an investigation to see if there is quasi-linear diffusion occurring in the nonlinear stages of development of the system. The mechanism for the development of the double layer needs to be fully understood. The critical currents, background

field strength, and collision rate for double layer formation need to be worked out theoretically and tested against the simulations as well as any future experiments.

Appendix A

Description of PIC Simulation

The simulation software used for a large portion of the analysis presented in the main body of the text is a one dimensional particle in cell (PIC) simulation written in Fortran 90. It is a timestepping code that keeps track of a manageable number of charged macro-particles. In each timestep, the simulation tracks the position of these macro particles, then iteratively solves Poisson's Equation for the self-consistent electrostatic potential on a uniform spatial grid. The potential is then used to calculate the electric field. Once the electric field for a given timestep is known then it is used to update the velocity of the macro particles. The velocity is used to update the particle's position. The boundary conditions for the simulation were chosen to be periodic for ease of implementation. The periodic boundary conditions are much easier to implement than inflow or outflow boundary conditions since the number of particles will be conserved automatically. This is a good setup for examining instabilities as it has some similar assumptions made that are used for linear stability analysis, i.e. the plasma is infinitely long and initially homogeneous.

Using a small set of macro-particles is an approximation that is used to

make the problem computationally tractable. It would be infeasible to simulate the true number of particles in the plasma. The macro-particle approximation and its accuracy have been developed for a number of years and is presented in great detail in [20].

A.1 Electrostatic field solver

One of the major steps in the algorithm is to solve for the self-consistent electric field produced by all of the particles in the system. This includes electrons, ions, and both polarities of dust grains.

A.1.1 Boltzmann approximation for electrons and ions

During the analysis of the case in the main body of the text the Boltzmann approximation was used to model the density profile of the electrons and ions:

$$n_{e,i} = n_{e0,i0} \exp \frac{-q_{e,i}\phi}{T_{e,i}}, \quad (\text{A.1})$$

which is expressed in normalized units. This approximation can be used in these simulation cases since the problem is in a regime where ion and electron streaming doesn't excite dust acoustic waves as discussed in Chapter 2 and Chapter 3.

A.1.2 Linearizing and solving the Poisson equation

The equation that needs to be solved is:

$$\nabla^2 \phi = \sum_j -4\pi \rho_j. \quad (\text{A.2})$$

In the case analyzed in this thesis Equation A.2 can be expanded into:

$$\nabla^2 \phi = -4\pi\rho_D + 4\pi|q_e|n_e - 4\pi|q_i|n_i, \quad (\text{A.3})$$

where ρ_D is the charge density calculated from the macro-particles that are tracked in the simulation. The charge of the electrons and ions is expressly written as q_e and q_i since it will be easier to deal with later when normalizing to Z_+ . Using the Boltzmann approximation listed in Equation A.1, writing $\phi = \phi_0 + \delta\phi$, and Taylor expanding the exponential of $\delta\phi$ one gets:

$$\begin{aligned} \nabla^2(\phi_0 + \delta\phi) = & -4\pi\rho_D + 4\pi|q_e|n_{e0} \exp\left(\frac{|q_e|\phi_0}{T_e}\right) \left(1 + \frac{|q_e|\delta\phi}{T_e}\right) \\ & - 4\pi|q_e|n_{i0} \exp\left(\frac{-|q_i|\phi_0}{T_i}\right) \left(1 - \frac{|q_i|\delta\phi}{T_i}\right). \end{aligned} \quad (\text{A.4})$$

After some algebraic manipulation and unit normalization one obtains:

$$\begin{aligned} \nabla^2 \delta\phi - \left[\frac{Z_e^2 n_{e0}}{T_e} \exp\left(\frac{|Z_e|\phi_0}{T_e}\right) + \frac{Z_i^2 n_{i0}}{T_i} \exp\left(\frac{-|Z_i|\phi_0}{T_i}\right) \right] \\ = -\nabla^2 \phi_0 - \rho_D + |Z_e|n_e \exp\left(\frac{|Z_e|\phi_0}{T_e}\right) - |Z_i|n_i \exp\left(\frac{-|Z_i|\phi_0}{T_i}\right). \end{aligned} \quad (\text{A.5})$$

In the previous expressions, ϕ_0 , $\delta\phi$, and ρ_D are all functions of x . Since the simulation is performed on a discrete grid, this turns into a system of equations that can be solved using linear algebra techniques. Using Equation A.5 one can start with an initial guess for ϕ_0 and iterate to a solution by solving for $\delta\phi$, adding it to ϕ_0 and continuing. This solver converges very rapidly, normally within 10 iterations. The initial guess on the first time step is $\phi_0 = 0$ for all x , for each subsequent time step the initial ϕ_0 is the solution to ϕ from the previous time step. The initial guess of zero works well because initially all of the species are spread throughout the domain and the net potential should be roughly zero. The Laplacian operator in one dimension is just $\frac{d^2}{dx^2}$, which can be calculated in a discrete system using a centered finite difference scheme:

$$\nabla^2 f(x) = \frac{f(x - \Delta x) - 2f(x) + f(x + \Delta x)}{\Delta x^2}. \quad (\text{A.6})$$

Applying Equation A.6 to Equation A.5 yields a tridiagonal system of equations with periodic boundary conditions, which can be solved quite efficiently on each iteration using common numerical techniques, some of which are discussed in [22].

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