LBL-12479 c.2
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted for publication

POLYNOMIAL CHROMODYNAMICS IN 1 + 1 DIMENSIONS

Herbert M. Ruck

January 1981

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

DISCLAIMER

LBL-12479

POLYNOMIAL CHROMODYNAMICS IN 1 + 1 DIMENSIONS

Herbert M. Ruck

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

POLYNOMIAL CHROMODYNAMICS IN 1 + 1 DIMENSIONS

Herbert M. Ruck

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

The forces that mold the strongly interacting particles out of gluons and quarks determine the interaction between these particles themselves.

Any theory of gluons and quarks that explains hadrons provides a natural foundation for the theory of nuclear physics.

The centerpiece of our model is the Lagrangian of two scalar fields ϕ_1 and ϕ_2 [1]:

$$L_g = (1/2)(\partial_\mu\phi_1)^2 + (1/2)(\partial_\mu\phi_2)^2 - [\lambda(\phi_1^2 + \phi_2^2)^2 - \nu(\phi_1^3 - 3\phi_1\phi_2^2) - \mu(\phi_1^2 + \phi_2^2) - \delta] \quad (1)$$

The internal symmetry group of this Lagrangian is Z(3) the cyclic group of order three. A transformation of the fields

$$\phi_1 \to \phi_1 \cos\theta n - \phi_2 \sin\theta n$$
$$\phi_2 \to \phi_1 \sin\theta n + \phi_2 \cos\theta n\ , \quad \theta=2\pi/3,\ n=0,1,2 \quad (2)$$

leaves the Lagrangian invariant.

This nonlinear Lagrangian was originally constructed as a field theory analog of the three states Potts model [2].

Examining the selfinteraction of the scalar fields we observe that the cubic and quartic terms are typical of the gluon field that provides the binding force between quarks [3].

Considering the ϕ_1 and ϕ_2 fields as scalar gluons we complete the system by adding two spinor fields ψ_1 and ψ_2 in the role of quark fields. The total Lagrangian is:

$$L = L_g + i\bar{\psi}_1\gamma_\mu\partial_\mu\psi_1 - m\bar{\psi}_1\psi_1 + i\bar{\psi}_2\gamma_\mu\partial_\mu\psi_2 - m\bar{\psi}_2\psi_2$$

$$- g(\bar{\psi}_1\psi_1 - \bar{\psi}_2\psi_2)\phi_1 - g(-\bar{\psi}_1\psi_2 - \bar{\psi}_2\psi_1)\phi_2 \quad . \qquad (3)$$

For invariance the ψ-fields have to transform in the same way as the ϕ-fields:

$$\psi_1 \to \psi_1\cos\theta n - \psi_2\sin\theta n$$

$$\psi_2 \to \psi_1\sin\theta n + \psi_2\cos\theta n, \quad \theta=2\pi/3, \quad n=0,1,2. \qquad (4)$$

The two components of the ψ-fields are associated with two different colors of the quarks. ϕ_2 changes the color of the quarks whereas ϕ_1 does not change any color.

The field equations for massless quarks are:

$$\Box\phi_1 = -4\lambda(\phi_1^2+\phi_2^2)\phi_1 + 3\nu(\phi_1^2-\phi_2^2) + 2\mu\phi_1 + g(\bar{\psi}_1\psi_1-\bar{\psi}_2\psi_2) \quad , \qquad (5)$$

$$\Box\phi_2 = -4\lambda(\phi_1^2+\phi_2^2)\phi_2 - 6\nu\phi_1\phi_2 + 2\mu\phi_2 - g(\bar{\psi}_1\psi_2+\bar{\psi}_2\psi_1) \quad , \qquad (6)$$

$$i\gamma_\mu\partial_\mu\psi_1 = g(\psi_1\phi_1 - \psi_2\phi_2) \, , \qquad (7)$$

$$i\gamma_\mu\partial_\mu\psi_2 = -g(\psi_2\phi_1 + \psi_1\phi_2) \, . \qquad (8)$$

I prove that due to the Z(3) symmetry any pair of the fermionic solutions, describing the quarks, is related algebraically:

$$\psi_2 = A\psi_1 \tag{9}$$

where $A = \eta\gamma_5$ in four dimensions and $A = i\eta\sigma_3$ in two dimensions. γ_5 and σ_3 are Dirac and Pauli matrices, $\eta = \pm 1$.

This property shows that the quarks cannot be separated from each other--when one field vanishes the other disappears too. This is the basis for confinement of the quarks at low energies.

Particular solutions of the field equations are obtained in 1+1 dimensions in a closed form when the effect of the quarks upon the gluons is neglected.

Then for the relation between the coupling constraints $\nu = (2/3)\lambda\phi_V$, $\mu = \lambda\phi_V^2$, $\delta = -(2/3)\lambda\phi_V^4$, ($\phi_V$ is the vacuum field) one set of soliton solutions is [4]:

$$\phi_1(x) = \phi_V(1 + 3 \tanh\alpha(x-x_0))/4 , \tag{10}$$

$$\phi_2(x) = 3^{1/2}\phi_V(1 - \tanh\alpha(x-x_0))/4, \tag{11}$$

and the localized solutions of the quarks moving in the above gluon fields are [5]:

$$\psi_1(t,x) = N \begin{pmatrix} 1 \\ \exp(-i\pi/6) \end{pmatrix} \exp(iEt - H(x)) , \tag{12}$$

$$\psi_2(t,x) = i\eta\sigma_3\psi_1(t,x) , \tag{13}$$

$$E = -(1/2)g\ \phi_V,\ H(x)=(2\lambda)^{-1/2}g\ \ln(\cosh\alpha(x-x_0)), \tag{14}$$

with $\alpha = (3\lambda/2)^{1/2}\phi_V$. The other sets of solutions are obtained by a symmetry transformation.

The total field energy generated by these solutions defines the mass of the ground state of a finite-size composite particle:

$$M = (3\lambda/2)^{1/2}\ \phi_V^3\ - g\phi_V\ . \tag{15}$$

The first part of (15) is the contribution of the gluons and the second part that of the quarks interacting with gluons. A perturbational calculation of the time oscillations of the gluon field gives the mass of the excited states of the entire system:

$$M^* = M + 3^{1/2}\alpha \quad \text{and } M^{**} = M + 2\alpha. \tag{16}$$

We can think of removing one quark. This process violates the field equations and theorem Eq. (9) and thus costs energy. The mass of this "colored" state is larger:

$$M^c = M + 7g\ \phi_V/8\ , \tag{17}$$

removing finally two quarks leaves a glueball behind with the even larger mass:

$$M^g = M + g\phi_V. \tag{18}$$

The removed quarks cannot travel as plane waves because this costs infinite energy. The quarks rather attach themselves as localized states to some neighboring particles. The mass spectrum is shown in Fig. 1.

The important issue in nuclear physics is the interaction between particles. Two-center fields are constructed by combining two gluon solutions localized at x_1 and x_2 in the following way:

$$\phi_1(x,x_1,x_2) = \phi_V(1 + 3\tanh\alpha(x-x_1))(-1 + 3\tanh\alpha(x-x_2))/8, \quad (19)$$

$$\phi_2(x,x_1,x_2) = 3^{1/2}\phi_V\ (1 - \tanh\alpha(x-x_1))(1 + \tanh\alpha(x-x_2))/8, \quad (20)$$

and the quark fields are superposed according to the Pauli exclusion principle [6] (applied to color):

$$\psi_1(x,x_1,x_2) = \psi_1(x-x_1) - \psi_1(x-x_2), \quad (21)$$

$$\psi_2(x,x_1,x_2) = \psi_2(x-x_1) - \psi_2(x-x_2), \quad (22)$$

The interaction potential of the two particles at a distance $d = x_1-x_2$ is defined as the difference between the field energy generated by the above fields and twice the mass of the ground state (16).

The potential shows a typical shallow attraction near the touching point of the particles and a strong repulsion at complete overlap (Fig. 2).

This construction is extended to many particle systems where the results show that the binding energy increases and the distance between particles at equilibrium decreases as the number of particles is increased. This is the phenomenon of saturation observed in nuclear physics.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

REFERENCES

1. F. Constantinuscu and H.M. Ruck; J. Math. Phys. 19 (1978) 2359 and Ann. Phys. 115 (1978) 474.
2. R.B. Potts, Proc. Cambr. Phil. Soc. 48 (1952) 106.
3. H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365.
4. H.M. Ruck, Nucl. Phys. B167 (1980) 320.
5. H.M. Ruck, Lecture Notes in Physics, vol. 135 (1980) 307, Springer Verlag, Berlin.
6. W. Pauli, Z. Physik 31 (1925) 765.
   Pauli Lecture Notes in Physics, vol. 5, The MIT Press, Cambridge, Mass. (1978).

FIGURE CAPTIONS

Figure 1. The energy spectrum of a composite particle. The mass of the ground state is chosen to be ten, to give an easy reference point for the rest of the spectrum. The coupling constants are $\lambda = 1$, $g = 5$, and the value of the vacuum field is $\phi_V = 2.6717$. The mass is measured in units of $\lambda^{1/2}$.

Figure 2. The interaction potential of two composite particles in their ground state of mass ten. The parameters are the same as in Fig. 1. the attraction is due to the gluons whereas the repulsion is due to the quarks. The distance is measured in units of $\lambda^{-1/2}$. The extension of this particle calculated as an average value is 0.387 also in units of $\lambda^{-1/2}$.

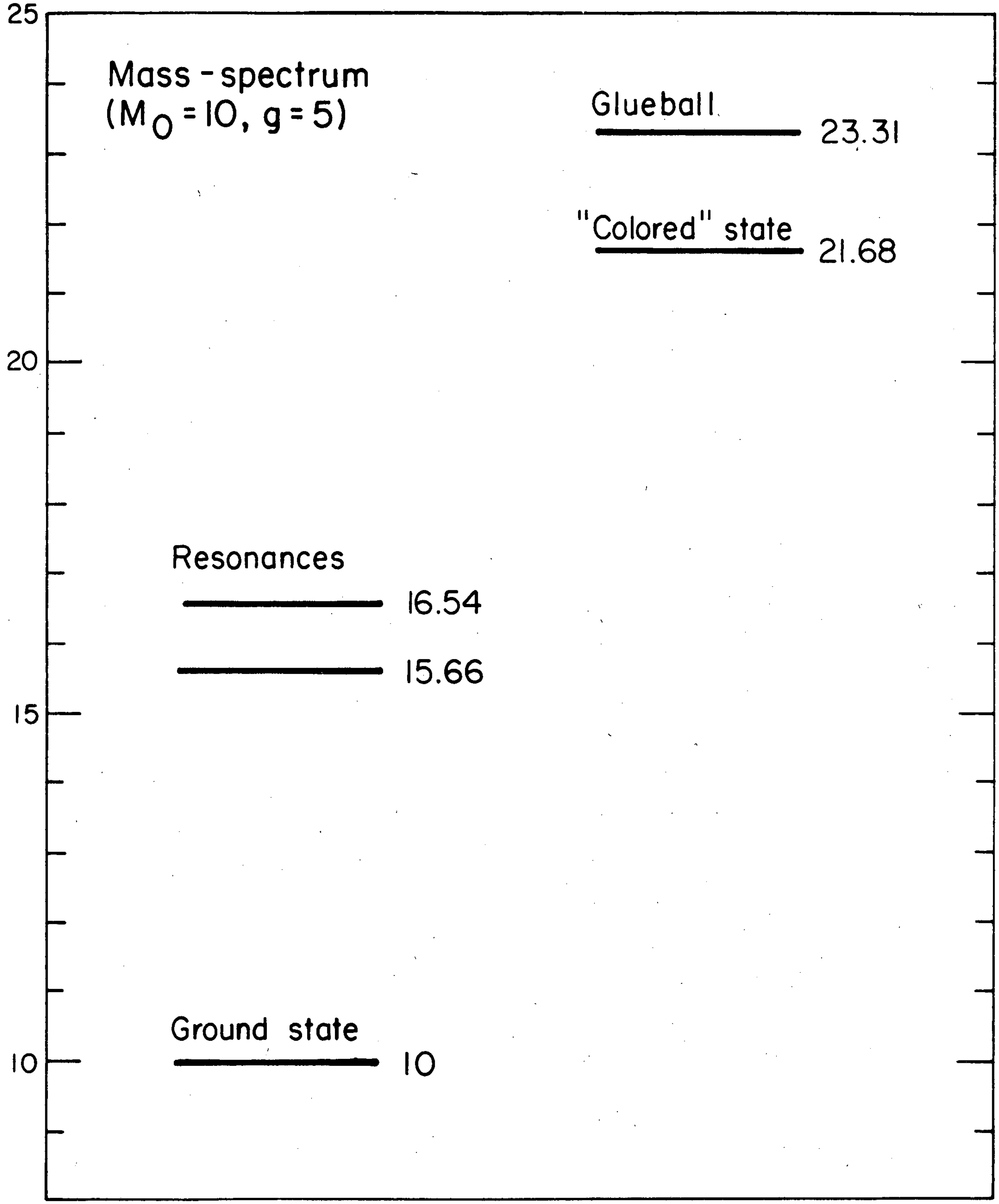
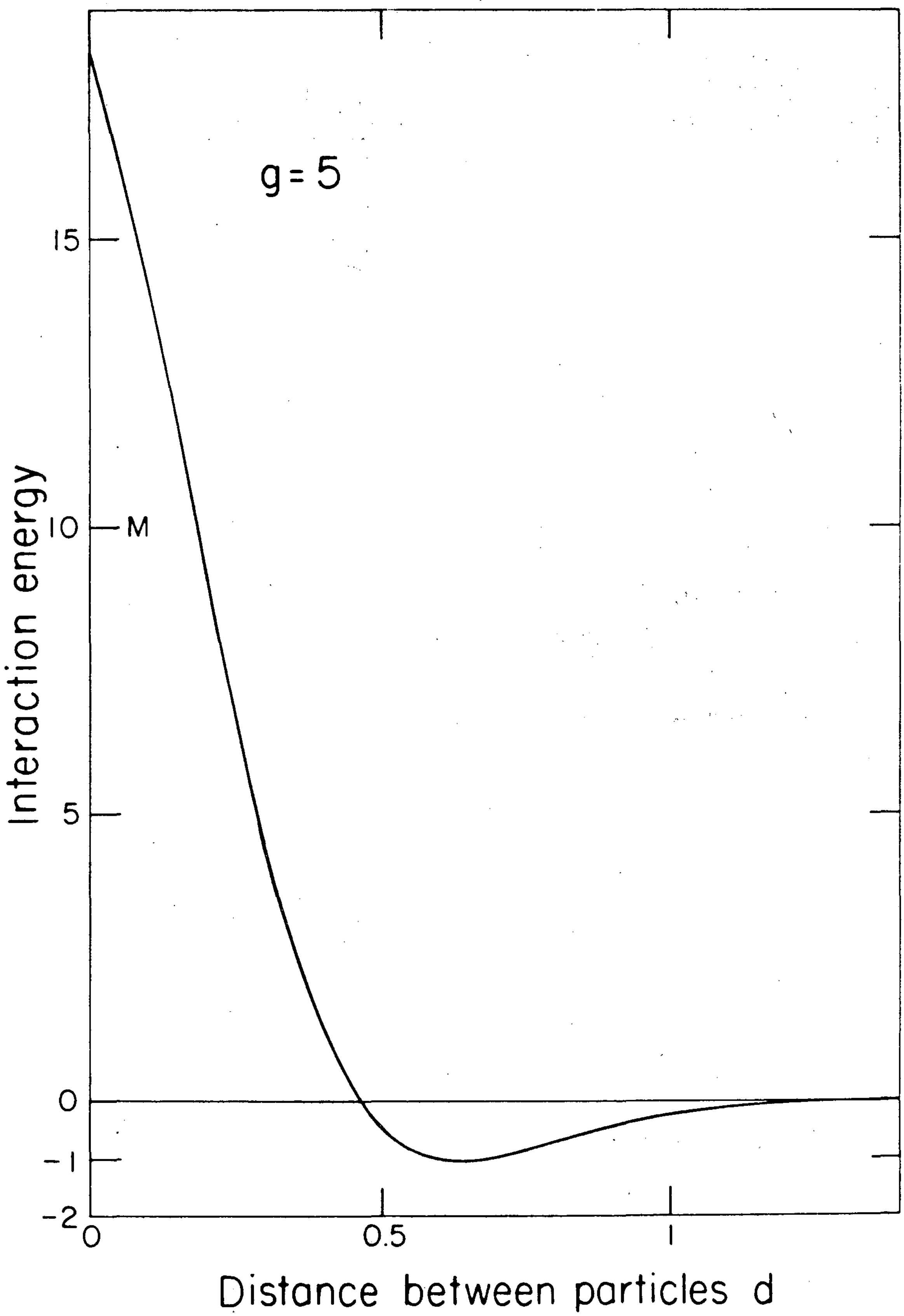


Fig. 1

XBL 814-2227



XBL 812 - 2162

Fig. 2

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720