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A SIMPLER APPROACH TO THE GEOMETRICAL EFFICIENCY OF A PARALLEL-DISK SOURCE AND DETECTOR SYSTEM

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Abstract

A little-known integral formulation, of considerable simplicity, has been used to compute representative values of the average solid angle subtended at a disk source by a coaxial parallel-disk detector. The advantage of the integral formulation is compared with a Monte Carlo calculation of the solid angle. It is concluded that the Monte Carlo calculation is of lesser advantage, but is more easily adaptable to complex geometries.

Introduction

Ruffle[1]) has reviewed the history of approximations made to the calculation of the average solid angle subtended at a disk source by a coaxial parallel-disk detector. He also recommends an evaluation in terms of several elliptic integrals. We should like to point out that a simpler method exists[2]) which does not appear to be widely known. In this method, the integrand of the average-solid-angle integral is transformed into a relatively simple expression involving Bessel functions of order unity. The latter expression is readily evaluated on any computer equipped with integrating and Bessel-function-calculating routines.

Theory

Consider a uniform source disk, assumed to emit isotropically, of area $S_1 = \pi R_s^2$, which is separated by a distance z from a coaxial parallel detector disk of area $S_2 = \pi R_d^2$. Let R denote the straight-line distance from an arbitrary element of surface area dS_1 to an arbitrary element of surface area dS_2. Let $\vec{n}_2$ be the normal vector from dS_2 directed towards S_1. Then, if the source intensity is S_0 per unit area of S_1, the number of particles per unit time emitted from S_1 which strike S_2 is

$$N = \frac{S_0}{4\pi} \int dS_2 \int dS_1 \quad \frac{\vec{n}_2 \cdot \vec{R}}{R^3} \quad .$$

The fractional solid angle, which is the ratio of the number of particles striking S_2 to the total emission from S_1, is

$$I = N/S_0 \pi R_S^2 \ .$$

In Ref. 2 it is shown that this integral can be transformed to one containing Bessel functions of order unity, namely

$$I = \frac{R_d}{R_s} \int_0^\infty \frac{dk}{k} e^{-kz} J_1(kR_s) J_1(kR_d).$$

For ease of calculation, the latter expression can be transformed to the equivalent form

$$I = F \int_0^\infty \frac{du}{u} e^{-uZ} J_1(u) J_1(uF),$$

where $u = kR_s$, $Z = z/R_s$, and $F = R_d/R_s$. We have calculated some representative values of the fractional solid angle as given by the integral formula described above, and also by the Monte Carlo approach to the problem as put forth by Williams, Craig, and Thompson[3,4]). The results are shown in Table 1.

For the integral calculation, the accuracy is determined by the choice of the convergence criterion in the integrating routine, which was taken to be 0.1%, but not to exceed 15 iterations; and by the choice of the finite integration interval, which was taken to be from $u = 0.001$ to $u = 80/F$. The maximum error, as indicated by the rate of convergence in the integral, did not exceed the 0.1%. The CDC 6600 computer time used was 77.8 sec.

For the Monte Carlo calculation, the principal source of error is the number of random particle trajectories considered, which was taken to be 10 000 for each fixed geometry. The maximum statistical error for any of the calculations was 2.8%, and decreased as the ratio R_d/R_s increased. As noted by the authors in Ref. 3, the convergence is slow for $R_d/R_s < 1$. This is especially evident for the point at $z = 0$ and $R_d/R_s = 0.5$, where the maximum statistical error occurs. With

50 000 trajectories for this point, instead of 10 000, the resulting value of the solid angle becomes 0.1484 with a statistical error of 1.0%, which may be compared to the correct value of 0.1250. The CDC 6600 computer time for the Monte Carlo calculations was 282 sec.

Discussion

The integral calculation discussed above appears to have a definite advantage over the Monte Carlo calculation, insofar as minimum error for a given amount of machine time is concerned. This is particularly the case for $R_d/R_s < 1$. Of course, for $R_d/R_s << 1$, the approximation formula as used, for example, by Wilkniss and Wynne[5]) can be just as useful as the integral formulation. The real advantage of the Monte Carlo method is that, in contrast to the integral formulation, it is more easily adaptable to geometries exhibiting less symmetry than coaxial parallel disks. In fact, the program described in Ref. 4 and used by us is also valid when the detector disk is not coaxial, but its axis intersects the source disk at its center.

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TABLE 1.

Calculation of Average Solid Angle for the Case of Unit Source Radius

z	R_D	Integral Method	Monte Carlo Method
0	0.5	0.1250	0.1507
0	1	0.4960	0.5000
0	2	0.5003	0.5000
0	4	0.5002	0.5000
0	8	0.5005	0.5000
1	0.5	0.3457×10^{-1}	0.3449×10^{-1}
1	1	0.1161	0.1117
1	2	0.2621	0.2631
1	4	0.3783	0.3745
1	8	0.4376	0.4370
2	0.5	0.1279×10^{-1}	0.1290×10^{-1}
2	1	0.4675×10^{-1}	0.4650×10^{-1}
2	2	0.1383	0.1377
2	4	0.2730	0.2739
2	8	0.3781	0.3785
3	0.5	0.6305×10^{-2}	0.6197×10^{-2}
3	1	0.2399×10^{-1}	0.2403×10^{-1}
3	2	0.8039×10^{-1}	0.7961×10^{-1}
3	4	0.1971	0.1979
3	8	0.3236	0.3232
4	0.5	0.3693×10^{-2}	0.3524×10^{-2}
4	1	0.1433×10^{-1}	0.1413×10^{-1}
4	2	0.5115×10^{-1}	0.5054×10^{-1}
4	4	0.1444	0.1448
4	8	0.2755	0.2747
5	0.5	0.2411×10^{-2}	0.2406×10^{-2}
5	1	0.9445×10^{-2}	0.9337×10^{-2}
5	2	0.3495×10^{-1}	0.3493×10^{-1}
5	4	0.1082	0.1081
5	8	0.2342	0.2344