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Authors

Ringland, Gordon A.
Thews, Robert L.

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PRODUCTION MECHANISMS AND RELATIONS BETWEEN

SPIN DENSITY MATRIX ELEMENTS*

Gordon A. Ringland[†] and Robert L. ThewsLawrence Radiation Laboratory
University of California
Berkeley, California

December 26, 1967

ABSTRACT

The spin structure of amplitudes for the production of a single baryon or meson resonance of arbitrary spin in a quasi-two-body final state has been investigated. If the dominant mechanism is the exchange of a single set of quantum numbers in the crossed channel, the spin density matrix elements are shown to satisfy bilinear relations. Typical relations are

$$(\text{Re } \rho_{m,n})^2 + (\text{Re } \rho_{m,-n})^2 = \rho_{mm} \rho_{nn}$$

for baryon production, and

$$(\text{Re } \rho_{m,n} \pm \text{Re } \rho_{m,-n})^2 = (\rho_{mm} \pm \rho_{m,-m})(\rho_{nn} \pm \rho_{n,-n})$$

for meson production. It is suggested that the use of these relations in data analysis is a good test for the presence of two or more exchange contributions.

The relations are used to analyze vector and tensor meson production from πN and KN initial states, and $\Delta(1236)$ production from πN and NN initial states. Two of the most interesting results concern Δ production. We find that the small-angle data on $NN \rightarrow N\Delta$

cannot be dominated by π exchange alone, and analysis of the data on $\pi N \rightarrow \pi \Delta$ at 8 GeV/c indicates the existence of contributions in addition to ρ exchange.

I. INTRODUCTION

A large amount of information has recently become available on high energy inelastic reactions of strongly interacting particles. One common feature of these reactions is that a large percentage of the events can be classified as quasi-two-body events, in which the final state is the result of the decay of one or two unstable particles. The differential cross sections often show a rapid decrease with increasing momentum transfer, indicating that a peripheral-model analysis might be appropriate. It is of interest to determine if the dominant mechanism is the exchange of a definite set of quantum numbers (a single particle or Regge trajectory) in the crossed channel.

The characteristic energy dependence $d\sigma/dt \sim s^{2\alpha(t)-2}$, where $\alpha(t)$ is the angular momentum exchanged, is not a very sensitive test for the presence of two or more exchanges with similar trajectory values. Polarization effects are a good test, since they depend only on interference terms, but no data are presently available on inelastic reactions with polarized targets or beams. However, an analysis of the angular distribution of decay products in an unpolarized experiment gives information on the real parts of the spin density matrix elements. These elements are directly related to the individual spin amplitudes. Requirements of parity and G-parity conservation give relations between different spin amplitudes, which may be transformed into relations between the spin density matrix elements. Relations of this type are necessary but not sufficient conditions for single particle or trajectory exchange.

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Restrictions such as these have been noted for certain reactions and have been used as constraints in data fitting.¹⁻⁵ They are valid for all energy and momentum transfer, and consequently are more useful than the obvious restriction of asymptotic energy independence outside the forward scattering region. We derive such relations in general for the production of a single meson or baryon resonance of arbitrary spin in a quasi-two-body meson-nucleon or nucleon-nucleon interaction. That such relations must exist for sufficiently high spin is obvious, since the number of independent spin amplitudes for the production of a spin J particle grows like J , while the number of spin density matrix elements grows like J^2 .

II. FORMALISM

Consider the s -channel process $A + B \rightarrow C + D$, with the subsequent decay of either C or D . For a two-body parity conserving decay, the angular distribution of the products may be written⁶

$$W(\theta, \phi) = N \sum_{\substack{mm' \\ \lambda_\alpha \lambda_\beta}} |M(\lambda_\alpha, \lambda_\beta)|^2 \rho_{mm'} e^{i(m-m')\phi} d_{m\lambda}^J(\theta) d_{m'\lambda}^J(\theta), \quad (1)$$

where λ_α and λ_β are the helicities of the decay particles, $\lambda = \lambda_\alpha - \lambda_\beta$, $M(\lambda_\alpha, \lambda_\beta)$ is the decay amplitude, and J the spin of the parent particle. The density matrix elements $\rho_{mm'}$ depend only on the production process. A convenient quantization axis for $\rho_{mm'}$ is the so-called t -channel axis, which is the direction of the incoming particle (A for C or B for D) as seen in the rest frame of the

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unstable particle. Then the density matrix elements may be expressed in terms of helicity amplitudes $F_{\lambda_B \lambda_D, \lambda_A \lambda_C}$ for the t-channel process $A + \bar{C} \rightarrow \bar{B} + D$. The relation is

$$\rho_{mm'}^C = \frac{\sum_{\lambda_A \lambda_B \lambda_D} F_{\lambda_B \lambda_D, \lambda_A m} F_{\lambda_B \lambda_D, \lambda_A m'}^*}{\sum_{\lambda_A \lambda_B \lambda_C \lambda_D} |F_{\lambda_B \lambda_D, \lambda_A \lambda_C}|^2} \quad (2)$$

for particle C, with a similar expression for particle D.

From parity conservation, the F's have the property

$$F_{-\lambda_B -\lambda_D, -\lambda_A -\lambda_C} = (-1)^{(\lambda_B - \lambda_D) - (\lambda_A - \lambda_C)} \frac{\eta_B \eta_D}{\eta_A \eta_{\bar{C}}} (-1)^{s_A + s_C - s_B - s_D} F_{\lambda_B \lambda_D, \lambda_A \lambda_C}, \quad (3)$$

where η_i and s_i are the intrinsic parity and spin of the particles.

Equations (2) and (3) imply

$$\rho_{mm'}^* = \rho_{m'm}, \quad (4)$$

$$\rho_{-m-m'} = (-1)^{m-m'} \rho_{mm'}, \quad (5)$$

and

$$\sum_m \rho_{mm} = 1. \quad (6)$$

Thus the independent measurable density matrix elements are,

$$\rho_{mm}, \quad 0 < m \leq J,$$

$$\text{Re } \rho_{mn}, \quad |n| < m \leq J,$$

and $\rho_{m, -m}$ if J is integral.

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The partial-wave expansion of the helicity amplitudes is⁷

$$F_{\lambda_B \lambda_D, \lambda_A \lambda_C}^{J}(t, x) = \sum_{J=\lambda_M}^{\infty} f_{\lambda_B \lambda_D, \lambda_A \lambda_C}^{J}(t) d_{\lambda \lambda'}^J(x), \quad (7)$$

where t is the total energy squared and x the cosine of the scattering angle in the t -channel center of mass, f^J is the partial-wave helicity amplitude, $\lambda = \lambda_A - \lambda_C$, $\lambda' = \lambda_B - \lambda_D$, and $\lambda_M = \text{Max}(|\lambda|, |\lambda'|)$. For the exchange of definite quantum numbers, the different partial-wave helicity amplitudes are related by parity restrictions. Since⁷

$$P|J \lambda_1 \lambda_2\rangle = P_1 P_2 (-1)^{J-s_1-s_2} |J, -\lambda_1 -\lambda_2\rangle, \quad (8)$$

where P_i and s_i are the intrinsic parity and spin of particle i , we obtain the relation

$$f_{\lambda_B \lambda_D, -\lambda_A -\lambda_C}^J = \sigma_E \sigma_A \sigma_C f_{\lambda_B \lambda_D, \lambda_A \lambda_C}^J, \quad (9)$$

where the J -parity $\sigma_i = P_i (-1)^{s_i}$ if i is a boson, or $\sigma_i = P_i (-1)^{s_i - \frac{1}{2}}$ if i is a fermion, and E represents the exchanged trajectory. Thus $f_{\lambda_B \lambda_D, 00}^J = 0$ if $\sigma_E \sigma_A \sigma_C = -1$. Note that the σ_i 's in (9) are for the particles in the s -channel reaction, not the antiparticles in the t -channel reaction.

If A and C or B and D are identical fermions, G parity brings in additional relations. A particular case of interest is the nucleon-antinucleon system. The results of combining parity and G -parity restrictions have been reported elsewhere⁴ and are summarized by the following rules. Isospin, parity, G -parity, and J -parity of the

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exchanged particle or trajectory are denoted by I, P, G, and σ .

1. For $\sigma = -1$, $PG(-1)^I = -1$, all amplitudes with $\lambda_1 \neq \lambda_2$ are zero.

2. For $\sigma = -1$, $PG(-1)^I = +1$, all amplitudes with $\lambda_1 = \lambda_2$ are zero.

3. For $\sigma = +1$, $PG(-1)^I = +1$, all amplitudes are nonzero.

4. For $\sigma = +1$, $PG(-1)^I = -1$, all amplitudes are zero.

In subsequent discussion, particles or trajectories with quantum numbers in 1, 2, and 3 above will be called π -type, A_1 -type, and ρ -type, respectively.

The symmetry properties of the rotation coefficients under interchange and reflection of helicities make it possible to transform the relations between partial-wave amplitudes into relations between the full helicity amplitudes. The useful properties are⁷

$$d_{\lambda\mu}^J(x) = d_{-\mu-\lambda}^J(x), \quad (10)$$

$$d_{\lambda\mu}^J(x) = (-1)^{\lambda-\mu} d_{\mu\lambda}^J(x), \quad (11)$$

and

$$d_{m,1}^J(x) + d_{m,-1}^J(x) = \begin{pmatrix} 1 \\ x \end{pmatrix} C_{m1}^J(x) + \text{lower order terms in } x, \quad (12)$$

for $m \geq 1$, where

$$C_{m1}^J(x) \equiv \frac{\Gamma(2J+1) \left(\frac{1-x^2}{4} \right)^{(m-1)/2} \left(\frac{x}{2} \right)^{J-m}}{\left[\Gamma(J+m+1)\Gamma(J-m+1)\Gamma(J)\Gamma(J+2)^{\frac{1}{2}} \right]}. \quad (13)$$

The assumption of the exchange of a single particle or trajectory in the t channel also assures that all helicity amplitudes will have the same phase.

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Since $\lambda_A = \lambda_C = 0$ in this reaction, we must have $\sigma_E = +1$ from (9). Denote the helicities of the $\frac{1}{2}^+$ and N^* by λ and m . Then parity conservation at the baryon vertex gives

$$f_{-\lambda-m}^J(t) = \sigma_{N^*} f_{\lambda m}^J(t). \quad (14)$$

With the use of (10) and (11), the parity restrictions on the helicity amplitudes may be written

$$F_{-\lambda-m} = \sigma_{N^*} (-1)^{\lambda-m} F_{\lambda m}. \quad (15)$$

In obtaining the sum rule, one writes out the measurable density matrix elements in terms of the independent helicity amplitudes. For simplicity in notation, all amplitudes are normalized by dividing by

$$\left[\sum_{\lambda m} |F_{\lambda m}|^2 \right]^{\frac{1}{2}} :$$

$$\rho_{mm} = \sum_{\lambda=\pm\frac{1}{2}} |F_{\lambda m}|^2 \quad (16)$$

$$\text{Re } \rho_{mn} = \sum_{\lambda} F_{\lambda m} F_{\lambda n} \quad (17)$$

$$\begin{aligned} \text{Re } \rho_{m, -n} &= \sum_{\lambda} F_{\lambda m} F_{\lambda, -n} \\ &= \sigma_{N^*} \sum_{\lambda} (-1)^{\lambda-n} F_{\lambda m} F_{-\lambda n}. \end{aligned} \quad (18)$$

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To eliminate the mixed m - n terms in (17) and (18), one may square and add, noting that the cross terms cancel:

$$\begin{aligned}
 (\text{Re } \rho_{mn})^2 + (\text{Re } \rho_{m,-n})^2 &= \sum_{\lambda\lambda'} F_{\lambda m} F_{\lambda' m} (F_{\lambda n} F_{\lambda' n} + (-1)^{\lambda-\lambda'} F_{-\lambda-n} F_{-\lambda'-n}) \\
 &= \sum_{\lambda} |F_{\lambda m}|^2 \sum_{\lambda} |F_{\lambda n}|^2 .
 \end{aligned}$$

The last expression is just the product of the m and n diagonal elements, so that the relation is

$$(\text{Re } \rho_{mn})^2 + (\text{Re } \rho_{m,-n})^2 = \rho_{mm} \rho_{nn} . \quad (19)$$

Note that this rule must be satisfied for the production of an N^* with any quantum numbers if it is dominated by the exchange of any single set of quantum numbers in the t channel. Note also that the two 0^- particles need not be identical, so that the relation should apply to processes like $\pi + N \rightarrow \eta + N^*$, $K + N \rightarrow \pi + Y^*$, etc.

If more than one exchanged particle or trajectory contributes to the process, the helicity amplitudes will not in general have the same phase, so that the relation may not be satisfied. However, we can express the terms which violate the relation in terms of the interference between amplitudes due to different exchanges, or equivalently in terms of the imaginary parts of the density matrix elements. The relevant expressions are

$$\rho_{mn} = F_{\frac{1}{2}m} F_{\frac{1}{2}n}^* + F_{-\frac{1}{2}m} F_{-\frac{1}{2}n}^* , \quad (20)$$

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$$\rho_{m, -n} = \sigma_{N^*} (-1)^{\frac{1}{2}-n} (F_{\frac{1}{2}m} F_{-\frac{1}{2}n}^* - F_{-\frac{1}{2}m} F_{\frac{1}{2}n}^*), \quad (21)$$

and

$$\rho_{m, -m} = \sigma_{N^*} (-1)^{m+\frac{1}{2}} 2i \operatorname{Im}(F_{\frac{1}{2}m} F_{-\frac{1}{2}m}^*). \quad (22)$$

The cross terms are again eliminated by squaring and adding, yielding

$$|\rho_{mn}|^2 + |\rho_{m, -n}|^2 = \rho_{mm} \rho_{nn} + (-1)^{m-n} \rho_{m, -m} \rho_{n, -n}. \quad (23)$$

The terms which violate the single-exchange rule (19) can be separated, to give the relation

$$\begin{aligned} \rho_{mm} \rho_{nn} - (\operatorname{Re} \rho_{mn})^2 - (\operatorname{Re} \rho_{m, -n})^2 &= (\operatorname{Im} \rho_{mn})^2 + (\operatorname{Im} \rho_{m, -n})^2 \\ &+ (-1)^{m-n} \operatorname{Im} \rho_{m-n} \operatorname{Im} \rho_{n, -n}. \end{aligned} \quad (24)$$

For production of a baryon of spin $K + \frac{1}{2}$, there are $2(K+1) - 1$ independent normalized helicity amplitudes. There are K independent diagonal elements ρ_{mm} and $K(K+1)/2$ independent $\operatorname{Re} \rho_{mn}$ for $n > 0$. There are also $K(K+1)/2$ elements $\operatorname{Re} \rho_{m, -n}$, constrained by the relation (19). However, the number of independent density matrix elements still grows faster than the number of helicity amplitudes, so there must be additional relations for high spin production. The number of these additional relations is

$$N = \frac{K(K-1)}{2} - 1. \quad (25)$$

Then for spin $7/2$ there are two additional relations, for $9/2$ five relations, and so on. The form of these relations is not immediately

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apparent from the general equations, so evidently they must be worked out individually for each spin. This could be done by trying to invert (16) - (18) and looking for singularities.

B. $N + N \rightarrow N + N^*$

The derivation of relations between spin density matrix elements for this reaction proceeds exactly as in the previous case, except for the complication of additional amplitudes due to nucleon spin. The parity relations for the partial-wave amplitudes are

$$f_{\lambda m, -\mu\delta}^J = \sigma_E f_{\lambda m, \mu\delta}^J \quad (26)$$

and

$$f_{-\lambda -m, \mu\delta}^J = \sigma_E \sigma_{N^*} f_{\lambda m, \mu\delta}^J \quad (27)$$

These are used to write the expressions for the density matrix elements in the form

$$\text{Re } \rho_{mn} = \sum_{\mu\delta\lambda} F_{\lambda m, \mu\delta} F_{\lambda n, \mu\delta}, \quad (28)$$

$$\begin{aligned} \text{Re } \rho_{m, -n} = & \sigma_E \sigma_{N^*} \sum_{\mu\lambda} (-1)^{\lambda+n} F_{\lambda m, \mu\mu} F_{-\lambda n, \mu\mu} \\ & - \sigma_{N^*} \sum_{\mu\lambda} (-1)^{\lambda+n} F_{\lambda m, \mu-\mu} F_{-\lambda n, -\mu\mu}. \end{aligned} \quad (29)$$

It is convenient to study separately the exchange of the three types of trajectories. For the π -type trajectory, only those amplitudes with $\mu = \delta$ will be nonzero, so that the second sum in (29) is absent. The same operations as in the previous case (with the additional sum over μ)

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yield the same result. The rule (19) is satisfied for the exchange of a single π -type trajectory.

For the exchange of an A_1 -type trajectory, only those amplitudes with $\mu \neq \delta$ will be nonzero. However, the parity relations (26), (27) cannot be directly taken over to the full helicity amplitudes, since the nonzero helicity flip appears also in the rotation coefficients. The methods of the previous case can be used to yield

$$\begin{aligned}
 & (\text{Re } \rho_{mn})^2 + (\text{Re } \rho_{m,-n})^2 - \rho_{mm} \rho_{nn} \\
 &= \sum_{\lambda\mu} F_{\lambda m, \mu-\mu} F_{-\lambda m, \mu-\mu} (F_{\lambda n, \mu-\mu} F_{-\lambda n, \mu-\mu} - F_{\lambda n, -\mu\mu} F_{-\lambda n, -\mu\mu}) \\
 &+ \sum_{\lambda\mu} F_{\lambda m, \mu-\mu} F_{\lambda m, -\mu\mu} (F_{\lambda n, \mu-\mu} F_{\lambda n, -\mu\mu} + F_{-\lambda n, \mu-\mu} F_{-\lambda n, -\mu\mu}) \\
 &- \sum_{\lambda\mu} F_{\lambda m, \mu-\mu}^2 \sum_{\lambda'} F_{\lambda' n, -\mu\mu}^2. \tag{30}
 \end{aligned}$$

The relation (12) must now be used in the form

$$d_{m, \pm 1}^J(x) = \left(\frac{1 \pm x}{2} \right) c_{m1}^J(x). \tag{31}$$

One can then show that the first sum in (30) is zero, and that the second and third sums cancel. Thus the rule (19) is also valid for the exchange of an A_1 -type trajectory.

For the exchange of a ρ -type trajectory, terms with both $\mu = \pm \delta$ are nonzero. Upon writing an expression analogous to (30) for this case, one sees that the second sum contains interference terms of the type

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$F_{\lambda m, \mu\mu} F_{\lambda' m, \mu' - \mu'} F_{-\lambda' n, -\mu' \mu'} F_{\lambda n, \mu\mu}$, which cannot be cancelled by any of the other terms. The result is that the rule (19) is not valid even for a single ρ -type trajectory exchange.

C. $O^- + \frac{1}{2}^+ \rightarrow V + \frac{1}{2}^+$

First consider the case when the meson V has natural spin-parity, $\sigma_V = +1$. From (9), we see that if $\sigma_E = +1$, all amplitudes with zero meson helicity are zero. As a consequence,⁶

$$\rho_{00} = 0, \quad (32a)$$

$$\text{Re } \rho_{m0} = 0. \quad (32b)$$

This is an immediate test of the necessity of $\sigma_E = -1$ exchanges in meson production reactions.^{3,4}

It is convenient to write out the density matrix elements for a combination of an arbitrary number of $\sigma_E = \pm 1$ exchanges. Denote the helicity amplitudes for each by $F_{m, \lambda\mu}^{A, B}$, respectively. The relations between various helicity amplitudes can be derived, using (8) - (12).

If we define

$$F_{m1}^{A, B} = f_{m, \frac{1}{2} - \frac{1}{2}}^{J_A, J_B} C_{m1}^{J_A, J_B}(x), \quad (33)$$

the spin density matrix elements may be written in the form

$$\begin{aligned} \text{Re } \rho_{mn} + (-1)^n \text{Re } \rho_{m, -n} &= 4 \text{Re} \left(F_{m, \frac{1}{2} \frac{1}{2}}^B F_{n, \frac{1}{2} \frac{1}{2}}^{B*} \right) + \text{Re} \left(F_{m1}^A F_{n1}^{A*} \right) \\ &+ x^2 \text{Re} \left(F_{m1}^B F_{n1}^{B*} \right) - x \text{Re} \left[F_{m1}^A F_{n1}^{B*} + F_{m1}^B F_{n1}^{A*} \right], \end{aligned} \quad (34)$$

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$$\begin{aligned}
\text{Re } \rho_{mn} - (-1)^n \text{Re } \rho_{m, -n} &= 4 \text{Re} \left(F_{m, \frac{11}{22}}^A F_{n, \frac{11}{22}}^{A*} \right) + \text{Re} \left(F_{ml}^B F_{nl}^{B*} \right) \\
&+ x^2 \text{Re} \left(F_{ml}^A F_{nl}^{A*} \right) - x \text{Re} \left[F_{ml}^A F_{nl}^{B*} + F_{ml}^B F_{nl}^{A*} \right].
\end{aligned}
\tag{35}$$

If a single $\sigma_E = +1$ exchange dominates, all F^B 's are zero, all F^A 's have the same phase, and from (34) we can derive the relation

$$\left(\text{Re } \rho_{mn} + (-1)^n \text{Re } \rho_{m, -n} \right)^2 = \left(\rho_{mm} + (-1)^m \rho_{m, -m} \right) \left(\rho_{nn} + (-1)^n \rho_{n, -n} \right).
\tag{36}$$

If a single $\sigma_E = -1$ exchange dominates, all F^A 's are zero, and (35) yields

$$\left(\text{Re } \rho_{mn} - (-1)^n \text{Re } \rho_{m, -n} \right)^2 = \left(\rho_{mm} - (-1)^m \rho_{m, -m} \right) \left(\rho_{nn} - (-1)^n \rho_{n, -n} \right).
\tag{37}$$

Note that (37) has no content for m or $n = 0$. If the $\frac{1}{2}^+$ particles are identical, G-parity conservation then distinguishes between the two types of $\sigma_E = -1$ exchanges. For a π -type exchange only $F_{m, \frac{11}{22}}^B$ are nonzero, and for A_1 -type exchange only F_{ml}^B are nonzero. One sees from (34) that in both cases the relation (36) is valid. Thus for a reaction such as $\pi N \rightarrow V N$, the relation (36) is valid for any single trajectory exchange, whereas for a reaction like $K N \rightarrow V \Sigma$, the relation is either (36) or (37), depending on the value of σ_E .

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The expressions (34) and (35) may be combined to give more information for $m = n$. Since $\left(\begin{smallmatrix} A, B \\ F_{m, \lambda \mu} \end{smallmatrix} \right)^2 \geq 0$, one can derive the inequality for any number of exchanges with the same σ_E ,

$$\rho_{mm}(1 - x^2) - \sigma_E(-1)^m \rho_{m, -m}(1 + x^2) \geq 0. \quad (38)$$

If this is combined with the constraint due to parity conservation alone that $|\rho_{m, -m}| \leq \rho_{mm}$, one has both an upper and lower bound,

$$\left(\frac{x^2 - 1}{x^2 + 1} \right) \leq \frac{|\rho_{m, -m}|}{\rho_{mm}} \leq 1, \quad (39)$$

along with the condition that $\rho_{m, -m}$ has the same sign as $\sigma_E(-1)^{m+1}$. Since $x^2 = 1$ for inelastic reactions in the forward direction, but increases rapidly away from the forward direction at high energy, (39) can be very restrictive and a good test for the presence of exchanges with different σ_E .

For the case where G-parity conservation differentiates between π - and A_1 -type exchanges, (34) and (35) can be used to give more information, even for $m \neq n$. One sees immediately that for any number of π -type exchanges,

$$\text{Re } \rho_{m, -n} = (-1)^n \text{Re } \rho_{mn}, \quad (40)$$

and for any number of A_1 -type exchanges

$$\text{Re } \rho_{m, -n} = (-1)^n \left(\frac{x^2 - 1}{x^2 + 1} \right) \text{Re } \rho_{mn} \quad (41)$$

for $n \neq 0$. If we insert (40) or (41) into the sum rule (36), it may be written in the compact form

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$$(\text{Re } \rho_{mn})^2 = \rho_{mm} \rho_{nn} . \quad (42)$$

For the production of V particles with $\sigma_V = -1$, the only change is due to the restriction of (9), which has the effect of switching the role of $\sigma_E = \pm 1$ exchanges. For example, (32a,b) are now valid for a $\sigma_E = -1$ exchange. For all the relations, the change can be made by inserting a factor σ_V before each $(-1)^m$ or $(-1)^n$ factor.

One can summarize the results as follows. Consider the reaction $PN \rightarrow VM$, where $P = 0^-$, $N, M = \frac{1}{2}^+$, V has J-parity σ_V . If a single trajectory with J-parity σ_E is exchanged, one has the relation

$$\begin{aligned} (\text{Re } \rho_{mn} + \sigma_E \sigma_V (-1)^n \text{Re } \rho_{m, -n})^2 &= (\rho_{mm} + \sigma_E \sigma_V (-1)^m \rho_{m, -m}) \\ &\times (\rho_{nn} + \sigma_E \sigma_V (-1)^n \rho_{n, -n}). \end{aligned} \quad (43)$$

In addition, the elements $\rho_{m, -m}$ have the property

$$\left(\frac{x^2 - 1}{x^2 + 1} \right) \leq \frac{|\rho_{m, -m}|}{\rho_{mm}} \leq 1 , \quad (44)$$

and

$$\rho_{m, -m} = \sigma_E \sigma_V (-1)^{m+1} |\rho_{m, -m}| \quad (45)$$

for any number of exchanges with the same σ_E . For the reaction $PN \rightarrow VN$, (43) - (45) above are true, with additional results from G-parity for $\sigma_E = -1$ exchanges. For a combination of π^- and A_1 -type exchanges, (43) - (45) also apply, with $\sigma_E = -1$. For a single π^- or A_1 -type exchange, (43) also holds with $\sigma_E = +1$. In addition, we have the relations

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$$\text{Re } \rho_{m, -n} = \sigma_V (-1)^n \text{Re } \rho_{mn} \quad (46)$$

for any number of π -type exchanges, and

$$\text{Re } \rho_{m, -n} = \sigma_V (-1)^n \left(\frac{x^2 - 1}{x^2 + 1} \right) \text{Re } \rho_{mn} \quad (47)$$

for any number of A_1 -type exchanges with $n \neq 0$. If these are inserted in (43) with $\sigma_E = +1$, the compact form (42) results.

As in the baryon resonance case, there may be additional relations for high-spin particle production. There are four cases to consider.

Let s be the spin of the meson resonance.

1. For $\sigma_V \sigma_E = +1$, there are $2s - 1$ independent normalized amplitudes, $s^2 + s - 1$ density matrix elements with $\frac{1}{2} s(s - 1)$ relations of the type (43). The number of additional relations is

$$N_1 = \frac{1}{2} s(s - 1), \quad (48)$$

which means that they start with spin-two production, in reactions such as $\pi N \rightarrow A_2 N$ with ρ exchange.

2. For $\sigma_V \sigma_E = +1$, but $\sigma_E = -1$ and G-parity restrictions, there are an additional $\frac{1}{2} s(s - 1)$ relations of the type (46) or (47). Then the number of additional relations is reduced to

$$N_2 = 0, \quad (49)$$

i.e., no additional relations are required for any spin.

3. For $\sigma_V \sigma_E = -1$, there are $2(s + 1) - 1$ independent normalized amplitudes, $s(s + 2)$ density matrix elements with $\frac{1}{2} s(s - 1)$ relations

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of the type (43). The number of additional relations must be

$$N_3 = \frac{1}{2} (s^2 + s - 2) . \quad (50)$$

The additional relations again start with spin-two production, in reactions such as $KN \rightarrow A_2 \Lambda$ with K exchange, and $\pi N \rightarrow B(2^-)N$ with ω exchange.

4. For $\sigma_V \sigma_E = -1$, but $\sigma_E = -1$ and G-parity restrictions, there are s more relations of the type (43), and $\frac{1}{2} s(s - 1)$ relations from (46) or (47), reducing the number of additional relations to

$$N_4 = -1 . \quad (51)$$

In cases one and three, the general form of the additional relations is not apparent, and evidently they must be treated individually. Note that in all cases, N_i represents the minimum number of additional relations. The detailed structure of the density matrix-helicity amplitude relationship could produce additional relations even when none are required by the counting of independent amplitudes.

D. $\pi^- p \rightarrow \rho^0 n$.

This reaction, although it falls into the previous category, requires special attention. This is due to the forward-backward asymmetry observed in ρ^0 decay, indicating that the pions do not decay from a pure 1^- state. The analysis of the $\pi\pi$ angular distribution is usually done with the assumption that it is a combination of ρ decay plus some S-wave two-pion state.⁹ There are five measurable density matrix elements in this case: the usual

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$\text{Re } \rho_{10}$ and $\rho_{1,-1}$ for the pure 1^- state, a combination of diagonal elements $\rho'_{00} \equiv \rho_{00} + \rho_{ss}$, and two interference elements $\text{Re } \rho_{s0}$ and $\text{Re } \rho_{s1}$. It is thought that pion exchange alone may account for this process, so we shall derive relations on this basis. We denote the helicity amplitudes for ρ production with helicity 0 and 1 by R_0 and R_1 , and the amplitude for s-wave production by R_2 . If we define $y \equiv R_1/R_0$ and $z \equiv R_2/R_0$, we can write

$$\rho'_{00} = N(1 + \frac{1}{3} z^2), \quad (52a)$$

$$\text{Re } \rho_{10} = N y, \quad (52b)$$

$$\text{Re } \rho_{s0} = N z, \quad (52c)$$

$$\rho_{1,-1} = -N y^2, \quad (52d)$$

and

$$\text{Re } \rho_{s1} = N y z, \quad (52e)$$

where

$$N \equiv \frac{1}{1 + 2y^2 + z^2}. \quad (52f)$$

One sees immediately that $\rho_{1,-1}$ must be negative, and that the product of the interference terms must have the same sign as $\text{Re } \rho_{10}$. To get relations between the matrix elements, we express y and z in terms of ρ'_{00} and $\text{Re } \rho_{s0}$, and then solve for the remaining elements. The results are

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$$(\text{Re } \rho_{10})^2 = \frac{(\text{Re } \rho_{s0})^2 (1 - \alpha\beta)}{3(\alpha - 1)}, \quad (53)$$

$$\rho_{1,-1} = -\rho'_{00} (1 - \alpha\beta)/\alpha, \quad (54)$$

and

$$(\text{Re } \rho_{s1})^2 = (\text{Re } \rho_{s0})^2 (1 - \alpha\beta), \quad (55)$$

where

$$\alpha \equiv \frac{3}{2} (\rho'_{00}/\text{Re } \rho_{s0})^2 \left(1 - \left[1 - \frac{4}{3} (\text{Re } \rho_{s0}/\rho'_{00})^2 \right]^{1/2} \right) \quad (56)$$

and

$$\beta \equiv (3\rho'_{00} - 1)/2\rho'_{00}. \quad (57)$$

From the reality condition, we get the additional constraint

$$\left| \frac{\text{Re } \rho_{s0}}{\rho'_{00}} \right| \leq \frac{3}{4}. \quad (58)$$

III. ANALYSIS OF DATA

The following comparisons with experiment are presented in the same order as the derivation of the constraints in Section II. We emphasize that while the failure of the data to satisfy the constraints demonstrates that more than one trajectory contributes, the converse does not hold.

A. $\underline{0^- + \frac{1}{2}^+ \rightarrow 0^- + N^*}$

Relation (19) is used to analyze data on the reactions $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ and $\pi^+ p \rightarrow \eta \Delta^{++}$. These reactions are of particular interest since only one established trajectory contributes in each case. We write

$$\rho_{33} \rho_{11} - (\text{Re } \rho_{31})^2 - (\text{Re } \rho_{3-1})^2 = \sum_{-\Delta\Sigma_-}^{+\Delta\Sigma_+}, \quad (59)$$

so that Σ measures the violation of the constraints by the data. This notation is used throughout the rest of this section.

A single trajectory exchange gives $\Sigma = 0$, and $\Delta\Sigma_{\pm}$ are the errors due to the experimental errors in the determination of the density matrix elements. The results are presented in Table I.

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Table I. Test of relation (19) for $\Delta(1236)$ production in π -N interactions.

Laboratory Momentum, P_{lab} , GeV/c	$-t, (\text{GeV}/c)^2$	Σ
$\pi^+p \rightarrow \pi^0\Delta^{++}$		
4.0^{10}	0.04	0.001
		+0.040
		-0.094
4.0^{10}	0.10	0.015
		+0.030
		-.081
4.0^{10}	0.20	0.030
		+0.044
		-0.078
8.0^{11}	0.03	0.050
		+0.007
		-0.038
8.0^{11}	0.11	0.052
		+0.023
		-0.037
8.0^{11}	0.20	0.006
		+0.039
		-0.070
$\pi^+p \rightarrow \eta\Delta^{++}$		
$3.0 - 4.0^{12}$	Average over all t	-0.010
		+0.035
		-0.052

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To understand the significance of the above results we must consider how badly constraint (19) can be violated. For the case of $J = 3/2$, assuming parity conservation, the maximum value of $\rho_{33}\rho_{11}$ is 0.0625, and the minimum value of $(\text{Re } \rho_{3-1})^2 + (\text{Re } \rho_{31})^2$ is, of course, zero. Therefore a value for Σ of 0.0625 represents the maximal violation of our constraint.

It is of considerable interest that the 4.0 GeV/c gives Σ compatible with zero, while the two lower-momentum transfer points in the 8.0-GeV/c data fail to satisfy (19) by approximately 1.5 standard deviations and the central value of Σ is approximately 80% of its maximum. We tentatively conclude that the reaction $\pi^+p \rightarrow \pi^0\Delta^{++}$ cannot be completely explained by ρ exchange alone. However, a more definite conclusion must await improved statistics. The above situation is similar to π -N charge exchange, where a nonvanishing polarization demonstrates that ρ exchange cannot be the only contribution to the reaction.

The analysis of $\pi^+p \rightarrow \eta\Delta^{++}$ shows that Σ is consistent with zero, and so gives no evidence of any exchange in addition to the A_2 .

B. $NN \rightarrow NN^*$

The 5.5-GeV/c $pp \rightarrow p\Delta^+$ data¹³ are analyzed by using (59), and the results presented in Table II. Recall that (19) is satisfied (i.e., $\Sigma = 0$ in (59)) for either single π or A_1 exchange.

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Table II. Test of relation (19) for $\Delta(1236)$ production in N-N interactions.

$P_{\text{lab}}, \text{ GeV/c}$	$-t, (\text{GeV/c})^2$	Σ
$pp \rightarrow p\Delta^+$		
5.5	0.004 - 0.084	+0.005
		-0.009
5.5	0.084 - 0.164	+0.016
		-0.027
5.5	0.164 - 0.406	+0.012
		-0.023
5.5	Average over all angles	+0.005
		-0.010

Clearly (19) is not satisfied by the smallest momentum transfer data, and indeed the value of Σ indicates approximately maximal violation. It has been proposed that π trajectory exchange alone could account for the small-angle data on $pp \rightarrow p\Delta^+$,¹⁴ and such a model can account satisfactorily for the shape and energy dependence of the differential cross section. However, the above result demonstrates that the π exchange model cannot possibly explain the spin structure of the scattering amplitudes near the forward direction. If π exchange is to play a significant role in this process, then there

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must exist a substantial contribution from another trajectory. Consideration of the energy dependence¹⁴ implies that this additional trajectory should have an intercept close to π at small t values. One notes that a conspirator for the π could fill this role.

C. Reactions of the Type $0^- + \frac{1}{2}^+ \rightarrow V + \frac{1}{2}^+$

As an example of the weakest constraints for $0^- + \frac{1}{2}^+ \rightarrow V + \frac{1}{2}^+$

we first discuss data on $K^- p \rightarrow \begin{pmatrix} \rho^0 \\ \phi \\ \omega \end{pmatrix} \Lambda$. In this case we have no constraints due to G-parity. The only constraints concerning 1^- production are $\begin{pmatrix} 3 & a \\ 2 & b \end{pmatrix}$ which can tell us if $\sigma_E = -1$ exchange is required, and constraints (44) and (45). The data considered are at 4.1 and 5.5 GeV/c,¹⁵ and the density matrix elements are the average over all angles. The results are presented in Table III.

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Table III. Test of relations (44) and (45) for vector meson production in K-N interactions.

$P_{\text{lab}}, \text{ GeV/c}$	ρ_{1-1}	ρ_{00}	$ \rho_{1-1} /\rho_{11}$	$(x^2 - 1)/(x^2 + 1)$ Average over all angles	
$K^- p \rightarrow \rho^0 \Lambda$					
4.1	0.00	+0.14	0.20 ± 0.16	+0.35	0.68
		-0.00		-0.00	
5.5	0.00	+0.14	+0.20	+0.31	0.71
		-0.00	-0.10	-0.00	
$K^- p \rightarrow \omega \Lambda$					
4.1	0.18 ± 0.16	0.10	+0.16	0.40 ± 0.36	0.70
			-0.10		
5.5	0.24 ± 0.16	0.20 ± 0.20	0.60 ± 0.43	0.74	
$K^- p \rightarrow \phi \Lambda$					
4.1	0.20	+0.10	0.60 ± 0.20	+0.71	0.64
		-0.14		-0.86	
5.5	0.23 ± 0.23	0.10	+0.30	+0.54	0.79
			-0.10	-0.51	

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$$\underline{K^- p \rightarrow \rho^0 \Lambda^0}$$

One sees immediately that $|\rho_{1-1}|/\rho_{11}$ fails by approximately two standard deviations to satisfy (44), $\frac{x^2 - 1}{x^2 + 1} \leq \frac{|\rho_{1-1}|}{\rho_{11}} \leq 1$, indicating more than one exchange contributes. The large errors on ρ_{00} do not allow us to determine through (32a) whether $\sigma_E = -1$ is required.

$$\underline{K^- p \rightarrow \omega \Lambda}$$

The sign of ρ_{1-1} , using (45), implies $\sigma = +1$ exchange, hence a nonzero value for ρ_{00} would demand both $\sigma = +1$ and $\sigma = -1$ exchange. However, ρ_{00} is not incompatible with zero. Finally we note that (44) is not violated by more than one standard deviation, and we therefore conclude that there is no definite evidence for the exchange of more than one trajectory.

$$\underline{K^- p \rightarrow \phi \Lambda}$$

As above, (44) is not violated. The 4.1 GeV/c results require $\sigma_E = -1$, which from (45) would give ρ_{1-1} negative. The positive central value of ρ_{1-1} implies the addition of $\sigma = +1$ exchange, but the large errors on ρ_{1-1} allow no firm conclusion. The 5.5 GeV/c results are consistent with a single contribution in the t channel.

Recall that $\frac{|\rho_{1-1}|}{\rho_{11}} > 1$ implies the existence of background, since

conservation of parity gives $|\rho_{1-1}|/\rho_{11} \leq 1$ for a pure $J = 1$ state.

For the remainder of this section we consider reactions with a nucleon in both the initial and final states. This, of course, means

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that we have stronger constraints than in the general case of $0^- + \frac{1}{2}^+ \rightarrow V + \frac{1}{2}^+$, because of the additional information given by conservation of G-parity.

$$\underline{\pi^- p \rightarrow A_2 p}$$

We write

$$(\rho_{22} + \rho_{2-2})(\rho_{11} - \rho_{1-1}) - (\text{Re } \rho_{21} - \text{Re } \rho_{2-1})^2 = \Sigma, \quad (60)$$

$$\frac{1}{2}\rho_{00}(\rho_{22} + \rho_{2-2}) - (\text{Re } \rho_{20})^2 = \Sigma, \quad (61)$$

$$\frac{1}{2}\rho_{00}(\rho_{11} - \rho_{1-1}) - (\text{Re } \rho_{10})^2 = \Sigma. \quad (62)$$

From (43), $\Sigma = 0$ in each of (60), (61), and (62) if a single trajectory dominates. The data analyzed are at 2.9 - 3.3 GeV/c,¹⁶ averaged over all production angles. We find in (60) and (61) that $\Sigma = 0.013 \begin{smallmatrix} +0.033 \\ -0.035 \end{smallmatrix}$

and $\Sigma = 0.011 \begin{smallmatrix} +0.023 \\ -0.024 \end{smallmatrix}$, respectively, clearly consistent with zero.

From (62), we obtain $\Sigma = 0.049 \begin{smallmatrix} +0.031 \\ -0.033 \end{smallmatrix}$, approximately 1.5 standard

deviations from zero. We tentatively conclude that a satisfactory model for the above reaction requires more than one trajectory to be exchanged. The errors on diagonal elements are such that (46) and (47) cannot give any indication of the value of σ_E required. When (32a) is used, the experimental value $\rho_{00} = 0.38 \pm 0.15$ shows $\sigma_E = -1$ is required

$$\underline{K^- p \rightarrow K^*(1400)^- p \text{ and } K^- p \rightarrow K^*(1400)^0 n}$$

The data for the above reactions are at 3.5 GeV/c,¹⁷ and the density matrix elements are the average over all production angles.

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The data are such that only (62), (32a), and (45) are relevant to the analysis. Applying (62), we obtain for $K^*(1400)^-$

$$\Sigma = 0.018 \begin{array}{l} +0.023 \\ -0.024 \end{array},$$

and for $K^*(1400)^0$

$$\Sigma = 0.120 \begin{array}{l} +0.038 \\ -0.040 \end{array}.$$

The violation of (43) by 3 standard deviations for the $K^*(1400)^0$ demonstrates that the reaction $K^-p \rightarrow K^*(1400)^0 n$ cannot proceed via single trajectory exchange. However, the data on $K^-p \rightarrow K^*(1400)^- p$ are quite compatible with $\Sigma = 0$ in (62). Since all contributions to the charge-exchange process also contribute to the non-charge-exchange case, the results for $K^-p \rightarrow K^*(1400)^0 n$ show that more than one trajectory must contribute to $K^-p \rightarrow K^*(1400)^- p$, even though the data satisfies (43). This situation underlines the warning given at the beginning of this section, that the satisfaction of our constraints does not imply the dominance of one trajectory.

We now consider the value of ρ_{00} , and the relative sign of ρ_{11} and ρ_{1-1} (32a) and (45).

$$\underline{K^*(1400)^-}$$

We have $\rho_{11} = 0.40 \pm 0.12$ and $\rho_{1-1} = 0.39 \pm 0.07$. From (45) this indicates a strong $\sigma_E = +1$ contribution. However, $\rho_{00} = 0.39 \pm 0.07$, showing that both $\sigma = +1$ and -1 trajectories are important.

$$\underline{K^*(1400)^0}$$

Here $\rho_{11} = 0.22 \pm 0.11$, and $\rho_{1-1} = -0.18 \pm 0.04$, and from (45) we see that $\sigma = -1$ exchange is required. The value of

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$\rho_{00} = 0.65 \pm 0.08$ clearly demands significant $\sigma = -1$ exchange, but from (45) we have no evidence for strong $\sigma = +1$ contribution. In comparing the two reactions we can say that from (43) both require multiple trajectory exchange, but that whereas both $\sigma_E = +1$ and $\sigma_E = -1$ are significant in $K^-p \rightarrow K^*(1400)^-p$, there is little evidence for $\sigma_E = +1$ in $K^-p \rightarrow K^*(1400)^0 n$. This suggests that the $\sigma_E = +1$ contribution to the former reaction is isoscalar.

$$\underline{K^-p \rightarrow K^*(890)^-p \text{ and } K^-p \rightarrow K^*(890)^0 n}$$

The data considered are those of the Argonne-Northwestern Collaboration at 4.1 and 5.5 GeV/c.¹⁸

$$\underline{K^-p \rightarrow K^*(890)^-p}$$

In Table IV we recall the data relevant to constraints (32 a) and (45), and we present the value of Σ for test (62). Note that the nonzero value of ρ_{00} , particularly near the forward direction, implies through (32 a) $\sigma = -1$ exchange. From (45), the sign of ρ_{1-1} indicates significant $\sigma_E = +1$ contribution. Σ is consistent with zero despite the necessity for the exchange of at least two trajectories. The data are compatible with the inequality (45). We are therefore unable to add anything to previous discussions¹⁹ in this case. The results are included only for completeness, and comparison with the charge exchange data.

Table IV. Test of relation (62) for $K^*(890)^-$ production in K-N interactions.

$-t, (\text{GeV}/c)^2$	ρ_{00}		ρ_{1-1}		Σ	
	4.1 GeV/c	5.5 GeV/c	4.1 GeV/c	5.5 GeV/c	4.1 GeV/c	5.5 GeV/c
0.0 - 0.1	0.400 ± 0.100	0.281 ± 0.073	0.300 ± 0.090	0.254 ± 0.060	$-0.0044^{+0.0083}_{-0.0036}$	$0.0109^{+0.0091}_{-0.0107}$
0.1 - 0.2	0.150 ± 0.072	0.057 ± 0.037	0.325 ± 0.067	0.379 ± 0.058	$0.0065^{+0.0054}_{-0.0066}$	$-0.0013^{+0.0032}_{-0.0045}$
0.2 - 0.4	0.132 ± 0.062	0.129 ± 0.046	0.356 ± 0.053	0.336 ± 0.044	$0.0040^{+0.0037}_{-0.0053}$	$0.0064^{+0.0029}_{-0.0035}$
0.4 - 1.0	0.226 ± 0.086	0.100 ± 0.070	0.329 ± 0.067	0.369 ± 0.074	$0.0064^{+0.0076}_{-0.0070}$	$0.0040^{+0.0037}_{-0.0050}$
0.0 - 1.0	0.224 ± 0.038	0.124 ± 0.027	0.325 ± 0.029	0.311 ± 0.027	$0.0062^{+0.0033}_{-0.0038}$	$0.0071^{+0.0019}_{-0.0023}$

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$$\underline{K^- p \rightarrow K^*(890)^0 n}$$

As in the above case, we present the results relevant to (32 a), (45), and (62) in a table.

The analysis of the data by use of (44),

$$\frac{x^2 - 1}{x^2 + 1} \leq \frac{|\rho_{1-1}|}{\rho_{11}} \leq 1,$$

is presented in Figures 1 and 2. We represent $\frac{x^2 - 1}{x^2 + 1}$, the lower bound

for $\frac{|\rho_{1-1}|}{\rho_{11}}$, by a dashed curve; and the upper bound, $\frac{|\rho_{1-1}|}{\rho_{11}} = 1$, by

the solid line. Recalling (46) and (47), we see that for any number of π -type exchanges alone, the data points lie on the line $\frac{|\rho_{1-1}|}{\rho_{11}} = 1$,

while for any number of A_1 -type exchanges the points lie on the dashed curve. Any number of $\sigma = +1$ exchanges, or a combination of π - and A_1 -type exchanges, give values for $\frac{|\rho_{1-1}|}{\rho_{11}}$ satisfying the inequality,

and from (45) the sign of ρ_{1-1} distinguishes the two cases. Data points below the dashed line in Figs. 1 and 2 fail to satisfy (44), and demonstrate the existence of a combination of $\sigma = +1$ and $\sigma = -1$ exchanges.

From Table V we see that the value of ρ_{00} indicates a substantial contribution from $\sigma = -1$ exchange. The values and errors for ρ_{1-1} at the two lower-momentum transfer points at both energies do not give any clear indication of $\sigma = +1$ exchange. However, we see

Table V. Test of relation (62) for $K^*(890)^0$ production in K-N interactions.

$-t, (\text{GeV}/c)^2$	ρ_{00}		ρ_{1-1}		Σ	
	4.1 GeV/c	5.5 GeV/c	4.1 GeV/c	5.5 GeV/c	4.1 GeV/c	5.5 GeV/c
0.0 - 0.1	0.550 ± 0.082	0.760 ± 0.088	-0.033 ± 0.067	0.047 ± 0.068	$0.0660^{+0.0101}_{-0.0135}$	$0.0009^{+0.0312}_{-0.0338}$
0.1 - 0.2	0.612 ± 0.112	0.500 ± 0.150	0.020 ± 0.099	0.100 ± 0.070	$0.0190^{+0.0354}_{-0.0420}$	$-0.0359^{+0.0392}_{-0.0500}$
0.2 - 0.4	0.378 ± 0.126	0.438 ± 0.124	0.095 ± 0.094	0.239 ± 0.099	$0.0336^{+0.0191}_{-0.0247}$	0. $^{+0.0240}_{-0.0109}$
0.4 - 1.0	0.360 ± 0.116	0.260 ± 0.110	0.179 ± 0.087	0.211 ± 0.089	$0.0248^{+0.0157}_{-0.0183}$	$0.0205^{+0.0116}_{-0.0140}$
0.0 - 1.0	0.500 ± 0.054	0.522 ± 0.060	0.044 ± 0.042	0.147 ± 0.041	$0.0487^{+0.0108}_{-0.0111}$	$0.0231^{+0.0115}_{-0.0124}$

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from Figs. 1 and 2 that there is clear evidence for $\sigma_E = +1$ in the first three momentum transfer bins at 4.1 GeV/c and in the second bin of the 5.5 GeV/c data.

D. $\pi^- p \rightarrow \rho^0 n$

The π exchange model for the reaction $\pi^- p \rightarrow \rho^0 n$, discussed in Section II D, is compared with experiment. We take the experimental values for ρ_{00} and $\text{Re } \rho_{S0}$, and using equations (53) to (57) inclusive, we predict the values of $\text{Re } \rho_{10}$, ρ_{1-1} , and $\text{Re } \rho_{S1}$. These results are presented in Table VI. The sign of $\text{Re } \rho_{10}$ was assumed to be negative. Most of the predictions are incompatible with experiment. The disagreement between the theoretical and experimental values for $\text{Re } \rho_{10}$ is particularly large. It is clear that this model, based on single π -trajectory exchange, is not consistent with the $\pi^- p \rightarrow \rho^0 n$ data. However it should be remembered that both the theoretical model and data analysis depend on the assumption that the asymmetry in the $\pi^+ n^-$ angular distribution (in the ρ mass region) is due to the interference of the ρ with an S-wave. If instead the requisite even partial wave is a D-wave, for instance, the interpretation of data would be modified, and our discussion in Section II D would, in its present form, be inappropriate.

Table VI. Test of relations (53) - (55).

P_{lab} (GeV/c)	$-t, (\text{GeV}/c)^2$	Re ρ_{10}		ρ_{1-1}		Re ρ_{s1}	
		Theoretical	Experimental	Theoretical	Experimental	Theoretical	Experimental
2.7^{20}	0.013-0.03	-0.29 $\pm$ 0.05	-0.16 $\pm$ 0.05	-0.11 $\pm$ 0.02	0.08 $\pm$ 0.05	-0.07 $\pm$ 0.03	-0.09 $\pm$ 0.03
	0.03 -0.05	-0.04 $\pm$ 0.06	-0.15 $\pm$ 0.05	-0.002 $\pm$ 0.003	0.14 $\pm$ 0.05	-0.01 $\pm$ 0.01	0.06 $\pm$ 0.03
	0.05 -0.085	-0.25 $\pm$ 0.07	-0.18 $\pm$ 0.05	-0.08 $\pm$ 0.04	0.02 $\pm$ 0.06	-0.06 $\pm$ 0.02	-0.03 $\pm$ 0.03
	0.085-0.12	-0.23 $\pm$ 0.07	-0.15 $\pm$ 0.06	-0.07 $\pm$ 0.04	0.09 $\pm$ 0.06	-0.08 $\pm$ 0.03	-0.06 $\pm$ 0.03
	0.12 -0.175	-0.33 $\pm$ 0.03	-0.12 $\pm$ 0.07	-0.19 $\pm$ 0.05	0.07 $\pm$ 0.07	-0.10 $\pm$ 0.05	-0.10 $\pm$ 0.03
	0.175-0.25	-0.35 $\pm$ 0.02	-0.18 $\pm$ 0.07	-0.22 $\pm$ 0.05	-0.03 $\pm$ 0.07	-0.07 $\pm$ 0.05	-0.09 $\pm$ 0.05
6.0^{21}	0. -0.05	-0.280 $\pm$ 0.017	-0.154 $\pm$ 0.025	-0.122 $\pm$ 0.019	-0.030 $\pm$ 0.030	-0.120 $\pm$ 0.014	-0.070 $\pm$ 0.020
	0.05 -0.10	-0.267 $\pm$ 0.019	-0.184 $\pm$ 0.030	-0.106 $\pm$ 0.019	-0.060 $\pm$ 0.030	-0.113 $\pm$ 0.014	-0.080 $\pm$ 0.010
	0.10 -0.20	-0.338 $\pm$ 0.011	-0.210 $\pm$ 0.040	-0.210 $\pm$ 0.030	0.025 $\pm$ 0.045	-0.089 $\pm$ 0.023	-0.065 $\pm$ 0.020
	0.20 -0.30	-0.301 $\pm$ 0.029	-0.250 $\pm$ 0.060	-0.130 $\pm$ 0.028	0.140 $\pm$ 0.030	-0.080 $\pm$ 0.027	-0.190 $\pm$ 0.050
	0.30 -0.50	-0.340 $\pm$ 0.020	-0.130 $\pm$ 0.050	-0.176 $\pm$ 0.035	0.180 $\pm$ 0.040	0.013 $\pm$ 0.031	0.085 $\pm$ 0.040

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IV. CONCLUSIONS

We have derived a set of relations between spin-density matrix elements which should prove very useful in the peripheral-model analysis of data on resonance production. They provide a simple method of testing for the minimum number and type of crossed-channel exchanges which is required to reproduce the spin structure of the reaction mechanism. Independent tests are possible at each energy and momentum transfer, thus avoiding the uncertainty due to normalization inherent in energy dependence tests, and also avoiding dependence on the parameterization of residue and trajectory functions. Since these relations are valid for arbitrary spin, they provide a convenient starting point for the analysis of future data on high spin resonance production. We emphasize again that the most useful information is obtained when the relations are not satisfied, indicating the presence of additional exchanges. Since interference terms violate the relations, relatively small contributions (to the cross section) from exchanges not associated with established particles may be detected. The analysis of data in the preceding section, while not exhaustive, contains examples of all the above points.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. Comparison of the 4.1-GeV/c $K^-p \rightarrow K^*(890)^0 n$ data (Ref. 18) with inequality (44).

Fig. 2. Comparison of the 5.5-GeV/c $K^-p \rightarrow K^*(890)^0 n$ data (Ref. 18) with inequality (44).

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Fig. 1

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Fig. 2

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