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Relativistic treatment of quark-antiquark spectra in quantum chromodynamics

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A fully relativistic bound-state equation is used to study the spectra (and leptonic widths) of charmonium, b-quarkonium, D mesons, F mesons, B mesons, and mesons composed of light quarks. The short-distance behavior of the interaction is that of asymptotically free QCD, while the large-distance interaction is a linear confining one whose Lorentz nature is either pure scalar or an equal mixture of vector and scalar. Multiquark states are accounted for in an approximate manner. An interesting dependence of the quark masses on the Lorentz nature of the long-distance interaction and on the overall mass scale of the system is obtained.

I. INTRODUCTION

In the study of quark-antiquark systems it has come to be recognized that even in the case of charmonium there is a need for a relativistic treatment beyond the v^2/c^2 terms of the Darwin-Breit Hamiltonian which give the spin-orbit and spin-spin couplings. In this paper we study the quark-antiquark systems with a relativistic treatment using an expansion of the field equations in intermediate[1–3] states and approximating the sum over intermediate states in the antiquark channel by a single on-shell antiquark. The resulting equation, after partial-wave analysis, is a single-variable integral equation in the one off-shell momentum as opposed to the Bethe-Salpeter equation which is a double-variable integral equation in both off-mass-shell momenta.

We apply this equation to quark-antiquark systems using a vector exchange at short distances with asymptotic-freedom effects included and a combination of scalar and vector exchanges which are the linear confining potentials effective at large distances. The potentials we choose as well as the range of parameters we explore are not completely free but *highly motivated by theoretical prejudices;* at short distances we use the full asymptotically free QCD vector potential whose only parameter is $\Lambda_{\rm QCD}$, and the large-distance linear confining potential is either an equal combination of vector and scalar exchanges, as motivated by the bag model,[4] or a pure scalar linear potential. Other works parametrized the QCD potential by an arbitrary multiplicative factor,[5] and when, in a relativistic treatment, a distinction between vector and scalar exchanges for the linear part was made, this exchange was taken only to be purely scalar in order to avoid problems with the Klein paradox.[6] We fit the parameters of the exchanges (with the above-mentioned theoretical restrictions in mind) and a cutoff representing inelastic channels to the charmonium mass spectra and to the leptonic decay widths which reflect the wave functions at short distances. We fit the ratios of the leptonic widths as well as the magnitudes modulo the unknown QCD corrections. The relativistic effects also show up in the ψ-η_c, ψ'-η_c', and P-wave splittings. We fit the ψ-η_c, P-wave splitting, and leptonic width successfully using a bag-model-motivated linear potential with equal vector and scalar strengths. The ψ'-η_c' splitting is too small, as in other fits, being about 50 MeV instead of the 90 MeV indicated by the newly discovered η_c'.

The potential with its parameters fixed is then applied to the b-quarkonium spectra with good results, including the splitting of the newly discovered $2P$ levels.[7] The ratio of the leptonic decay widths reproduces the experimental results and the magnitudes are closer to experiment than in the charmonium case. This is to be expected since the QCD corrections at higher mass are smaller. Predictions are made of the location of the η_b and $\eta_{b'}$.

The relativistic equation and potential are then applied to the heavy-light-quark systems of the $\bar{c}u$, $\bar{c}d$, and $\bar{c}s$ mesons. The D mass and D-D^* splitting favor the use of very-light-mass u and d quarks in the case of equal vector and scalar linear potentials and masses of the order of several hundred MeV in the case of a purely scalar linear potential.

We then turn to the light-light-quark states. The equal-vector-and-scalar linear long-range choice does not give states below a GeV for any quark mass below 500 MeV and reasonable cutoff. We also tried a purely scalar long-range linear part. This will give an approximate fit to the light-meson states provided we take large u-, d-, and s-quark masses at this larger-distance scale. This may be motivated by QCD mass-renormalization effects. The light-mass mesons π, K, and η are not given correctly due to the lack of inclusion of chiral-symmetry effects in the bound-state equation. Likewise the masses of the light quarks differ from those in the fits to the heavy-light-quark states. We shall discuss these effects through the use of renormalization-group arguments.

The integral equation that we use has the following appealing properties:

(a) Upon performing a partial-wave analysis it becomes a one-variable integral equation.

(b) Since one leg is on shell it is current conserving and even the one-gluon exchange is gauge invariant.

(c) In the nonrelativistic limit it reduces to the Darwin-Breit Hamiltonian, which includes the v^2/c^2 effects.

(d) It has the Dirac equation as a limit as the on-shell quark becomes extremely heavy relative to the off-shell quark (which should be helpful in the heavy-quark—light-quark systems).

(e) It includes retardation effects which cannot be reproduced by spatial potentials.

(f) As it is a momentum-space equation it is straightforward to incorporate the renormalization-group momentum dependence of the QCD coupling strength.

This equation was formulated by Greenberg[1] in the N-quantum approximation and applied to the deuteron problem by Gross.[2] It has been applied to the problem of deeply bound composites by[8] Bander, Chiu, Shaw, and Silverman.

In Sec. II the relativistic bound-state equation is formulated and a partial-wave analysis is performed. An expression for the leptonic decay width is also derived. In Sec. III the short- and long-range QCD potentials are formulated. In Sec. IV the fit to the charmonium spectra is presented. In Sec. V we discuss the b-quarkonium spectra predictions and in Sec. VI the heavy-light-quark fits are discussed. Section VII describes the light-light-quark fits, and in Sec. VIII we present a summary of our results.

II. FORMULATION OF RELATIVISTIC EQUATIONS FOR QUARK-ANTIQUARK SYSTEMS

A. Derivation of the bound-state equation

The relativistic bound-state equation we shall use is based on the equation for a quark field $\psi(x)$ coupled to a gauge potential A_μ:

$$(i\partial\!\!\!/ - m_1)\psi(x) = g A\!\!\!/(x)\psi(x) \ . \tag{2.1}$$

m_1 is the mass of the quark. The bound-state equation is obtained from the matrix element of (2.1) between the bound state of momentum B and an antiquark state of mass m_2, momentum $\vec{p}$, and spin λ (for the moment we do not indicate the spin quantum numbers of the bound state):

$$(i\partial\!\!\!/ - m_1)\langle \vec{p},\lambda \,|\, \psi(x) \,|\, \vec{B}\rangle = g\sum_n \langle \vec{p},\lambda \,|\, A\!\!\!/(x) \,|\, n\rangle\langle n \,|\, \psi(x) \,|\, \vec{B}\rangle \ . \tag{2.2}$$

A complete set of states has been inserted into the right-hand side of (2.2). Up to this point the equations are exact. The approximation we have in mind consists of keeping only the antiquark state in the sum. The justification for this approximation is that, at least at large distances, the valence quark model appears to work quite well; in this model the mesons are made up of only a quark and an antiquark. The effect of the inclusion of multiquark states or states composed of quarks and gluons will be discussed at the end of this and in subsequent sections. In this approximation (2.2) becomes a linear equation for the matrix element of the quark field between the bound state and an antiquark state. We define

$$\Psi_\alpha(\vec{p},\lambda) = (2\pi)^3 \left[\frac{2\omega_B\omega(p)}{m_2}\right]^{1/2} \langle \vec{p},\lambda \,|\, \psi_\alpha(0) \,|\, \vec{B}\rangle \ , \tag{2.3}$$

where $\omega_B = (B^2+M^2)^{1/2}$, $\omega(p) = (p^2+m_2{}^2)^{1/2}$, m_2 is the antiquark mass, and M is the unknown bound-state mass. The potential for this problem is obtained from the matrix element of the gauge potential or its current in the antiquark state. The simplest form of the bound-state equation is obtained by taking the perturbative result for this matrix element:

$$(B\!\!\!/ - p\!\!\!/ - m_1)\Psi(p,\lambda) = \sum_{\lambda'} \int \frac{d^3p' m_2}{(2\pi)^3\omega(p')} \frac{g^2}{(p'-p)^2} \gamma_\mu \Psi(p',\lambda')\bar{v}(p',\lambda')\gamma^\mu v(p,\lambda) \ . \tag{2.4}$$

We wish to extend this result to more general interactions in order to be able to include the effects of asymptotic freedom and linear confinement which involve both vector and scalar exchanges. Detailed forms for these exchanges will be discussed in Sec. III. The generalization of (2.4) that we shall use is

$$(B\!\!\!/ - p\!\!\!/ - m_1)\Psi(p,\lambda) = \sum_{\lambda'} \int \frac{d^3p' m_2}{(2\pi)^3\omega(p')} [\, V_V((p'-p)^2)\gamma_\mu \Psi(p',\lambda')\bar{v}(p',\lambda')\gamma^\mu v(p,\lambda) + V_S((p'-p)^2)\Psi(p',\lambda')\bar{v}(p',\lambda')v(p,\lambda)] \ . \tag{2.5}$$

Equation (2.5) is a linear eigenvalue problem for the bound-state mass M. The interaction is given by the as yet unspecified V_V and V_S. In the limit $m_2 \to \infty$ this equation reduces to the Dirac equation

$$[i\partial\!\!\!/ - m_1 - \gamma_0 V_V(r) - V_S(r)]\psi(x) = 0 \ , \tag{2.6}$$

where V_V and V_S are the corresponding potentials in position space.

B. Angular-momentum decomposition

As usual, Eq. (2.5) is solved by first performing an angular-momentum decomposition. It is useful to introduce a 4×4-component wave function Φ:

$$\Phi_{\alpha\beta}(p)=\sum_{\lambda}\Psi_{\alpha}(p,\lambda)\bar{v}_{\beta}(p,\lambda)\ . \tag{2.7}$$

The wave equation for Φ is

$$(\not{B}-\not{p}-m_1)\Phi(p)=\int\frac{d^3p'}{(2\pi)^3 2\omega(p')}[V_V((p'-p)^2)\gamma_\mu\Phi\gamma^\mu(\not{p}-m_2)+V_S((p'-p)^2)\Phi(\not{p}-m_2)]\ . \tag{2.8}$$

In terms of the $\vec{\alpha}$ and β matrices this equation takes the form

$$[M-\omega(p)-\vec{\alpha}\cdot\vec{\mathrm{p}}-\beta m_1]\Phi(p)=\int\frac{d^3p'}{(2\pi)^3 2\omega(p')}\{V_V((p'-p)^2)[\Phi(p')+\vec{\alpha}\cdot\Phi(p')\vec{\alpha}]$$
$$+V_S((p'-p)^2)\beta\Phi(p')\beta\}[\omega(p)-\vec{\alpha}\cdot\vec{\mathrm{p}}-\beta m_2]\ . \tag{2.9}$$

As in the case of the spinor Dirac equation we introduce the analogs of large and small components; in this situation we have four 2×2 submatrices:

$$\Phi=\begin{bmatrix}\tilde{G}_u & \tilde{G}_d\\ \tilde{F}_u & \tilde{F}_d\end{bmatrix}\ . \tag{2.10}$$

In the nonrelativistic limit, $\tilde{G}_d$ is the large component, $\tilde{G}_u$ and $\tilde{F}_d$ are of order v/c, and $\tilde{F}_u$ is of order $(v/c)^2$. From the definition of Φ, Eq. (2.7), it follows that

$$\Phi(p)(\not{p}+m_2)=0 \tag{2.11}$$

and all the components of (2.10) are not independent. Equation (2.11) expresses the fact that the antiquark is on the mass shell. As independent components we shall choose $\tilde{G}_d$ and $\tilde{F}_d$. The other ones are then given by

$$\tilde{G}_u=-\frac{\tilde{G}_d\vec{\sigma}\cdot\vec{\mathrm{p}}}{\omega(p)+m_2},\quad \tilde{F}_u=-\frac{\tilde{F}_d\vec{\sigma}\cdot\vec{\mathrm{p}}}{\omega(p)+m_2}\ . \tag{2.12}$$

The equations for the two independent components are

$$\begin{bmatrix}[M-\omega(p)-m_1]\tilde{G}_d+\vec{\sigma}\cdot\vec{\mathrm{p}}\tilde{F}_d\\ [M-\omega(p)+m_1]\tilde{F}_d+\vec{\sigma}\cdot\vec{\mathrm{p}}\tilde{G}_d\end{bmatrix}$$
$$=\int\frac{d^3p'}{(2\pi)^3 2\omega(p')}\left[V_V((p'-p)^2)\begin{bmatrix}[\omega(p')+m_2](\tilde{G}_d+\vec{\sigma}\cdot\tilde{F}_u\vec{\sigma})-\tilde{G}_u\vec{\sigma}\cdot\vec{\mathrm{p}}-\vec{\sigma}\cdot\vec{\mathrm{p}}\tilde{F}_d+i\vec{\mathrm{p}}\cdot(\vec{\sigma}\times\tilde{F}_d\vec{\sigma})\\ [\omega(p')+m_2](\tilde{F}_d+\vec{\sigma}\cdot\tilde{G}_u\vec{\sigma})-\tilde{F}_u\vec{\sigma}\cdot\vec{\mathrm{p}}-\vec{\sigma}\cdot\vec{\mathrm{p}}\tilde{G}_d+i\vec{\mathrm{p}}\cdot(\vec{\sigma}\times\tilde{G}_d\vec{\sigma})\end{bmatrix}\right.$$
$$\left.+V_S((p'-p)^2)\begin{bmatrix}-[\omega(p)+m_2]\tilde{G}_d-\tilde{G}_u\vec{\sigma}\cdot\vec{\mathrm{p}}\\ [\omega(p)+m_2]\tilde{F}_d+\tilde{F}_u\vec{\sigma}\cdot\vec{\mathrm{p}}\end{bmatrix}\right]\ . \tag{2.13}$$

It will be easier to perform the angular-momentum decomposition by going to the direct-product representation for the quark and antiquark spinors which results in moving all the Pauli matrices to the left (remembering that those presently on the right-hand side of the G and F matrices refer to the antiquark). Defining

$$\begin{bmatrix}G_d\\ F_d\\ G_u\\ F_u\end{bmatrix}=\begin{bmatrix}\tilde{G}_d\\ \tilde{F}_d\\ \tilde{G}_u\\ \tilde{F}_u\end{bmatrix}\sigma_y \tag{2.14}$$

and noting that $\vec{\sigma}^{\,T}=-\sigma_y\vec{\sigma}\sigma_y$ we obtain

$$\begin{bmatrix} [M-\omega(p)-m_1]G_d+\vec{\sigma}_1\cdot\vec{p}F_d \\ [M-\omega(p)+m_1]F_d+\vec{\sigma}_1\cdot\vec{p}G_d \end{bmatrix}$$

$$= \int \frac{d^3p'}{(2\pi)^3 2\omega(p')} \left[V_V((p'-p)^2) \begin{bmatrix} [\omega(p')+m_2](G_d-\vec{\sigma}_1\cdot\vec{\sigma}_2F_u)+\vec{\sigma}_2\cdot\vec{p}G_u-(\vec{p}\cdot\vec{\sigma}_1+i\vec{p}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2))F_d \\ [\omega(p')+m_2](F_d-\vec{\sigma}_1\cdot\vec{\sigma}_2G_u)+\vec{\sigma}_2\cdot\vec{p}F_u-(\vec{p}\cdot\vec{\sigma}_1+i\vec{p}\cdot(\vec{\sigma}_1\times\vec{\sigma}_2))G_d \end{bmatrix} \right.$$

$$\left. + V_S((p'-p)^2) \begin{bmatrix} \vec{\sigma}\cdot\vec{p}G_u-[\omega(p)+m_2]G_d \\ -\vec{\sigma}\cdot\vec{p}F_u+[\omega(p)+m_2]F_u \end{bmatrix} \right] . \tag{2.15}$$

We may note in passing that the nonrelativistic limit of Eq. (2.15) for G (after substituting for F), to order $(v/c)^2$, coincides with the Darwin-Breit Hamiltonian.

In order to proceed with the angular-momentum decomposition we expand the functions G and F in terms of total-angular-momentum eigenstates. Let us do the natural-parity case first. For a bound state of total angular momentum J and z component M we have

$$\begin{aligned} G_d(\vec{p}) &= g_-(p)\,|\,J,M;L=J-1,S=1\rangle+g_+(p)\,|\,J,M;L=J+1,S=1\rangle\ , \\ F_d(\vec{p}) &= f_0(p)\,|\,J,M;L=J;S=0\rangle+f_1(p)\,|\,J,M;L=J,S=1\rangle\ . \end{aligned} \tag{2.16}$$

With $\alpha=[J/(2J+1)]^{1/2}$ and $\beta=[(J+1)/(2J+1)]^{1/2}$ and with the angular-momentum decompositions of the potentials

$$\begin{aligned} V_{V,S}((p-p')^2) &= \sum \frac{2l+1}{4\pi} P_l(\hat{p}\cdot\hat{p}')V^J_{V,S}(p,p')\ , \\ K^J_{V,S}(p,p') &= \frac{p'^2}{(2\pi)^3 2\omega(p')} V^J_{V,S}(p,p')\ , \end{aligned} \tag{2.17}$$

we obtain the desired integral equations for the angular-momentum components of the wave functions

$$T(p)\begin{bmatrix} g_- \\ g_+ \\ f_0 \\ f_1 \end{bmatrix}(p) = \int dp' K(p,p') \begin{bmatrix} g_- \\ g_+ \\ f_0 \\ f_1 \end{bmatrix}(p')\ . \tag{2.18}$$

Using $k=p/[\omega(p)+m_2]$ and $k'=p'/[\omega(p')+m_2]$ the kinetic- and potential-energy matrices are given by

$$T(p)=\begin{bmatrix} M-\omega(p)-m_1 & 0 & \alpha p & -\beta p \\ 0 & M-\omega(p)-m_1 & -\beta p & -\alpha p \\ \alpha p & -\beta p & M-\omega(p)+m_1 & 0 \\ -\beta p & -\alpha p & 0 & M-\omega(p)+m_1 \end{bmatrix}, \tag{2.19a}$$

$$K(p,p')=[\omega(p)+m_2]\begin{bmatrix} (K_V^{J-1}+K_S^{J-1})+kk'(K_V^J-K_S^J) & 0 & -3\alpha kK_V^J+\alpha k'K_V^{J-1} & \beta kK_V^J+\beta k'K_V^{J-1} \\ 0 & K_V^{J+1}+K_S^{J+1}+kk'(K_V^J-K_S^J) & 3\beta kK_V^J-\beta k'K_V^{J+1} & \alpha kK_V^J+\alpha k'K_V^{J+1} \\ \alpha kK_V^{J-1}-3\alpha k'K_V^J & -\beta kK_V^{J+1}+3\beta k'K_V^J & K_V^J-K_S^J+kk'[\alpha^2(K_V^{J-1}+K_S^{J-1})+\beta^2(K_V^{J+1}+K_S^{J+1})] & \alpha\beta kk'[K_V^{J-1}+K_S^{J-1}-K_V^{J+1}-K_S^{J+1}] \\ \beta kK_V^{J-1}+\beta k'K_V^J & \alpha kK_V^{J+1}+\alpha k'K_V^J & \alpha\beta kk'[K_V^{J-1}+K_S^{J-1}-K_V^{J+1}-K_S^{J+1}] & K_V^J-K_S^J+kk'[\beta^2(K_V^{J-1}+K_S^{J-1})+\alpha^2(K_V^{J+1}+K_S^{J+1})] \end{bmatrix} \tag{2.19b}$$

These form a set of four coupled integral equations in four unknown functions. The $J=0$ case simplifies in that there are only two unknown functions. The unnatural-parity equations are obtained by interchanging f and g and letting $m_1 \to -m_1$ and $K_S^J \to -K_S^J$.

It may at first seem surprising that we obtain (in the natural-parity case) an f_1 contribution as it appears to violate charge-conjugation invariance for the case in which quark and antiquark are mutual antiparticles. We are, however, treating the particle and antiparticle on different footings in that the antiparticle is kept on shell. We are not dealing with a wave function but rather with the matrix element of a quark field between a bound state and an antiquark. Charge conjugation relates this matrix element to one of the charge-conjugate fields between the bound state and a quark state.

C. Normalization

Normalization is provided by ensuring that the charge operator $\int \psi^+\psi\, d\vec{x}$ has the correct value, namely one, when evaluated in the bound state. We again insert a complete set of states between the two field operators and, consistent with the approximation above, keep only the antiquark state. This results in

$$\frac{m_2{}^2}{(2\pi)^3 M}\int \frac{d^3p}{\omega(p)}\frac{1}{\omega(p)+m_2}\mathrm{Tr}(G_d^\dagger G_d+F_d^\dagger F_d)=1\ , \tag{2.20}$$

which in terms of the partial-wave amplitudes becomes

$$\frac{m_2{}^2}{(2\pi)^3 M}\int \frac{p^2dp}{\omega(p)}\frac{1}{\omega(p)+m_2} \times(|g_-|^2+|g_+|^2+|f_0|^2+|f_1|^2)=1\ . \tag{2.21}$$

Equation (2.19) is Hermitian with respect to the measure of (2.21). The transformation

$$(f,g)\to p\left[\frac{\omega(p)+m_2}{2\omega(p)}\right]^{1/2}(f,g)$$

brings it to an explicitly symmetric form since $V_{V,S}^J(p,p')$ in (2.1) is symmetric.

D. Leptonic widths

In addition to the energy levels we shall also be interested in the leptonic decay widths of the 1^- states. This is governed by the matrix element of the electromagnetic current between the bound state and the vacuum. With

$$\langle 0 \,|\, j_\mu \,|\, \vec{B},\lambda \rangle = \frac{\epsilon_\mu(\lambda) F}{(2\pi)^{3/2}\sqrt{2\omega_B}} , \tag{2.22}$$

the leptonic width is

$$\Gamma = \frac{4\pi\alpha^2 Q^2 F^2}{M^3} . \tag{2.23}$$

In order to evaluate F we again insert a complete set of states and keep only the usual antiquark contribution. We obtain

$$\epsilon_\mu F = \int \frac{d^3p}{(2\pi)^3} \frac{m_2}{\omega(p)} \mathrm{Tr}(\gamma_\mu \Phi) , \tag{2.24}$$

which, in terms of the partial-wave amplitudes, becomes

$$F = \frac{\sqrt{8\pi}}{(2\pi)^3} m_2 \int \frac{p^2 dp}{\omega(p)} \left[g_-(p) + \frac{p}{\omega(p)+m_2} [(\tfrac{1}{3})^{1/2} f_0(p) + (\tfrac{2}{3})^{1/2} f_1(p)] \right] . \tag{2.25}$$

In the nonrelativistic limit this is proportional to the position-space wave function at the origin.[9]

E. Momentum cutoff

The leptonic widths involve the wave functions at the origin. In the case of the Dirac equation we know that these behave as $r^{(1-\alpha^2)^{1/2}-1}$ making Eq. (2.23) unusable. Due to asymptotic freedom we expect milder behavior; our numerical studies indicate, however, that these functions are still very large, possibly infinite, at the origin and would lead to unacceptably large (by orders of magnitude) widths. This problem could be cured by taking into account pair creation. However, we expect other multiparticle states to become important before this pair creation does. For example, in the case of bound $\bar{c}$-c states a Dirac-type equation would become unstable when the potential became strong enough to produce an extra $\bar{c}$-c pair; however, the inelasticity starts as soon as a pair of light quarks can be produced. Properly, this problem should be treated in a multichannel formalism. We shall approximate these effects by introducing a large-momentum, or equivalently a small-distance, cutoff. We shall introduce into the kernel of Eq. (2.18) a cutoff function $S(\Lambda,p)$ which approaches zero for $p \gg \Lambda$ and approaches one as $\Lambda \to \infty$.

Equation (2.18) is Hermitian. In order to maintain this Hermiticity after the introduction of the momentum cutoff we must modify the normalization condition. The expression for F, Eq. (2.25), will likewise be modified. These modifications are straightforward. The normalization condition becomes

$$\frac{m_2{}^2}{(2\pi)^3 M} \int \frac{p^2 dp}{\omega(p)} \frac{S(\Lambda,p)}{\omega(p)+m_2} (g_-{}^2 + g_+{}^2 + f_0{}^2 + f_1{}^2) = 1 , \tag{2.26}$$

while the modification of (2.15) is

$$F = \frac{\sqrt{8\pi}}{(2\pi)^3} m_2 \int \frac{p^2 dp}{\omega(p)} S(\Lambda,p) \left[g_- + \frac{p}{\omega(p)+m_2} [(\tfrac{1}{3})^{1/2} f_0 + (\tfrac{2}{3})^{1/2} f_1] \right] . \tag{2.27}$$

III. POTENTIAL FOR RELATIVISTIC CALCULATIONS AND CUTOFFS

The QCD nonrelativistic potential is generally considered to have a short-range Coulomb part modified by an asymptotically free coupling strength and a linear long-distance part. These have been combined by Richardson[10] in the three-momentum-transfer form:

$$V_R(\vec{q}^{\,2}) = -\frac{4\pi(\frac{4}{3})(4\pi/9)}{\vec{q}^{\,2} \ln(1+\vec{q}^{\,2}/\Lambda_R{}^2)} . \tag{3.1}$$

In converting this potential to a relativistic form of an exchange propagator, its Lorentz-transformation properties must be specified. While the large-$\vec{q}^{\,2}$ modified Coulomb part is due to a vector exchange, the long-range infinitely rising part cannot be pure vector since it would lead to a Klein paradox. In order to avoid this paradox, the scalar linear part must equal or exceed the vector linear part. Thus, we are led to subtract off from the Richardson potential its long-range or low-$\vec{q}^{\,2}$ part and add in separate linear vector and scalar parts. The bag model[4] favors an equal mixture of vector and scalar linear confining potentials. We explored this case as well as the situation where the confining potential is purely scalar. As mentioned in the Introduction, our parameters were highly motivated by theory, and as a result we did not attempt to find the best arbitrary combination of vector and scalar exchange potentials. The overall slope or string tension was determined by a fit to the spectra.

For two reasons we cut off the linearly rising potential beyond a large distance. First, we do not expect the confining linear potential to rise forever. At some point it becomes favorable to create a pair of light quarks out of the vacuum resulting in a leveling off of the potential. In a nonrelativistic treatment the eigenvalues of levels well below this leveling off are very insensitive to the exact position of this ramp. Had we been dealing with a pure vector linear potential, such a leveling off would have been crucial for the avoidance of the Klein paradox. With equal vector and scalar exchanges there would be no Klein paradox in the Dirac case. The second reason is computational. In order to regulate the Fourier transform of the linearly rising potential a cutoff is likewise needed. We

leveled off the potential at a distance b or equivalently at a height κb. In the limit $b \to \infty$, the potential and the solutions to our equations are independent of b. In practice, we chose ramps at values of around 3 GeV and the results were very insensitive to the precise value of this height. We did not treat this ramp height as a parameter. The spatial form and Fourier transform of the linear ramp which rises linearly to $r=b$ with slope κ and levels off at a height κb is

$$V_L(r)=\kappa r\theta(b-r)+\kappa b\theta(r-b)\ , \tag{3.2}$$

$$V_L(\kappa,q=|\vec{q}\,|)=(2\pi)^3\kappa b\delta^3(\vec{q}) + \frac{4\pi\kappa}{q^4}[bq\sin(bq)+2\cos(bq)-2]\ . \tag{3.3}$$

The δ function arises from the part of the potential that is constant out to large distances. The apparent divergence of this term as $b \to \infty$ is exactly canceled when integrated over $\vec{q}$ by the same divergence of opposite sign present in the non-δ-function terms of (3.3).

To convert these potentials to relativistic functions of four-momentum transfer q^2 we replace $\vec{q}^{\,2}$ by $-q^2$ and include them in the relativistic equations as vector or scalar exchanges. Since the antiquark is always on its mass shell, q^2 is always negative. We can thus take over the Richardson form, which avoids pseudopoles at $\vec{q}^{\,2}$ positive, directly into a relativistic form with $\vec{q}^{\,2} \to -q^2$. This form includes the effects of asymptotic freedom in the high-q^2 limit. The relativistic form of the linear part is not known. However, since it is important only at smaller $\vec{q}^{\,2}$, where the retardation in converting to q^2 is only a v^2/c^2 correction, the difference does not have a significant effect on the calculations, i.e., for $p,p' \ll m_2$ we have

$$q^2 \cong -\vec{q}^{\,2}+(p-p')^2(p+p')^2/4m_2{}^2\ . \tag{3.4}$$

In extending the large-distance-cutoff procedure, explained below (3.3), from $\vec{q}^{\,2}$ to $-q^2$, we note that as $|\vec{q}\,| \to 0$ so does $-q^2$, giving

$$-q^2 \to \vec{q}^{\,2}[1-(p^2/\omega^2)\cos^2\theta_{pq}]\ . \tag{3.5}$$

We consider a potential $V(\vec{q}^{\,2})$ which has a long-range-constant part V_0 giving a $(2\pi)^3V_0\delta(\vec{q})$ term as in (3.3). In an integral equation over the potential $V(-q^2)$, in the large-b or small-$|\vec{q}\,|$ limit we have

$$\begin{aligned}\int d^3q\, V(-q^2)&=2\pi\int(-q^2)d(-q^2)^{1/2}\int_{-1}^{1}d\cos\theta[1-(p^2/\omega^2)\cos^2\theta]^{-3/2}V(-q^2)\\ &=\frac{\omega}{m}4\pi\int(-q^2)d(-q^2)^{1/2}V(-q^2)\ .\end{aligned} \tag{3.6}$$

If the $V_0\delta(\vec{q})$ term canceled for the nonrelativistic potential [substituting $(-q^2)^{1/2}=|\vec{q}\,|$ above] then the cancellations of the V_0-dependent terms as $V_0 \to \infty$ will now only occur in the relativistic form if we include a factor of ω/m in front of the δ function. This gives for the relativistic form of (3.3)

$$V_L(\kappa,q=(-q^2)^{1/2})=(2\pi)^3\kappa b(\omega/m)\delta^3(\vec{q})+(4\pi\kappa/q^4)[bq\sin(bq)+2\cos(bq)-2]\ . \tag{3.7}$$

In the relativistic form of the short-range vector Coulomb potential modified by the asymptotic-freedom coupling strength we use the Richardson form with $\vec{q}^{\,2} \to -q^2$. We subtract out the $q^2 \to 0$ part that would give the linear rise and replace it later with both vector and scalar parts as in Eq. (3.7). The resulting short-range vector potential is then

$$V_C(q^2)=-4\pi(16\pi/27)\times\left[\frac{1}{-q^2\ln(1-q^2/\Lambda_R{}^2)}-\frac{\Lambda_R{}^2}{(q^2)^2}\right]\ . \tag{3.8}$$

The limit of $V_C(q^2)$ as $q^2 \to 0$ is now the same as a long-range Coulomb potential, behaving as $1/q^2$. This avoids any infrared singularities in the partial-wave analysis and in the integral equation.

We now put the short- and long-range parts of the potentials together (in their relativistic forms). For the vector part we have the QCD short-range exchange and a linear potential with slope κ_V as given in Eq. (3.7):

$$V_V(q^2)=V_C(q^2)+V_L(\kappa_V,q)\ . \tag{3.9}$$

For the scalar-exchange potential we have only a linear part with slope κ_S in the form of Eq. (3.7):

$$V_S(q^2)=V_L(\kappa_S,q)\ . \tag{3.10}$$

As was discussed in Sec. II E, our equations without a momentum cutoff would lead to wave functions with very large values at the origin and in turn to unacceptably large widths. This problem may be traced to the neglect of other states, beyond the on-shell antiquark state in the approximation to Eq. (2.2). A proper treatment of these effects would require the solution of a full field theory; we approximate it by introducing a large-momentum or equivalently a small-distance cutoff.

We use two forms of the cutoff (one being the square of the other) to test whether the results of the fits are sensitive to the form of the cutoff:

$$\begin{aligned}S_1(\Lambda,p)&=\Lambda^2/(\Lambda^2+p^2)\ ,\\ S_2(\Lambda,p)&=\Lambda^4/(\Lambda^2+p^2)^2\ .\end{aligned} \tag{3.11}$$

These have been chosen quadratic in p^2 so that in the nonrelativistic limit the correction is explicitly of order p^2/Λ^2 which is much smaller than the usual v^2/c^2 correction of p^2/m^2. The parameter Λ is determined by the phenomenology and depends on which quark system we study.

IV. CHARMONIUM SPECTRA AND PARAMETERS OF THE POTENTIALS AND CUTOFFS

In this section we will present fits to the charmonium spectra using the parameters of the potential or exchanges and the integral-equation cutoff. We will present fits using both the bag $\kappa_V=\kappa_S$ ansatz and the pure scalar linear ansatz. For each of these cases we will test both the single-power and double-power cutoffs of the high momenta in the integral equation. In the following sections we will apply the parameters of the potential to fit the b-quarkonium spectra and the spectra of lighter mesons.

First we should discuss the limitations of the validity of fitting the charmonium spectra using only the integral-equation results. From the study of mixing and shifting of levels by coupling to D-$\bar{D}$, D-$\bar{D}^*$, and D^*-$\bar{D}^*$ channels by Eichten, Gottfried, Kinoshita, Lane, and Yan,[11] it is known that different L levels can shift on the order of 100 to 200 MeV from the "bare" masses given by the short-range and confining potentials. These mixings to the D-$\bar{D}$ set of states will occur in the N-quantum equations in the higher-mass intermediate state in Eq. (2.2) consisting of more q-$\bar{q}$ pairs. The scattering in the resulting state will be dominated by the D-$\bar{D}$ set of bound states or resonances. That significant mixing of S-wave and D-wave states occur through these states is known by the potential model's inability to account for the size of the leptonic width of the D-wave ψ'' state.

Another problem is the ratio of splittings of the P-wave states. In contrast to the atomic case where a single-power-law potential leads to a unique ratio of the splittings $({}^3P_2-{}^3P_1)/({}^3P_1-{}^3P_0)$, independent of radial matrix elements, in the quarkonia case the ratio depends on the relative sizes of the expectation values of (r^{-3}) for the Coulomb part versus (r^{-1}) for the linear part.[12] In quarkonia then, the ratio is not unique and does not test the power law of the potential. Phenomenologically, the P waves may be moved on the order of 100 MeV by mixing. If the relative movement of the ${}^3P_1$ state differs as little as 25 MeV relative to the other P states, the splitting ratio could move from 0.5 to 1.0.

Another phenomenological problem is the complicated structure of the 4100-MeV region. In addition, the QCD corrections to the leptonic decay widths are not completely determined as higher orders seem as important as lower ones and the correct mass scale or scheme in which to evaluate the coupling strength is still an open question.[13] The ratio of the decay widths is considered to be a more firm criteria for fitting. In general we have constrained the widths, where possible, to be within a factor of 2 of the experimental ones and constrained the fits only to the ratios of the widths.

For momenta $p \ll mc$ the equation reduces to the Schrödinger equation and if for charmonium this is a fair approximation, we expect that the values of Λ_R and total $\kappa=\kappa_V+\kappa_S$ will be close to those found by Richardson (whose fits, however, did not include the splitting of levels of a given L). This is indeed the case.

The short-distance or v^2/c^2 effects of the ψ-η_c, ψ'-η_c', and P-wave splittings as well as the leptonic decay widths are roughly proportional to the cutoff Λ_c whereas the spacings of the centers of gravity of the levels are roughly independent of Λ_c. Since we will be comparing our charmonium results to 15 experimental quantities while fitting only 4 parameters we cannot expect to fit all of them. Thus, there are many presentable fits depending on one's preferences. The single fit which we present for each case is thus not uniquely best but only illustrative and consistent with the points discussed above.

The fits to the charmonium spectra are presented in Table I. First we present in Table I the fitted parameters for the potential, Λ_R, and $\kappa_S+\kappa_V$, and then the parameters for charmonium, Λ_c, and m_c. The columns $VS1$ and $VS2$ contain parameters and results of fits with $\kappa_V=\kappa_S$ and with single and double powers of the cutoff, respectively. The columns $S1$ and $S2$ are fits with a purely scalar linear confining potential and with single or double powers of the cutoff. Below the parameters are the data and the results of the four fits.

The fits to the S-, P-, and D-wave states are comparable to those of others[14] which use only a minimal number of parameters. In comparing fits $S1$ and $S2$ we see that the power of the cutoff is not crucial in the spectra and can be accommodated by the adjustment of Λ_c. The crucial difference is in the leptonic widths which are reduced by about a factor of 2 in going from a double- to a single-power cutoff for the scalar linear case. The range of 8 to 9 keV for Γ_ψ is the same as that obtained by Schrödinger-equation fits. For the equal-scalar-and-vector linear fits $VS1$ and $VS2$, a change in Λ_c and also Λ_R gives about the same fits for the single- or double-cutoff cases.

Scalar linear fits with a lower Λ_R of around 0.15 GeV can give the general charmonium energy levels and a better P-wave-splitting ratio of about 0.5 to 0.6, but the leptonic widths of the $2S$ to $4S$ states then become close to or greater than the $\psi(1S)$ leptonic width. In view of the comments at the start of this section, we have presented the fits with the successful ratios of leptonic widths.

The main spectral discrepancies of the results of using only the bound-state equation without taking into account mixing with the D-$\bar{D}$ states as discussed above are in the $3S$ and $4S$ states in the fits $VS1$ and $VS2$, or in the $2D$ state for the fits $S1$ and $S2$.

In all four fits the predicted ψ'-η_c' splitting is about 40 MeV compared to the 94-MeV splitting[15] obtained for the newly observed η_c'.

V. b-QUARKONIUM SPECTRA AND LEPTONIC DECAY WIDTHS

The parameters of the potential found in the charmonium fits, namely κ_V, κ_S, and Λ_R, in the four cases studied, will now be applied to the b-quarkonium spectrum and leptonic widths. The mass of the b quark and the cutoff Λ_c representing the effect of higher-quanta intermediate states in the b-$\bar{b}$ channel must be fitted. The two new parameters will be found by fitting five spectral values and four leptonic decay widths.

The results of the fits and predictions of as-yet-unfound spectral levels are presented in Table II. It is apparent that the four cases all lead to the same spectral results, reflecting the nonrelativistic nature of b-quarkonium. The

TABLE I. Charmonium results. Masses in MeV, Γ_ψ (leptonic decay width) in keV.

Parameters for cases:		*VS*1	*VS*2	*S*1	*S*2
Λ_R (GeV)		0.5	0.4	0.4	0.4
$\kappa_S+\kappa_V$ (GeV^2)		0.15	0.15	0.15	0.15
Λ_c (GeV)		2.4	4.0	4.0	6.0
m_c (GeV)		1.52	1.51	1.57	1.58
Data	**Expt.**	***VS*1**	***VS*2**	***S*1**	***S*2**
$\psi(1S)$	3097	3097	3097	3097	3097
$\psi'(2S)$	3686	3699	3693	3683	3684
$\psi''(1D)$	3768	3728	3726	3747	3748
$\psi(3S)$	4030±5	4125	4125	4088	4082
$\psi(2D)$	4160±20	4137	4140	4113	4115
$\psi(4S)$	4415±6	4487	4496	4411	4414
$1^3P_{\text{c.o.g}}$	3523	3492	3488	3492	3493
ψ-η_c	113±4	99	99	102	102
ψ'-η_c'	94±5	41	42	38	36
$^3P_2$-$^3P_0$	141	154	155	136	133
$R_P=\dfrac{^3P_2\text{-}^3P_1}{^3P_1\text{-}^3P_0}$	0.48	1.03	1.04	0.74	0.77
Γ_ψ	4.8±0.6	6.31	8.26	9.00	15.3
$\Gamma_{\psi'}/\Gamma_\psi$	0.40	0.43	0.41	0.51	0.39
$\Gamma(3S)/\Gamma_\psi$	0.16	0.28	0.25	0.34	0.24
$\Gamma(4S)/\Gamma_\psi$	0.09	0.20	0.18	0.27	0.18

TABLE II. *b*-quarkonium results. Masses in MeV, Γ_Υ in keV.

Parameter		*VS*1	*VS*2	*S*1	*S*2
Λ_R (GeV)		0.5	0.4	0.4	0.4
$\kappa_S+\kappa_V$ (GeV^2)		0.15	0.15	0.15	0.15
Λ_c (GeV)		4.5	9	7	10
m_b (GeV)		4.89	4.89	4.90	4.90
Data	**Expt.**	***VS*1**	***VS*2**	***S*1**	***S*2**
$\Upsilon(2S)$-$\Upsilon(1S)$	561	568	558	563	562
$\Upsilon(3S)$-$\Upsilon(1S)$	890	887	874	873	872
$\Upsilon(4S)$-$\Upsilon(1S)$	1116	1118	1119	1118	1116
$1\,^3P_{\text{c.o.g}}$-$\Upsilon(1S)$	446	421	424	431	429
$2\,^3P_{\text{c.o.g.}}$-$\Upsilon(1S)$	787	790	785	788	786
Γ_Υ	1.22	1.08	1.55	1.63	1.77
$\Gamma_{\Upsilon'}/\Gamma_\Upsilon$	0.42	0.39	0.41	0.41	0.40
$\Gamma_{\Upsilon''}/\Gamma_\Upsilon$	0.31	0.30	0.28	0.28	0.27
$\Gamma_{\Upsilon'''}/\Gamma_\Upsilon$	0.24	0.14	0.11	0.11	0.10
$2\,^3P_2$-$2\,^3P_0$	38	38	45	40	39
$^3D_1$-$\Upsilon(1S)$		656	661	673	670
$\Upsilon(1S)$-η_b		49	57	60	58
$\Upsilon(2S)$-$\eta_{b'}$		18	21	22	21
$1\,^3P_2$-$1\,^3P_0$	41	62	69	62	61
R_{2P}	0.85±0.1	0.90	0.96	0.82	0.86
R_{1P}	0.93±0.1	0.88	0.86	0.77	0.74

ratio of leptonic widths is also approximately the same, but the magnitude of the widths has some differences. The problem of the QCD corrections to the leptonic-width magnitude does not yet make this a testable prediction.

VI. MESONS COMPOSED OF HEAVY AND LIGHT QUARKS

The potentials and cutoff methods developed for charmonium and b-quarkonium may now be applied with the same relativistic equation to the case of mesons with one heavy quark and one light quark. We now take the heavy constituent as the one which is kept on the mass shell (as an antiquark). The equation then approaches the Dirac equation for the light quark, but still needs a cutoff for reasons discussed earlier. The parameters of the potentials, namely κ_V, κ_S, and Λ_R, will remain the same as in the charmonium and b-quarkonium cases, but the different nature of the higher-quanta states will necessitate a resetting of the parameter Λ_c which effectively takes these into account. Fitting the D and D^* states will give us the mass or mass limit for the light u and d quarks, and fitting the F and F^* states will give the mass of the s quark. The fits give different results for the u-, d-, and s-quark masses in the $VS1$ and $VS2$ cases from the masses in the $S1$ and $S2$ cases.

A. D and D^* mesons

The results of calculations for the $D(0^-)$ and $D^*(1^-)$, $L=0$ mesons with charged masses 1869 and 2010 MeV, respectively, are given in Table III. The results are in terms of the center of gravity of the 0^- and 1^- system of 1975 MeV and of the D^*-D splitting of 141 MeV.

First we mention the results of the direct calculation of the D and D^* mesons using the same Λ_c from the charmonium fit assuming a very-light-mass u or d quark of 50 MeV. (The calculation varies little for quarks lighter than this.) In all four cases we find a D^*-D splitting of about 200 MeV indicating that the cutoff Λ_c must be lowered for an accurate fit. Remarkably, in the cases $VS1$ and $VS2$ the center of gravity comes out correct, indicating that very light u- and d-quark masses are appropriate for these cases. For the cases $S1$ and $S2$, the center of gravities are low, requiring heavier u and d quarks.

By varying Λ_c and the u or d mass we obtain the accurate fits given in Table III. These use a very light mass for the $VS1$ and $VS2$ cases and a mass of about 250 or 220 MeV for u or d quarks in the $S1$ and $S2$ cases.

In cases $S1$ and $S2$, setting the larger u and d masses allows an exact fit to both the splitting and the center of gravity. In the cases $VS1$ and $VS2$, however, when Λ_c is set to account for the D^*-D splitting, the centers of gravity come out high by about 30 MeV, even with very light u- and d-quark masses. Extrapolating the light-quark mass from 10 MeV to zero only lowers the D and D^* states by 4 MeV and would not significantly reduce the 30-MeV difference.

This mismatch by 30 MeV may be an indication of the approximation of dropping higher-quanta states and replacing them simply by a cutoff with parameter Λ_c. We note that the use of a single or double power of the cutoff in the $VS1$ or $VS2$ case leads to the same results.

B. F and F^* mesons

In the case of the F and F^* mesons there are only two known states and two parameters to be fit, namely Λ_c and the mass of the strange quark m_s. The results for these parameters for the F and F^* states are given in Table III. Having found m_s we can make predictions of further spin-parity states of c and $\bar{s}$ quarks.

TABLE III. Heavy-light-quark-meson results.

Cases	$VS1$	$VS2$	$S1$	$S2$
	Parameters from charmonium and b-quarkonium			
Λ_c (GeV)	0.5	0.4	0.4	0.4
(κ_S,κ_V) (GeV^2)	(0.075,0.075)	(0.075,0.075)	(0.15,0)	(0.15,0)
m_c (GeV)	1.52	1.51	1.57	1.58
m_b (GeV)	4.89	4.89	4.90	4.90
	D-meson fits			
Λ_c (GeV)	1.4	2.3	3.0	4.5
$m_{u,d}$ (MeV)	10	10	250	220
D^*-D (MeV) (expt:141)	143	141	141	139
$(D,D^*)_{\text{c.o.g.}}$ (MeV) (expt:1974)	2008	2004	1979	1962
	$F^*(2140)$,$F(2020)$ fit			
Λ_c (GeV)	1.2	2.0	2.7	4.1
m_s (GeV)	210	210	410	410
	$B(5274)$ fit			
Λ_c (GeV)	2.1	3.8	3.7	5.0
$(B^*$-$B)$ (MeV) predict	54	55	50	47

C. *B* mesons

The recently discovered B meson[16] allows us to fit one parameter in the b—light-quark system. This will be the cutoff Λ_c for this system since we already know the light-quark masses to use for each case. This then allows us to predict the B^* mass. The fitted cutoff and predicted B^* mass for each case are given at the bottom of Table III.

While the few heavy-light-quark states have mainly allowed us to set cutoff parameters and find u-, d-, and s-quark masses without testing the complete spectra of the system, at least no glaring inconsistencies have been found.

VII. SPECTRA OF LIGHT-LIGHT QUARKS

In this section we apply the field-theory-expansion bound-state equation to the s-$\bar{s}$ meson sector, to the $\bar{s}$-u or $\bar{s}$-d (to be called $\bar{s}$-l for a light-mass u or d quark) sector, and to the l-$\bar{l}$ quark mesons. In the comparison of the $L=0$ and $L=1$ levels of possible J's from the q-$\bar{q}$ equation versus experiment we take a perspective based on many careful studies by theorists of multichannel hadron dynamics in the light-meson sectors.[17] First of all, almost complete sets of $L=0$ and $L=1$ levels of $J^P=0^-$, 1^-, 2^+, 1^+, and 0^+ have been found including both $I=0$ and $I=1$ in the u,d-quark sector (see Table IV). The $J^P=0^+$ levels $\delta(980)$ and $\epsilon(1300)$ are known to be lowered in energy by multichannel mixing and we will not try to fit them here (see the minireviews[17] by the Particle Data Group and references therein). With these shifts of hundreds of MeV, we may expect other levels to be shifted by smaller but still significant amounts. On the theoretical side, the multichannel effects are contained in the $qq\bar{q}\,\bar{q}$ intermediate states in the N-quantum expansion which we have not included. Also, at these lower-mass scales, the q^2 dependence of the quark masses can increase the quark masses from those found at the charmonium or b-quarkonium scales and the q^2 dependence of $\alpha_{\rm QCD}$ will be more important and depend on a function more detailed than that in the Richardson potential. It is useful, however, to study these light-quark systems from the basis of a relativistic equation since they are relativistic.

First we study the $\kappa_V=\kappa_S$ fits $VS1$ using the strange-quark mass of 210 MeV of the F-meson fits. Using a value of Λ_c of 3.0 GeV, which is even larger than that used for charmonium, only brings the ϕ meson down to 1500 MeV. Freeing the strange-quark mass and using 500 MeV, and Λ_c of 3.0 GeV, still only gives a ϕ of 1320 MeV and an f' of 2000 MeV. Similar results occur for the equal-linear-vector-and-sector case $VS2$.

The excessive energies of order 500 MeV for the $\kappa_V=\kappa_S$ case can be seen by simply using the Klein-Gordon equation with a Coulomb-plus-linear vector potential and a linear scalar potential. The quadratic term in the vector potential gives a cross term of $\alpha\kappa/(E+m)$. Using an effective α of about 2 for the Richardson form and E of order half the ϕ mass gives about 500 MeV excess energy with a small quark mass m.

We now discuss the fits to the light-quark sectors. We do not include the $J^P=0^-$ states as they are strongly affected by chiral-symmetry breaking or large mixings. In the rest of the states we neglect the small amount of mixing away from the description of the states by the number of strange-quark constituents. In Table IV are shown the known 1^{--}, 2^{++}, 1^{++}, 0^{+-}, and 1^{+-} states for the s-$\bar{s}$, l-$\bar{s}$, and l-$\bar{l}$ quark sectors and the leptonic decay widths for the ρ and ϕ mesons. A certain regularity is shown in the 2^{++} and 1^{--} states of a decrease of 100—120 MeV for each replacement of a strange quark by a u or d quark. Also apparent is the lowering of the 0^{++} states relative to the rest of the $L=1$ multiplet as discussed above.

When we fit with the scalar-linear-potential cases and use the s- and u- or d-quark masses determined in the fits to the D and F mesons, we still have the parameter Λ_c adjustable for each light-quark sector. For values of Λ_c in the range of 1.5 to 2.0 GeV, the states are in the correct range but the splitting of the 2^+ and 1^- states is only about 400—420 MeV in contrast to the experimental splitting of 500—550 MeV. Further increases in Λ_c increase this splitting but further drive down the states below the 1^- and 2^+ states. To achieve the larger splitting range

TABLE IV. Light-quark sector results ($S1$). Meson masses in MeV, quark masses in GeV.

			$\Lambda_R=0.4$ GeV, $\kappa_S=0.15$ GeV^2, $\kappa_V=0.0$ GeV^2				
	States		1^{--}	2^{++}	1^{++}	0^{++}	1^{+-}
	$\bar{s}s$ Expt.:		1020	1520	1418	975	
m_s	m_s	Λ_c					
0.52	0.52	3.0	1056	1524	1390	1314	
	$\bar{s}l$ Expt.:		892	1434	1414	$\sim$1350	1270
m_s	m_l	Λ_c					
0.55	0.35	3.5	916	1390	1215	1123	1257
	$\bar{l}l$ Expt. ($I=1$):		770	1318	1275	983	1238
	Expt. ($I=0$):		783	1273	1283	$\sim$1300	1190±60
m_l	m_l	Λ_c					
0.35	0.35	2.0	832	1264	1167	1139	1186

then requires not only larger Λ_c but also larger quark masses. The increase of the quark masses with decreasing energy scale can be justified as due to QCD renormalization-group effects. Since an accurate formulation of their variation does not exist at this low an energy scale, we will treat them as arbitrary parameters to be determined phenomenologically. This then improves the 2^+ to 1^- splitting, but an upper limit is still set on Λ_c. A representative set of fits is shown in Table IV with their parameters. One problem with these fits is that the 2^+ to 1^+ splitting also increases with Λ_c and is much larger than the experimental splitting in the *l-s* and *l-l* quark sectors. We also mention that in the fits the leptonic widths are rapidly varying functions of Λ_c and therefore should not be considered as crucial parameters to test bound-state calculations in the light-quark sectors.

VIII. SUMMARY

It has been recognized for a long time that meson states are described adequately as composites of a quark and an antiquark. Those made out of the heavy *c* or *b* quarks have their general features accounted for by a nonrelativistic treatment with first-order v^2/c^2 corrections accounting for some of the spin-orbit and spin-spin effects. An extension to mesons made up of lighter quarks will make relativistic effects play a more important role. In this work we have used a relativistic bound-state formalism, the *N*-quantum approximation, in order to make a simultaneous study of all meson systems from those composed of only heavy quarks to those made up of only light quarks; as our treatment was fully relativistic, spin-orbit, spin-spin, and retardation effects were taken into account automatically without recourse to a v/c expansion. Multiquark states, especially at short distances, were treated in an approximate manner.

We did not attempt a "best fit" to experiment but limited our choice of parameters by theoretical considerations. The potential was described at short distances by a full asymptotically free QCD exchange with its overall strength fixed by $\Lambda_{\rm QCD}$. The large-distance part is given by a $1/q^4$ exchange and no extra constant terms are added. The nature (vector or scalar) of the long-distance linearly confining potential is not fixed by theory; the MIT bag model predicts an equal amount of scalar and vector exchanges and such a prescription fits the heavy-quark systems reasonably well; however, this becomes untenable for the description of mesons involving lighter quarks where pure scalar linear exchange gives a much better fit. Even for heavy-quark systems a scalar exchange is marginally more successful. Our results, summarized below, may indicate that the ratio of scalar to vector confinement increases as we go down in quark mass.

The summary of what we have learned is as follows.

(a) The gross features of the quark-antiquark spectrum can be reproduced by the use of a relativistic bound-state formalism with theoretically reasonable exchanges. The masses of mesons composed of light quarks can be reproduced to an accuracy of 200 MeV (except for the lightest ones for which chiral-symmetry effects are expected to play a significant role). The masses of the heavier mesons are reproduced to a few MeV and good values are obtained for ratios of leptonic decay widths.

(b) The inclusion of multiparticle effects is necessary for the detailed understanding of the fine structure and of the leptonic widths. Our crude treatment of these effects through the use of a cutoff is crucial but not precise enough.

(c) The properties of mesons made up only of heavy quarks is not very sensitive to the vector or scalar nature of the long-distance confining potential. For mesons containing light quarks, a confining potential with equal vector and scalar parts seems to be excluded. When using equal-vector-and-scalar confinement on the heavy-light-quark mesons, the light-quark masses needed to fit the data are close to their current-algebra values, whereas when using a scalar linear confining potential the constituent quark masses must then be used in order to obtain reasonable fits. For mesons consisting of light quarks and using pure scalar confinement the values of these masses must be somewhat further increased. This suggests that a fuller treatment will have to take into account the renormalization-group-directed q^2 variation of quark masses analogous to the q^2 variation of the gauge coupling constant. An increase in quark masses may reflect itself in an increase in the importance of the scalar exchange.

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[1]O. W. Greenberg, Phys. Rev. 139, B1038 (1965); O. W. Greenberg and R. J. Genolio, *ibid.* 150, 1070 (1966); O. W. Greenberg, in *Field Theory in Elementary Particles,* proceedings of Orbis Scientiae 1982, edited by A. Perlmutter (Plenum, New York, 1983), pp. 379—391.

[3]K. Johnson, Phys. Rev. D 4, 1101 (1972).

[4]P. Hasenfratz, R. R. Horgan, J. Kuti, and J. M. Richard, Phys. Lett. 95B, 299 (1980).

[5]D. Beavis, S.-Y. Chu, B. R. Desia, and P. Kaus, Phys. Rev. D 20, 2345 (1979); R. McClary and N. Byers, *ibid.* 28, 1692 (1983).

[6]A. B. Henriques, B. H. Kellett, and R. G. Moorhouse, Phys. Lett. 64B, 85 (1976).

[7]K. Han *et al.*, Phys. Rev. Lett. 1612 (1982); G. Eiegen *et al.*, *ibid.* 49, 1616 (1982); F. Pauss *et al.*, Phys. Lett. 130B, 439 (1983).

[8]M. Bander, T.-W. Chiu, G. Shaw, and D. Silverman, Phys. Rev. Lett. 47, 549 (1981).

[9]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50A, 617

(1967).

[10]J. L. Richardson, Phys. Lett. 82B, 272 (1979); R. D. Carlitz and D. B. Creamer, Ann. Phys. (N.Y) 118, 429 (1979); R. Levine and Y. Tomozawa, Phys. Rev. D 21, 840 (1979). For spectroscopy with a more exact asymptotic-freedom form see W. Buchmüller, G. Grunberg, and S.-H. H. Tye, Phys. Rev. Lett. 45, 103 (1980).

[11]E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D 21, 203 (1980).

[12]T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York, 1981).

[13]S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, Phys. Rev. D 28, 228 (1983).

[14]W. Buchmüller and S.-H. H. Tye, Phys. Rev. D 24, 132 (1981); S. Ono, *ibid.* 23, 1119 (1981); A. Martin, Phys. Lett. 110B, 511 (1981); G. Bhanot and S. Rudaz, *ibid.* 78B, 119 (1978); H. Kraseman and S. Ono, Nucl. Phys. B154, 283 (1979); see also Refs. 10 and 11.

[15]A. Martin and J. M. Richard, Phys. Lett. 115B, 323 (1982).

[16]S. Behrends *et al.*, Phys. Rev. Lett. 50, 881 (1983).

[17]Particle Data Group, Phys. Lett. 111B, 1 (1982), minireviews on $\delta(980)$ and $\epsilon(1300)$.