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Decentralization in Replicated Club Economies with Multiple Private Goods

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Abstract

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Abstract

We show that "exhaustion of blocking opportunities" is a sufficient condition such that every allocation in the core of a replicated club economy can be decentralized as a competitive equilibrium, and that a related condition "efficient scale" is a necessary condition such that any allocation in the core can be decentralized. Efficient scale is defined with respect to the economy as a whole, and not with respect to individual club sizes. These decentralization results do not require the Euclidean structure and monotonicity assumed in Lindahl equilibrium, do not require convexity of preferences or costs, and do not require the strong assumption that private goods are "esssential".

1 Introduction

Club theory focuses on the idea, introduced by Buchanan (1965) and Tiebout (1956), that it is optimal to provide public goods in groups that are small relative to the economy. Economies where agents are partitioned into small groups for the purpose of sharing public goods are called *club economies*. In studying whether efficient and core allocations for club economies can be decentralized as competitive equilibria, most authors have assumed there is only one private good. This assumption is justified if the relative prices of private goods are fixed or do not depend on the clubs that form.

Decentralization with one private good[1] overlooks a complication that can only be addressed in a model with several private goods; namely, there may be complementarities between private and club goods in the sense that the consumer's ranking of clubs depends on the consumption of private goods, or alternatively, in the sense that the marginal rates of substitution between private goods depend on the club goods, by which we mean public facilities provided by the club, and also the crowding externalities within the club. We show by example that complementarities between private goods and club goods introduce an important complexity to the decentral-

[1] For early discussions of decentralization with one private good see Berglas (1976),(1981), Berglas and Pines (1980 and 1981), Boadway (1982), Brueckner (1994), Cornes and Sandler (1986), Manning (1993a), Oakland (1972), Pines (1991), and Sandler and Tschirhart (1980). For more explicit treatments of the core when there is one private good, see Scotchmer and Wooders (1987a, b), and Scotchmer (1993,1994), Conley and Wooders (1994), and Engl and Scotchmer (1992).

ization problem, namely that optimality might require that agents with the same preferences and endowments occupy clubs of different sizes. It follows that there is an efficient scale for the economy as a whole, rather than for clubs themselves.

Our main contribution is to identify *exhaustion of blocking opportunities* as a sufficient condition to decentralize all allocations in the core as competitive equilibria. Blocking opportunities are exhausted when a larger economy does not contain any trading group that could achieve higher equal-treatment utilities than are already available to some trading group in the economy. The related condition of *efficient scale* - i.e., the economy as a whole can achieve utilities that cannot be exceeded by any trading group in any (larger) economy - is necessary for decentralization in the sense that otherwise no allocation in the core can be decentralized. We also show that efficient scale is necessary and sufficient for existence of competitive equilibrium, whether or not preferences for private goods are convex. We develop these notions for both replicated club economies and replicated exchange economies.

We decentralize the core using a concept of price-taking equilibrium where private goods and admissions to all potential clubs are priced. Previous papers in this vein include Ellickson (1979), Bewley (1981), and Cole and Prescott (1994). The latter studied an even richer model where agents trade lotteries for memberships to clubs. Our admission prices will not be anonymous in the sense of Ellickson, who required that all members of a club pay the same admission price, or Bewley, who required that the admission prices are linked only to an individual's endowment. Since we permit that agents care not only about the number of other agents that share their clubs, but also about their types, the price anonymity required by Bewley and Ellickson is doomed to fail: agents who impose positive externalities will typically have lower admission prices, while agents who impose negative externalities must pay higher prices for admission.[2]

The assumptions we use in our decentralization theorems are less restrictive than in most of club theory in three ways. First, we avoid convexity assumptions. Of

[2]Externality pricing is formalized for the case of transferable utility by Engl and Scotchmer (1992) who show that admission prices can be decomposed as a linear sum of the externality-producing attributes, with each club member paying the same amount per unit externality contributed.

course there may be problems of existence, but we trace them to failures of efficient scale, and not to failures of convexity. It is to highlight the role and meaning of efficient scale in club economies that we show analogous theorems also for exchange economies with nonconvex preferences.

Second, we avoid the linear structure in public goods that is imposed in Lindahl equilibrium. Following Samuelson (1954) and Foley (1970), public goods are often modeled as points in a Euclidean space and preferences are assumed to be monotone. Hence, agents rank levels of each public good according to the natural Euclidean structure: more is preferred to less. This ranking is independent of the private goods consumed, and all agents agree on the ranking. In contrast, Mas-Colell (1980) modeled public goods using an unstructured set. Without linear structure, monotonicity cannot be defined, and consequently there is no implicit hypothesis that all agents have the same ranking. We follow Mas-Colell in avoiding linear structure, although a Euclidean structure on the set of public goods is not excluded by our assumptions.

Third, we avoid the assumption that private goods are essential. *Essentiality* is an assumption introduced by Mas-Colell (1980) that many subsequent authors have employed in decentralization results.[3] It requires that, given an allocation, a consumer can be made indifferent between that allocation and any club structure by letting him consume an appropriate amount of private goods. It follows that at zero consumption of private goods consumers must be indifferent between all club structures. While convenient for writing proofs, the essentiality assumption is very restrictive. For example, unless utility approaches minus infinity as private goods consumption becomes small, it excludes separability between private and club goods. Our analysis shows that such a strong assumption is not required.

In Section 2 we present a motivating example to show the role of replication in a club economy with many private goods, and in Sections 3 and 4 we present the decentralization results. Section 5 shows how exhaustion of blocking opportunities and efficient scale are interpreted in exchange economies, particularly ones where

[3] See, for example, Manning (1993b), Assumption 3, Manning (1993a), Assumption 5, and Wooders (1989) where the assumption is called "overriding desirability", and Wooders (1993), assumption (e), called "substitution".

preferences are not convex, and shows how our theorems are related to known results. Section 6 concludes with interpretive remarks.

2 An Example: The Optimal Size of an Economy

Much of club theory is devoted to the homogeneous case where all agents are identical. A central question from the point of view of club theory is how to define the optimal club size, usually referred to as n^*. The optimal size n^* is usually defined as $\text{argmax}_n\, U(n, y(n), w - \frac{c(y(n),n)}{n})$, where U is the utility function of each agent, n is the number of club members, $y(n)$ is the optimal provision of public goods with n members, w is each agent's endowment of private goods, and c is the cost function for the public goods. The question is how to extend this definition of optimal club size to the case of many private goods.

The following example shows two consequences of the multiplicity of private goods that might be surprising:

- It is not necessarily optimal to partition the population into identical clubs, even when all citizens are identical, irrespective of how large the economy is. Optimality might require trade between identical agents in different clubs.

- There is an efficient scale for the economy which is not the same as an optimal scale for clubs, and multiples of that scale are both necessary and sufficient for decentralization of every core allocation as a competitive equilibrium.

Example 1

We consider a club economy with identical agents, no public facilities, but externalities among agents. There are two private goods, and initial endowments are $w = (1,1)$ for each agent. We assume that utility depends on the consumption of private goods and the number of agents n in an agent's club. Let the utility functions be given by

$$U(x, n) = \quad 0 \qquad \text{for } \; n > 2 \;\; , x \in \mathbb{R}^2_+$$

$$U(x,2) = \quad x_1 + x_2 \quad \text{for} \quad x \in \mathbb{R}^2_+$$
$$U(x,1) = \quad \tfrac{4}{3}x_1 + \tfrac{1}{2}x_2 \quad \text{for} \quad x \in \mathbb{R}^2_+$$

where n is the number of members of a club.

If there are $N = 2$ agents in the economy, it is efficient to put them in a club of size $n = 2$, where each can achieve utility 2. However, suppose the economy is replicated so that there are $N = 4$ agents in the population. Contrary to what one might expect, it is not optimal simply to replicate the club with $n = 2$ members. The efficient allocation is to have two of agents in a club with $n = 2$, and two agents in singleton clubs. The optimal distribution of private goods that equalizes utility is given by $\tilde{x} = (\frac{2}{7}, 2)$ in clubs of size $n = 2$ and $\hat{x} = (\frac{12}{7}, 0)$ in clubs of size $n = 1$, with utility given by

$$u = U((\tfrac{2}{7}, 2), 2) = U((\tfrac{12}{7}, 0), 1) = 2\tfrac{2}{7}.$$

No other club structure and distribution of private goods can provide higher per-capita utility in the economy with $N = 4$ agents. However, even though the allocation is efficient and in the core, it is not possible to support the allocation as an equilibrium.

Here we define competitive equilibrium such that agents trade private goods at price vector p, and in addition they pay prices $V(n)$ for admission to a club of size n. The admission prices must be such that every club in equilibrium breaks even and out-of-equilibrium clubs earn nonpositive profits. Since there are no public goods to pay for, this implies that $nV(n) \leq 0$ for all n, with equality for clubs in the equilibrium club structure. Further, if agents receive utility u in equilibrium, it must be the case that $p \cdot x + V(n) > p \cdot w$ if $U(x, n) > u$. That is, a preferred consumption bundle is unaffordable.

To support the optimal allocation with $N = 4$ agents, the private goods prices must satisfy $p = (1, 1)$, and the zero-profit conditions imply that $V(1) = 2V(2) = 0$. All agents receive the same income $p \cdot w = 2$. But the private-goods expenditure required for the equal-treatment utility $u = 2\frac{2}{7}$ in clubs of size $n = 2$ is $2\frac{2}{7}$ while the expenditure required in clubs of size $n = 1$ is $1\frac{5}{7}$. Thus to support the efficient allocation as an equilibrium the admission prices must satisfy $V(1) > 0$ and $V(2) < 0$, a contradiction. Hence, even though the allocation described above is efficient and in the core, it is not a competitive equilibrium.

Suppose the population is replicated still further to $N = 14$. $N = 14$ turns out to be an ***efficient scale*** for the economy, i.e., there is no other population size N for which the economy can achieve higher per-capita utility than is available to $N = 14$ people, namely $u' = 2\frac{1}{3}$. An optimal allocation is to have 6 agents in clubs of size $n = 2$ (three clubs), with each member consuming $\tilde{x}' = (0, \frac{7}{3})$, and 8 singleton clubs in which each agent consumes $\hat{x}' = (\frac{7}{4}, 0)$. Not only is $N = 14$ an efficient scale, but in addition competitive equilibrium exists. Prices are $p' = (\frac{4}{3}, 1)$, per-capita income is $1\frac{4}{3}$, and admissions prices are $V(n) = 0$ for all club sizes n. Furthermore, multiples of $N = 14$ are the *only* scales for which competitive equilibrium exists. □

It is no mere coincidence that existence of competitive equilibrium coincides with the efficient scales in Example 1. The next section develops the mathematical structure underlying this fact. In addition to ***efficient scale*** we develop a notion called ***exhaustion of blocking opportunities***, which implies that *all* allocations in the core can be decentralized. If there is only one type of agent as in Example 1, an economy of efficient scale exhausts blocking opportunities and an economy that exhausts blocking opportunities contains a subeconomy with efficient scale. However in general an economy can have efficient scale without exhausting blocking opportunities.

An efficient scale for the economy is not always finite. We can modify the example such that no finite economy has efficient scale even if clubs are bounded in size:

Example 1 (revisited)

Modify the above example so that $U(x, 1) = \sqrt{2}\ x_1 + \frac{1}{2}x_2$. Then the maximum available utility is $1 + \sqrt{2}$, and it is achieved when a fraction $\frac{1}{1+\sqrt{2}}$ of the people are in clubs of size $n = 2$. But since $\frac{1}{1+\sqrt{2}}$ is irrational, the maximum utilities cannot be achieved in any finite economy. Hence, there is no economy in which there exist competitive equilibria. □

Although the theorems below make no reference to convexity, Example 1 shows that a club economy might not have efficient scale even if preferences for private goods are convex. This contrasts with exchange economies. Exchange economies with convex preferences always have efficient scale, and equilibrium exists.

3 Decentralization

Let $\mathbf{T} = \{1, \cdots, T\}$ be an initial set of *types*. If agents are of the same type, they have the same preferences and the same initial endowments of private goods. If we replicate the type set m times, we denote the agent set by $m\mathbf{T} = \{1_1, \ldots, 1_m, 2_1, \ldots, T_m\}$, and all agents with indices $t_r \in m\mathbf{T}$, $r \in \{1, \cdots, m\}$ for a given $t \in \mathbf{T}$, are of the same type. We say an agent $a \in m\mathbf{T}$ is *type-t* if $a = t_r$ for some $r \in \{1, \ldots, m\}$, and we refer to the type of a by $t(a) \in \mathbf{T}$. A coalition, say E, is a subset of $m\mathbf{T}$. We use $\eta(E) = (\eta_1(E), \ldots, \eta_T(E)) \in \mathbb{N}^T$ to mean that the coalition E contains $\eta_t(E)$ members of type-t, $t \in \mathbf{T}$.

The main feature of a club economy is that the agents are partitioned into a coalition structure. For each $A \subset m\mathbf{T}$, a finite collection $\mathbf{C} \subset 2^A$ is a *partition of* A if the coalitions in $\mathbf{C}$ are pairwise disjoint, and if $\cup_{E \in \mathbf{C}} E = A$. The collection of partitions of $A \subset m\mathbf{T}$ is indicated by $\mathcal{C}^A$.

The main features of a club are that it provides its members with certain public facilities and it creates externalities between members. Following Mas-Colell (1980), Mas-Colell and Silvestre (1989), Diamantaras and Gilles (1994), Diamantaras, Gilles and Scotchmer (1994), and Hahn and Gilles (1994), we model public goods as an unstructured set of public projects $\mathcal{Y}$. Since the set $\mathcal{Y}$ is not structured, one cannot assume monotonicity or convexity of preferences in public goods. The Samuelsonian setup is a special case where $\mathcal{Y} = \mathbb{R}^k_+$, and k is the number of public goods available to a club.

A *club* is a coalition endowed with certain public facilities (E, y), $E \subset m\mathbf{T}$, $y \in \mathcal{Y}$. For a set of agents $A \subset m\mathbf{T}$, a *club structure for* A, say K, is a collection of clubs that partitions A, i.e., $K = \{(E, y) \mid E \in \mathbf{C} \text{ and } y \in \mathcal{Y}\}$, where $\mathbf{C} \in \mathcal{C}^A$ is a partition of A.

It is assumed that there are $\ell \in \mathbf{N}$ private commodities in the economy, and that endowments are described by $w = \{w_t\}_{t \in \mathbf{T}}$ where $w_t \in \mathbb{R}^\ell_+ \setminus \{0\}$ denotes the endowment of type t, $t \in \mathbf{T}$. For $n \in \mathbb{N}^T$ we let $w(n) \equiv \sum_{t \in \mathbf{T}} n_t w_t$. Finally, $w(A) \equiv w(\eta(A))$ represents the total endowment of a coalition $A \subset m\mathbf{T}$.

The cost of providing facilities might depend on the coalition. We interpret the notion of types to mean that two agents of the same type have the same effect on the cost of public facilities, and affect other agents' utility in the same way. Instead of depending on the specific coalition E, cost therefore depends only on $\eta(E)$. Costs are thus introduced as a set-valued mapping C that assigns to every $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$ a closed set in $\mathbb{R}^\ell_+$, consisting of input vectors for production of the public project y in the club (n, y). We assume that if $c \in C(n, y)$ and $c' > c$, then $c' \in C(n, y)$.

For agent $a \in m\mathbf{T}$ we define preferences over his private goods consumption, the types of agents in the club of which the agent is a member, and the public facilities provided by that club. The preferences are represented by utility functions $U = \{U_t\}_{t \in \mathbf{T}}$ with $U_t \colon \mathbb{R}^\ell_+ \times \mathbb{N}^T \times \mathcal{Y} \to \mathbb{R}$.

In what follows we frequently use additional assumptions regarding the preferences of the agents in the economy. For every $t \in \mathbf{T}$ we call t's preferences

- *strictly monotone* if for every $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$ and all $x', x'' \in \mathbb{R}^\ell_{++}$: $x' \geq x''$ and $x' \neq x''$ implies that $U_t(x', n, y) > U_t(x'', n, y)$,

- *continuous* if U_t is continuous in its first argument (private goods).

A *club economy*, indicated by $\mathbb{E}^m$, is an m-fold replica of $\mathbb{E} \equiv \langle \mathbf{T}, U, w, \mathcal{Y}, C \rangle$. If the set of agents is comprised of a subset $A \subset m\mathbf{T}$ we refer to the subeconomy of $\mathbb{E}^m$ restricted to A as $\mathbb{E}^m(A)$.

Let $A \subset m\mathbf{T}$. An *allocation* for the subeconomy $\mathbb{E}^m(A)$ is (K, c, x) where K is a club structure of A, $x \colon A \to \mathbb{R}^\ell_+$ is the allocation of private goods, and $c \colon K \to \mathbb{R}^\ell_+$ represents an input vector for each $(E, y) \in K$. The allocation is *feasible* if $c(E, y) \in C(\eta(E), y)$ for each $(E, y) \in K$ and

$$\sum_{a \in A} x(a) + \sum_{(E,y) \in K} c(E, y) \leq w(A)$$

For $(E, y) \in K$ and every $a \in E$ we define

$$u(a; (K, c, x)) \equiv U_{t(a)}(x(a), \eta(E), y)$$

and say that an allocation (K, c, x) *achieves* utilities $u(a; (K, c, x))$, $a \in A$. We say that utilities $u: A \to \mathbb{R}$ are *achievable* in an economy $\mathbb{E}^m(A)$ if there is a feasible allocation in $\mathbb{E}^m(A)$ that achieves them. We will say that $u: A \to \mathbb{R}$ are *equal-treatment utilities* if there exists $\tilde{u} \in \mathbb{R}^T$ such that $u(a; (K, c, x)) = \tilde{u}_{t(a)}$, $a \in A$. We say that equal treatment utilities $\tilde{u} \in \mathbb{R}^T$ are *achievable* in $\mathbb{E}^m(A)$ if there are utilities $u: A \to \mathbb{R}$, which are achievable in $\mathbb{E}^m(A)$ and have the property that $u(a) = \tilde{u}_{t(a)}$ for every $a \in A$. In case $A = m\mathbf{T}$ all definitions extend to $\mathbb{E}^m$.

A coalition $A \subset m\mathbf{T}$ is said to *block* an allocation (K, c, x) in $\mathbb{E}^m$ if the subeconomy $\mathbb{E}^m(A)$ can achieve utilities $\tilde{u}: A \to \mathbb{R}$ such that $\tilde{u}(a) \geq u(a; (K, c, x))$, $a \in A$, with strict equality for at least one such a. The *core* of an economy $\mathbb{E}^m$ is a set of feasible allocations, each with the property that no coalition $A \subset m\mathbf{T}$ can block it. An allocation (K, c, x) is in the *equal-treatment core* of an economy $\mathbb{E}^m$ if it is in the core and achieves equal-treatment utilities.

Given $m \in \mathbb{N}$, let $W(m)$ be the set of equal-treatment utilities that coalitions of the economy $\mathbb{E}^m$ can achieve, i.e.,

$$W(m) = \left\{u \in \mathbb{R}^T \mid \exists\, A \subset m\mathbf{T},\ A \neq \emptyset, \text{ such that } u \text{ is achievable in } \mathbb{E}^m(A)\right\}$$

$$W = \bigcup_{m \in \mathbf{N}} W(m)$$

$$\overline{W} = \{u \in W \mid \ \nexists\, u' \in W \text{ such that } u' \gg u\ \}$$

$\overline{W}$ is the efficient, or upper, boundary of W.

Definition 1 *An economy* $\mathbb{E}^m$ **exhausts blocking opportunities** *if* $W(m) = W$.

Definition 2 *An economy* $\mathbb{E}^m$ *has* **efficient scale** *if there exists* $u \in \overline{W}$ *such that* $\mathbb{E}^m$ *can achieve* u.

If an economy $\mathbb{E}^m$ exhausts blocking opportunities, then every u in the interior of W can be blocked. If the equal-treatment core is nonempty, an allocation in the equal-treatment core achieves utilities in $\overline{W}$. An economy with efficient scale might or might not exhaust blocking opportunities, but every economy that exhausts blocking opportunities contains a subeconomy of efficient scale.

In Example 1 we have $W(14) = W = \{u \in \mathbb{R} \mid u \leq 2\frac{1}{3}\}$. Economies larger than $N = 14$ exhaust blocking opportunities, and the only efficient scales are multiples of 14. The equal-treatment utility achieved in the core under efficient scale is $2\frac{1}{3}$.

In the following definition private goods prices are in the price simplex

$$\Delta \equiv \left\{ p \in \mathbb{R}^{\ell}_{+} \;\middle|\; \sum_{i=1}^{\ell} p_i = 1 \right\}.$$

Definition 3 *A feasible allocation (K, c, x) is a* **competitive equilibrium** *for $\mathbb{E}^m$ if there exists a price vector for private goods $p \in \Delta$ and admission prices $V: \mathbb{N}^T \times \mathcal{Y} \to \mathbb{R}^T$ such that:*

(i) *Each club's budget is balanced, i.e., for each $(E, y) \in K$,* $\eta(E) \cdot V(\eta(E), y) = p \cdot c(E, y)$.

(ii) *No alternative club can make positive profit, i.e., for every $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$ and $c \in C(n, y)$, $n \cdot V(n, y) \leq p \cdot c$.*

(iii) *For every $(E, y) \in K$ and $a \in m\mathbf{T}$, $V_{t(a)}(\eta(E), y) + p \cdot x(a) = p \cdot w_{t(a)}$, and for every $g \in \mathbb{R}^{\ell}_{+}$, $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$:*

$$U_{t(a)}(g, n, z) > u(a; (K, c, x)) \implies V(n, y) + p \cdot g > p \cdot w_{t(a)}.$$

Definition 4 *A feasible allocation (K, c, x) is a* **quasi-equilibrium** *for $\mathbb{E}^m$ if there exists a price vector for private goods $p \in \Delta$ and admission prices $V: \mathbb{N}^T \times \mathcal{Y} \to \mathbb{R}^T$ such that (i) and (ii) of Definition 1 hold, and also*

(iv) *For every $(E, y) \in K$ and $a \in m\mathbf{T}$, $V_{t(a)}(\eta(E), y) + p \cdot x(a) = p \cdot w_{t(a)}$, and for every $g \in \mathbb{R}^{\ell}_{+}$, $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$:*

$$U_{t(a)}(g, n, z) \geq u(a; (K, c, x)) \implies V(n, y) + p \cdot g \geq p \cdot w_{t(a)}.$$

Before showing that exhaustion is a sufficient condition for decentralization of the core, we make two remarks.

First, the profit-maximization condition (ii) and the consumers' optimization condition (iii) or (iv) apply for any $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$. Most of the literature gives a less demanding definition of equilibrium in which equilibrium choices are only preferred among the clubs for which $n \leq \eta(m\mathbf{T}) \equiv (m, \cdots, m)$, i.e., only those clubs that are feasible in the economy.[4] We argue that the more demanding conditions (ii), (iii) and (iv) are more consistent with the basic idea of competitive equilibrium. A basic premise of price-taking is that agents only consider what is feasible in their budget sets, and not what is feasible in the economy, in making their choices. In an ordinary exchange economy a consumer deciding between two bundles x and x' in his budget set does not ask whether the aggregate endowment permits those trades or whether he could find someone to trade with. Similarly here: When an agent chooses between two consumption patterns (x, n, y) and (x', n', y'), he only asks whether they are affordable, but not whether the economy contains the agents required to compose such clubs, or whether they would join him.

Our second remark concerns the circumstances in which the core can be supported as a quasi-equilibrium but not as an equilibrium. At a quasi-equilibrium each agent is minimizing the cost of achieving his equilibrium utility level. Equilibrium imposes the stronger requirement that each agent is maximizing utility subject to the budget constraint. A quasi-equilibrium of an exchange economy is not an equilibrium if there are strictly preferred bundles that cost the same as the equilibrium bundle. The latter may occur if some prices are zero and some agents have zero income. In order to avoid this problem we assume that preferences are strictly monotone and continuous, and that the costs of establishing clubs in a core allocation are less than the total endowment.

In club economies there is another reason, in addition to the problem that there may be zero prices and incomes, that a quasi-equilibrium might not be an equilibrium. The additional problem arises because transfers of utility within prospective clubs are constrained whenever some member's consumption of private goods is zero. We

[4] Since the earliest papers on clubs used concepts of "utility-taking" equilibrium rather than a "price-taking" equilibrium, they are not phrased such that this distinction can be made. Among later papers, our more demanding condition is used also by Scotchmer and Wooders (1987a) for the special case of anonymous crowding, Scotchmer and Wooders (1987b) and Scotchmer (1993) for the special case of nonanonymous crowding and transferable utility. The less demanding condition is used by Scotchmer (1994), Conley and Wooders (1994), and Wooders (1978, 1989, 1993), who only require nonpositive profit for coalitions that are subsets of the economy.

refer the reader to Cole and Prescott (1994) for an example in which there might not be an equal-treatment allocation in the core of a club economy because enough private goods cannot be transferred to equalize utilities. The following example shows something stronger:

Example 2

A quasi-equilibrium might not be an equilibrium even when prices are positive, incomes are positive, and the allocation is in the equal-treatment core.

Suppose there are two private goods, three possible public projects $\mathcal{Y} = \{y_0, y_1, y_2\}$, and two types $\mathbf{T} = \{1, 2\}$. Suppose that endowments are given by $w_1 = w_2 = (1, 1)$, and preferences can be represented by

$$\begin{aligned}
U_t(x,(1,1),y_1) &= \sqrt{x_1 x_2} && \text{for } t = 1,2 \\
U_1(x,(1,1),y_2) &= .25\ x_1 + .01\ x_2 && \\
U_2(x,(1,1),y_2) &= 2 + .01\ x_1 + .01\ x_2 && \\
U_t(x,n,y) &= .01\ x_1 + .01\ x_2 && \text{for all other } (n,y) \in \mathbb{N}^T \times \mathcal{Y},\ t = 1,2
\end{aligned}$$

Suppose that $C(n,y_0) = C(n,y_1) = C(n,y_2) = \mathbb{R}^{\ell}_{+}$, so that either club good can be provided without inputs. We assume there are equal numbers of type-1 and type-2 agents. An allocation (K,c,x) in the core of every replica economy has a club structure K consisting only of clubs $((1,1),y_1)$, produced at cost $c((1,1),y_1) = (0,0)$, and each agent consumes private goods $(1,1)$. Each agent receives utility 1.

We now construct prices that support the core allocation as a quasi-equilibrium. The prices of private goods must be $p = (1,1)$ and $V_1((1,1),y_1) = V_2((1,1),y_1) = 0$. Each agent's income is 2. In quasi-equilibrium, prices must be such that an agent of type-1 cannot achieve utility 1 in another club while spending less than his income. This implies that in quasi-equilibrium, $V_1((1,1),y_2) \geq -2$. The nonprofit condition for a hypothetical club $((1,1),y_2)$ is $V_1((1,1),y_2) + V_2((1,1),y_2) - p \cdot c \leq 0$ for all $c \in \mathbb{R}^{\ell}_{+}$. The latter two conditions imply $V_2((1,1),y_2) \leq 2$. But if $V_2(1,1),y_2) \leq 2$, then a type-2 agent can achieve more than his equilibrium utility 1 without spending all his income, namely by purchasing membership in the club $((1,1),y_2)$ and consuming no private goods. Thus, no quasi-equilibrium of the economy is an equilibrium. □

Theorem 1(b) below gives sufficient conditions on endowments that exclude the anomalous case illustrated by Example 2. The first condition says that endowments are large enough so that private goods are consumed in positive amount after paying for the club goods. The second condition is less standard.

We say that *endowments are large relative to the value of club goods* if for each t, $U_t(w_t, e^t, y_0) > U_t(0, n, y)$ for all $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$, where e^t is the t-th unit vector and y_0 is an element of $\mathcal{Y}$ that requires no inputs ($C(n, y_0) = \mathbb{R}^\ell_+$ for all $n \in \mathbb{N}^T$), and is interpreted to mean that the club produces no public project. This condition means that a consumer would rather be a singleton consuming only his private endowment than participate in a club in which he consumes no private goods. It excludes Example 2, where $U_2(0, (1, 1), y_2) > U_2(w_2, e^2, y_0)$.

Theorem 1 [**Sufficiency of Exhaustion for Decentralization**]
Suppose that preferences are strictly monotone and continuous and the club economy $\mathbb{E}^m$ exhausts blocking opportunities. Let (K, c, x) be in the equal-treatment core of $\mathbb{E}^m$.

(a) *The allocation (K, c, x) can be supported as a quasi-equilibrium.*

(b) *Suppose in addition that $\sum_{(E,y)\in K} w(E) \gg \sum_{(E,y)\in K} c(E, y)$, and that endowments are large relative to the value of club goods. Then (K, c, x) can be supported as a competitive equilibrium.*

Theorem 2(a) which follows states that efficient scale is a *necessary* condition to decentralize the core as a competitive equilibrium. Here we use conditions (ii) and (iii) of the definition of competitive equilibrium, namely that they hold for all (n, y), and not just for those that are feasible. Under a less demanding definition of equilibrium, where conditions (ii) and (iii) hold only for (n, y) such that $n \leq \eta(m\mathbf{T}) = (m, \ldots, m)$, the core might be decentralizable as a competitive equilibrium even without efficient scale.[5]

[5]See, for example, Scotchmer (1994), Proposition 1, who shows core/competitive equivalence without an exhaustion hypothesis in a club economy with one private good, but with a less demanding definition of competitive equilibrium. In Scotchmer (1993) it is shown for the special case of transferable utility and one private good that with exhaustion the definition of competitive equilibrium can be strengthened as above.

Theorem 2 [**Necessity and Sufficiency of Efficient Scale for Existence of Competitive Equilibrium**]

(a) *If any allocation (K, c, x) in the equal-treatment core of the economy $\mathbb{E}^m$ can be supported as a competitive equilibrium, then the economy $\mathbb{E}^m$ has efficient scale.*

(b) *Suppose that preferences are strictly monotone and continuous and that endowments are large relative to the value of club goods. If an economy $\mathbb{E}^m$ has efficient scale, then there exists an allocation (K, c, x) in the core of $\mathbb{E}^m$. If in addition it holds that $\sum_{(E,y)\in K} w(E) \gg \sum_{(E,y)\in K} c(E, y)$, then the allocation (K, c, x) can be decentralized as a competitive equilibrium.*

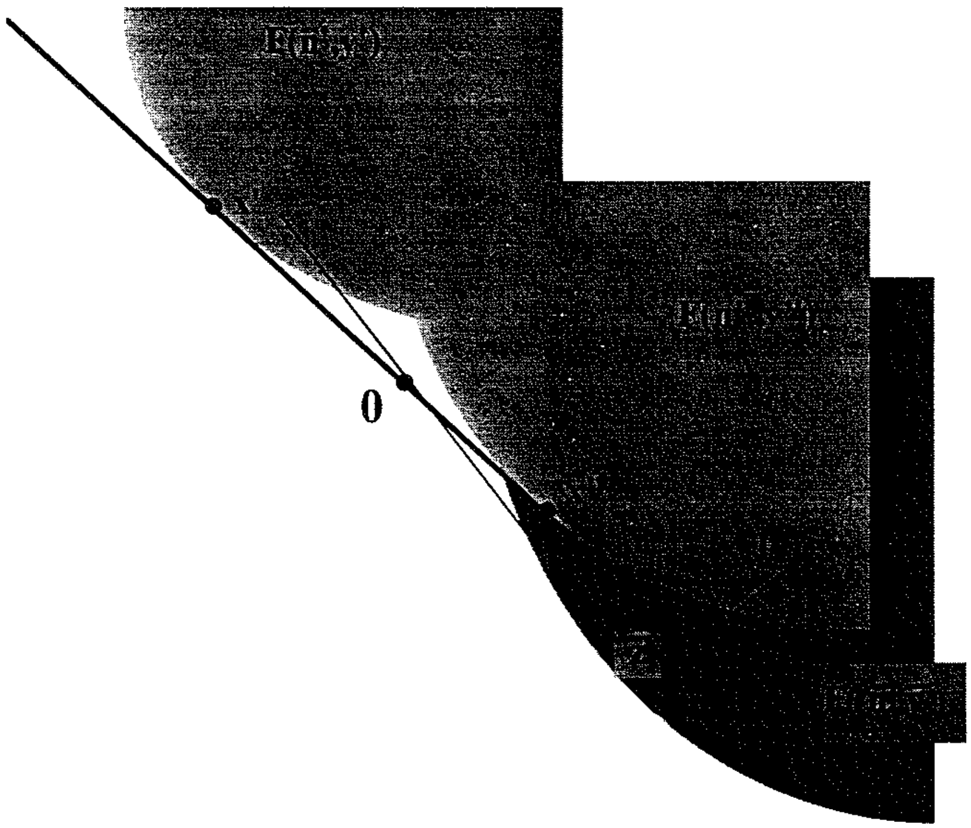
The next theorem completes the core/competitive equivalence argument.

Theorem 3 *Every competitive equilibrium in $\mathbb{E}^m$ is an equal-treatment core allocation.*

4 Proofs of Theorems

Before giving the proof of Theorem 1 we summarize its architecture. We first construct private-goods prices p and then construct admission prices V. Figure 1 illustrates the argument. Consider an allocation (K, c, x) with two types of clubs (n', y') and (n'', y'') in equal numbers that make net trades x' and x'' which sum to zero, and are therefore feasible. Let $\{F(n, y) \mid (n, y) \in \mathbb{N}^T \times \mathcal{Y}\}$ represent preferred net trades for the members of a club (n, y). Suppose that for every (n, y) with $n \leq \eta(m\mathbf{T}) \equiv (m, \ldots, m)$, the preferred set $F(n, y)$ lies above the bounding hyperplane with normal vector p. Then there is no partition of the economy and no preferred net trades such that the sum of net trades is nonpositive. Therefore the allocation (K, c, x) is in the core of the economy.

If the allocation in the core can be decentralized as a competitive equilibrium, the normal vector p represents the prices at which the members of the clubs (n', y') and (n'', y'') must trade private goods. Since x' is on the boundary of $F(n', y')$ and x'' is


0

on the boundary of $F(n'', y'')$ both types of clubs make zero profit. However, suppose there exists $(\overline{n}, \overline{y})$, $\overline{n}_t > m$ for some t, such that the hyperplane p intersects $F(\overline{n}, \overline{y})$ as shown. Then there exists $\overline{z} \in F(\overline{n}, \overline{y})$ such that $p \cdot \overline{z} < 0$, and it follows that such a club could make positive profit. Hence the core allocation is not a competitive equilibrium.

That the potential club $(\overline{n}, \overline{y})$ is profitable implies that the convex hull of $F(\overline{n}, \overline{y})$ and $F(n', y')$ contains 0. A sufficiently large economy contains a subeconomy that can be partitioned into clubs of types $(n', y'), (\overline{n}, \overline{y})$, and can make net trades that achieve higher utilities than those achieved in (K, c, x). Thus we can conclude from the fact that (K, c, x) is not a competitive equilibrium that the economy $\mathbb{E}^m$ does not exhaust blocking opportunities.

The argument using the convex hull of preferred sets is reminiscent of Debreu's and Scarf's (1963) proof that the core of an exchange economy converges to Walrasian equilibrium. (The same idea is applied to an atomless economy by Aumann (1964) and Vind (1964).)

Insert Figure 1 here.

Proof of Theorem 1 (a)

Let $u \in \mathbb{R}^T$ be the equal-treatment utilities achieved by the core allocation (K, c, x). For $(n, y) \in \mathbf{N}^T \times \mathcal{Y}$, we define the following sets. (These sets depend on the core utility vector $u \in \mathbb{R}^T$, but we omit that parameter from the notation for conciseness.) The set $\mathbf{T}^H(n, y)$ is the set of types who dislike (n, y) so much that they cannot achieve the core payoff irrespective of how much private goods they receive. The set $\mathbf{T}^L(n, y)$ is the set of types who prefer (n, y) to the core payoff even if given no private goods.

$$\mathbf{T}^H(n, y) \equiv \left\{ t \in \mathbf{T} \;\middle|\; n_t > 0,\ U_t(g, n, y) < u_t \text{ for every } g \in \mathbb{R}^\ell_+ \right\},$$

$$\mathbf{T}^L(n, y) \equiv \{ t \in \mathbf{T} \mid n_t > 0,\ U_t(0, n, y) > u_t \},$$

For (n, y) such that $\mathbf{T}^H(n, y) \neq \mathbf{T}$ we define

$$F(n,y) \equiv \left\{ [x+c-w(n)] \in \mathbb{R}^{\ell} \;\middle|\; \begin{array}{c} x = \sum_{t\in \mathbf{T}\setminus \mathbf{T}^H(n,y)} \; n_t\, x_t, \;\; c \in C(n,y), \\ U_t(x_t,n,y) > u_t \;\text{ for each} \\ t \in \mathbf{T} \setminus \mathbf{T}^H(n,y) \text{ with } n_t > 0 \end{array} \right\}$$

For (n,y) such that $\mathbf{T}^H(n,y) = \mathbf{T}$ we define $F(n,y) \equiv \emptyset$. Remark that $\mathbf{T}^H(\eta(E),y) = \mathbf{T}^L(\eta(E),y) = \emptyset$ for any club $(E,y) \in K$. Now let

$$\Gamma(u) \equiv \text{ conv } \left\{ \bigcup_{\{(n,y)\in \mathbb{N}^T \times \mathcal{Y} \mid \mathbf{T}^H(n,y)=\emptyset\}} F(n,y) \right\},$$

where "conv" refers to the convex hull. From the assumptions on preferences, $\Gamma(u)$ is convex, open, nonempty, and bounded from below.

Claim 1 *If $\mathbb{E}^m$ exhausts blocking opportunities, then $0 \notin \Gamma(u)$.*

PROOF:
We will show that if $0 \in \Gamma(u)$, then $\mathbb{E}^m$ does not exhaust blocking opportunities.

If $0 \in \Gamma(u)$, then by Carathéodory's theorem for convex hulls in Euclidean spaces there exists a finite collection $D \subset \{(n,y) \in \mathbb{N}^T \times \mathcal{Y} \mid \mathbf{T}^H(n,y) = \emptyset\}$, a collection $\{z(n,y) \in F(n,y) \mid (n,y) \in D\}$, and nonnegative weights $\{\lambda(n,y) \mid (n,y) \in D\}$ such that $\sum_D \lambda(n,y) = 1$ and $\sum_D \lambda(n,y)\, z(n,y) \leq 0$. Since $\mathbf{T}^H(n,y) = \emptyset$ for each $(n,y) \in D$, there exists $\{z_t(n,y) \in \mathbb{R}^{\ell}_{+} \mid t \in \mathbf{T},\; (n,y) \in D\}$ and $\{c(n,y) \in C(n,y) \mid (n,y) \in D\}$ such that $\sum_{t\in\mathbf{T}} \; n_t z_t(n,y) + c(n,y) - w(n) = z(n,y)$ and $U_t(z_t(n,y),n,y) > u_t$. By continuity of the preferences we can assume without loss of generality that the weights λ are rational. Let $\{\Lambda(n,y) \mid (n,y) \in D\}$ be nonnegative integers such that for each $(n',y') \in D$, $\frac{\Lambda(n',y')}{\sum_D \Lambda(n,y)} = \lambda(n',y')$. Let m' be large enough so that for each $(n,y) \in D$, the economy $\mathbb{E}^{m'}$ contains at least $\Lambda(n,y)$ coalitions E such that $\eta(E) = n$. Then $\sum_D \Lambda(n,y)\; [\sum_{t\in\mathbf{T}} \; n_t z_t(n,y) + c(n,y) - w(n)] \leq 0$ so that utilities $(U_1(z_t(n,y),n,y), \cdots, U_T(z_T(n,y),n,y) \; > \; u$ for each $(n,y) \in D$ are feasible for a subeconomy of $\mathbb{E}^{m'}$. Since u is in the core of $\mathbb{E}^m$, it follows that the constructed utilities are not feasible for any subeconomy of $\mathbb{E}^m$, i.e., the economy $\mathbb{E}^m$ does not exhaust blocking opportunities. □

Since $\Gamma(u)$ is convex and bounded from below, by Claim 1 and Minkowski's separating hyperplane theorem (see, e.g., Rockafellar (1970) and Hildenbrand (1974)),

$$B \equiv \{p \in \Delta \mid p \cdot z \geq 0 \text{ for all } z \in \Gamma(u)\} \neq \emptyset.$$

Claim 2 *Suppose the economy $\mathbb{E}^m$ exhausts blocking opportunities and $p \in B$. Then*

1. *p is the norm of a supporting hyperplane to $\Gamma(u)$ at 0.*

2. $x(a) \in \operatorname{argmin}_x \{p \cdot x \mid U_t(x, \eta(E), y) \geq u_t\}$ for $(E, y) \in K$, $a \in E$

3. $c(E, y) \in \operatorname{argmin}_c \{p \cdot c \mid c \in C(\eta(E), y)\}$ for $(E, y) \in K$

PROOF:
(1) For each $(E, y) \in K$, $z(E, y) \equiv [\sum_{a \in E} x(a) + c(E, y) - w(E)] \in \text{cl } F(\eta(E), y) \subset \text{cl } \Gamma(u)$ and by feasibility $\sum_{(E,y) \in K} z(E, y) = 0$, which implies that $0 \in \text{cl } \Gamma(u)$. Therefore the separating hyperplane between the negative orthant and $\Gamma(u)$ includes 0. (2) and (3) follow because otherwise there exists $[\sum_{a \in E} \tilde{x}(a) + c - w(E)] \in F(\eta(E), y)$ such that $p \cdot [\sum_{a \in E} \tilde{x}(a) + c - w(E)] < 0$, and since $[\sum_{a \in E} \tilde{x}(a) + c - w(E)] \in \Gamma(u)$, this would contradict $p \in B$. □

We remark that if $\mathbf{T}^H(n, y) \neq \emptyset$, then it can occur that $p \cdot z < 0$ for $z \in F(n, y)$, $p \in B$.

Next we choose a particular $p \in B$ and define admission prices V with reference to p. We first define $\gamma_t(n, y) \in \mathbb{R}_+$, $t \in \mathbf{T}$, $(n, y) \in \mathbb{N}^T \times \mathcal{Y}$, as follows. Let

$$\gamma_t(n, y) \equiv \begin{cases} \inf_x \{p \cdot x \mid U_t(x, n, y) \geq u_t\} & \text{if } (n, y) \in \mathbb{N}^T \times \mathcal{Y}, \\ & t \in \mathbf{T} \setminus [\mathbf{T}^L(n, y) \cup \mathbf{T}^H(n, y)],\ n_t > 0 \\ 0 & t \in \mathbf{T}^L(n, y) \cup \mathbf{T}^H(n, y) \text{ or } n_t = 0 \end{cases}$$

$$\gamma(n, y) \equiv \sum_{t \in \mathbf{T}} n_t \, \gamma_t(n, y)$$

$$n^L(n, y) \equiv \sum_{t \in \mathbf{T}^L(n,y)} n_t \quad \text{and} \quad n^H(n, y) \equiv \sum_{t \in \mathbf{T}^H(n,y)} n_t$$

$$V_t(n, y) \equiv \begin{cases} p \cdot w_t - \gamma_t(n, y) & \text{if } t \notin \mathbf{T}^H(n, y) \\ -(n^H(n, y))^{-1} \sum_{t \notin \mathbf{T}^H(n,y)} [n_t \, p \cdot w_t - n_t \, \gamma_t(n, y)] & \text{if } t \in \mathbf{T}^H(n, y) \end{cases}$$

Next we check the conditions for quasi-equilibrium.

CONDITION (II)

Using Claim 1 and $p \in B$, if $\mathbf{T}^H(n,y) = \emptyset$, $n \cdot V(n,y) - p \cdot c = p \cdot w(n) - \gamma(n,y) - p \cdot c \le 0$ for all $c \in C(n,y)$, since $\gamma(n,y) + p \cdot [c - w(n)] = \inf\{p \cdot x \mid x \in \text{cl } F(n,y)\}$. If $\mathbf{T}^H(n,y) \neq \emptyset$, then $n \cdot V(n,y) - p \cdot c \le -\sum_{t \notin \mathbf{T}^H(n,y)} [n_t p \cdot w_t - n_t \gamma_t(n,y)] + \sum_{t \notin \mathbf{T}^H(n,y)} [n_t p \cdot w_t - n_t \gamma_t(n,y)] - p \cdot c = -p \cdot c \le 0$ for all $c \in C(n,y)$.

CONDITION (I)

Since $-[w(E) - \sum_{a \in E} x(a) - c(E,y)] \in \text{cl } \Gamma(u)$ for all $(E,y) \in K$, $p \cdot [w(E) - \sum_{a \in E} x(a) - c(E,y)] \le 0$. Further, since (K,c,x) is feasible, $\sum_{(E,y) \in K} [w(E) - \sum_{a \in E} x(a) - c(E,y)] = 0$, hence $p \cdot \sum_{(E,y) \in K} [w(E) - \sum_{a \in E} x(a) - c(E,y)] = 0$. Now for each $(E,y) \in K$, $p \cdot [w(E) - \sum_{a \in E} x(a) - c(E,y)] = 0 = \eta(E) \cdot V(\eta(E),y) - p \cdot c(E,y)$, where the latter follows from Claim 2 and the definition of V, since $p \cdot \sum_{a \in E} [w_{t(a)} - x(a)] = \sum_{t \in \mathbf{T}} \eta_t(E) [p \cdot w_t - \gamma_t(\eta(E),y)] = \eta(E) \cdot V(\eta(E),y)$.

CONDITION (IV)

If $t \in \mathbf{T}^H(n,y)$, then there does not exist $g \in \mathbb{R}^\ell_+$ such that $U_t(g,n,y) \ge u_t$, so the condition is satisfied trivially. If $t \in \mathbf{T} \setminus \mathbf{T}^H(n,y)$ and $U_t(g,n,y) \ge u_t$ then $V_t(n,y) + p \cdot g = p \cdot w_t - \gamma_t(n,y) + p \cdot g \ge p \cdot w_t$ by definition of $\gamma_t(n,y)$ in the case of $t \notin \mathbf{T}^L(n,y)$, and by $p \ge 0$ and $g \ge 0$ in the case of $t \in \mathbf{T}^L(n,y)$ where by definition $\gamma_t(n,y) = 0$. □

In the quasi-equilibrium constructed above, if $t(a) \in \mathbf{T}^L(n,y)$, then agent a strictly prefers $(0,n,y)$ to his equilibrium consumption bundle, and both bundles have the same cost. However, if endowments are large relative to the value of club goods, then $\mathbf{T}^L(n,y) = \emptyset$, so this case does not arise. That is the basis of the following proof.

Proof of Theorem 1 (b)

With (p,V) constructed above, we verify that we have a competitive equilibrium.

Claim 3 *Suppose that preferences are strictly monotone and continuous, and that* $\sum_{a \in A} w_{t(a)} \gg \sum_{(E,y) \in K} c(E,y)$. *Then if* $p \in B$, $p \gg 0$.

PROOF:

Suppose instead that $p_i = 0$ and $p_j > 0$ for some $i \neq j$. By feasibility of x and the hypothesis of the claim there exists at least one $a \in A$ such that $x_j(a) > 0$. Let e^i and e^j be the i^{th} and j^{th} unit vectors respectively. By strict monotonicity and continuity of preferences there exist positive numbers b_i, b_j such that $x = [x(a)+b_i\, e^i - b_j\, x_j(a)\, e^j] > 0$, $U_{t(a)}(x, \eta(E), y) > U_{t(a)}(x(a), \eta(E), y)$, and $p \cdot x < p \cdot x(a)$, which contradicts Claim 2(2) above. $\square$

The arguments for conditions (i) and (ii) are the same as above.

CONDITION (III)

Suppose that $U_t(x, n, y) > u_t$. Then $t \notin \mathbf{T}^H(n, y)$ by definition, and since $\mathbf{T}^L(n, y) = \emptyset$ by the assumption that endowments are large relative to club goods, $t \in \mathbf{T} \setminus [\mathbf{T}^L(n, y) \cup \mathbf{T}^H(n, y)]$. For $t \in \mathbf{T} \setminus \mathbf{T}^H(n, y)$ we show that $p \cdot x > \gamma_t(n, y)$, and therefore condition (iii) holds, since $V_t(n, y) + p \cdot x > V_t(n, y) + \gamma_t(n, y) = p \cdot w_t$. We have $\gamma_t(n, y) > 0$ by our assumption that endowments are large relative to the value of club goods. If $p \cdot x \leq \gamma_t(n, y)$, then by continuity there exists $x' \leq x$, with $x'_j < x_j$ for some j, such that $U_{t(a)}(x', n, y) > u_t$, and since $p \gg 0$, $p \cdot x' < \gamma_t(n, y)$, which contradicts the definition of $\gamma_t(n, y)$.

Proof of Theorem 2

Proof of (a)

Suppose that (K, c, x) is in the equal-treatment core, achieves utilities $u \in \mathbb{R}^T$, and that there exist (p, V) such that (K, c, x) is a competitive equilibrium. If $\mathbb{E}^m$ does not have efficient scale, then $u \notin \overline{W}$, and there exists $m' \in \mathbb{N}$, $A \subset m'\mathbf{T}$, and a feasible allocation $(\tilde{K}, \tilde{c}, \tilde{x})$ for $\mathbb{E}^{m'}(A)$ that achieves equal-treatment utilities $\tilde{u} \gg u$. Since (K, c, x) and (p, V) are a competitive equilibrium, conditions (i), (ii) and (iii) hold. In particular

$$\eta(E) \cdot V(\eta(E), y) - p \cdot c \leq 0 \quad \text{for all } (E, y) \in \tilde{K},\ c \in C(\eta(E), y) \tag{1}$$

$$p \cdot \tilde{x}(a) + V_{t(a)}(\eta(E), y) > p \cdot w_{t(a)} \quad \text{for} \quad (E, y) \in \tilde{K},\ a \in A \tag{2}$$

since the economy $\mathbb{E}^m$ contains at least one agent of type $t(a)$. Hence,

$$\sum_{a\in A} \left[p \cdot \tilde{x}(a) + V_{t(a)}(\eta(E), y)\right] > p \cdot \sum_{a\in A} w(a) \tag{3}$$

Since $(\tilde{K}, \tilde{c}, \tilde{x})$ is feasible for $\mathbb{E}^{m'}(A)$,

$$\sum_{(E,y)\in\tilde{K}} \left[\tilde{c}(E, y) + \sum_{a\in E} \tilde{x}(a)\right] \leq \sum_{a\in A} w(a)$$

which implies that

$$p \cdot \sum_{(E,y)\in\tilde{K}} \left[\tilde{c}(E, y) + \sum_{a\in E} \tilde{x}(a)\right] \leq p \cdot \sum_{a\in A} w(a) \tag{4}$$

Subtracting (4) from (3), we have a contradiction to (1):

$$\sum_{(E,y)\in\tilde{K}} [\eta(E) \cdot V(\eta(E), y) - p \cdot \tilde{c}(E, y)] > 0$$

Proof of (b)

Let (K, c, x) be an allocation that achieves $u \in \overline{W}$. We claim that if $u \in \overline{W}$, then $u_t \geq U_t(w_t, e^t, y_0)$ for all t.

Suppose, contrary to the claim, that $u \in \overline{W}$ and for some t', $u_{t'} < U_{t'}(w_{t'}, e^{t'}, y_0)$, where $C(e^{t'}, y_0) = \mathbb{R}^\ell_+$. We make no restrictions on u_t, $t \neq t'$, other than $u \in \overline{W}$. Let ε satisfy $0 < \varepsilon < U_{t'}(w_{t'}, e^{t'}, y_0) - u_{t'}$. We argue that the vector $u' = u + \varepsilon(1, .., 1)$ is in $W(m)$ for every m. Namely, take the singleton coalition $E = \{a\}$ with $t(a) = t'$. The utility vector u' is achievable for E. Hence $u' \gg u$ is in W, and therefore u is not in $\overline{W}$. This is a contradiction.

Now we show that (K, c, x) is in the core. Suppose that a coalition $A \subset m\mathbf{T}$ can block (K, c, x). Then there exists $(\tilde{K}, \tilde{c}, \tilde{x})$ that is feasible for $\mathbb{E}^m(A)$ and achieves $\tilde{u}: A \to \mathbb{R}$ such that $\tilde{u}(a) \geq u_{t(a)}$ for all $a \in A$, with strict inequality for some such a. We will show that there exists $(\tilde{K}, \tilde{c}, \hat{x})$ that achieves equal treatment utilities $\hat{u} > u$ for $\mathbb{E}^m(A)$, and that will contradict the hypothesis that $u \in \overline{W}$. To construct $\hat{x}$, first note that for every $(E, y) \in \tilde{K}$ and $a \in E$, $U_{t(a)}(0, \eta(E), y) < U_{t(a)}(w_{t(a)}, e^{t(a)}, y_0) \leq u_{t(a)} \leq U_{t(a)}(\tilde{x}(a), \eta(E), y)$. Hence, by strict monotonicity and continuity there is some $\lambda_a \in [0,1]$ with $U_{t(a)}(\lambda_a\,\tilde{x}(a), \eta(E), y) = u_{t(a)}$. Since $\tilde{u}(a) > u_{t(a)}$ for some $a \in A$, by strict monotonicity $\sum_{a\in A} \lambda_a\,\tilde{x}(a) < \sum_{a\in A} \tilde{x}(a)$. By strict monotonicity and continuity, there exist $\hat{u} \in \mathbb{R}^T$, and $\{\varepsilon_t(E,y) \in \mathbf{R}_{++} \mid t \in \mathbf{T},\ (E,y) \in \tilde{K}\}$ such that $\sum_{a\in A} \lambda_a\,\tilde{x}(a) + \sum_{(E,y)\in\tilde{K}} \sum_{t\in\mathbf{T}}\ \eta_t(E)\ \varepsilon_t(E,y) = \sum_{a\in A} \tilde{x}(a)$ and $U_{t(a)}(\lambda_a\,\tilde{x}(a) + \varepsilon_{t(a)}(E,y), E, y) = \hat{u}_{t(a)}$ for each $a \in E$, $(E,y) \in \tilde{K}$. Let $\hat{x}(a) \equiv \lambda_a\,\tilde{x}(a) + \varepsilon_{t(a)}(E,y)$ for each $a \in E$, $(E,y) \in \tilde{K}$. Then the allocation $(\tilde{K}, \tilde{c}, \hat{x})$ is feasible in $\mathbb{E}^m(A)$, achieves equal-treatment utilities $\hat{u} \gg u$, and $\hat{u} \in W(m)$, which contradicts the hypothesis that $u \in \overline{W}$. Hence no coalition can block, and (K,c,x) is in the core of $\mathbb{E}^m$.

Now we observe that $0 \in \text{cl}\,\Gamma(u) \setminus \Gamma(u)$, as defined in the proof of Theorem 1(a). Otherwise there exists $m' \in \mathbb{N}$ and $A \subset m'\mathbf{T}$, such that $\mathbb{E}^{m'}(A)$ can achieve $u' > u$, which contradicts that $u \in \overline{W}$. Hence (K,x,c) can be decentralized as a competitive equilibrium using the prices (p,V) constructed in the proof Theorem 1(a) above.

Proof of Theorem 3

Suppose to the contrary that $A \subset m\mathbf{T}$ can block (K,c,x). Then there exists an allocation $(\tilde{K}, \tilde{c}, \tilde{x})$ for the economy $\mathbb{E}^m(A)$ such that for each $a \in E$, $(E,y) \in \tilde{K}$,

$$U_{t(a)}(\tilde{x}(a), \eta(E), y) \geq u(a; (K,c,x)),$$

with strict inequality for some $a \in A$. Hence,

$$V_{t(a)}(\eta(E), y) + p \cdot \tilde{x}(a) \geq p \cdot w_{t(a)}$$

with strict inequality for some $a \in A$. Since $(\tilde{K}, \tilde{c}, \tilde{x})$ is feasible for $\mathbb{E}^m(A)$,

$$\sum_{(E,y)\in\tilde{K}} [\eta(E) \cdot V(\eta(E), y) + p \cdot \sum_{a\in A} \tilde{x}(a)] >$$

$$> p \cdot \sum_{(E,y)\in\tilde{K}} w(E) \geq p \cdot \left[\sum_{a\in A} \tilde{x}(a) + \sum_{(E,y)\in\tilde{K}} \tilde{c}(E,y)\right],$$

which implies

$$\eta(E) \cdot V(\eta(E), y) - p \cdot \tilde{c}(E,y) > 0 \quad \text{for some } (E,y) \in \tilde{K}$$

The latter contradicts (ii) of the definition of competitive equilibrium. Furthermore, it is obvious that (K, c, x) achieves equal treatment utilities.

5 Decentralization in Exchange Economies

In order to bridge club economies and exchange economies, in this section we state the analogous theorems for standard exchange economies. The "sufficiency" theorem is a modification of Debreu's and Scarf's (1963) theorem for finite economies. To our knowledge the "necessity" theorem for exchange economies has not previously been stated. We comment more on the implications of these theorems in the conclusion.

We maintain all the previous definitions, except that we redefine preferences, allocations and feasibility to the corresponding notion of an exchange economy. Using our previously introduced notation we let $\mathbb{E} \equiv \langle \mathbf{T}, (U_t, w_t)_{t\in\mathbf{T}} \rangle$ represent an exchange economy, where every agent of type-t has utility function U_t and endowments $w_t \in \mathbb{R}^{\ell}_{+} \setminus \{0\}$. We refer to its m-fold replication by $\mathbb{E}^m \equiv \langle m\mathbf{T}, (U_t, w_t)_{t\in\mathbf{T}} \rangle$. Let $A \subset m\mathbf{T}$ be a coalition in $\mathbb{E}^m$. The related *subeconomy* $\mathbb{E}^m(A)$ of $\mathbb{E}^m$ is $\langle A, (U_t, w_t)_{t\in\mathbf{T}} \rangle$.

Let $\mathbb{E}^m(A)$ be some subeconomy. An *allocation* is a function $x: A \to \mathbb{R}^{\ell}_{+}$.[6] An allocation x is *feasible* for $\mathbb{E}^m(A)$ if $\sum_{a\in A} x(a) = \sum_{t\in\mathbf{T}} \eta_t(A)\, w_t$. An allocation x is an *equal treatment allocation* if there exists $u \equiv (u_1, \cdots, u_T) \in \mathbb{R}^T$ with $U_t(x(a)) = u_t$ for every $a \in A$ of type-t, and we say that the allocation x *achieves* the equal-treatment utilities $u \equiv (u_1, \cdots, u_T) \in \mathbb{R}^T$. These definitions extend to the economy $\mathbb{E}^m$ itself.

In the same fashion as given for club economies we introduce for every $m \in \mathbb{N}$ the set of achievable utility values $W(m) \subset \mathbb{R}^T$. Similarly we introduce the related sets W and $\overline{W}$, and adopt the above definitions of exhaustion of blocking opportunities and efficient scale.

An allocation x is in the *core* of an economy $\mathbb{E}^m$ if there is no coalition $A \subset m\mathbf{T}$ and allocation g that is feasible for $\mathbb{E}^m(A)$ such that $U_t(g(a)) \geq U_t(x(a))$ for every

[6] In the case of non-convex preferences the equal treatment property shown by Debreu and Scarf (1963) for the core allocations in exchange economies with convex preferences does *not* have to hold. Therefore, we do not restrict ourselves to allocations in which agents of the same type consume the same private goods.

$a \in A$ of type-t, with strict inequality for some a. An allocation x is in the *equal treatment core* of $\mathbb{E}^m$ if it is in the core of $\mathbb{E}^m$ and achieves equal-treatment utilities.

We assume as above that private goods prices p are normalized in the ℓ-simplex Δ. A pair (x, p) is a *competitive equilibrium* for an economy $\mathbb{E}^m$ if x is a feasible allocation, if for each $a \in m\mathbf{T}$, $p \cdot \left(x(a) - w_{t(a)}\right) = 0$, and $U_{t(a)}(z) > U_{t(a)}(x(a))$ implies that $p \cdot z > p \cdot w_{t(a)}$.

The next theorem is analogous to Theorem 1 and, since it elaborates the Debreu-Scarf (1963) core convergence theorem for replica economies, suggests that our Theorem 1 is the analogue for club economies to that theorem.

Theorem 4 *Suppose that the economy $\mathbb{E}^m$ exhausts blocking opportunities. If preferences are strictly monotone and continuous and $\sum_{t \in \mathbf{T}} w_t \gg 0$, then for every equal-treatment allocation x in the core of the economy $\mathbb{E}^m$, there are suitable prices $p \in \Delta$ such that (x, p) is a competitive equilibrium.*

PROOF:
First, we state a claim that relates exhaustion of blocking opportunities to Debreu's and Scarf's (1963) proof that the core converges to the set of competitive equilibria. Their argument uses the fact that an allocation in the core of all replica economies implies that the convex hull of the union of the better sets excludes 0.

Suppose that an equal-treatment allocation x in the core of $\mathbb{E}^m$ achieves $u \in \mathbb{R}^T$ in $\mathbb{E}^m$. Let

$$F(t; u) \equiv \left\{ z \in \mathbb{R}^\ell \,\middle|\, U_t(z + w_t) > u_t \right\}$$

$$\Gamma(u) \equiv \operatorname{conv} \left\{ \bigcup_{t \in \mathbf{T}} F(t; u) \right\}.$$

Claim 4 *If $\mathbb{E}^m$ exhausts blocking opportunities, then $0 \notin \Gamma(u)$.*

PROOF:
We will show that if $0 \in \Gamma(u)$, then the economy does not exhaust blocking opportunities. Suppose $0 \in \Gamma(u)$. Then there exist $\{z_t \mid t \in \mathbf{T}\}$ with $z_t \in F(t; u)$ and

nonnegative rational weights λ_t, $t \in \mathbf{T}$ such that $\sum_{t\in\mathbf{T}} \lambda_t = 1$ and $\sum_{t\in\mathbf{T}} \lambda_t\, z_t \leq 0$. Let Λ_t, $t \in \mathbf{T}$ be nonnegative integers such that for each $t \in \mathbf{T}$, $\lambda_t = \frac{\Lambda_t}{\sum_{t\in\mathbf{T}} \Lambda_t}$. Let m' be large enough so that the economy $\mathbb{E}^{m'}$ contains at least Λ_t agents of type t for each $t \in \mathbf{T}$. Then since $\sum_{t\in\mathbf{T}} \Lambda_t\; z_t \leq 0$ the utilities $u' \equiv (U_1(z_1), \cdots, U_T(z_T)) \gg u$ are achievable for a subeconomy of $\mathbb{E}^{m'}$. But then $u' \in W(m')$, so $u' \in W$. But if there were a subseconomy of $\mathbb{E}^m$ that could achieve u', then x would not be in the core of $\mathbb{E}^m$. Hence $u \notin W(m)$, and therefore $\mathbb{E}^m$ does not exhaust blocking opportunities.

□

Using Claim 4 and Minkowski's separation theorem, there is a separating hyperplane with normal vector $p \in \Delta$ between $\mathbb{R}^\ell_-$ and $\Gamma(u)$ such that $p \cdot z \geq 0$ for all $z \in \Gamma(u)$. By feasibility of x we have $\sum_{a\in m\mathbf{T}} x(a) = \sum_{t\in\mathbf{T}} m\, w_t$ and from strict monotonicity and continuity it follows that $(x(a) - w_{t(a)}) \in \mathrm{cl}\, F(t; u)$ for every $a \in m\mathbf{T}$. Hence,

$$\frac{1}{mT} \sum_{a\in m\mathbf{T}} \left[x(a) - w_{t(a)}\right] = 0 \in \mathrm{cl}\, \Gamma\,(u)\,.$$

Thus p is the normal vector of a supporting hyperplane to $\Gamma\,(u)$ at 0. Thus, if $U_{t(a)}(z) \geq u_{t(a)}$, then $(z - w_{t(a)}) \in \mathrm{cl}\; F(t; x) \subset \mathrm{cl}\; \Gamma(u)$, from which it follows that $p \cdot (z - w_{t(a)}) \geq 0$.

We now show that (p, x) is a competitive equilibrium. First, we show that $p \cdot (x(a) - w_{t(a)}) = 0$ for all $a \in m\mathbf{T}$. We have $\frac{1}{mT}\left[\sum_{a\in m\mathbf{T}} \left(x(a) - w_{t(a)}\right)\right] = 0$, so $\frac{1}{mT}\left[\sum_{a\in m\mathbf{T}}\; p \cdot \left(x(a) - w_{t(a)}\right)\right] = 0$. If $p \cdot (x(a') - w_{t(a')}) > 0$ for some $a' \in m\mathbf{T}$ then there exists $b \in m\mathbf{T}$ such that $p \cdot (x(b) - w_{t(b)}) < 0$. But this is a contradiction, since $(x(b) - w_{t(b)}) \in \mathrm{cl}\; F(b; x) \subset \mathrm{cl}\; \Gamma(u)$, hence $p \cdot (x(b) - w_{t(b)}) \geq 0$.

Second, $p \gg 0$. Suppose to the contrary that $p_i = 0$ and $p_j > 0$ for some $i \neq j$. By the feasibility of x and the hypothesis that the economy's total endowment is strictly positive, $x_j\,(a) > 0$ for some $a \in m\mathbf{T}$. Then by strict monotonicity and continuity of a's preferences there are $\beta_i, \beta_j > 0$ with $z \equiv \left[x(a) + \beta_i\, e^i - \beta_j\, x_j(a)\, e^j\right] > 0$, $U_{t(a)}(z) > U_{t(a)}\,(x(a))$ and $p \cdot z < p \cdot x(a)$, where e^i (e^j) respectively are the i-th (j-th) unit vector in $\mathbb{R}^\ell$. But this contradicts the conclusion above. Hence $p \gg 0$.

By standard arguments $p \gg 0$ implies that (x, p) is a competitive equilibrium.

□

Theorem 5 [Necessity and Sufficiency of Efficient Scale for Existence of Competitive Equilibrium]

(a) *If any allocation x in the core an exchange economy $\mathbb{E}^m$ can be decentralized, then the economy has efficient scale.*

(b) *Suppose that the utility functions are strictly increasing and continuous. If the economy $\mathbb{E}^m$ has efficient scale, then there exists an allocation x in its core and x can be decentralized as a competitive equilibrium.*

PROOF

(a) Suppose that x is a competitive equilibrium. Hence, it is in the core (Debreu and Scarf (1963), Theorem 1). If the economy does not have efficient scale, then there exist $m' \in \mathbb{N}$, $A \subset \mathbb{E}^{m'}$, and an equal-treatment allocation $\tilde{x}$ that is feasible for $\mathbb{E}^{m'}(A)$ such that $U_{t(a)}(\tilde{x}_{t(a)}) \geq U_t(x(a))$ for $a \in A$, with strict inequality for some a'. Since the economy $\mathbb{E}^m$ contains an agent of each type $t(a)$, $a \in A$, it follows that if (x,p) is a competitive equilibrium, $p \cdot (\tilde{x}(a) - w_{t(a)}) > 0$, and $p \cdot \sum_{a \in A}(\tilde{x}(a) - w_{t(a)}) > 0$. Let $\tilde{x}_j(a)$ be the j^{th} element of $\tilde{x}(a)$ and let $w_{t(a),j}$ be the j^{th} element of $w_{t(a)}$. It follows from $p > 0$ and $p \cdot \sum_{a \in A}(\tilde{x}(a) - w_{t(a)}) > 0$ that $\sum_{a \in A}(\tilde{x}_{t(a)}(a) - w_{t(a),j}) > 0$ for some good j, which contradicts the requirement that $\tilde{x}$ is feasible for $\mathbb{E}^m(A)$.

(b) Let x be an equal-treatment allocation that achieves $u \in \overline{W}$. Suppose that $A \subset m\mathbf{T}$ can block x; i.e., there is an allocation $\tilde{x}$ that is feasible for $\mathbb{E}^m(A)$ such that $U_t(\tilde{x}(a)) \geq u_{t(a)}$ with strict inequality for some $a \in A$. Using strict monotonicity and continuity of preferences, one can show, analogously to our proof of Theorem 2)(b) above, that there exists $\hat{x} : \mathbf{T} \to \mathbb{R}^{\ell}_+$ such that $U_t(\hat{x}_t) > u_t$ for each $t \in \mathbf{T}$, and $\sum_{t \in \mathbf{T}} \eta_t(A)\hat{x}_t = \sum_{a \in A} \tilde{x}(a)$. Then $(U_1(\hat{x}_1), \cdots, U_T(\hat{x}_T)) \in W(m)$, which contradicts that $u \in \overline{W}$. Observe that $0 \in \text{cl}\,\Gamma(u) \setminus \Gamma(u)$, since otherwise there exist $\hat{m} \in \mathbb{N}$ and $\hat{A} \subset \hat{m}\mathbf{T}$ such that $\hat{u} \gg u$ is achievable for $\mathbb{E}^{\hat{m}}(\hat{A})$, which contradicts that $u \in \overline{W}$. Thus x can be decentralized with the price p constructed in the proof of Theorem 4.

□

An exchange economy with convex, monotone preferences always has efficient scale, and hence Theorem 5 has no bite. However the following example shows that when preferences are not convex the economy might not have efficient scale, and it might be impossible to decentralize any allocation in the core. However, if the economy exhausts blocking opportunities and the core is nonempty, then the economy has efficient scale, and the core can be decentralized even without convex preferences. In the following example this occurs for $m = 2, 4, 6, \cdots$.

Example 3

In an exchange economy without efficient scale, no core allocation can be decentralized as a competitive equilibrium, but decentralization is possible in an economy with efficient scale even without convex preferences.

Suppose there is one consumer and two goods, and that the endowment is $w = (1, 1)$. Suppose that preferences are $U(x) = x_1^2 + x_2^2$. Then the economy with one person, $\mathbb{E}^1$, has an allocation in the core in which the consumer consumes his endowment and receives utility 2. But the core is not a competitive equilibrium, since at any prices the consumer prefers to specialize in consumption. Further, the economy does not have efficient scale, since the larger economy $\mathbb{E}^2$ can achieve per-capita utility 4. Any economy $\mathbb{E}^m$, $m \geq 2$, exhausts blocking opportunities, and if the core is nonempty, it can be decentralized as a competitive equilibrium. For any $m = 1, 3, 5, \ldots$, the economy $\mathbb{E}^m$ does not have efficient scale and competitive equilibria do not exist. For $m = 3, 5, \ldots$, the core is empty. For any even $m = 2, 4, 6, \cdots$, the economy $\mathbb{E}^m$ has efficient scale, and the core can be decentralized with prices $p = (1, 1)$, with each consumer specializing in consumption of one commodity.

6 Conclusion

Theorems 2(a) and 5(a) ("necessity") have implications for existence of competitive equilibrium. For both club and exchange economies, competitive equilibria are in the core of the economy, and therefore if no allocation in the core can be decentralized, competitive equilibrium does not exist. Theorems 2 and 5 imply that competitive equilibrium exists if and only if the economy has efficient scale. In exchange economies with convex preferences, every economy has efficient scale. However, Example 1 shows that a well-behaved club economy might not satisfy efficient scale except for certain replicas. Example 3 shows the same thing for exchange economies when preferences are not convex. Example 1 (revisited) shows it might be the case that *no* finite club economy has efficient scale, and the same is true for exchange economies with nonconvex preferences.

The scale problem for club economies has been dealt with in various ways. An early recognition was by Pauly (1967, 1970), who discussed the efficient scale of a club rather than the efficient scale of the economy. The same idea has been carried over to the context of multiple private goods on the assumption that it will not be optimal for identical agents with identical endowments to trade. See, e.g., Wooders (1978) who calls the optimal club scale a "distinguished number". Our example in Section 2 shows that such an assumption is unwarranted. The scale problem is assumed away in the theorems of Bewley (1981), and Ellickson (1979), where there is either no crowding in consumption or no scale effects in public goods production. Much of the literature simply ignores the existence problem, or the "integer problem", as it is described, by implicitly or explicitly assuming a continuum economy in which any departure from a multiple of the efficient scale is negligible; see for example Cole and Prescott (1994). Another approach that may work for a finite economy is to define equilibrium such that a small percentage of consumers are permitted not to optimize (Scotchmer (1993)), or to recognize that firms of optimal scale in a finite economy will not be price-takers, and hence to study Nash equilibrium instead of price-taking equilibrium (Scotchmer (1985a,b)).

Theorems 1 and 4 ("sufficiency") report that all allocations in the core can be decentralized in both types of economies if the economy exhausts blocking opportunities. By definition limit economics ($m \to \infty$) exhaust blocking opportunities, and therefore the core converges to the set of competitive equilibria in the limit. Thus, Theorem 4 is a reinterpretation of Debreu's and Scarf's (1963) Theorem 3. Our Theorem 1 leaves open the same question for club economies that Debreu and Scarf left open for exchange economies, namely, how to define an appropriate notion that an allocation in the core is close to a competitive allocation, and correspondingly, the rate at which the core converges. See Anderson (1991) for a discussion of subsequent work on rate of convergence in exchange economies.

For club economies there is an advantage to expressing core/competitive equivalence in terms of exhaustion of blocking opportunities, namely that it has a natural interpretation in the case of one private good. Core/competitive equivalence in exchange economies with one private good is trivial, in contrast to club economies. Club theory has focussed on the case of one private good precisely because it isolates the feature of club economies that is different from exchange economies: namely the partition of agents into clubs. The premise of club theory is that optimal clubs are small relative to the economy. For example Scotchmer and Wooders (1987a,b) express this idea by saying that, whatever the relative numbers of members of different types in a club, the optimal club must be smaller than some bound. A club economy exhausts blocking opportunities whenever it is large enough to accommodate any such group, and, thus, finite economies can exhaust blocking opportunities. Example 1 (revisited) shows that this feature may vanish when there are many private goods.

Finally, we comment on the nature of trading groups. Our decentralization theorems assume that there are no barriers to trade across clubs. If clubs are interpreted as firms, academic departments, or household groups, then such an assumption is warranted. However, if clubs are interpreted as jurisdictions such as countries or trading blocks, there might be barriers to trade (high transaction costs) such that most trade takes place within jurisdictions, and prices are therefore localized. We refer the reader to our (1995) paper for decentralization with local trade.

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